Transport Coefficients from pQCD to the Hadron Resonance Gas at finite BSQ densities
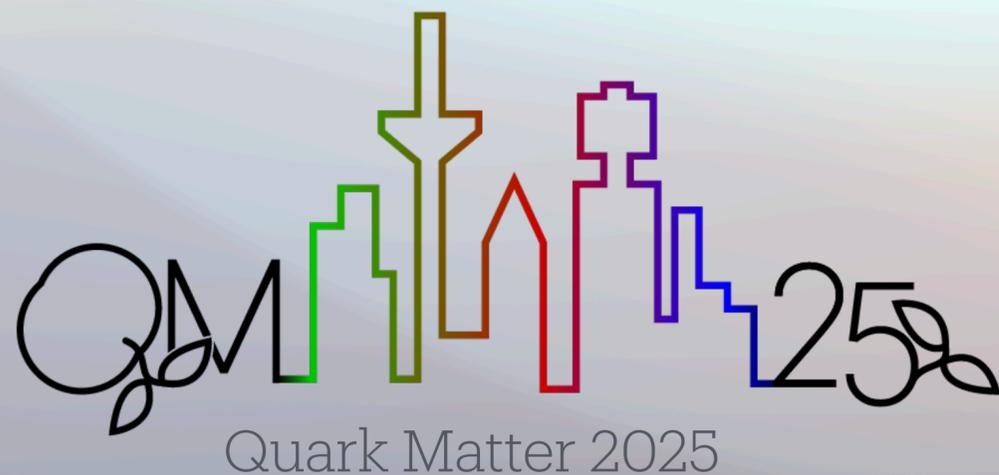
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In collaboration with Guy Moore, Jaki Noronha-Hostler, and Jordi Salinas San Martin.

Based on arXiv:2406.04968 and arXiv:2408.00524

Quark Matter 2025

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UNIVERSITY OF ILLINOIS
URBANA-CHAMPAIGN

Introduction

Quark-Gluon Plasma

Primordial Universe ($<10^{-6}$ s)

Hadronic gas

Phase Transition

Neutron Stars

"Solid" state

"Ordinary" state

Net baryonic density (normalised, d/d_0)

2011 CERN, for the benefit of the ALICE Collaboration

- 90% credible region [arXiv:2106.05019]
- Posterior median
- Nature Physics 15, (2019)
- $1/(4\pi)$

- State-of-the-art Bayesian analyses using relativistic viscous hydrodynamics estimate the value of $\eta/s \sim 0.08 - 0.2$.

  Nature Phys. 15, 1113 (2019), Phys. Rev. C 103, 054904 (2021), Phys. Rev. C 103, 054909 (2021), Phys. Rev. C 104, 054904 (2021)

- Bayesian analyses of η/s are not based on microscopic calculations.

- It is not yet possible to directly calculate η/s in the strongly interacting regime directly from lattice QCD.

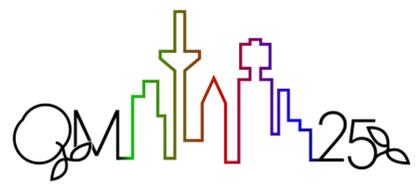 Phys. Rev. Lett. 94, 170201 (2005), Rept. Prog. Phys. 81, 084301 (2018)

- Preliminary Bayesian analyses suggest that $\eta T/w$ should increase with μ_B, but there is no clear functional form.

  Phys. Rev. C 97, 044905 (2018), Phys. Rev. Lett. 132, 072301 (2024)

- We compute $\eta T/w$ for $\tilde{\mu}(\mu_B, \mu_S, \mu_Q)$ in two limits:

  - pQCD with 3 flavors.

  - HRG with an excluded-volume.

Hadron Resonance Gas

- HRG model has been used to successfully extract thermodynamic information at freeze-out of the strongly interacting medium.
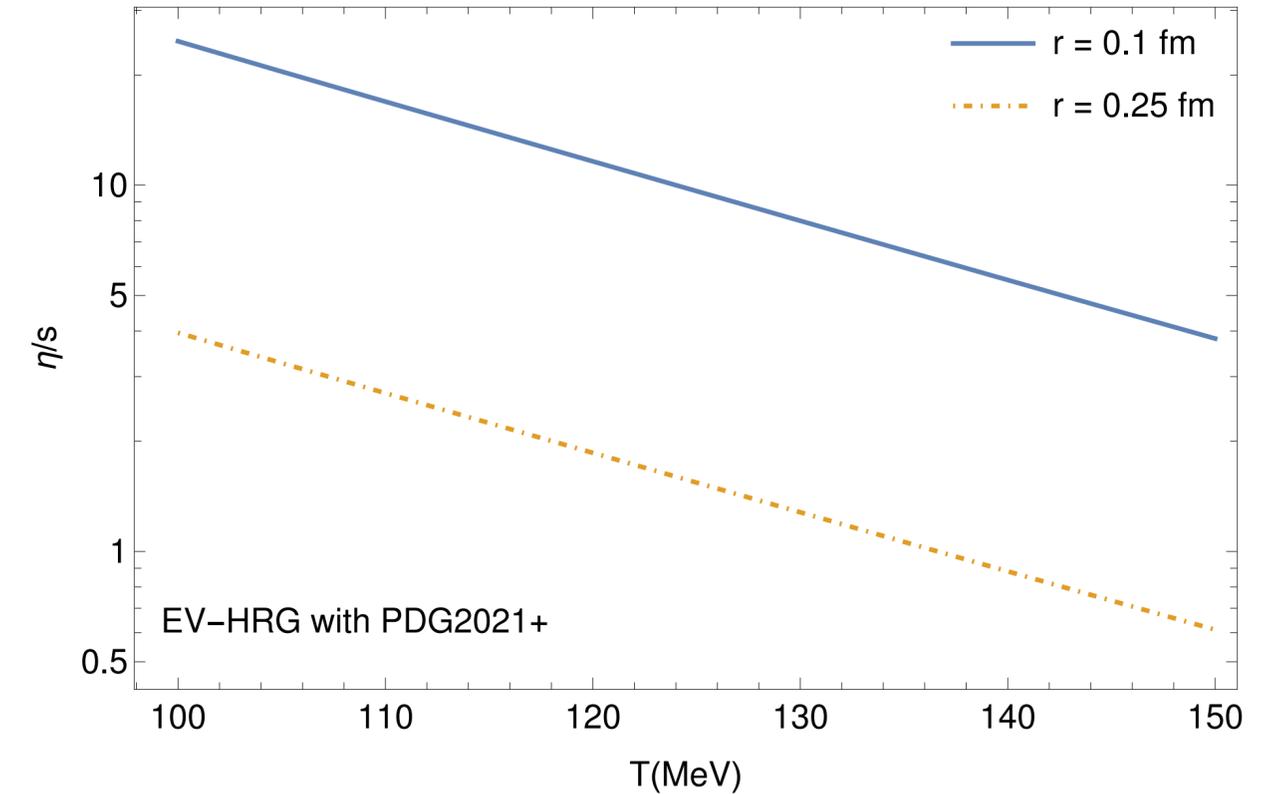
  Rapp and Shuryak, Phys. Rev. Lett. 86, 2980 (2001). Greiner and S. Leupold, J. Phys. G 27, L95 (2001).
  Greiner, Koch-Steinheimer, Liu, Shovkovy, and Stoecke, J. Phys. G 31, S725 (2005).
  Noronha-Hostler, Greiner, Beitel, and Shovkovy, Phys. Rev. C 81, 054909 (2010)
  Noronha-Hostler, Greiner, and Shovkovy, Phys. Rev. Lett. 100, 252301 (2008)
  Beitel, Greiner, and Stoecker, Phys. Rev. C 94, 021902 (2016).

- We include repulsive interactions through an excluded-volume (EV-HRG) such that the hard-core volume of each hadron i is,

$$v_i = 4 \cdot \frac{4}{3}\pi r_i^3,$$

- The shear viscosity at finite $\tilde{\mu}$ using an EV-HRG is:

$$\eta^{\text{HRG}} = \frac{5}{64\sqrt{8}}\frac{1}{r^2}\frac{1}{n_{\text{tot}}^{\text{id}}}\sum_i n_i^{\text{id}} \frac{\int_0^\infty k^3 \exp\left(\frac{-\sqrt{k^2+m_i^2}+\tilde{\mu}_i}{T}\right)dk}{\int_0^\infty k^2 \exp\left(\frac{-\sqrt{k^2+m_i^2}+\tilde{\mu}_i}{T}\right)dk}$$

EV–HRG with PDG2021+

- The normalization of η^{HRG} at finite $\tilde{\mu}$, requires an EV p^{ex} and ε^{ex},

$$\varepsilon^{\text{ex}}(T,\mu) = \frac{\varepsilon^{\text{id}}(T,\tilde{\mu})}{\exp\left[vp^{\text{ex}}(T,\tilde{\mu})/T\right] + vn_{\text{tot}}^{\text{id}}(T,\tilde{\mu})}$$

G. D. Yen et all, Phys. Rev. C 56, 2210 (1997)
Gorenstein, Kostyuk, and Krivenko,, J. Phys. G 25, L75 (1999)
Rischke, Gorenstein, Stoecker, and Greiner, Z. Phys. C 51, 485 (1991)

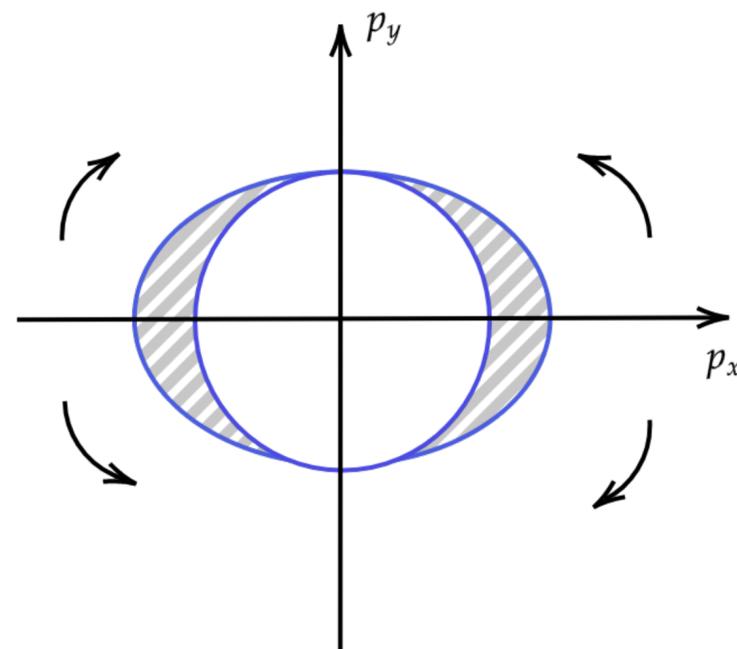# Kinetic Theory for weakly coupled QCD

- We focus on the weakly coupled picture $\longrightarrow$ gas of weakly interacting particles.

- This can be described using the Boltzmann equation:
$$\left[\vec{v}_p \cdot \frac{\partial}{\partial \vec{x}}\right] f^a(\vec{p}, \vec{x}, t) = -\mathscr{C}^a[f]$$

- This system has a non-equilibrium distribution that is given by: $f^a(\vec{k}, \vec{x}) = f_0^a(\vec{k}, \vec{x}) + f_0^a(1 \pm f_0^a) f_1^a(\vec{k}, \vec{x})$

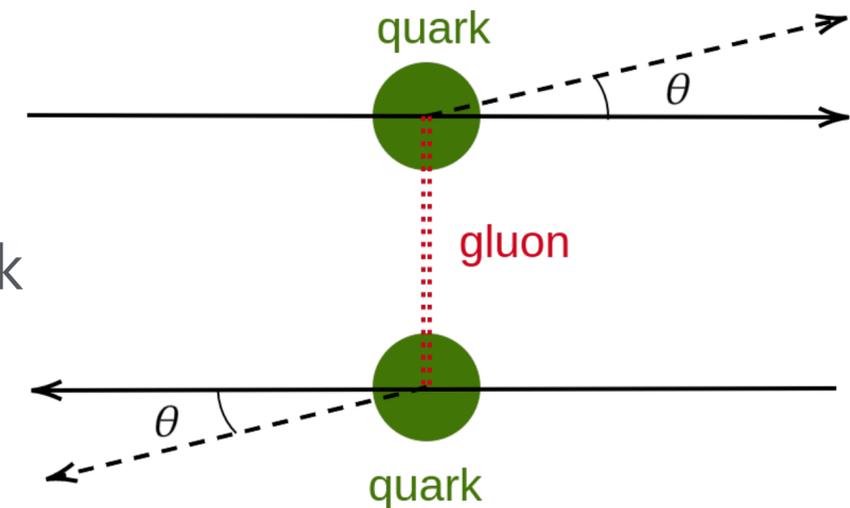 (a is the species - gluon, quark, or anti-quark)

The LHS:

- Describes propagation.

- leads an inhomogeneous system out of equilibrium

The RHS:

- Scatterings.

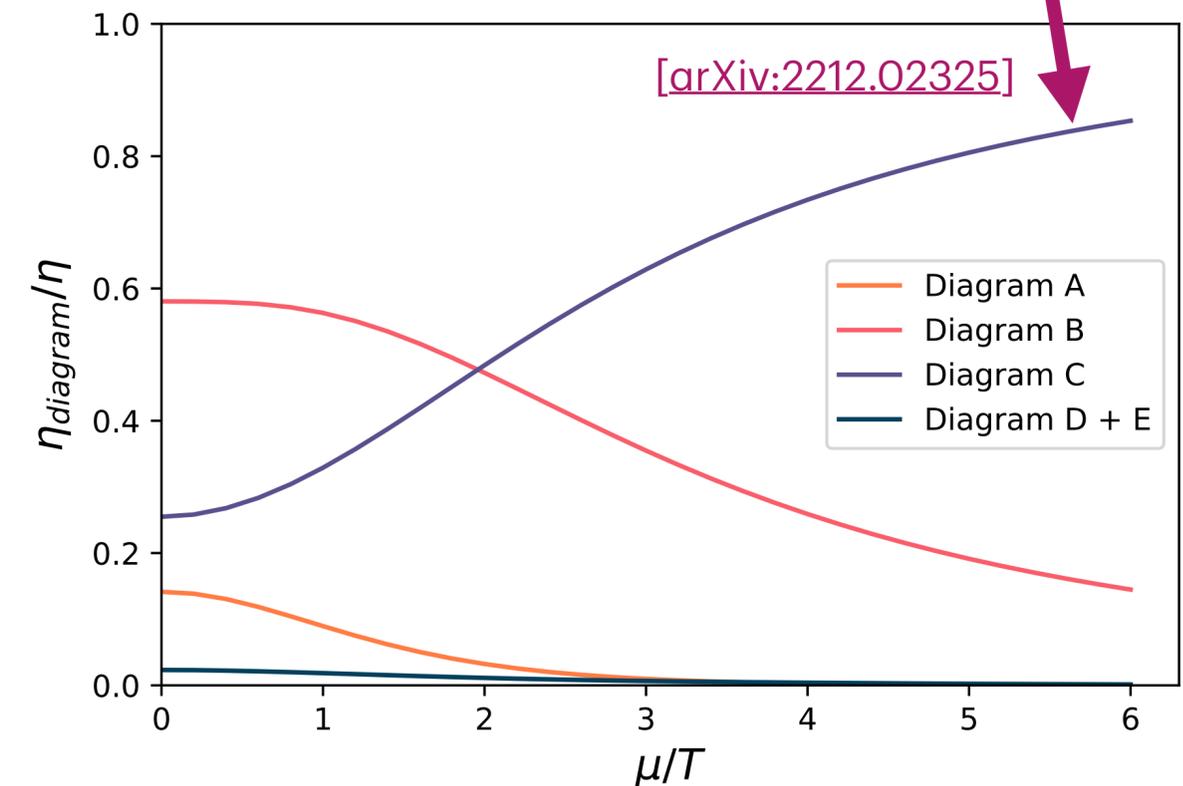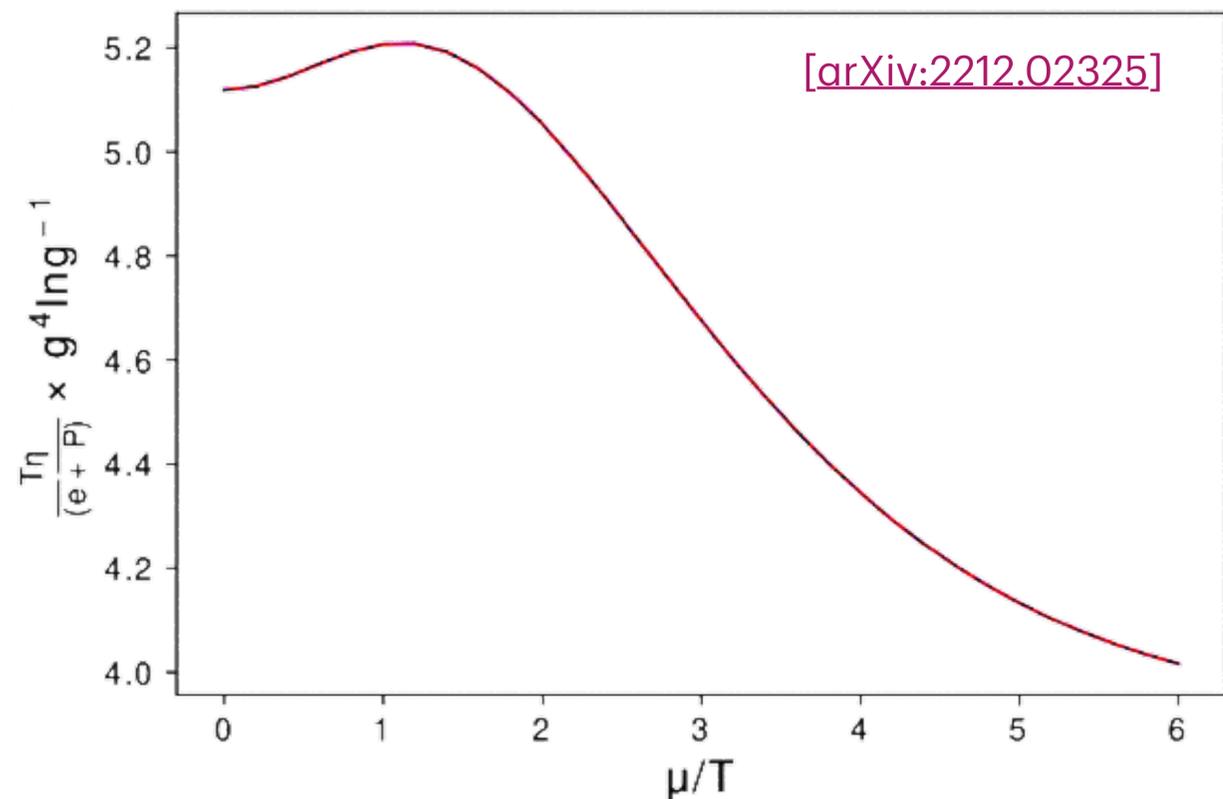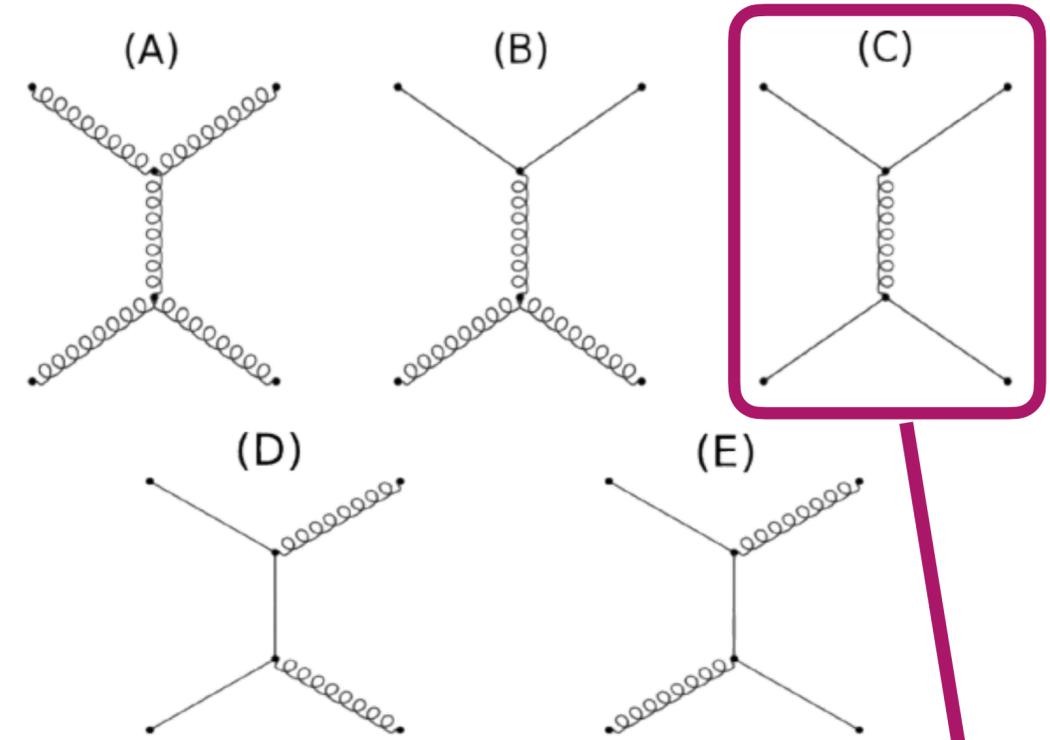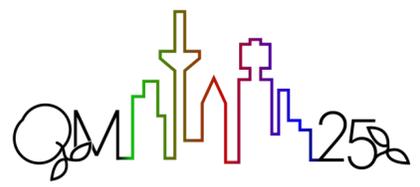- Bring the system back toward equilibrium

Shear viscosity: leading log description

- Our goal is to obtain the size of the contribution of each diagram → perturbation theory.

- High density ⇒ high chemical potential μ.

- Since high μ means more quarks, diagram (C) is the interesting one.

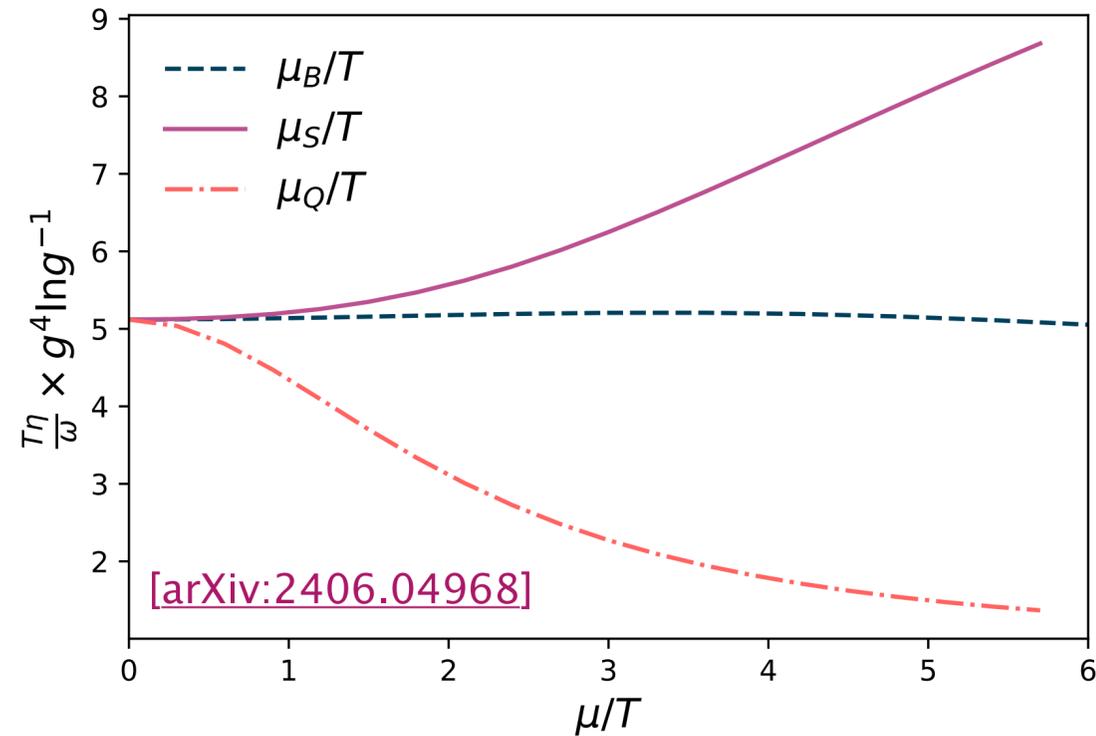[arXiv:2212.02325]

[arXiv:2212.02325]

Multiple conserved charges (μ_B, μ_S, μ_Q)

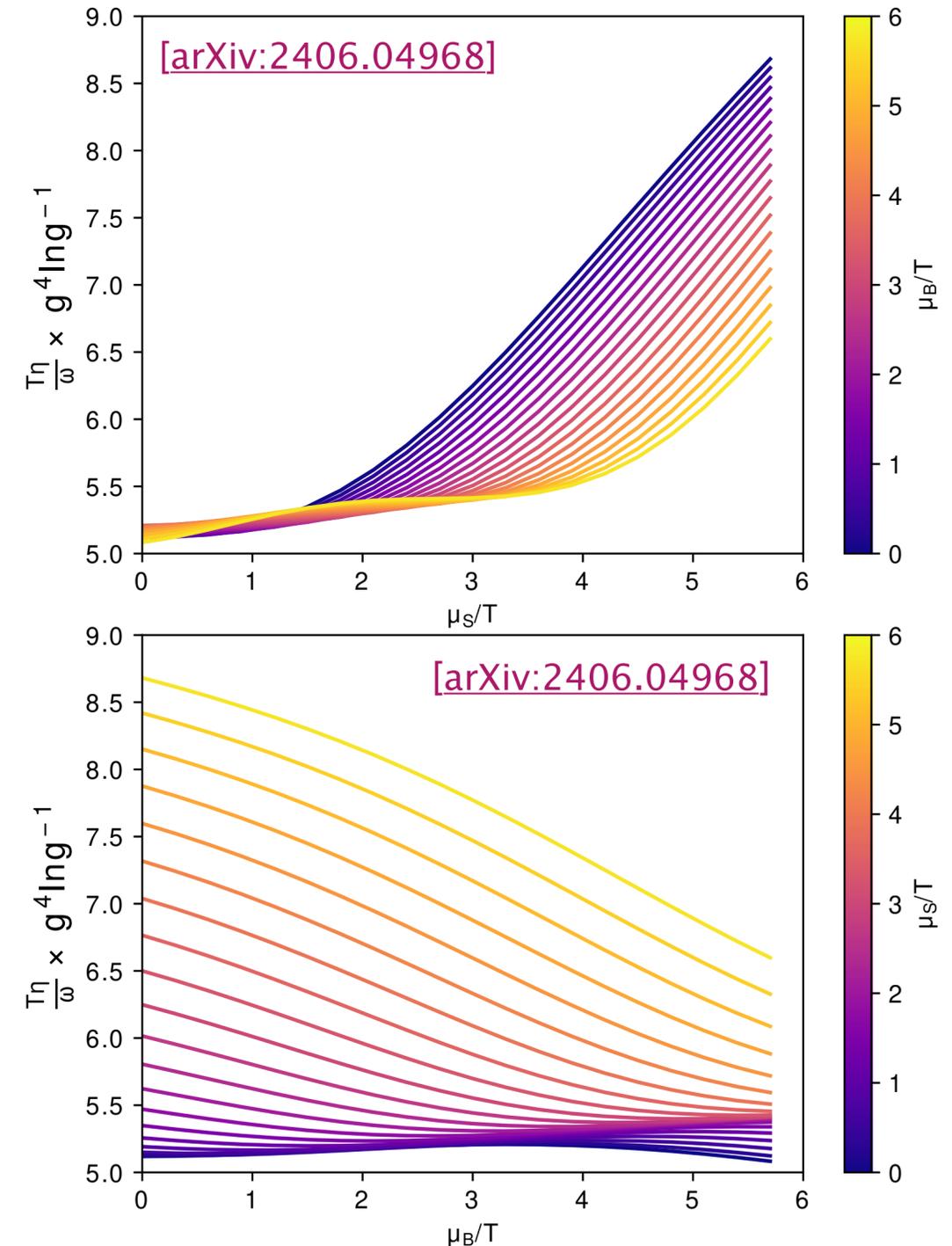- η depends on an effective chemical potential:

$$\tilde{\mu}_i = B_i\frac{\mu_B}{T} + S_i\frac{\mu_S}{T} + Q_i\frac{\mu_Q}{T}$$

[arXiv:2406.04968]

- Interplay between different chemical potentials:

[arXiv:2406.04968]

[arXiv:2406.04968]

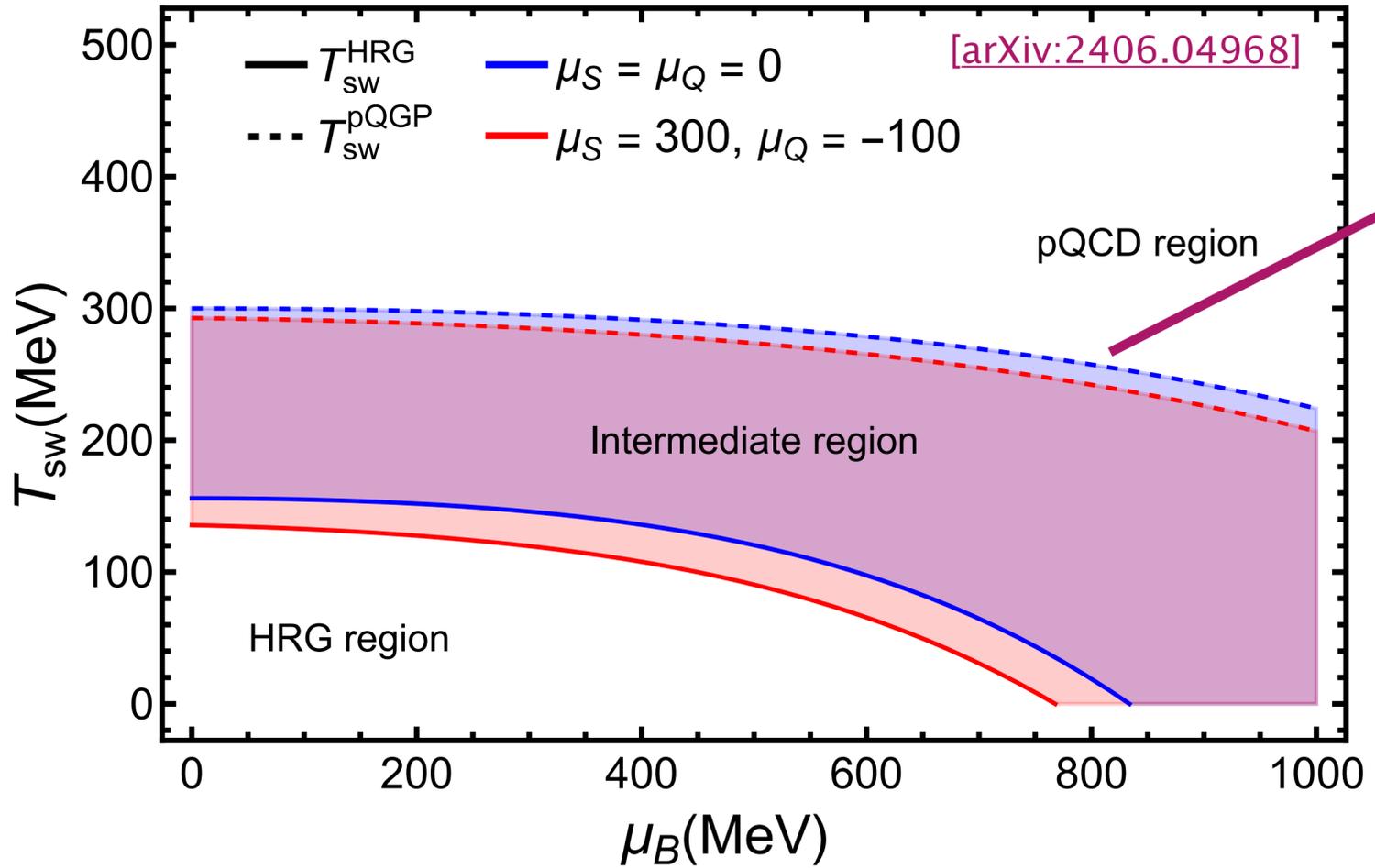- This change implies that the scattering matrix will evolve differently as a function of each chemical potential.

Shear Viscosity across the QCD phase diagram
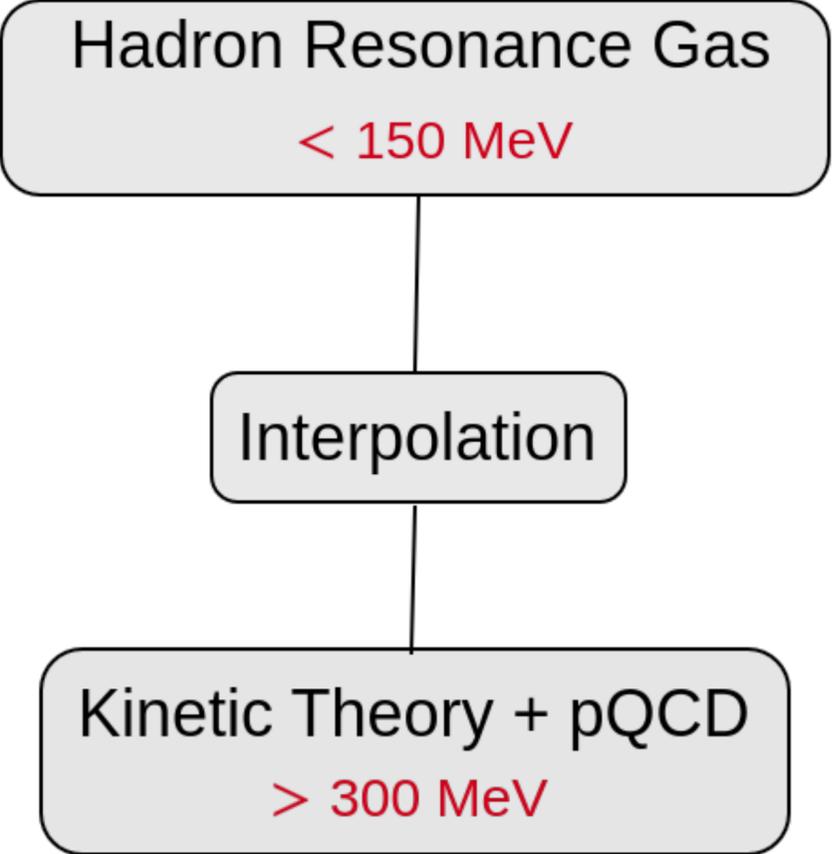
- We consider the chiral transition line with three conserved charges: A. Bazavov et al. (HotQCD), Phys. Lett. B 795, 15 (2019)

$$\frac{T_{sw}(\mu_B, \mu_S, \mu_Q)}{T_{sw,0}} = \left\{ 1 - \sum_{X=B,S,Q} \left[\kappa_2^X \left(\frac{\mu_X}{T_{sw,0}} \right)^2 + \kappa_4^X \left(\frac{\mu_X}{T_{sw,0}} \right)^4 \right] + 2\kappa_{BS} \frac{\mu_B \mu_S}{T^2} \right\}$$

- The region close to the first-order transition line cannot be described by pQCD or an HRG model.

$T_{sw}(\mu_B, \mu_S, \mu_Q)$ is a free parameter.

We use 300 MeV.

Hadron Resonance Gas
< 150 MeV

Interpolation
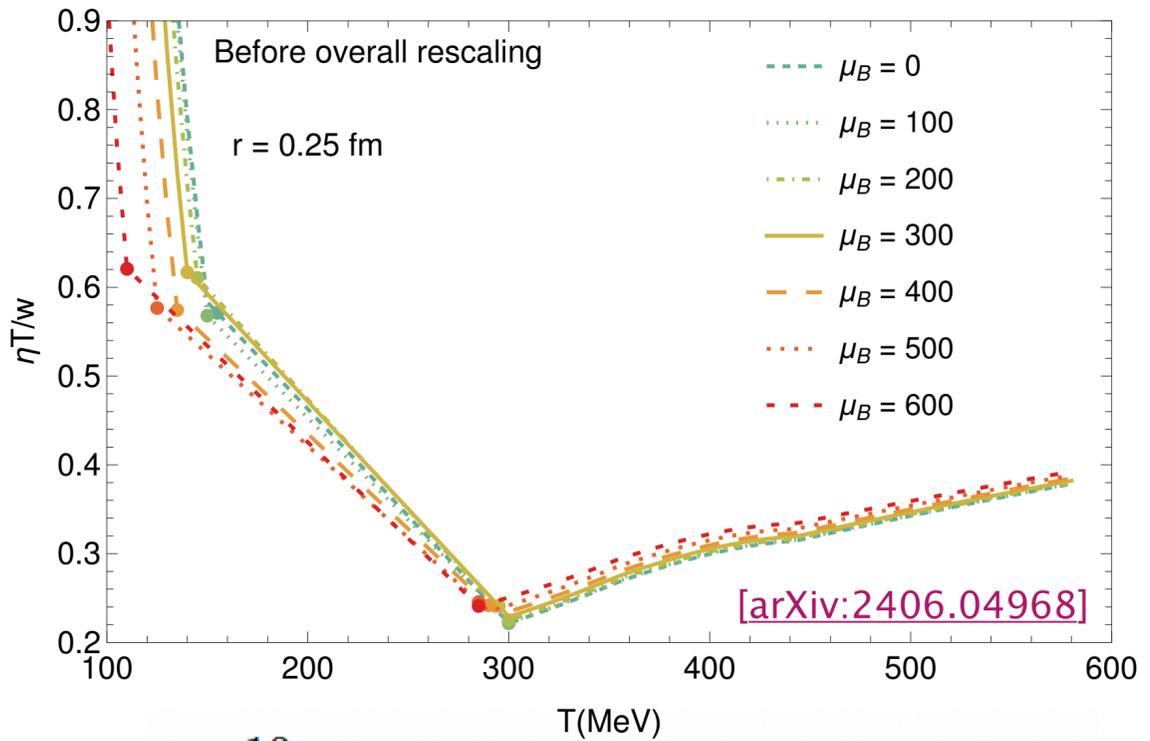
Kinetic Theory + pQCD
> 300 MeV

Shear Viscosity across the QCD phase diagram

- Our shear viscosity algorithm is:

$$\left(\frac{\eta T}{w}\right)_{\text{tot}}(T, \tilde{\mu}) = g_{\text{norm}} \begin{cases} \left(\frac{\eta T}{w}\right)_{\text{HRG}} & T \leq T_{\text{sw}}^{\text{HRG}}, \\ \left(\frac{\eta T}{w}\right)_{\text{intermediate}} & T_{\text{sw}}^{\text{HRG}} < T < T_{\text{sw}}^{\text{pQCD}}, \\ g_{\text{GMT}} \left(\frac{\eta T}{w}\right)_{\text{pQCD}} & T \geq T_{\text{sw}}^{\text{pQCD}}, \end{cases}$$
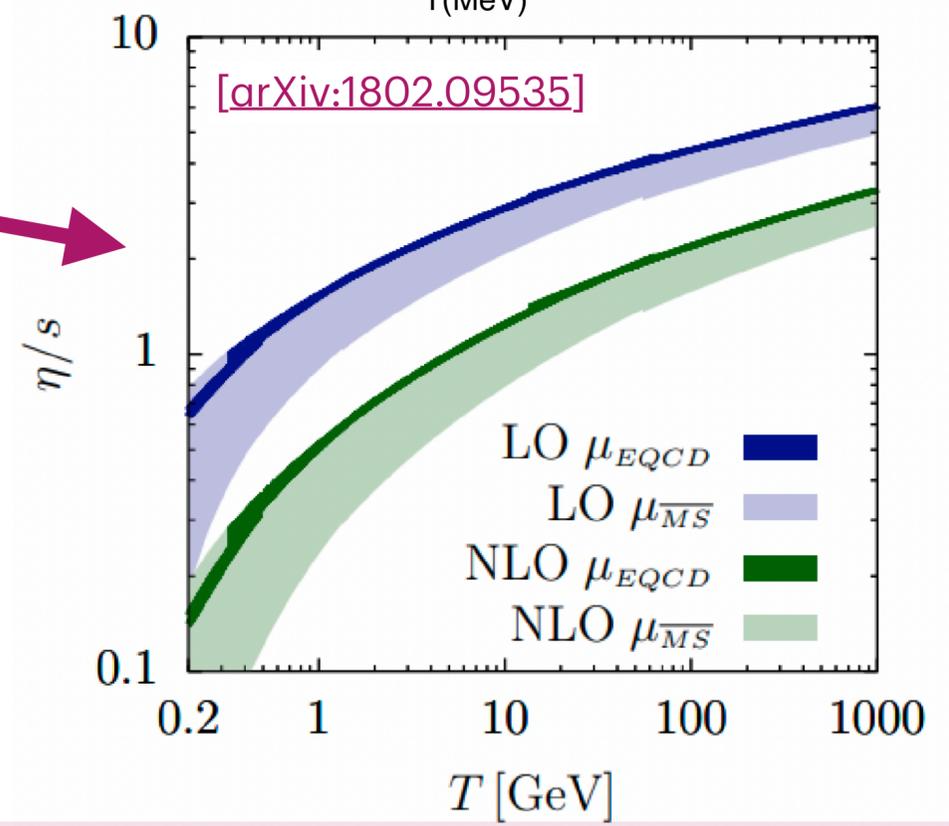

Before overall rescaling

r = 0.25 fm

μ_B = 0
μ_B = 100
μ_B = 200
μ_B = 300
μ_B = 400
μ_B = 500
μ_B = 600

[arXiv:2406.04968]

g_{GMT} ➡ scaling factor to reproduce the pQCD results from GMT at $\tilde{\mu} = 0$

Ghiglieri, Moore, and Teaney, JHEP 03, 179

[arXiv:1802.09535]

LO μ_{EQCD}
LO $\mu_{\overline{MS}}$
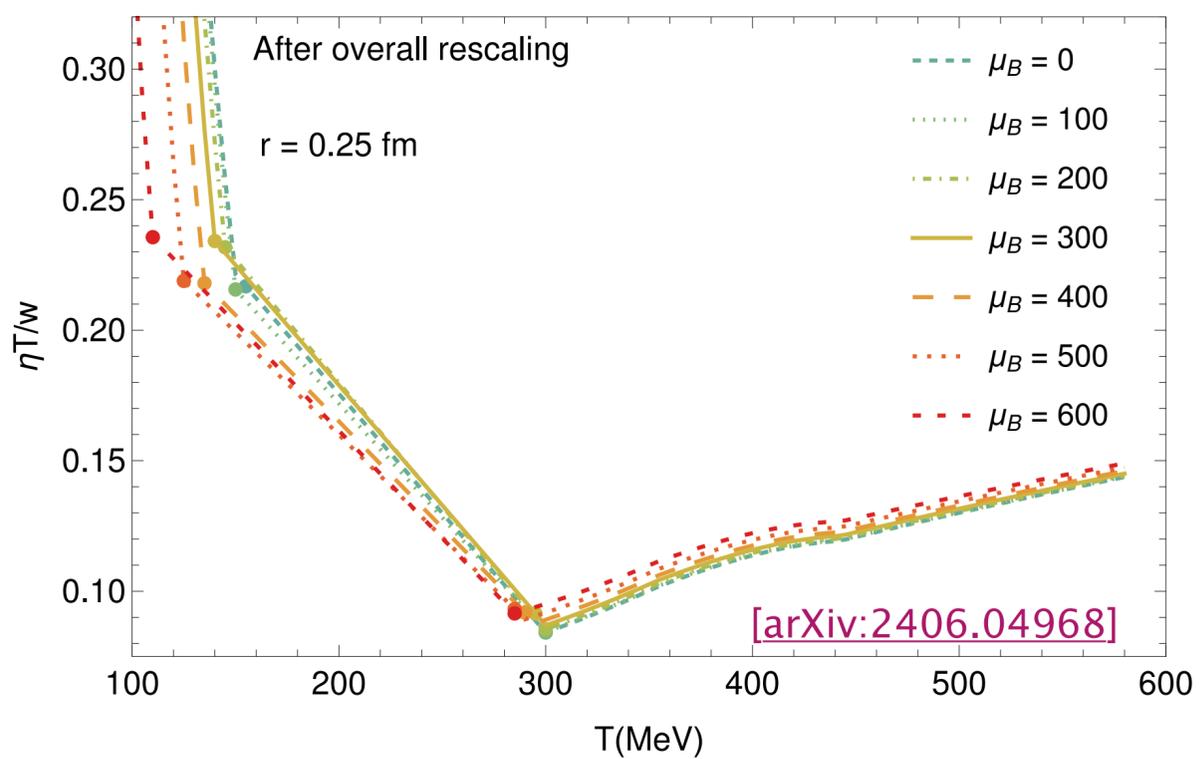NLO μ_{EQCD}
NLO $\mu_{\overline{MS}}$

- Our procedure is:

Find $(\eta/s)_{\text{min}}$ for $\mu_B = 0$ ➡ T_{min} temperature where $(\eta/s)_{\text{min}}$ has its minimum
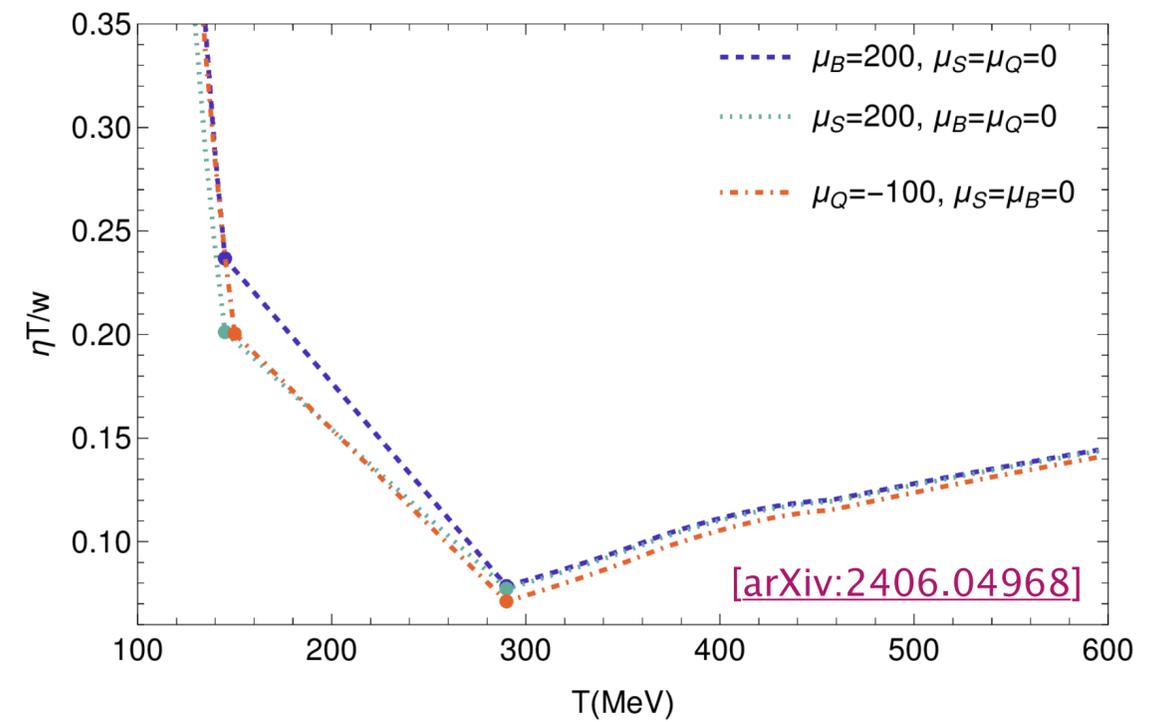
Then choose our new minimum

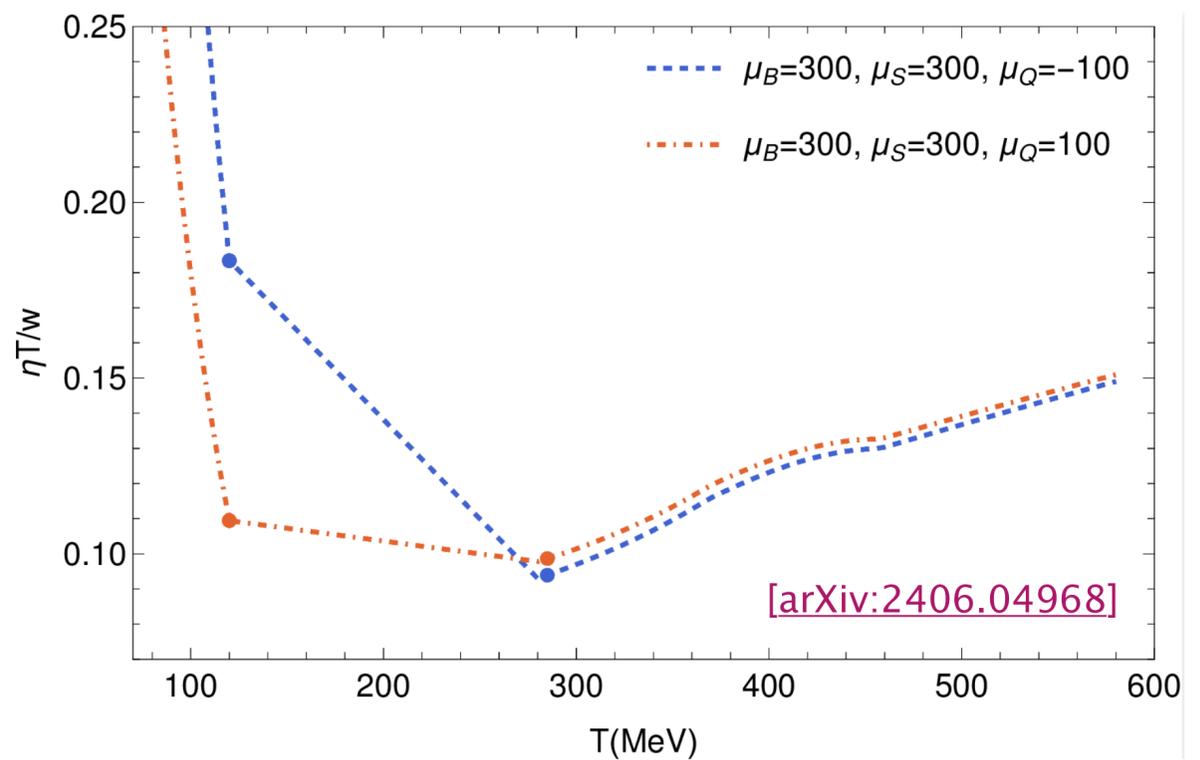$(\eta/s)_{\text{set}} \equiv 0.08$ ➡ $g_{\text{norm}} = \dfrac{(\eta/s)_{\text{set}}}{(\eta/s)_{\text{min}}}$,

Shear Viscosity across the QCD phase diagram

After overall rescaling

r = 0.25 fm

- $\mu_B = 0$
- $\mu_B = 100$
- $\mu_B = 200$
- $\mu_B = 300$
- $\mu_B = 400$
- $\mu_B = 500$
- $\mu_B = 600$

[arXiv:2406.04968]

- Our setup shows that the HRG has a stronger μ_B dependence than the pQCD phase.

- $\mu_B=200, \mu_S=\mu_Q=0$
- $\mu_S=200, \mu_B=\mu_Q=0$
- $\mu_Q=-100, \mu_S=\mu_B=0$

[arXiv:2406.04968]

- $\mu_B=300, \mu_S=300, \mu_Q=-100$
- $\mu_B=300, \mu_S=300, \mu_Q=100$

[arXiv:2406.04968]

- $\mu_Q < 0$ has a larger $\eta T/\omega$ than $\mu_Q > 0$ in the HRG phase, and the opposite happens in the pQCD phase.

- The HRG results find that a finite μ_S leads to a suppression of $\eta T/w$ in the HRG phase.

- For $T < 300$ MeV, a finite μ_Q, and a finite μ_S leads to the smallest $\eta T/w$.
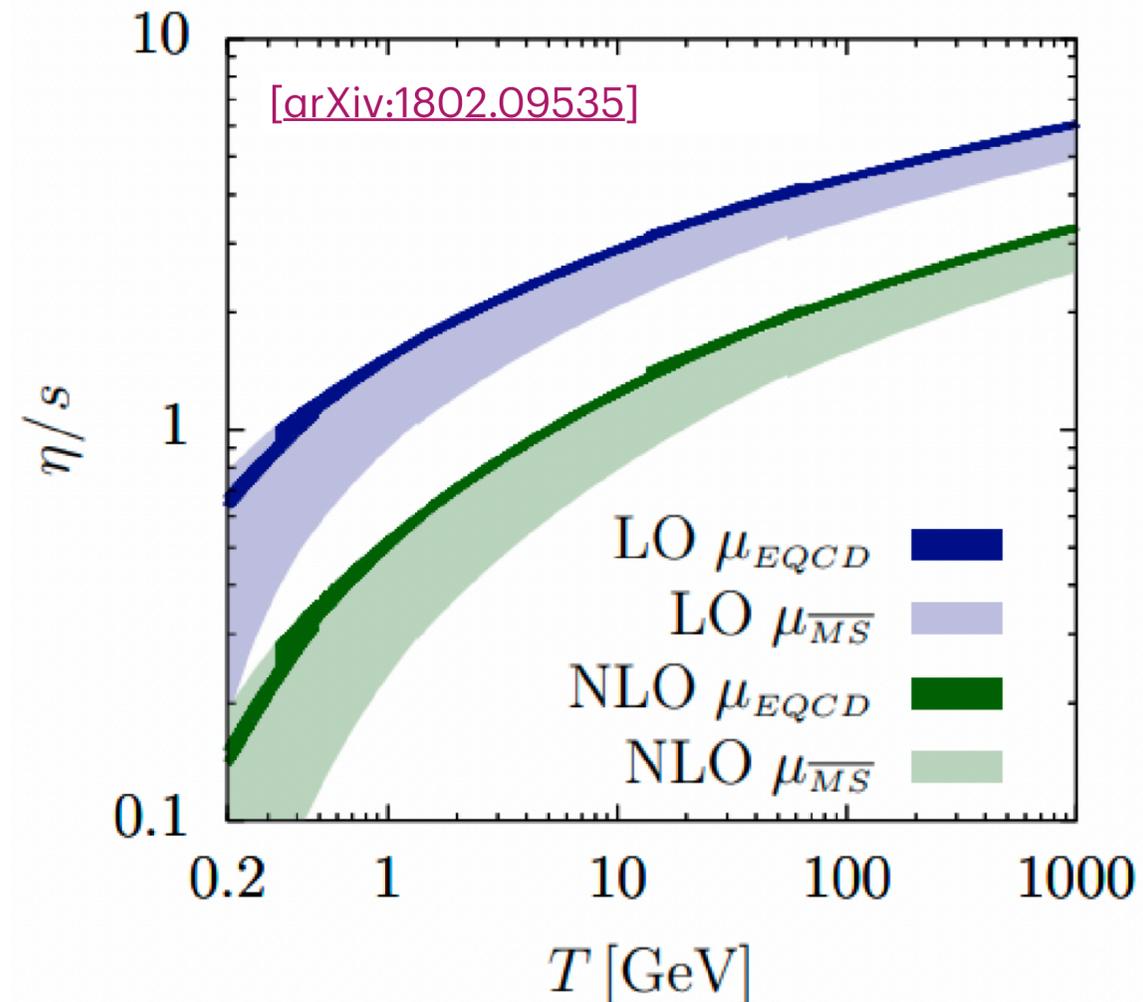
Next-to-leading order corrections for QCD

- Ghiglieri, Moore, and Teaney investigated NLO effects and found large corrections at vanishing μ_B.
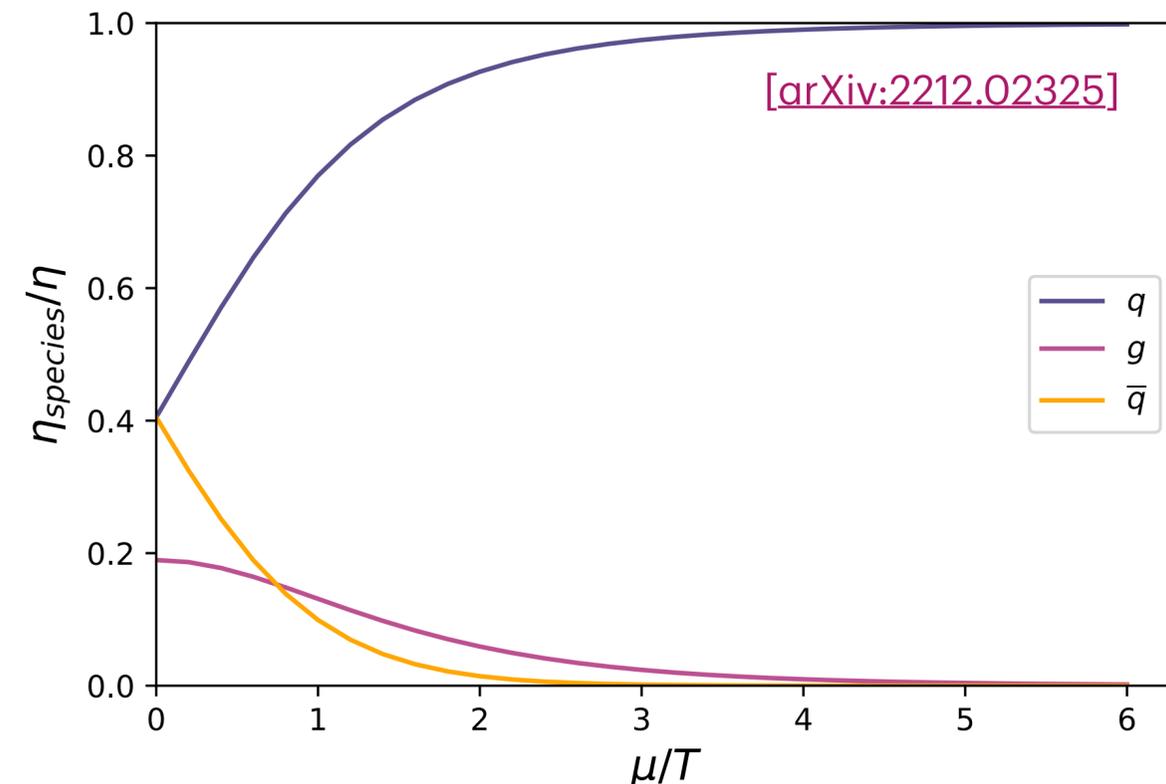
  Ghiglieri, Moore, and Teaney, JHEP 03, 179

- This suggests that η/s is smaller than the LO perturbative estimate.

[arXiv:1802.09535]

- Signals severe convergence problems in the perturbative expansion, even for large T or small g.

- These NLO corrections are gluonic in nature.

- Quark scattering dominates at large $\mu \to$ large NLO effects will be less important.

[arXiv:2212.02325]
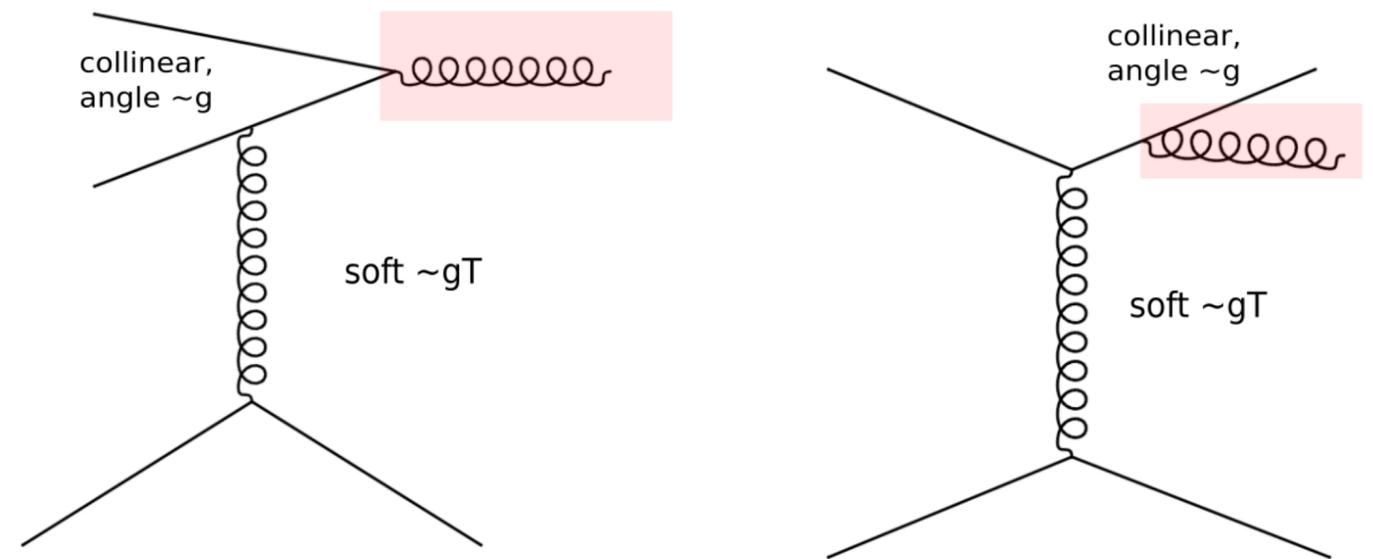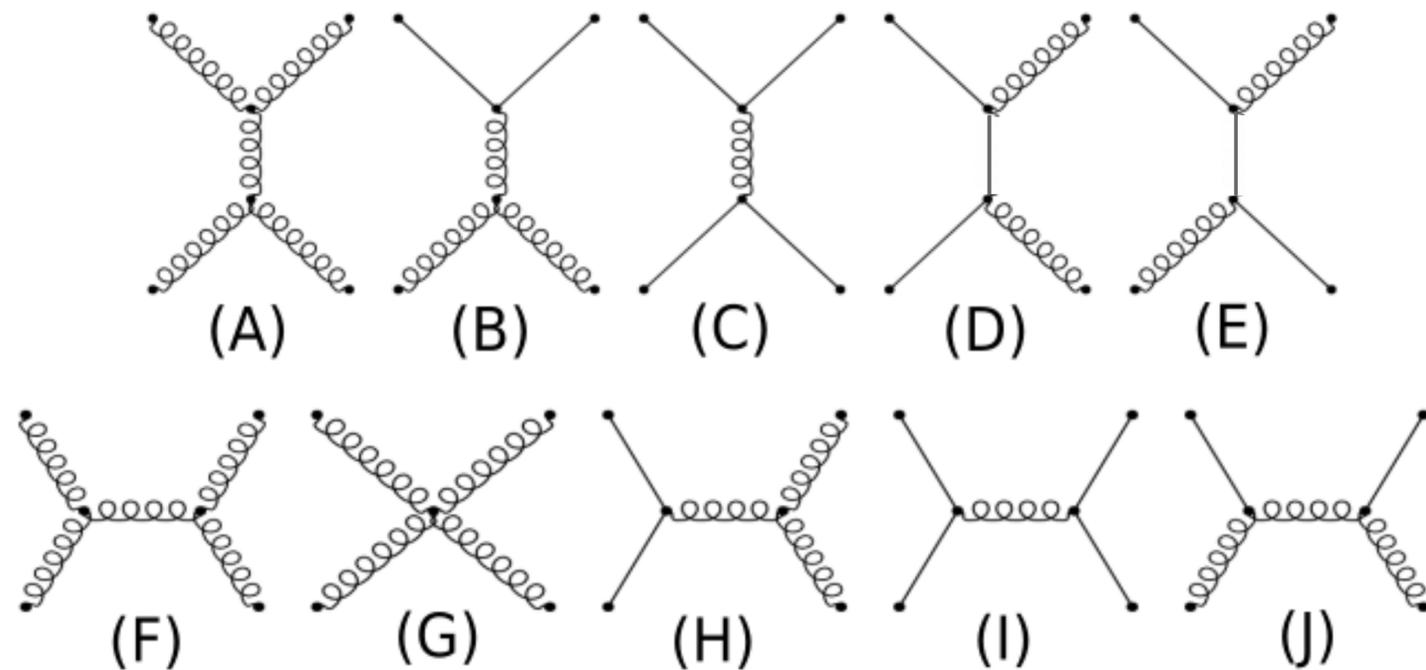
Leading order description for QCD

Arnold, Moore and Yaffe, JHEP 05 (2003) 051.

- The leading-order collision operator encodes the contribution of tree-level scattering processes, with Hard-Loop resummed propagators, and the effect of splitting caused by any number of soft scatterings:

$$\mathscr{C}_a[f] = \mathscr{C}_a^{2\leftrightarrow2}[f] + \mathscr{C}_a^{1\leftrightarrow2}[f]$$

- $\mathscr{C}_a^{2\leftrightarrow2}[f]$ involves diagrams A-J, as well as the interference terms:

- $\mathscr{C}_a^{1\leftrightarrow2}[f]$ contributions involve $q \leftrightarrow qg$, $\bar{q} \leftrightarrow \bar{q}g$, $g \leftrightarrow \bar{q}q$, and $g \leftrightarrow gg$.

(A) (B) (C) (D) (E)

(F) (G) (H) (I) (J)

collinear, angle ~g

soft ~gT

collinear, angle ~g

soft ~gT

Danhoni and Moore, JHEP 09 (2024) 075
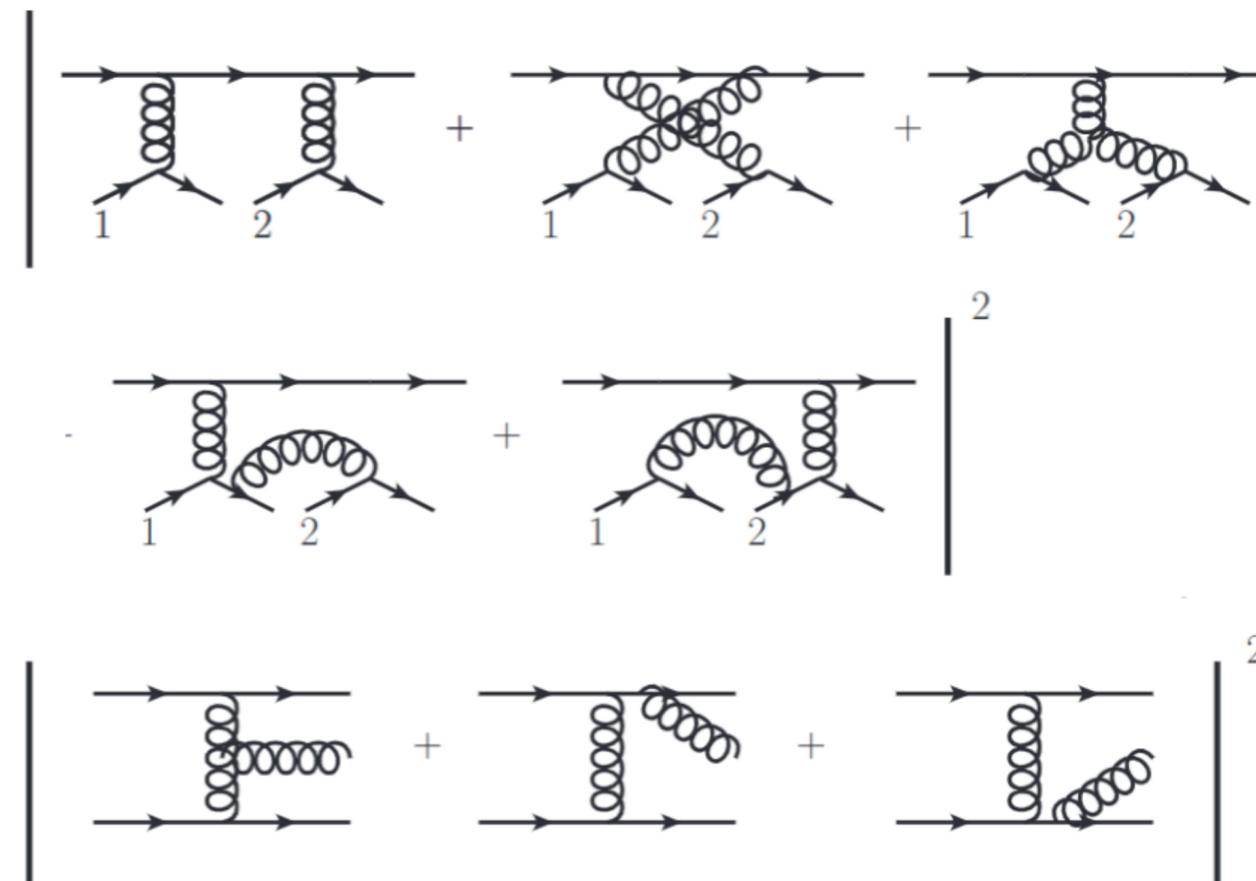
Next-to-leading order description

- Small-momentum-exchange effects.

- Diagrams (A) and (B) will have large corrections if the gluon carries soft momentum.

- The interference between multiple soft scatterings, soft gluon emission during scattering, and nonabelian interactions between scattering processes.

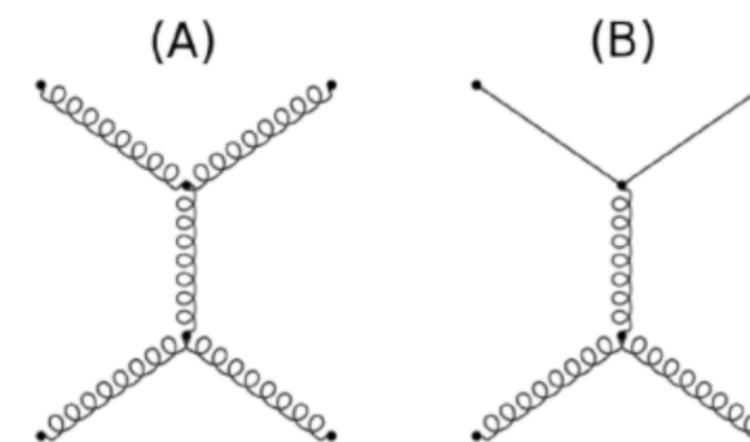- At NLO, the collision term will be given as,

$$\left(f_1, \mathscr{C}_{NLO} f_1\right) = \left(f_1, \mathscr{C}_{LO} f_1\right) + \left(f_1, \delta\mathscr{C} f_1\right)$$

(A)   (B)

- δC represents the NLO corrections,

$$\left(f_1, \delta\mathscr{C} f_1\right) = \left(f_1, \mathscr{C}^{\delta\hat{q}} f_1\right) - \left(f_1, \mathscr{C}^{2\leftrightarrow2}_{\mathcal{O}(g)^{finite}} f_1\right) + \left(f_1, \mathscr{C}^{semi} f_1\right) + \left(f_1, \delta\mathscr{C}^{1\leftrightarrow2} f_1\right)$$

Next-to-leading order results

- Note that the chemical potential μ we refer to is that for a quark.

- We have used the EQCD-inspired scale setting by choosing $\mu_{EQCD} = 2.7T$.

- The relevant physics is much more dominated by quarks and less by gluons, and the strong mutual interactions between gluons are the cause of large NLO effects.

- For higher densities, soft gluon contributions are less important, and LO effects again dominate the collision operator, but still with a big contribution from NLO.

- These calculations confirm the behavior found earlier for LL.

[arXiv:2408.00524]

LO	NLO
$\mu/T = 0$	$\mu/T = 0$
$\mu/T = 3$	$\mu/T = 3$
$\mu/T = 6$	$\mu/T = 6$

[arXiv:2408.00524]

$\mu/T = 0$	
$\mu/T = 3$	
$\mu/T = 6$	

Summary and outlook

- We used pQCD results with three conserved charges, a hadron resonance gas picture, and a state-of-the-art list of resonances to study the QCD shear viscosity at large baryon densities.

- We have applied a phenomenological approach to produce curves of $\eta T/w(T, \mu_B, \mu_S, \mu_Q)$ across the QCD phase diagram, which can used to feed relativistic viscous hydrodynamic codes simulating collisions at energies covered by the RHIC Beam Energy Scan or BSQ fluctuations of conserved charges at the LHC ⟶ See Kevin Pala's poster about CCAKE

- The combination of all three chemical potentials and the effect of the transition region leads to non-monotonic changes in $\eta T/w$ across the phase diagram.

- We have extended the investigation of shear viscosity for hot QCD with μ_B to both leading and (almost) next-to-leading order.

- Perturbation theory does not converge at experimentally achievable combinations of (μ_B, T), but it comes closer at high μ_B than at small μ_B.