

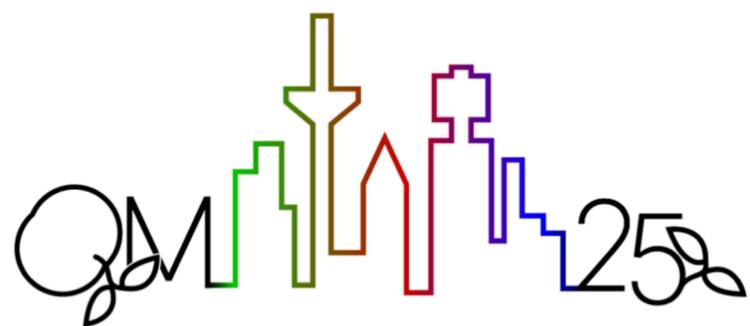
Finite temperature expansion of the dense matter equation of state

Débora Mroczek

Illinois Center for Advanced Studies of the Universe (ICASU)

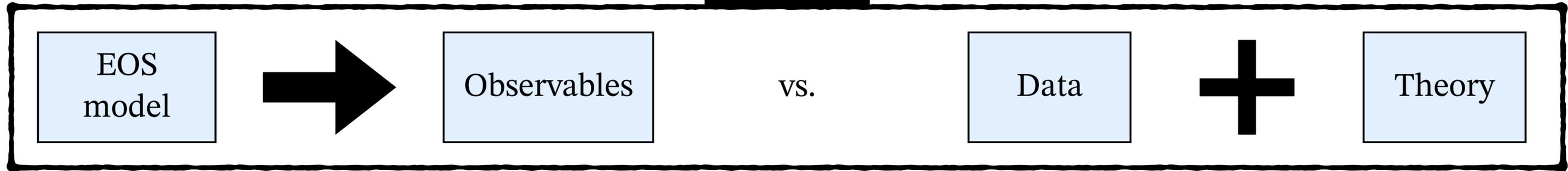
University of Illinois at Urbana-Champaign

arXiv (v2 out soon, improved theory disc.): [2404.01658](https://arxiv.org/abs/2404.01658), [D. Mroczek](https://arxiv.org/abs/2404.01658), N. Yao, K. Zine, and J. Noronha-Hostler (UIUC/ICASU), V. Dexheimer (Kent State), A. Haber (U. Southampton), L. Brodie (Wash. U.), E. Most (Caltech).

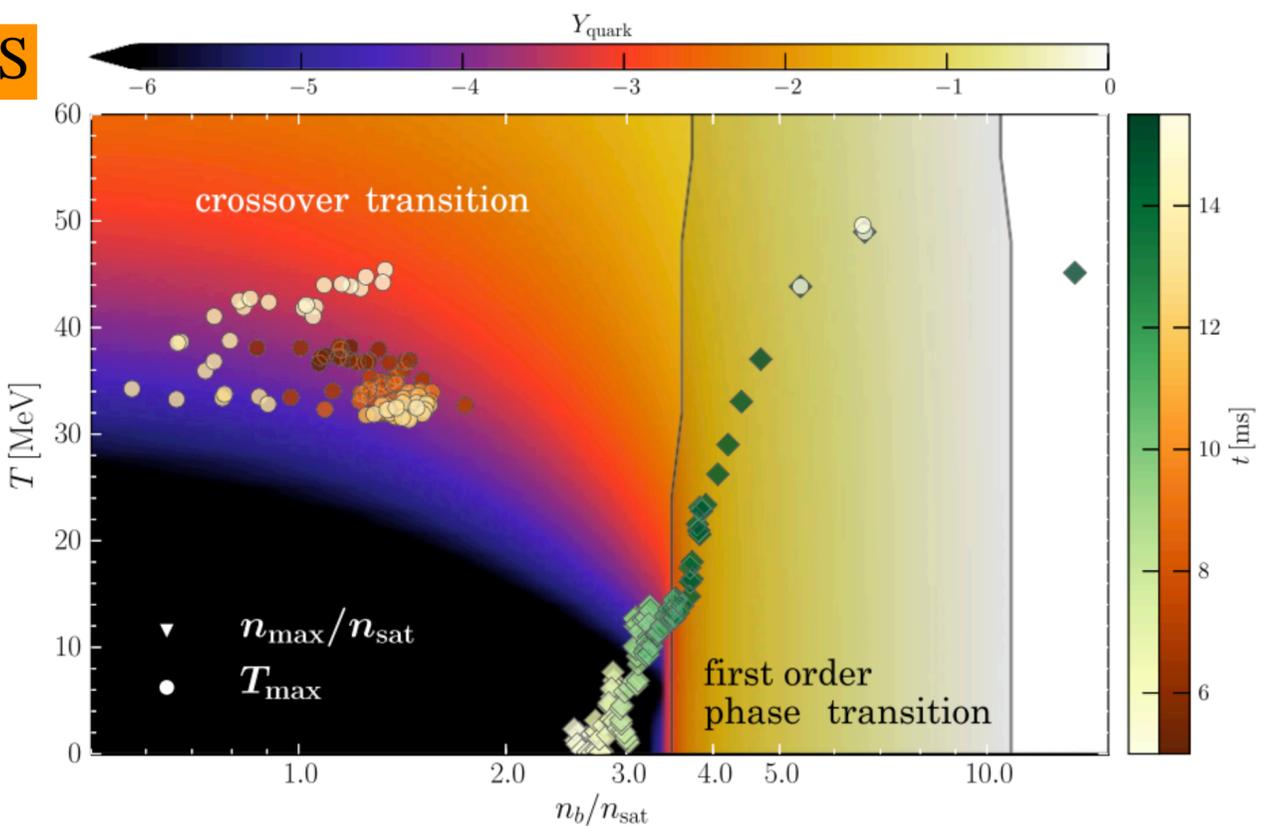


Why do we need a finite temperature expansion of the dense matter EOS?

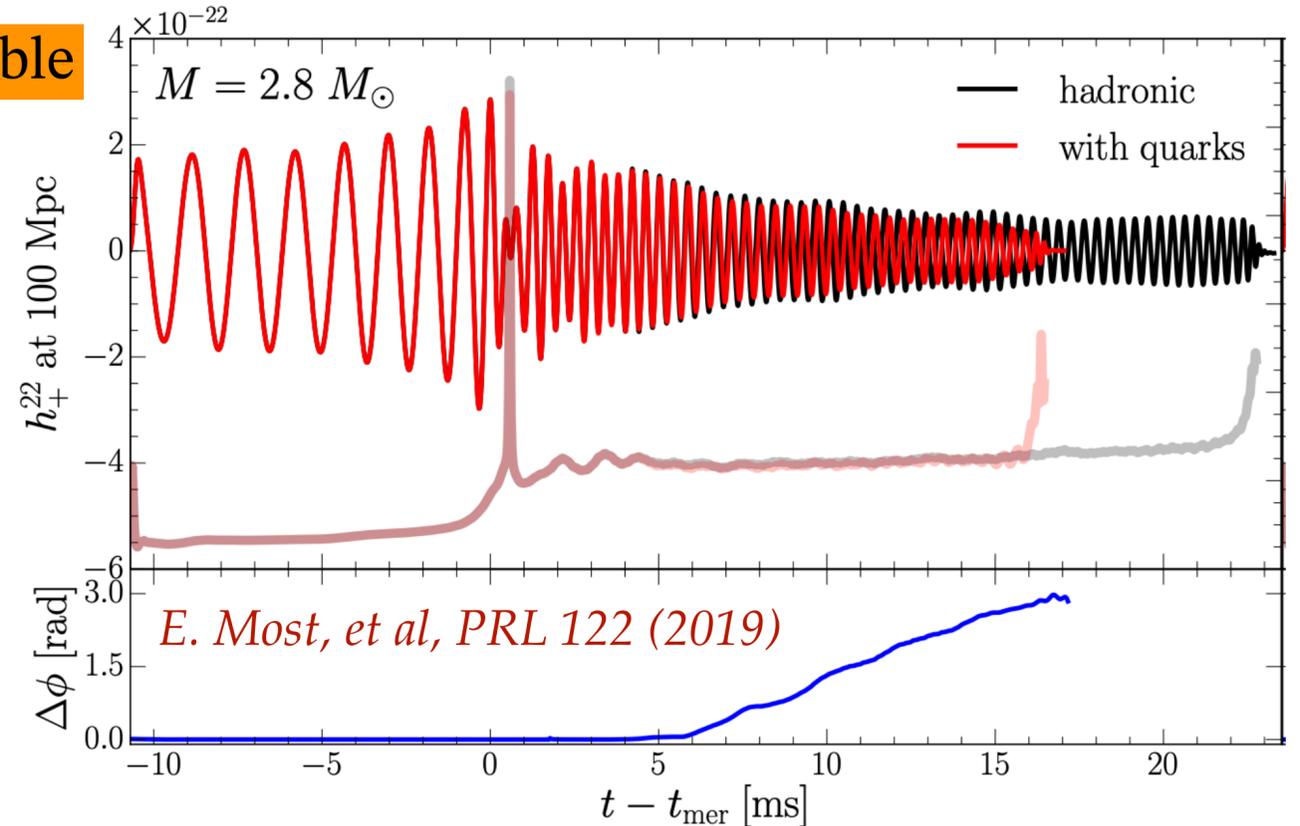
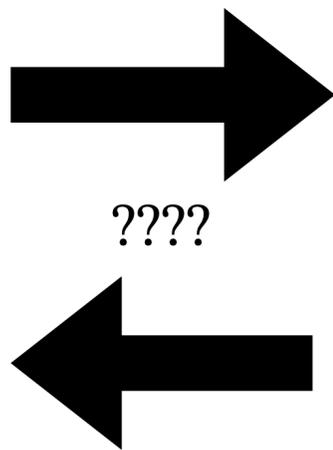
Inference



EOS



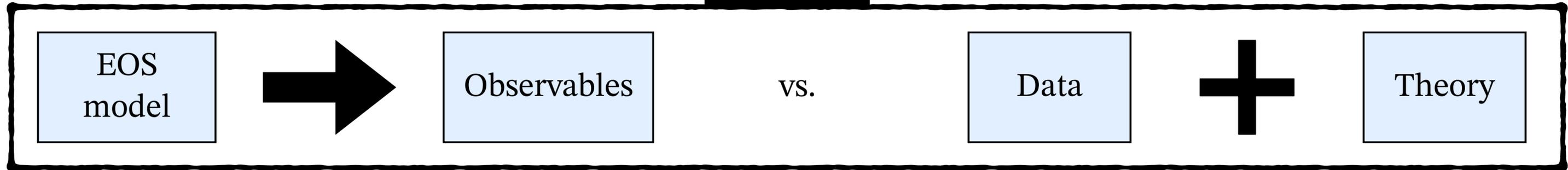
Observable



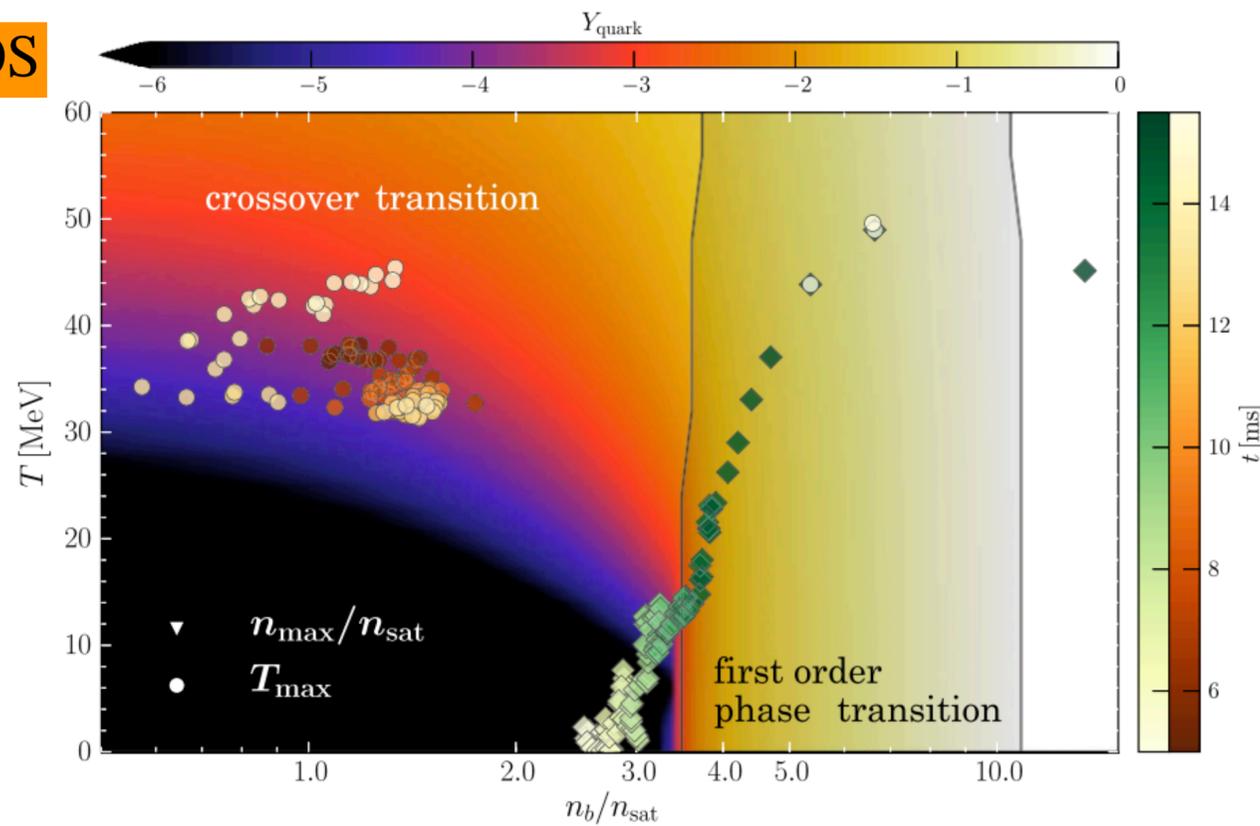
- **Dynamical** description of neutron star formation and their mergers requires a **3D** EOS
- Current tools are **limited/oversimplified**

Why do we need a finite temperature expansion of the dense matter EOS?

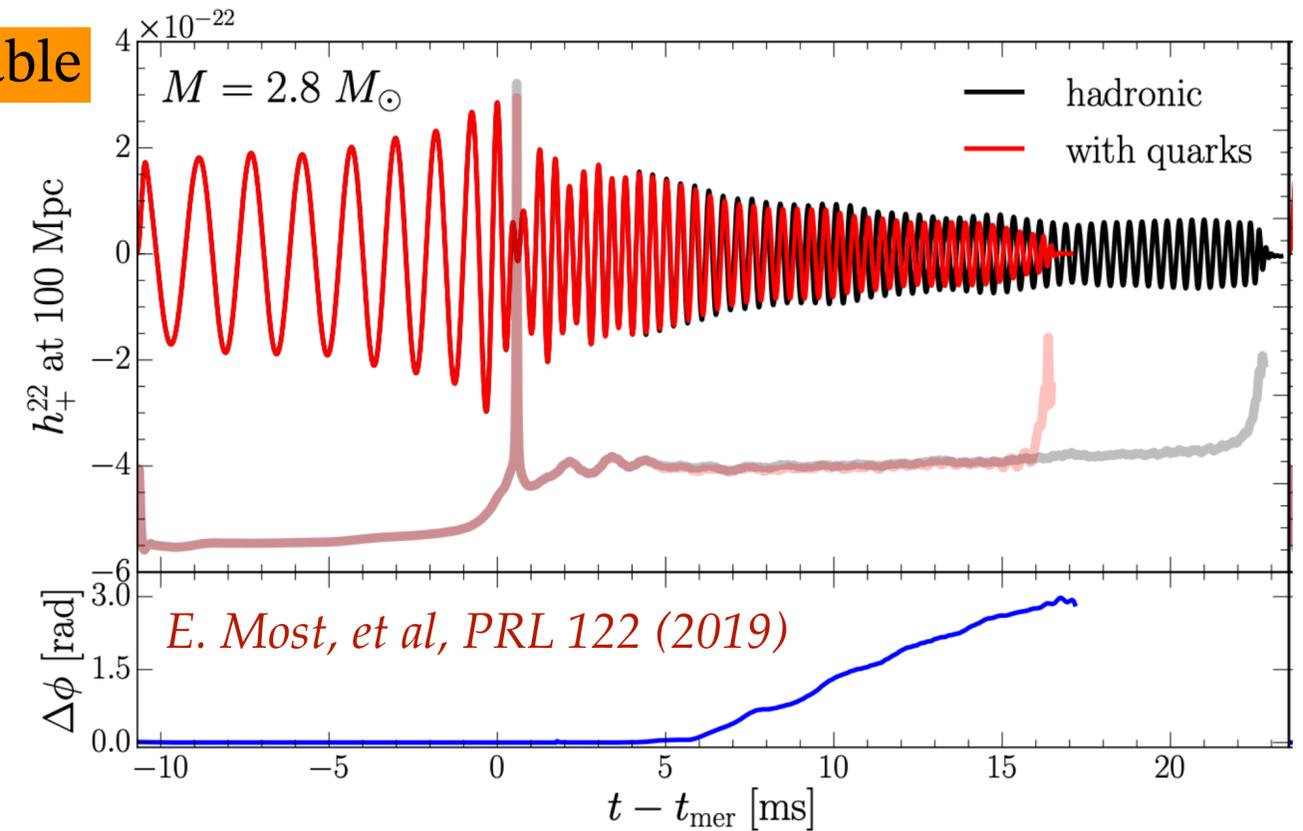
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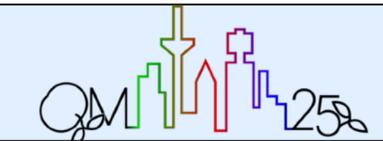
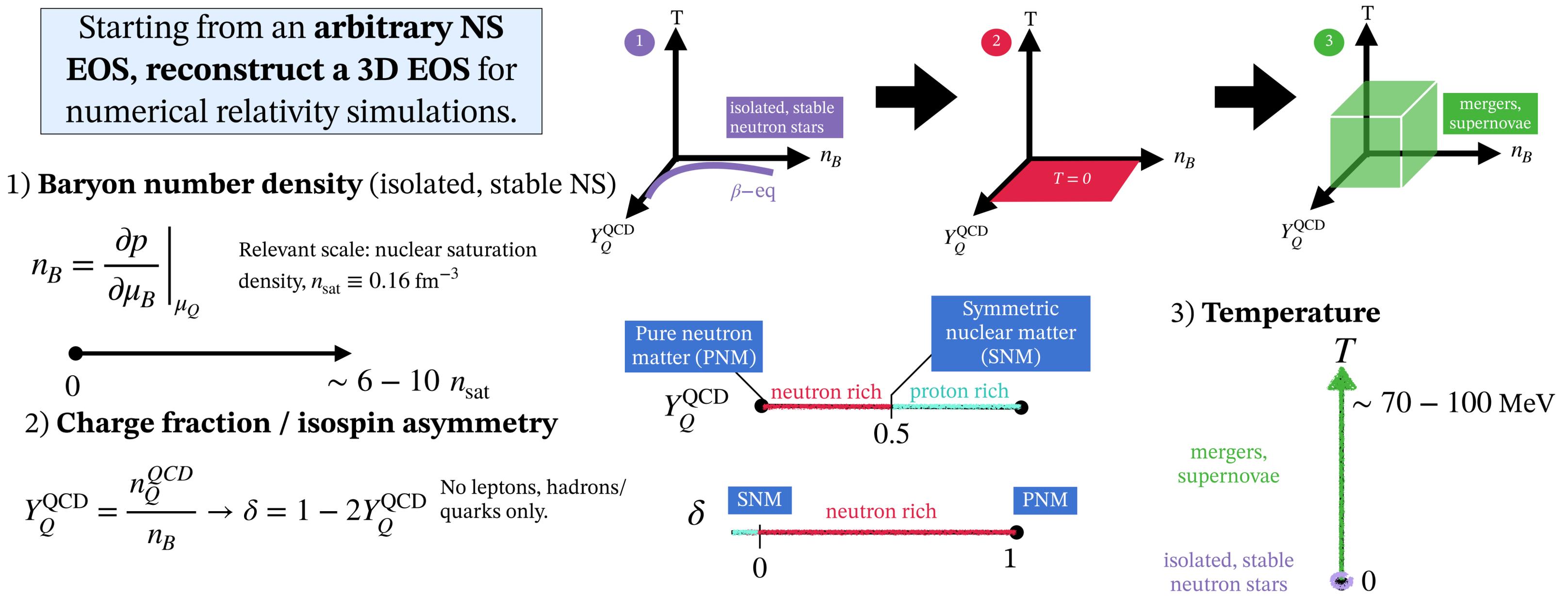


- **Dynamical** description of neutron star formation and their mergers requires a **3D** EOS
- Current tools are **limited/oversimplified**

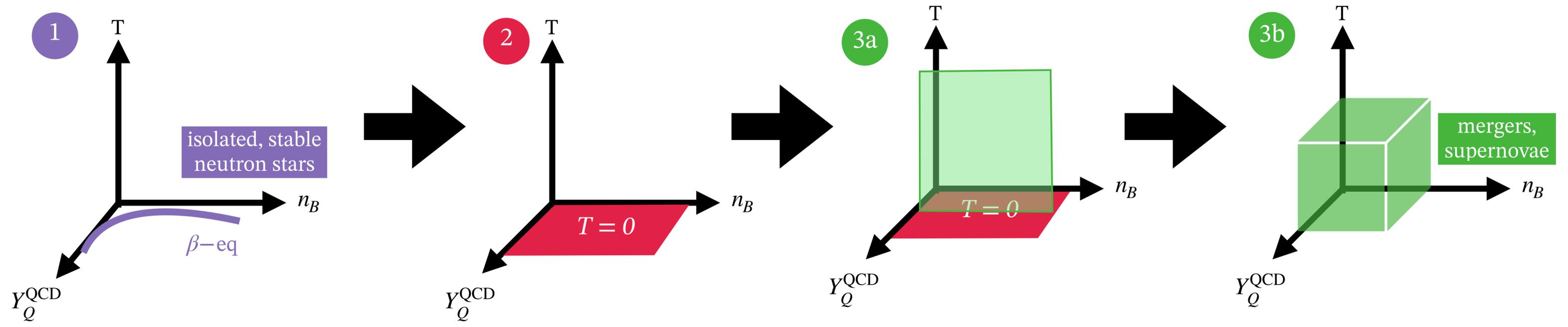
What is needed from a temperature expansion of the dense matter EOS?

Dense matter (in this work) → **hadron/quark** state of matter with **no strange degrees of freedom** in the regime relevant for **neutron stars**.

Starting from an **arbitrary NS EOS**, **reconstruct a 3D EOS** for numerical relativity simulations.



What is needed (pt. 2) and our approach



- Thermodynamically consistent
- Beyond n+p degrees of freedom
- Connection to available experiments, observations, and theory predictions

Lab.

Obs.

Theory

1 → 2: Expansion of the symmetry energy about NS EOS

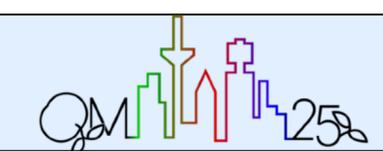
Yao et al, PRC 109 (2024)

2 → 3a: Finite temperature expansion at fixed μ_Q

New!

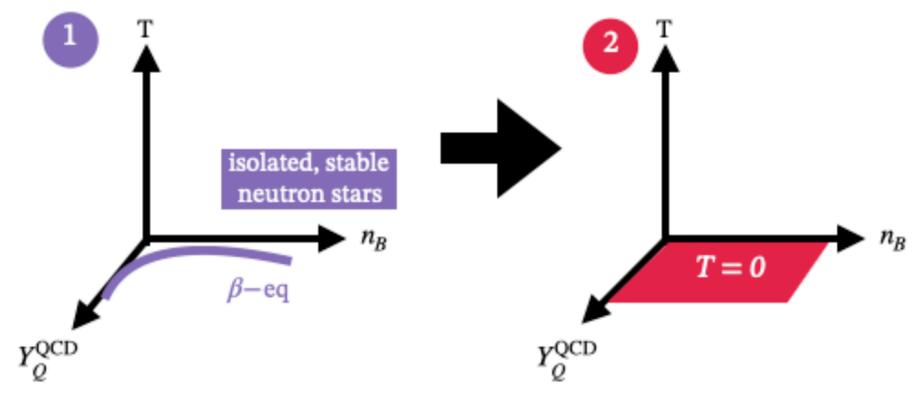
3a → 3b: Expansion of charge fraction dependence of finite temperature effects

New!

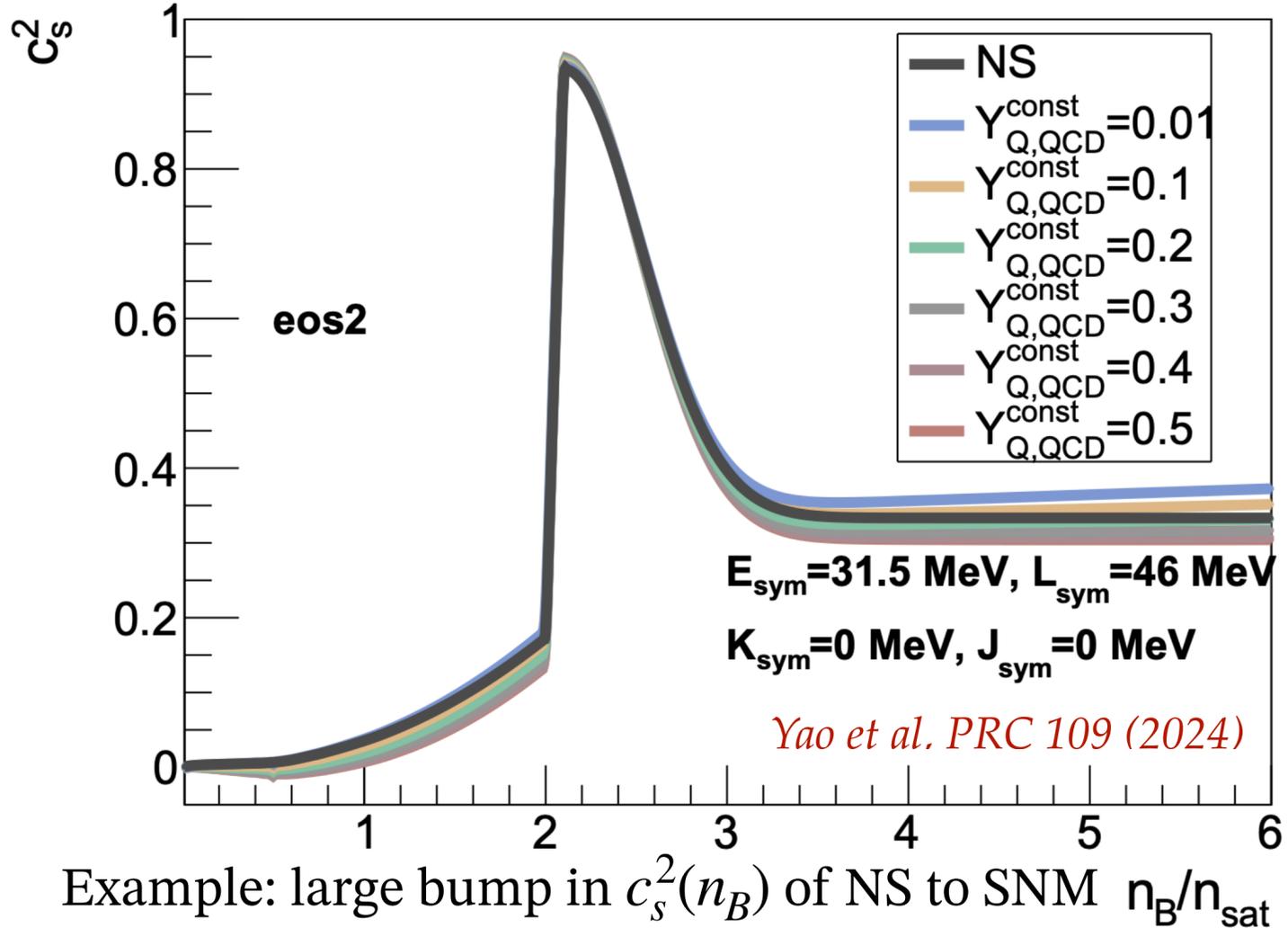


From β -equilibrium to arbitrary charge fraction

- Symmetry energy expansion derived in Bombaci and Lombardo (1991), modified in Yao et al. (2024):



$$\frac{E_{\text{HIC,sym}}}{N_B} = \frac{E_{\text{NS,QCD}}}{N_B} - \left[E_{\text{sym,sat}} + \frac{L_{\text{sym,sat}}}{3} \left(\frac{n_B}{n_0} - 1 \right) + \frac{K_{\text{sym,sat}}}{18} \left(\frac{n_B}{n_0} - 1 \right)^2 + \frac{J_{\text{sym,sat}}}{162} \left(\frac{n_B}{n_0} - 1 \right)^3 \right] (1 - 2Y_{Q,QCD})^2$$

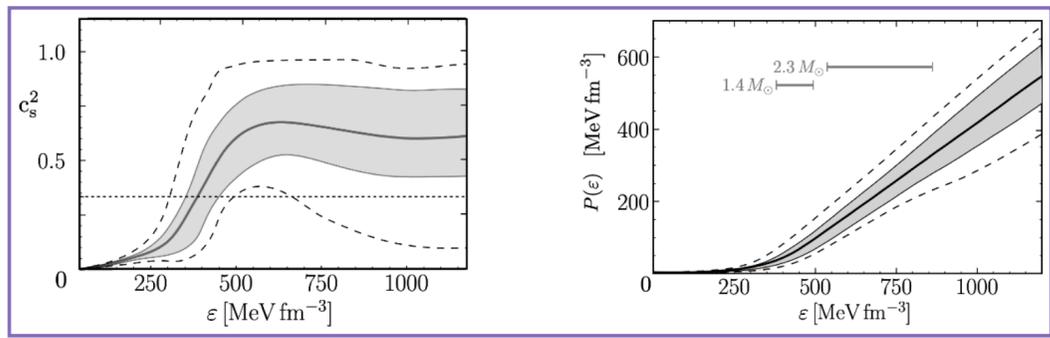


Example: large bump in $c_s^2(n_B)$ of NS to SNM n_B/n_{sat}

Input:

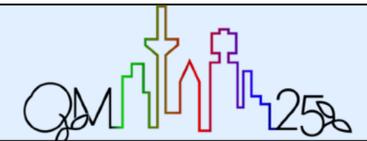
- NS EOS
- Symmetry energy coefficients

Brandes & Weise, PRD 111 (2025)



Coefficient	Definition	Range	References
$E_{\text{sym,sat}}$	$\left(\frac{E_{\text{PNM}} - E_{\text{SNM}}}{N_B} \right)_{n_{\text{sat}}}$	31.7 ± 3.2 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$L_{\text{sym,sat}}$	$3n_{\text{sat}} \left(\frac{dE_{\text{sym},2}}{dn_B} \right)_{n_{\text{sat}}}$	58.7 ± 28.1 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$K_{\text{sym,sat}}$	$9n_{\text{sat}}^2 \left(\frac{d^2E_{\text{sym},2}}{dn_B^2} \right)_{n_{\text{sat}}}$	106 ± 37 [MeV]	PREXII [122, 123]
$J_{\text{sym,sat}}$	$27n_{\text{sat}}^3 \left(\frac{d^3E_{\text{sym},2}}{dn_B^3} \right)_{n_{\text{sat}}}$	-120^{+80}_{-100} [MeV]	Bayesian analyses inferred from GW170817 and PSR J0030+0451[124]
		300 ± 500 [MeV]	Many-body nuclear theory [125]

Yao et al. PRC 109 (2024)



From $T = 0$ to finite T

- Taylor expansion about $p(T = 0, \mu_B, \mu_Q)$ **New!**

$$p(T, \vec{\mu}) = p_{T=0} + \left. \frac{\partial p}{\partial T} \right|_{T=0, \vec{\mu}} T + \frac{1}{2} \left. \frac{\partial^2 p}{\partial T^2} \right|_{T=0, \vec{\mu}} T^2 + \dots$$

Entropy!

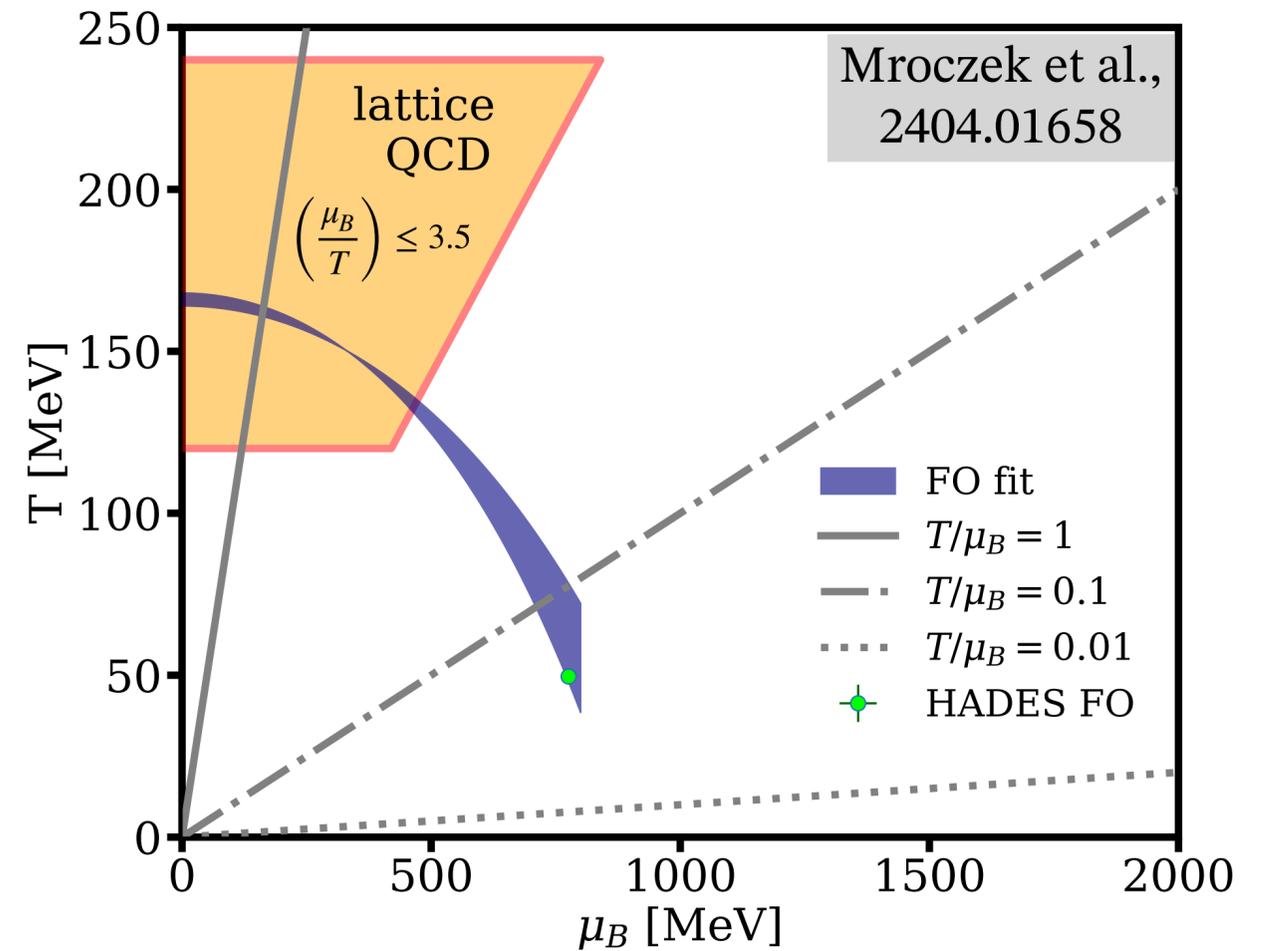
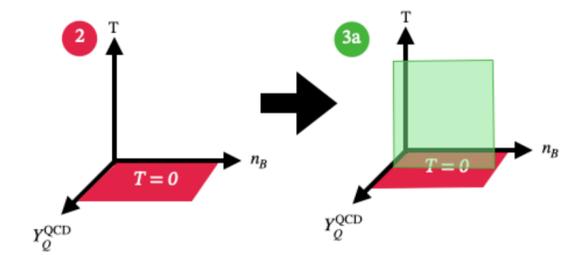
$$s(T = 0) = 0$$

Heat capacity $\left. \frac{\partial s}{\partial T} \right|_{T=0} > 0$

$$p(T, \vec{\mu}) \approx p_{T=0} + \frac{1}{2} \left. \frac{\partial s}{\partial T} \right|_{T=0, \vec{\mu}} T^2$$

- Special case: Sommerfeld (1928) expansion
- Ideal Fermi systems at $T \ll T_F$,

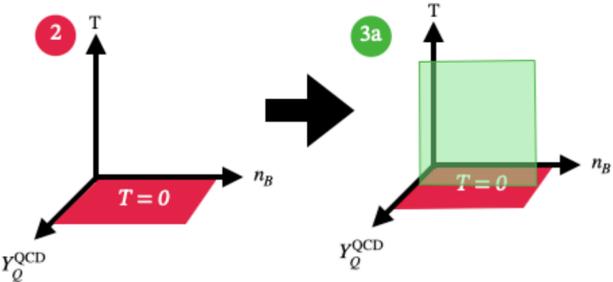
$$p \approx p_{T=0} + aT^2 + bT^4 + \dots$$
- Fermionic **quasi-particles**



- * Physical motivation
- * Expansion parameter $(T/\mu_B) < 0.1$ in relevant regime
- * Overlap with few-GeV $\sqrt{s_{NN}}$ freeze-out (FO)

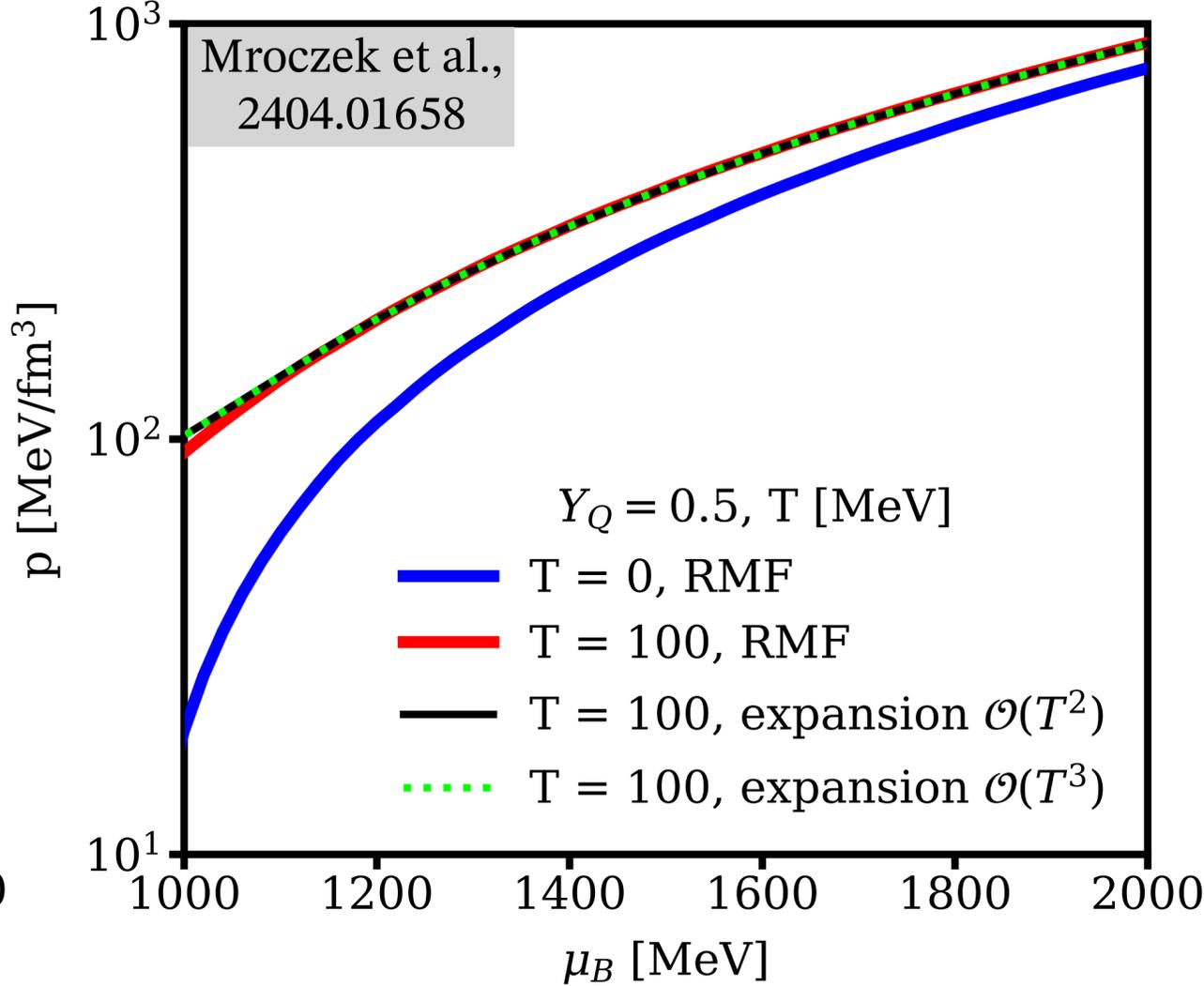
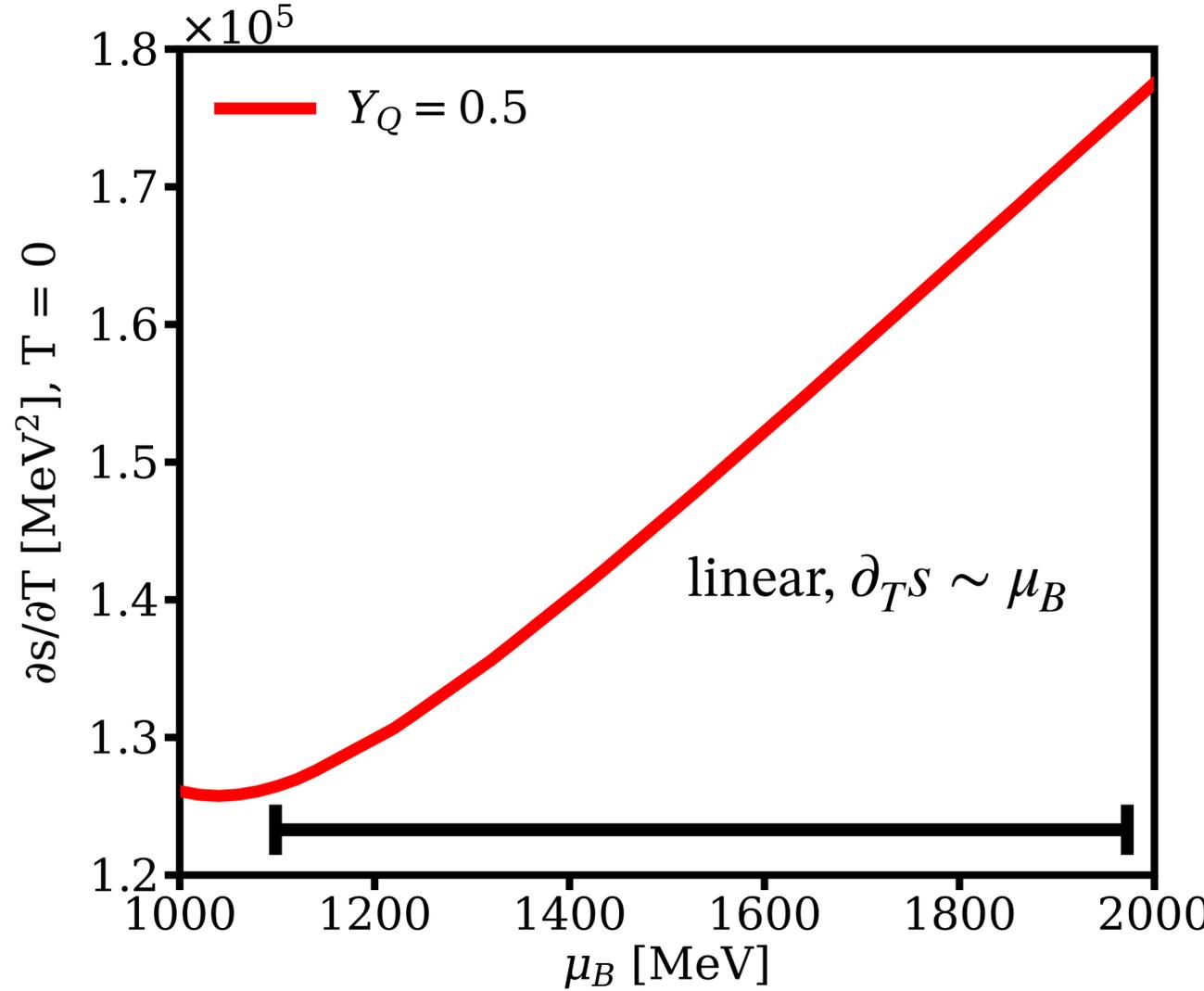
FO fit from Cleymans et al, PRC 73 (2006), HADES FO from Harabasz et al, PRC 102 (2020)

From $T = 0$ to finite T , test with microscopic model



- Numerical tests with relativistic mean-field (RMF) theory (n+p) well suited for the expansion

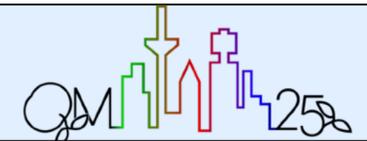
T^2 term captures the finite temperature behavior of the pressure to high accuracy when $\partial s/\partial T$ is known



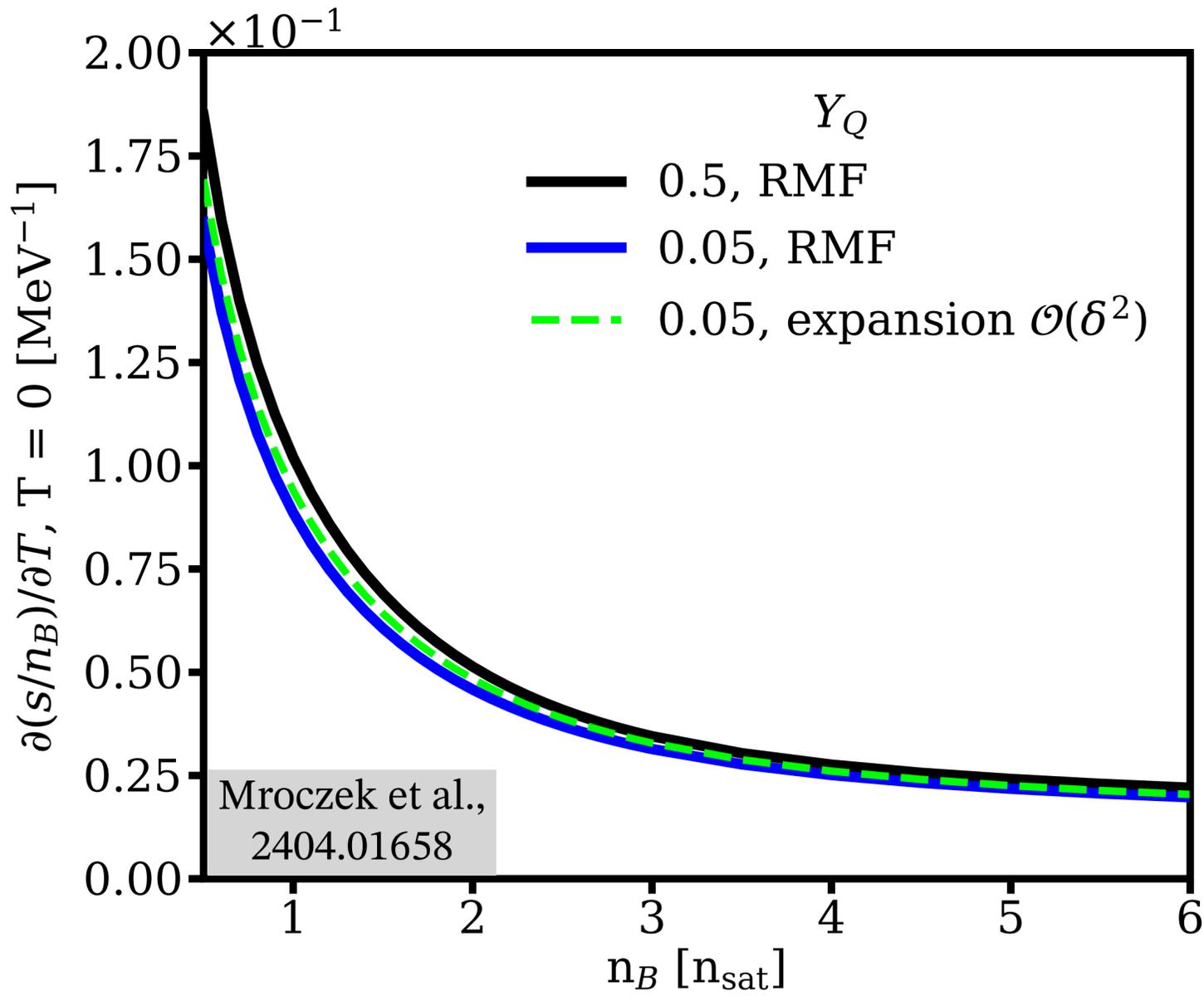
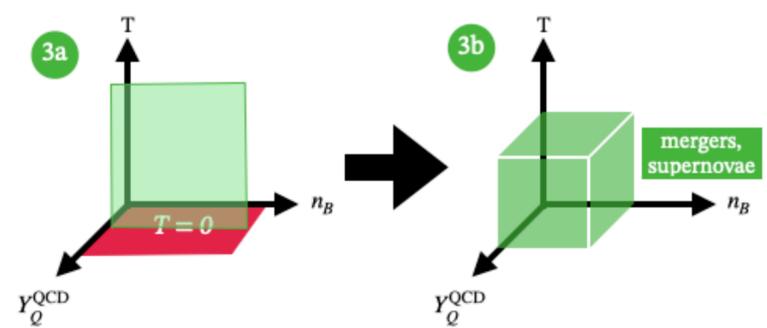
- Breakdown near liquid-gas PT
- Linear coefficient → easy to parametrize
- T^2 term dominates

But: must know $\partial_T S$ for all μ_B, μ_Q

Microscopic model: RMF theory from Alford et. al PRC 106, (2022)



Charge fraction dependence of finite temperature effects



Heat capacity across all $\vec{\mu}$ can be extracted from microscopic models

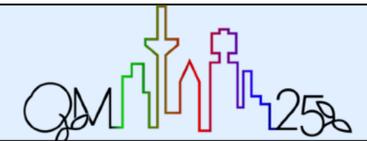
- Motivation: s/n_B for a given $(Z/A, \sqrt{s_{NN}})$ can be extracted from thermal fits of particle yields
- Expand $\partial_T(s/n_B)$ about SNM assuming isospin symmetry

• **New expansion:**

Heat capacity at $Y_Q^{QCD} = 0.5$

$$\left. \frac{\partial \tilde{S}(T, n_B, Y_Q)}{\partial T} \right|_{T=0} = \frac{1}{n_B} \left. \frac{\partial s_{SNM}(T, n_B, Y_Q)}{\partial T} \right|_{T=\delta=0} + \frac{1}{2} (1 - 2Y_Q)^2 \left. \frac{\partial^3 \tilde{S}_{SNM,2}(T, n_B, \delta = 0)}{\partial T \partial \delta^2} \right|_{T=\delta=0}$$

Heat capacity dependence on Y_Q

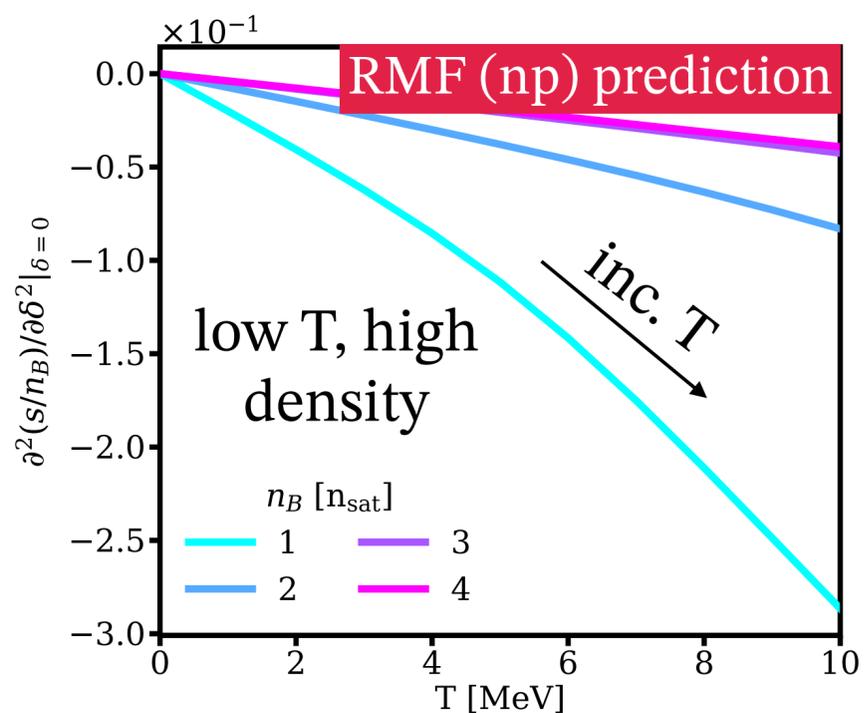
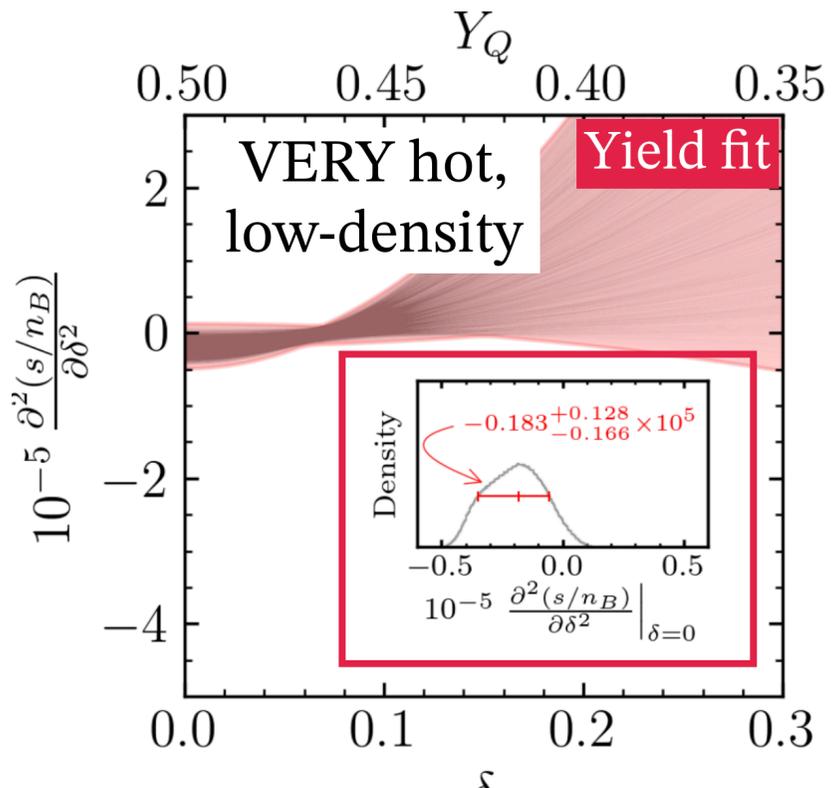


Connection to heavy-ion collisions: **system scan**

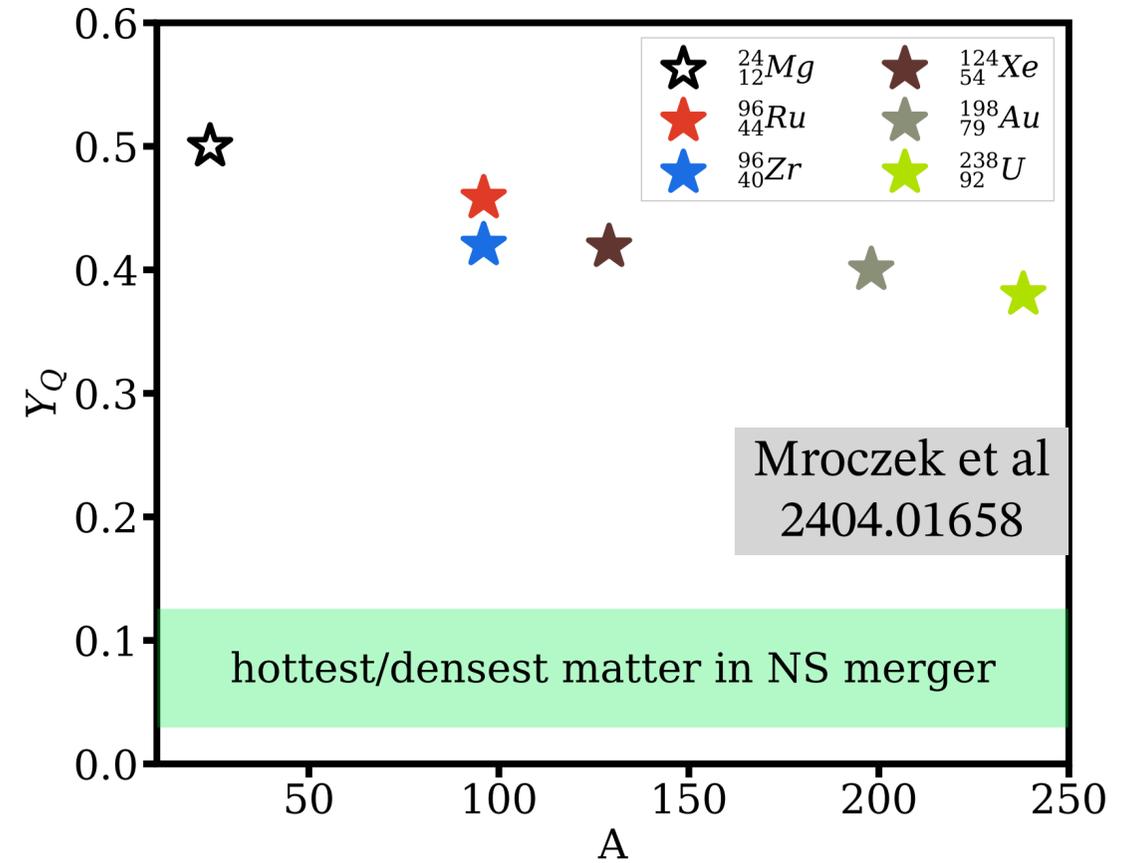
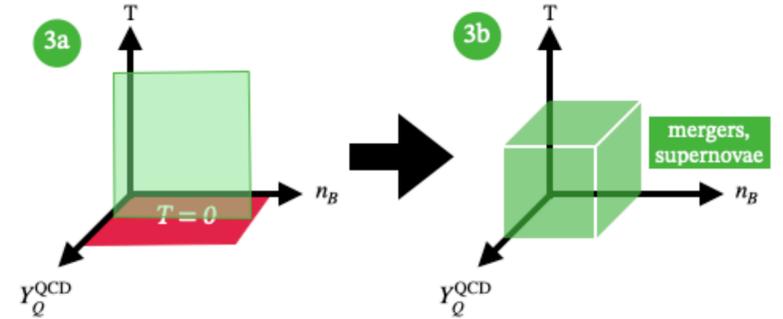
- Nana et al extracted $\partial^2(s/n_B)/\partial\delta^2$ from particle yields across different colliding species, central collisions at $\sqrt{s_{NN}} = 200$ GeV

System	Z	A	Y_Q	Published yield data?
O+O	8	16	0.500	no
Cu+Cu	29	63	0.460	yes
Ru+Ru	44	96	0.458	no*
Zr+Zr	40	96	0.417	no*
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes

Fits predict a **large and negative** value for $\partial^2(s/n_B)/\partial\delta^2$ at $T_{FO} \sim 145$ MeV, $n_B \sim 0.025 n_{sat}$, in **qualitative agreement** with RMF (n+p) results



Mroczek et al 2404.01658



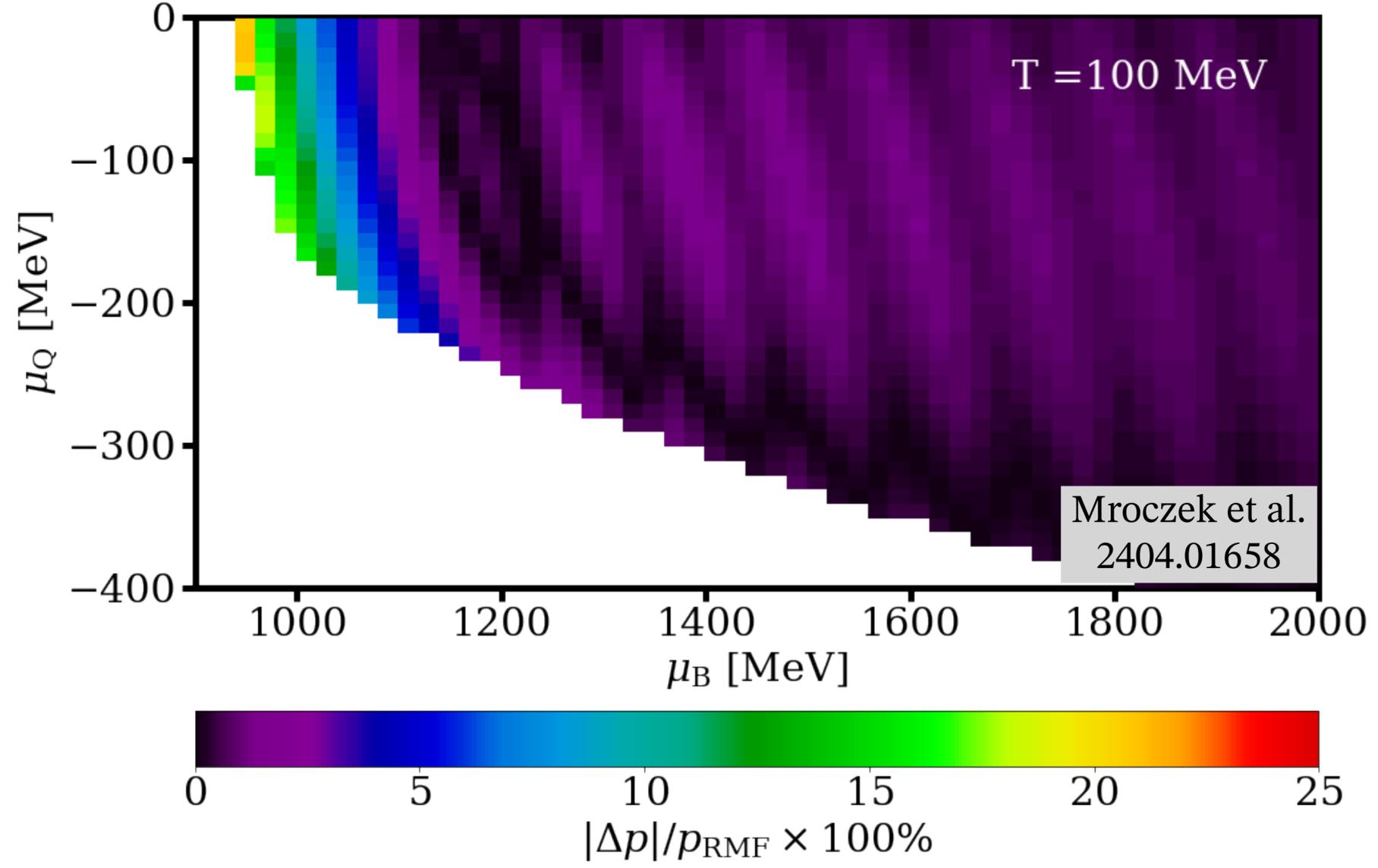
- Needed: **system + energy scan**
- Symmetric nuclei**, e.g., O+O, crucial for extracting the expansion coefficient at $\delta = 0$

LHC, CBM @ FAIR?

F. Nana, J. Salinas San Martín, and J. Noronha-Hostler, 2411.03705

Proof-of-principle with a microscopic EOS

- Expansions 2) and 3) tested against an RMF EOS
- Error introduced by finite $T + s/n_B$ expansions **below 5%** across almost all μ_B, μ_Q at $T=100$ MeV
- Larger error: liquid-gas phase transition near n_{sat}
- Did not account for uncertainty in expansion coefficients:



$$\left. \frac{\partial s_{\text{SNM}}(T, n_B, Y_Q)}{\partial T} \right|_{T=\delta=0}, \quad \left. \frac{\partial^3 \tilde{S}_{\text{SNM},2}(T, n_B, \delta)}{\partial T \partial \delta^2} \right|_{T=\delta=0}$$



✓ definitely



● hopefully

Note: n+p RMF is used as a proof-of-principle, 100 MeV is arbitrary. More realistic T for mergers is 50 MeV where we achieve <1% error.

Summary

- Proposed: **two new expansions** for obtaining finite T, Y_Q equation of state
- Allows for beyond np degrees of freedom, path for incorporating **theoretical + experimental + observational information** → HIC system/energy scan !
- Reproduce a microscopic EOS up to $T=100$ MeV for $\mu_B \gtrsim 1100$ MeV ($\sim 1 - 2 n_{\text{sat}}$) within 5% error
- Clear method for **uncertainty quantification**

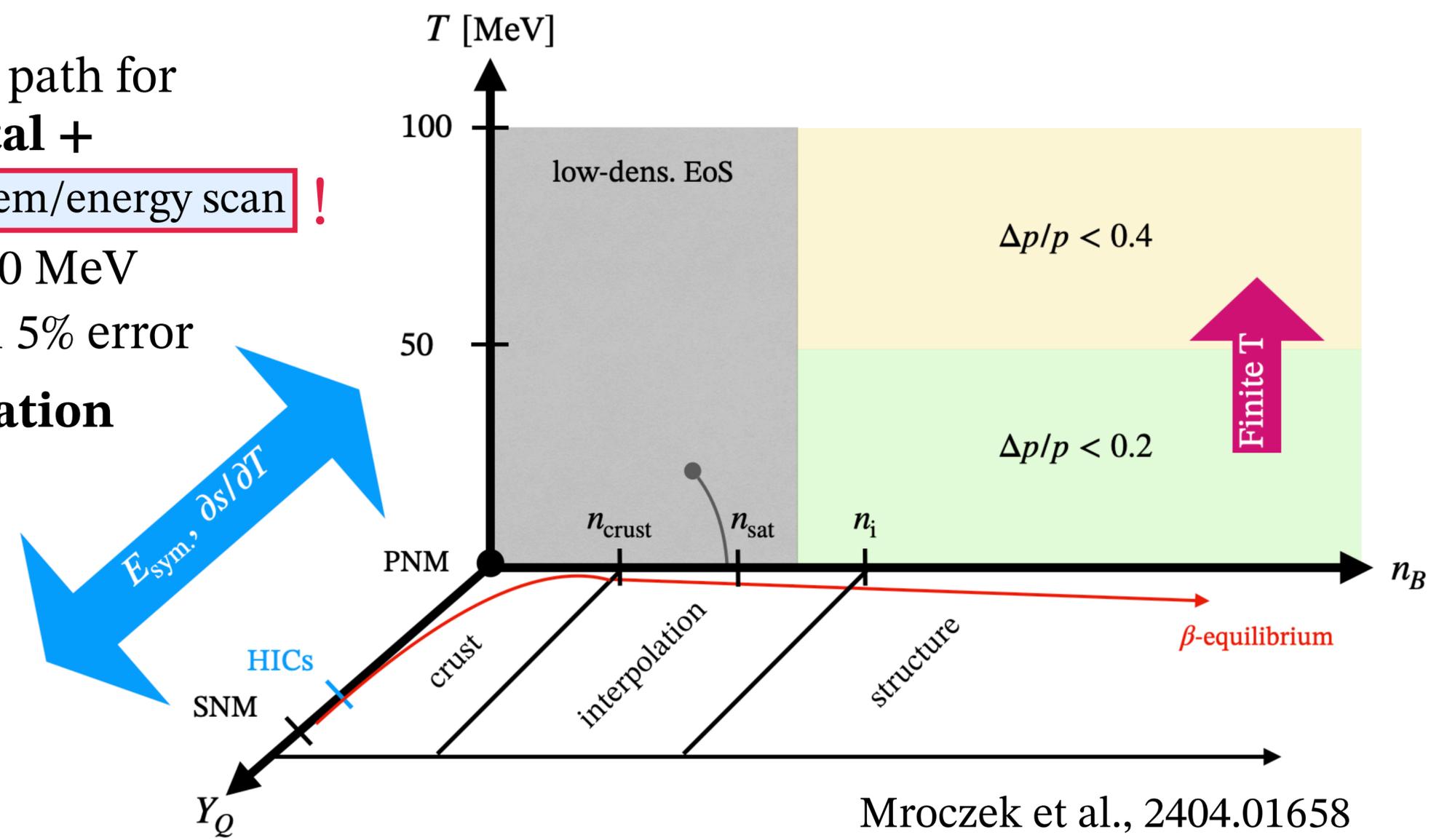
Outlook

- Caveats: no strangeness, no phase transitions → both solvable
- Future study: reducing numerical error, **low-density EOS** at finite T, Y_Q (e.g. hadron resonance gas)



problem:

$$\beta\text{-equilibrium } \{p(n_B), Y_Q(n_{n_B})\} \rightarrow 3\text{D EOS } (T, n_B, Y_Q)$$

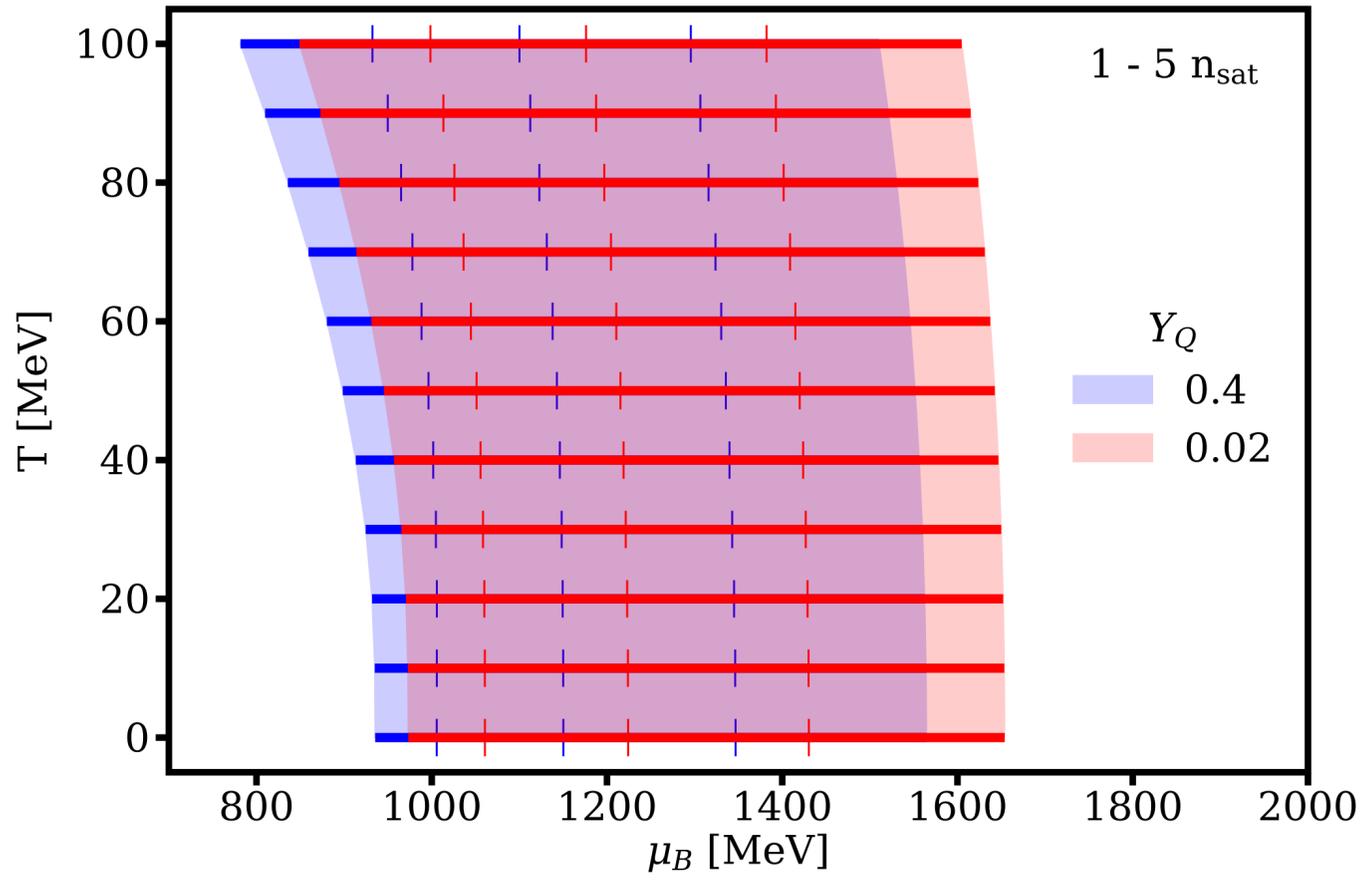


Numerical error quantification

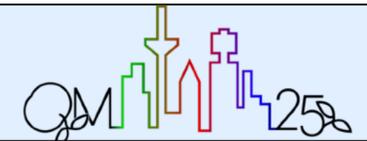
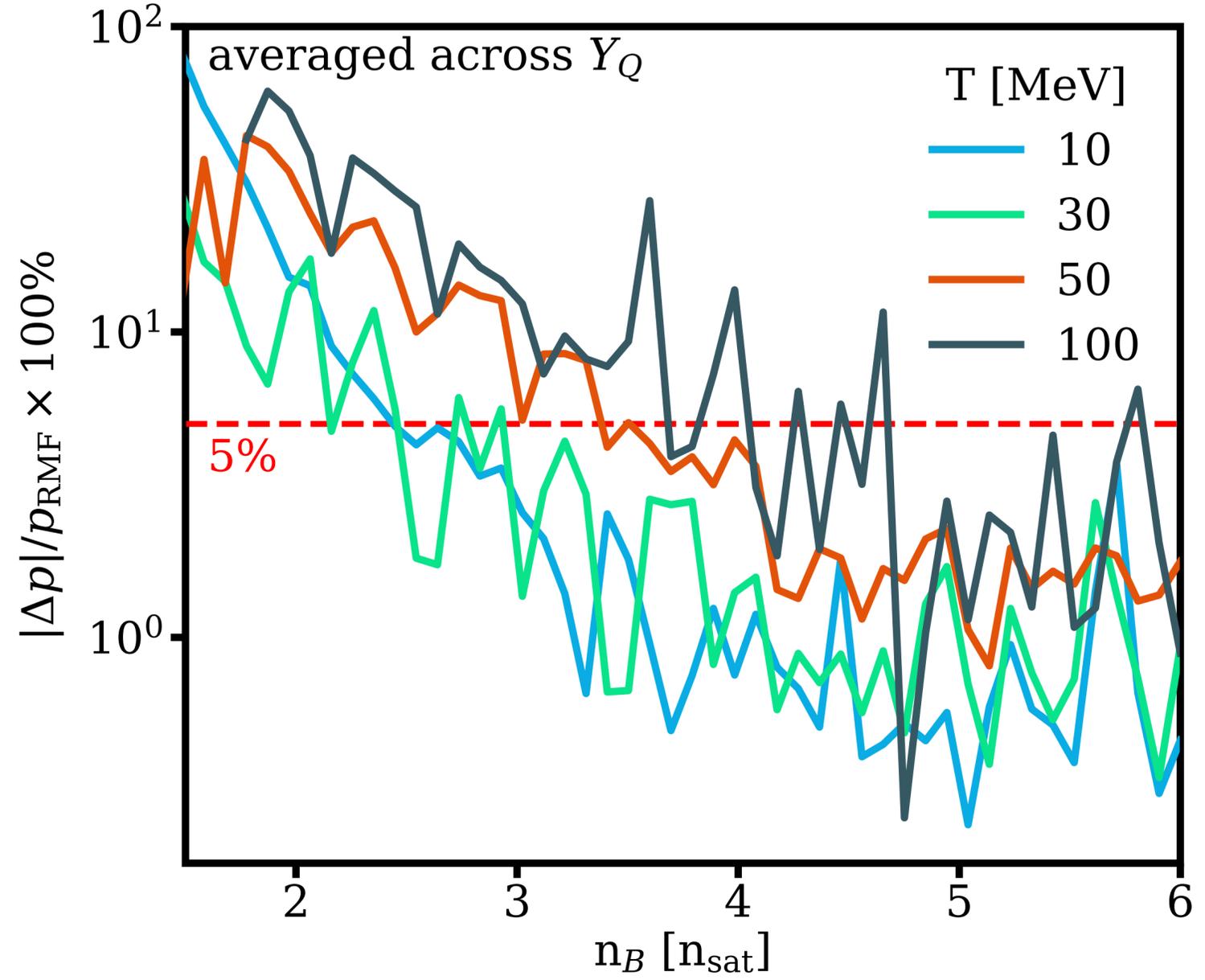
Biggest source of numerical error:

$$(T, \vec{\mu}) \rightarrow (T, n_B, Y_Q)$$

1. Requires numerical derivatives of $p(T, \vec{\mu})$
2. $n_i(\vec{\mu})$ is dependent on Y_Q



Mroczek et. al
2404.01658



$Y_Q(n_B)$ for arbitrary EOS in the symmetry energy expansion

$$Y_p = \frac{1}{16} \left[8 - \frac{\pi^{4/3} n_B}{2^{1/3} X} + \left(\frac{\pi}{2} \right)^{2/3} \frac{X}{E_{\text{sym}}^3} \right]$$

$$X = \left(-24E_{\text{sym}}^6 n_B + \sqrt{2} \sqrt{288E_{\text{sym}}^{12} n_B^2 + \pi^2 E_{\text{sym}}^9 n_B^3} \right)^{1/3}$$

