



# Dependence of the bulk viscosity of neutron star matter on the nuclear symmetry energy

Yumu Yang

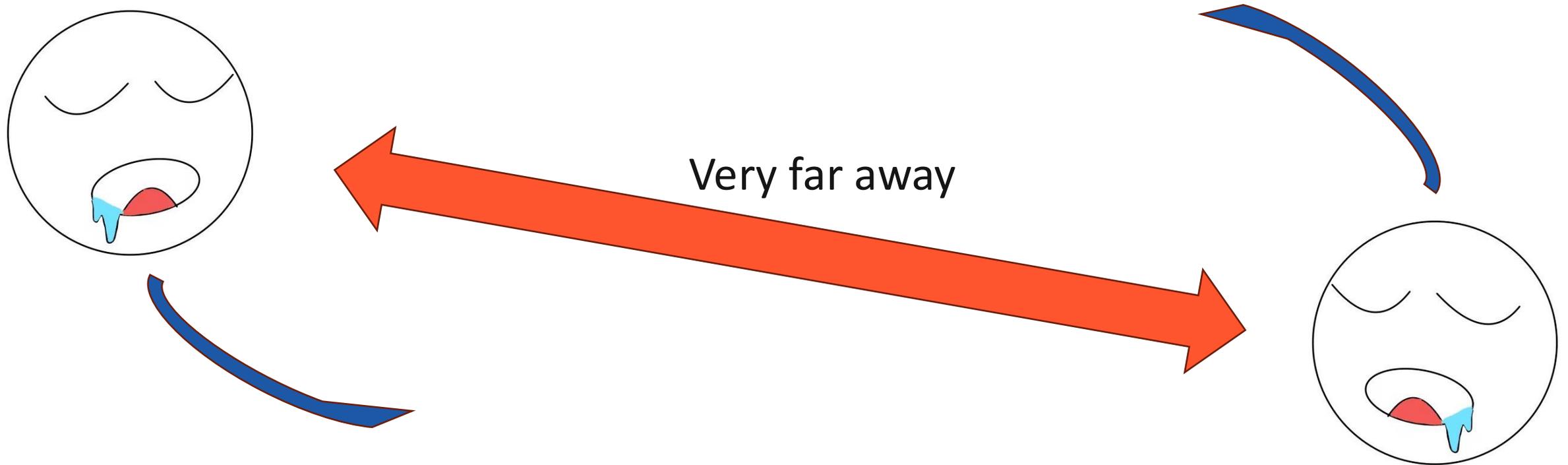
Collaboration with: Mauricio Hippert, Enrico Speranza, Jorge Noronha

Quark Matter 2025

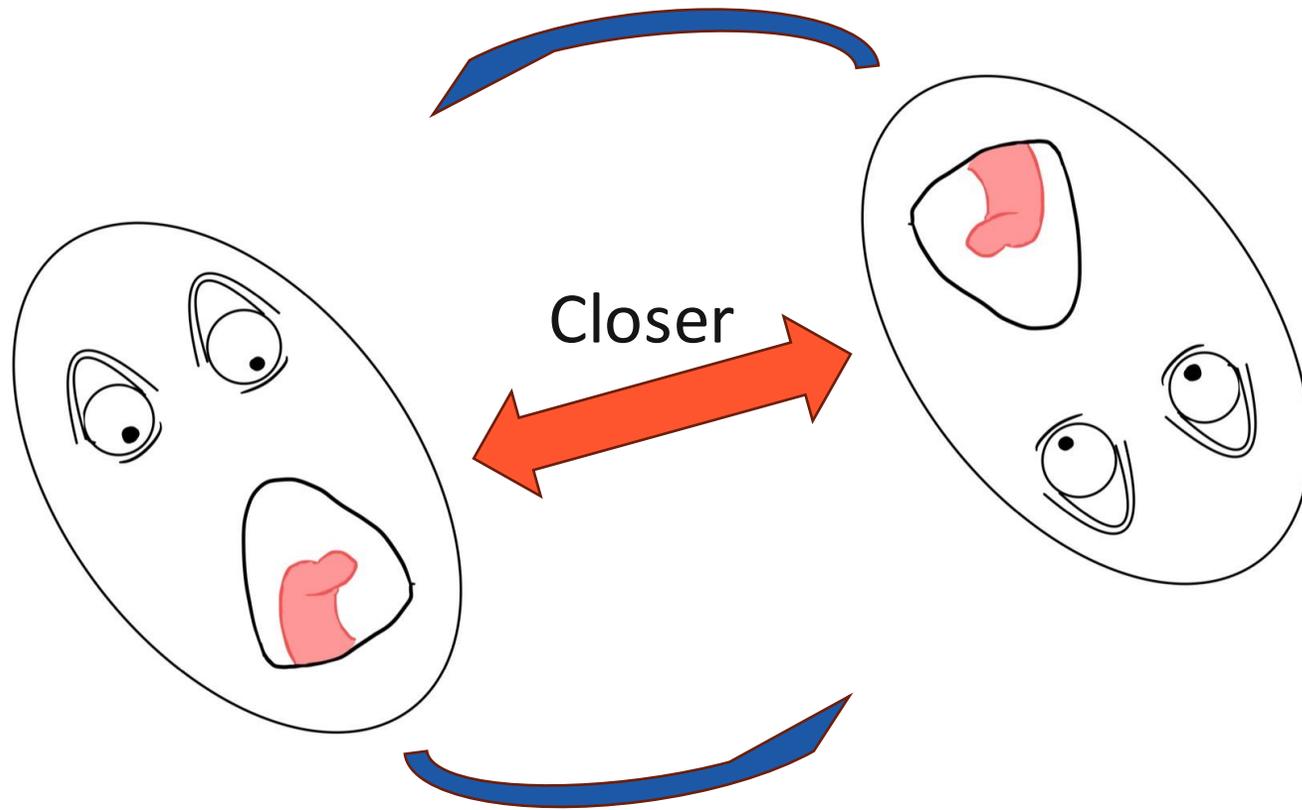
YY, Hippert, Speranza, Noronha, arXiv 2504.07805

# How do neutron stars get out of chemical equilibrium?

# Consider a neutron star binary Initially, chemical equilibrium...



# Deformed by tidal forces...



Change in density



Out of chemical equilibrium

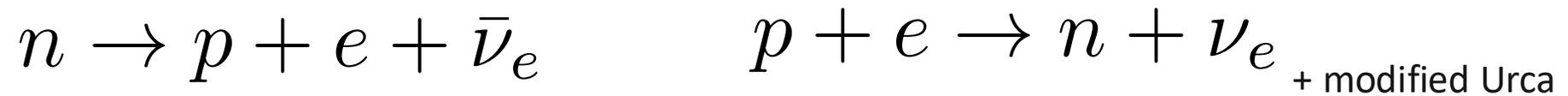


Equilibrated through weak interactions

# How is the system equilibrated?

Sawyer, PRD (1989)

- Consider neutrino-transparent npe matter: neutron, proton, and electron
- When the density changes, flavor content needs to be adjusted



Out of equilibrium physics from chemical imbalance

- Beta equilibrium:  $\delta\mu = \mu_n + \mu_{\nu_e} - \mu_p - \mu_e = 0$

# How to describe this out-of-equilibrium dynamics?

# Reactive fluid

(coupled with Einstein's equations)

- Baryon conservation

$$\nabla_{\mu} (n_B u^{\mu}) = 0$$

- Energy-momentum conservation

$$\nabla_{\mu} [(\varepsilon + P)u^{\mu}u^{\nu} - Pg^{\mu\nu}] = 0$$

- **Non-conserved** electron current

$$\nabla_{\mu} (n_e u^{\mu}) = \Gamma_e$$

Reaction rates  
(weak interactions)

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Gavassino et al., Class. Quant. Grav. (2021)  
Celora et al. PRD (2022)  
Gavassino, Noronha, PRD (2024)  
YY, Hippert, Speranza, Noronha, PRC (2024)

# Bulk-viscous fluid

- Total pressure

$$P = P_{eq} + \Pi$$

bulk scalar  $\Pi = \frac{\partial P}{\partial \delta\mu} \delta\mu$

Israel-Stewart like Equation!

$$u^{\mu} \nabla_{\mu} \Pi = -\frac{\Pi}{\tau_{\Pi}} - \frac{\zeta}{\tau_{\Pi}} \nabla_{\mu} u^{\mu}$$

Israel, Stewart, Annals of Physics (1979)

# Reactive fluid

(coupled with Einstein's equations)



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Neutron star mergers/inspirals are  
bulk-viscous systems

This Talk: Given the rates, transport  
coefficients fully determined by  
experiments (S, L) at saturation!



# Nuclear Symmetry Energy

- Energy difference between pure neutron matter and symmetric nuclear matter

$$\frac{\varepsilon}{n_B} = \frac{\varepsilon}{n_B} \Big|_{Y=1/2} + E_{sym}(n_B) (1 - 2Y)^2 + \mathcal{O}((1 - 2Y)^3)$$

- Can be expanded around nuclear saturation density

$$E_{sym}(n_B) = S + \frac{L}{3} \left( \frac{n_B}{n_{sat}} - 1 \right) + \mathcal{O} \left( \left( \frac{n_B}{n_{sat}} - 1 \right)^2 \right)$$

symmetry energy  $S$ , and its slope  $L$  at  $n_{sat}$   Experimentally measured!

**How to connect the symmetry energy  
with the transport coefficients?**

# Symmetry energy fixes chemical imbalance

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- At  $T \rightarrow 0$ , first law of thermodynamics gives

$$\delta\mu = -\frac{1}{n_B} \frac{\partial \varepsilon}{\partial Y}$$

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$$\delta\mu = -\frac{1}{n_B} \frac{\partial\varepsilon}{\partial Y}$$


- Besides the rates, transport coefficients depend on the symmetry energy

$$\tau_{\Pi} \sim \frac{1}{E_{sym}(n_B)} \quad \zeta \sim \left( \frac{\partial E_{sym}(n_B)}{\partial n_B} / E_{sym}(n_B) \right)^2$$

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- At saturation, simple behavior emerges

$$\tau_{\Pi}(n_{sat}) \sim \frac{1}{S} \quad \zeta(n_{sat}) \sim \left(\frac{L}{S}\right)^2$$

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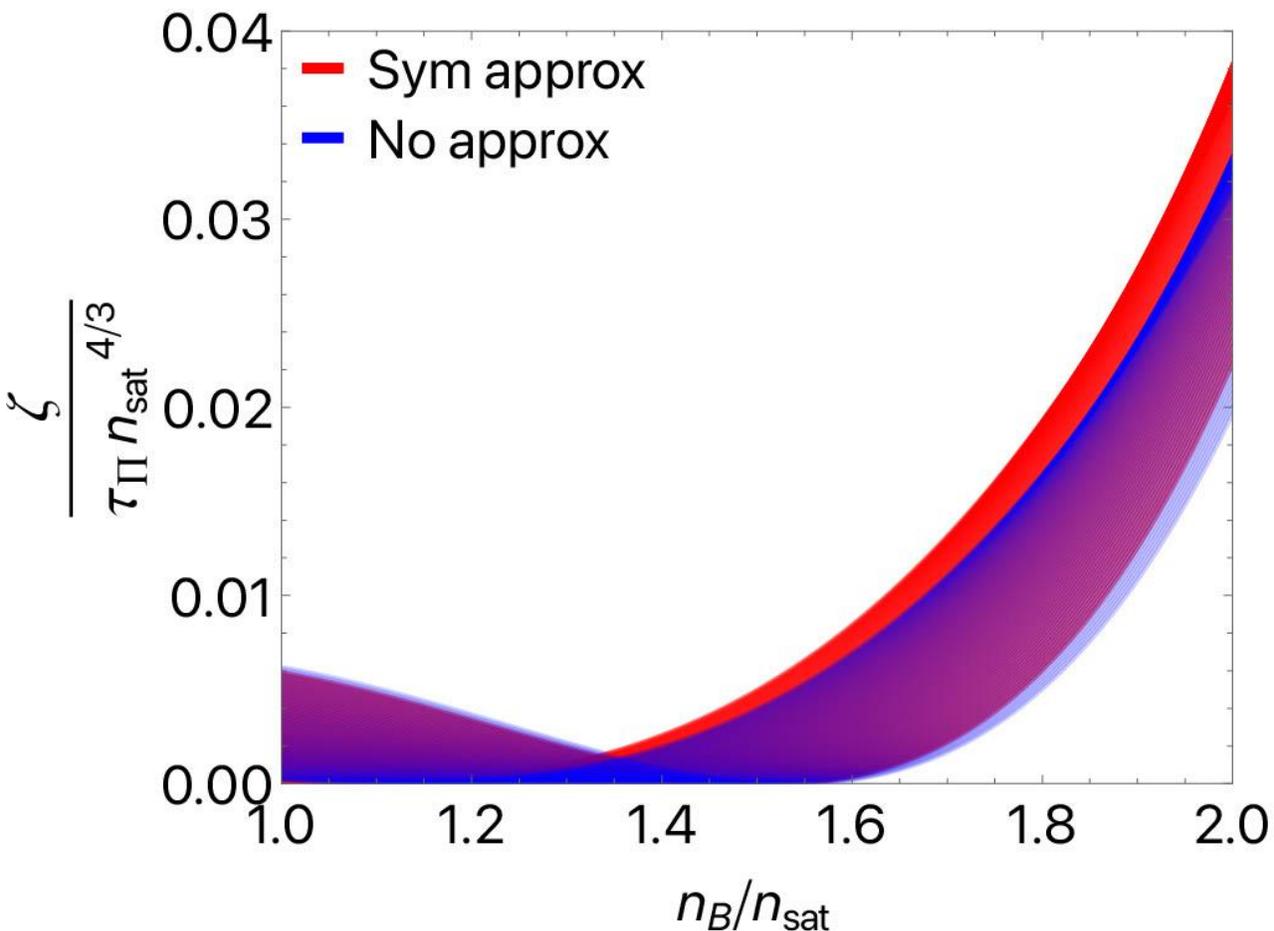
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Experimentally measured observables

# Validation using chiral EFT

YY, Hippert, Speranza, Noronha, arXiv 2504.07805



- Blue: Calculated directly from chiral EFT parameterizations

Tews et al., *Astrophys J* (2018)

Hippert, Noronha, Romatschke, *Phys. Lett. B* (2025)

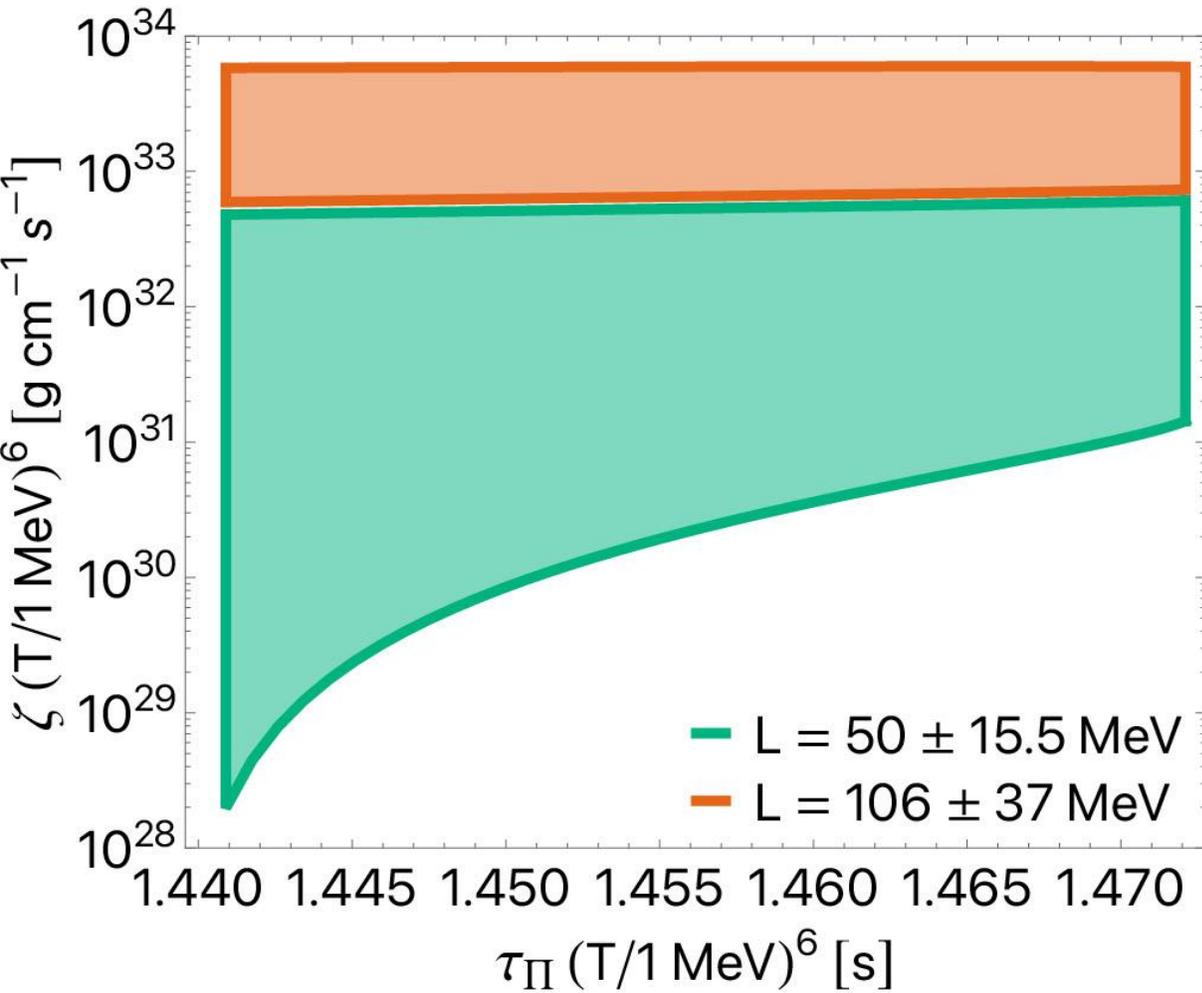
Hebeler et al. *PRL* (2010)

Bedaque, Steiner, *PRL* (2015)

- Red: Calculated using the respective symmetry energy of the chiral EFT parametrizations

# Dependence on symmetry slope $L$

YY, Hippert, Speranza, Noronha, arXiv 2504.07805



- Fan, Dong, Zuo, PRC 2014:

$$L = 50 \pm 15.5 \text{ MeV}$$

Li et al., PRL (2019)

Similar results

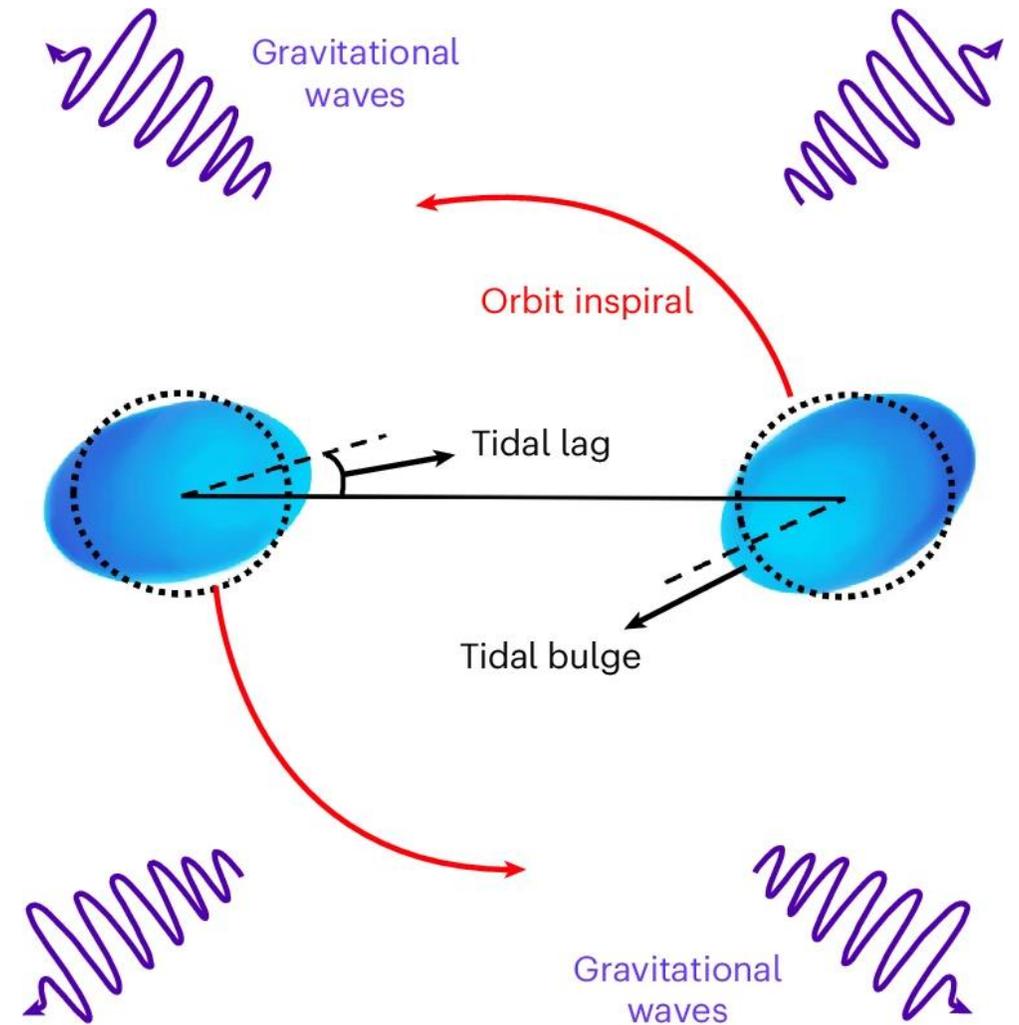
Reinhard, Roca-Maza, Nazarewicz, PRL (2021)

Reed et al., PRL (2021)

- Reed, Fattoyev, Horowitz, Piekarewicz  
PRL 2021:

$$L = 106 \pm 37 \text{ MeV}$$

# Could dissipative effects be relevant already in the inspiral phase?



Ripley et al., Nature Astronomy (2024)

# For npe matter near saturation

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- As  $T \rightarrow 0$

$$\tau_{\Pi} \rightarrow \infty$$

$$\zeta \rightarrow \infty$$

$$\frac{\zeta}{\tau_{\Pi}} \sim \text{const}$$

- What are the consequences?
- How does npe matter under these conditions react to compression/expansion?

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YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- As  $T \rightarrow 0$

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$$\frac{\zeta}{\tau_{\Pi}} \sim \text{const}$$

- Resulting EoM respects time-reversal symmetry

$$u^{\mu} \nabla_{\mu} \Pi = -\cancel{\frac{\Pi}{\tau_{\Pi}}} - \frac{\zeta}{\tau_{\Pi}} \nabla_{\mu} u^{\mu} \quad \rightarrow \quad u^{\mu} \nabla_{\mu} \Pi = -\frac{\zeta}{\tau_{\Pi}} \nabla_{\mu} u^{\mu}$$

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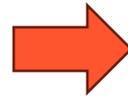
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No entropy production!

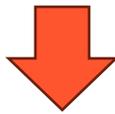
npe matter would behave like a relativistic elastic medium, not a fluid!

# Elastic regime in the inspiral phase?

- Introduce frequency dependence by metric perturbation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu} \quad \delta g^{\mu\nu} \propto e^{i\omega t} \quad \delta T^{\mu\nu} \sim G_R \delta g^{\mu\nu}$$

YY, Hippert, Speranza, Noronha, PRC(2024)


$$\Pi = \frac{\zeta}{\tau_{\Pi}} \frac{i\tau_{\Pi}\omega}{1 - i\tau_{\Pi}\omega} \eta^{\alpha\beta} \delta g_{\alpha\beta}$$

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YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$\tau_{\pi}\omega \gg 1$$

Elastic

$$\tau_{\pi}\omega \approx 1$$

Resonant

$$\tau_{\pi}\omega \ll 1$$

Navier-Stokes

# For typical inspiral orbital periods

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$\begin{array}{|l} \tau_{\pi} \omega \gg 1 \\ \text{Elastic} \end{array} \quad \rightarrow \quad \begin{array}{|l} \text{Early Inspiral} \quad T \approx 10^5 \text{ K} \quad \omega \approx 2\pi \times 400 \text{ Hz} \\ \tau_{\Pi} \omega|_{n_{\text{sat}}} \approx 3.8 \times 10^{26} \gg 1 \end{array}$$

- npe matter at saturation in the inspiral phase behaves like an elastic medium during compression/expansion
- system not in beta equilibrium, but no entropy production

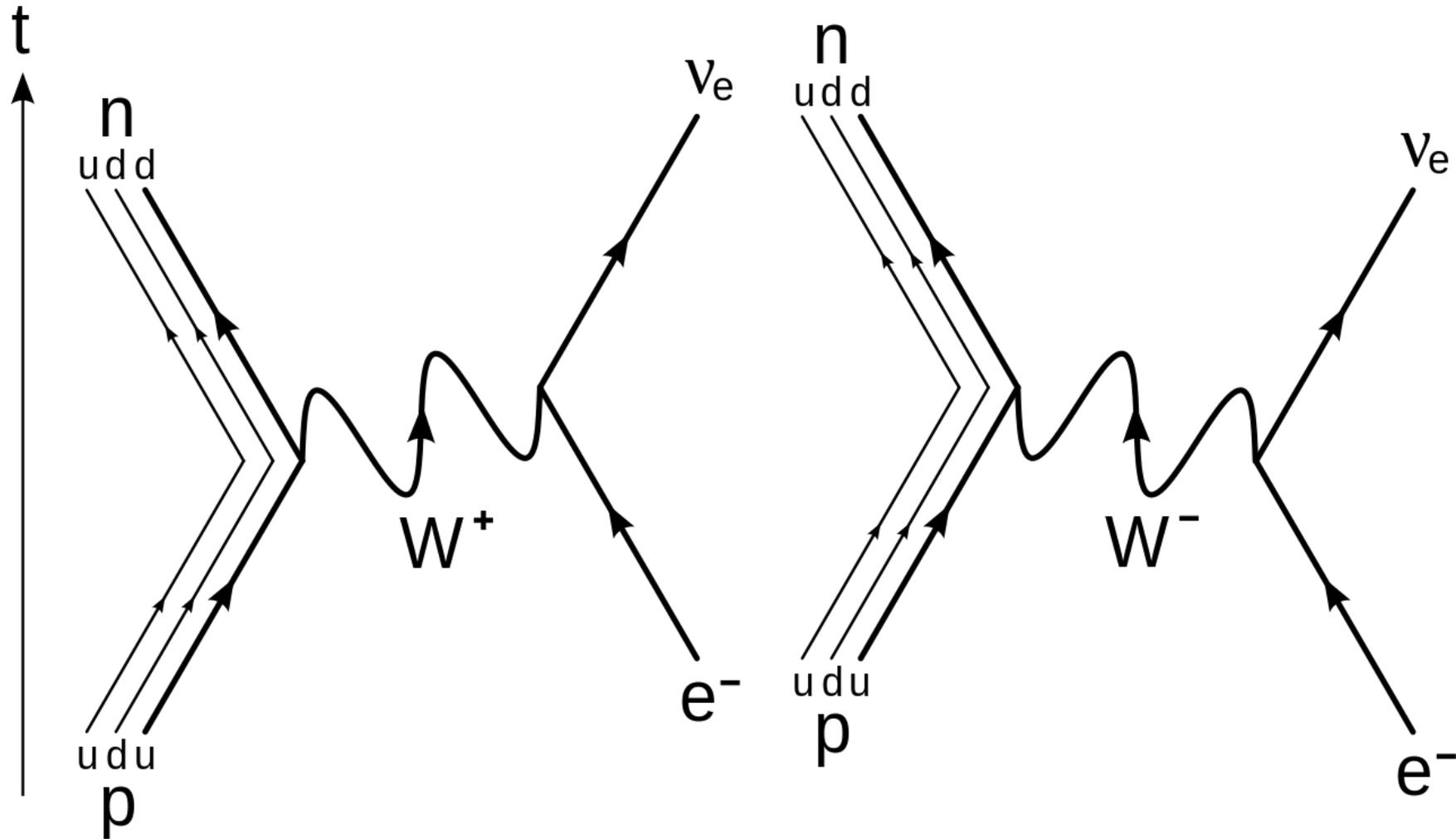
# Conclusions

- npe bulk viscous transport coefficients at saturation universally given by  $S$  and  $L$  (and the rates)
- Factor of 2 difference in  $L$  (50 vs. 100 MeV): Orders of magnitude change in bulk viscosity at saturation
- At saturation, npe matter in inspirals displays bulk elastic response

# EXTRA SLIDES



# Direct Urca



# Modified Urca

- Chemical reactions with a spectator  $X$



# Effect of equilibration timescale

Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL (2018)

- Slow equilibration: composition is fixed, and the process is reversible
- Fast equilibration: mixture is instantaneously equilibrated
- Same order as the pressure: **Phase Lag -> Bulk Viscosity!**

# How does it get out of equilibrium?

- Change in electron fraction is related to the Urca rates

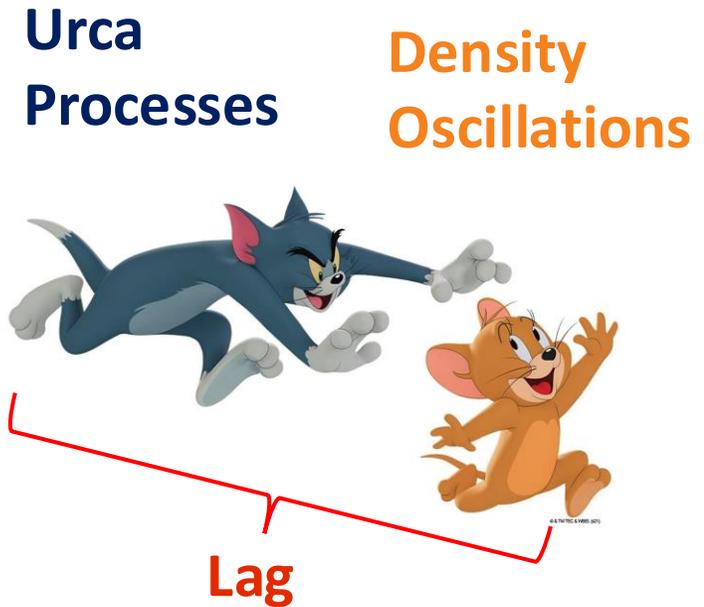
$$\Gamma_{\bar{\nu}} - \Gamma_{\nu} = n_B \frac{d(\delta Y_e)}{dt} = \Gamma_e$$

- Urca processes & density oscillations: same timescale



Deviation from beta equilibrium

- Energy loss from compression/expansion: bulk viscosity?

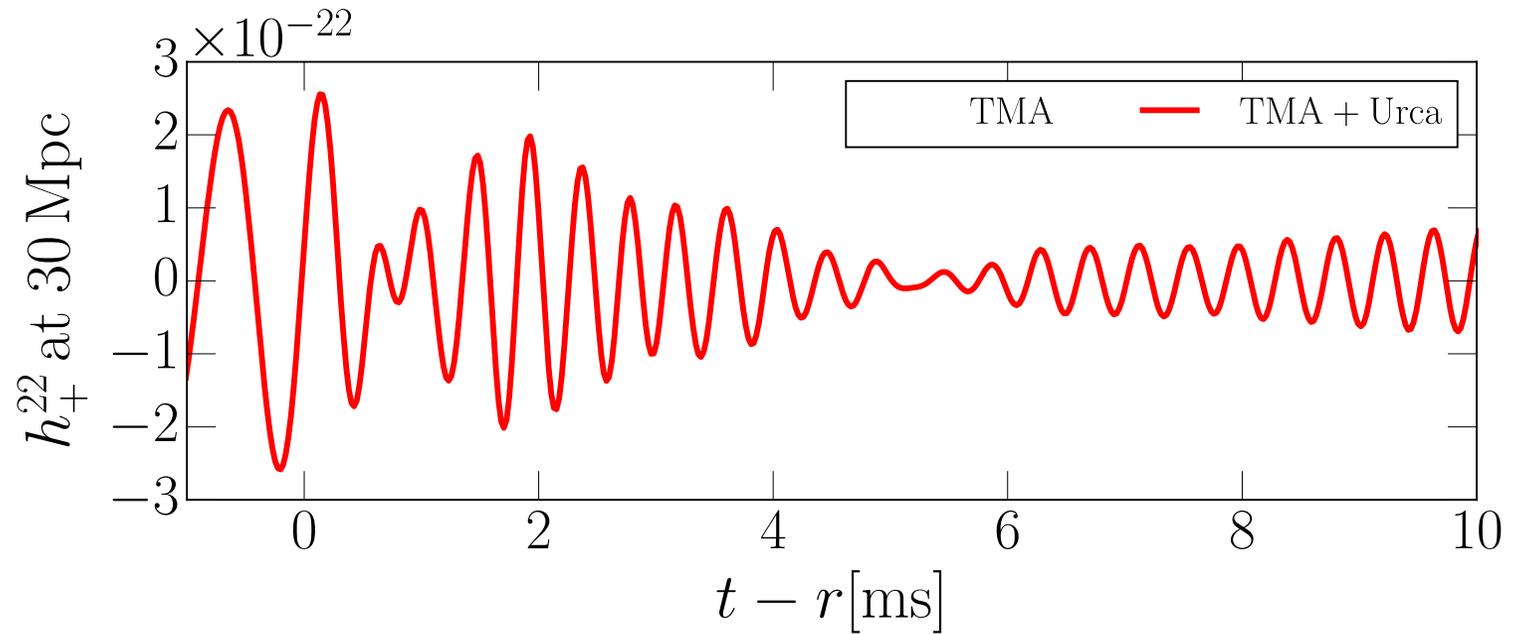
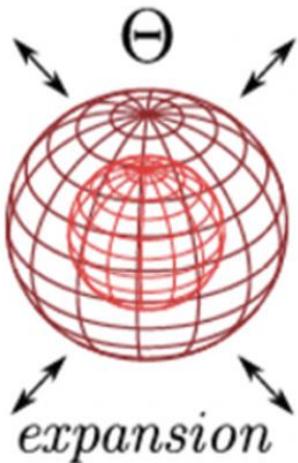


# Bulk viscosity in neutron star mergers?

Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL (2018)

- Density oscillations + weak interactions: deviations from beta equilibrium  
→ realistic simulations with URCA rates

Bulk viscosity  $\zeta$



Most et al, arXiv:2207.00442

# Uncertainties of symmetry energy

$S$ (MeV)	$31.7 \pm 3.2$	Li et al., EPJ (2019)
$L$ (MeV)	$58.7 \pm 28.1$	Li et al., EPJ (2019)
	$106 \pm 37$	Reed et al., PRL (2021)
$K_{sym}$ (MeV)	$(-400, 100)$	Li et al., EPJ (2019)

# Chiral EFT parametrizations

- Linearly interpolating between the uncertainty bands

$$(E/A)_\sigma = (1 - \sigma)(E/A)_{\text{up}} + \sigma(E/A)_{\text{low}}$$

- Fit the curve with the following parametrization

Tews et al., *Astrophys J* (2018)

Hippert, Noronha, Romatschke, *Phys. Lett. B* (2025)

Hebeler et al. *PRL* (2010)

Bedaque, Steiner, *PRL* (2015)

$$\begin{aligned} \frac{E}{A}(n_B, Y) - m_B = T_0 & \left[ \frac{3}{5} \left( Y^{3/5} + (1 - Y)^{3/5} \right) \left( \frac{2n_B}{n_{\text{sat}}} \right)^{2/3} - [(2\alpha - 4\alpha_L)Y(1 - Y) + \alpha_L] \frac{n_B}{n_{\text{sat}}} \right. \\ & \left. + [(2\eta - 4\eta_L)Y(1 - Y) + \alpha_L] \left( \frac{2n_B}{n_{\text{sat}}} \right)^\gamma \right] \end{aligned}$$

# Dissipative tidal deformability

Conservative tidal deformability

Tidal lag time

$$\Xi_A = \frac{2}{3} k_{2,A} \left( \frac{1}{C_A^6} \right) \left( \frac{c\tau_{d,A}}{R_A} \right)$$

Compactness

Radius

$$\Xi_A = \frac{c^3}{G} \frac{p_{2,A}}{C_A} \frac{\langle \zeta \rangle}{\langle \varepsilon \rangle m_A}$$

Ripley et al., Nature Astronomy (2024)

# Constraints on bulk viscosity transport coefficients

$$\left[ \frac{\zeta}{\tau_{\Pi}} + n_B \frac{\partial P}{\partial n_B} \Big|_{\varepsilon, Y_e = Y_e^{eq}} \right] \frac{1}{\varepsilon + P} \leq 1 - \frac{\partial P}{\partial \varepsilon} \Big|_{n_B, Y_e = Y_e^{eq}}$$

Bemfica, Disconzi, Noronha, PRL (2019)

# Bulk viscosity from the phase lag

Alford et al, JPGNPP (2010)

Sa'd, Schaffner-Bielich, arXiv 0908.4190 (2010)

- Assume

$$n_i = n_i(\mu_i)$$

$$Y_e = Y_{e,0} + \text{Re}(\delta Y_{e,0} e^{i\omega t})$$

- Energy density dissipation

$$\langle \dot{\mathcal{E}}_{diss} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2 = \frac{n_{B,0}}{\tau} \int_0^\tau (P + \delta P) \frac{d}{dt} (V + \delta V) dt$$

Bulk viscosity

$$\zeta = \frac{-\gamma C^2}{\omega^2 + (\gamma B / n_B)^2}$$