Monte-Carlo sampling of nucleon positions in the nuclear shell model for heavy-ion collisions

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Motivation

- Examination of N-body density distributions is crucial for better understanding deeply inelastic scattering and nucleus-nucleus collisions
- New sampling algorithm takes on account **quantum correlations** of the nuclear many-body wave function
- Usage of the **nuclear shell model** [1] permits the extension of the algorithm to large systems, such as Au and Pb
- Initial state observables sensitive to the N-body densities can be explored, such as the initial eccentricities, extracted from the McDIPPER model [2]

Sampling from a N-body wave function

- Nucleons move independently in an average potential
  – harmonic oscillator potential
- **Single particle wavefunction** with spin-orbit coupling:

$$\Phi_{nljm_j}(\vec{x},s) = \sum_{m_l,m_s} C^{jm_j}_{lm_l,sm_s}\chi_{nlm_l}(\vec{x})$$

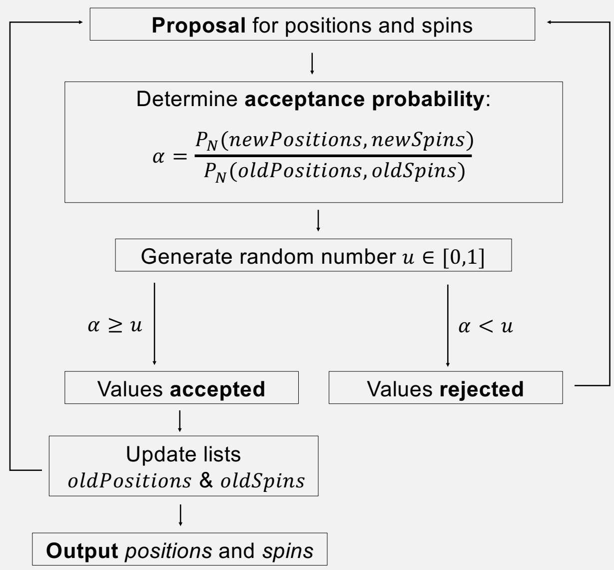
- **N-body wavefunction** using $\tilde{x}_i = (\vec{x}_i, s_i)$:

$$\psi(\tilde{x}_1,\dots\tilde{x}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix}\Phi_1(\tilde{x}_1) & \cdots & \Phi_N(\tilde{x}_1)\\ \vdots & \ddots & \vdots \\ \Phi_1(\tilde{x}_N) & \cdots & \Phi_N(\tilde{x}_N)\end{vmatrix}$$

- Markov Chain Monte Carlo method is used to generate nucleon positions and spins from **total Probability** P_N

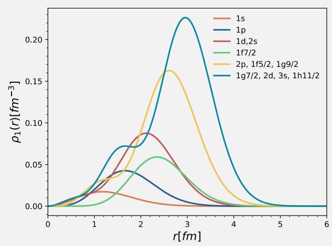
Metropolis Algorithm

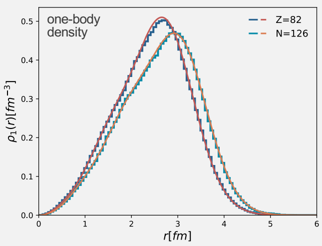
- Target distribution: $P_N(\tilde{x}_1,\dots\tilde{x}_N) = |\psi(\tilde{x}_1,\dots\tilde{x}_N)|^2$

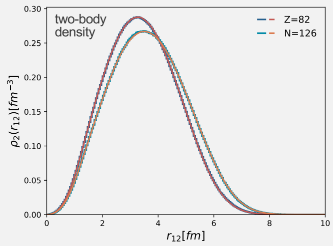


Case of interest: ^{208}Pb

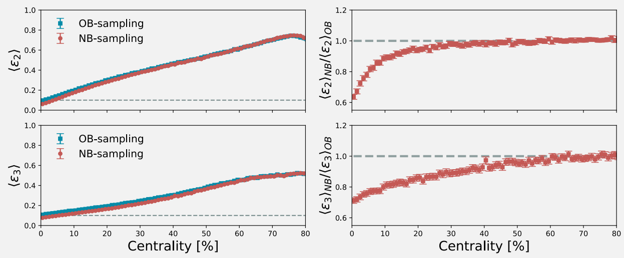
- Analytical one-body densities for each individual shell show **composition of the one-body density** for e.g. a lead nucleus with 82 protons
- Sampled **one- and two-body densities** (for $Z = 82$ & $N = 126$) agree with analytical calculations



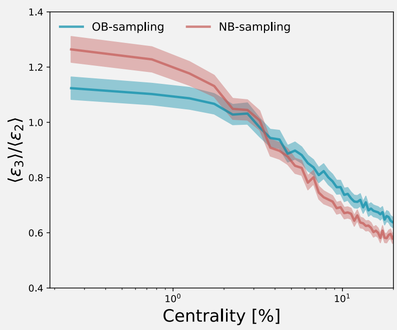




Effect on initial state phenomenology



- Usage of the **McDIPPER model** to explore the initial eccentricities ϵ_n
- **Decrease of eccentricities** at ultra-central collisions due to the inclusion of N-body correlations
- **Ratio of $\langle\epsilon\rangle_3/\langle\epsilon\rangle_2$ is increased** as compared to sampling from a one-body nucleon density



Conclusion & Outlook

- Algorithm to generate positions of nucleons including spin and spatial correlations for large systems
- Nuclear structure plays a critical role for the ratio of $\langle\epsilon\rangle_3/\langle\epsilon\rangle_2$ in ultra-central events

- Extension to calculation for non-closed shell nuclei
- Extendable to explore correlations in large nuclei using more complex Hamiltonians

References:
[1] P. Ring and P. Schuck, *The nuclear many-body problem*, pp. 40-44 , 1980
[2] O. Garcia-Montero, H. Elfner and S. Schlichting, *Phys.Rev.C* 109 (2024) 4, 044916