Energy correlators for a gluon splitting to heavy quarks

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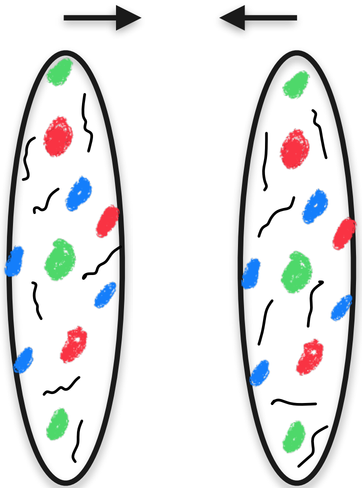
UNIVERSIDAD DE GRANADA

High Energy Theory

European Research Council
Established by the European Commission

Quark-gluon plasma in HICs

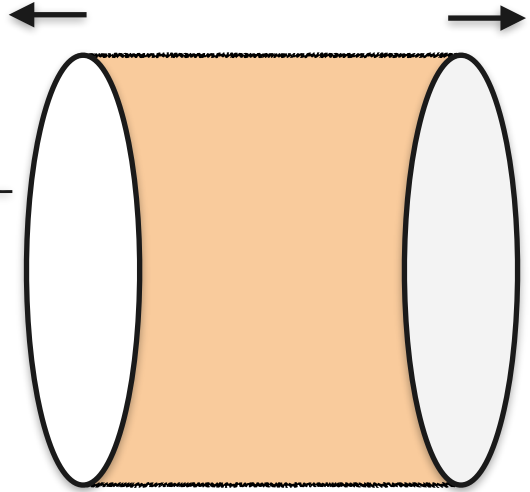
Initial nuclei



Hot and dense medium of deconfined quarks and gluons produced ~ 1 fm/c (10^{-24} s) after a HIC!

Driven to equilibrium $\longrightarrow$ τ

QGP



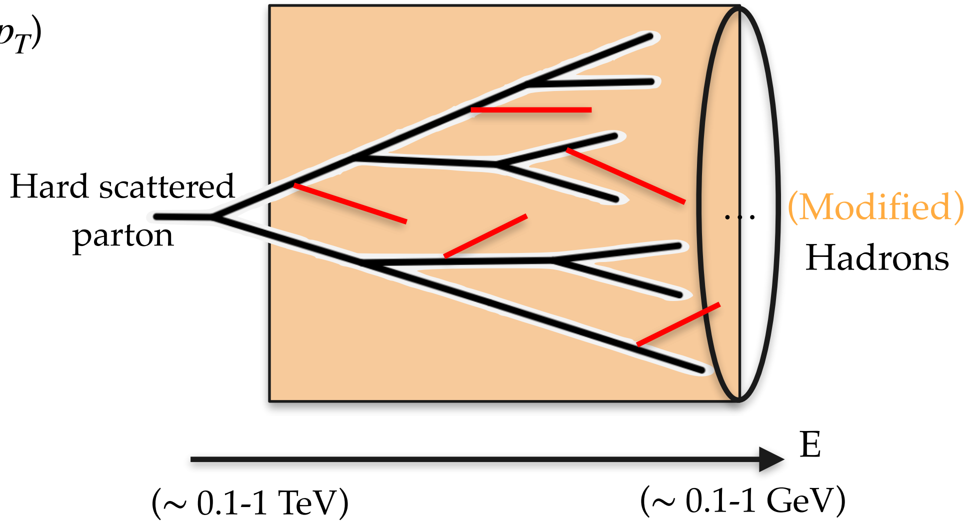
(Near) local thermal eq.

nuclear matter lifetime ~ 10 fm $\longrightarrow$ **use hard probes!**

Probing the QGP with jets

jets - high energy, collimated QCD cascades

- ✦ Jets are produced **early** in the collision ($\tau \sim 1/p_T$)
- ✦ Jets are extended in time and **evolve simultaneously with the QGP.**
- ✦ Jets carry **imprints of medium interactions** (energy loss, substructure modifications, medium response, etc) - **jet quenching.**



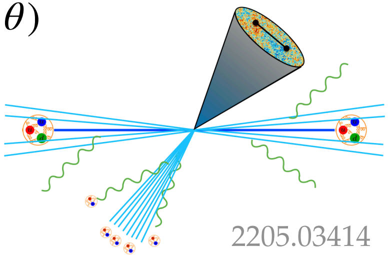
By **comparing heavy-ion jets** with their vacuum counterparts (**p-p jets**), <u>dedicated observables</u> can be used to access the **QGP's transport properties**.

Energy-correlators inside jets

see e.g., [Sveshnikov et al., arXiv:hep-ph/9512370]
[Hofman et al., arXiv: 0803.1467]
[Chen et al., arXiv: 2004.113811]

✦ Energy-correlators (ENCs) are **angular projections** of correlation functions of **energy flow operators:**

$$\mathscr{E}(\vec{n}) = \lim_{r \to +\infty} \int dt\, r^2\, n^i \underbrace{T^{0i}(t, r\vec{n})}_{\text{EM tensor}} \xrightarrow{\text{EEC}} \frac{d\Sigma^{(2)}}{d\theta} = \int_{\vec{n}_1, \vec{n}_2} \frac{\langle \mathscr{E}(\vec{n}_1) \mathscr{E}(\vec{n}_2) \rangle}{p_t^2} \delta(\vec{n}_1 \cdot \vec{n}_2 - \cos\theta)$$



2205.03414

e.g. Kudinoor (17:00 Mon, Pa.5), Holguin (17:20 Mon, Pa.5), Sirimanna (11:50 Tue, Pa.18), Soto-Ontoso (14:45 Thu, Pl.5)

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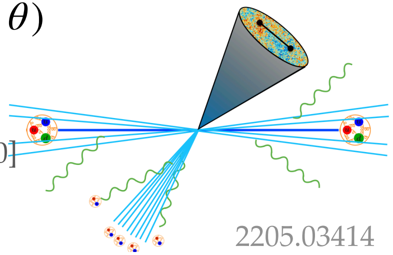


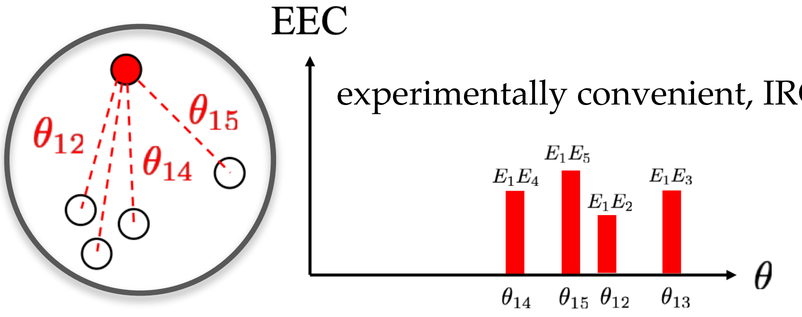
[Dixon et al, arXiv: 1905.01310]

2205.03414

✦ The **collinear limit of EEC** is particularly useful in accessing jet substructure.

$$\boxed{\frac{d\Sigma^{(2)}}{d\theta} = \int_0^1 dz\, z(1-z)\frac{d\sigma}{\sigma dz d\theta}}$$

can be **systematically** calculated at higher orders in perturbation theory



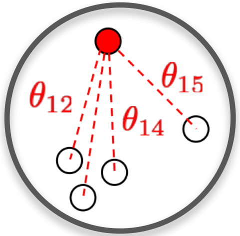
e.g. Kudinoor (17:00 Mon, Pa.5), Holguin (17:20 Mon, Pa.5), Sirimanna (11:50 Tue, Pa.18), Soto-Ontoso (14:45 Thu, Pl.5)

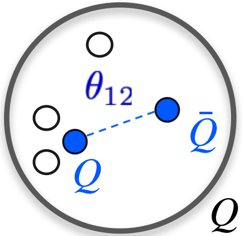
Heavy flavour hadron EEC inside the QGP

see e.g. ENCs in QGP
[Andres et al., 2307.15110]
[Barata et al., 2308.01294]
[Bossi et al., 2407.13818]
[Barata et al., 2503.13603]

- EECs organize different physics into **small/large angle information.**
  - → Useful to **sort out QGP-induced modifications!**
- Focus on EEC for $g \to Q\bar{Q}$ inside heavy-ion jets. Why?
  - → $m_Q \gg T_{QGP}\ ,\ \Lambda_{QCD}$ → produced in hard scattering or in high energy $g \to Q\bar{Q}$
  - → Access splitting kinematics by tagging a $Q\bar{Q}$ pair in jets with **two heavy flavour hadrons.**

all jet constituents





$Q\bar{Q}$ pair

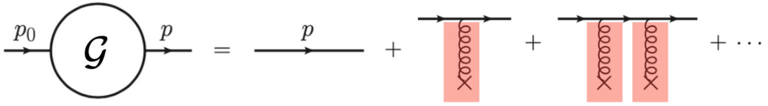
Heavy flavour hadron EEC provides unique opportunity to probe **medium modifications** to the $g \to Q\bar{Q}$ splitting.

This talk: **partonic** $g \to Q\bar{Q}$ EEC

Partonic splittings inside the QGP

How can one describe partonic splittings in a *dense* QCD medium?

Perturbative QCD + **Medium model**



$$\boxed{\langle \mathscr{A}^-_a(x^+, \boldsymbol{x}), \mathscr{A}^{*-}_b(y^+, \boldsymbol{y}) \rangle}$$

Resummation of *single* gluon exchanges with the medium (***BDMPS-Z***)
$(p^+ \gg |\boldsymbol{p}|, |\Delta \boldsymbol{p}|)$
+ color precession

Stochastic gauge field in light-cone gauge
Gaussian white noise model

see e.g. E. Weitz (10:50 Tue, Pa.19), A. Cordeiro (9:00 Wed, Pa.28), X. Mayo López (9:20 Thu, Pa.8)

Partonic splittings inside the QGP: medium model

- Some assumptions reflected on the **two-point correlator** of the medium field:

$$\langle \mathscr{A}_a^-(x^+,\boldsymbol{x}), \mathscr{A}_b^{*-}(y^+,\boldsymbol{y})\rangle = \delta^{ab} n(x^+)\delta(x^+ - y^+)\gamma(\boldsymbol{x},\boldsymbol{y})$$

matter density **collision kernel**

- $n(x^+) = n$ (***static***)
- $\gamma(\boldsymbol{x},\boldsymbol{y}) = \gamma(\boldsymbol{y}-\boldsymbol{x})$ (***homogeneous,*** i.e., translation invariant)
- $\gamma(\boldsymbol{y}-\boldsymbol{x}) = \gamma(|\boldsymbol{y}-\boldsymbol{x}|)$ (***isotropic,*** i.e., rotationally invariant)

$$\boxed{\gamma(0) - \gamma(\boldsymbol{r}) \sim \hat{q}\boldsymbol{r}^2} + \mathcal{O}(\boldsymbol{r}^2 \log \boldsymbol{r}^2) \longrightarrow$$

$\hat{q} \sim$ accumulated $k_\perp^2$ per mean free path

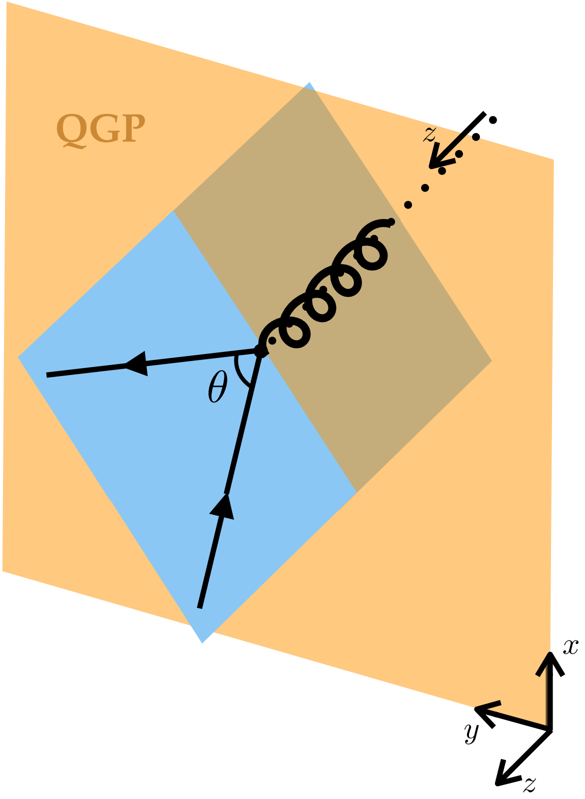
(multiple soft scattering approximation)

$g \to Q\bar{Q}$ splitting in the QGP

[Attems et al., arXiv: 2203.11241]
[Barata et al., arXiv: 2407.04774]

$$\frac{d^2\sigma}{dz d\theta}(L, \hat{q}, \ldots)$$

L - length of the medium
$\hat{q}$ - average $k_\perp^2$ acquired in a mean free path
θ - opening angle of $Q\bar{Q}$ antenna



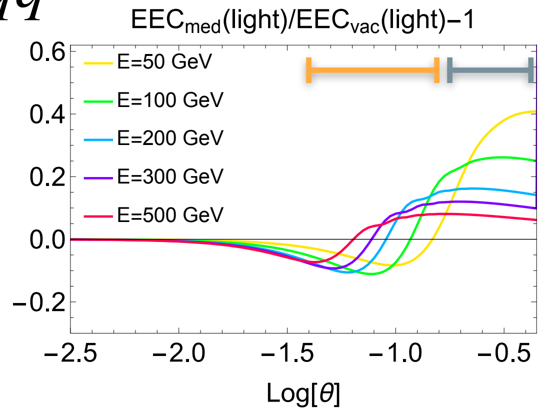
- More energetic $Q\bar{Q}$ pairs are less modified **(antenna formation time dependence)**
- Depletion at small to intermediate $k_\perp \sim E\theta$ **(final state broadening)**

In-medium EEC for $g \to Q\bar{Q}$

[in prep. J. Barata, J. Brewer, K. Lee, JMS]

Depletion at intermediate θ **(broadening effect)** and enhancement at large θ

$q\bar{q}$



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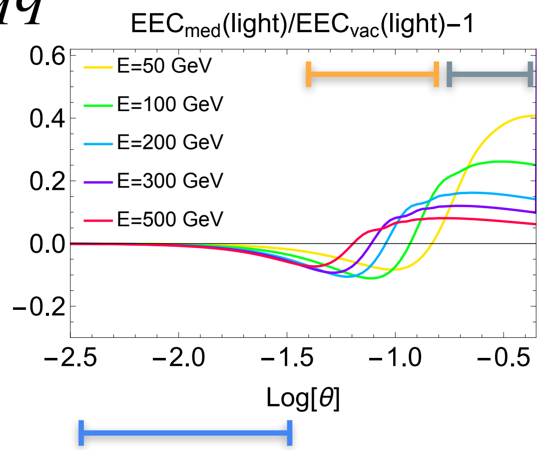

No modification at small θ

(~ light $q\bar{q}$ produced outside the medium)

$$\tau_{q\bar{q}} \sim \frac{z(1-z)E}{m_Q^2 + z^2(1-z)^2E^2\theta^2} \sim \frac{1}{z(1-z)E\theta^2}$$

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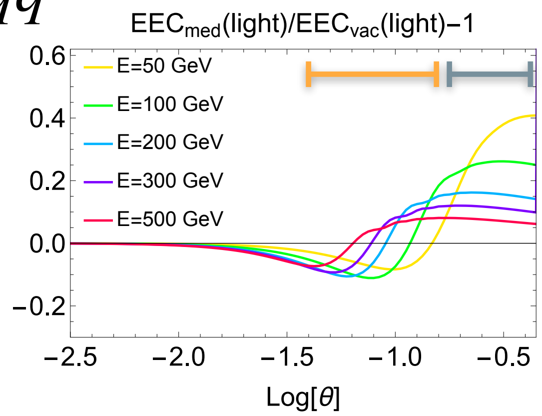
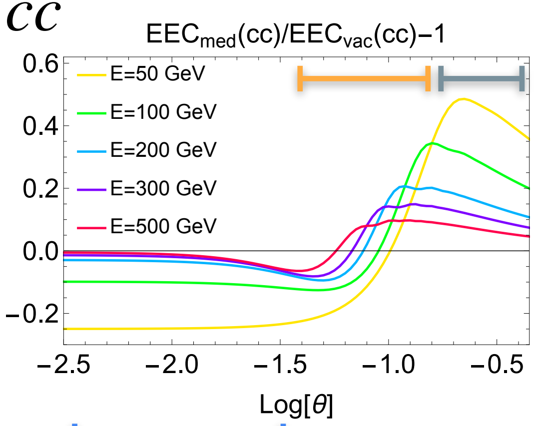
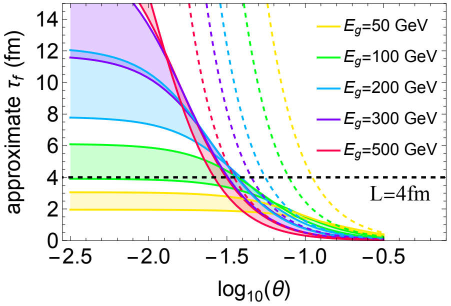


$c\bar{c}$



No modification at small θ
(~ light $q\bar{q}$ produced outside the medium)

Suppression at small θ
(~ heavy $Q\bar{Q}$ can be produced inside the medium for all angles)



$\tau_{c\bar{c}}$

$$\tau_{Q\bar{Q}} \sim \frac{z(1-z)E}{m_Q^2 + z^2(1-z)^2E^2\theta^2}$$

see J. Brewer (10:20 Thu, Pa.7)

In-medium EEC for $g \to Q\bar{Q}$

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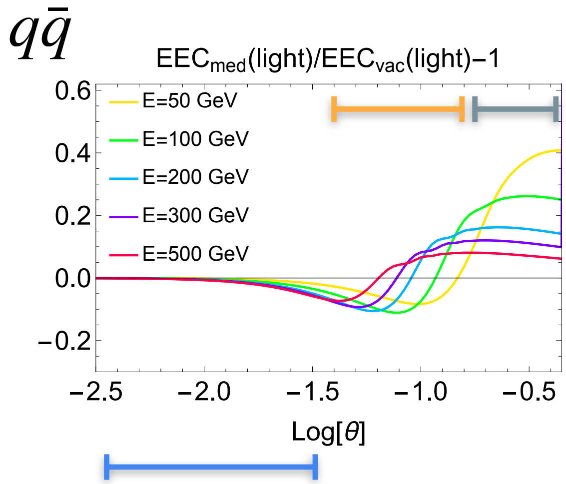

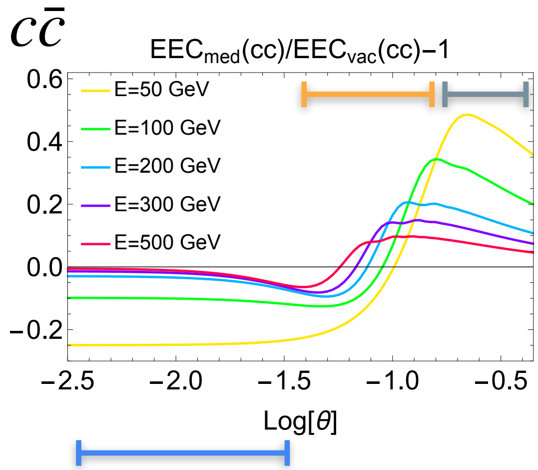

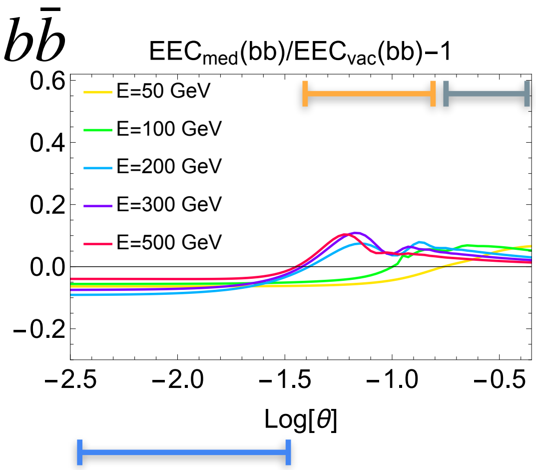

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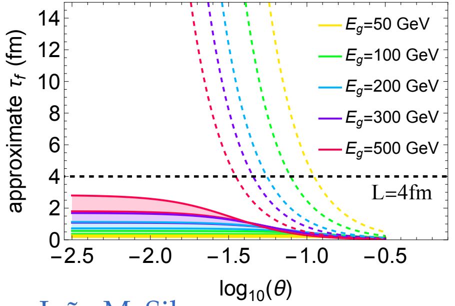


$\tau_{b\bar{b}}$

$$\tau_{Q\bar{Q}} \sim \frac{z(1-z)E}{m_Q^2 + z^2(1-z)^2E^2\theta^2}$$

see J. Brewer (10:20 Thu, Pa.7)

Anisotropies in a pre-QGP state

[Hauksson, Iancu, 2303.03914] (applied to $g \to gg$ splitting)

The state **preceeding** the QGP is expected to be **far from local thermal eq**.
In principle, it is **highly anisotropic**, e.g.:

expansion of the medium along the collision axis: $\longrightarrow$ anisotropic momentum distribution of medium constituents

This generates *jet-medium momentum exchanges* which have a *preferred direction*

$$\tilde{\gamma}(\boldsymbol{q}) \neq \tilde{\gamma}(|\boldsymbol{q}|) \longrightarrow \boxed{\gamma(0) - \gamma(\boldsymbol{r}) \sim \hat{q}_x r_x^2 + \hat{q}_y r_y^2} + \mathcal{O}(\boldsymbol{r}^2 \log \boldsymbol{r}^2)$$

$\longrightarrow$ relevant anisotropy is in the **plane transverse to jet axis**

Anisotropic broadening has been approached in:

Glasma
e.g. [Ipp et al., 2009.14206]

Kinetic theory
e.g. [Boguslavski et al., 2303.12595]

Hard thermal loop
e.g. [Hauksson et al., 2109.04575]

…

If this state evolves **slowly to isotropy**, a non-negligible anisotropy strength may persist at large times - may be **relevant for heavy-ion jet pheno**.

Lindenbauer (18:20 Mon, Pa.4), Schimdt (18:40 Mon, Pa.4), Mäntysaari (19:00 Mon, Pa.4), Lu (11:10 Tue, Pa.16), Cabodevilla (17:30, Tue, Pa. 24)

$g \to Q\bar{Q}$ splitting in an anisotropic medium

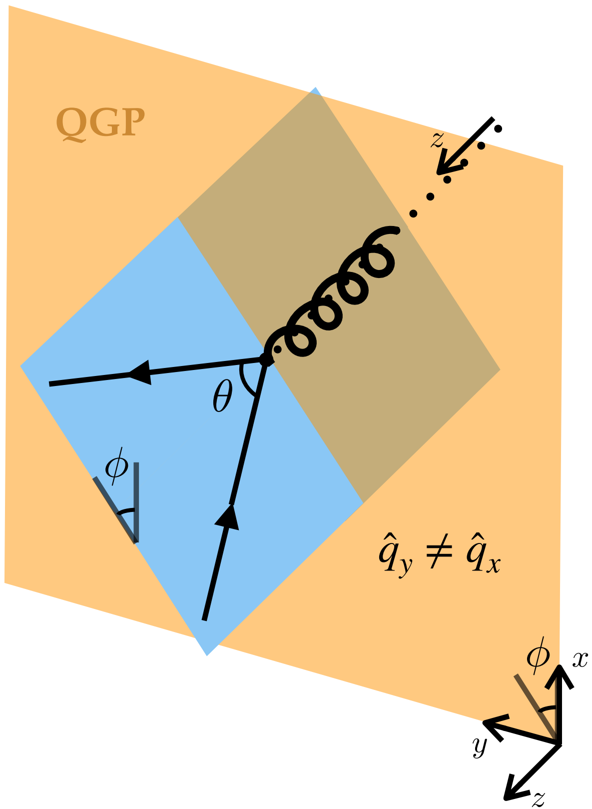
[Barata, Salgado, JMS; arXiv: 2407.04774]

$$\frac{d\sigma}{dz\,d\theta\,d\phi}(L, \hat{q}_i, \dots) \qquad \hat{q}_y \neq \hat{q}_x$$

anisotropy introduces a non-trivial angular modulation in ϕ

L - length of the medium
$\hat{q}_i$ - average $k^2_{\perp,i}$ acquired in a mean free path
ϕ - azimuthal orientation of plane spanned by $q\bar{q}$

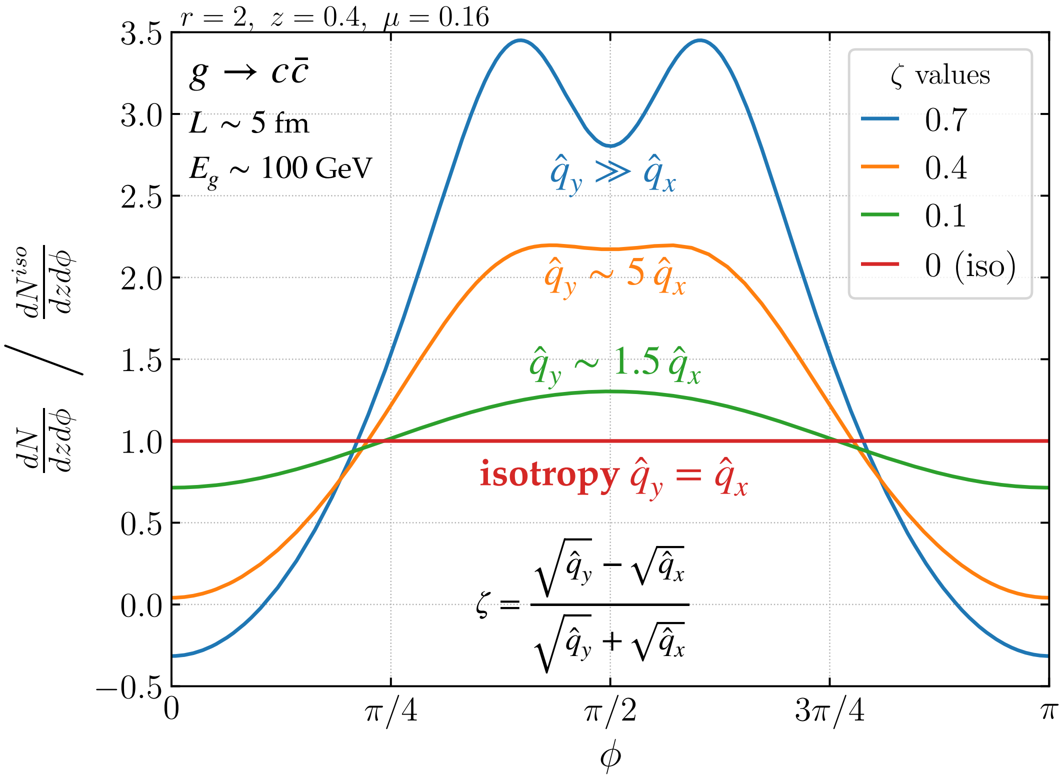
$$\left(\boldsymbol{P}^{rel} = (1-z)\boldsymbol{p_1} - z\boldsymbol{p_2} \quad \tan\phi = \frac{\boldsymbol{P}^{rel}_y}{\boldsymbol{P}^{rel}_x} \right)$$



Anisotropies in the azimuthal spectrum

Ratio wrt. isotropic distribution ($dN \equiv d\sigma/\sigma$)

[Barata, Salgado, JMS; arXiv: 2407.04774]



- **Isotropic** dN is flat in ϕ.
- **Angular modulation** gets stronger the **larger the anisotropy** strength.
- For **small anisotropies**, it is simply described by an **elliptic** type modulation:

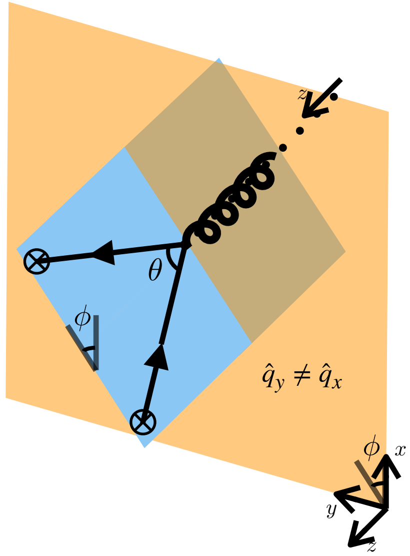
$$\frac{2\pi}{dN/dz}\frac{dN}{dzd\phi}\bigg|_{\zeta\ll 1} = 1 + \zeta v_2^{jet}\cos(2\phi)$$

- For **larger anisotropies**, higher harmonics play a role.

Azimuth dependent EEC

[in prep. J. Barata, J. Brewer, K. Lee, JMS]

- Azimuth dependent EECs were applied in e.g. 2104.00009 (Chen, Moult, Zhu), to study correlations between planes of successive splittings in vacuum.
- In our case, we correlate a physical direction from the medium (e.g. the direction of maximal anisotropy) and the $Q\bar{Q}$ splitting plane.



$$\frac{d\Sigma^{\text{med}}_{g\to Q\bar{Q}}}{d\theta d\phi} = \frac{1}{\sigma}\int_0^1 dz\, z(1-z)\frac{d\sigma^{\text{med}}_{g\to Q\bar{Q}}}{dz d\theta d\phi}$$

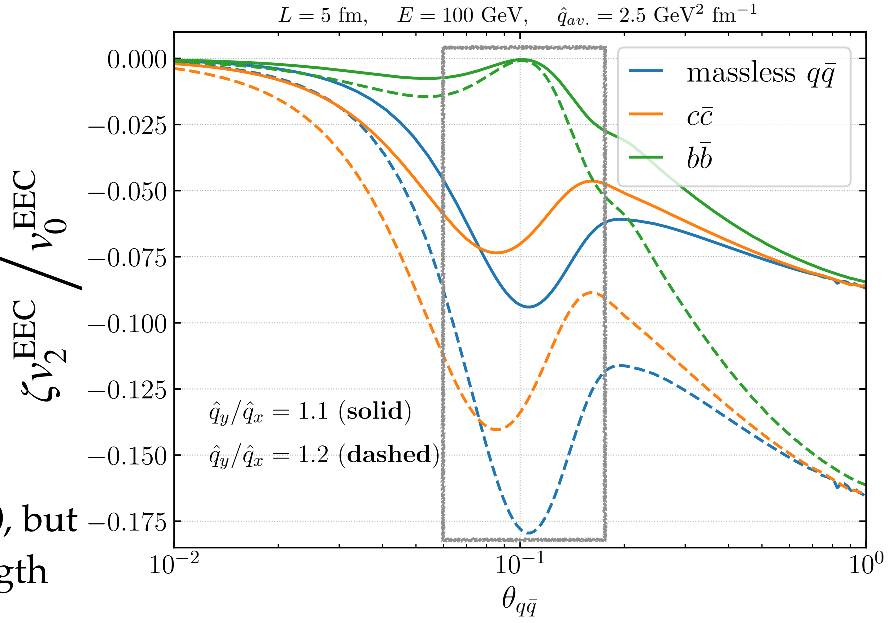
Anisotropies in the azimuth dependent EEC

[in prep. J. Barata, J. Brewer, K. Lee, JMS]

As we saw, for small anisotropies ($\zeta \ll 1$) we have:

$$\frac{d\Sigma}{d\theta d\phi} \sim v_0^{\mathrm{EEC}}(\theta) + \zeta v_2^{\mathrm{EEC}}(\theta)\ \cos 2\phi$$

$$\left(\zeta = \frac{\sqrt{\hat{q}_y} - \sqrt{\hat{q}_x}}{\sqrt{\hat{q}_y} + \sqrt{\hat{q}_x}} \right)$$



Mass dependent anisotropy effect

Heavier mass seems to suppress the sensitivity to $\zeta \neq 0$, but $g \to c\bar{c}$ still shows a non-negligible modulation strength when compared to the isotropic EEC (v_0^{EEC}).

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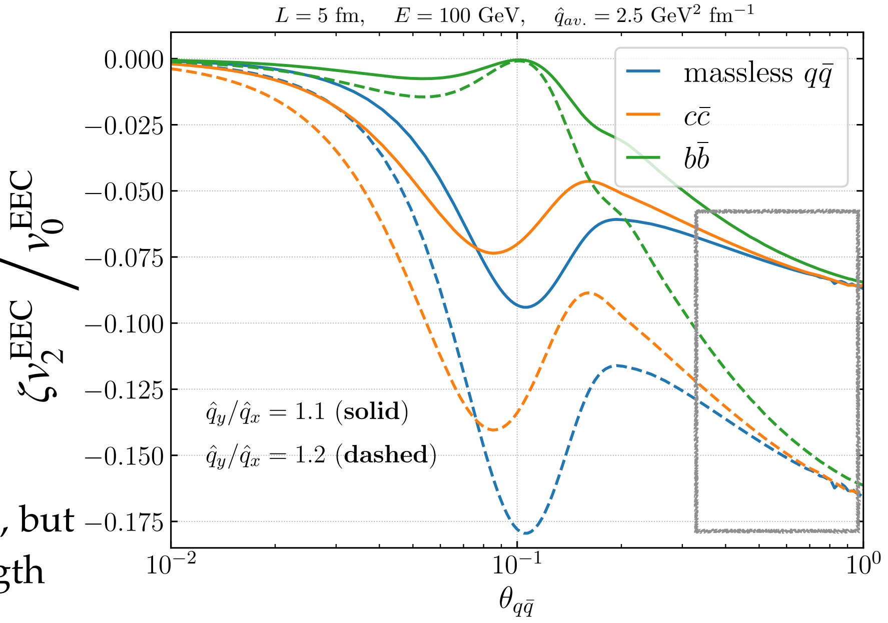

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Mass independent anisotropy effect

Summary and outlook

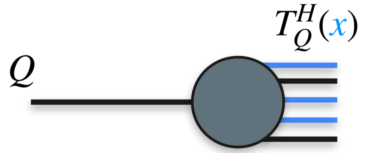
Summary:

- EEC for gluon splitting to a massive $Q\bar{Q}$ provides insight into:
  - formation time dependent modifications induced by the QGP;
  - interplay between heavy quark mass and anisotropies in pre-QGP state.

Outlook:

- Comparison between analytic results and medium modified shower with hadronization:
  - Heavy flavour hadron EEC obtained from partonic EEC by multiplying by track function moments.

[Chang et al., 1303.6637]
[Chen et al., 2210.10058]

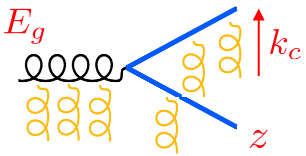


THANKS!

Back up slides

Estimating the modification of $g \to c\bar{c}$ splittings in jets

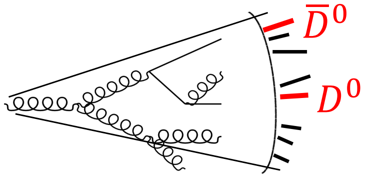
- Calculated medium-modified $g \to c\bar{c}$ splitting function



$$P_{g\to c\bar{c}}(E_g, k_c^2, z) = P^{\mathrm{vac}}_{g\to c\bar{c}}(k_c^2, z) + P^{\mathrm{med}}_{g\to c\bar{c}}(E_g, k_c^2, z)$$

Attems, JB, Innocenti, Mazeliauskas, Park, van der Schee, Wiedemann *JHEP 01 (2023) 080* [2203.11241]

- Estimate the modification inside jets



Get kinematics of g $\to c\bar{c}$

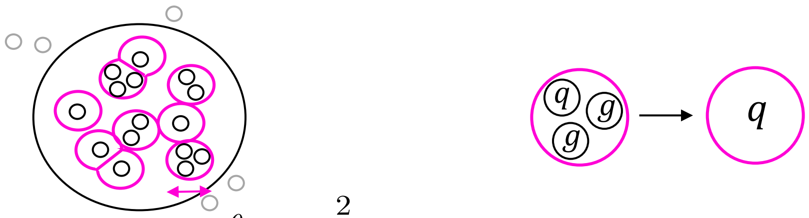
Reweight each splitting by $w^{med}_{g\to c\bar{c}}(E_g, k_c^2, z) = 1 + \dfrac{\left(\frac{1}{Q^2} P_{g\to c\bar{c}}\right)^{\mathrm{med}}(E_g, k_c^2, z)}{\left(\frac{1}{Q^2} P_{g\to c\bar{c}}\right)^{\mathrm{vac}}(k_c^2, z)}$

Attems, JB, Innocenti, Mazeliauskas, Park, van der Schee, Wiedemann *Phys.Rev.Lett.* 132 (2024) 21 [2209.13600]

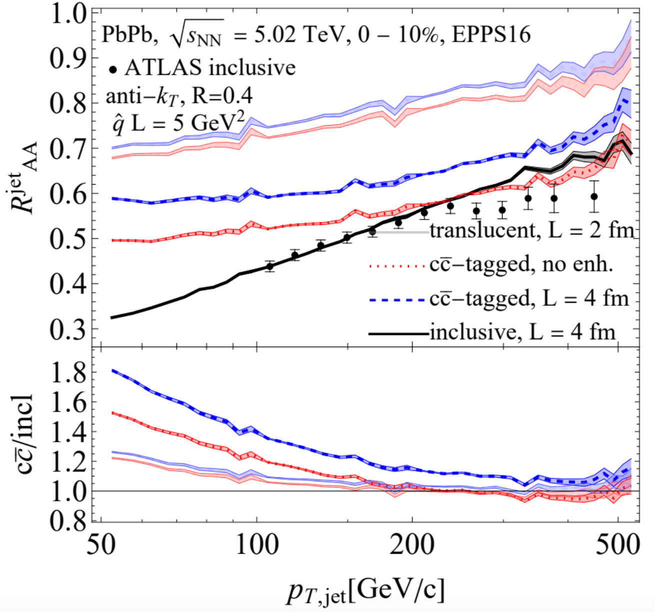
- Calculate energy loss for jet as a whole
  - parton-level jets in Pythia with nuclear pdfs
  - assume coherence of particles within θ_c

jet constituents → "clusters" of radius θ_c

cluster flavor assigned from parton content



$$\theta_c = \frac{2}{\sqrt{\hat{q}L^3}}$$



- Jet energy loss from quenching each cluster with weights

Barata, Caucal, Soto-Ontoso, Szafron [2312.12527]

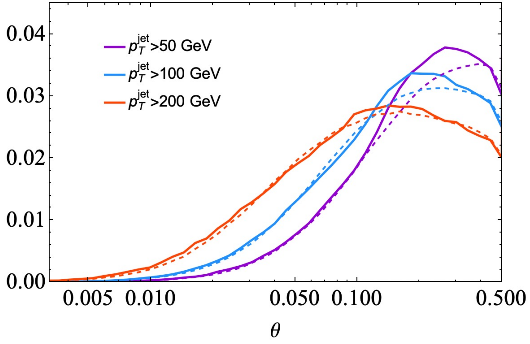
$$Q_i = \exp\left[-\int d\omega \int d^2\mathbf{k} \frac{d\mathcal{P}_i^{\rm med}}{d\omega d^2\mathbf{k}}\left(1-e^{-\frac{n\omega}{p_t}}\right)\right] = \exp\left[\underbrace{-\int_T^{\omega_s} d\omega \int d^2\mathbf{k} \frac{d\mathcal{P}_i^{\rm med}}{d\omega d^2\mathbf{k}}\left(1-e^{-\frac{n\omega}{p_t}}\right)}_{\text{rapid turbulent thermalization; } \omega \ll \omega_c} \underbrace{- \int_{\omega_s}^{\infty} d\omega \int d^2\mathbf{k} \frac{d\mathcal{P}_i^{\rm med}}{d\omega d^2\mathbf{k}}\left(1-e^{-\frac{n\omega}{p_t}}\right)}_{\text{semi-hard perturbative gluon emission}}\right]$$

Quenching weights

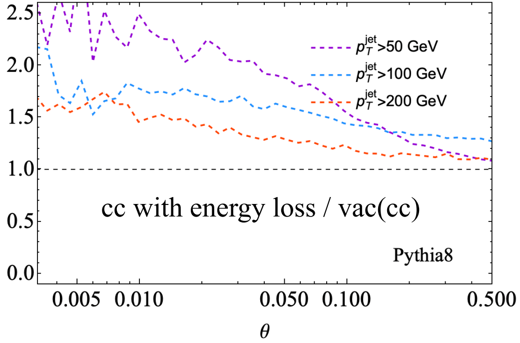
Effects of energy loss on energy correlators of jets with two heavy quarks

Energy loss shifts the EEC toward smaller angles…

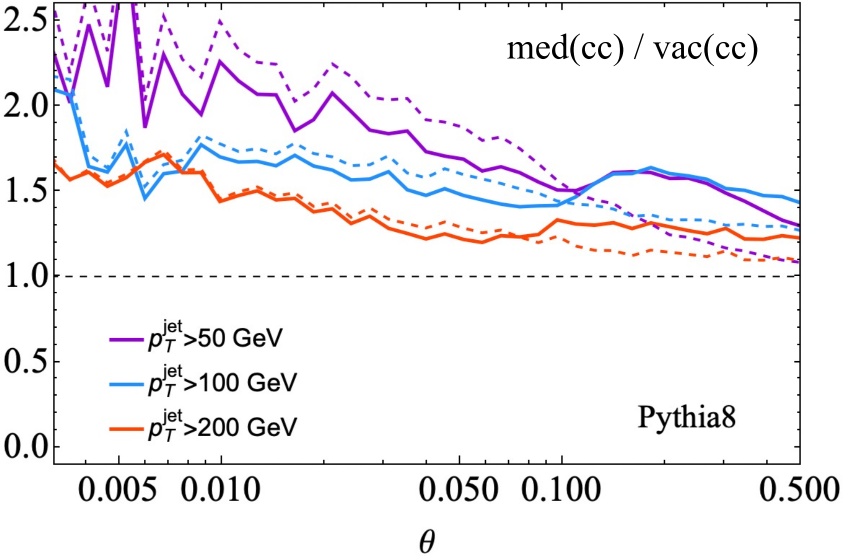
unmodified (dashed), modified (solid)



…and enhances the charm yield*



Putting it together: interplay of medium-modified $g \to c\bar{c}$ splitting with energy loss



To dig out formation time effects, would like new ways to reduce energy loss effects

*(relative to the total number of jets)

JB, Milhano, Thaler [1812.05111], Andres, Holguin, Elayavalli, Viinikainen [2409.07514]