Advancing our understanding of in-medium QCD splittings

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Introduction

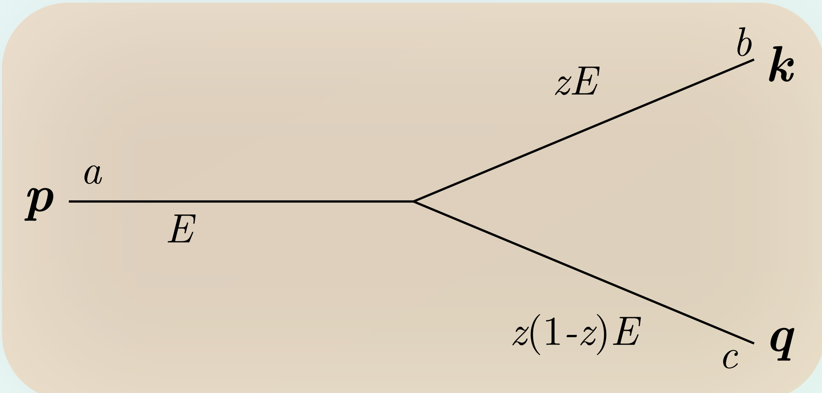
The problem: to understand medium induced radiation patterns of a jet propagating in the hot and dense **Quark-Gluon Plasma (QGP)**, produced in heavy-ion collisions.

Approach: study a single **parton splitting** in a background field (the QGP medium), using the **BDMPS-Z formalism** [1].

Cross-section for an in-medium splitting:

$$\int_{\boldsymbol{P}} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_k \mathrm{d}\Omega_q} = \frac{\mathrm{d}I}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}\frac{\mathrm{d}\sigma_0}{\mathrm{d}E}$$

$$\frac{\mathrm{d}^3 I}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}} = \frac{\mathrm{d}^3 I^{\mathrm{vac}}}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}(1+F_{\mathrm{med}})$$



Vac. radiation spectrum

Medium modification factor
Available at the eikonal limit

Goal: Extract the medium modification factor with **exact kinematics**.

Preliminaries

In-medium splittings are encapsulated in a system of **Schrodinger-like equations** [2] of the form

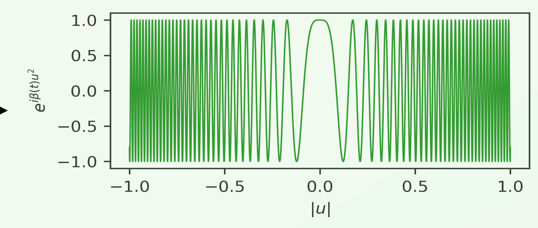
$$i\frac{\partial \mathcal{F}_\sigma(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}},t)}{\partial t} = \left(-\frac{\partial^2_{\boldsymbol{u}}-\partial^2_{\boldsymbol{v}}}{2\omega}\delta^{\sigma'}_{\sigma} + i\mathrm{M}^{\sigma'}_{\sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\right)\mathcal{F}_{\sigma'}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}},t) + \mathcal{B}_\sigma(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}},t)$$

For instance, in the Harmonic Oscillator (HO) approximation for the interaction potential:

$$\tilde{\mathcal{F}}_\sigma(\vec{\boldsymbol{p}},t) = \int_{\boldsymbol{u},\boldsymbol{v}} \mathcal{F}_\sigma(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}},t)e^{-i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{u}}-\vec{\boldsymbol{v}})}, \qquad F_{\mathrm{med}}(t) = \mathrm{Re}\left[\frac{\theta^2}{4}\tilde{\mathcal{F}}_2(\omega\theta\hat{\boldsymbol{x}},t) - 2\left(1-e^{-i\frac{\tan(\Omega L)}{2\Omega}\omega\theta^2}\right)\right]$$

Extremely complex Schrödinger-like PDE due to **high dimensionality**, **highly oscillatory source term** and **non-Hermitian Hamiltonian**.

$$\mathcal{B}_\sigma \sim e^{i\beta(t)\boldsymbol{u}^2}$$



Original proposed method (Euler/RK4) is **unconditionally unstable**, i.e., it will eventually explode at a certain time, and very **inaccurate**

This requires more **sophisticated numerical algorithms!**

Towards a novel algorithm

Proof of concept: 1D Toy Model of our problem:

$$i\frac{\partial F(x,t)}{\partial t} = -\frac{1}{2\omega}\frac{\partial F(x,t)}{\partial x} + iV(x)F(x,t) + s(x,t)$$

A **basis projection** (e.g. discretization) converts the PDE into a **system of coupled ODEs** of the form:

$$i\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t} = \mathbb{H}\boldsymbol{f}(t) + \boldsymbol{s}(t)$$

And the recursive relation for the solution is

$$\boldsymbol{f}(t+\delta t) = e^{-i\mathbb{H}\delta t}\boldsymbol{f}(t) - i\int_t^{t+\delta t} e^{i(t-t')}\boldsymbol{s}(t)$$
$$\simeq e^{-i\mathbb{H}\delta t}\left[\boldsymbol{f}(t) - \frac{i}{2}\boldsymbol{s}(t)\delta t\right] + -\frac{i}{2}\boldsymbol{s}(t+\delta t)\delta t\,.$$

How to compute the action of the **non-Hermitian** evolution operator on a vector?

Faber polynomial expansion!

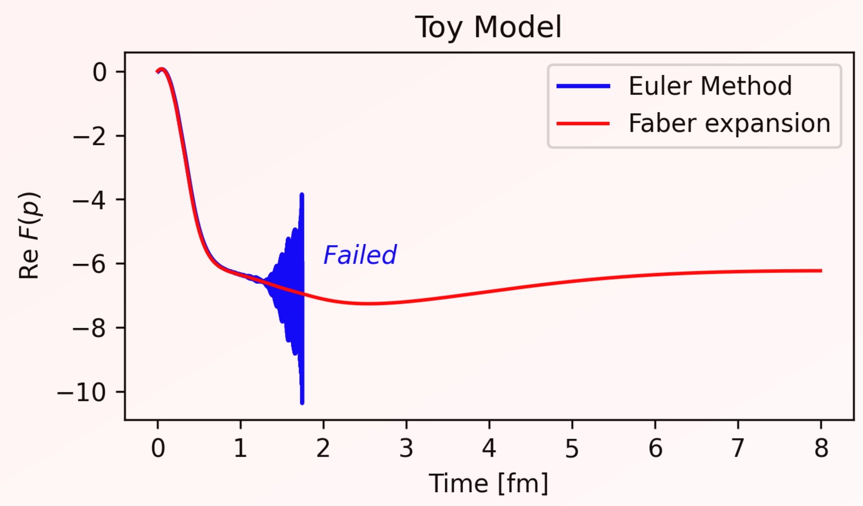
The Faber expansion [3] of the evolution operator is

$$e^{-i\mathbb{H}\delta t}\boldsymbol{f}(t) \simeq \sum_n c_n(\delta t)[F_n(\mathbb{H})\boldsymbol{f}(t)]$$

Depends on **spectral properties** of H

Computed w/ **recursion relation** of Faber pol.

Provides a **very stable** time evolution!



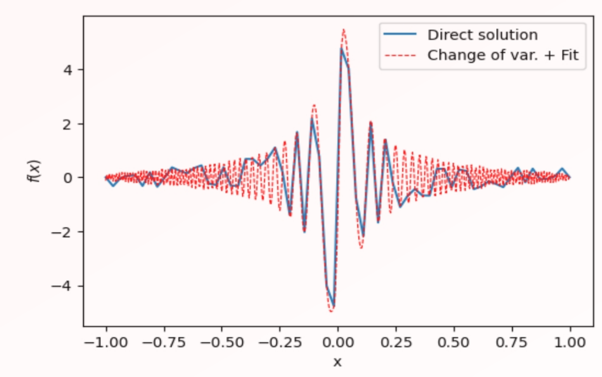
What about source term **oscillations**?

Consider $s(x,t) = k(x,t)\, e^{i\beta(t)x^2}$ — non-oscillatory

Change of variable: $G(x,t) = F(x,t)e^{i\beta(t)x^2}$ cancels oscillations

Proposed strategy:

- **solve** the PDE for $\boldsymbol{G(x,t)}$
- **fit** the function G (e.g. with **Neural Networks**)
- **multiply by oscillating factor** and get results with arbitrary precision.



Challenges:

- **time dependent** Hamiltonian
- $G(x,t)$ may be **hard to fit**

work in progress...

First results

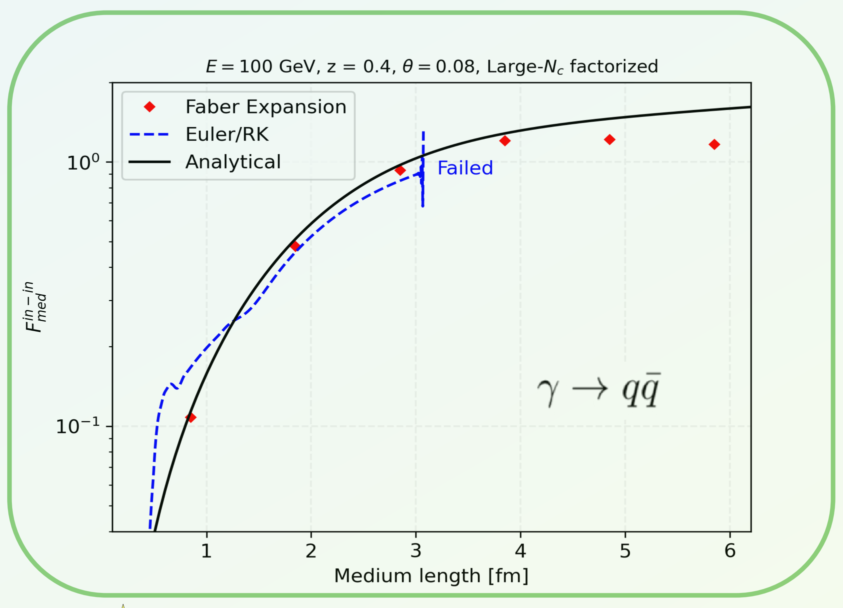
Modification factor can be splitted in

$$F_{\mathrm{med}} = F^{\mathrm{in-in}}_{\mathrm{med}} + F^{\mathrm{in-out}}_{\mathrm{med}}$$

Analytical for large Nc factorized

100% Analytical for HO

Test of time evolution (**medium length**) of large Nc fac. with Faber vs Euler/RK, for $\gamma \to q\bar{q}$



Excellent time stability!

Future Work

- **Improve** the Neural Network **fitter** for the PDE solution pre-oscillations
- **Develop** a mechanism to treat **high dimensionality** more efficiently
- **Accelerate** the algorithm with CUDA in C++
- **Obtain results** for other QCD vertices
- **Go beyond** the HO approximation for the interaction potential

[1] arXiv:hep-ph/9607355
[2] arXiv:2303.12119
[3] arXiv:physics/0512074

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