Precision Physics in ZH/ZZ/HH

Marco Vitti (Karlsruhe Institute of Technology, TTP and IAP)

1st COMETA General Meeting, Izmir, Feb 29 2024

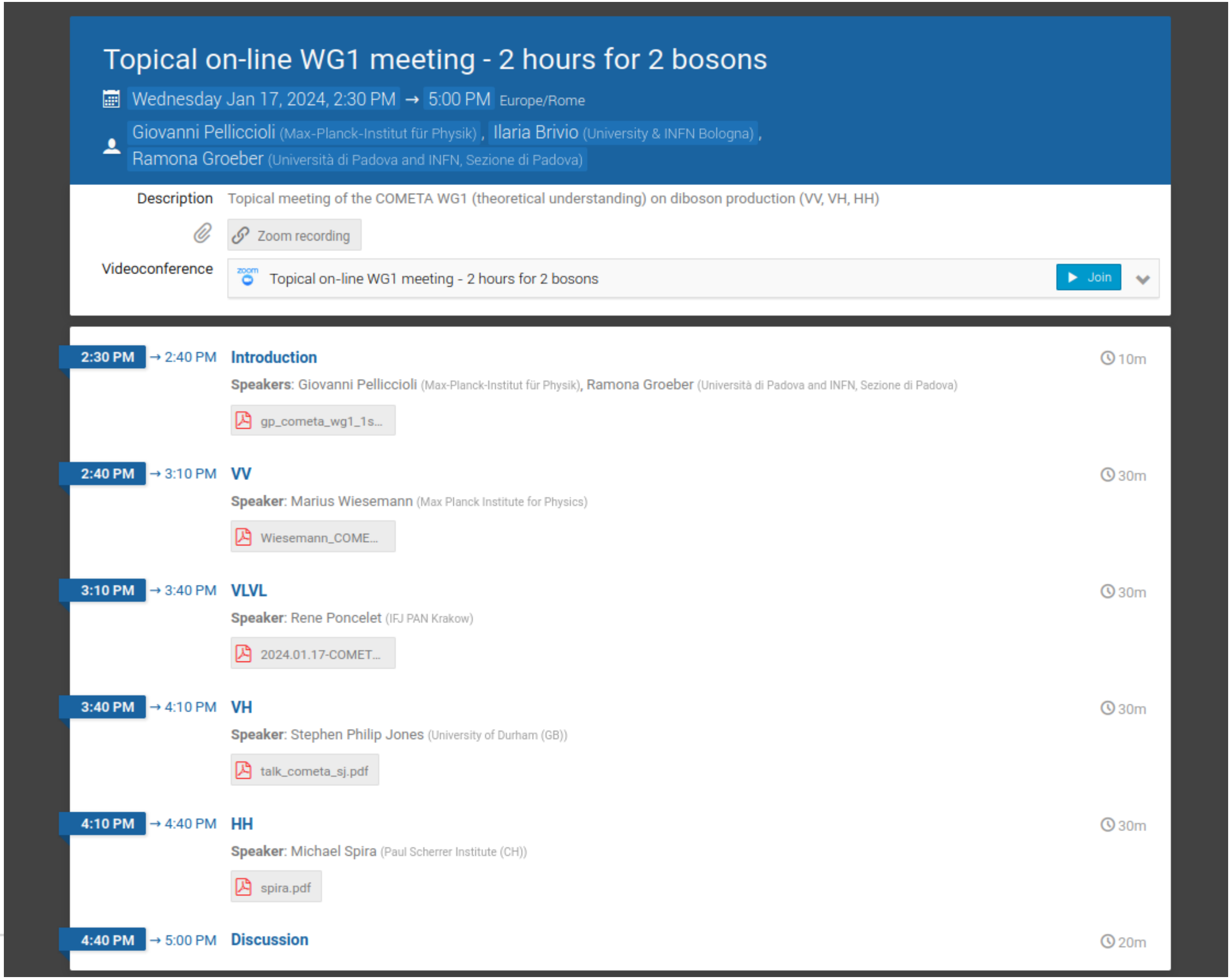
Overview

- Recent developments in SM predictions
- Emphasis on fixed-order
- Massive loops in *gg*-initiated processes

- See

  https://indico.cern.ch/event/1360356/

  for dedicated discussions



Topical on-line WG1 meeting - 2 hours for 2 bosons

Wednesday Jan 17, 2024, 2:30 PM → 5:00 PM Europe/Rome

Giovanni Pellicciolі (Max-Planck-Institut für Physik), Ilaria Brivio (University & INFN Bologna), Ramona Groeber (Università di Padova and INFN, Sezione di Padova)

Description: Topical meeting of the COMETA WG1 (theoretical understanding) on diboson production (VV, VH, HH)

Zoom recording

Videoconference: Topical on-line WG1 meeting - 2 hours for 2 bosons — Join

2:30 PM → 2:40 PM **Introduction** — 10m
Speakers: Giovanni Pellicciolі (Max-Planck-Institut für Physik), Ramona Groeber (Università di Padova and INFN, Sezione di Padova)
gp_cometa_wg1_1s...

2:40 PM → 3:10 PM **VV** — 30m
Speaker: Marius Wiesemann (Max Planck Institute for Physics)
Wiesemann_COME...

3:10 PM → 3:40 PM **VLVL** — 30m
Speaker: Rene Poncelet (IFJ PAN Krakow)
2024.01.17-COMET...

3:40 PM → 4:10 PM **VH** — 30m
Speaker: Stephen Philip Jones (University of Durham (GB))
talk_cometa_sj.pdf

4:10 PM → 4:40 PM **HH** — 30m
Speaker: Michael Spira (Paul Scherrer Institute (CH))
spira.pdf

4:40 PM → 5:00 PM **Discussion** — 20m

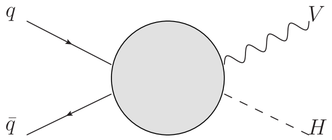
ZH

VH Production at the LHC

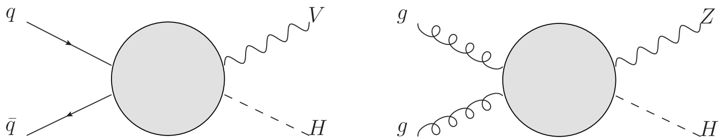
$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

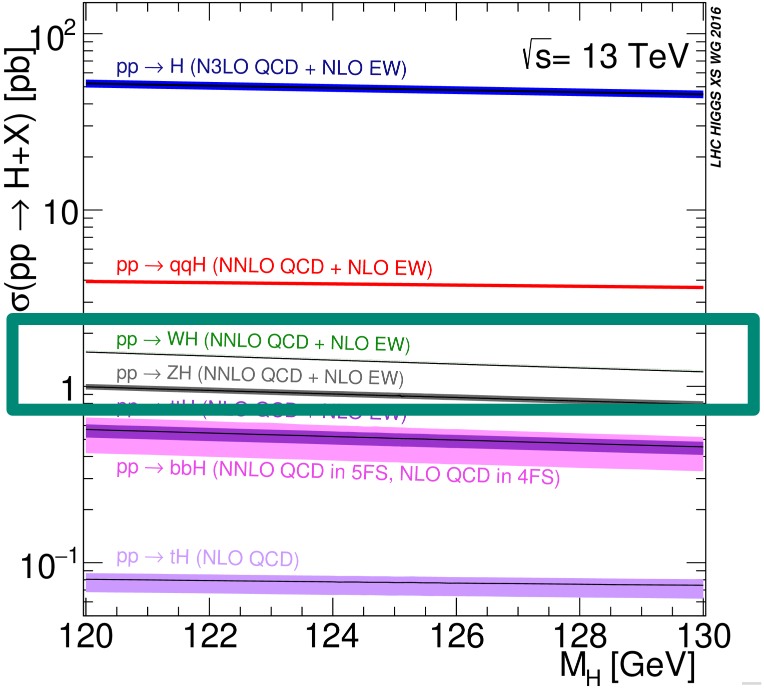
- Work in progress on $H \to c\bar{c}$
  [ATLAS-2201.11428, CMS-2205.0555]
- Probe of VVH coupling
- Partonic channels

$pp \to WH$



$pp \to ZH$





VH Production at the LHC

Karlsruhe Institute of Technology

$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Work in progress on $H \to c\bar{c}$
  [ATLAS-2201.11428, CMS-2205.0555]
- Probe of VVH coupling
- Larger scale uncertainties in *ZH*

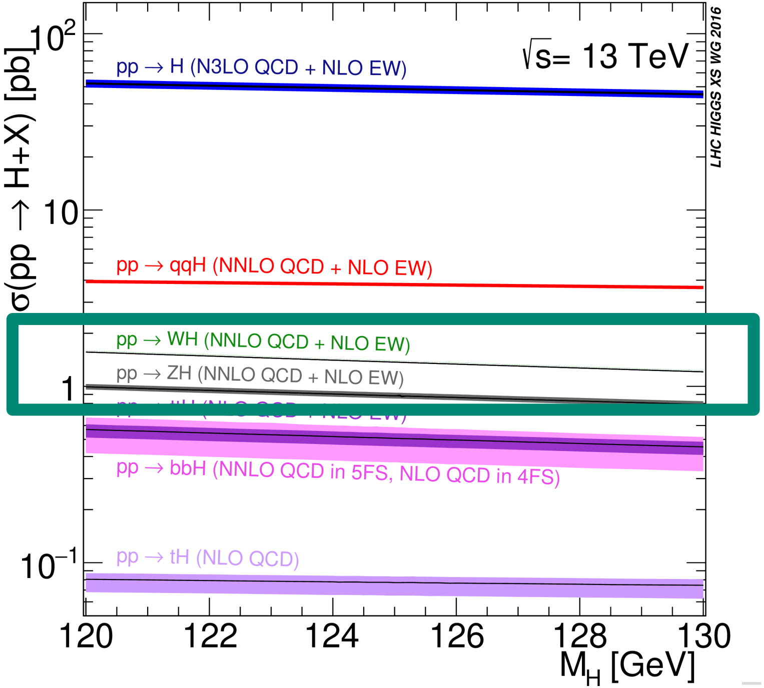
$pp \to WH$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	1.358	$^{+0.51}_{-0.51}$	1.35
14	1.498	$^{+0.51}_{-0.51}$	1.35
27	3.397	$^{+0.29}_{-0.72}$	1.37

$pp \to ZH$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.880	$^{+3.50}_{-2.68}$	1.65
14	0.981	$^{+3.61}_{-2.94}$	1.90
27	2.463	$^{+5.42}_{-4.00}$	2.24

[Cepeda et al. - 1902.00134]

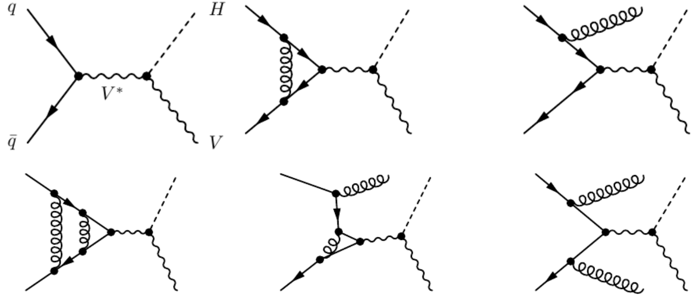


Theoretical Predictions for $pp \to ZH$

LO: quark-initiated tree-level contribution (purely EW)

QCD effects: mainly due to Drell-Yan (DY) production followed by $Z^* \to ZH$ decay

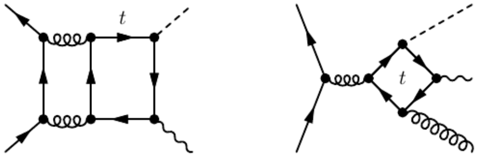
- **Drell-Yan:** Known through N3LO **(+30% wrt LO)**



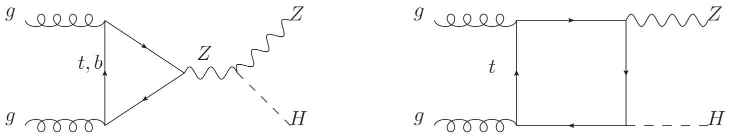
[Han, Willenbrock ('91) ; Hamberg, van Neerven, Matsuura ('92) ;
Brein, Djouadi, Harlander – 0307206;
Baglio, Duhr, Mistlberger, Szafron - 2209.06138]

- **Non Drell-Yan - quark-initiated** O(1%) wrt LO

[Brein, Harlander, Wiesemann, Zirke - 1111.0761]



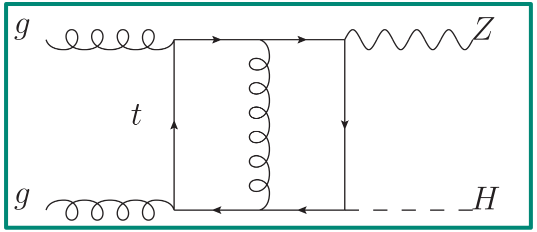
- **Non Drell-Yan - gluon-initiated**



EW corrections: through NLO (-(5-10%) wrt LO)

[Ciccolini, Dittmaier, Krämer - 0306234]

$gg \to ZH$ @ NLO QCD



Main problem in the virtual NLO calculation

Multi-scale (m_Z, m_H, m_t, s, t) two-loop box integrals

No full analytic results

Three (almost) independent calculations in agreement

- Small-mass expansion $m_Z, m_H \to 0$ [Wang, Xu, Xu, Yang - 2107.08206]

  Elliptic integrals evaluated numerically

- Sector decomposition $\oplus$ High-Energy expansion [Chen et al. - 2011.12325; Davies et al. - 2011.12314; Chen, et al. - 2204.05225;]

  $$m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$$

- pT expansion $\oplus$ High-Energy expansion [Alasfar et al. - 2103.06225; Bellafronte et al. – 2202.12157; Degrassi, Gröber, MV, Zhao - 2205.02769]

  $$m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s} \qquad m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$$

$gg \to ZH$ @ NLO QCD

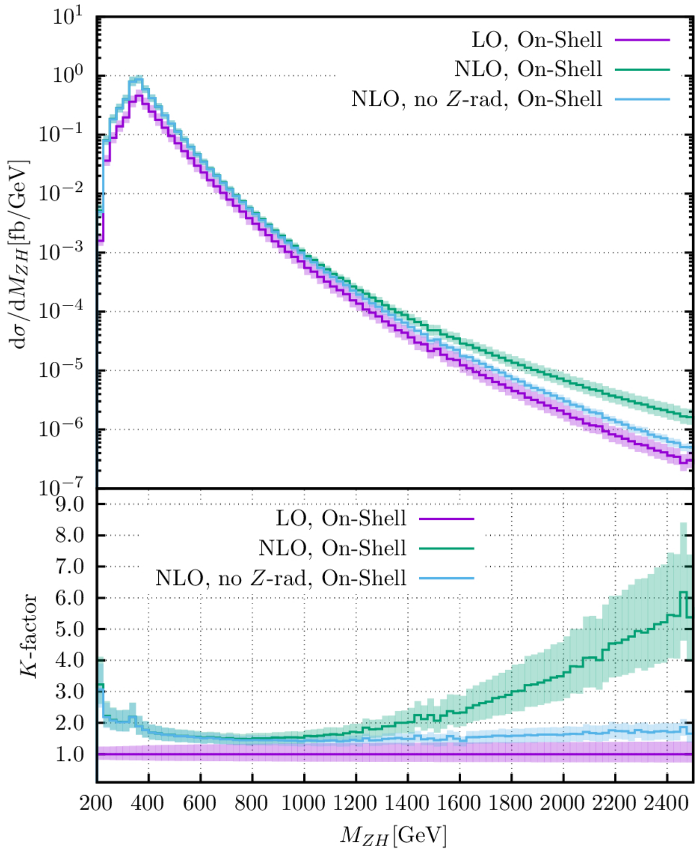
Inclusive cross section $\sqrt{s} = 13\text{TeV}$ $\mu_r = \mu_f = M_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 30% wrt LO

Invariant-mass distribution

- K-factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail $(M_{ZH} > 1\,\text{TeV})$



[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \to ZH$ @ NLO QCD

Inclusive cross section

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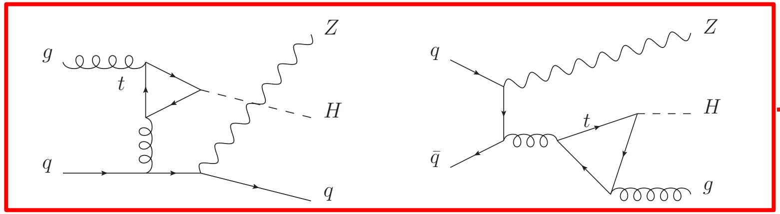
Invariant-mass distribution



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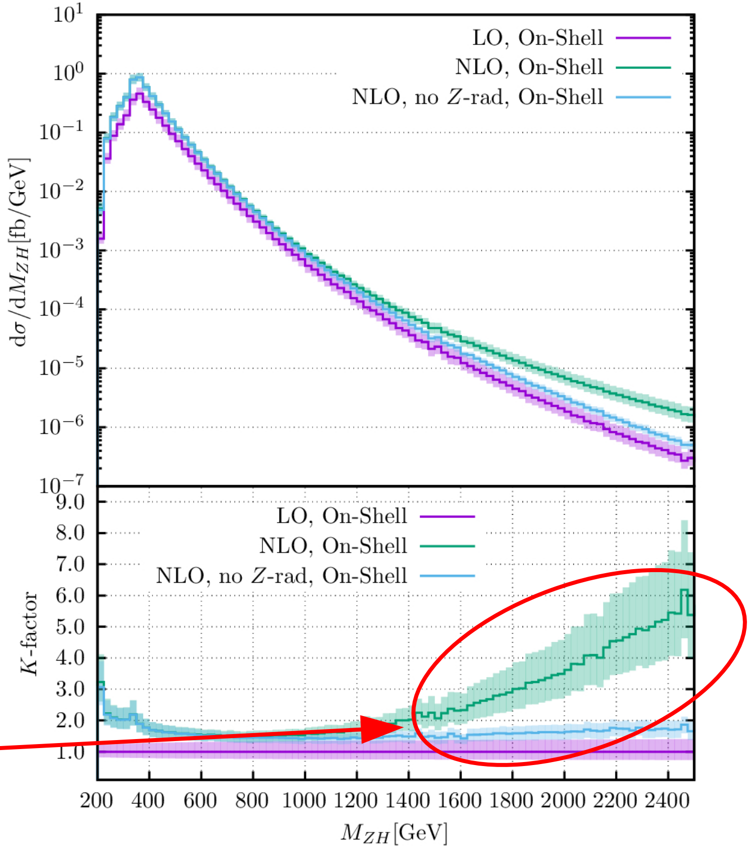


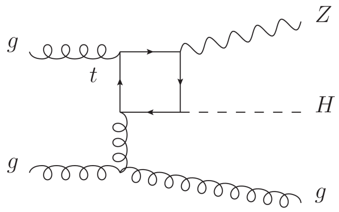
[Degrassi, Gröber, MV, Zhao - 2205.02769]

High-Energy Tails – pT Distributions

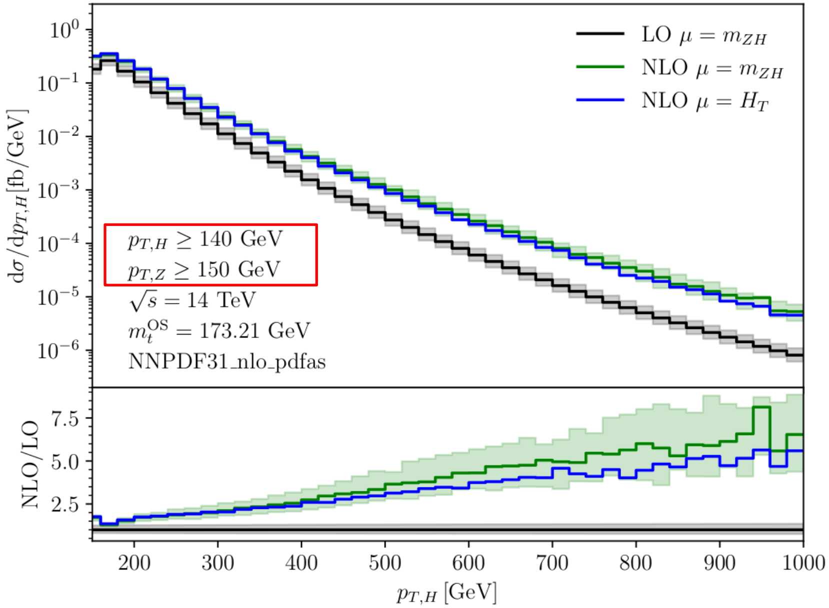
- Very large (~20) NLO corrections for $p_{T,H} > 400\,\mathrm{GeV}$
- Still K-factor of ~5 after pT cuts

- The pT cuts remove 2→3 configurations with a hard jet and a soft Z

  These are very likely in the high-energy region



Observed in [Hespel, Maltoni, Vryonidou -1503.01656]



[Chen, Davies, Heinrich, Jones, Kerner, Mishima, Schlenk, Steinhauser - 2204.05225]

Top Mass Scheme Uncertainty

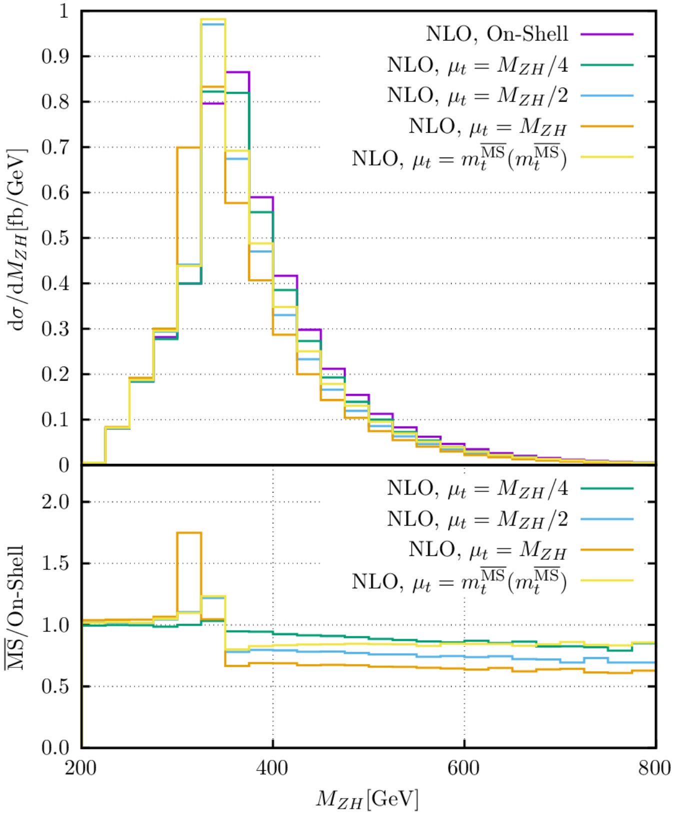
- Envelope of deviations of $\overline{\text{MS}}$ schemes wrt OS result
  Same method already used for HH production
  [Baglio et al. - 1811.05692, 2003.03227]

- Uncertainty sensitive to the binning of top-pair threshold peak

Avoid overestimate of uncertainty

Bin Width [GeV]	LO	NLO
1	$64.01^{+15.6\%}_{-35.9\%}$	$118.6^{+17.2\%}_{-27.0\%}$
5	$64.01^{+15.3\%}_{-35.6\%}$	$118.6^{+14.7\%}_{-24.9\%}$
25	$64.01^{+14.0\%}_{-33.1\%}$	$118.6^{+10.9\%}_{-20.8\%}$
100	$64.01^{+2.0\%}_{-25.3\%}$	$118.6^{+0.6\%}_{-13.7\%}$
∞	$64.01^{+0\%}_{-23.1\%}$	$118.6^{+0\%}_{-12.9\%}$

- Top-mass uncertainty ~ scale uncertainty

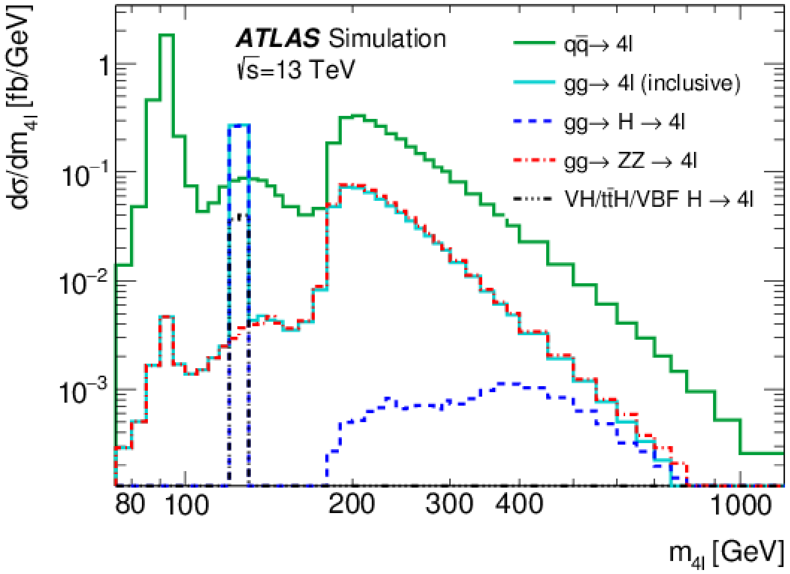


[Degrassi, Gröber, MV, Zhao - 2205.02769]

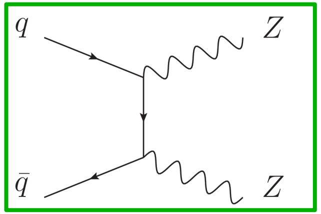
ZZ

pp → *ZZ* at the LHC

- Probe of EW theory: VVV couplings, Higgs production, polarisations measurements...
- Indirect access to Higgs width from off-shell measurements [Kauer, Passarino – 1206.4803] [Caola, Melnikov – 1307.4935] [Campbell. Ellis, Williams - 1311.3589]



[ATLAS – 1902.05892]



QCD: known through NNLO

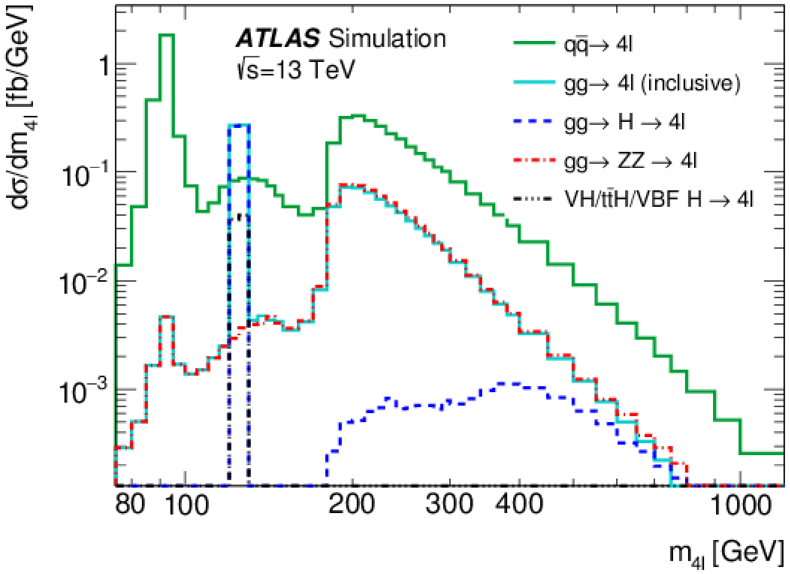
[Cascioli et al. - 1405.2219; Heinrich et al. - 1710.06294; Gehrmann et al. - 1404.4853; Caola et al. - 1408.6409; Gehrmann et al. - 1503.04812; Grazzini et al. - 1507.06257; Kallweit, Wiesemann - 1806.05941]

EW: known through NLO

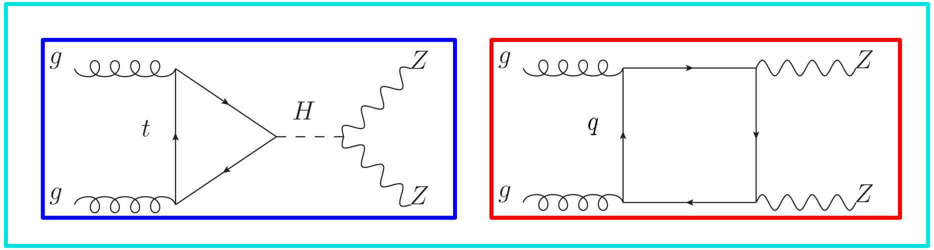
[Bierweiler et al. – 1305.5402; Baglio, Ninh, Weber – 1307.4331; Chiesa et al. - 2005.12146]

$pp \to ZZ$ at the LHC

- Probe of EW theory: VVV couplings, Higgs production, polarisations measurements...
- Indirect access to Higgs width from off-shell measurements [Kauer, Passarino – 1206.4803] [Caola, Melnikov – 1307.4935] [Campbell. Ellis, Williams - 1311.3589]



[ATLAS – 1902.05892]



Higgs-mediated: NLO QCD
[Spira et al. - 9504378 ; Aglietti et al. - 0611266 ; Harlander, Kant - 0509189; Anastasiou et al. - 0611236]

Continuum *ZZ* production: NLO QCD (light quark) [von Manteuffel, Tancredi – 1503.08835; Caola et al. - 1509.06734]

Impact of *gg*→*ZZ*

[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

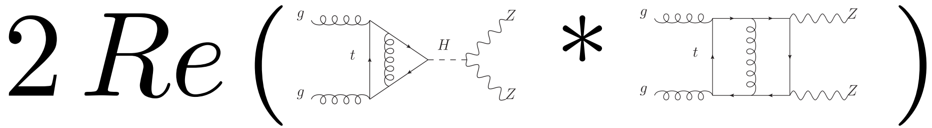
$\sqrt{s}$	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\text{NLO}}-1$	
LO	$8.1881(8)^{+2.4\%}_{-3.2\%}$	$13.933(1)^{+5.5\%}_{-6.4\%}$	-27.5%	-29.8%
NLO	$11.2958(4)^{+2.5\%}_{-2.0\%}$	$19.8454(7)^{+2.5\%}_{-2.1\%}$	0%	0%
$q\bar{q}$NNLO	$12.09(2)^{+1.1\%}_{-1.1\%}$	$21.54(2)^{+1.1\%}_{-1.2\%}$	$+7.0\%$	$+8.6\%$
	σ [fb]		$\sigma/\sigma_{\text{ggLO}}-1$	
ggLO	$0.79355(6)^{+28.2\%}_{-20.9\%}$	$2.0052(1)^{+23.5\%}_{-17.9\%}$	0%	0%
$gg\text{NLO}_{gg}$	$1.4787(4)^{+15.9\%}_{-13.1\%}$	$3.626(1)^{+15.2\%}_{-12.7\%}$	$+86.3\%$	$+80.8\%$
ggNLO	$1.3892(4)^{+15.4\%}_{-13.6\%}$	$3.425(1)^{+13.9\%}_{-12.0\%}$	$+75.1\%$	$+70.8\%$
	σ [fb]		$\sigma/\sigma_{\text{NLO}}-1$	
NNLO	$12.88(2)^{+2.8\%}_{-2.2\%}$	$23.55(2)^{+3.0\%}_{-2.6\%}$	$+14.0\%$	$+18.7\%$
nNNLO	$13.48(2)^{+2.6\%}_{-2.3\%}$	$24.97(2)^{+2.9\%}_{-2.7\%}$	$+19.3\%$	$+25.8\%$

Top-quark loops → permille level in inclusive cross section → negligible

BUT important effects at high invariant masses

Interference @ NLO: massless vs massive quarks

Two-loop boxes are a problem (again)



- Light-quark (~massless) known fully analytically
  [Caola et al. - 1509.06734]

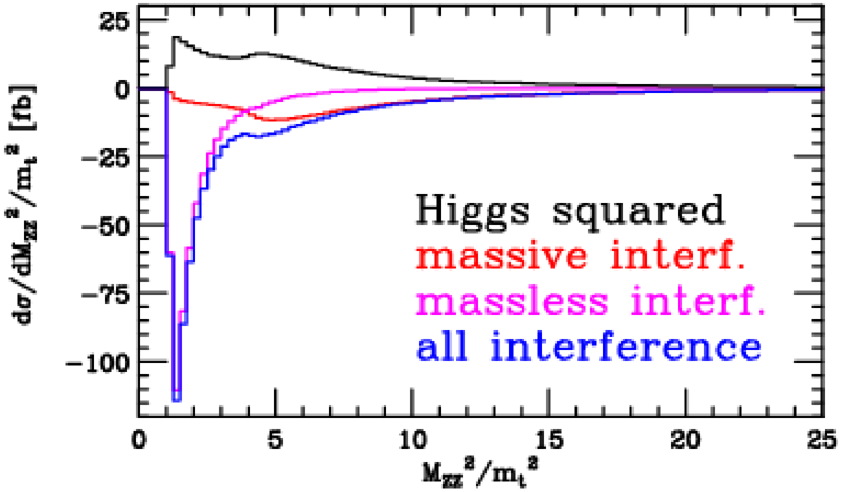
- Heavy quarks

  → **Exact numerical** results available
  [Agarwal, Jones, von Manteuffel - 2011.15113 ; Brønnum-Hansen, Wang - 2101.12095]

  → **Analytic approximations**:

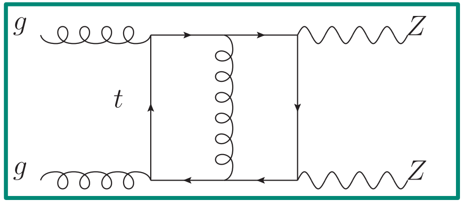
  - $m_t \to \infty$ [Melnikov, Dowling - 1503.01274 ; Gröber, Maier, Rauh - 1605.04610]

  -High-Energy exp [Davies et al. - 2002.05558]
  $$m_Z^2 \ll m_t^2 \ll \hat{s}, \hat{t}$$



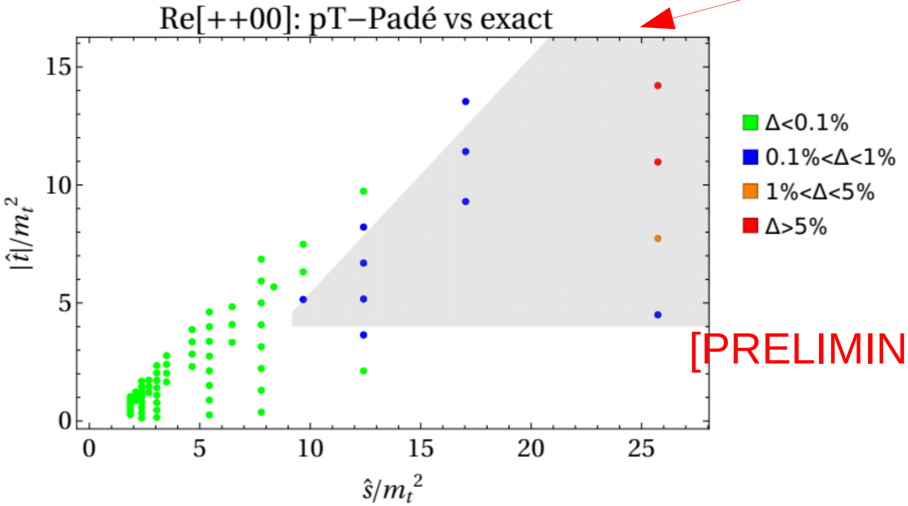
[Campbell et al. - 1605.01380]

gg→*ZZ*: Top Quark Loops @NLO QCD

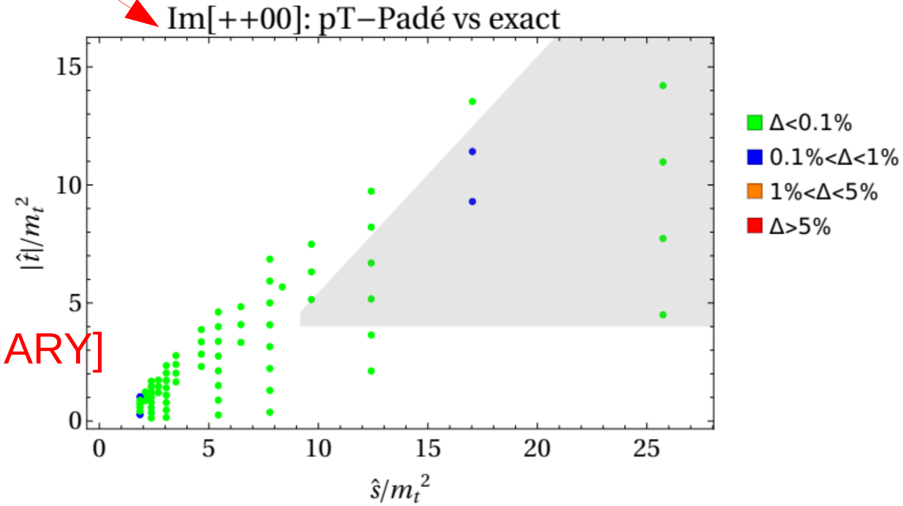


Comparison with helicity amplitudes of [Agarwal, Jones, von Manteuffel - 2011.15113]

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \mathcal{O}(\alpha_s^3)$$



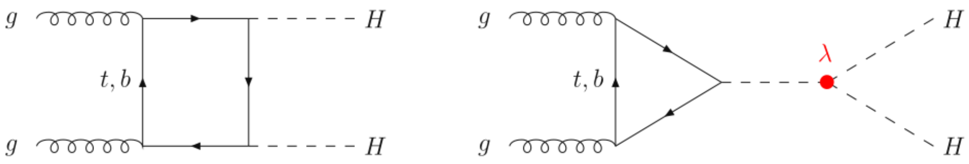
[PRELIMINARY]



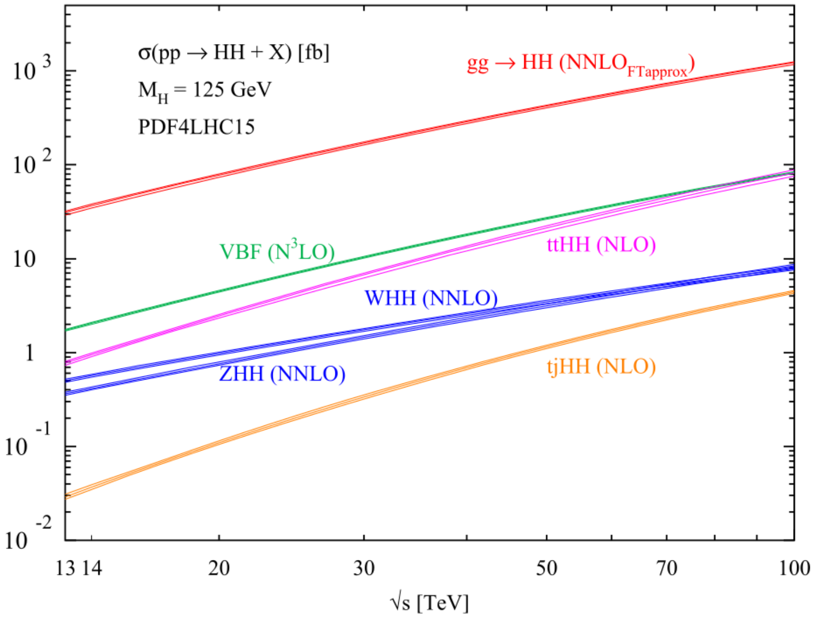
[Degrassi, Gröber, MV – in preparation]

HH

HH Production @LHC



- Best shot at measurement of Higgs trilinear self-coupling
- Gluon-initiated channel dominant



[Di Micco et al. - 1910.00012]

HH QCD corrections

$m_t \to \infty$ limit: N3LO

[De Florian, Mazzitelli 1305.5206 and 1309.6594; . Grigo, Melnikov and Steinhauser – 1408.2422; Chen et al. - 1909.06808 and 1912.13001;]

Finite $1/m_t$ effects

[Grigo, Hoff, Steinhauser - 1508.00909; Davies, Steinhauser – 1909.01361; Davies et al. 2110.03697]

Full top-mass dependence: NLO

- Numerical evaluation

  [Borowka et al. - 1604.06447, 1608.04798; Baglio et al. - 1811.05692]

- Analytic approximations

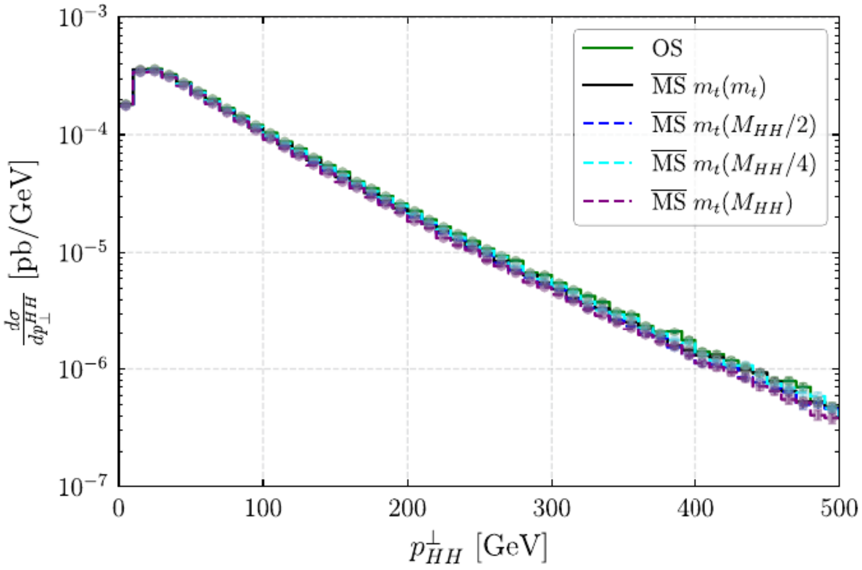
  [Davies et al. - 1811.05489; Bonciani et al. - 1806.11564; Wang et al. - 2010.15649]

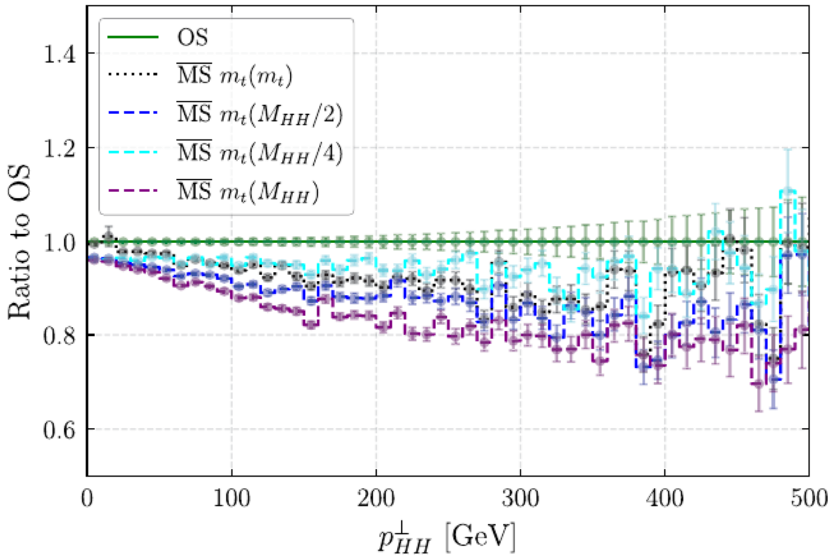
  (High-energy exp) (pT expansion) Small-mass expansion

Full phase space covered in [Bellafronte et al. – 2202.12157; Davies et a. - 2302.01356]

HH @NLO QCD (+ PS)

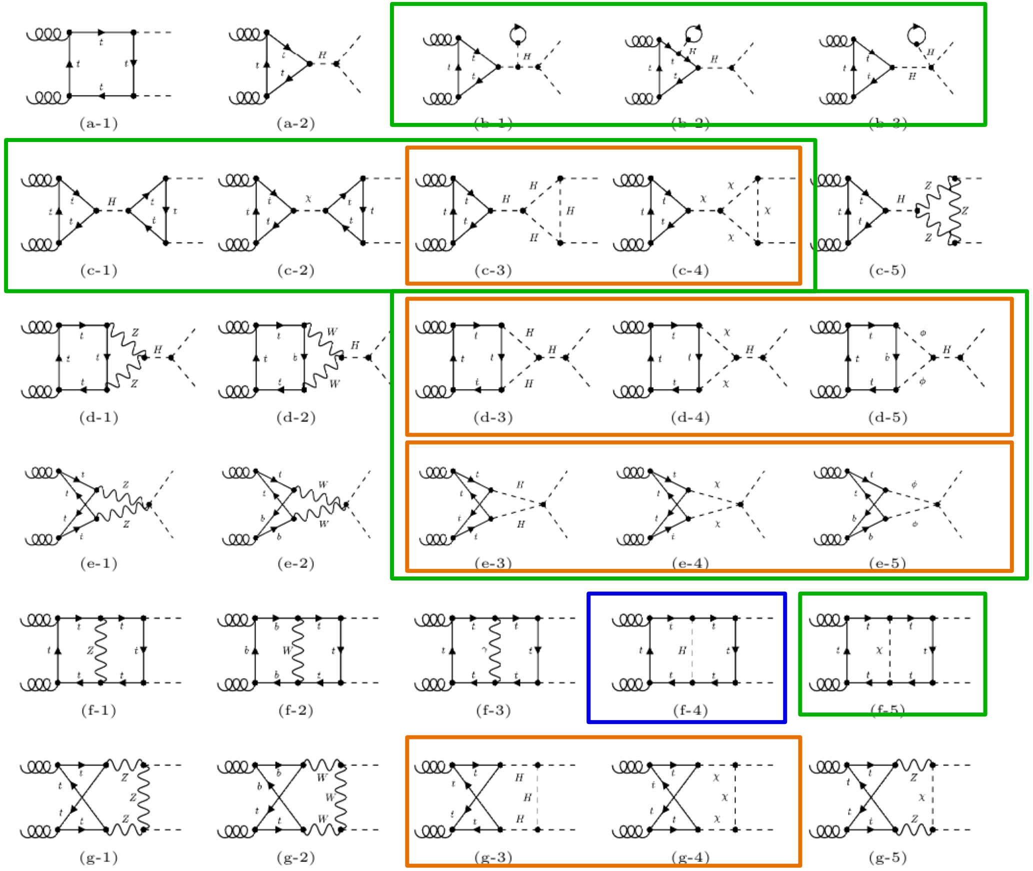
- Full flexibility over choice of trilinear and top mass
- Top mass scheme uncertainty studied differentially (see also [Baglio et al. - 2008.11626])





[Bagnaschi, Degrassi, Gröber - 2309.10525]

HH NLO EW

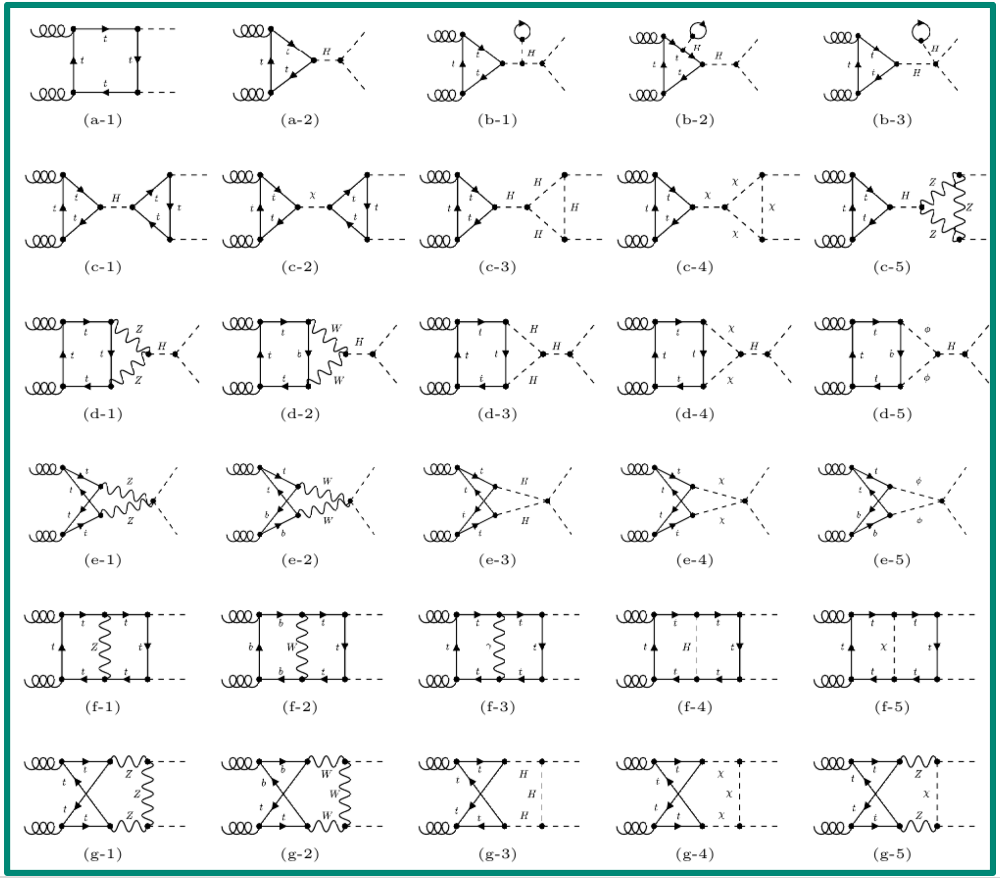


Trilinear and quadrilinear contributions at two loops (Sector Decomposition)
[Borowka et al. - 1811.12366]

Top-Yukawa-induced corrections (Heavy-Top EFT)
[Mühlleitner, Schlenk, Spira – 2207.02524]

Two loop boxes with virtual H (High-Energy expansion)
[Davies, et al. - 2207.02587]

HH NLO EW

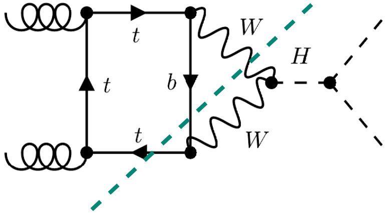


Complete NLO EW in Large Top Expasion

$$m_t^2 \gg \xi_W m_W^2, \xi_Z m_Z^2 \gg \hat{s}, \hat{t}, m_W^2, m_Z^2, m_H^2$$

- Typically valid for $\sqrt{\hat{s}} \lesssim 2m_t \sim 350\text{GeV}$

  Compromised by diagrams with t-W-b cut



$$\sqrt{\hat{s}} \lesssim m_t + m_W + m_b \sim 250\text{GeV}$$

[Davies, Schönwald, Steinhauser, Zhang - 2308.01355]

HH NLO EW

Full numerical evaluation

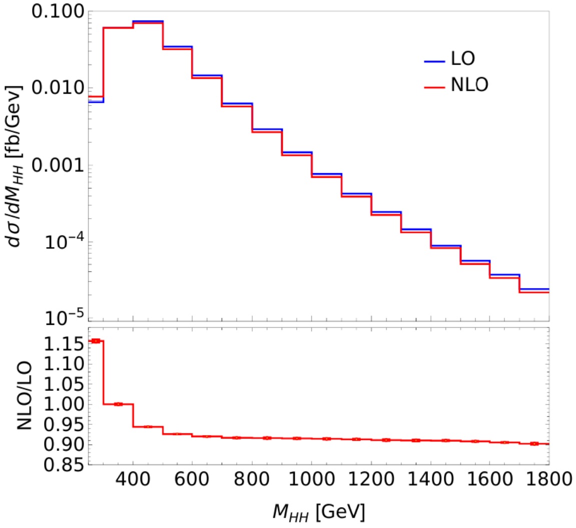
Inclusive cross-section
-4%, unaffected by choice of parameters

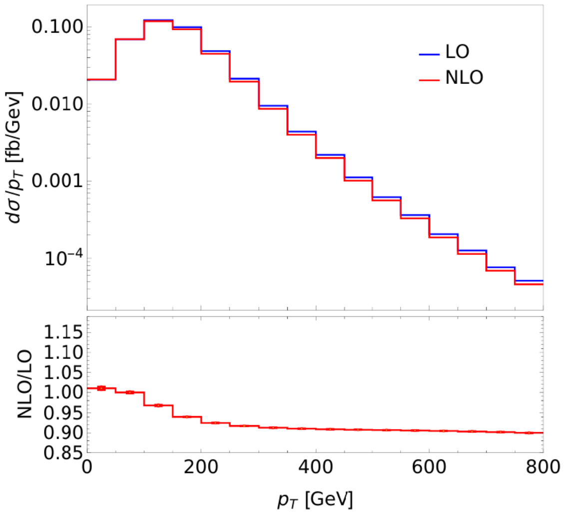
μ	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)

[Bi, Huang, Huang, Ma, Yu - 2311.16963]

Invariant-mass
- Large positive corrections at threshold
- -10% at high M_{HH}

Negative corrections (-10%) at high pT





Conclusions

- ZH and ZZ: NNLO accuracy well established
- *gg*-initiated channel important ingredient for N3LO
- Top mass effects and relative scheme uncertainties are important

- HH: NLO QCD with full top dependence

  NLO EW recently achieved

- Getting closer to theorists' goal of 1% accuracy

Thank you for your attention

Backup

ZH

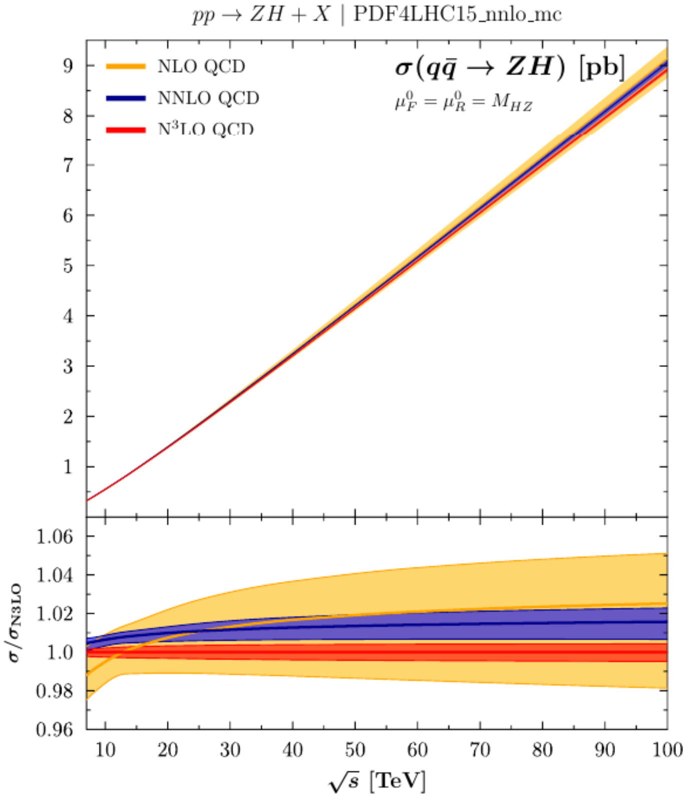
Drell-Yan @ N3LO for ZH

[Baglio, Duhr, Mistlberger, Szafron - 2209.06138]

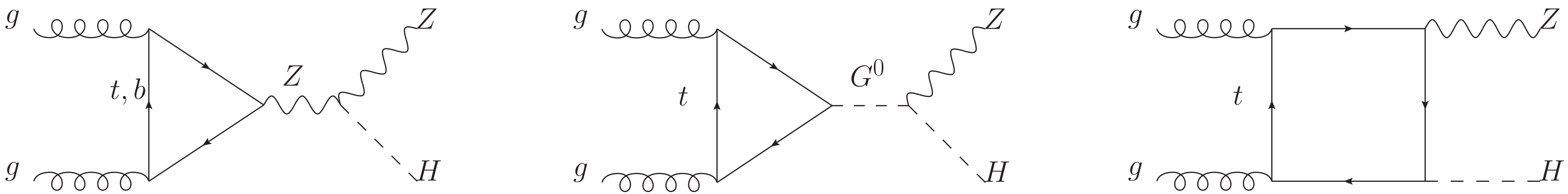
$\sqrt{s} = 13\text{TeV}$ $\mu_0 = M_{VH}$

Process	σ^{LO} [pb]	σ^{NLO} [pb]	K^{NLO}	σ^{NNLO} [pb]	K^{NNLO}	$\sigma^{\text{N}^3\text{LO}}$ [pb]	$\text{K}^{\text{N}^3\text{LO}}$
W^+H	$0.758^{+2.43\%}_{-3.13\%}$	$0.883^{+1.38\%}_{-1.20\%}$	1.16	$0.891^{+0.28\%}_{-0.34\%}$	1.18	$0.884^{+0.27\%}_{-0.30\%}$	1.17
W^-H	$0.484^{+2.50\%}_{-3.26\%}$	$0.560^{+1.34\%}_{-1.23\%}$	1.16	$0.564^{+0.27\%}_{-0.34\%}$	1.17	$0.559^{+0.30\%}_{-0.33\%}$	1.16
ZH	$0.678^{+2.40\%}_{-3.11\%}$	$0.786^{+1.33\%}_{-1.16\%}$	1.16	$0.792^{+0.25\%}_{-0.32\%}$	1.17	$0.786^{+0.26\%}_{-0.29\%}$	1.16

- N3LO corrections larger than NNLO corrections
- No reduction in scale uncertainties at N3LO
- Fixed $(\mu_0 = m_V + m_H)$ and dynamical $(\mu_0 = M_{VH})$ scale choice yield comparable results
- NNLO and N3LO scale bands do not overlap



$gg \to ZH$ @ LO



- Top-quark loops give dominant contribution [Kniehl ('90) - Dicus, Kao ('88)]
- O(α_s^2) correction to $\sigma(pp \to ZH)$
- NNLO suppression wrt to $q\bar{q} \to ZH$ compensated by larger gluon luminosity
- Contributes to ~ 6% of $\sigma(pp \to ZH)$ for $\sqrt{s} = 14$ TeV
- Only LO included in MC → scale variation leads to **25%** relative uncertainties

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.123	$^{+24.9}_{-18.8}$	4.37
14	0.145	$^{+24.3}_{-19.6}$	7.47
27	0.526	$^{+25.3}_{-18.5}$	5.85

[Cepeda et al. - 1902.00134]

$gg \rightarrow ZH$ @ NLO in QCD - Ingredients

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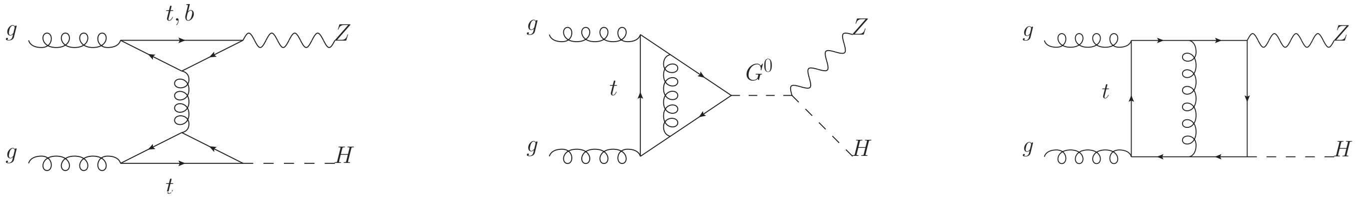
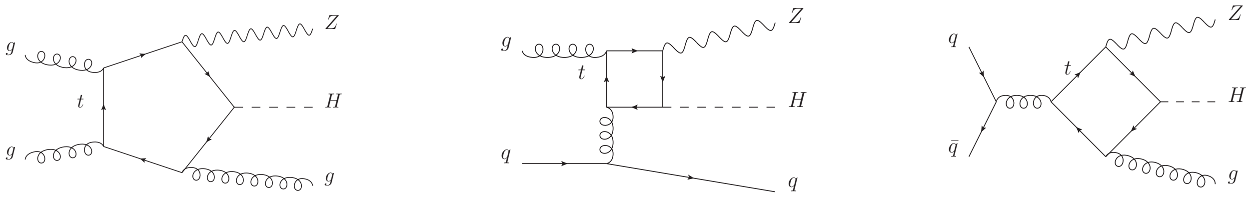
Virtual corrections (2 → 2, two loops) - interference with LO



Real emission (2 → 3, one loop) - squared amplitudes



Two-loop Massive Boxes for $gg \to ZH$

Karlsruhe Institute of Technology

Numerical Evaluation [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]

- Exact results
- Demanding in terms of computing resources and time
- Issues with flexibility

Analytic Approximations: exploit **hierarchies** of masses/kinematic invariants

- Reduce the number of scales in Feynman integrals
- Proliferation of integrals
- Restricted to specific phase-space regions

- Limit $m_t \to \infty$
  [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.50]
- Large mass expansion
  [Hasselhuhn, Luthe, Steinhauser - 1611.05881]
- High-energy expansion: $m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$
  [Davies, Mishima, Steinhauser - 2011.12314]
- Small-mass expansion: $m_Z, m_H \to 0$
  [Wang, Xu, Xu, Yang - 2107.08206]
- pT expansion: $m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}$
  [Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]

pT Expansion - Calculation Overview

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1. Generation of Feynman diagrams - O(100 diags) (FeynArts [Hahn - 0012260])

2. Lorentz decomposition of the amplitude: **projectors** and **scalar form factors** (FeynCalc [Mertig et al. ('91) ; Shtabovenko et al. - 1601.01167]): contractions, Dirac traces...

$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^{6} \mathcal{P}^{(i)}_{\mu\nu\rho} F^{(i)} \qquad F^{(i)} = \sum_{i=1}^{n} C^{(i)}\, I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

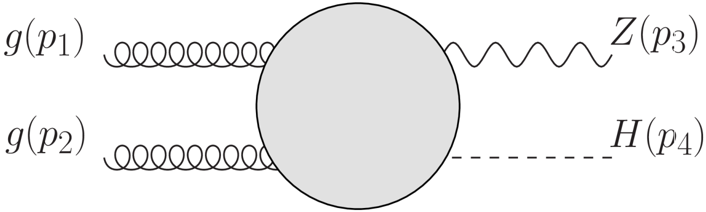
3. Expansion of the form factors in the limit of small pT

4. Decomposition of scalar integrals using integration-by-parts (IBP) identities (LiteRed [Lee - 1310.1145])

5. Evaluation of master integrals

Steps implemented in **Mathematica** code on a **desktop machine**

pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1+p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1 \qquad \frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at integrand level

- Now scalar loop integrals depend on fewer scales

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2)$$

- The new scalar integrals are decomposed in MIs using IBP relations
- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **only one scale**

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2) \to \mathrm{MI}(\hat{s}/m_t^2)$$

- 52 MIs already known in the literature
  SAME MIs FOR $gg \to HH$, $gg \to ZH$, $gg \to ZZ$

Comparing Validity Ranges

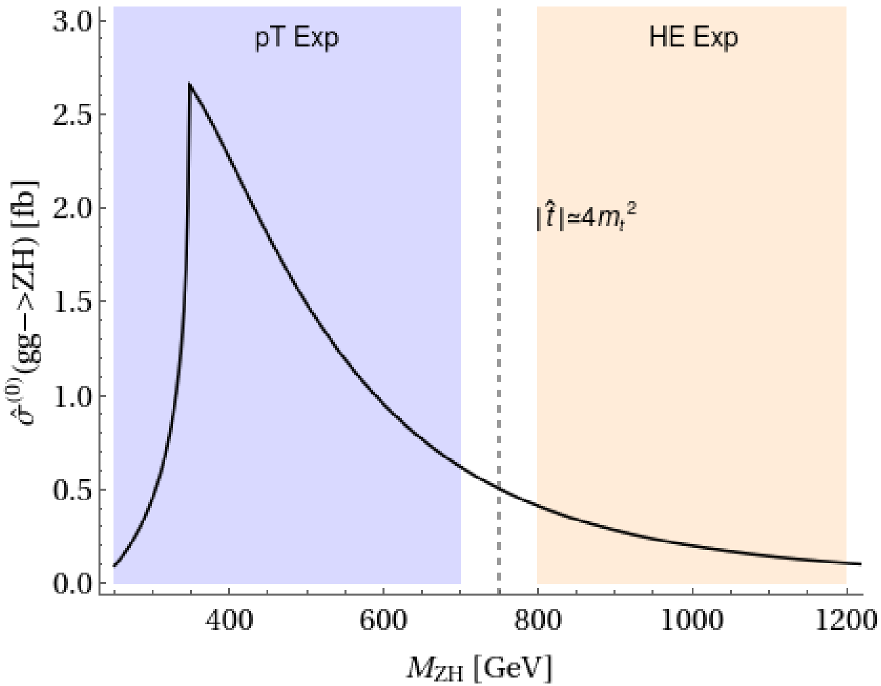
- p_T exp: valid for

$$p_T^2 \lesssim 4m_t^2$$

or

$$|\hat{t}| \lesssim 4m_t^2$$

[Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]



- High-Energy exp:

$$|\hat{t}| \gtrsim 4m_t^2$$

[Davies, Mishima, Steinhauser - 2011.12314]

The two expansions can be combined
(Needed refinement using Padé approximants)
[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

- Accuracy at % level or below $\Rightarrow$ OK for phenomenology
- Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo

Merging pT and HE Expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \quad (q = m+n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

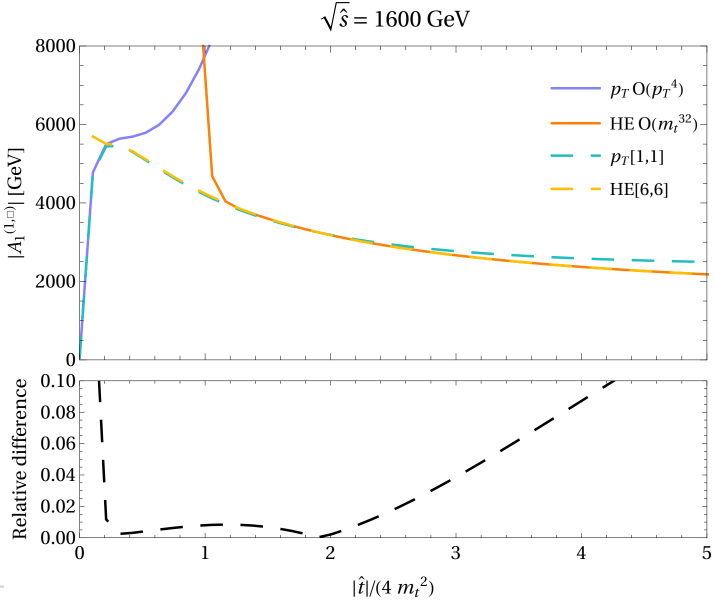
- For each FF we merged the following results

  - pT exp improved by [1/1] Padé

  - HE exp improved by [6/6] Padé

- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2 \rightarrow$ can switch without loss of accuracy (% level or below)

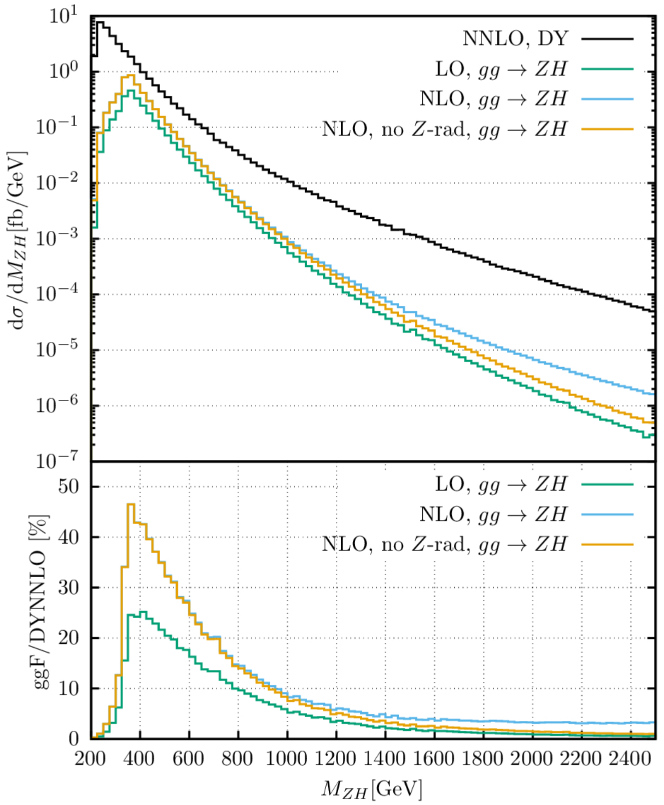
- Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo



$gg \to ZH$ @ vs Drell-Yan contribution

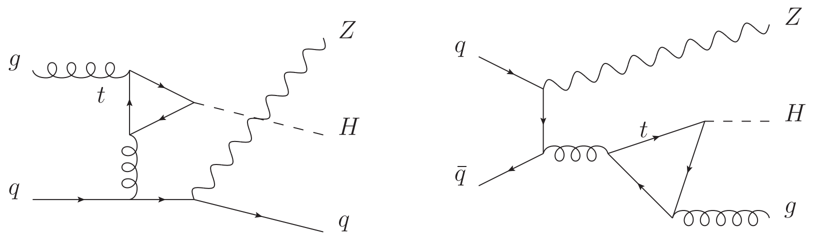
- $gg \to ZH$ is almost 50% of DY near $M_{ZH} \sim 2\, m_t$

  DY obtained using **vh@nnlo**
  [Harlander et al - 1802.04817]



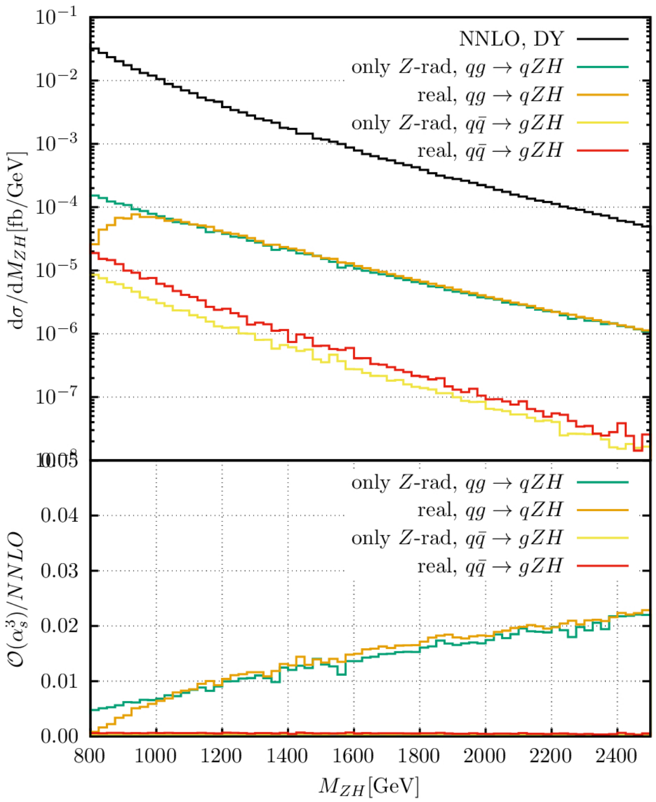
[Degrassi, Gröber, MV, Zhao - 2205.02769]

High-Energy Tails II – Z Radiation



In the high-energy tail (M_{ZH} > 1 TeV)

- **qg → ZHq channel**
  - Z-radiated diagrams dominate
  - Non-negligible contribution (up to 2% wrt DY)
- **$q\bar{q}$ → ZHg channel**
  - Z-radiated diagrams dominate
  - Negligible (PDF suppression)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \to ZH$ @ NLO QCD – Inclusive Cross Section

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[Wang, Xu, Xu, Yang - 2107.08206]

$\sqrt{s} = 13\text{TeV}$

$\mu_r = \mu_f$	σ_{LO}^{gg}	σ_{NLO}^{gg}	$\sigma_{pp\to ZH}^{\text{no }gg}$	$\sigma_{pp\to ZH}$	$\sigma_{\text{NLO}}^{gg,m_t\to\infty}$	$\sigma_{pp\to ZH}^{m_t\to\infty}$
$M_{ZH}/3$	73.56(7)	129.4(3)	784.0(7)	913.4(7)	133.6(6)	917.6(9)
M_{ZH}	51.03(5)	101.7(2)	781.1(7)	882.9(7)	106.0(4)	887.2(8)
$3M_{ZH}$	36.62(4)	80.4(2)	780.7(8)	861.1(8)	84.0(3)	864.8(9)

[Chen et al. - 2204.05225]

$\mu_r = \mu_f = M_{ZH}$

$\sqrt{s}$	LO [fb]	NLO [fb]
13 TeV	$52.42^{+25.5\%}_{-19.3\%}$	$103.8(3)^{+16.4\%}_{-13.9\%}$
13.6 TeV	$58.06^{+25.1\%}_{-19.0\%}$	$114.7(3)^{+16.2\%}_{-13.7\%}$
14 TeV	$61.96^{+24.9\%}_{-18.9\%}$	$122.2(3)^{+16.1\%}_{-13.6\%}$

- General agreement between the three results when same inputs are adopted

[Degrassi, Gröber, MV, Zhao - 2205.02769]

$\sqrt{s} = 13\text{TeV}$

$\mu_r = \mu_f = M_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

$gg \to ZH$ @ NLO QCD – Inclusive Cross Section

Karlsruhe Institute of Technology

[Wang, Xu, Xu, Yang - 2107.08206]

$\sqrt{s} = 13\text{TeV}$

$\mu_r = \mu_f$	σ^{gg}_{LO}	σ^{gg}_{NLO}	$\sigma^{\text{no } gg}_{pp\to ZH}$	$\sigma_{pp\to ZH}$	$\sigma^{gg,m_t\to\infty}_{\text{NLO}}$	$\sigma^{m_t\to\infty}_{pp\to ZH}$
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- NLO corrections are the same size as LO

[Degrassi, Gröber, MV, Zhao - 2205.02769]

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- Scale uncertainties reduced by ~30% wrt LO

[Degrassi, Gröber, MV, Zhao - 2205.02769]

$\sqrt{s} = 13\text{TeV}$
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