Symmetry breaking and magnetic fields

Tanmay Vachaspati

Cosmology Initiative

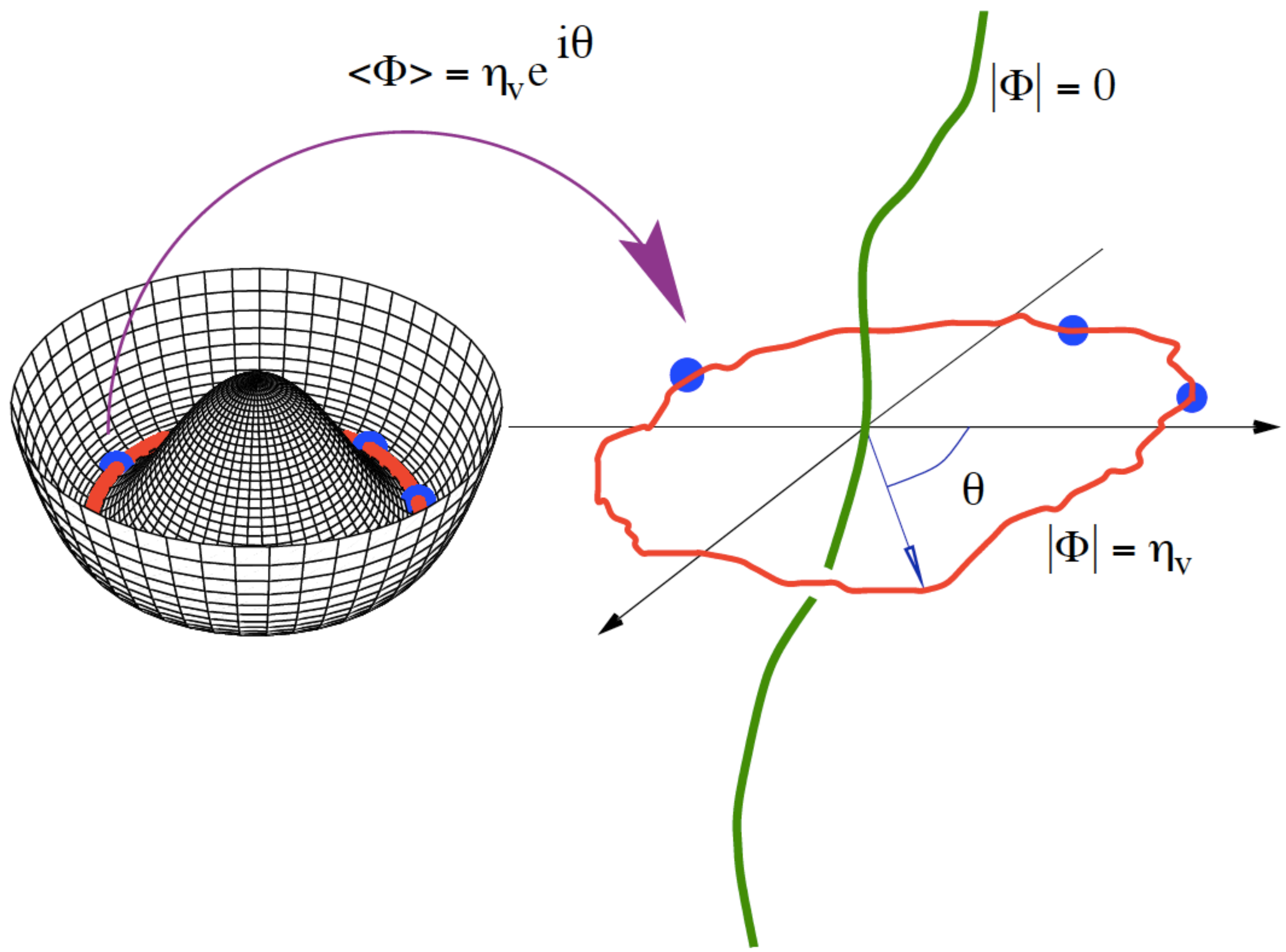
Arizona State University

Bernoulli Center; 12 April 2024

(Check out 2010.10525)

Spontaneous symmetry breaking and defects

An example — strings.



Non-trivial topology implies zeros of the order parameter but does not imply existence of solutions or the stability of solutions.

String formation and evolution

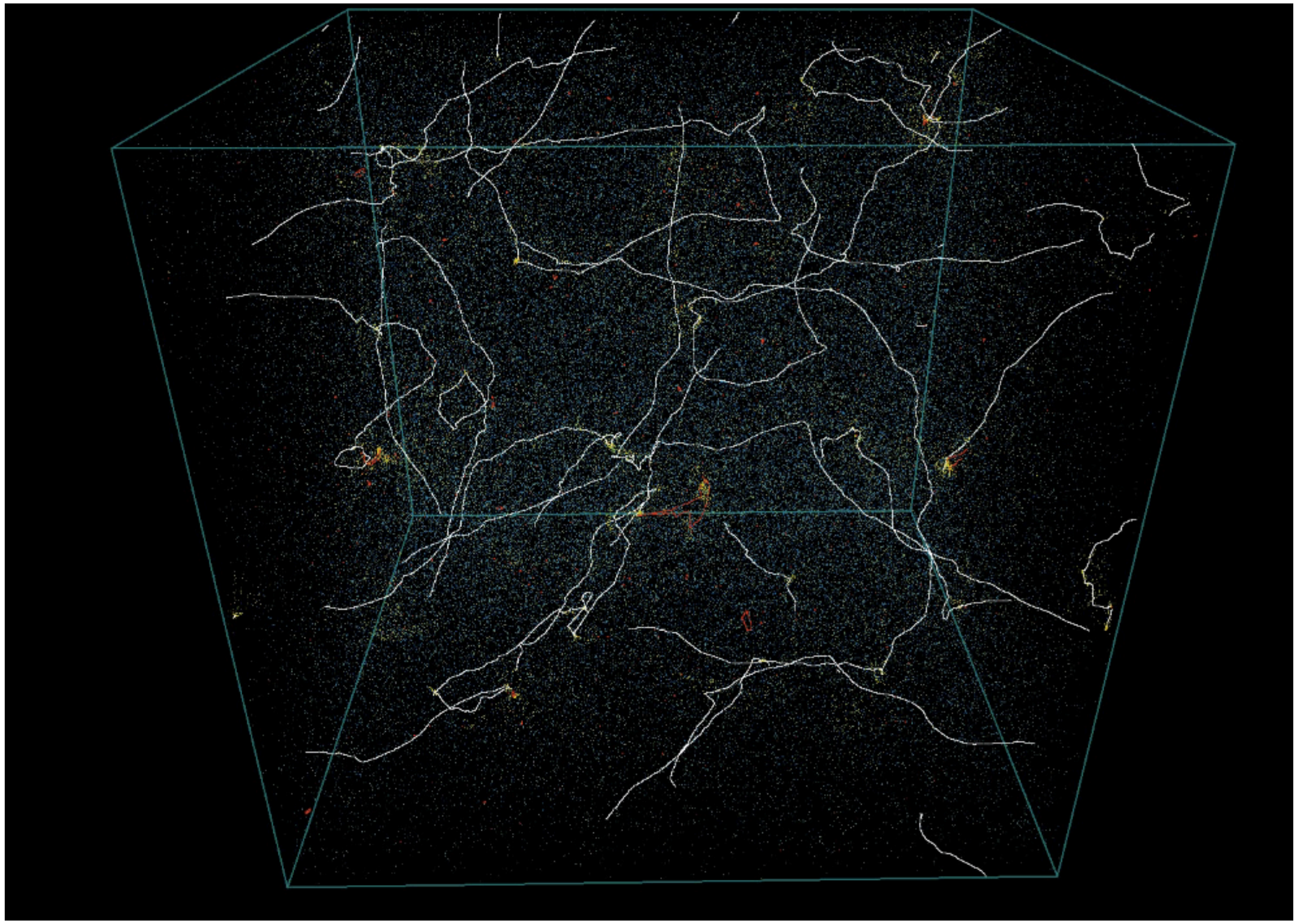


Image provided by Christoph Ringeval

Key feature: most of the string length (~80% at formation) is in infinite strings. If only small loops were formed, strings would rapidly decay and not survive.

TV & Vilenkin, 1984

Electroweak symmetry breaking

Order parameter:

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Higgs field

Vacuum manifold:

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$$

Hopf parametrization:

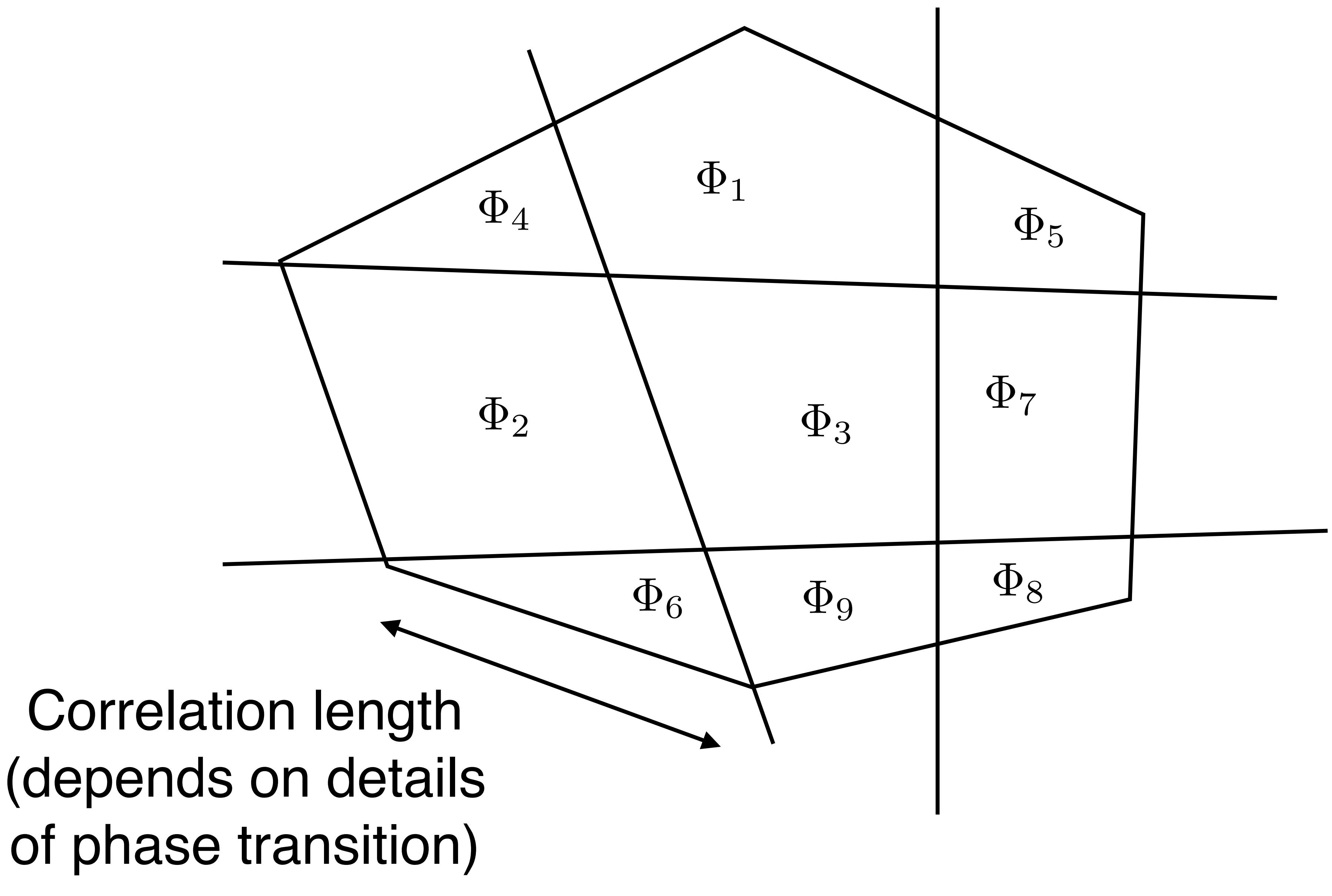
$$\Phi = \eta \begin{pmatrix} \cos\alpha \; e^{i\beta} \\ \sin\alpha \; e^{i\gamma} \end{pmatrix}$$

“angular coordinates on a three-sphere”

Kibble mechanism

Kibble, 1976

Finite size domains of ~constant order parameter.



Topology

$$\langle \Phi \rangle \in S^3$$

$\pi_1(S^3) = 1, \quad \pi_2(S^3) = 1$ but consider: $\hat{n} = -\frac{\Phi^\dagger \vec{\sigma} \Phi}{\Phi^\dagger \Phi}$

The n-vector is also distributed in domains. But it lives on a two-sphere. Wrappings of n-vector on the two-sphere give zeros of Phi and carry electromagnetic magnetic charge (magnetic monopoles).

Kephart & TV, 1996; Patel & TV, 2022

Composite nature of n-vector implies that monopoles are connected to anti-monopoles by Z-strings.

Nambu, 1977

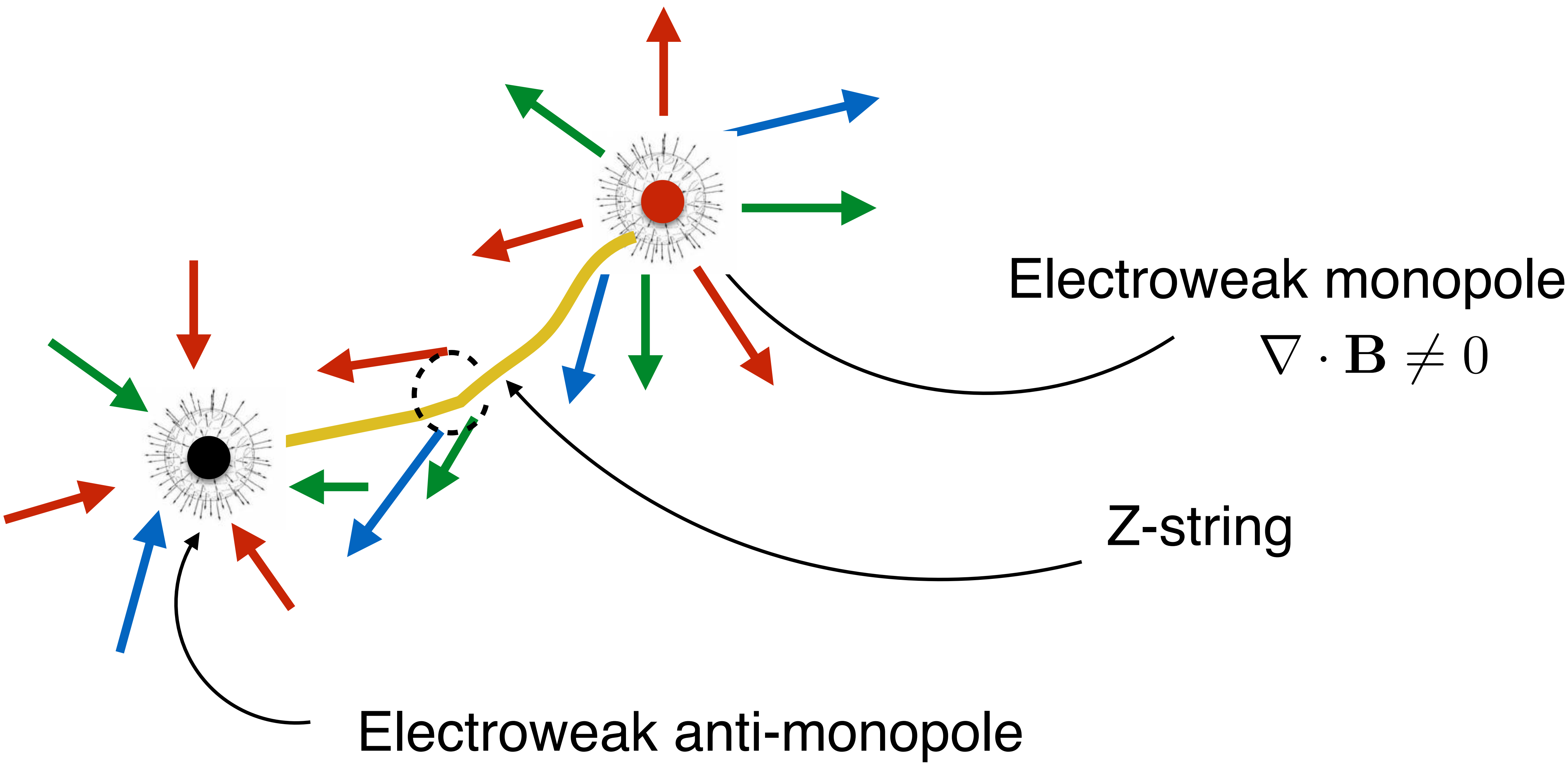
Vacuum manifold is better thought of as $S^2 \times S^1$ (Hopf fibered S^3).

TV & Achucarro, 1991

Gibbons, Ortiz, Ruiz Ruiz & Samols, 1992

Electroweak Dumbbells

Nambu, 1977



Arrows indicate points on S^2, colors indicate points on S^1.

Electromagnetism

Unbroken symmetry (electromagnetism) generator Q is given by:

$$Q\Phi = 0$$

Associated gauge field is the electromagnetic gauge field,

$$A_\mu = \sin\theta_w \hat{n}^a W^a_\mu + \cos\theta_w Y_\mu$$

What is the field strength?

't Hooft, 1974

Two guiding principles — definition should be gauge invariant and definition should reduce to usual Maxwell definition in "unitary gauge" (Phi=constant).

$$A_{\mu\nu} \stackrel{?}{=} \sin\theta_w \partial_\mu(\hat{n}^a W^a_\nu) + \cos\theta_w \partial_\mu Y_\nu - (\mu \leftrightarrow \nu)$$

not gauge invariant

$$A_{\mu\nu} \stackrel{?}{=} \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu}$$

doesn't reduce to Maxwell in unitary gauge

Magnetic field definition

TV, 1991

$$A_{\mu\nu} = \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu} - i\frac{2\sin\theta_w}{g\eta^2}(D_\mu\Phi^\dagger D_\nu\Phi - D_\nu\Phi^\dagger D_\mu\Phi)$$

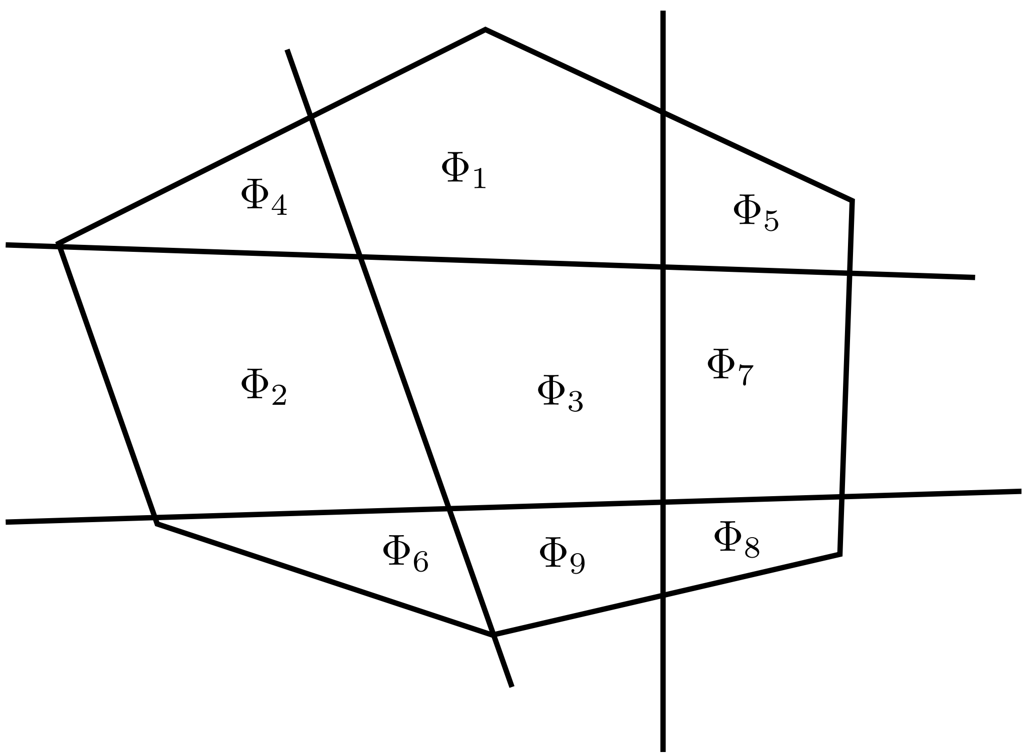
$$= \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g\eta^2}(\partial_\mu\Phi^\dagger\partial_\nu\Phi - \partial_\nu\Phi^\dagger\partial_\mu\Phi) \qquad (|\Phi| = \eta)$$

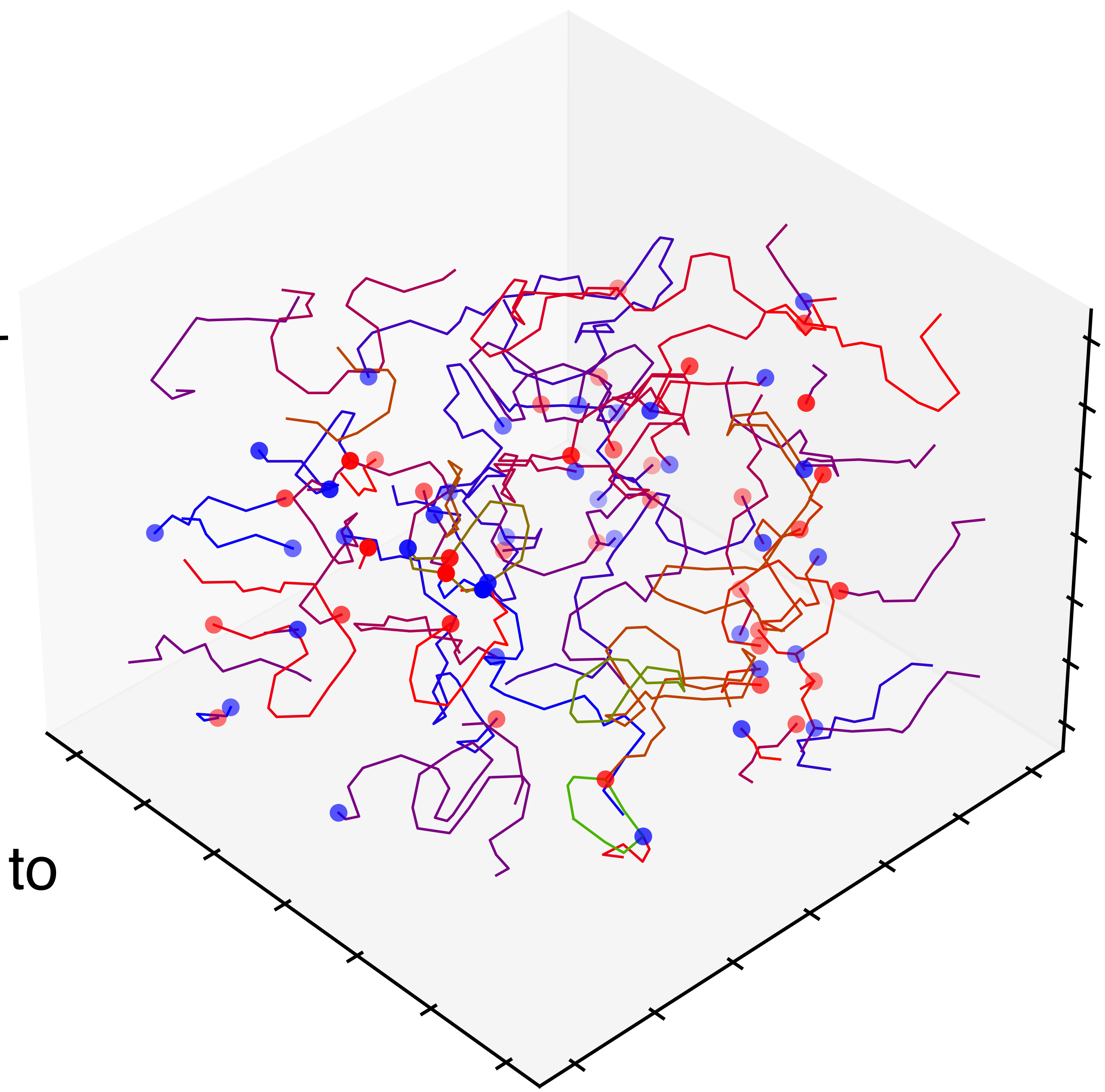
$$\mathbf{B} = \nabla\times\mathbf{A} - i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger\times\nabla\Phi$$

Example: $\Phi = \eta\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{i\phi}\end{pmatrix}$ ⟶ $\mathbf{B} \sim \frac{\hat{r}}{r^2}$

Magnetic charge distribution

Teerthal Patel & TV, 2021





(Kibble simulation applied to the standard model)

Dumbbells at large Weinberg angle, small Higgs mass

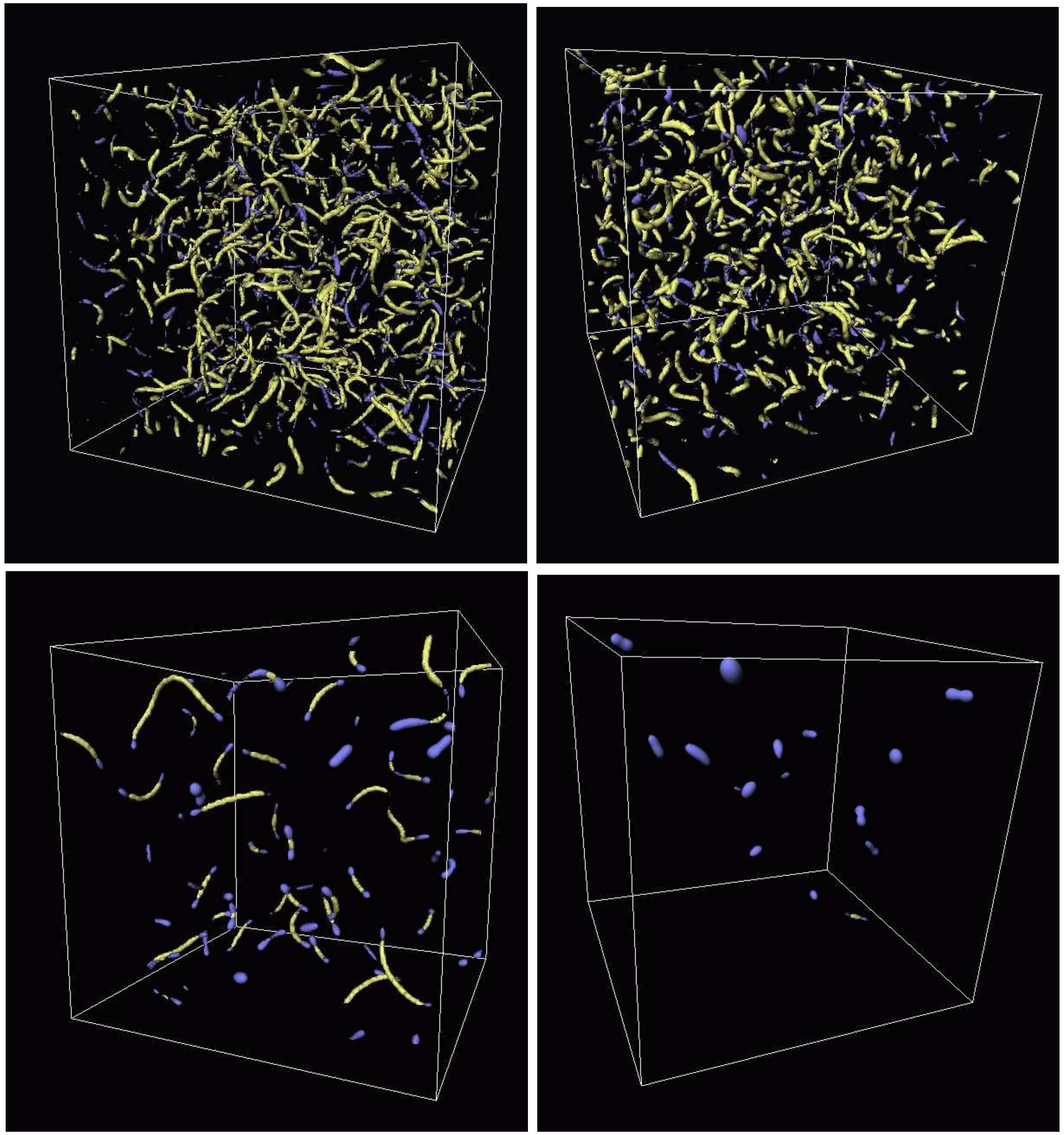
Urrestilla, Achucarro, Borrill & Liddle, 2002

Standard model but with:

$m_H \lesssim m_Z$

$\sin^2 \theta_w \approx 0.995$

yellow=Z magnetic
blue=A "magnetic" (w/o Higgs term)



Scaling of B from monopoles

TV, 2021

Monopole contribution: $\mathbf{B} \to -i\frac{2\sin\theta_w}{g}\nabla\hat{\Phi}^\dagger \times \nabla\hat{\Phi}$

Volume-averaged magnetic field:

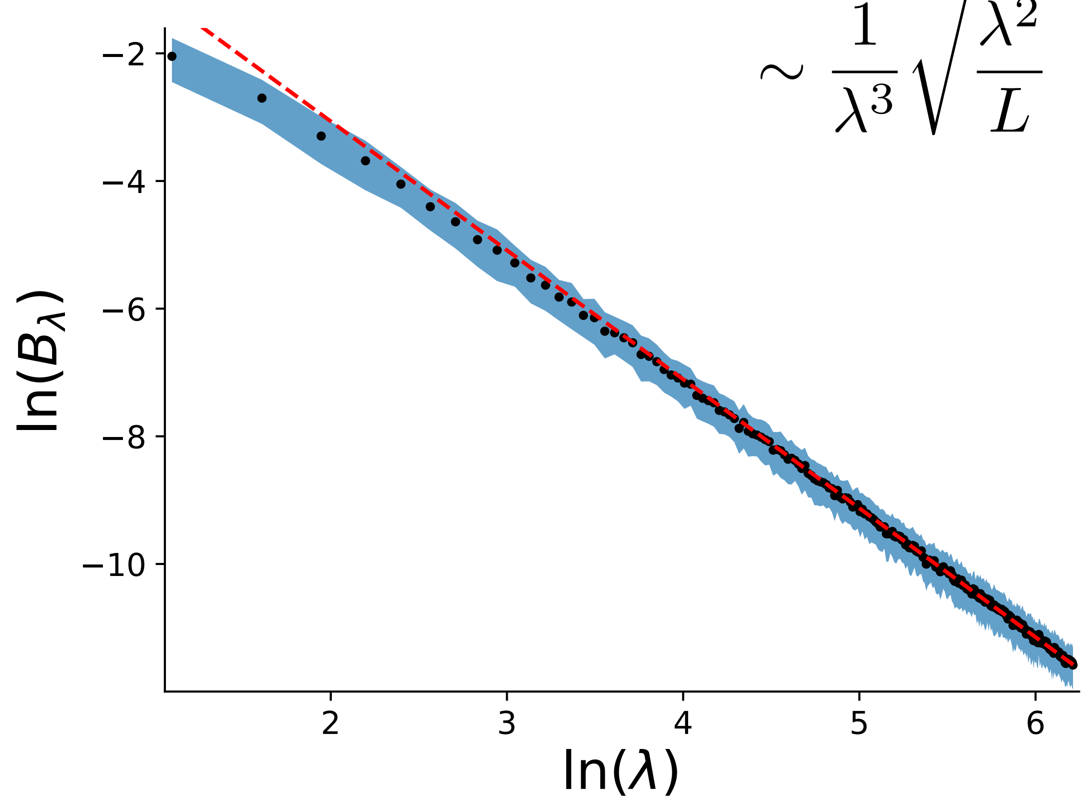
$$\langle \mathbf{B}\rangle_V = \frac{1}{V}\int_V d^3x\,\mathbf{B} = -i\frac{2\sin\theta_w}{gV}\int_{\partial V} d\mathbf{S}\times(\hat{\Phi}^\dagger\nabla\hat{\Phi})$$

$$\sim \frac{1}{\lambda^3}\sqrt{\frac{\lambda^2}{L}}$$

volume~$\boldsymbol{\lambda}^3$

area~$\boldsymbol{\lambda}^2$

domain size~L



$$\langle \mathbf{B}\rangle_{\lambda^3} \propto \frac{1}{\lambda^2}$$

Power spectrum from monopoles

"power spectrum" "helicity power spectrum"

$$\langle b_i(\mathbf{k}) b_j^*(\mathbf{k}')\rangle = \left[\frac{E_M(k)}{4\pi k^2} p_{ij} + i\epsilon_{ijl} k^l \frac{H_M(k)}{8\pi k^2}\right] \times (2\pi)^6 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

Monin & Yaglom, 1975

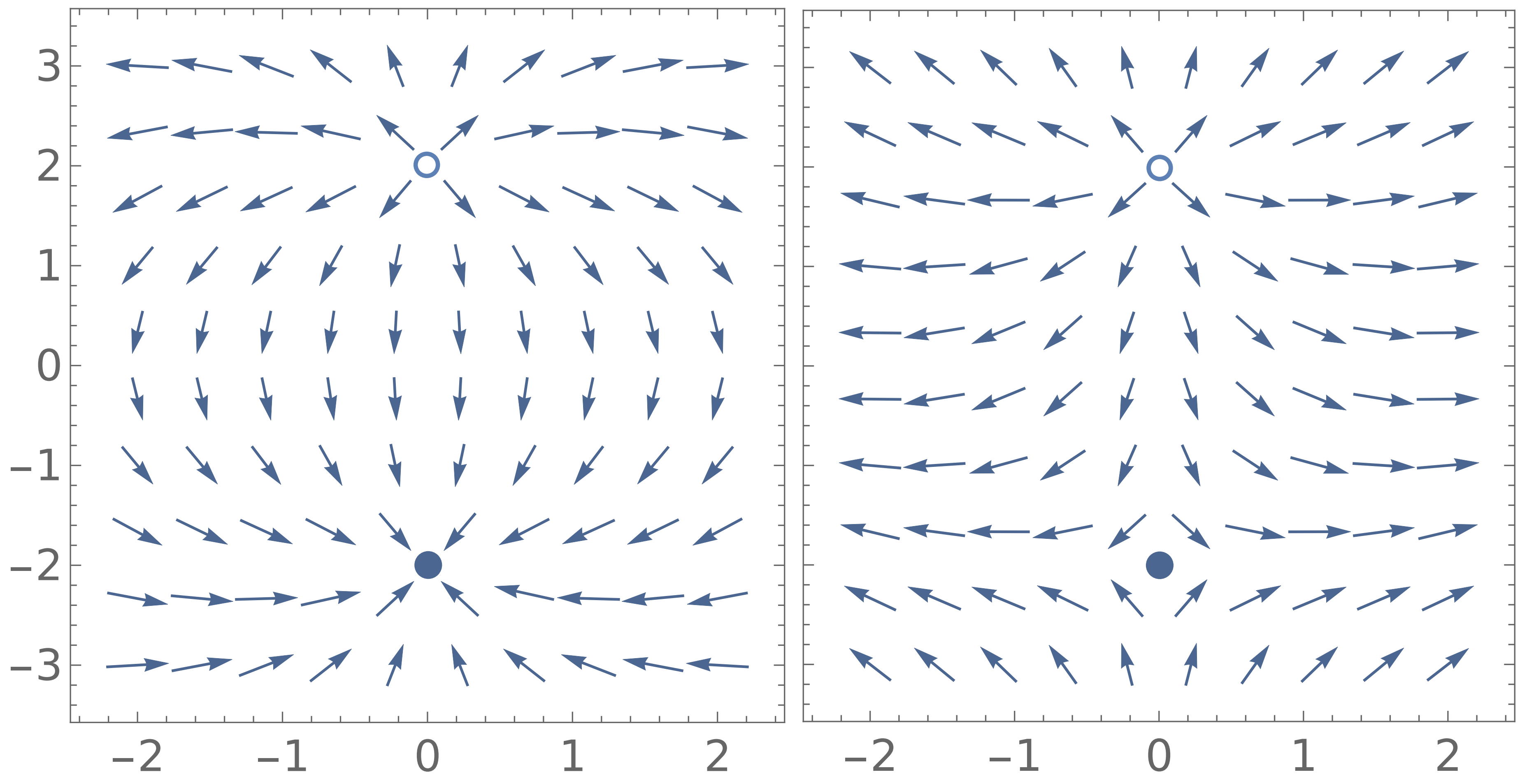
$$E_M(k) \sim \frac{1}{k} B_{V,\lambda}^2 \propto k^3$$

$$E_M(k) = \frac{4\rho_{\mathrm{EM},B}}{k_*}\left(\frac{k}{k_*}\right)^3, \quad k \leq k_*$$

Twisted dumbbells

(plots of n-vector in a plane)

Ayush Saurabh & TV, 2017



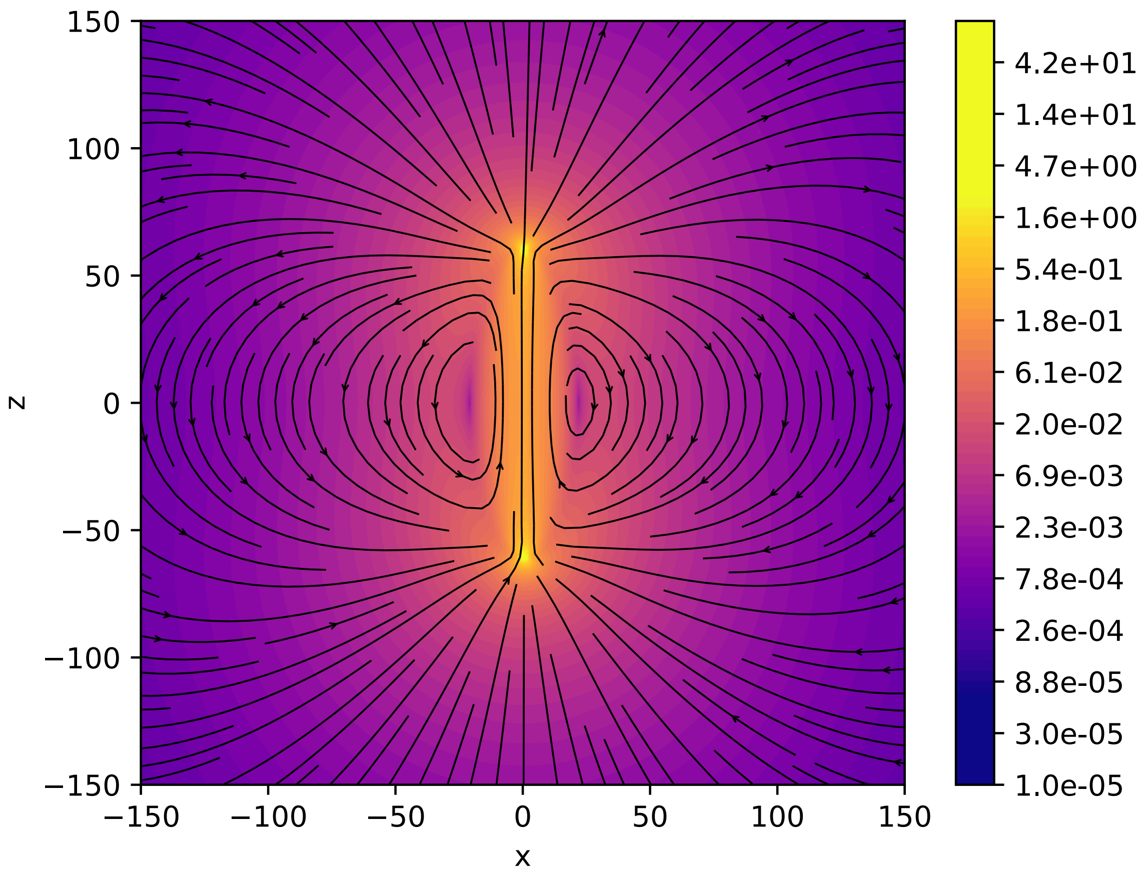
twist=pi

twist=0

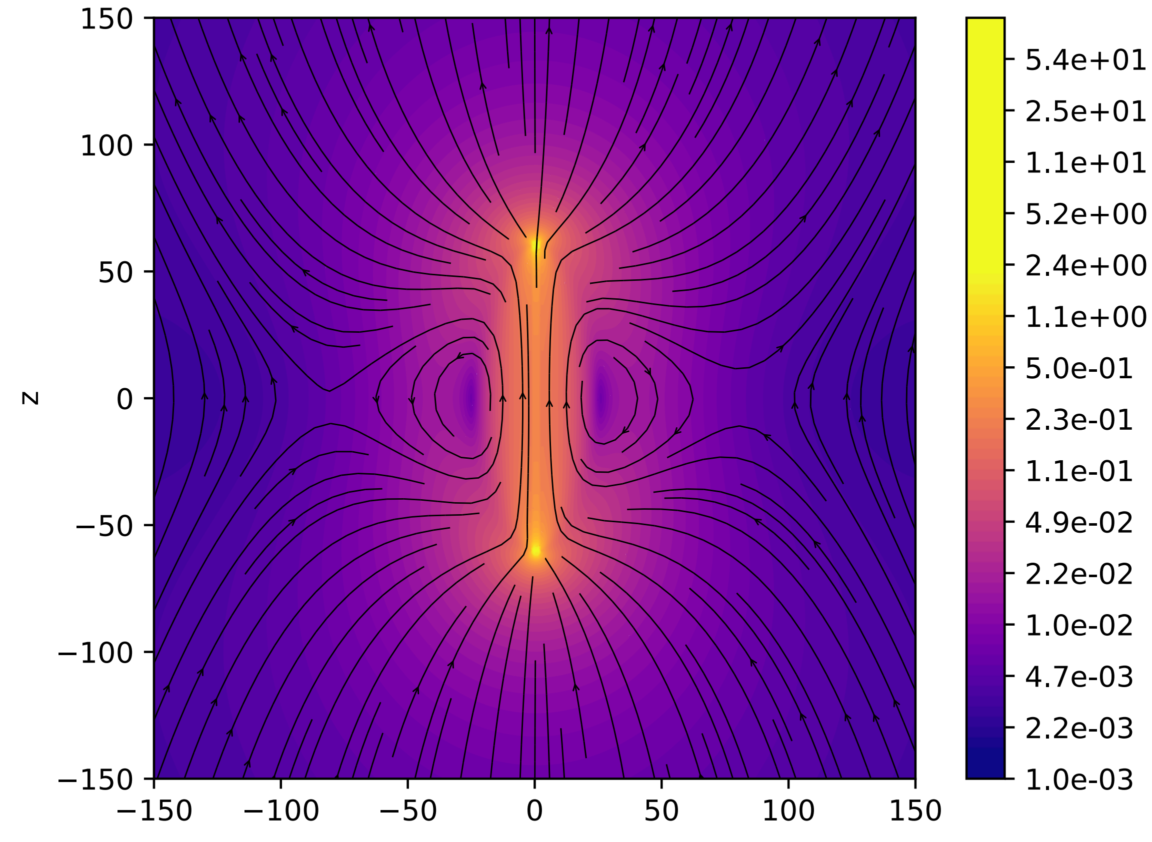
Shape of magnetic fields

Teerthal Patel & TV, 2023

Dumbbells aren’t just magnetic dipoles.



twist=0

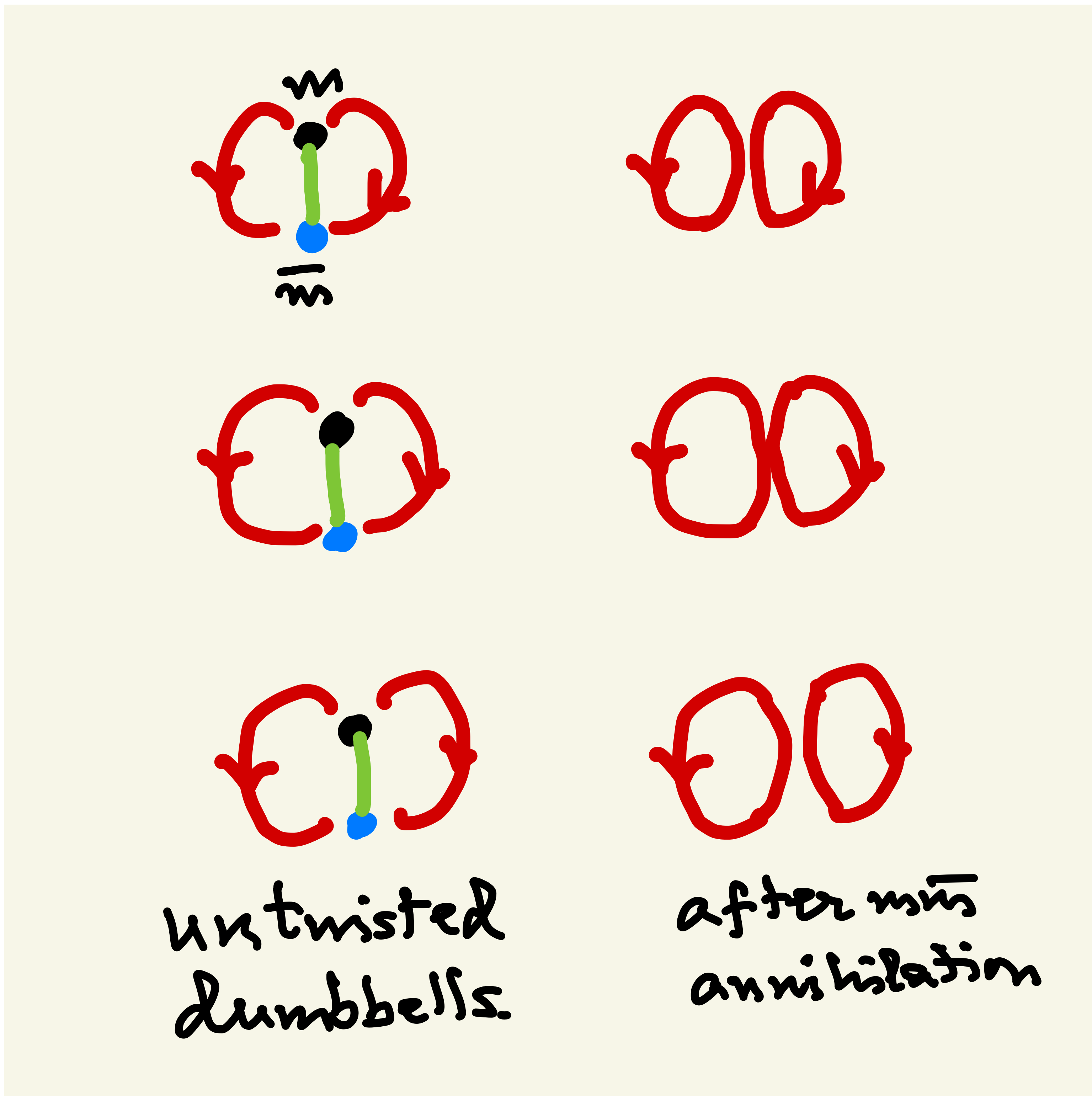


twist=pi

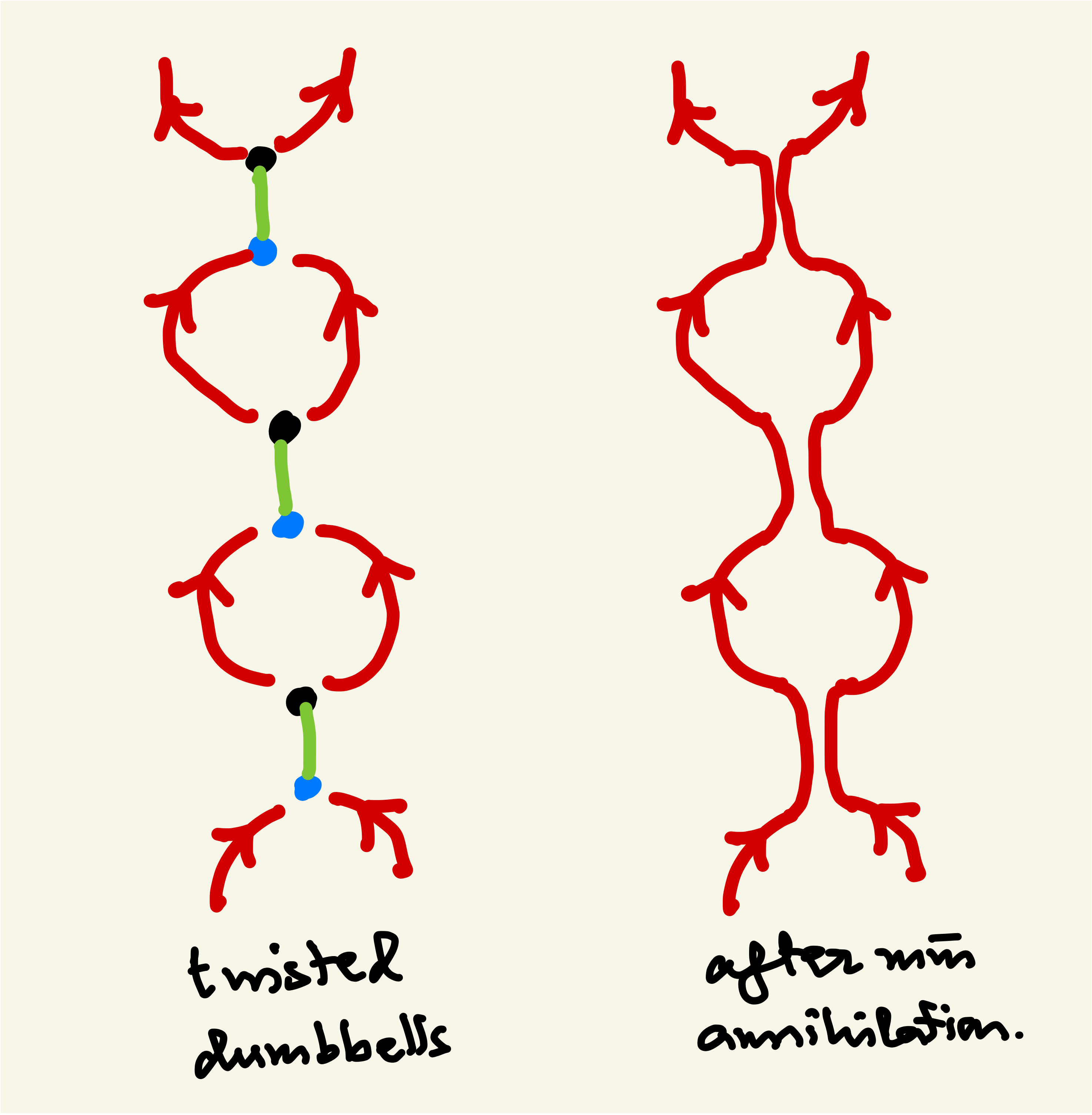
$$B \propto \frac{1}{r^3}$$

$$B \propto \frac{\cos\theta}{r^2}$$

Gas of untwisted dumbbells

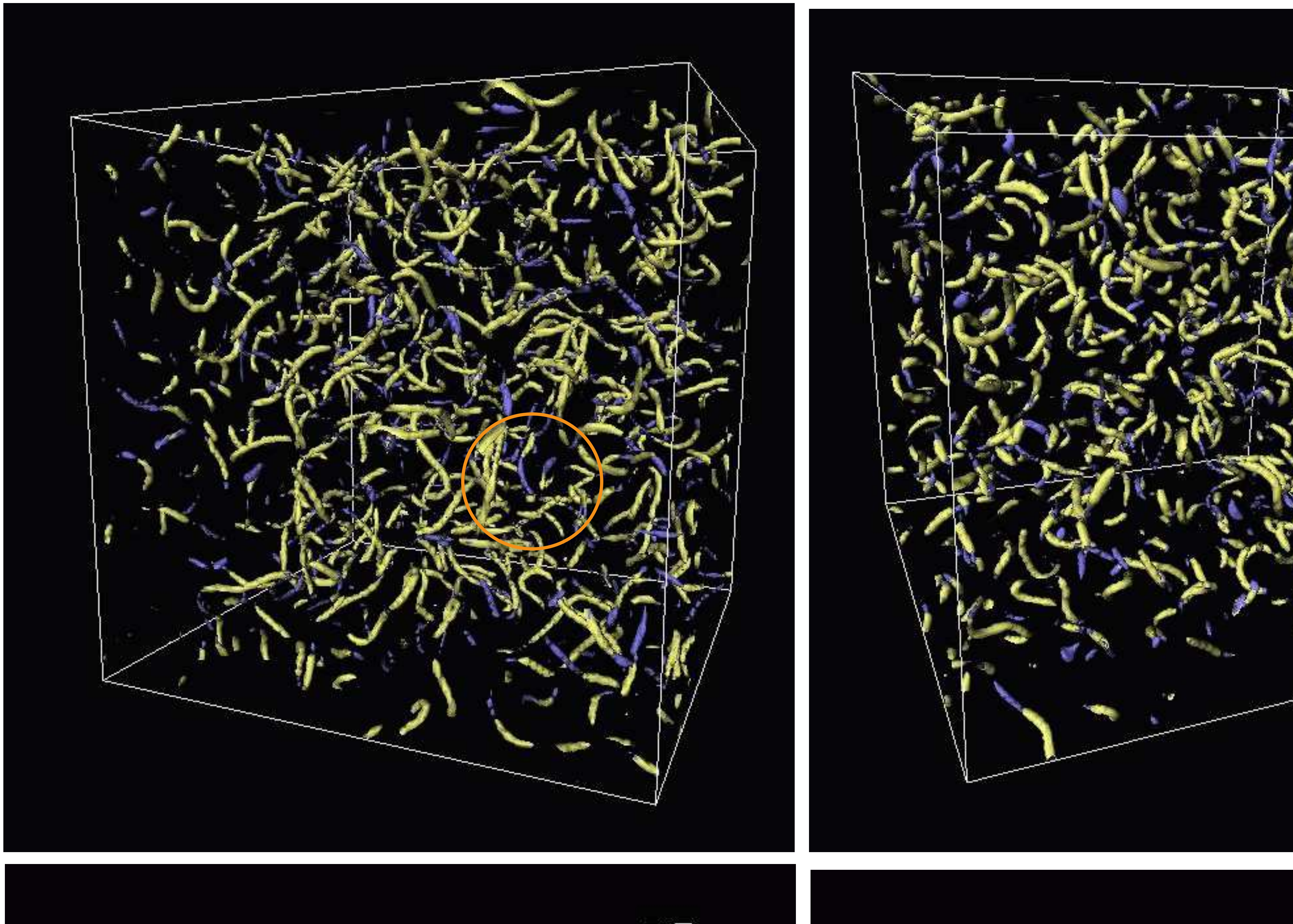


Gas of twisted dumbbells



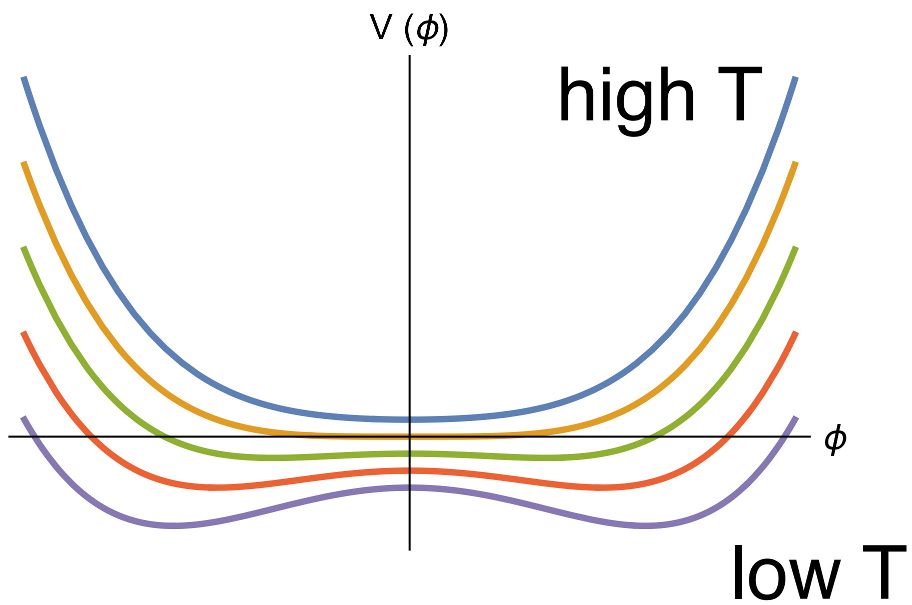
Network of magnetic fields

Urrestilla, Achucarro, Borrill & Liddle, 2002



EWSB in thermal state

Zhou-Gang Mou, Teerthal Patel, Paul Saffin & TV (ongoing)



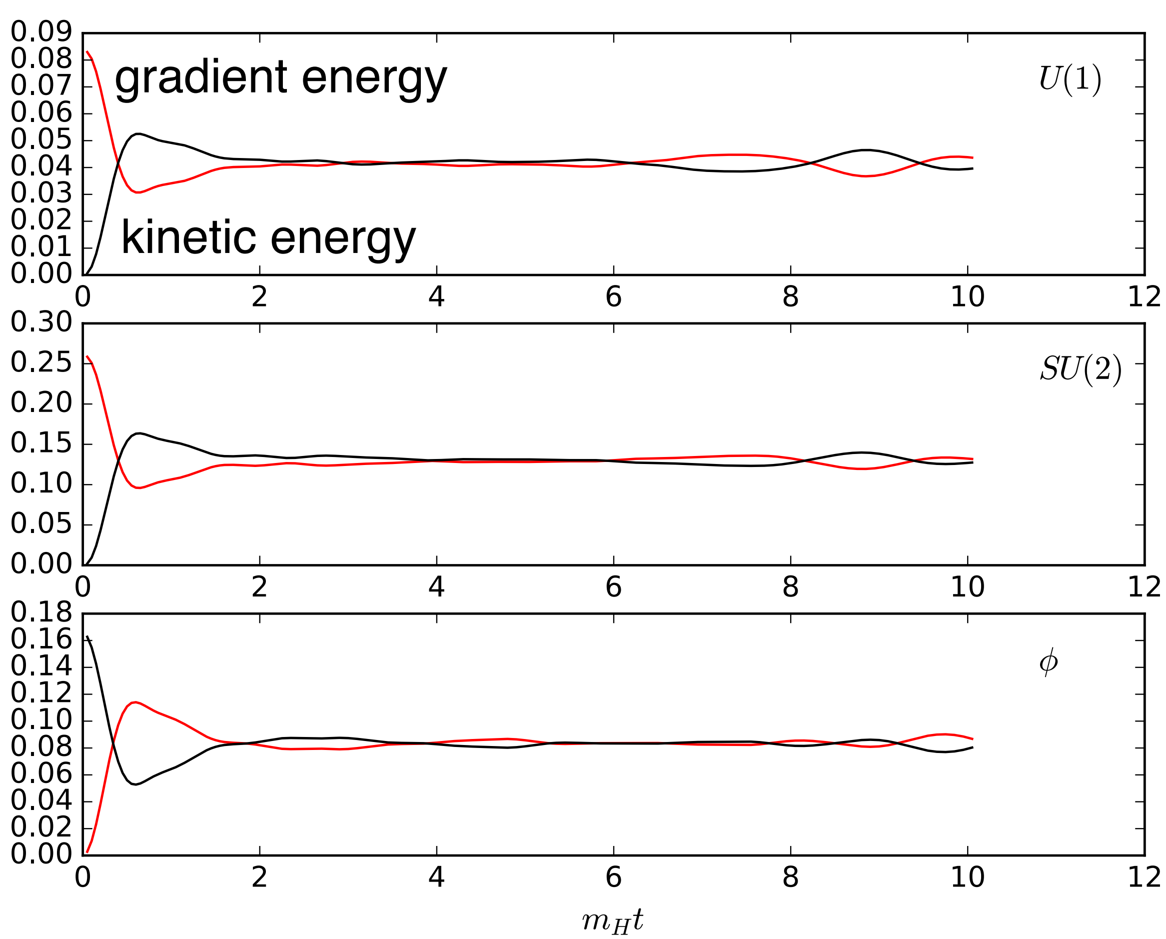
Set up “half” thermal initial conditions of Higgs and gauge fields at high temperature (T) in temporal gauge.

$$\Phi = \mathrm{Bose-Einstein\ distribution\ of\ } \Phi_{\mathbf{k}}$$

$$W_i^a = \mathrm{Bose-Einstein\ distribution\ of\ W^a_{i,\mathbf{k}}}$$

$$Y_i = \mathrm{Bose-Einstein\ distribution\ of\ Y_{i,\mathbf{k}}}$$

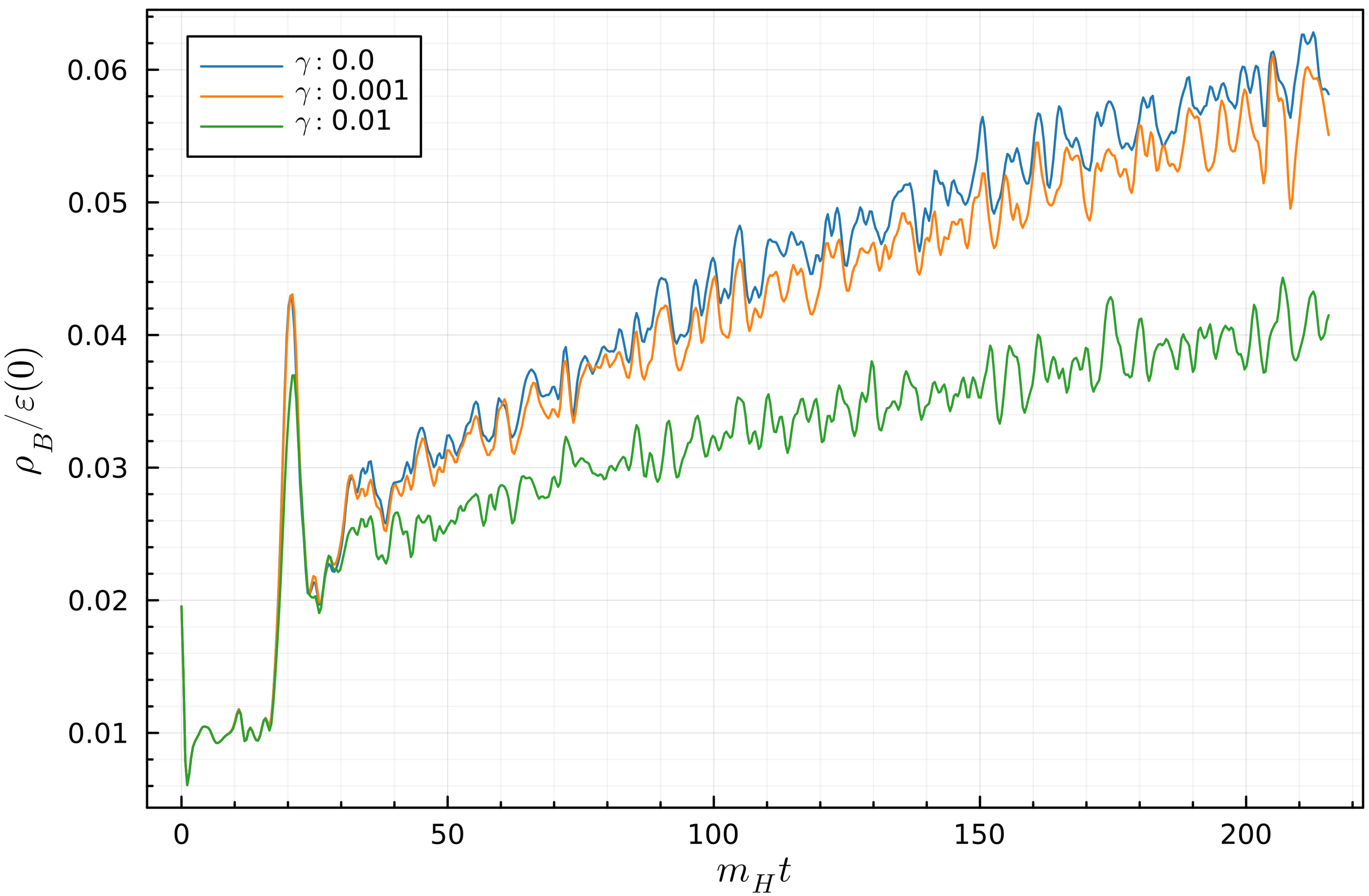
$$\dot{\Phi} = 0 = \dot{W}_i^a = \dot{Y}_i$$



Evolve so that the system completely thermalizes.

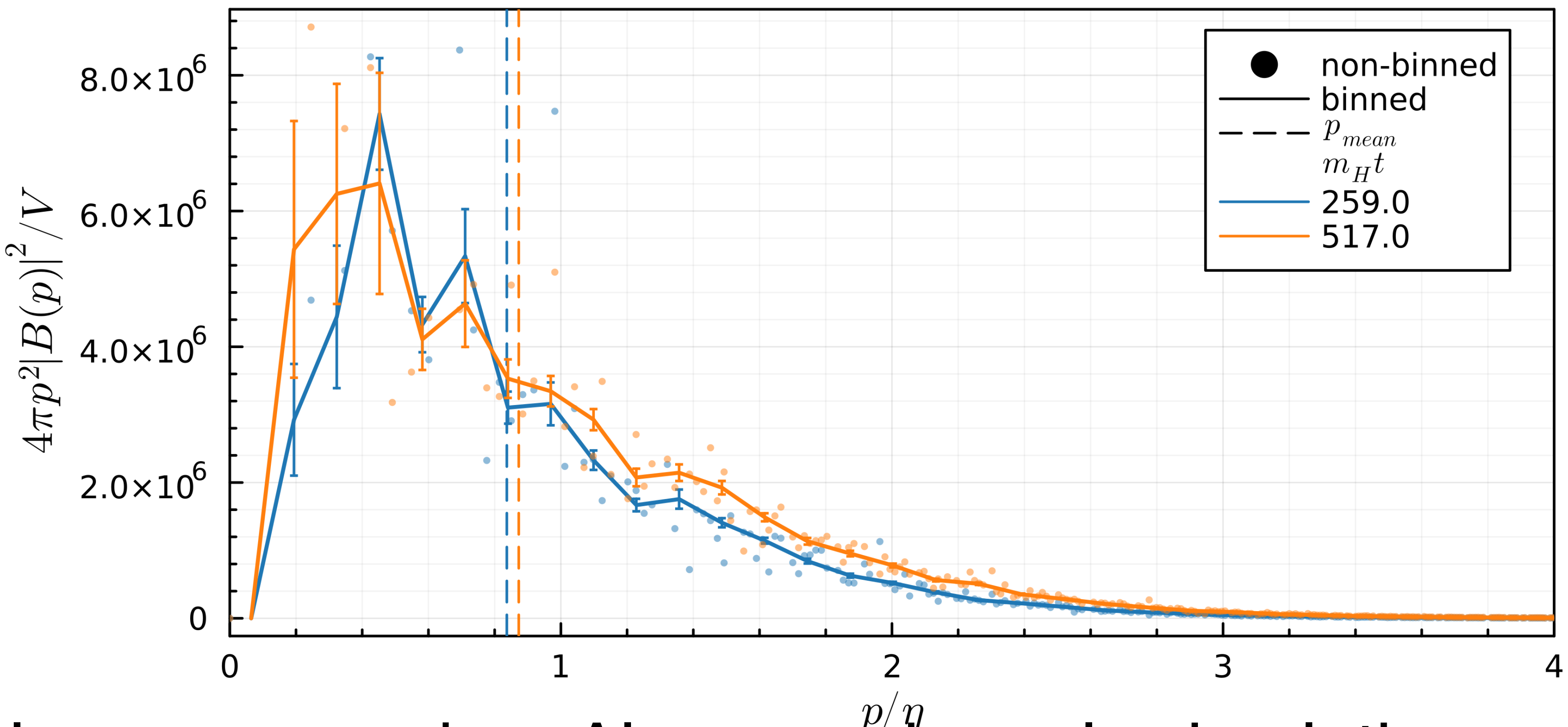
EWSB dynamics

Turn off thermal corrections to Higgs potential and evolve the equations of motion. Track the energy in electromagnetic magnetic fields for different damping parameters:



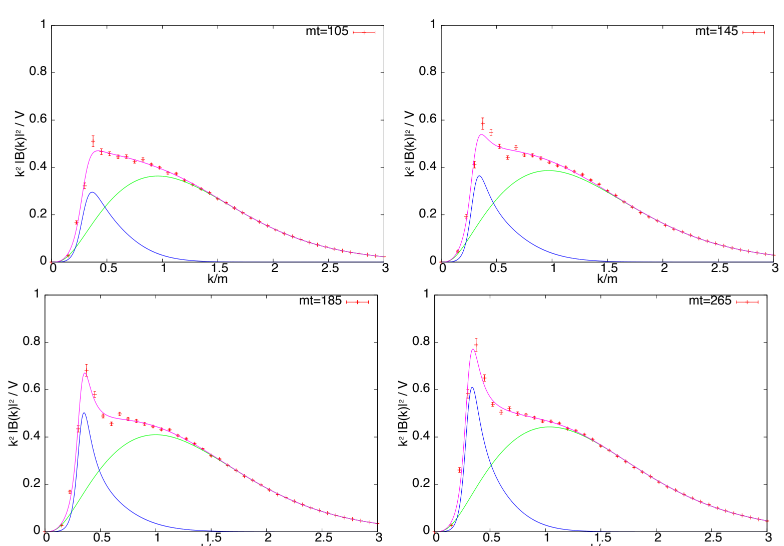
Order 5% energy density in B (but still growing).

Magnetic field power spectrum

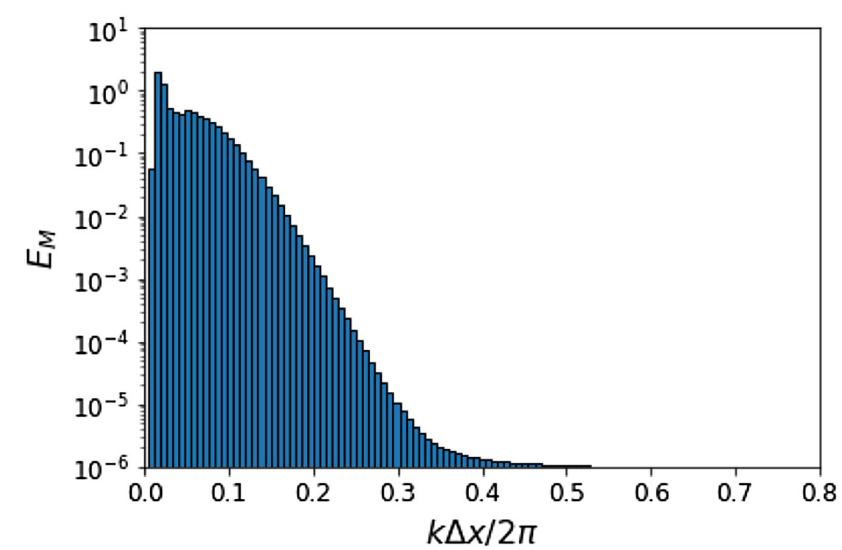


Peak at low wavenumber. Also seen in early simulations using different (non-thermal) initial conditions.

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo, 2008

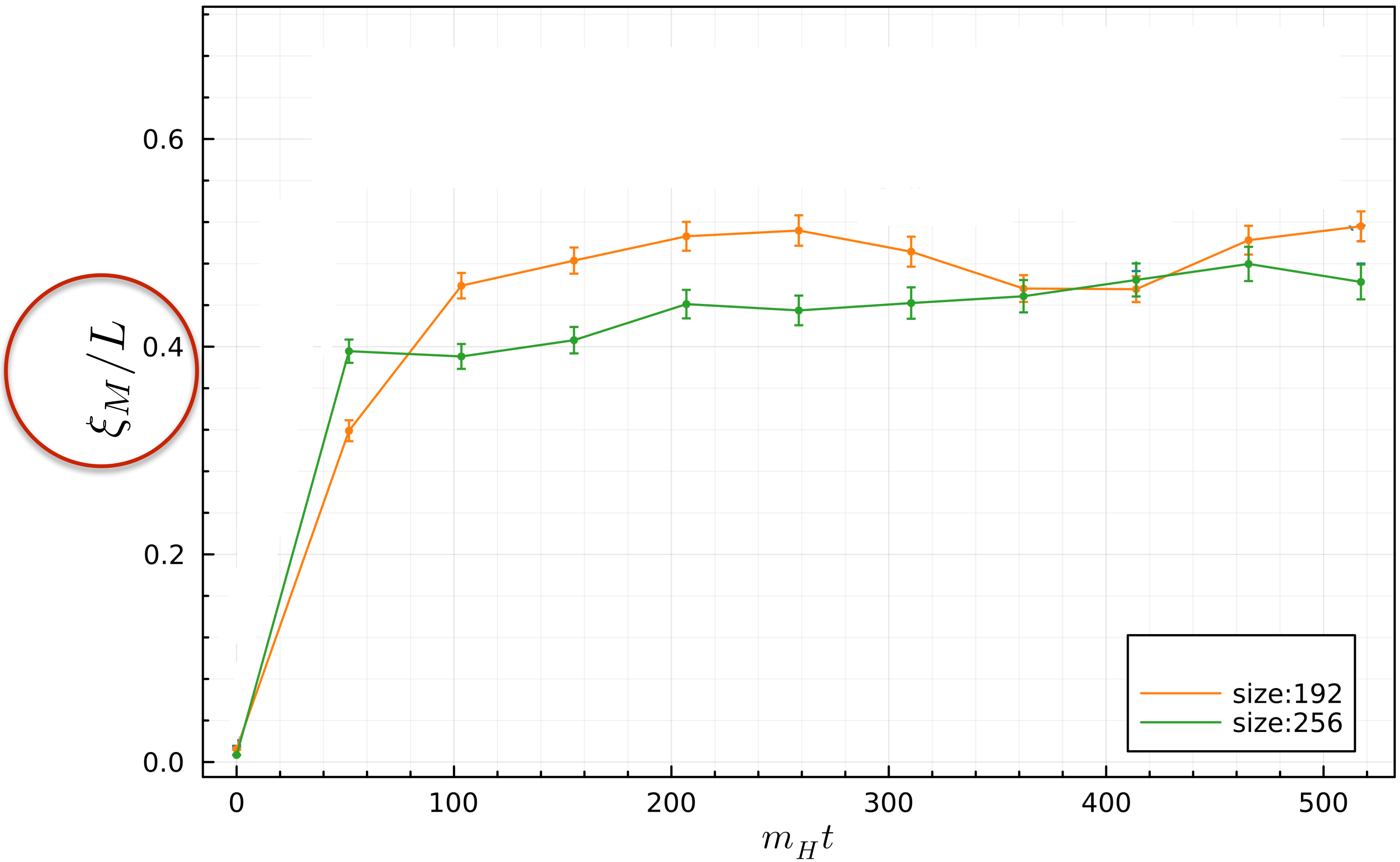


Zhang, Ferrer & TV, 2019



Coherence scale

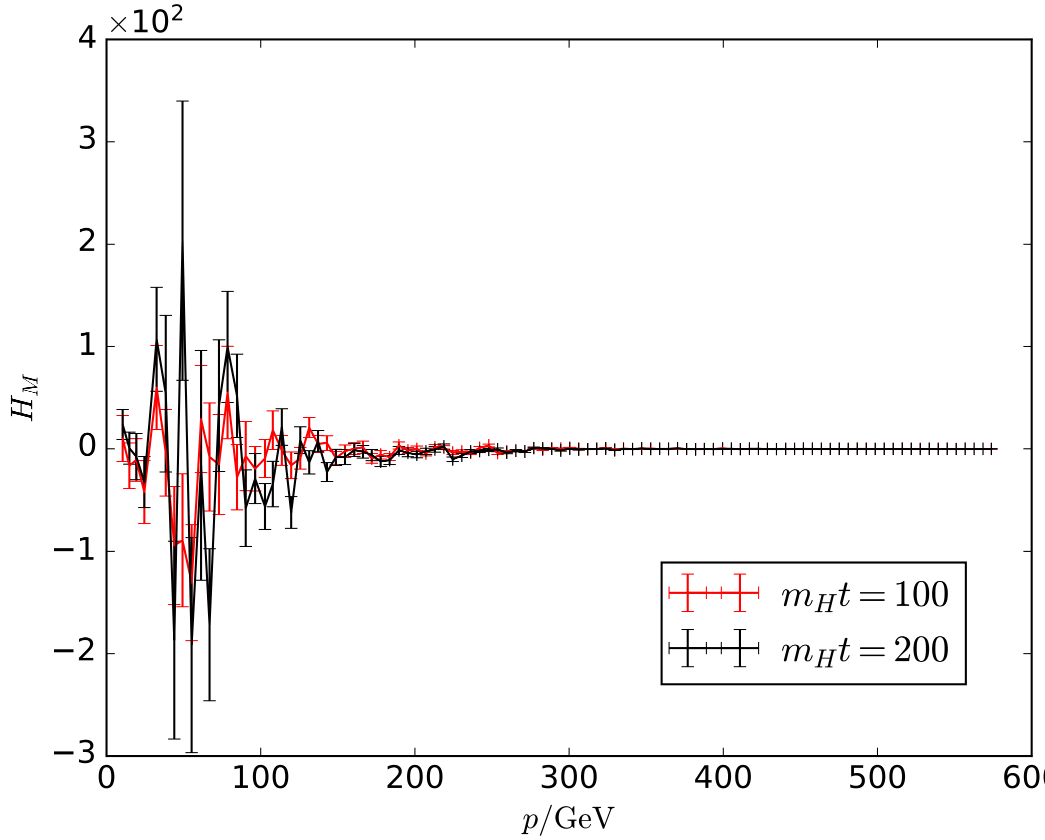
$$\xi_M = \frac{\int dk\ (2\pi/k) E_M(k)}{\int dk\ E_M(k)}$$

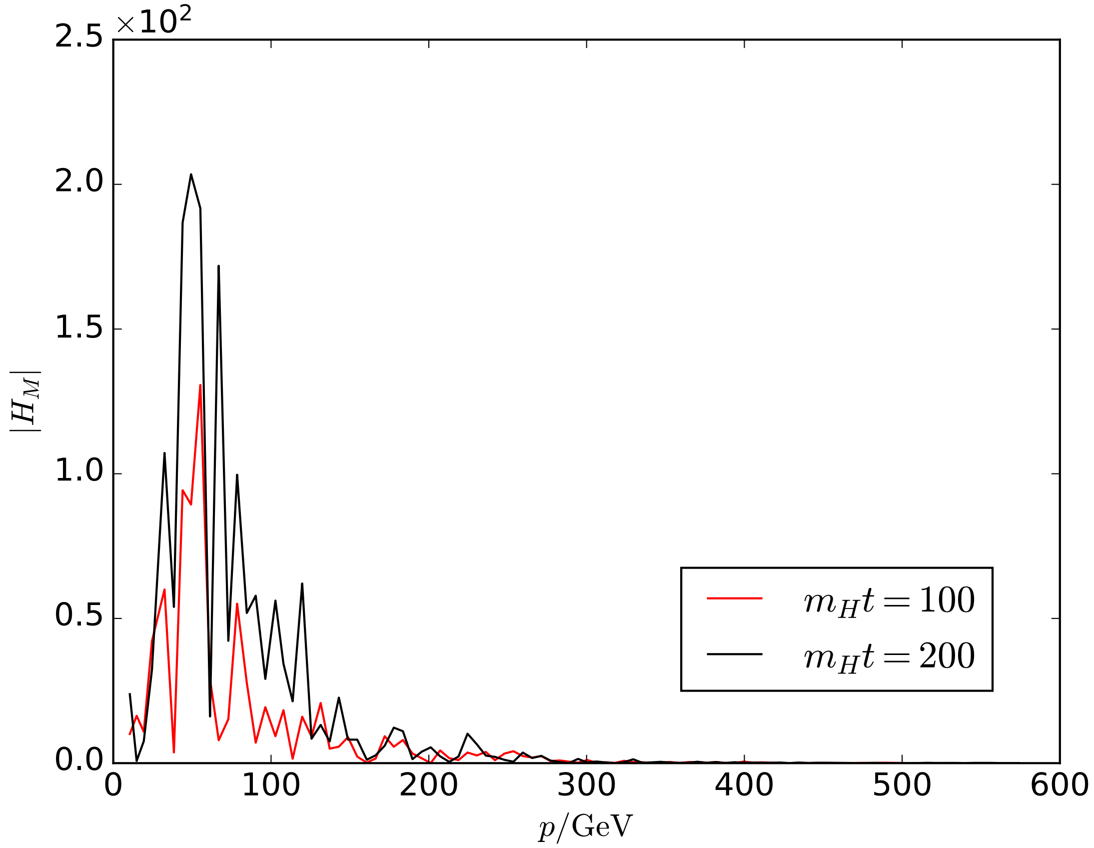


(Simulations running in yet larger volumes.)

Helicity

$$\langle b_i(\mathbf{k}) b_j^*(\mathbf{k}')\rangle = \left[\frac{E_M(k)}{4\pi k^2} p_{ij} + i\epsilon_{ijl}k^l \frac{H_M(k)}{8\pi k^2}\right] \times (2\pi)^6 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$





Magnetic Field Evolution

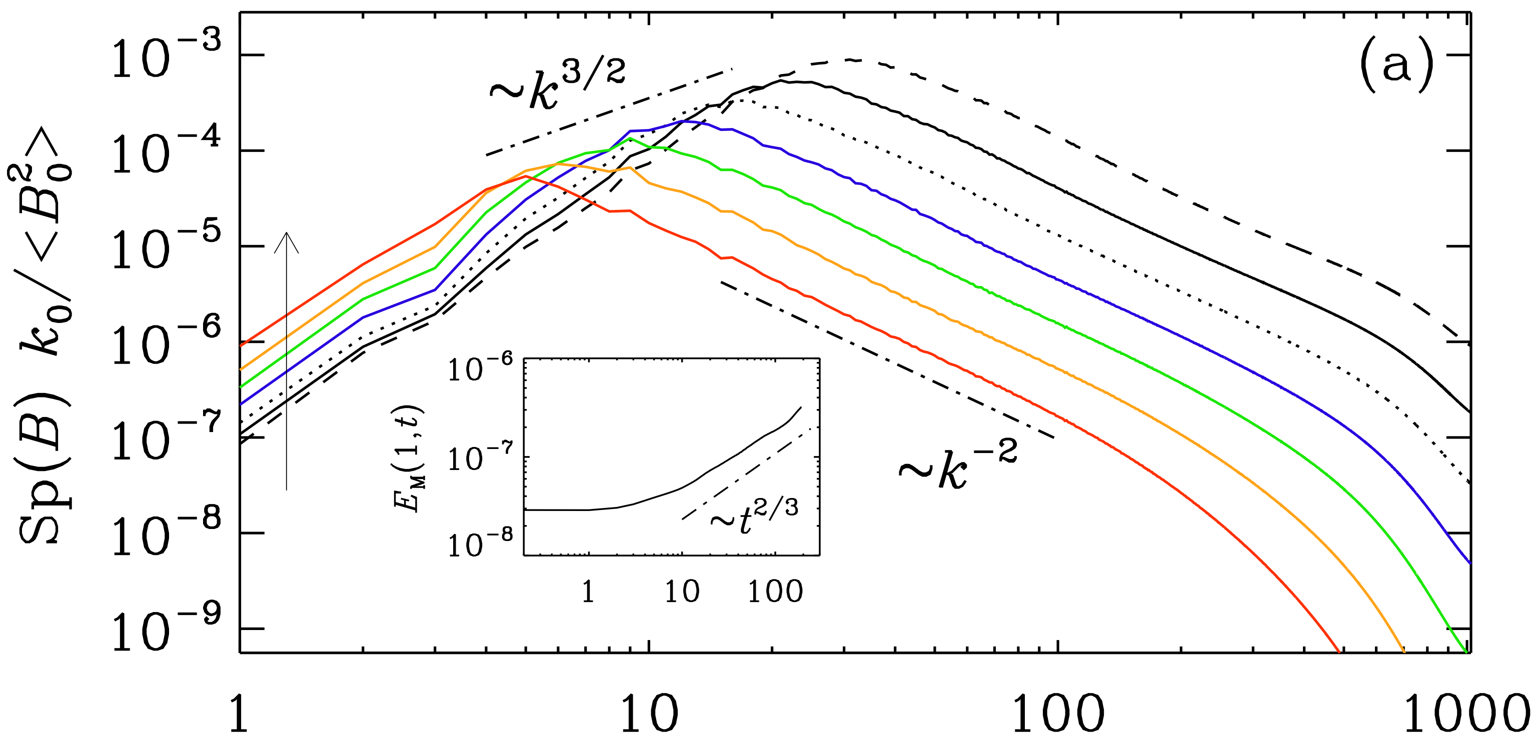
Many experts: Banerjee, Brandenburg, Hosking, Jedamzik, Kahniashvili, Schekochihin, Sigl, Subramanian,…

k^3 evolution

(Conformal variables. Non-helical.)

Hosking integral conserved.

Brandenburg, Sharma & TV, 2023



$$E_M(k,t) = \xi_M^{-\beta}\phi(\xi_M k) \sim \xi_M^{\alpha-\beta} k^\alpha \sim t^{(\alpha-\beta)q} k^\alpha$$

Brandenburg & Kahniashvili, 2017
Olesen, 1997

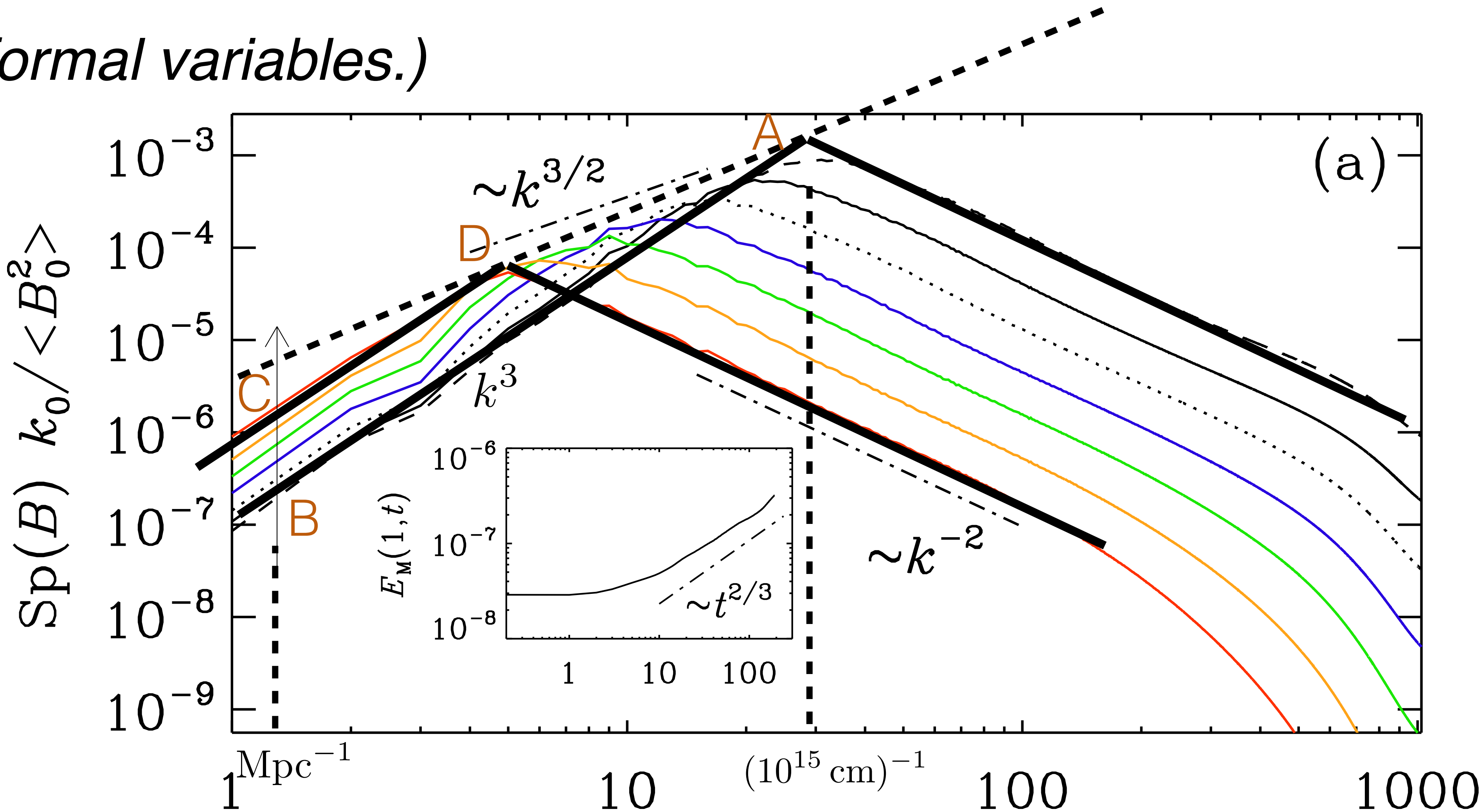
$$\alpha = 3 \text{ (spectrum)},\ \beta = 3/2 \text{ (simulation)},\ q = 2/(\beta+3) = 4/9 \text{ (self}-\text{similarity)}$$

$$\Longrightarrow\ E_M(1,t) \propto t^{2/3} \text{ growth}$$

("t" is conformal time.)

Estimating present B

(Conformal variables.)



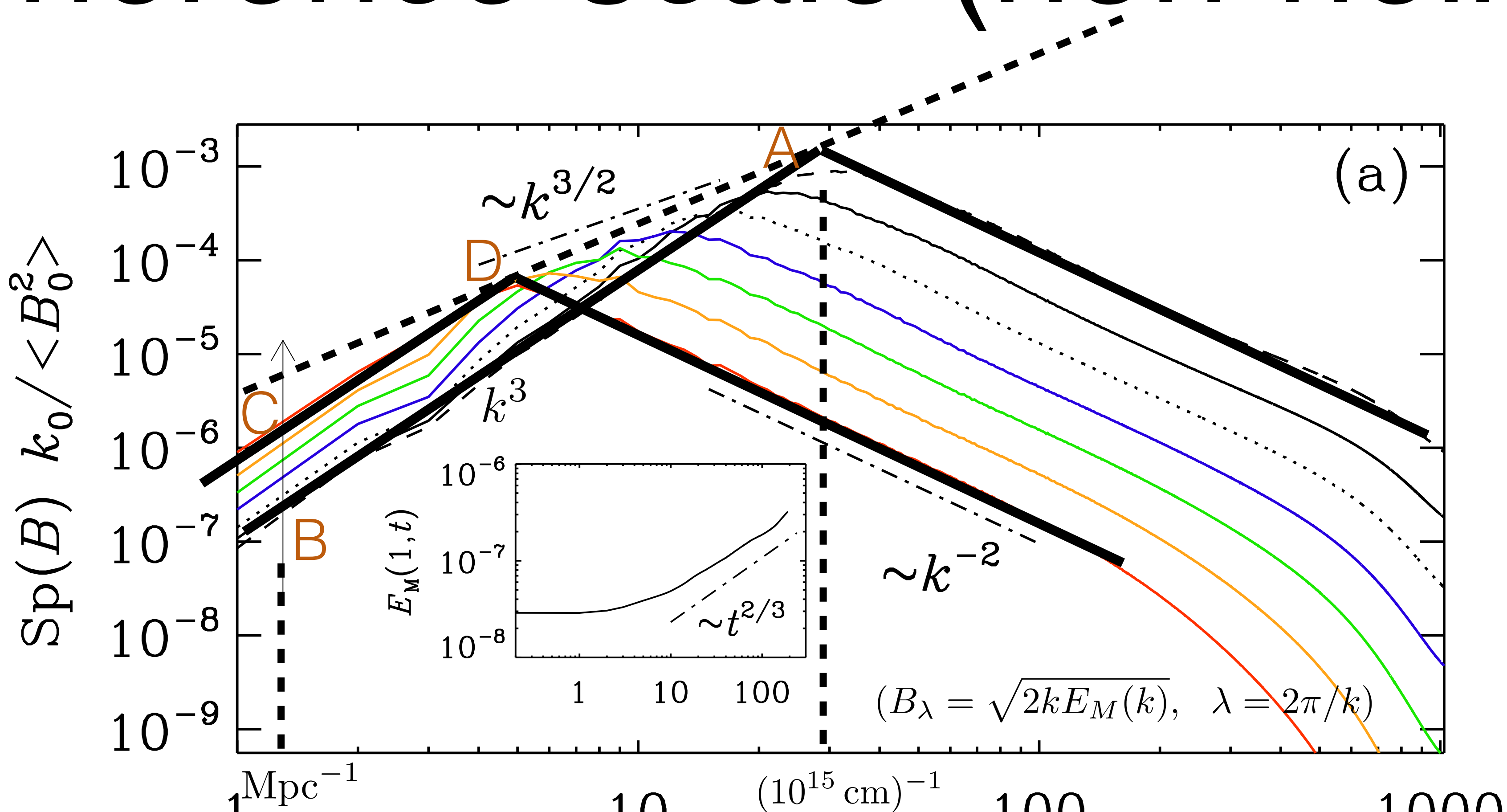
$$B_B = B_A (k_B/k_A)^2 \qquad (B_\lambda = \sqrt{2kE_M(k)}, \quad \lambda = 2\pi/k)$$

$$B_C = B_B(\tau_C/\tau_B)^{1/3} = B_A(k_B/k_A)^2(\tau_C/\tau_B)^{1/3}$$ (“τ” is conformal time.)

$$B_D = B_C(k_D/k_C)^2 = B_A(k_B/k_A)^2(\tau_C/\tau_B)^{1/3}(k_D/k_C)^2$$

k_D?

Coherence scale (non-helical)



k_D determined by intersection of lines CD and AD.

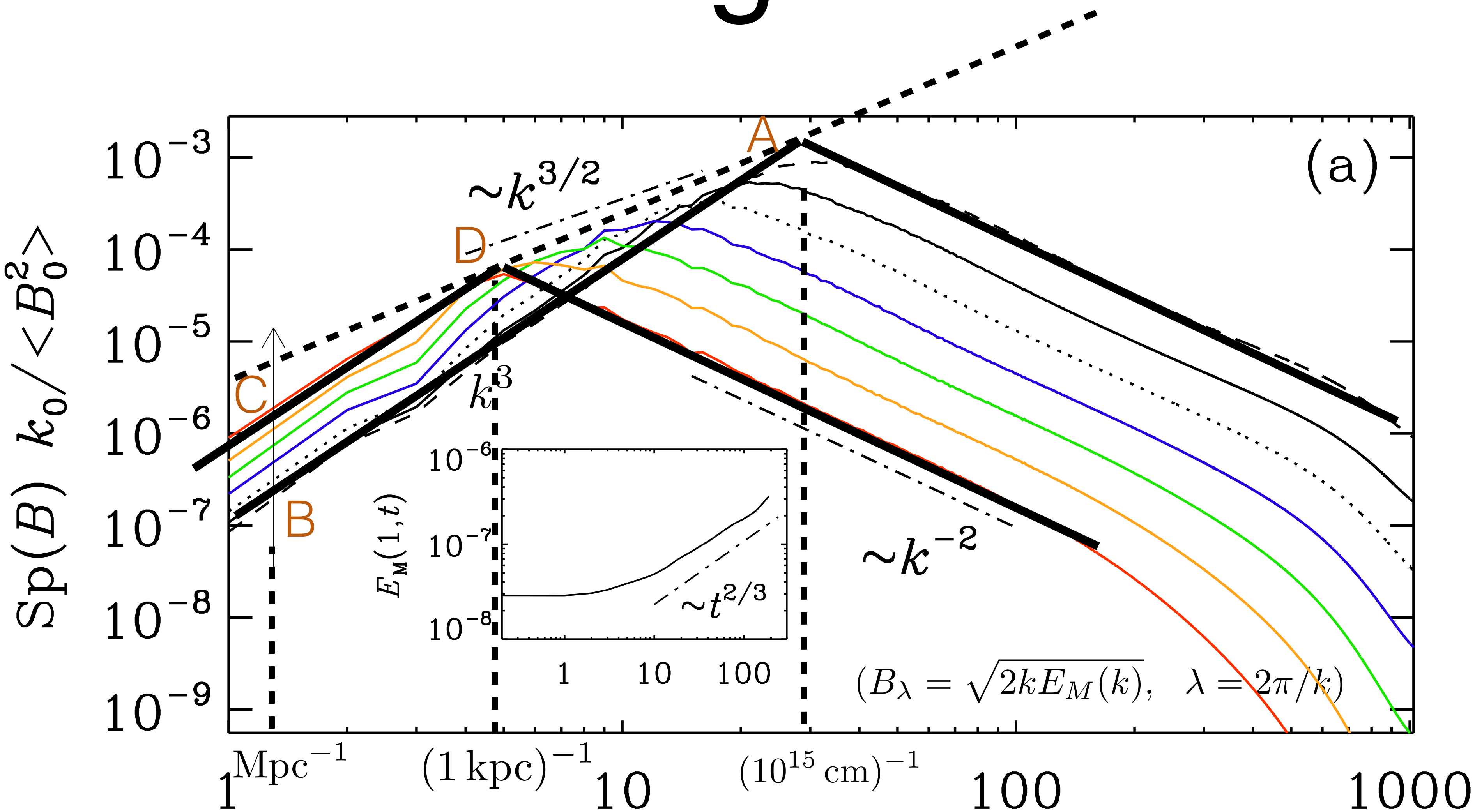
$$y_D - y_C = 3(x_D - x_C) \qquad\qquad y_A - y_D = 3(x_A - x_D)/2$$

Therefore, $x_D = x_C + 2(y_A - y_C)/3 - (x_A - x_C)$

Some more algebra gives, (ignoring some cosmological events for simplicity)

$$k_D = k_A\left(\frac{\tau_B}{\tau_C}\right)^{4/9} = k_A\left(\frac{T_{\rm eq}}{T_{\rm EW}}\sqrt{\frac{T_0}{T_{\rm eq}}}\right)^{4/9} \sim 10^{-6}k_A \sim (1\ \mathrm{kpc})^{-1}$$

Non-helical magnetic field today



Initial B on EW horizon scale (A): $\sim 10^{24}\,\mathrm{G}$

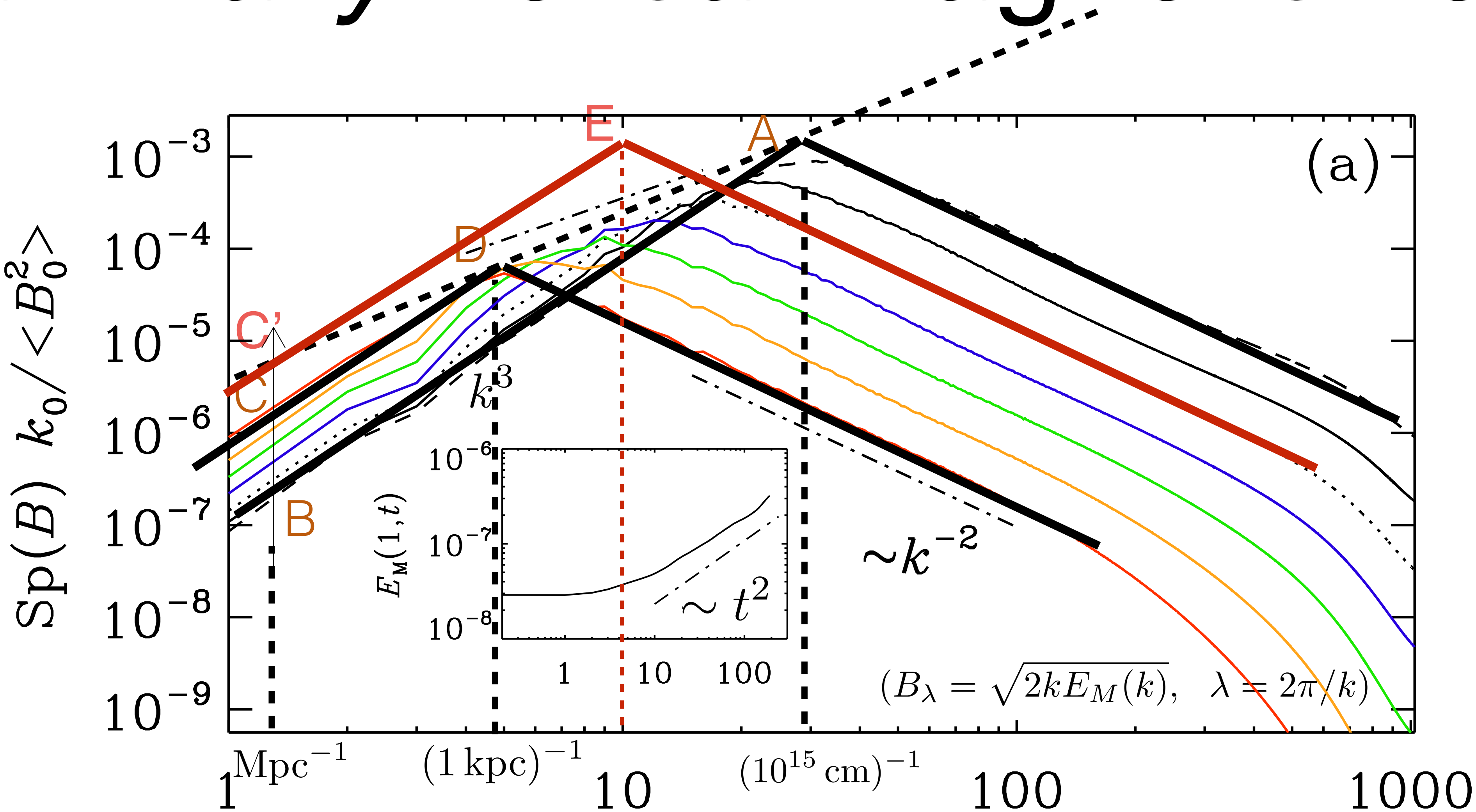
$T_{\mathrm{EW}} \sim 10^{11}\,\mathrm{eV},\quad T_{\mathrm{eq}} \sim 1\,\mathrm{eV},\quad T_0 \sim 10^{-4}\,\mathrm{eV}$

Initial B on Mpc scales (B): $B_{\mathrm{Mpc}}(t_{\mathrm{EW}}) \sim B_{\mathrm{A}} \left(\frac{10^{15}\,\mathrm{cm}}{10^{24}\,\mathrm{cm}}\right)^2 \mathrm{G} \sim 10^6\,\mathrm{G}$ $\quad (1\,\mathrm{Mpc} \sim 10^{24}\,\mathrm{cm})$

Final B on Mpc scales (C): $B_{\mathrm{Mpc}}(t_0) = B_{\mathrm{Mpc}}(t_{\mathrm{EW}}) \left(\frac{T_0}{T_{\mathrm{EW}}}\right)^2 \left(\frac{T_{\mathrm{EW}}}{T_{\mathrm{eq}}}\sqrt{\frac{T_{\mathrm{eq}}}{T_0}}\right)^{1/3} \sim 10^{-20}\,\mathrm{G}$

Final non-helical B (D; $l_{\mathrm{D}}\sim 1$ kpc): $B_{1\,\mathrm{kpc}}(t_0) \sim B_{\mathrm{C}} \left(\frac{1\,\mathrm{Mpc}}{1\,\mathrm{kpc}}\right)^2 \sim 10^{-14}\,\mathrm{G}$

Maximally helical magnetic field



$$\alpha = 3\ (\text{spectrum}),\ \beta = 0\ (\text{max.hel.}),\ q = 2/(\beta+3) = 2/3\ (\text{self}-\text{similarity})$$

$$\Longrightarrow\ E_M(1,t) \propto t^2\ \text{growth}$$

Final B on Mpc scales (C'): $B_{\mathrm{Mpc}}(t_0) = B_{\mathrm{Mpc}}(t_{\mathrm{EW}}) \left(\frac{T_0}{T_{\mathrm{EW}}}\right)^2 \left(\frac{T_{\mathrm{EW}}}{T_{\mathrm{eq}}}\sqrt{\frac{T_{\mathrm{eq}}}{T_0}}\right) \sim 10^{-11}\,\mathrm{G}$

For maximally helical fields: $E_{M,A} = E_{M,E}$

$\frac{E_{M,E}}{E_{M,C'}} = \left(\frac{k_E}{k_{C'}}\right)^3$ which gives, $k_E \sim (10\,\mathrm{Mpc})^{-1}$ and $B_E = B_{C'}\left(\frac{k_E}{k_{C'}}\right)^2 \sim 10^{-13}\,\mathrm{G}$

(drawing not to scale: C' should be much higher)

EWSB & magnetic helicity

The actual helicity is probably somewhere between zero helicity and maximal helicity.

- electroweak baryogenesis $h \approx \frac{n_b}{\alpha}$ **Cornwall, 1997** **TV, 2001**
- chiral medium **Joyce & Shaposhnikov, 1997** **Tashiro, TV & Vilenkin** **Boyarsky, Frohlich & Ruchayskiy, 2021** **Schober et al, 2017** **…**
    - e.g. tau lepton decays $n_\chi \sim \frac{\eta_B m_\tau^2}{\alpha m_e^2} \frac{T}{m_P} n_\gamma$ **TV & Vilenkin, 2021**
- new interactions $|\Phi|^2 W\tilde{W},\ |\Phi|^2 Y\tilde{Y}$ **Mou, Patel, Saffin & TV (ongoing)**

Future directions

- Hosking integral from EWSB.
- Effect of CP violating interactions.
- Fermions (?).
- Chiral effects (?).

Summary

- Electroweak symmetry breaking predicts a magnetized Universe on *very general* grounds.
- **During EWSB:** Significant (~10%) vacuum energy goes into magnetic fields during EWSB. Magnetic power spectrum peaks at small wave numbers and magnetic coherence corresponds to the largest simulated length scales.
- **Evolution:** Estimates for the present day magnetic field are in the same ball-park as the magnetic field lower bounds from TeV blazar observations.
- Observation of magnetic field helicity may inform CP violation in particle physics.