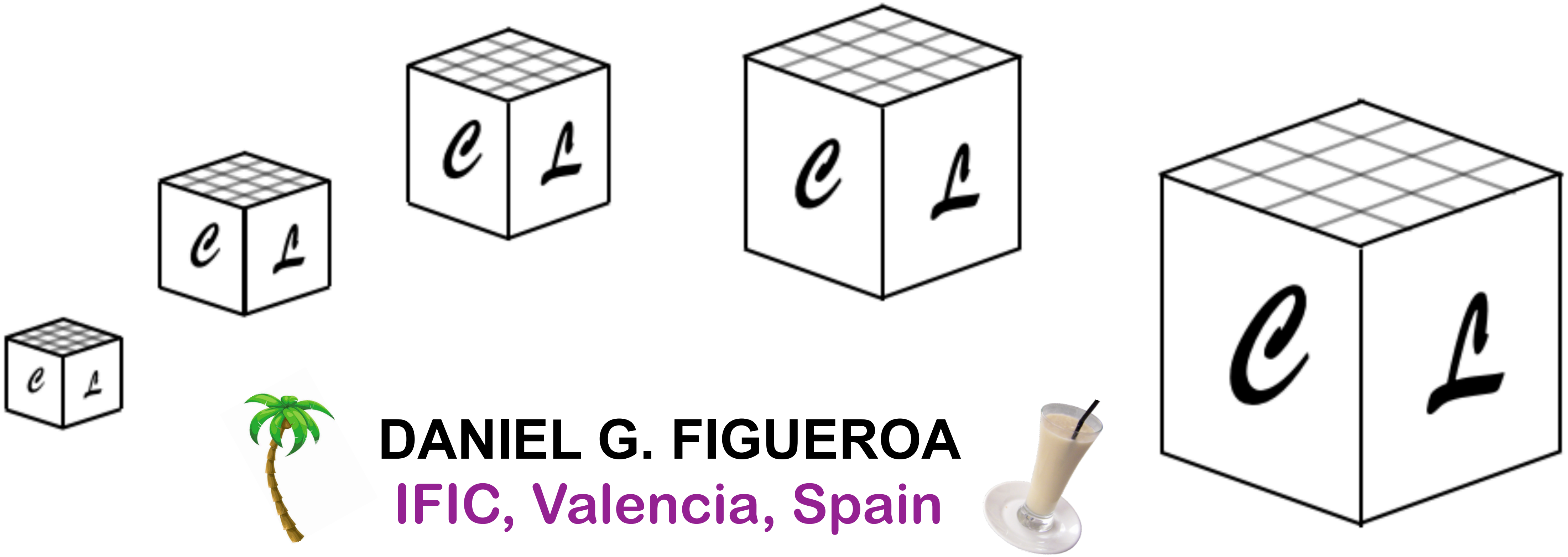
The Non-Linear Early Universe



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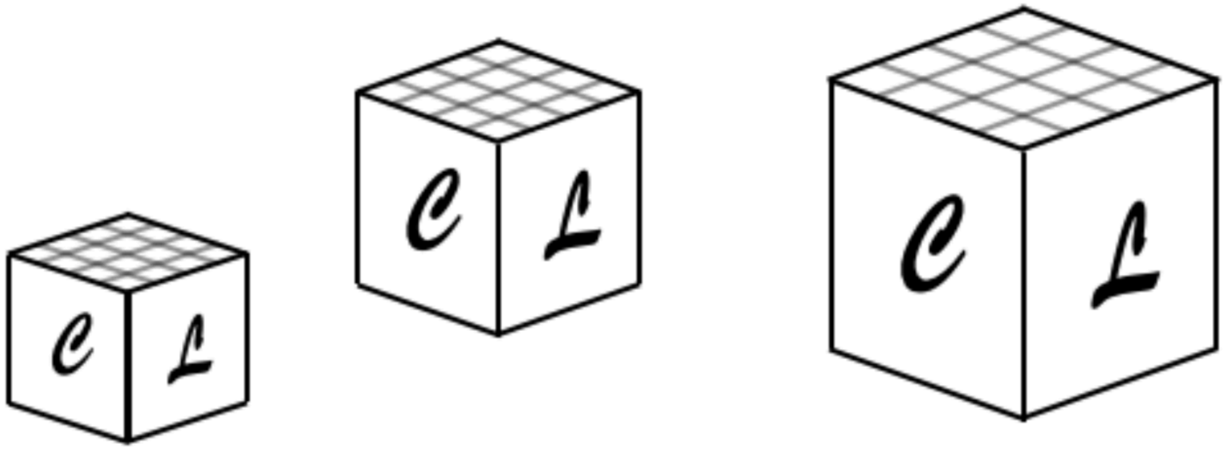
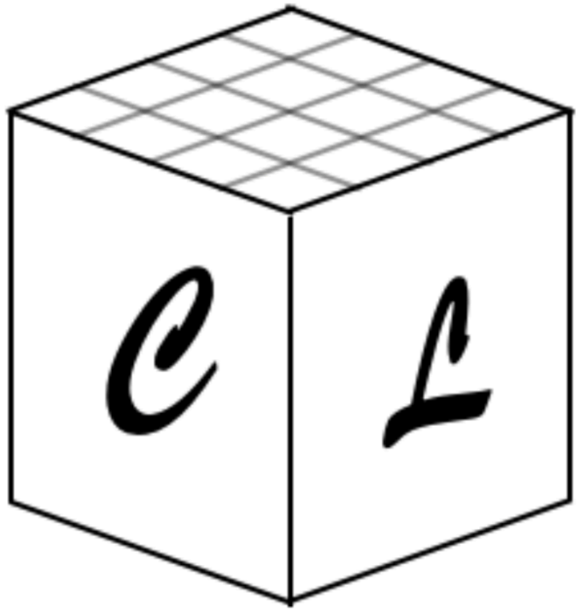
IFIC, Valencia, Spain

"All" about Magnetic Fields, Bernoulli Center, EPFL, Lausanne, Switzerland

The Numerical Early Universe





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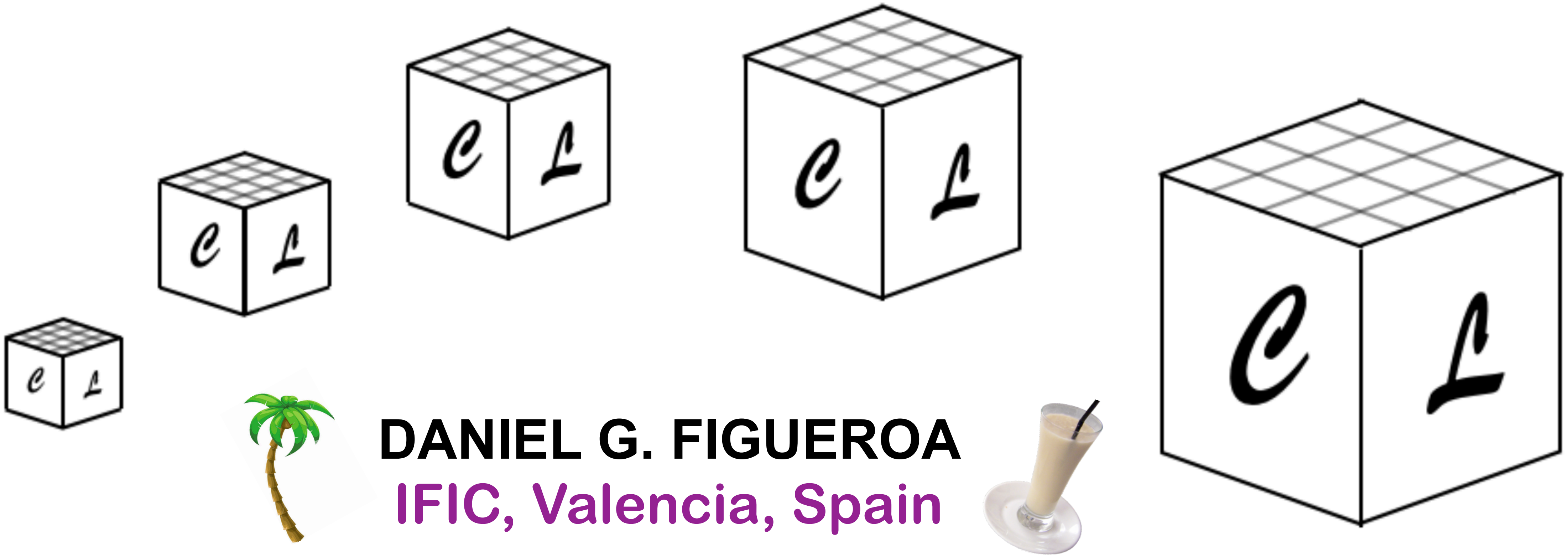
Lattice Techniques in Cosmology

DANIEL G. FIGUEROA

IFIC, Valencia, Spain

"All" about Magnetic Fields, Bernoulli Center, EPFL, Lausanne, Switzerland

LATTICE COSMOLOGY



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LATTICE COSMOLOGY

The Art of Simulating the Early Universe

"All" about Magnetic Fields, Bernoulli Center, EPFL, Lausanne, Switzerland

LATTICE COSMOLOGY

The Art of Simulating the Early Universe

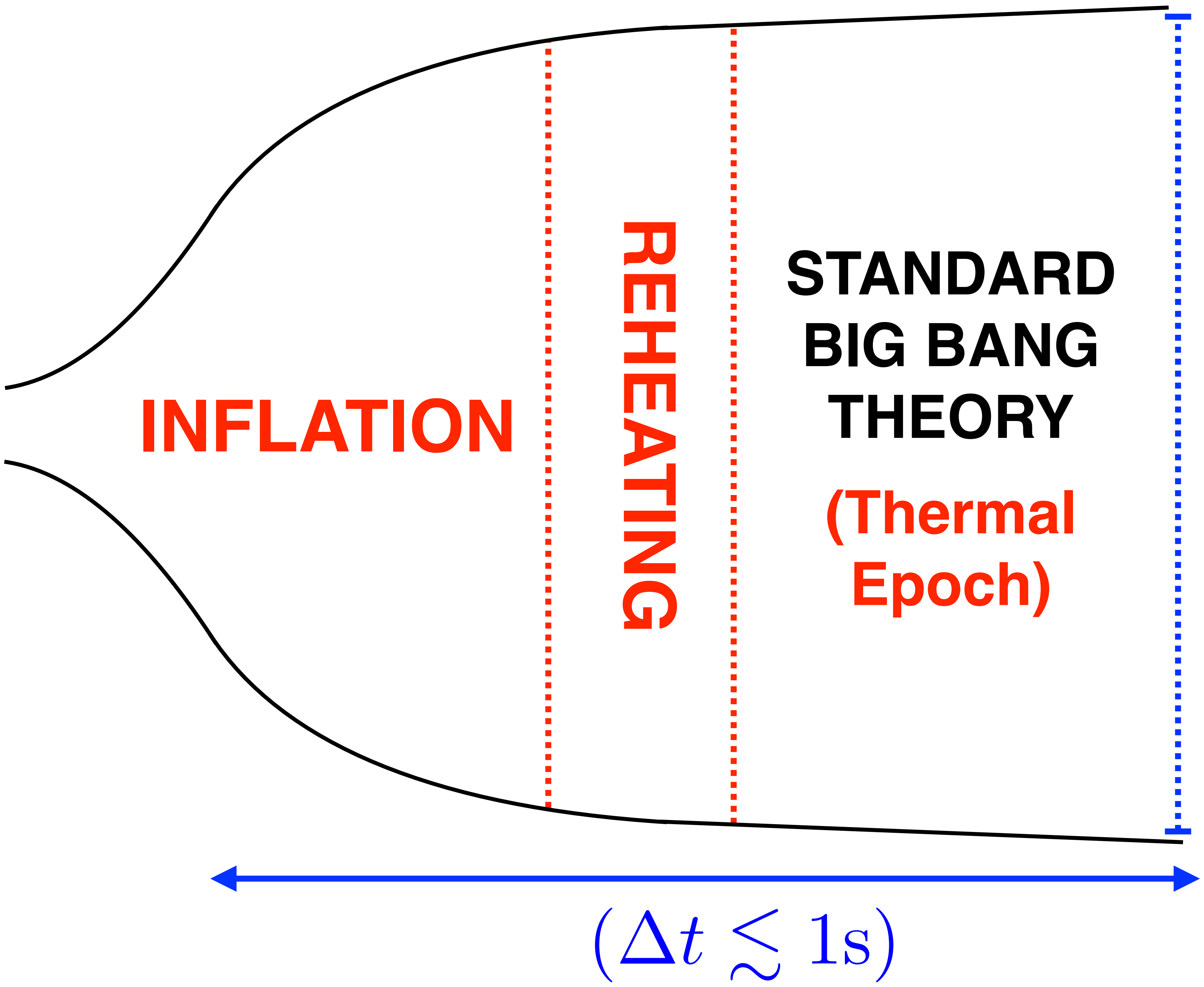
(When do we need
to simulate it ?)

LATTICE COSMOLOGY

The Art of Simulating the Early Universe

When things get complicated:
non-linear, strong coupling,
non-perturbative, etc

The Early Universe



INFLATION

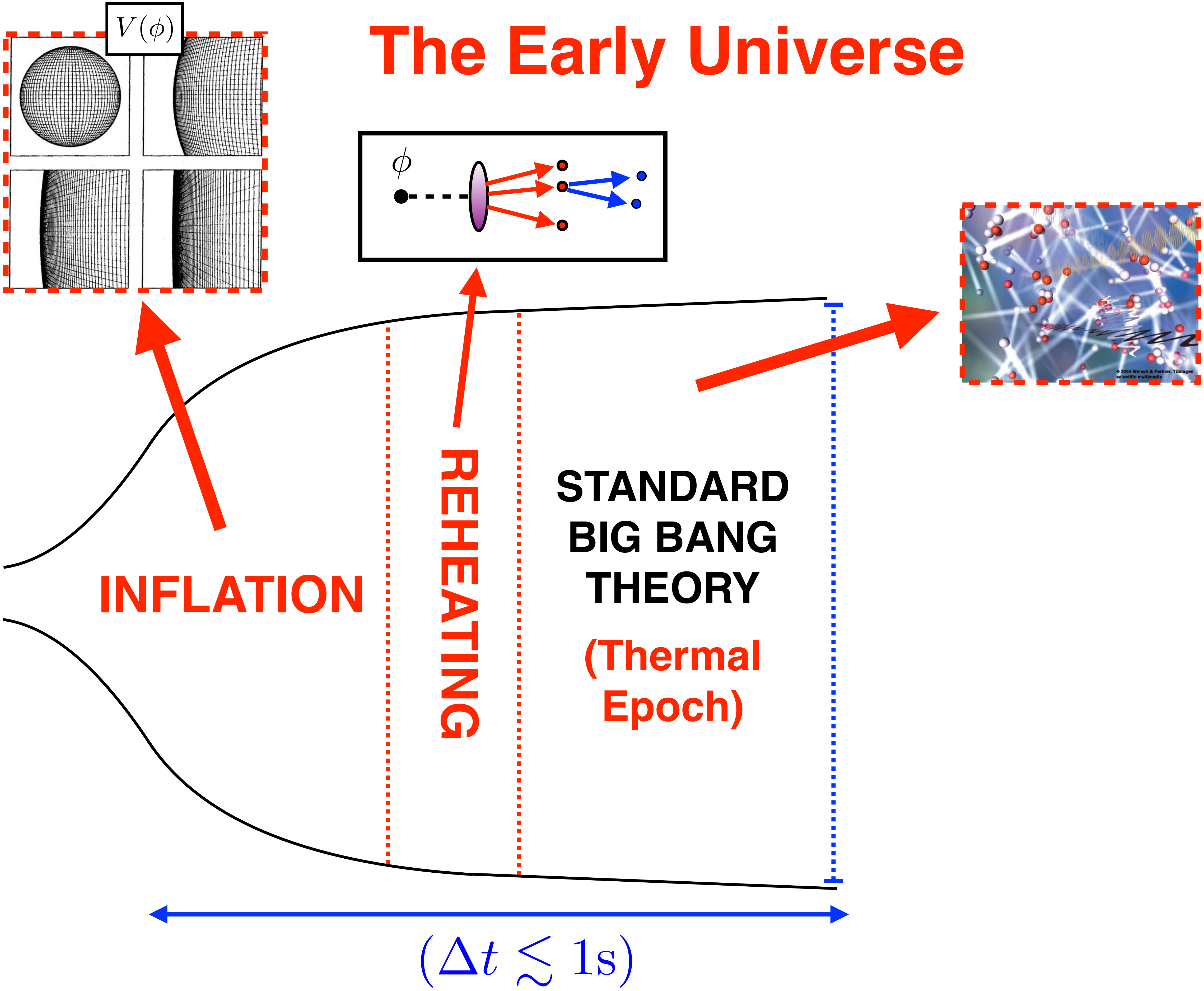
REHEATING

STANDARD
BIG BANG
THEORY

(Thermal
Epoch)

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



INFLATION

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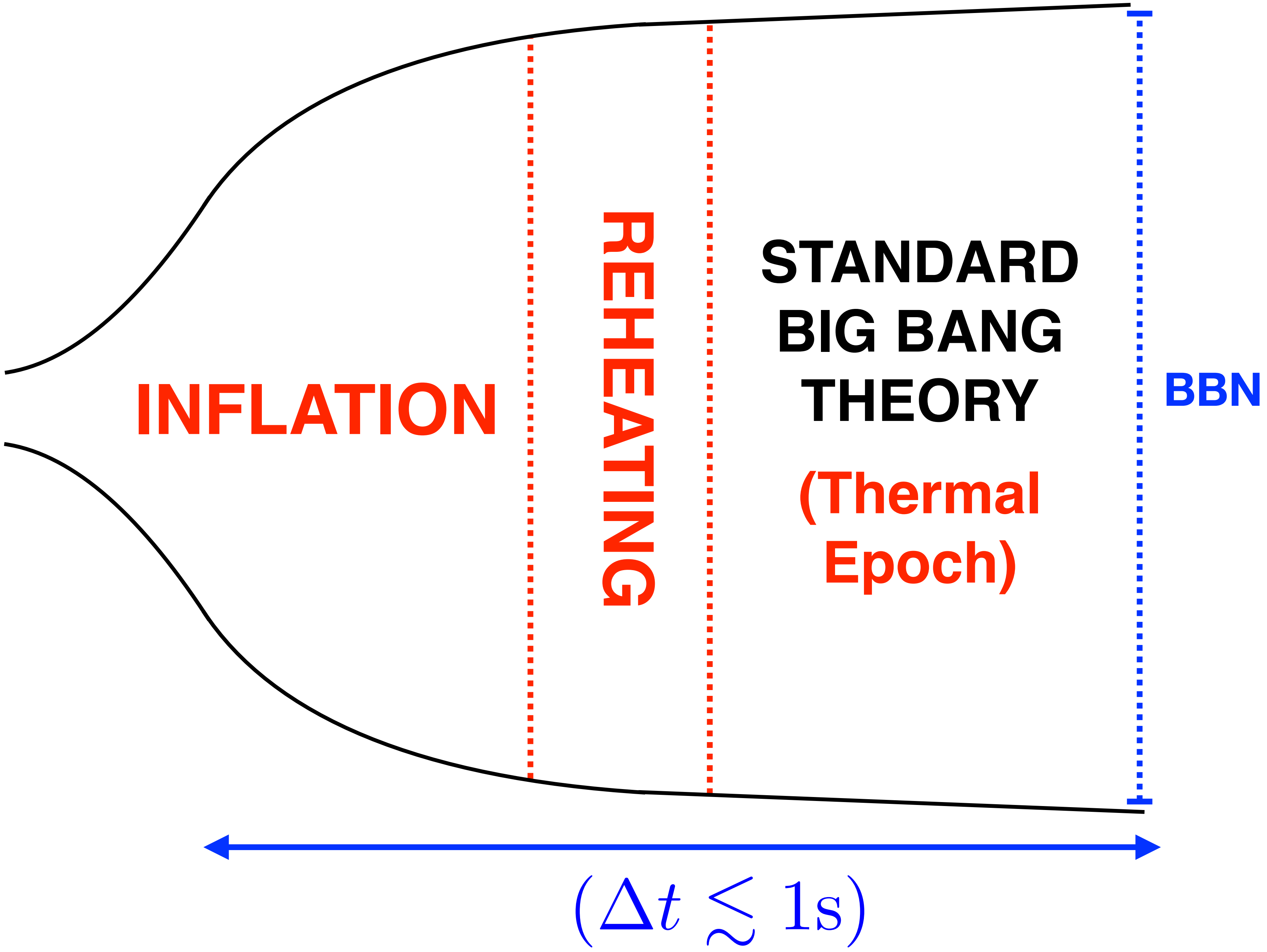
STANDARD BIG BANG THEORY

(Thermal Epoch)

$(\Delta t \lesssim 1\text{s})$

The Early Universe



INFLATION

REHEATING

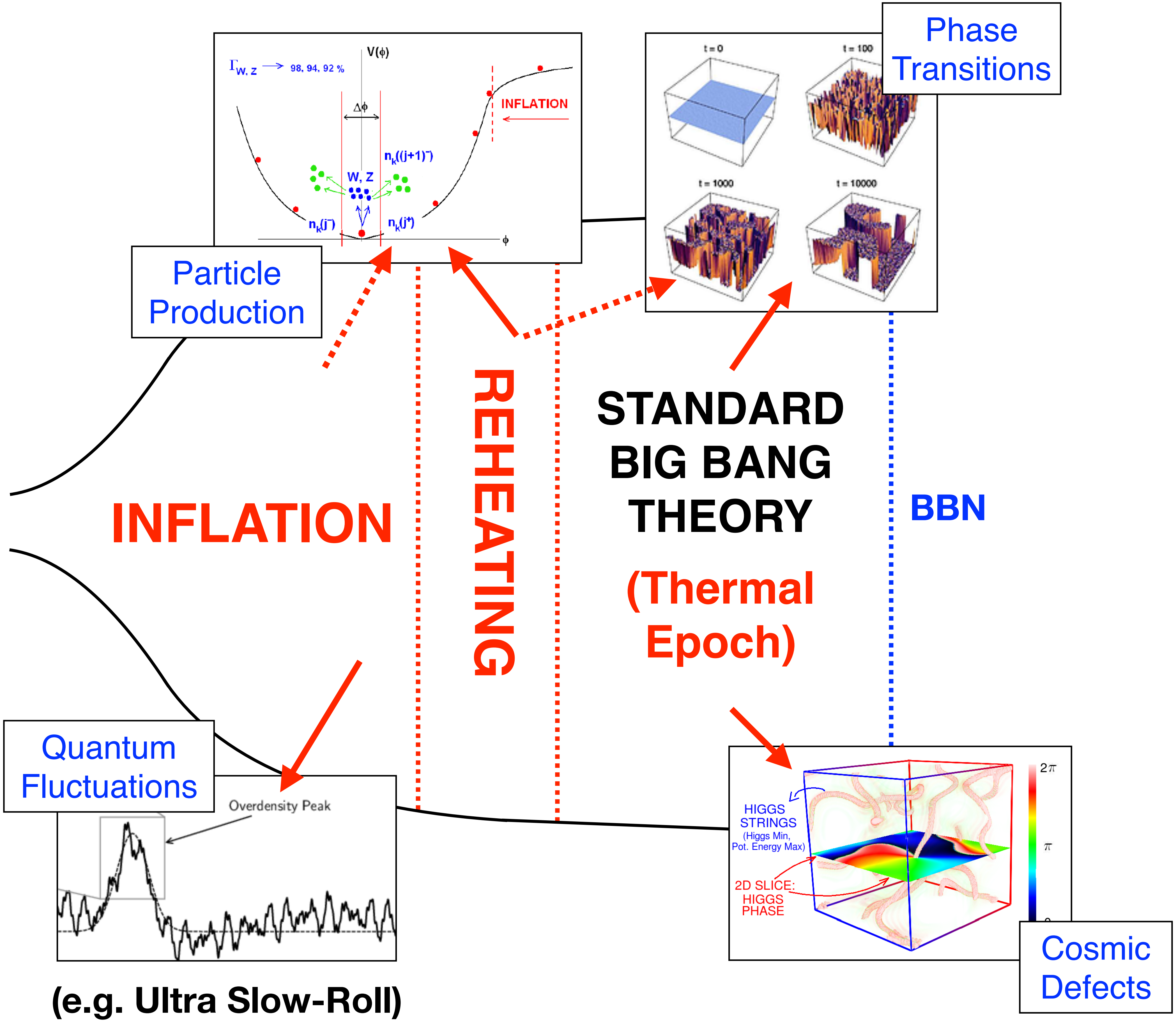
**STANDARD
BIG BANG
THEORY**

(Thermal
Epoch)

BBN

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Phase Transitions

Particle Production

INFLATION

REHEATING

STANDARD BIG BANG THEORY

(Thermal Epoch)

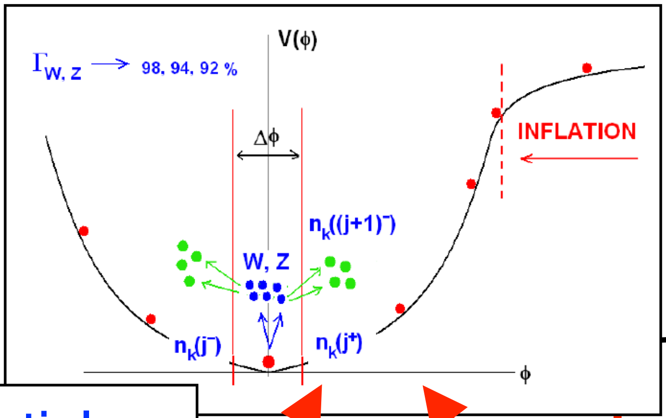
BBN

Quantum Fluctuations

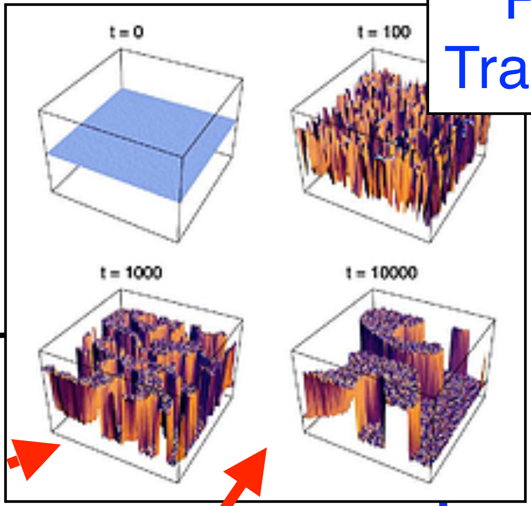
(e.g. Ultra Slow-Roll)

Cosmic Defects

The Early Universe



Particle Production



Phase Transitions

INFLATION

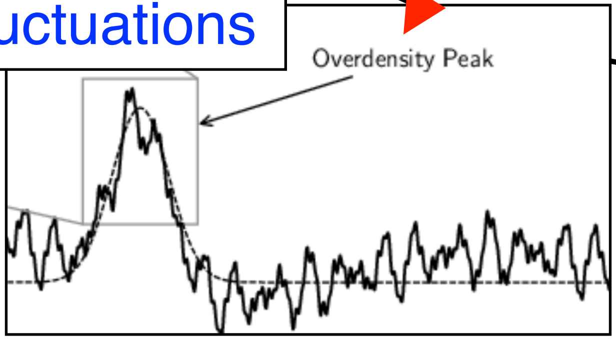
REHEATING

STANDARD BIG BANG THEORY

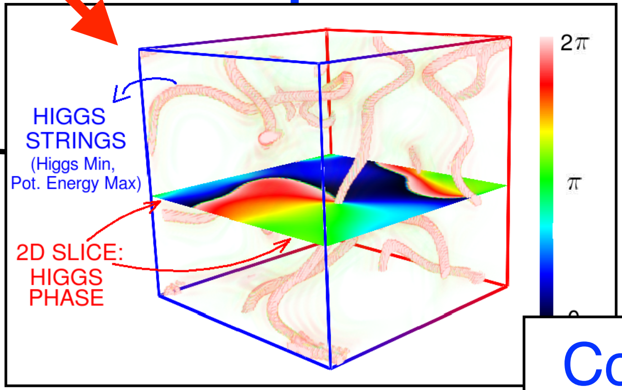
(Thermal Epoch)

Common Factor ?

Quantum Fluctuations

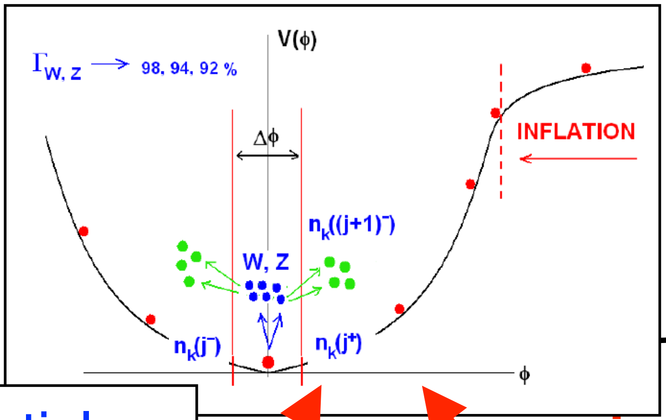


(e.g. Ultra Slow-Roll)

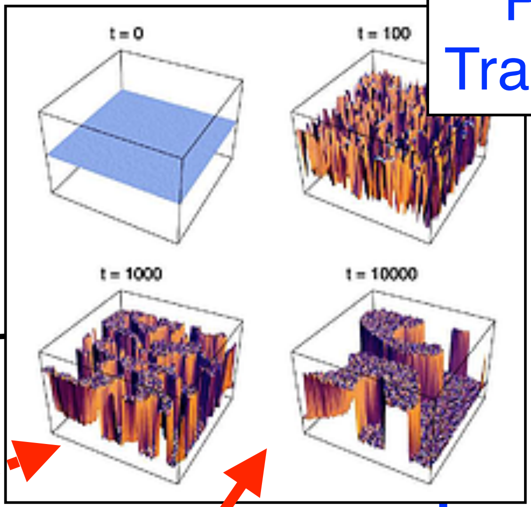


Cosmic Defects

The Early Universe



Particle Production



Phase Transitions

INFLATION

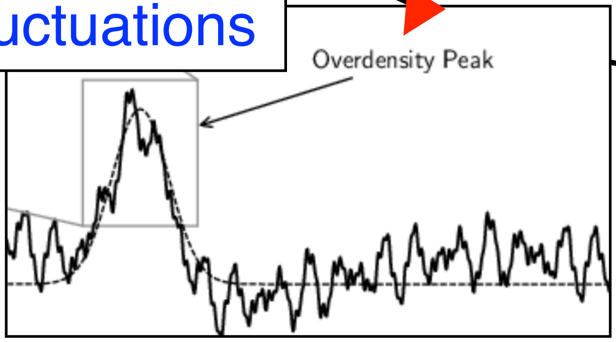
REHEATING

STANDARD BIG BANG THEORY

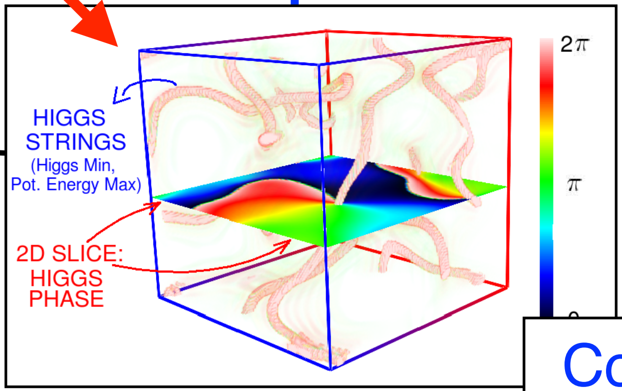
(Thermal Epoch)

Non-linear field dynamics

Quantum Fluctuations



(e.g. Ultra Slow-Roll)



Cosmic Defects

The Early Universe

Particle Production

Phase Transitions

Non-linear field dynamics

Curvature Fluctuations

Cosmic Defects

The Early Universe

Particle Production

Phase Transitions

Non-linear field dynamics

Curvature Fluctuations

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-linear field dynamics

Curvature Fluctuations

Cosmic Defects

The Early Universe

Particle Production

Phase Transitions

Gravitational waves

Baryo-genesis

Non-linear field dynamics

Curvature Fluctuations

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Baryo-genesis

Non-linear field dynamics

Magneto-genesis

Curvature Fluctuations

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Baryogenesis

Non-linear field dynamics

Magnetogenesis

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-linear field dynamics

Magneto-genesis

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryogenesis

Non-linear field dynamics

Non-minimal Kinetic Theories

Magnetogenesis

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-linear field dynamics

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation ….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

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….

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Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-linear

≡ Numerical simulations

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation ….

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The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-linear ≡ Lattice simulations

Non-minimal Kinetic Theories

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Turbulence Thermalisation ….

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Cosmic Defects

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Latticeeasy / Clustereasy, Defrost,

CUDAEasy, HLattice, PyCOOL,

GABE, PSpectRe, Stella, GFiRe, …

Non-minimal Kinetic Theories

Magneto-genesis

+ GRChombo, GABERel, …

Turbulence Thermalisation ….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 10-25 yrs

The Early Universe

Gravitational waves

Particle Production

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Non-minimal Kinetic Theories

Magneto-genesis

+ GRCho

(Scalar) Field Theory Codes

Curvature Fluctuations

Bla
For

~ 10-25 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Latticeeasy / Clustereasy, Defrost,

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Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation ….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 10-25 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryogenesis

Latticeeasy / Clustereasy, Defrost,
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Non-minimal Kinetic Theories

Magnetogenesis

+ GRChombo, GABERel, …

Turbulence
Thermalisation
….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 5-10 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Gen. Rel. (GR) Codes

Non-minimal Kinetic Theories

Magneto-genesis

+ GRChombo, GABERel, …

Turbulence Thermalisation ….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 5-10 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryogenesis

Latticeeasy / Clustereasy, Defrost,
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Non-minimal Kinetic Theories

Magnetogenesis

+ GRChombo, GABERel, …

Turbulence Thermalisation
….

+ PENCIL, …

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 5-10 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

MHD Codes
(Fluid Dynamics)

Non-minimal Kinetic Theories

Magneto-genesis

+ GRChombo, GABERel, …

Turbulence Thermalisation
….

Curvature Fluctuations

+ PENCIL, …

Black Hole Formation

Cosmic Defects

~ 5-10 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

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Non-minimal Kinetic Theories

Magnetogenesis

+ GRChombo, GABERel, …

Turbulence Thermalisation
….

+ PENCIL, …

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 10-25 yrs

The Early Universe

Lattice Cosmology

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation
….

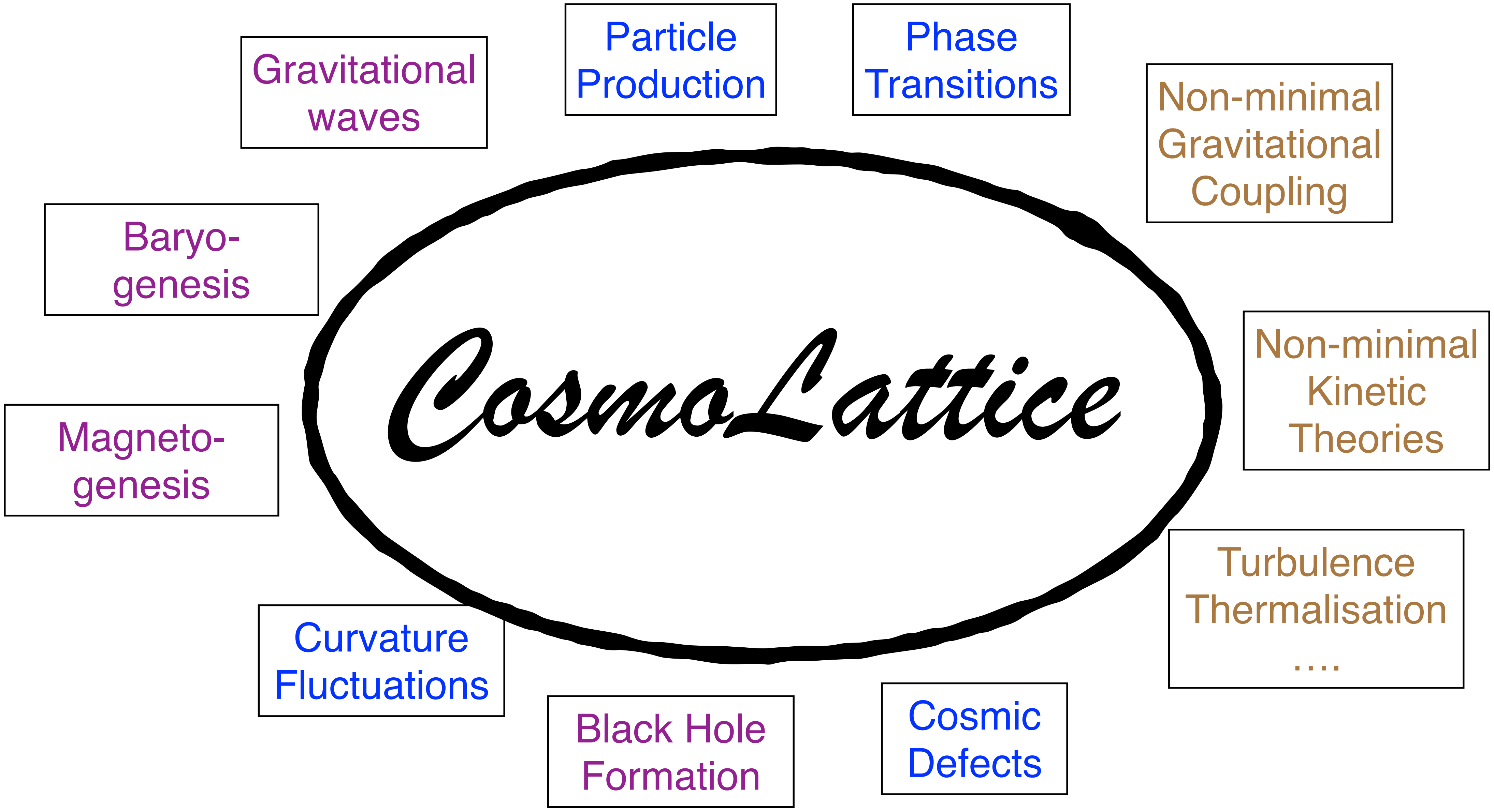
Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 10-25 yrs

The Early Universe



CosmoLattice

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation
….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 3 yrs

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

CosmoLattice

+ CosmoLattice-MHD

+ CosmoLattice-GR

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation
….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 3 yrs

[+ upcoming]

The Early Universe

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryogenesis

CosmoLattice

Non-minimal Kinetic Theories

Magnetogenesis

Turbulence Thermalisation
….

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

~ 3 yrs

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

Lattice Theory: arXiv: 2006.15122 (+100 pages)

Code Manual: arXiv: 2102.01031 (+100 pages)

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

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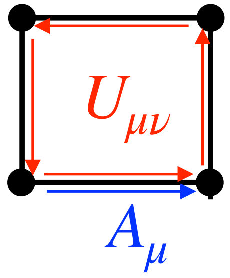
Code Manual: arXiv: 2102.01031

- Simulates **scalar-gauge field dynamics** [w. **self-consistent** expanding background]

[U(1) x SU(2)]

Links & **plaquettes**
(~ **lattice-QCD**)



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- Written in **C++**, with **modular structure** separating physics (CosmoInterface library) and technical details (TempLat library).

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- Written in **C++**, with **modular structure** separating physics (CosmoInterface library) and technical details (TempLat library).
- **Parallellized** in multiple spatial dimensions (**but you write in serial !**)
- **Family** of evolution **algorithms**, accuracy ranging from $\delta\mathcal{O}(\delta t^2)$ - $\delta\mathcal{O}(\delta t^{10})$

  [LeapFrog, Verlet, Runge-Kutta, Yoshida, …]

CosmoLattice

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http://www.cosmolattice.net/

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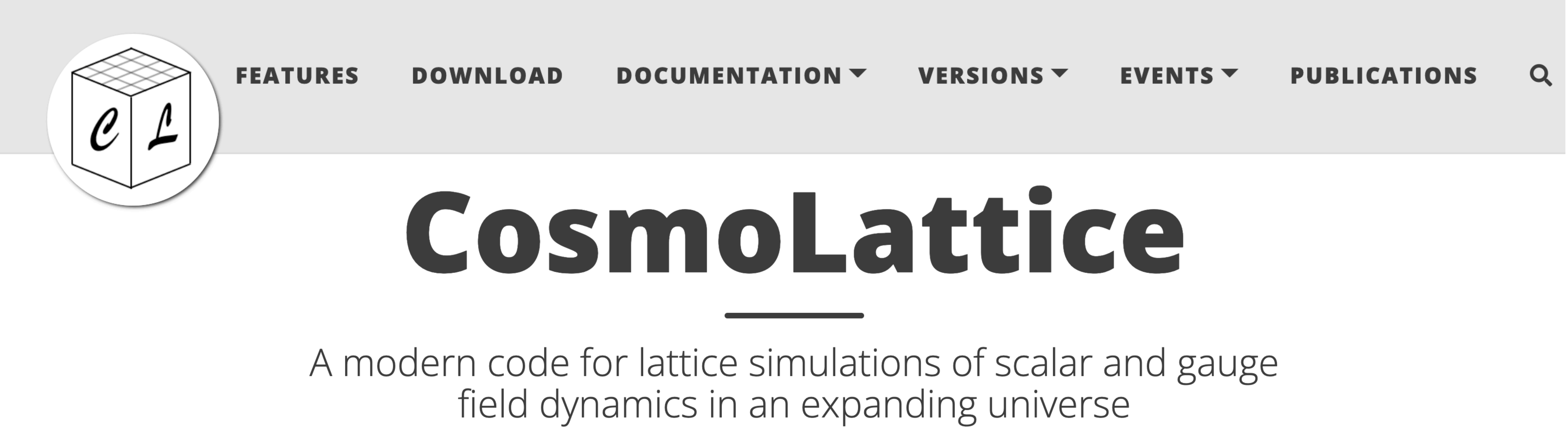

CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

CosmoLattice – Default Field Content

- **Matter content:**

$$\boxed{S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (D_\mu^A\varphi)^*(D_A^\mu\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} + V(\phi,|\varphi|,|\Phi|)\right\}}$$

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$$\boxed{\phi \in \mathscr{R}e}$$

Scalar sector

$$\boxed{\begin{gathered}\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1)\\ D_\mu^A \equiv \partial_\mu - iQ_A^{(\varphi)}g_{_A}A_\mu\\ F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu\end{gathered}}$$

U(1) gauge sector

$$\boxed{\begin{gathered}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_0 + i\varphi_1\\ \varphi_2 + i\varphi_3\end{pmatrix}\\ D_\mu \equiv \mathscr{I}D_\mu^A - ig_BQ_BB_\mu^aT_a\\ G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu]\end{gathered}}$$

SU(2) gauge sector

Scalar potential

CosmoLattice – Default Field Content

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$$\boxed{\phi \in \mathscr{Re}}$$

Scalar sector

$$\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1)$$

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SU(2) gauge sector

Scalar potential

- **Background Metric:**

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx^i dx^j \left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

- **Self-consistent expansion** (Friedmann equations)
- **Fixed power-law background** $a(t) \sim t^{\frac{2}{3(1+w)}}$

CosmoLattice – Default Field Content

- **Matter content:**

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CosmoLattice – Equations of Motion

- **Hamiltonian scheme**: coupled first-order differential equations

- **Scalar fld example**

$$\frac{d^2\phi}{dt^2} - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial\phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}}$$

KICK: $(\pi_\phi)' = -a^{3+\alpha}\frac{\partial V}{\partial\phi} + a^{1+\alpha}\nabla^2\phi$

DRIFT: $\phi' \equiv \pi_\phi a^{\alpha-3}$

CosmoLattice – Equations of Motion

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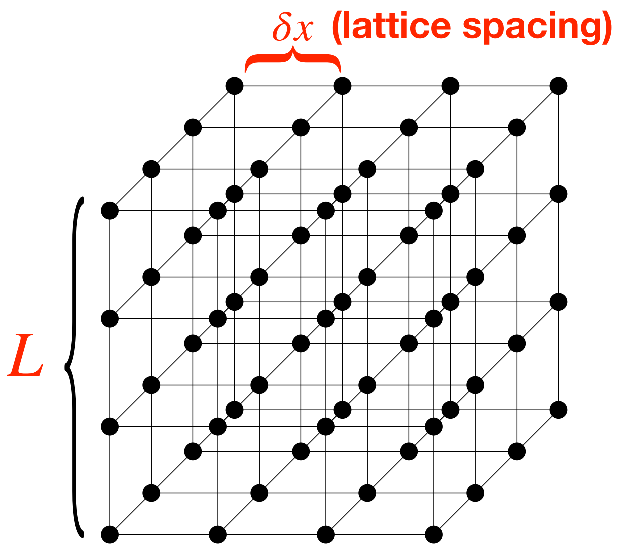
- **Scalar Fields and momenta** are defined in the **lattice sites**



N : number of points/dimension

$L = N \cdot \delta x$: length side

δt : time step

Minimum and maximum momenta:

$$k_{\min} = \frac{2\pi}{L} \qquad k_{\max} = \frac{\sqrt{3}}{2} N k_{\min}$$

CosmoLattice – Equations of Motion

- **Hamiltonian scheme**: coupled first-order differential equations

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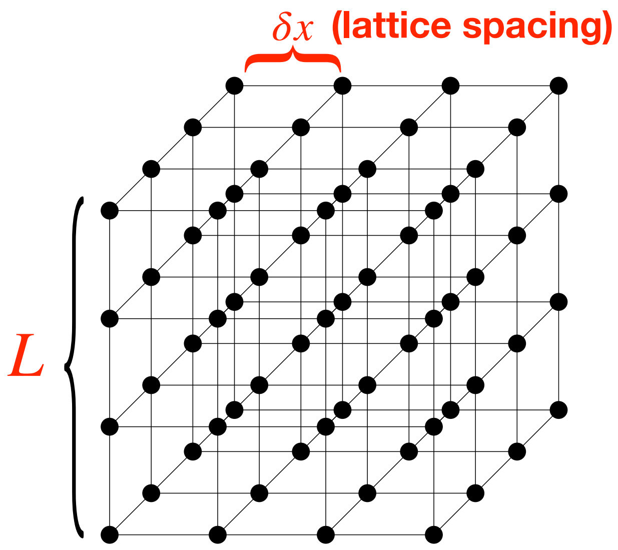

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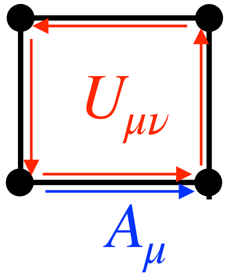
δt : time step

Minimum and maximum momenta:

$$k_{\min} = \frac{2\pi}{L} \qquad k_{\max} = \frac{\sqrt{3}}{2} N k_{\min}$$

- **Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)



CosmoLattice – Expansion Evolution

- Algorithms use **second Friedmann equation** to **evolve the scale factor**.
- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2}\langle(\alpha-2)(K_\phi+K_\varphi+K_\Phi)+\alpha(G_\phi+G_\varphi+G_\Phi)+(\alpha+1)V + (\alpha-1)(K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)})\rangle$$

$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}\langle K_\phi+K_\varphi+K_\Phi+G_\phi+G_\varphi+G_\Phi+K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)}+V\rangle$$

$\langle ... \rangle$ represents volume averaging

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$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}\langle K_\phi + K_\varphi + K_\Phi + G_\phi + G_\varphi + G_\Phi + K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)} + V\rangle$$

$\langle ... \rangle$ represents volume averaging

$$\begin{aligned} K_\phi &= \tfrac{1}{2a^{2\alpha}}\phi'^2 \\ K_\varphi &= \tfrac{1}{a^{2\alpha}}(D_0^A\varphi)^*(D_0^A\varphi) \;; \\ K_\Phi &= \tfrac{1}{a^{2\alpha}}(D_0\Phi)^\dagger(D_0\Phi) \end{aligned} \qquad \begin{aligned} G_\phi &= \tfrac{1}{2a^2}\textstyle\sum_i(\partial_i\phi)^2 \\ G_\varphi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i^A\varphi)^*(D_i^A\varphi) \;; \\ G_\Phi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i\Phi)^\dagger(D_i\Phi) \end{aligned} \qquad \begin{aligned} K_{U(1)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_i F_{0i}^2 \\ K_{SU(2)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_{a,i}(G_{0i}^a)^2 \\ G_{U(1)} &= \tfrac{1}{2a^4}\textstyle\sum_{i,j<i} F_{ij}^2 \\ G_{SU(2)} &= \tfrac{1}{2a^4}\textstyle\sum_{a,i,j<i}(G_{ij}^a)^2 \end{aligned}$$

(Kinetic-Scalar) (Gradient-Scalar) (Electric & Magnetic)

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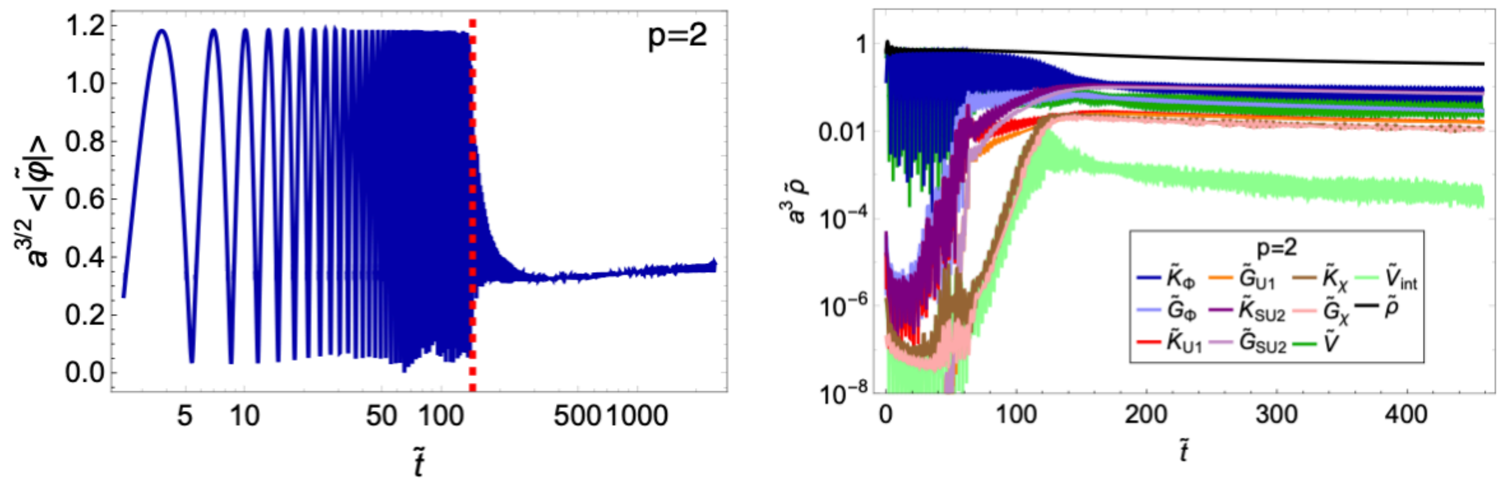
(Kinetic-Scalar) (Gradient-Scalar) (Electric & Magnetic)

CosmoLattice – Output / Observables

Output Types

Volume averages: variance, energies, etc



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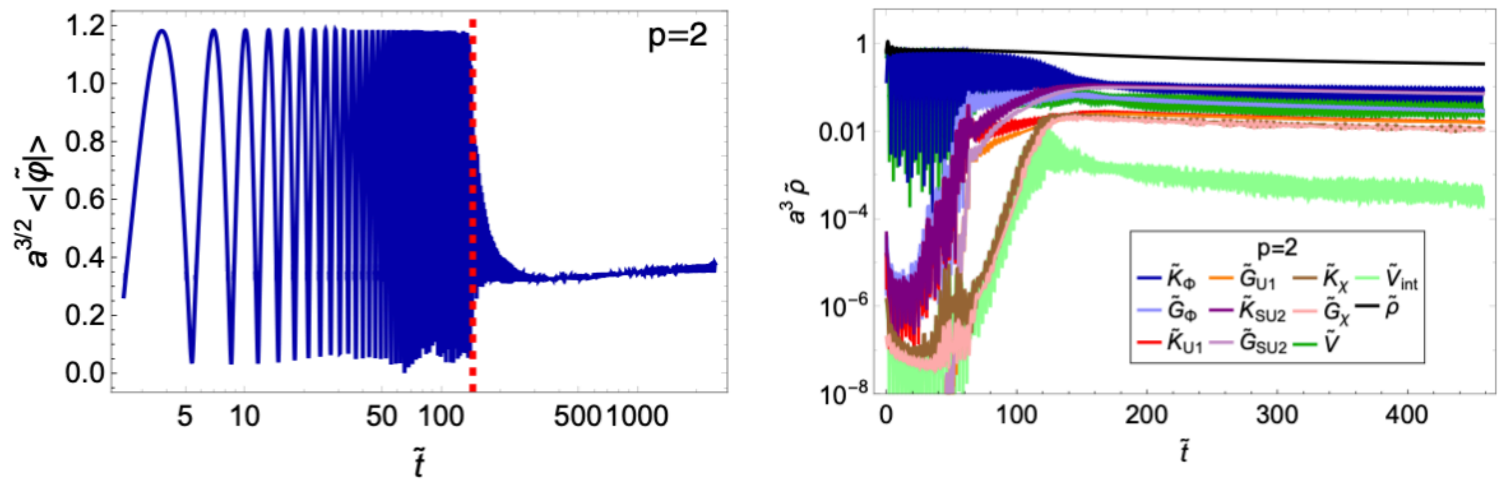
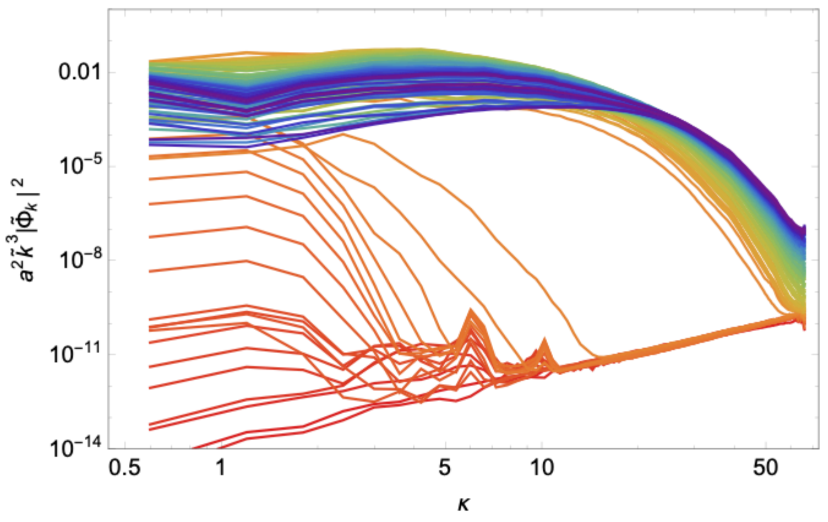


Fld Spectra: Raw/Binned



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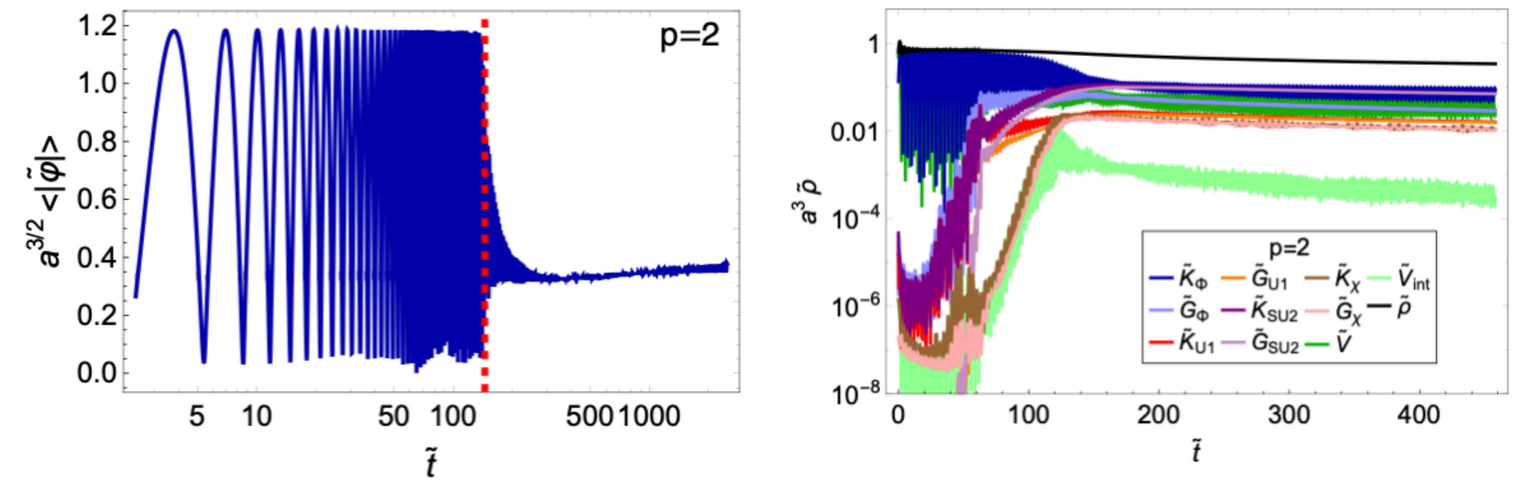
Output Types

Volume averages: variance, energies, etc



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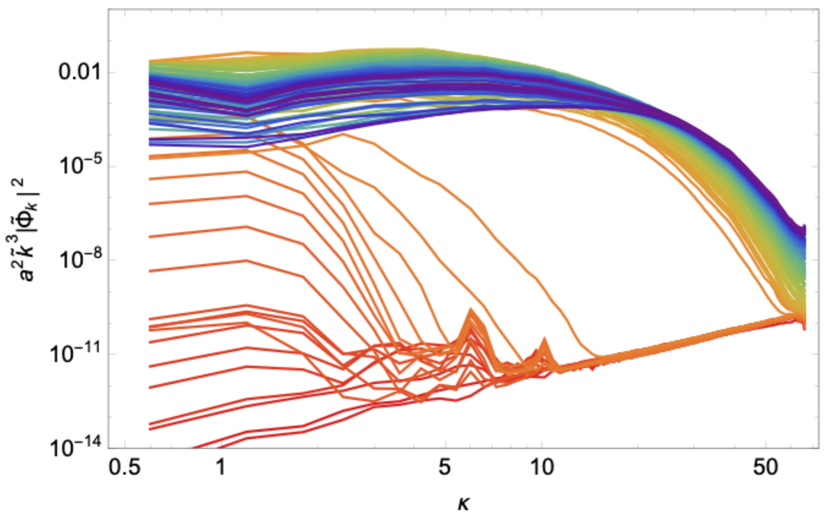
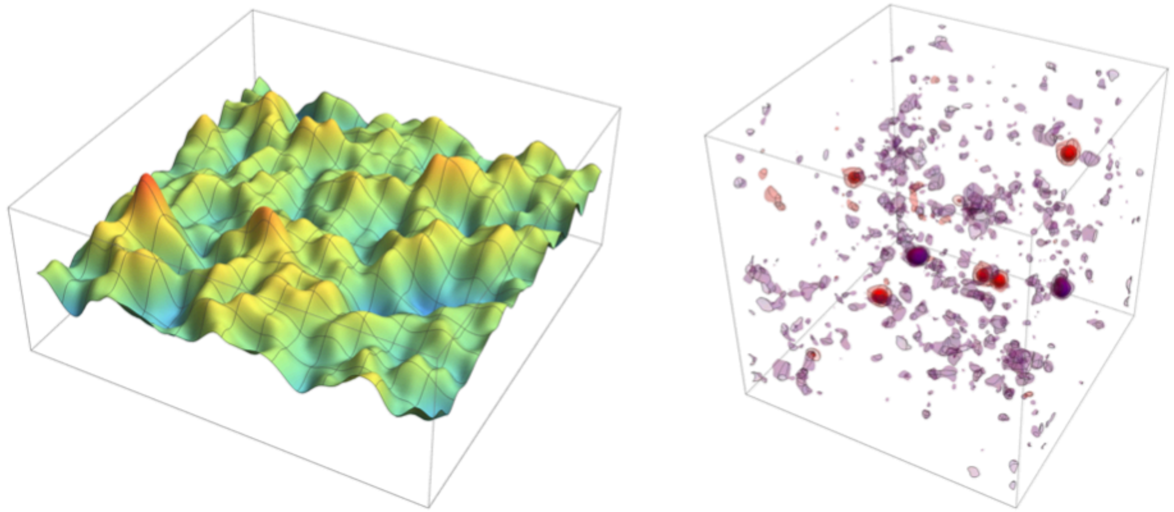


Snapshots: 2D/3D distribution



In summary …

CosmoLattice

Theory Review
arXiv: 2006.15122

http://www.cosmolattice.net/

Code Manual
arXiv: 2102.01031

Field Th. Problem

- * Init Conditions
- * Eqs. of Motion

(
Field Objects
Field Algebra
)

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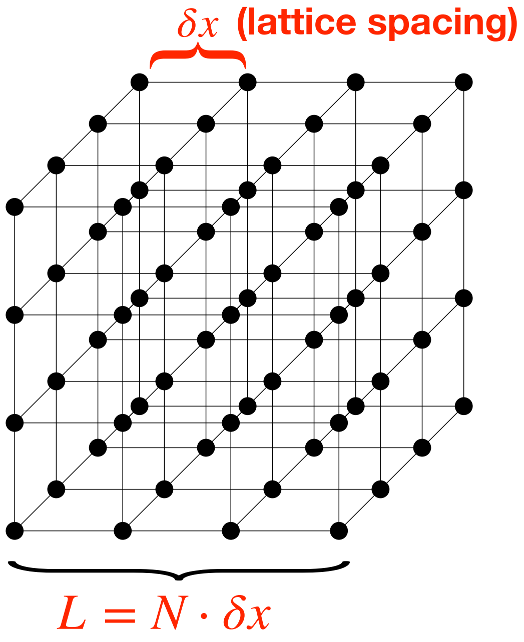
CosmoLattice

Field Th. Problem

- * Init Conditions
- * Eqs. of Motion

- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\mathcal{O}(\delta t^n)$
- * Choose Param: g, m, ...
- * Choose Observables



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CosmoLattice

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Algorithms

- *Staggered LeapFrog (LF)*
- *Position-Verlet (PV2)*
- *Velocity-Verlet (VV2)*
- *Runge-Kutta (RK2, RK3, RK4)*
- *Yoshida (VV4, VV6, VV8, VV10)*

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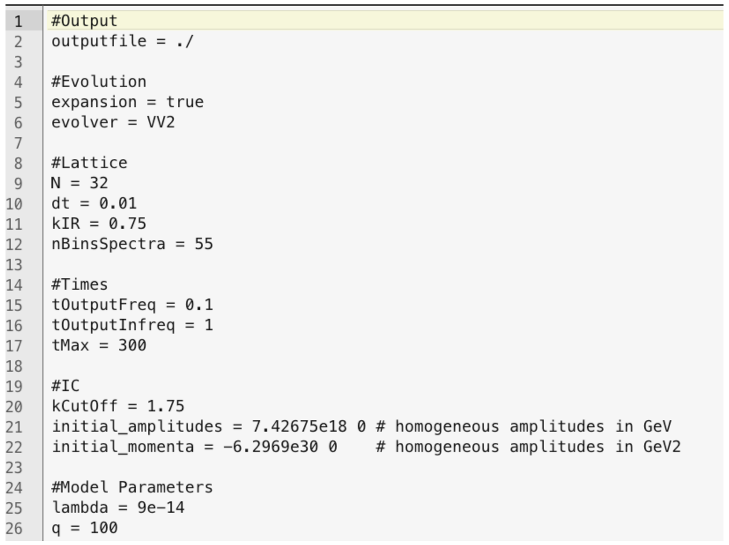
- **Choose Observables**

$$\lambda_1\,,\,\lambda_2\,,\,\ldots,\,g_1^2\,,\,g_2^2\,,\,\ldots$$

$$m_\phi^2\,,\,m_\psi^2\,,\,\ldots,v^2,\,\Phi_*,\,\ldots$$



```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = −6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e−14
q = 100
```

➤ **Parameters** via **input file**

(no need to re-compile !)

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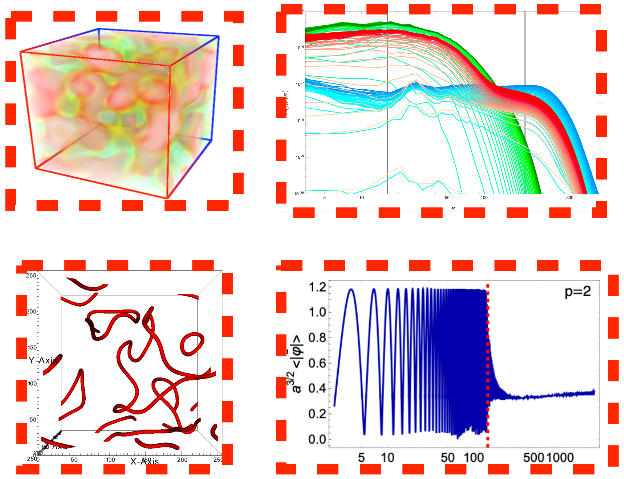

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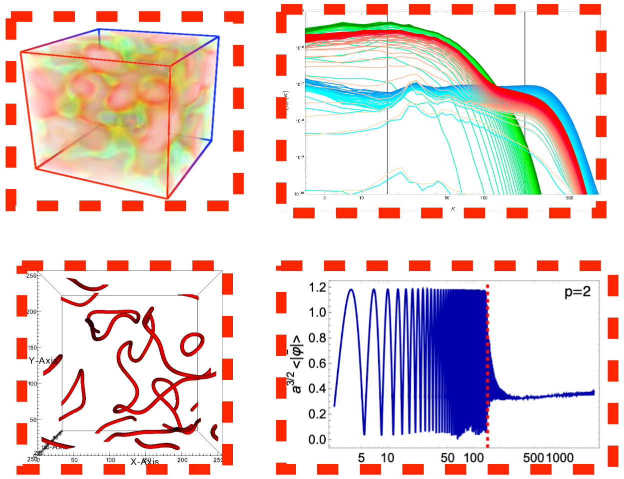

CL is a platform for field theories
You choose the problem to solve !

CosmoLattice

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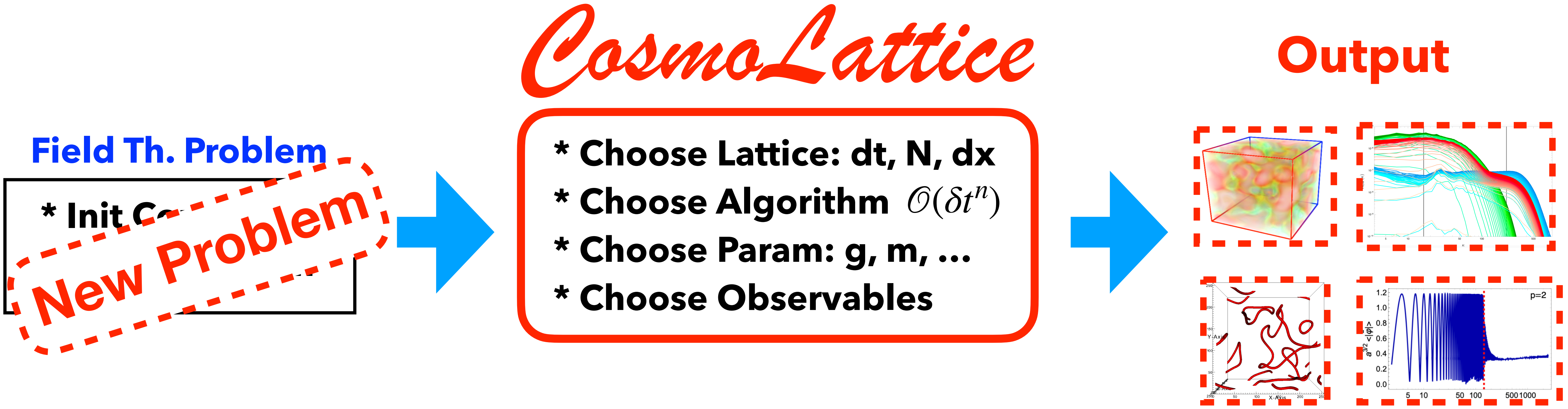

Field Th. Problem

* Init Co

New Problem

CosmoLattice

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- * Choose Algorithm $\mathcal{O}(\delta t^n)$
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Output

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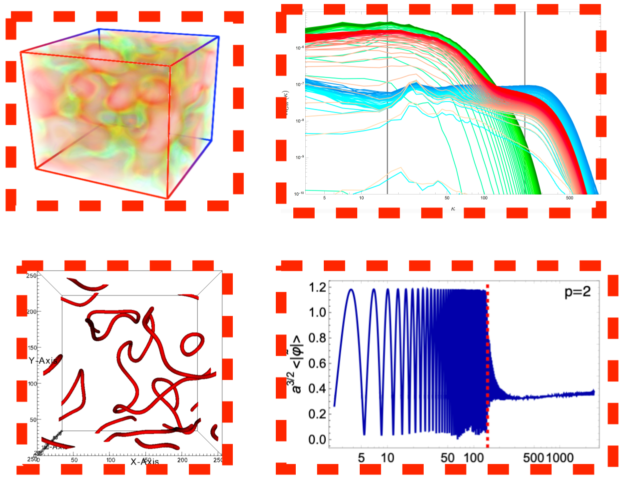

- **CL so far (v1.0, Public):**
  - Global scalar field dynamics
  - U(1) scalar-gauge dynamics
  - SU(2) scalar-gauge dynamics

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CosmoLattice

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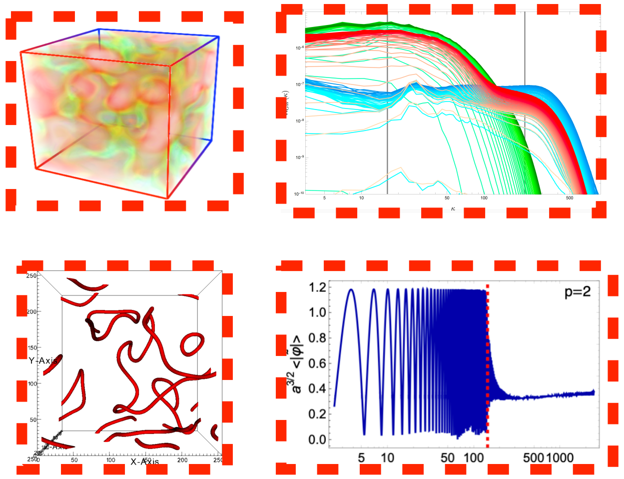

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- ➤ **CL update (v2.0, to be released by ~2024)**
  - ➤ Gravitational waves $\Box h_{ij} = 2\Pi_{ij}^{\rm TT}$
  - ➤ Axion-like couplings $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$
  - ➤ Non-minimal coupling $\xi\phi^2 R$
  - ➤ Cosmic String Networks

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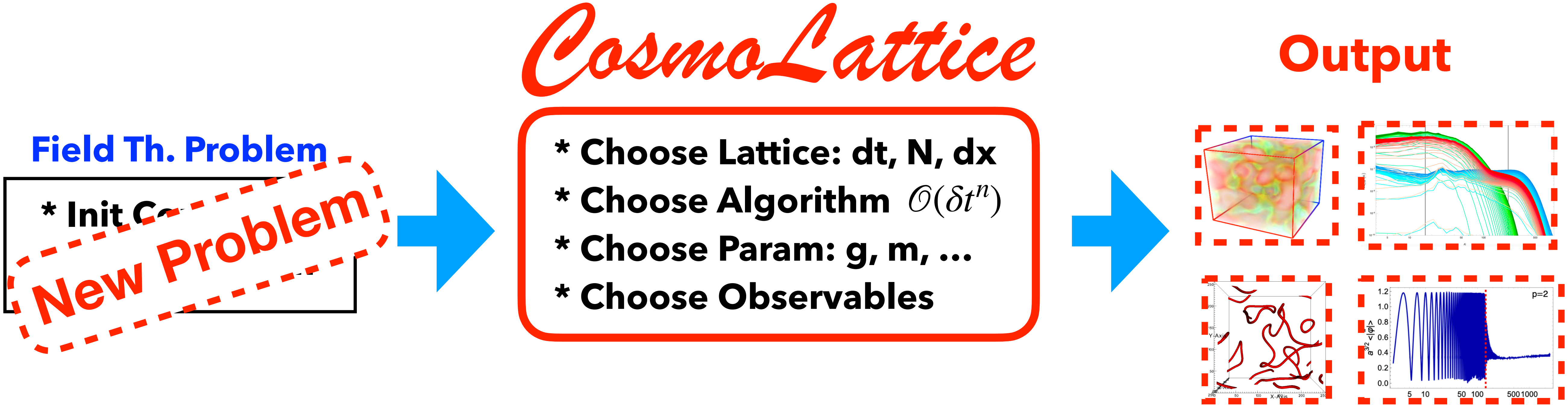

Field Th. Problem

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New Problem

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Released in 2022/23 !

New Modules

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic string networks
* Non-minimal Grav. coupling

New
Modules

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
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Magneto Hydro-dynamics (MHD)

$$T^{\mu\nu} = (p+\rho)U^{\mu}U^{\nu} - pg^{\mu\nu}$$

$$D_{\nu}T^{\mu\nu} = \partial_{\nu}T^{\mu\nu} + \Gamma^{\mu}_{\sigma\nu}T^{\sigma\nu} + \Gamma^{\nu}_{\nu\sigma}T^{\mu\sigma} = \frac{1}{\sqrt{-g}}\partial_{\nu}(\sqrt{-g}\,T^{\mu\nu}) = 0,$$

$$\boxed{\begin{aligned} \partial_{\eta}\tilde{T}^{00} + \partial_{i}\tilde{T}^{0i} &= \tilde{S}^{0}[\phi, A_k, \{\tilde{T}_{lk}\}]\,, \\ \partial_{\eta}\tilde{T}^{0i} + \partial_{j}\tilde{T}^{ij} &= \tilde{S}^{i}[\phi, A_k, \{\tilde{T}_{lk}\}]\,, \end{aligned}}$$

Work in progress … key to GWs from PhT's !

**(w/ K. Marschall, A. Midiri,
and A. Roper Pol)**

New
Physics

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic string networks
* Non-minimal Grav. coupling

Axion-gauge interactions

$$\mathcal{S}_{\rm ax} = -\int \mathrm{d}x^4 \sqrt{-g}\,\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_\Lambda}{4}\frac{\phi}{m_p}F_{\mu\nu}\tilde{F}^{\mu\nu}\}$$

$$\left.\begin{aligned}
\ddot{\phi} &= -3H\dot{\phi} + \tfrac{1}{a^2}\vec{\nabla}^2\phi - V_{,\phi} + \tfrac{\alpha_\Lambda}{a^3 m_p}\,\overrightarrow{E}\cdot\overrightarrow{B}\,, \\
\dot{\vec{E}} &= -H\vec{E} - \tfrac{1}{a^2}\vec{\nabla}\times\vec{B} - \tfrac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\overrightarrow{B} + \overrightarrow{\nabla}\phi\cdot\overrightarrow{E}\right), \\
\ddot{a} &= -\tfrac{a}{3m_p^2}\left(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}\right), \\
\vec{\nabla}\cdot\vec{E} &= -\tfrac{\alpha_\Lambda}{a m_p}\overrightarrow{\nabla}\phi\cdot\overrightarrow{B}\,, & \texttt{[Gauss law]} \\
H^2 &= \tfrac{1}{3m_p^2}\left(\rho_{\rm K} + \rho_{\rm G} + \rho_{\rm V} + \rho_{\rm EM}\right), & \texttt{[Hubble law]}
\end{aligned}\right\}$$

Used in *Phys. Rev. Lett. 131 (2023) 15, 151003*

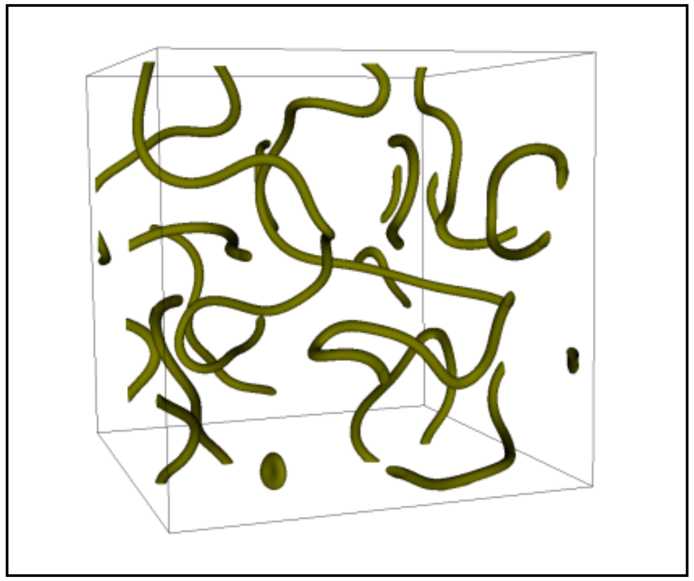
(Topic I)

New Physics

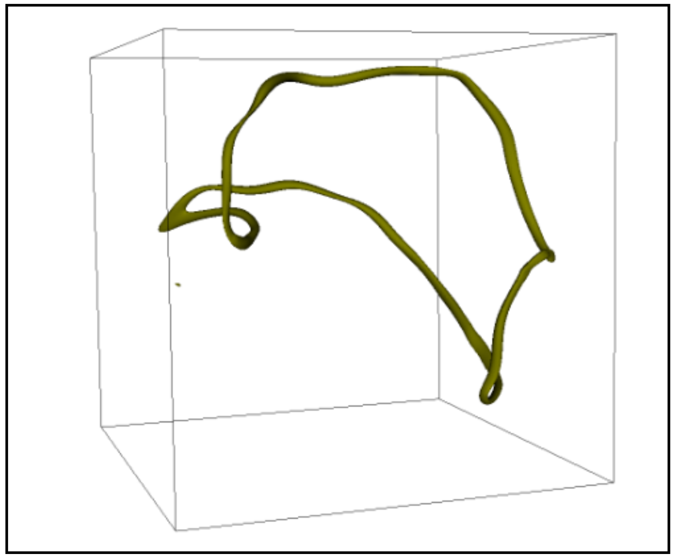
* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic defect networks
* Non-minimal Grav. coupling

Cosmic Defect Networks

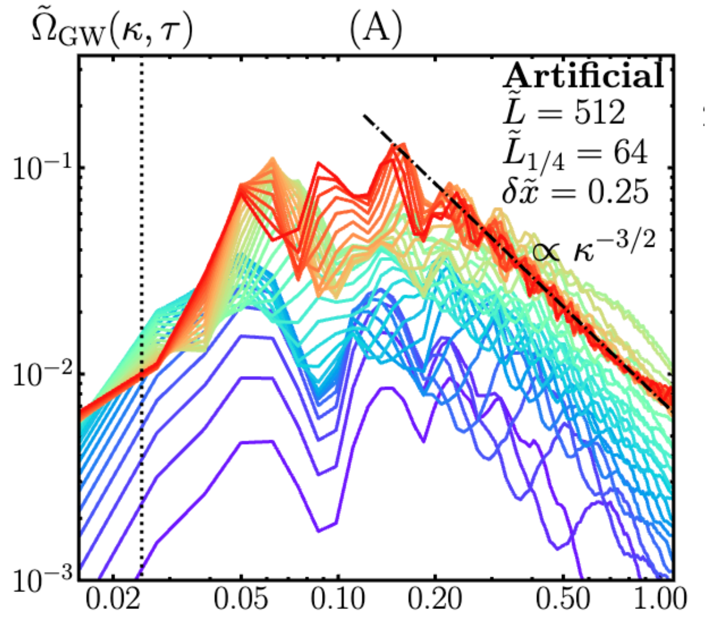
Particle & GW Emission



(Networks)



(Loops isolated)



(GW emission)

Used in *arXiv:2308.08456 ; Phys. Rev. Lett. (submitted)*

(Topic II)

New
Physics

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic string networks
* Non-minimal Grav. coupling

Non-minimal Grav. coupling

$$\mathcal{S}_{\rm NMC} = -\int d^4x\sqrt{-g}\left(\frac{1}{2}\xi R\phi^2+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi,\{\varphi_{\rm m}\})\right)$$

$$[\texttt{Non}-\texttt{minimally coupled}]\begin{cases}\phi' = a^{\alpha-3}\pi_\phi\,,\\ \pi'_\phi = a^{1+\alpha}\,\nabla^2\phi - a^{3+\alpha}\left(\xi R\phi + V_{,\phi}\right)\,,\end{cases}$$

$$[\texttt{Expanding background}]\begin{cases}a' = a^{\alpha-1}\pi_a\,,\\ \pi'_a = \frac{a^{2+\alpha}}{6}R\,,\end{cases}$$

with

$$R = \frac{1}{m_p^2}\left[\frac{2\left(1-6\xi\right)\left(E_G^\phi - E_K^\phi\right) + 4\langle V\rangle - 6\xi\langle\phi\, V_{,\phi}\rangle + \left(\rho_{\rm m} - 3p_{\rm m}\right)}{1+\left(6\xi-1\right)\xi\langle\phi^2\rangle/m_p^2}\right]\,,$$

(w/ B. Stefanek, T. Opferkuch, and A. Florio)

New
Physics

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic string networks
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To be released in 2024/25 !

New
Physics

* Magneto Hydro-dynamics (MHD)
* Axion-gauge interactions
* Cosmic string networks
* Non-minimal Grav. coupling
* Grav. Pert. Th / Full GR

(w/ N. Loayza & R. Flauger)

To be released in 202X?

Applications

1) Non-linear inflation dynamics
2) GW from non-linear dynamics
3) Preheating & Equation of State after inflation
4) Cosmic string networks (axions, AH, …)
5) Single string loop dynamics
6) Non-minimal gravitational Interactions
7) Phase transitions
….
X) Your project !

Applications

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2) GW from non-linear dynamics

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5) Single string loop dynamics

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….

X) Your project !

Applications

I) Non-linear inflation dynamics

II) Single string loop dynamics

Applications

I) Non-linear inflation dynamics

II) Single string loop dynamics

Applications

I) Non-linear inflation dynamics

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Applications

I) Non-linear inflation dynamics (e.g Axion-inflation)

II) Single string loop dynamics

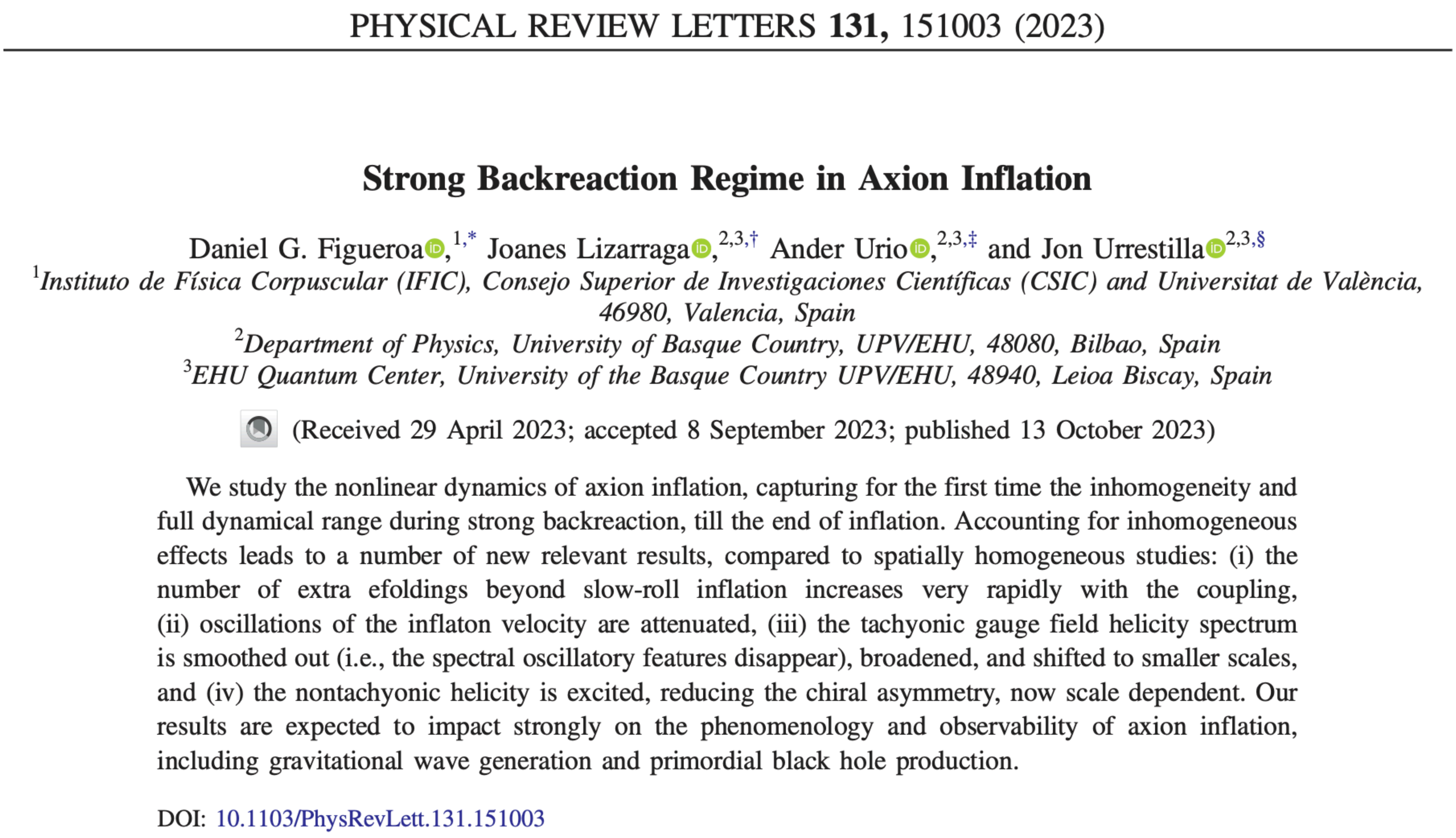
Applications

I) Non-linear inflation dynamics (e.g Axion-inflation)

II) Single string loop dynamics } (If time permits)

Example I

(Non-Linear) Field Dynamics of Axion Inflation



PHYSICAL REVIEW LETTERS **131,** 151003 (2023)

Strong Backreaction Regime in Axion Inflation

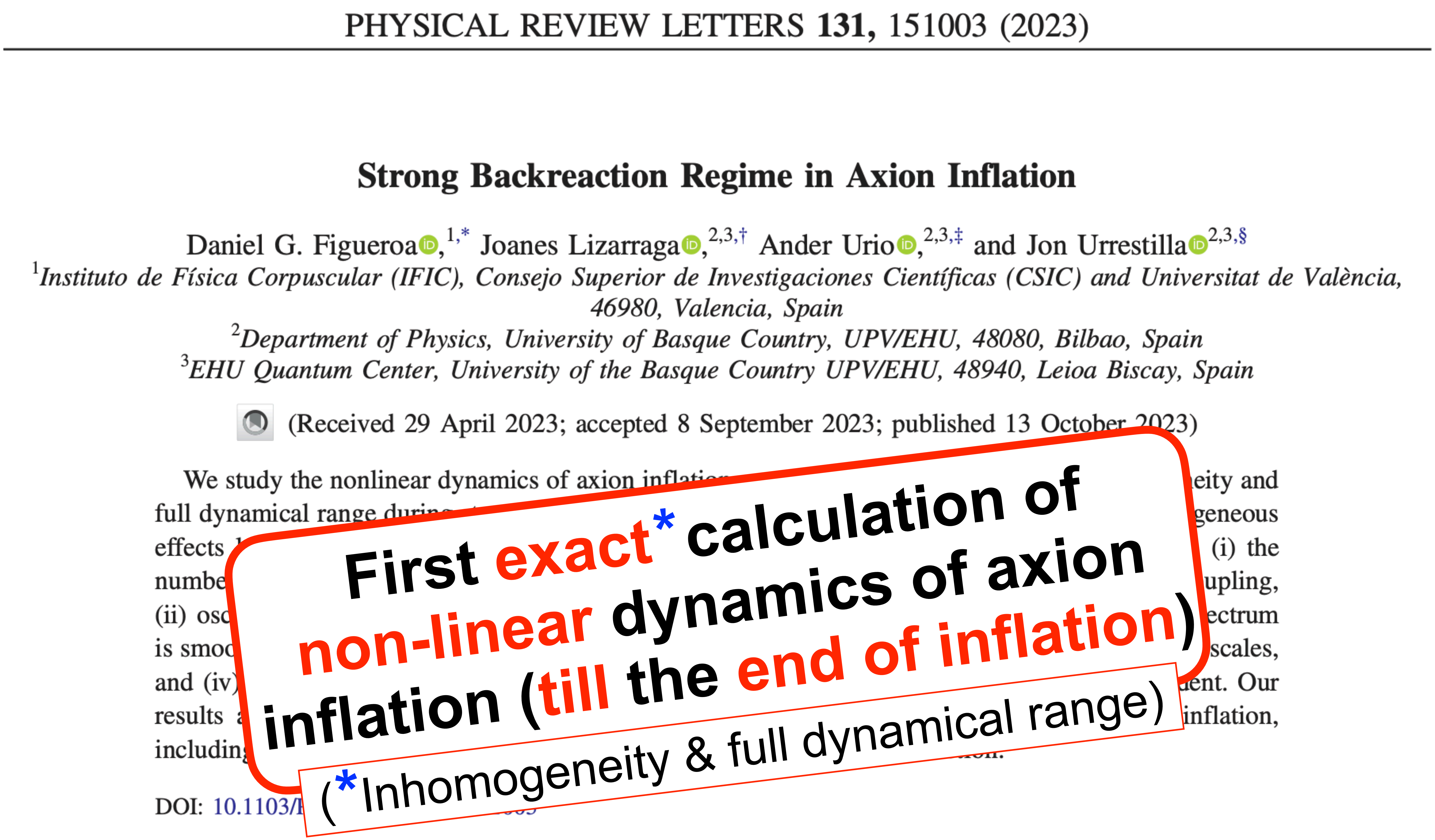
Daniel G. Figueroa,[1,*] Joanes Lizarraga,[2,3,†] Ander Urio,[2,3,‡] and Jon Urrestilla[2,3,§]
[1]*Instituto de Física Corpuscular (IFIC), Consejo Superior de Investigaciones Científicas (CSIC) and Universitat de València, 46980, Valencia, Spain*
[2]*Department of Physics, University of Basque Country, UPV/EHU, 48080, Bilbao, Spain*
[3]*EHU Quantum Center, University of the Basque Country UPV/EHU, 48940, Leioa Biscay, Spain*

(Received 29 April 2023; accepted 8 September 2023; published 13 October 2023)

We study the nonlinear dynamics of axion inflation, capturing for the first time the inhomogeneity and full dynamical range during strong backreaction, till the end of inflation. Accounting for inhomogeneous effects leads to a number of new relevant results, compared to spatially homogeneous studies: (i) the number of extra efoldings beyond slow-roll inflation increases very rapidly with the coupling, (ii) oscillations of the inflaton velocity are attenuated, (iii) the tachyonic gauge field helicity spectrum is smoothed out (i.e., the spectral oscillatory features disappear), broadened, and shifted to smaller scales, and (iv) the nontachyonic helicity is excited, reducing the chiral asymmetry, now scale dependent. Our results are expected to impact strongly on the phenomenology and observability of axion inflation, including gravitational wave generation and primordial black hole production.

DOI: 10.1103/PhysRevLett.131.151003

(e-Print: 2303.17436 [astro-ph.CO])



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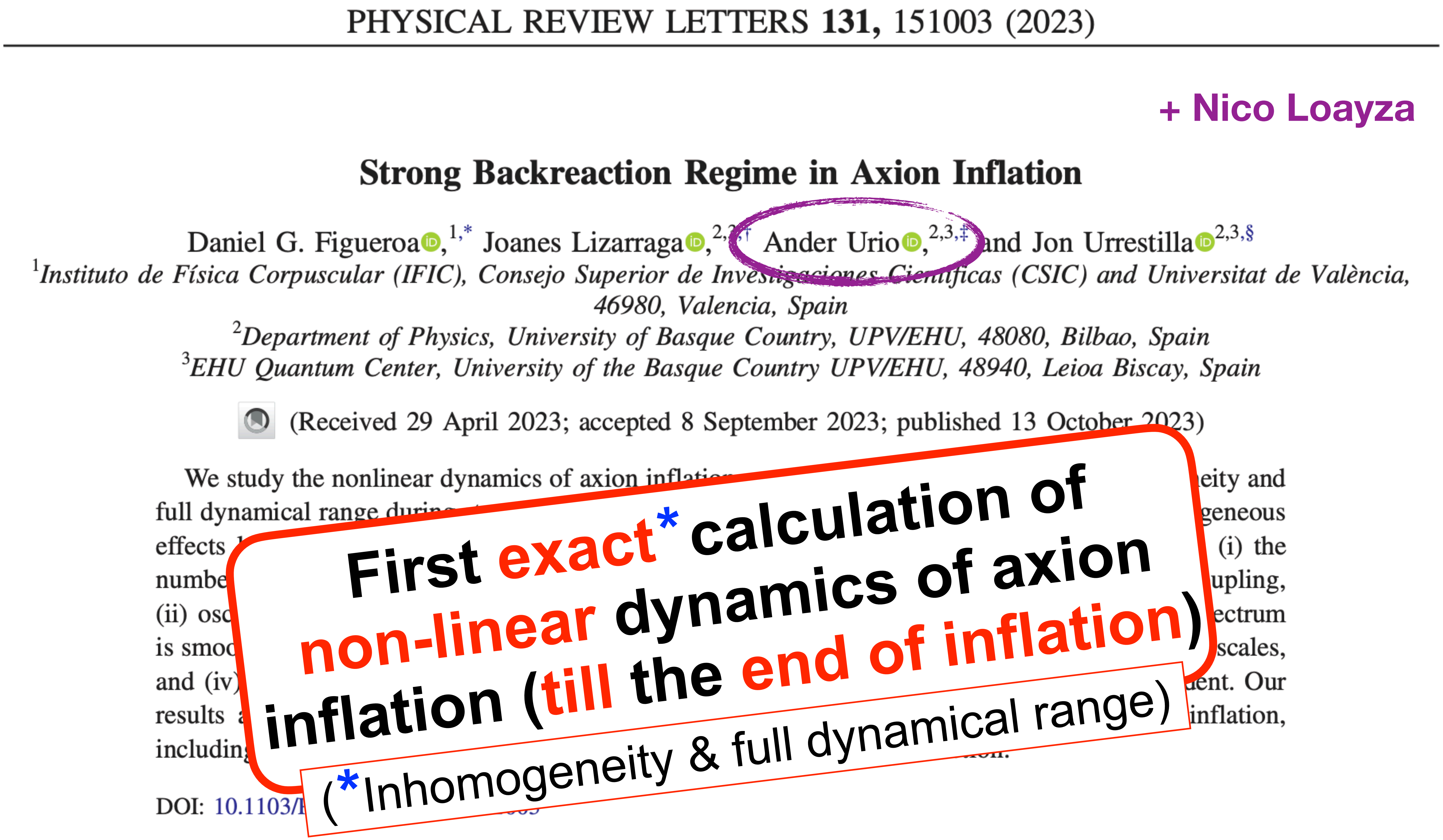
DOI: 10.1103/P … 003

First exact* calculation of non-linear dynamics of axion inflation (till the end of inflation)

(*Inhomogeneity & full dynamical range)

(e-Print: 2303.17436 [astro-ph.CO])



PHYSICAL REVIEW LETTERS **131,** 151003 (2023)

+ Nico Loayza

Strong Backreaction Regime in Axion Inflation

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Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

Axion

=

Inflaton

Axion-Inflation

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$$\frac{\phi}{4\Lambda} F\tilde{F}$$

(Shift Symm.)

$$\phi \longrightarrow \phi + const\,.$$

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Gauge Sector

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Potential

$$V(\phi) + \frac{\phi}{4\Lambda} F\tilde{F}$$

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Due to
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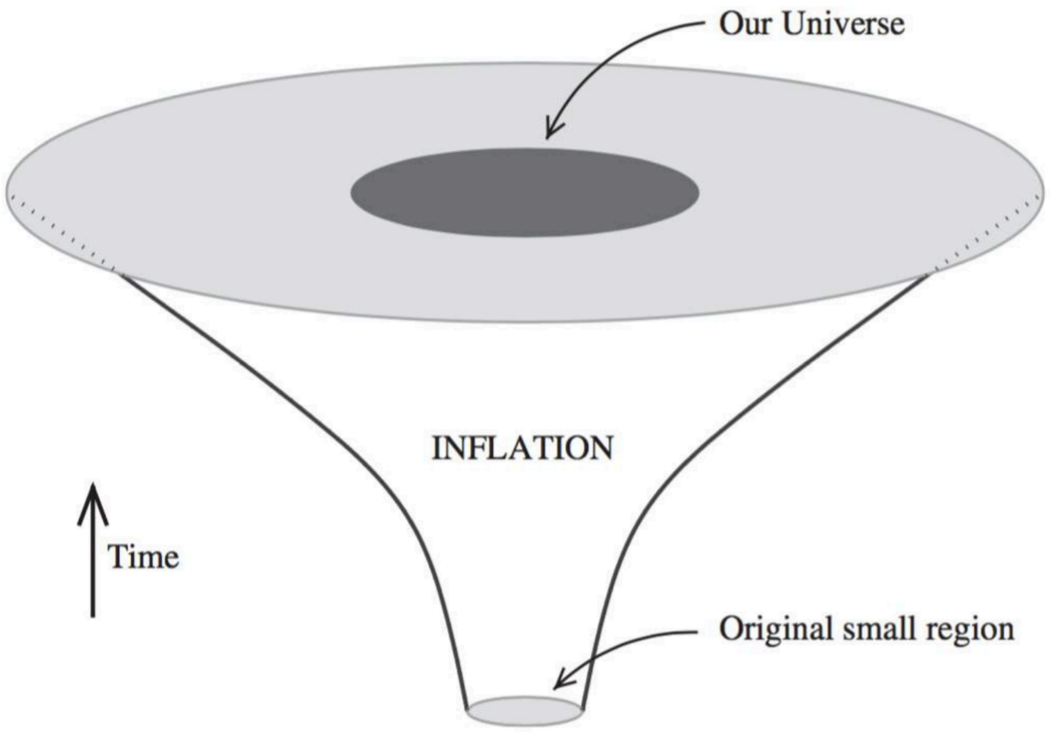

Inflation

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$$V(\phi) + \frac{\phi}{4\Lambda} F\tilde{F} \qquad \Rightarrow \qquad A''_{\pm} + \left(k^2 \pm \frac{k\dot{\phi}}{\Lambda H \tau}\right) A_{\pm} = 0$$

(Shift Symm.)

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Gauge field dynamics during inflaton

Axion-Inflation

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L.Sorbo et al 2006-2012

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Chiral instability

$$A_+ \propto e^{\pi\xi}, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified

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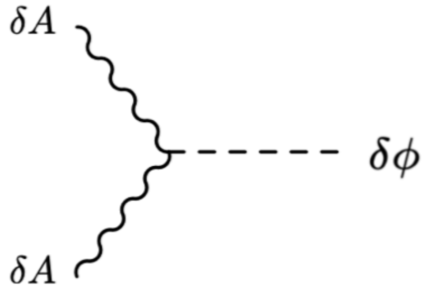
Inflaton perturbations $\delta\phi$
through inverse decay
(highly non-Gaussian)



Barnaby, Peloso '10
Planck '15

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Amplitude $\delta\phi$ must be bounded

Otherwise too many

Primordial Black Holes (PBH) !

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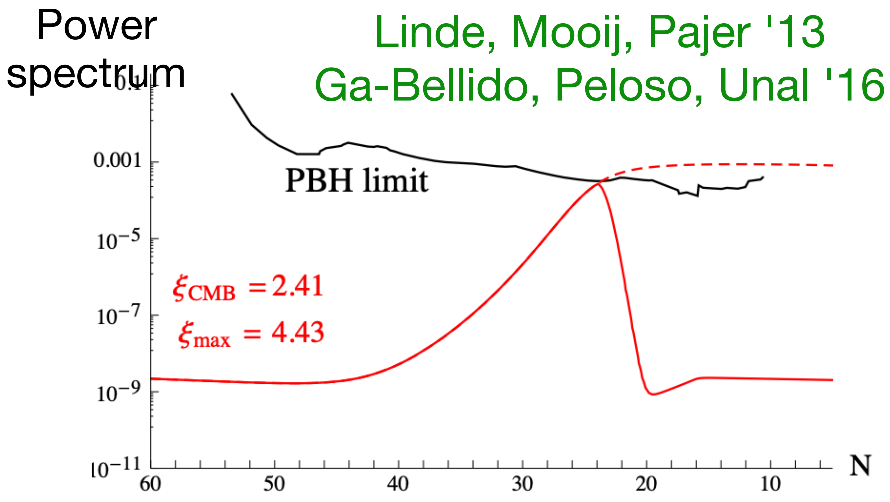

Linde, Mooij, Pajer '13
Ga-Bellido, Peloso, Unal '16

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one chirality
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then… chiral GWs !

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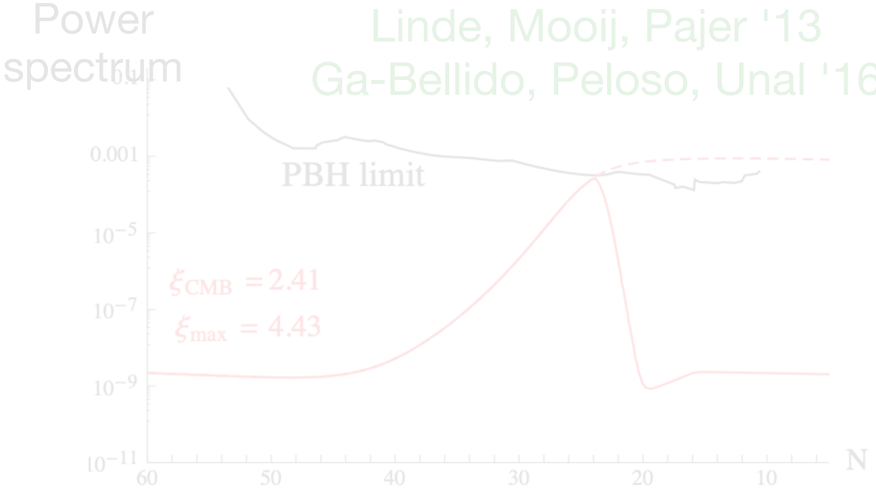

Linde, Mooij, Pajer '13
Ga-Bellido, Peloso, Unal '16

$$\{E_i E_j + B_i B_j\}^{\rm TT}$$

$h_L \quad , \quad \cancel{h_R}$

Cook & Sorbo '11
Amber & Sorbo ‘12

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of gauge field
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Can we trust current pheno calculations ?

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Problem ?

As $A_+ \propto e^{\dot{\phi}}$, pNG, PBH, GW very sensitive
to choice of $V(\phi)$ and calculation details

Problem ?

Axion-Inflation

PROBLEM: **PNG, GW and PBH** ⟶ **Approximations (e.g. Analytical)**

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ Approximations (e.g. Analytical)

Let’s have a look to the full problem !

$$\left(V(\phi) = \frac{1}{2} m^2 \phi^2 \right)$$

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$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

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EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\ \\ \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

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Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \Big\rangle$$

$$\left(\begin{gathered} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{gathered} \right)$$

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→ **Non-Linear dynamics**

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EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde\pi}_\phi = \cancel{a\vec\nabla^2\phi} - a^3 m^2\phi \cancel{+ \frac{1}{a\Lambda}\tilde{\vec E}\cdot\vec B}\ , \\ \dot{\tilde{\vec E}} = -\frac{1}{a}\vec\nabla\times\vec B - \frac{1}{a^3\Lambda}\tilde\pi_\phi\vec B + \cancel{\frac{1}{a\Lambda}\vec\nabla\phi\times\tilde{\vec E}}\ . \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\right\rangle$$

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EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

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Back-Reaction

Hom. EoM $(\mathbf{t})$

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Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

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Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$$\begin{tikzcd}
 & \lambda = +\,,\text{if}\phi > 0 \\
\lambda = \pm \arrow[ur] \arrow[dr] & \\
 & \lambda = -\,,\text{if}\phi < 0
\end{tikzcd}$$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases}
\dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}}\cdot\vec{B} \right\rangle \\[2ex]
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\end{cases}$$

Hom. (t) Approx.

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Hom. EoM (t)

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\left(\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \right\rangle - \left\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A} \right\rangle\right)$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\,, \quad V \equiv \frac{1}{2}m^2\phi^2\,, \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde{E}_i^2}{a^4}\,, \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{array}\right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

$$\langle \vec{E}\vec{B}\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A\rangle \equiv \left\langle \frac{E^2+B^2}{2}\right\rangle = \frac{1}{a^4}\int\frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$$\begin{tikzcd} & \lambda = +\,,\mathrm{if}\phi > 0 \\ \lambda = \pm \arrow[ur] \arrow[dr] & \\ & \lambda = -\,,\mathrm{if}\phi < 0 \end{tikzcd}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\langle\vec{\tilde{E}}\cdot\vec{B}\rangle \\[2em] \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}\,.} \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V}\big\rangle - \big\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\big\rangle\Big)$$

DallAgata et al 2019, Domcke et 2020 → **Elaborated Iterative scheme !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$\lambda = \pm$

$\lambda = +\,,\, \text{if} \dot\phi > 0$

$\lambda = -\,,\, \text{if} \dot\phi < 0$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}}\cdot\vec{B} \right\rangle \\[2em] \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0 \quad \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\left(\left\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V}\right\rangle - \left\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\right\rangle\right)$$

Gorbar et al 2021 → **Gradient Expansion Formalism (GEF)**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

Can we do better than homogeneous backreaction ?

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \frac{1}{a\Lambda}\langle\tilde{\vec{E}}\cdot\vec{B}\,\rangle \\ \frac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0 \; \cancel{\frac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}} \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V}\big\rangle - \big\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\big\rangle\Big)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \boxed{\frac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}} \\ \frac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \ \cancel{\frac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\ . \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \right\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec\nabla^2\phi} - a^3 m^2 \phi + \frac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\ \dot{\tilde{\vec{E}}} = -\frac{1}{a}\vec\nabla\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\frac{1}{a\Lambda}\vec\nabla\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \right\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → Approximations (e.g. Analytical)

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \frac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B} \\ \dot{\vec{\tilde{E}}} = -\frac{1}{a}\vec{\nabla}\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}\ . \end{cases}$$

Back-Reaction (Fully Local)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle -\underset{\text{(Kin)}}{2K_\phi} + \underset{\text{(Pot)}}{V} - \left(\underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\right)\right\rangle$$

Axion-Inflation

PROBLEM: **PNG, GW and PBH** ⟶ **Approximations (e.g. Analytical)**

Let's "latticize" the system of EOM !

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\ \\ \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\,. \end{cases}$$

Back-Reaction (Fully Local)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - (\underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A})\right\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → Approximations (e.g. Analytical)

Let's "latticize" the system of EOM !

DGF, Shaposhnikov 2017
Canivete, DGF 2018

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ Approximations (e.g. Analytical)

Let's "latticize" the system of EOM !

DGF, Shaposhnikov 2017
Canivete, DGF 2018

1. Lattice Gauge Inv: $A_\mu \longrightarrow A_\mu + \Delta_\mu^+ \alpha$
2. Cont. Limit to $\mathcal{O}(dx^2)$
3. Lattice Bianchi Identities: $\Delta_i^-(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}) = 0\,, \ldots$
4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L = \Delta_\mu^+ K^\mu$ (**CS current**)

$[\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu \,]$

Exact Shift Sym. on the lattice !

Axion-Inflation

PROBLEM: **PNG, GW and PBH** ⟶ **Approximations (e.g. Analytical)**

Let's "latticize" the system of EOM !

DGF, Shaposhnikov 2017
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1. Lattice Gauge Inv: $A_\mu \longrightarrow A_\mu + \Delta^+_\mu \alpha$

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   $[\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu \,]$

   Exact Shift Sym. on the lattice !

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations (e.g. Analytical)**

Let's "latticize" the system of EOM !

DGF, Shaposhnikov 2017
Canivete, DGF 2018

We show now **our recent work**

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(CosmoLattice)

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \alpha_\Lambda = 18\right)$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

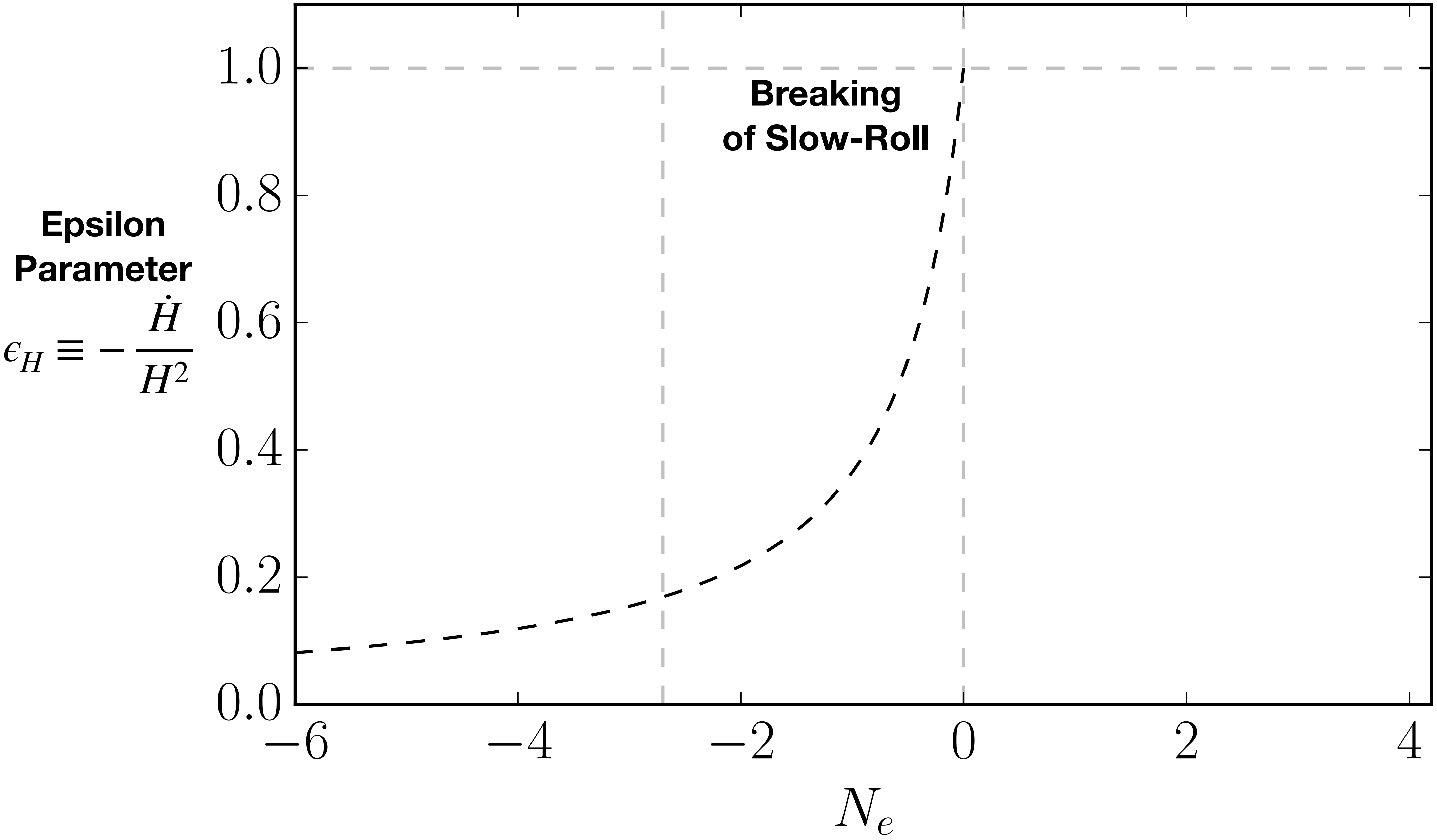
$$\alpha_\Lambda \equiv \frac{m_p}{\Lambda}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \alpha_\Lambda = 18$)

$$\alpha_\Lambda \equiv \frac{m_p}{\Lambda}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18$)

Linear regime (– – –)

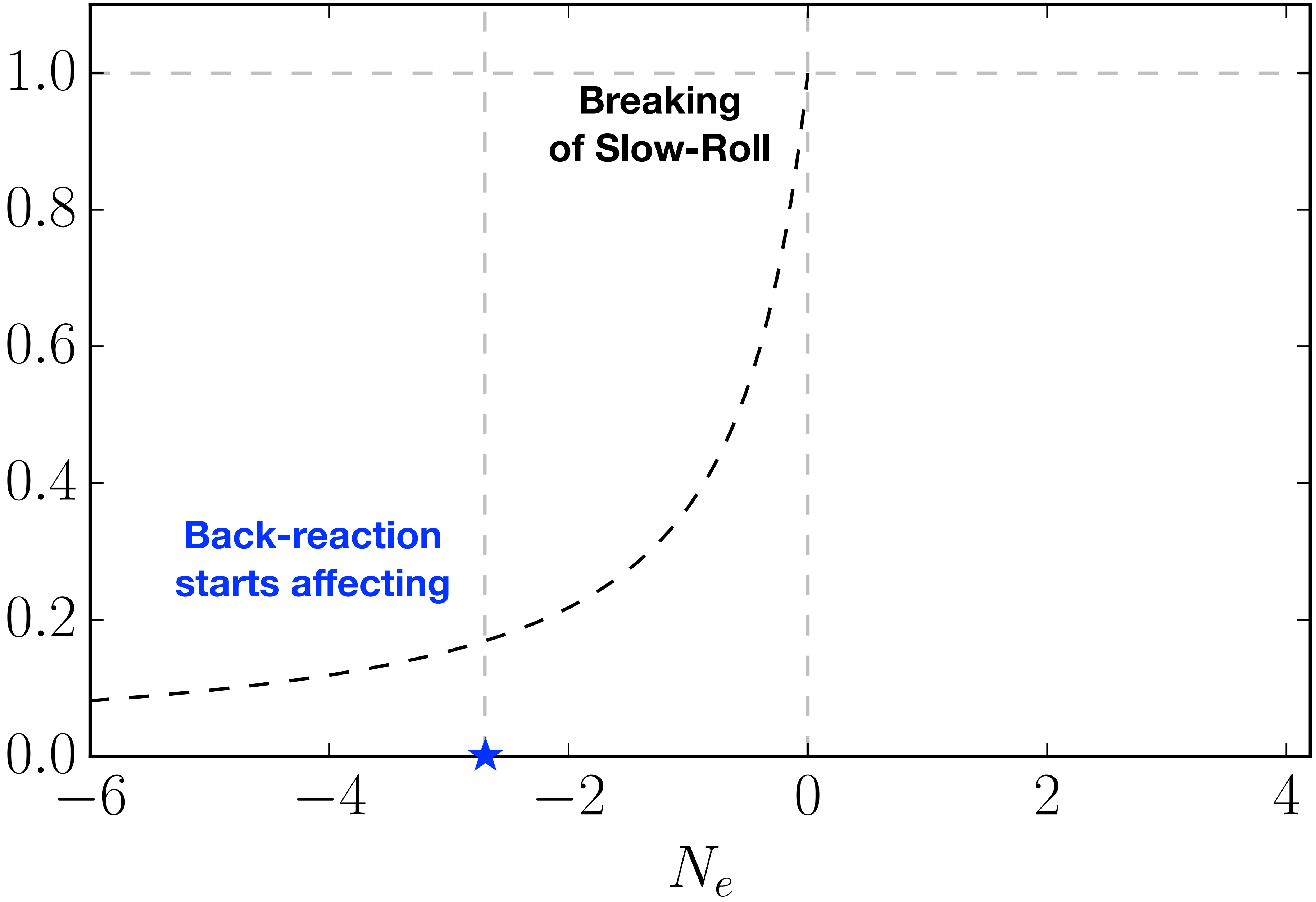


Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

Linear regime (– – –)

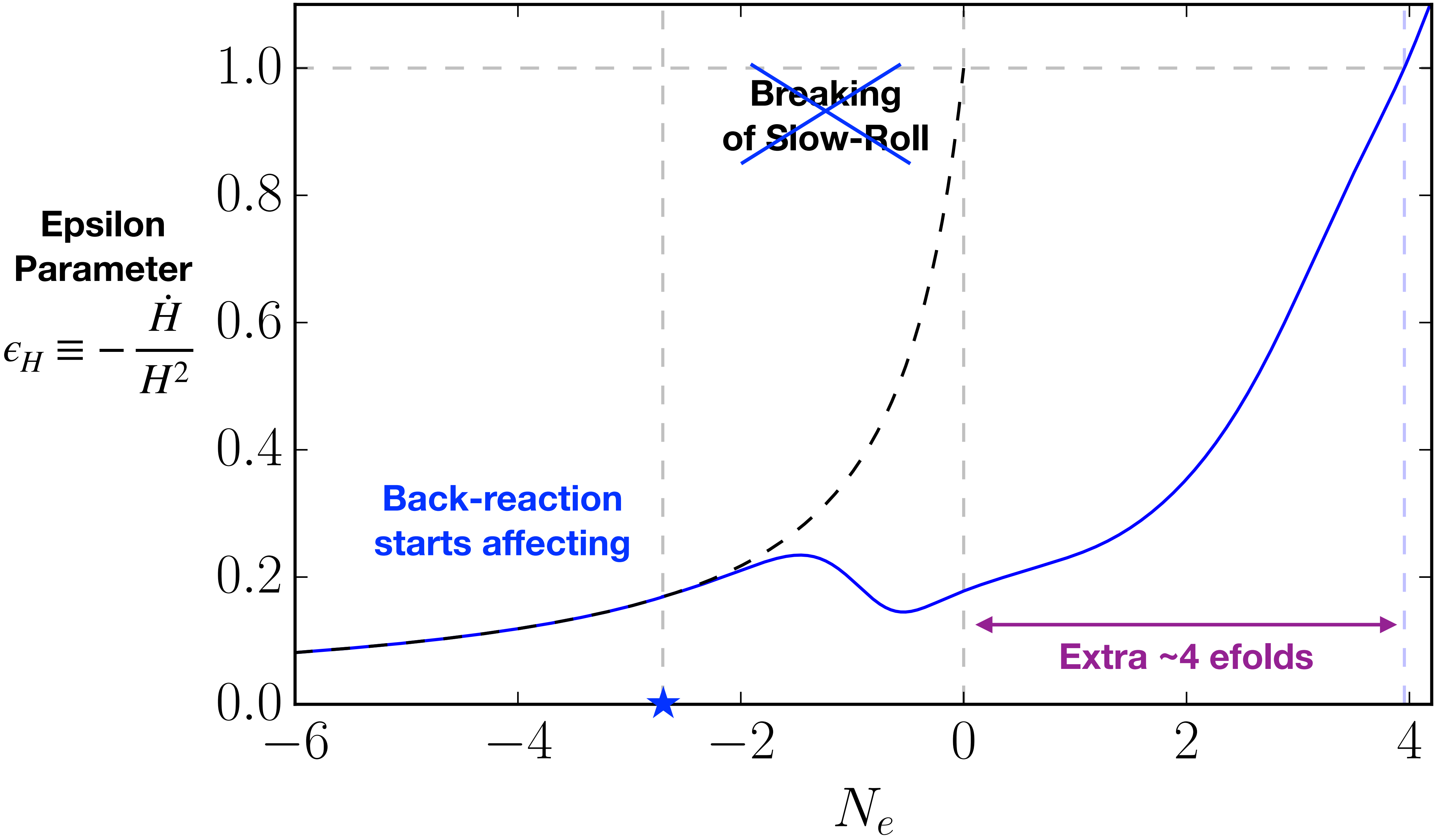
Epsilon Parameter

$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$

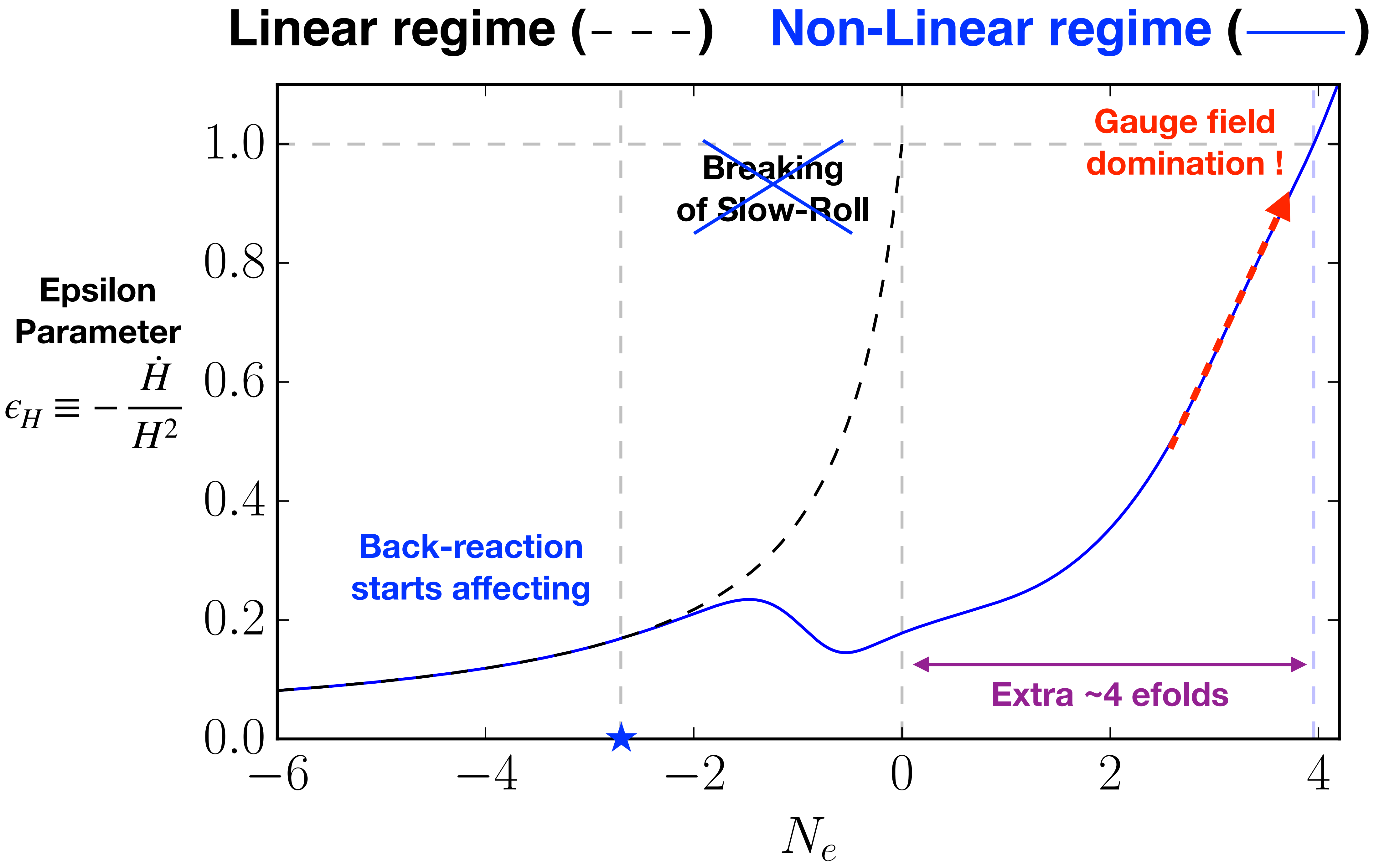


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

Linear regime (- - -) **Non-Linear regime (——)**



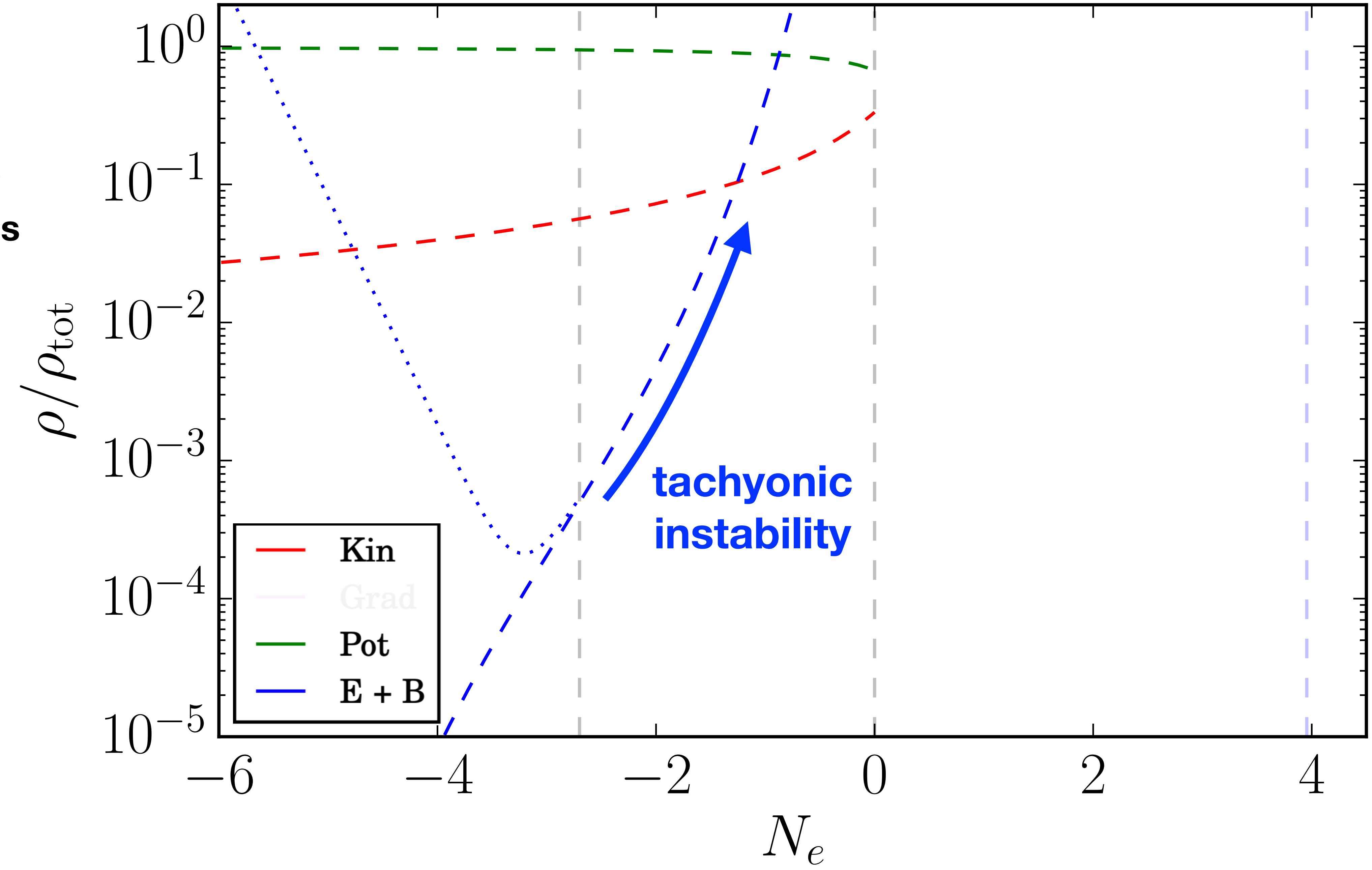
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18$)



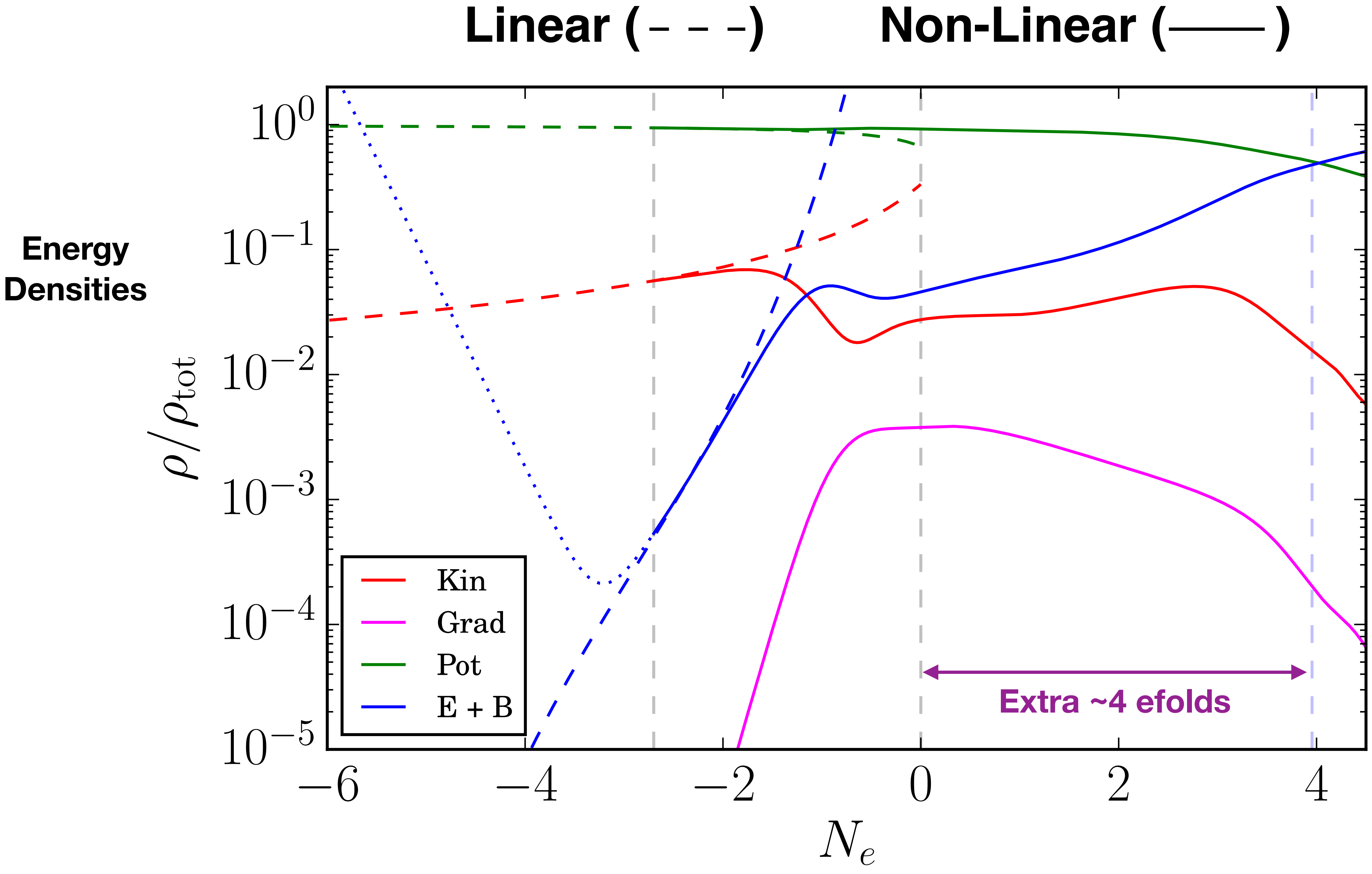
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

Linear (- - -)

Energy Densities



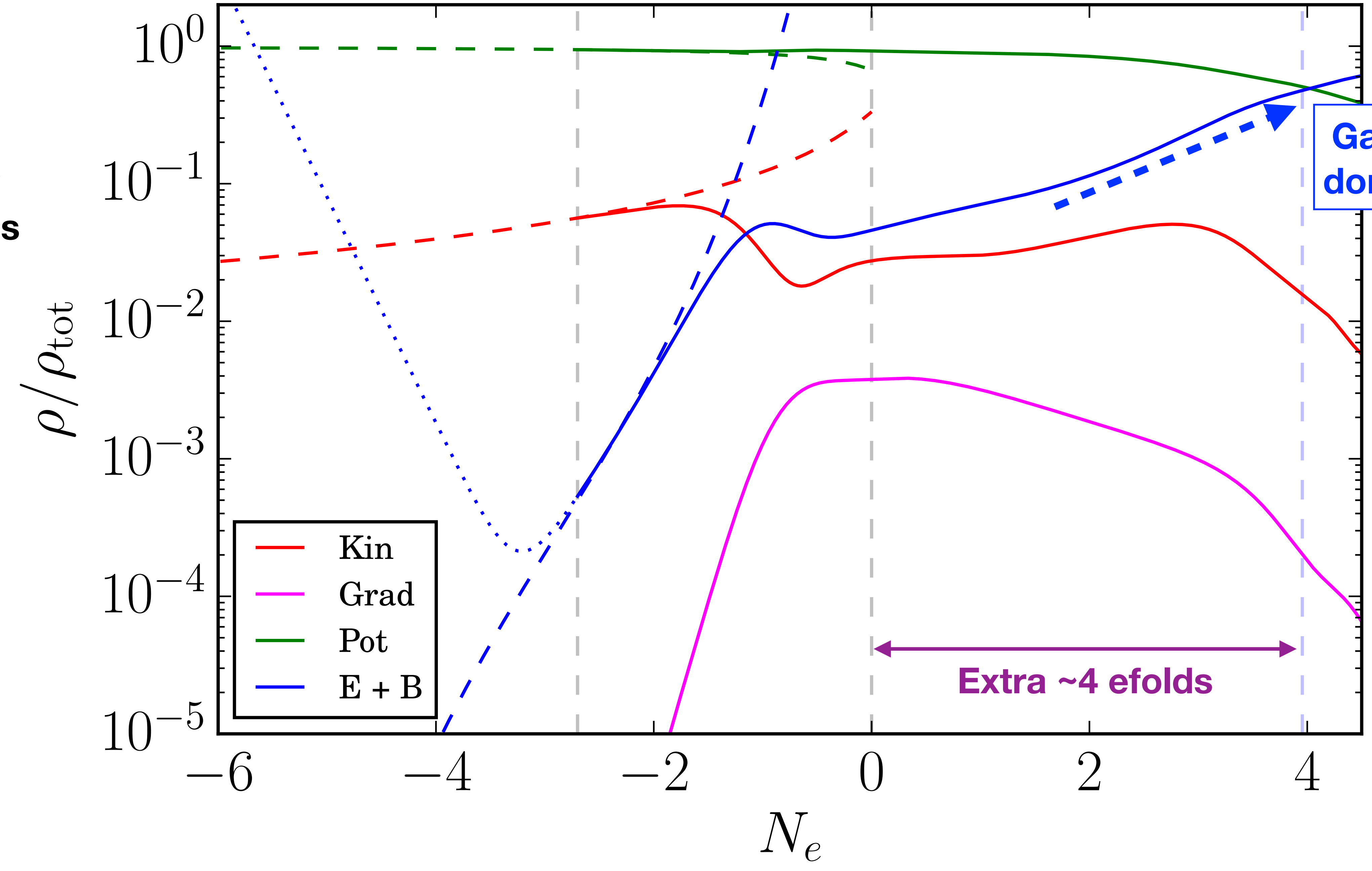
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

Linear (- - -) **Non-Linear (——)**

Energy Densities



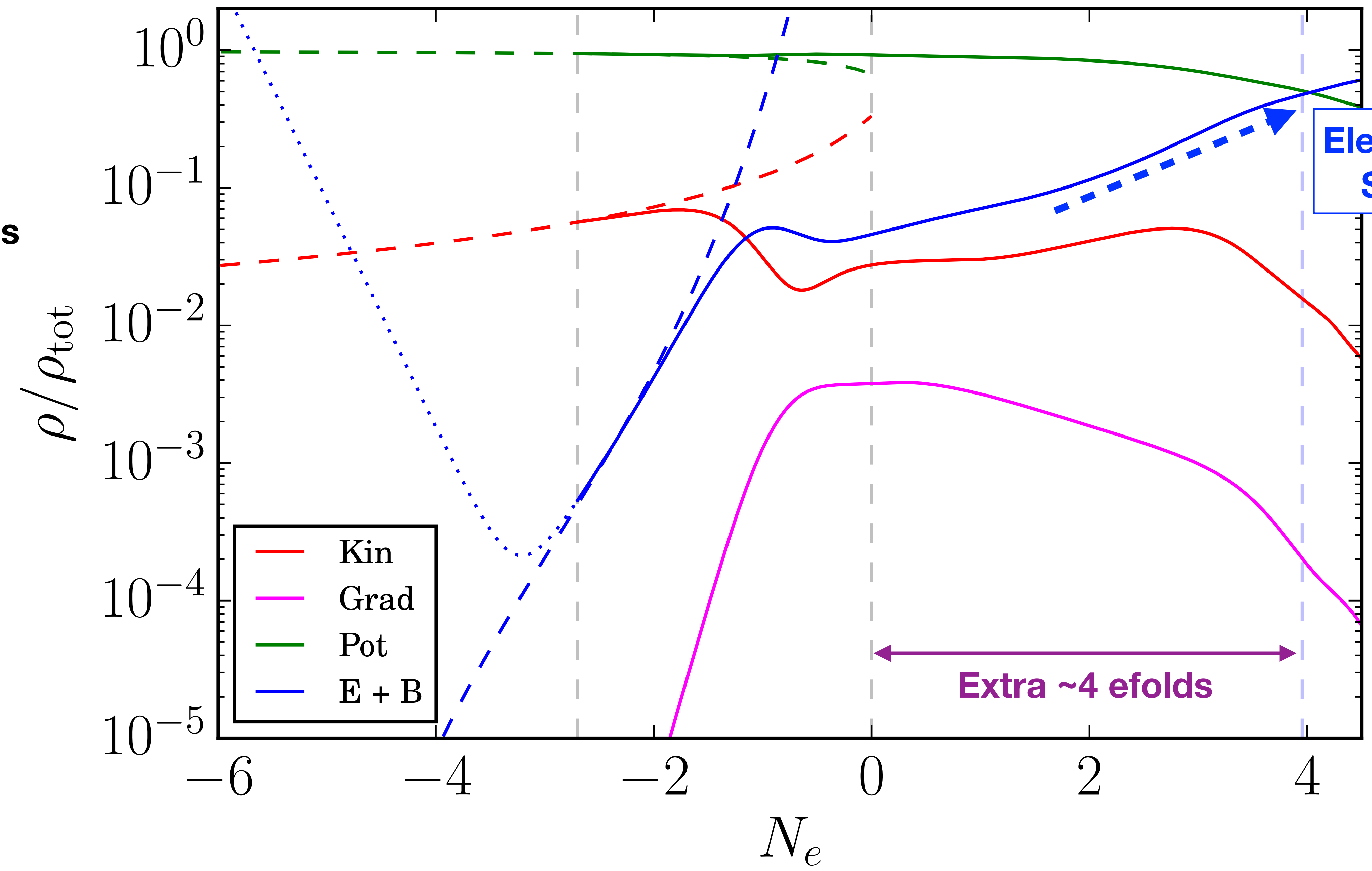
Gauge field domination !

Extra ~4 efolds

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

Linear (- - -) **Non-Linear (——)**

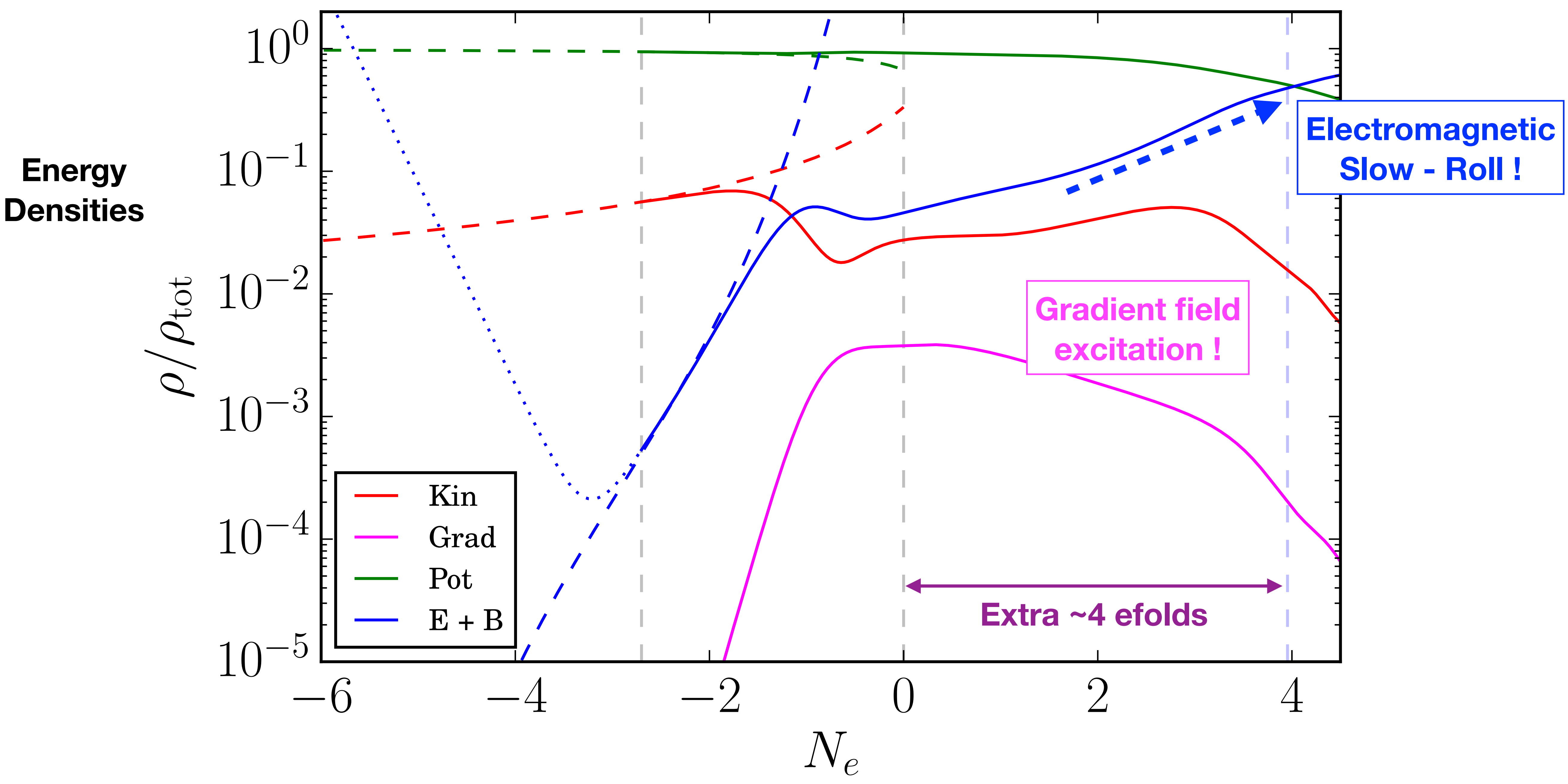
Energy Densities



Electromagnetic Slow - Roll !

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

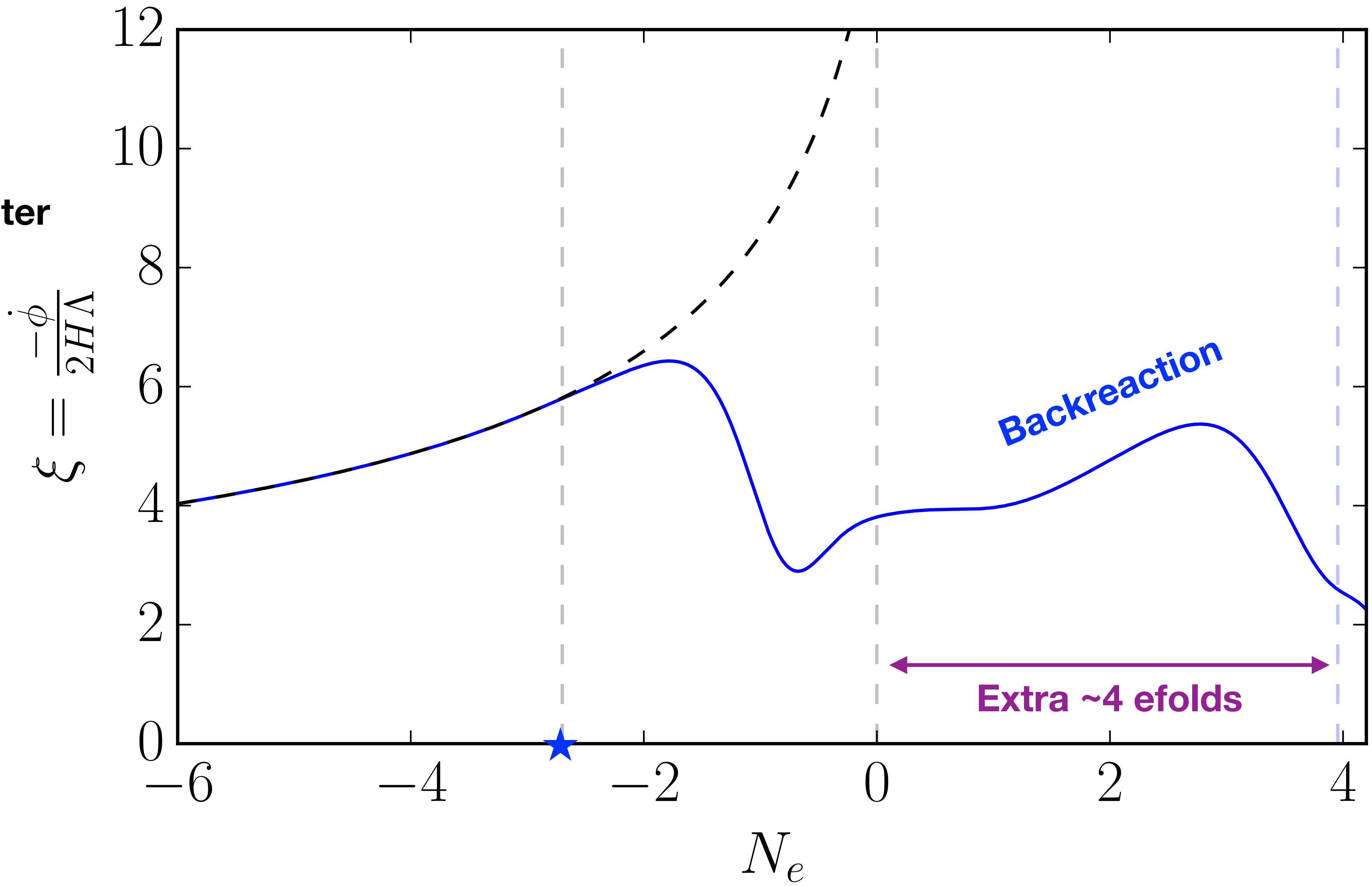
Linear (- - -) **Non-Linear (——)**



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

Linear (- - -) **Non-Linear** (——)

Xi parameter

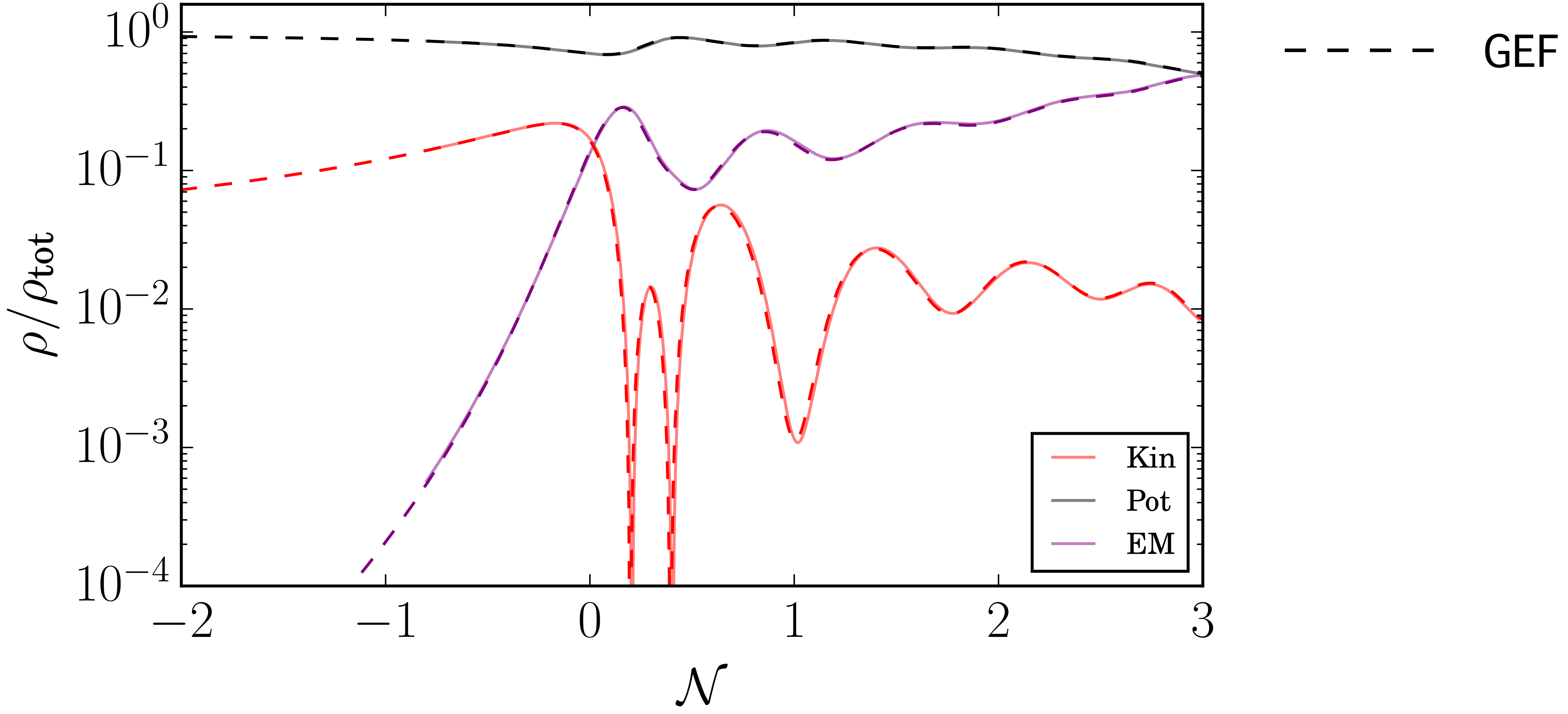


Comparison to Homogeneous Backreaction

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 15\right)$

Comparison to GEF (Homogeneous backreaction)

GEF (O. Sobol, R. von Eckardstein, K. Schmitz)



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 15\right)$

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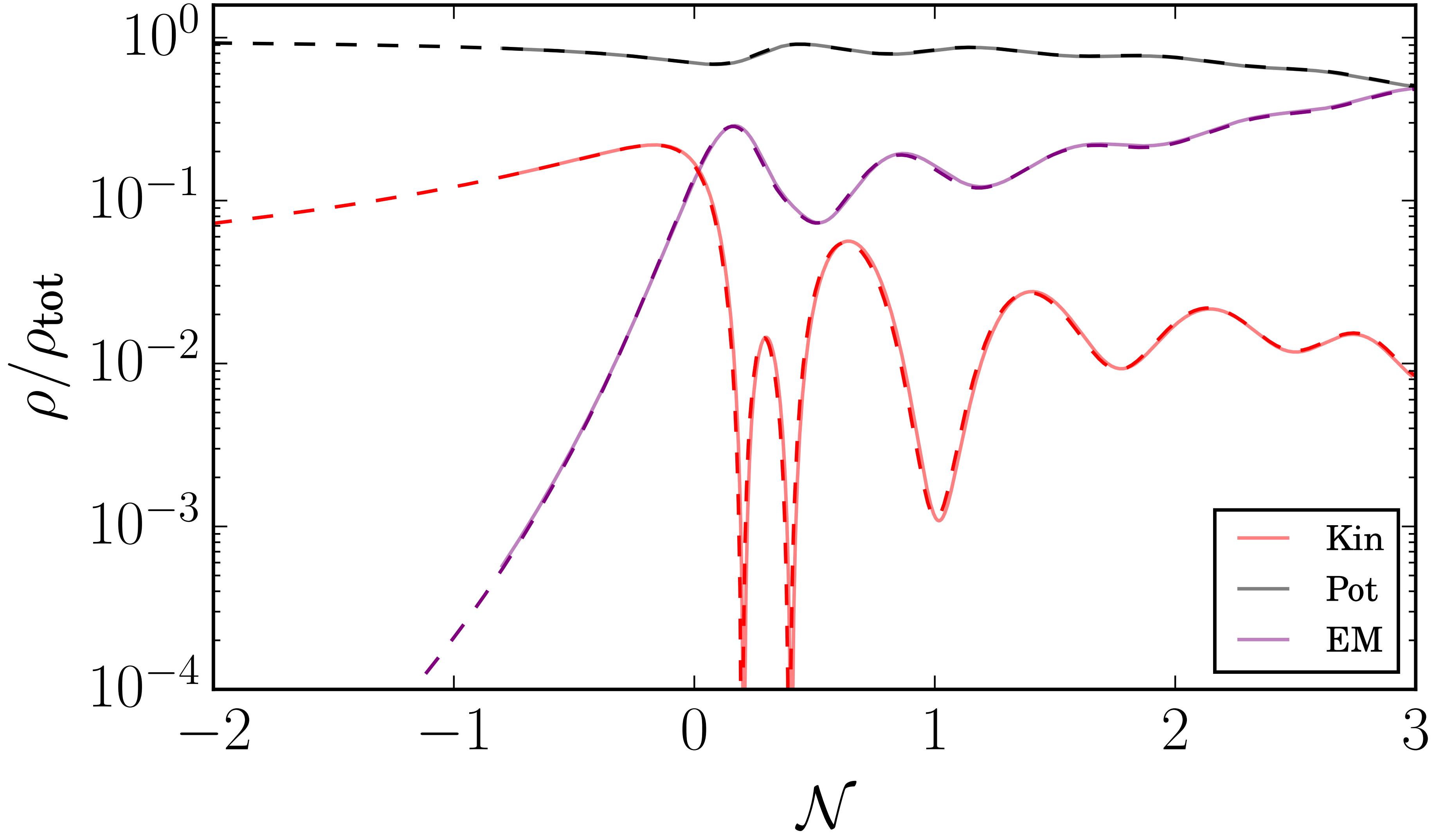


- - - - GEF

—— Lattice*

* Lattice:

$\nabla\phi \longrightarrow 0$

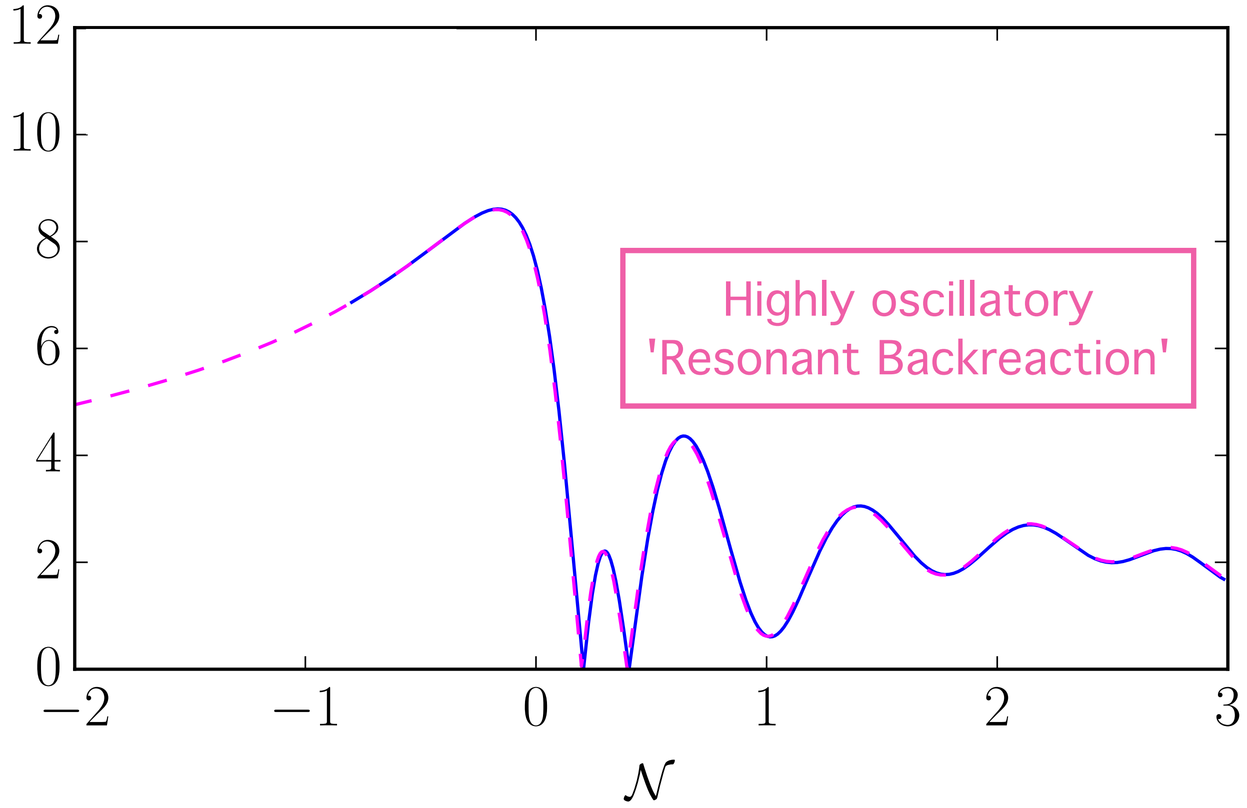
$\vec{E}\cdot\vec{B} \longrightarrow <\vec{E}\cdot\vec{B}>$

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 15\right)$

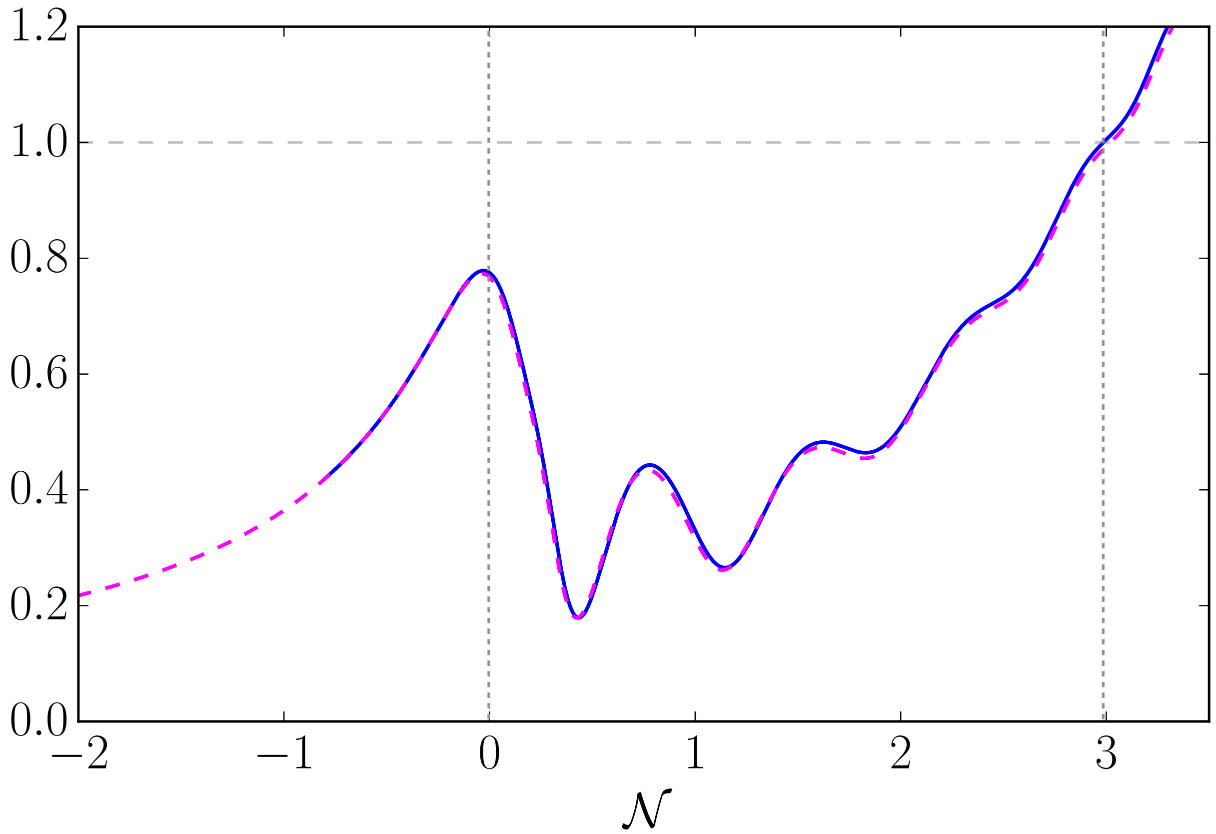
Comparison to GEF (Homogeneous backreaction)

GEF (O. Sobol, R. von Eckardstein, K. Schmitz)

$$\xi \equiv \frac{|\dot{\phi}|}{2H\Lambda}$$



$$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$$



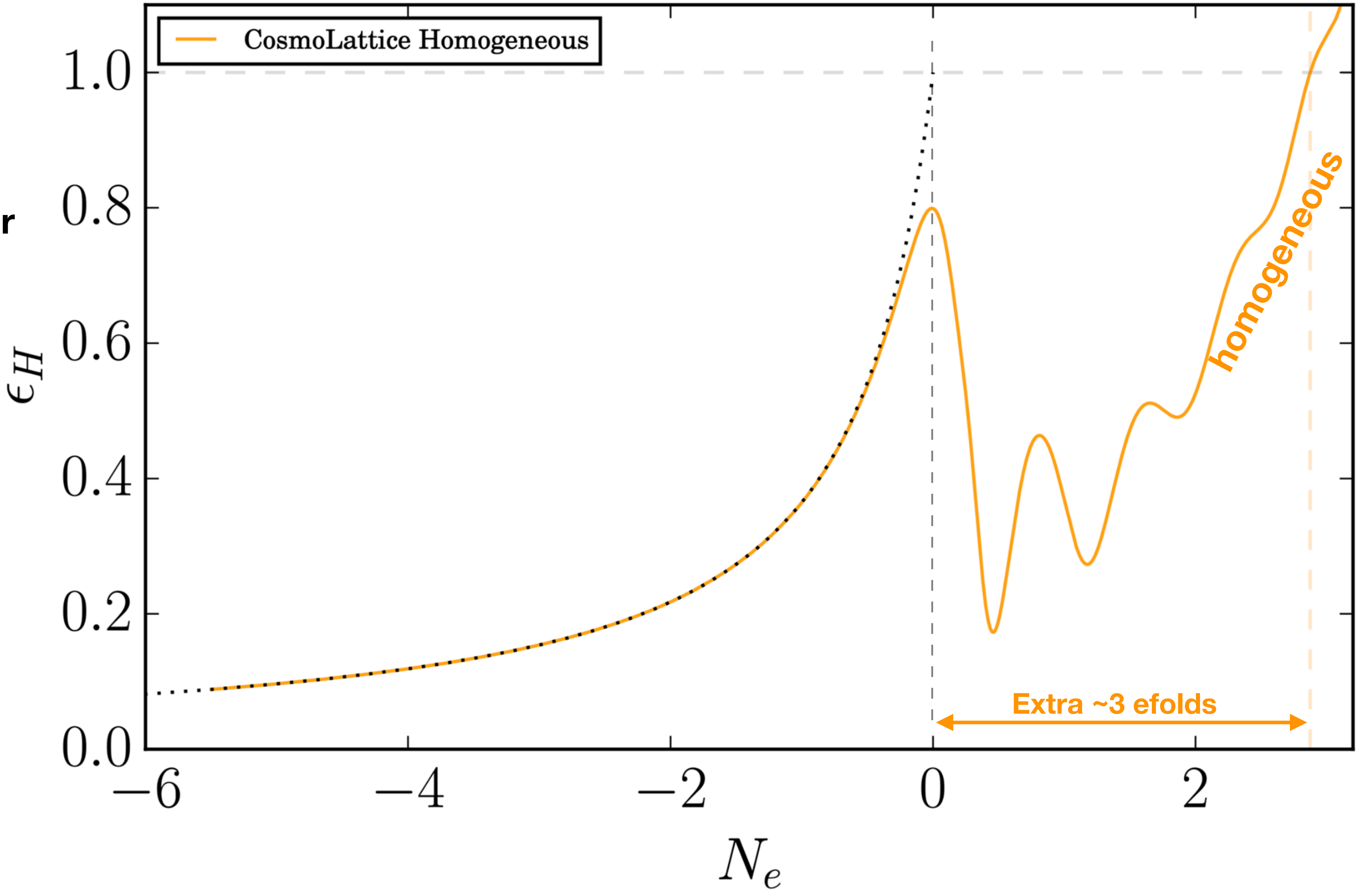
(GEF

Lattice*: $\nabla\phi \longrightarrow 0 \, , \; \vec{E}\cdot\vec{B} \longrightarrow \, < \vec{E}\cdot\vec{B} >$)

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \alpha_\Lambda = 15\right)$

Linear (········) **Non-Linear (——)**

Epsilon Parameter

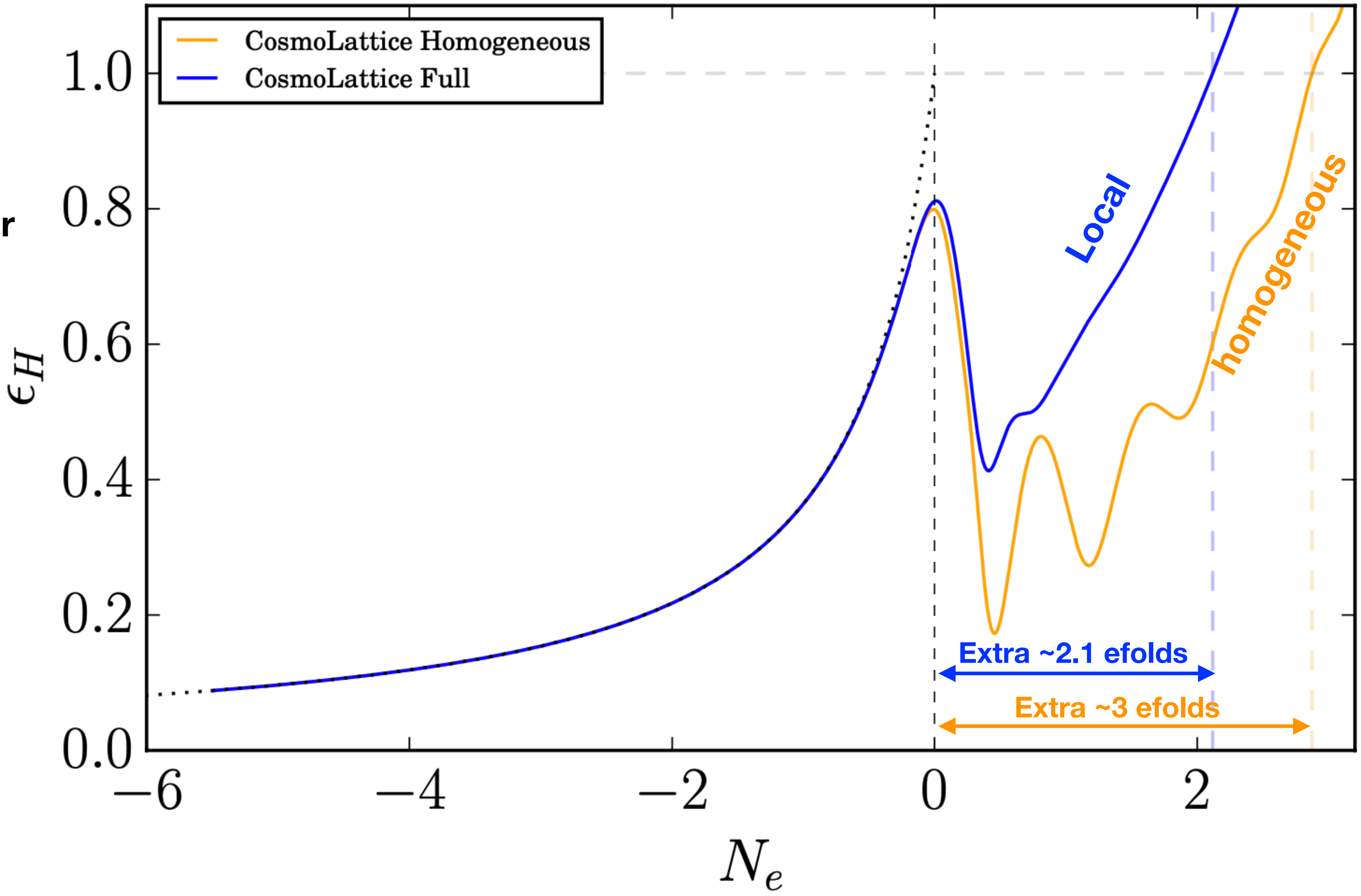


$\boxed{\alpha_\Lambda \equiv 15}$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 15$)

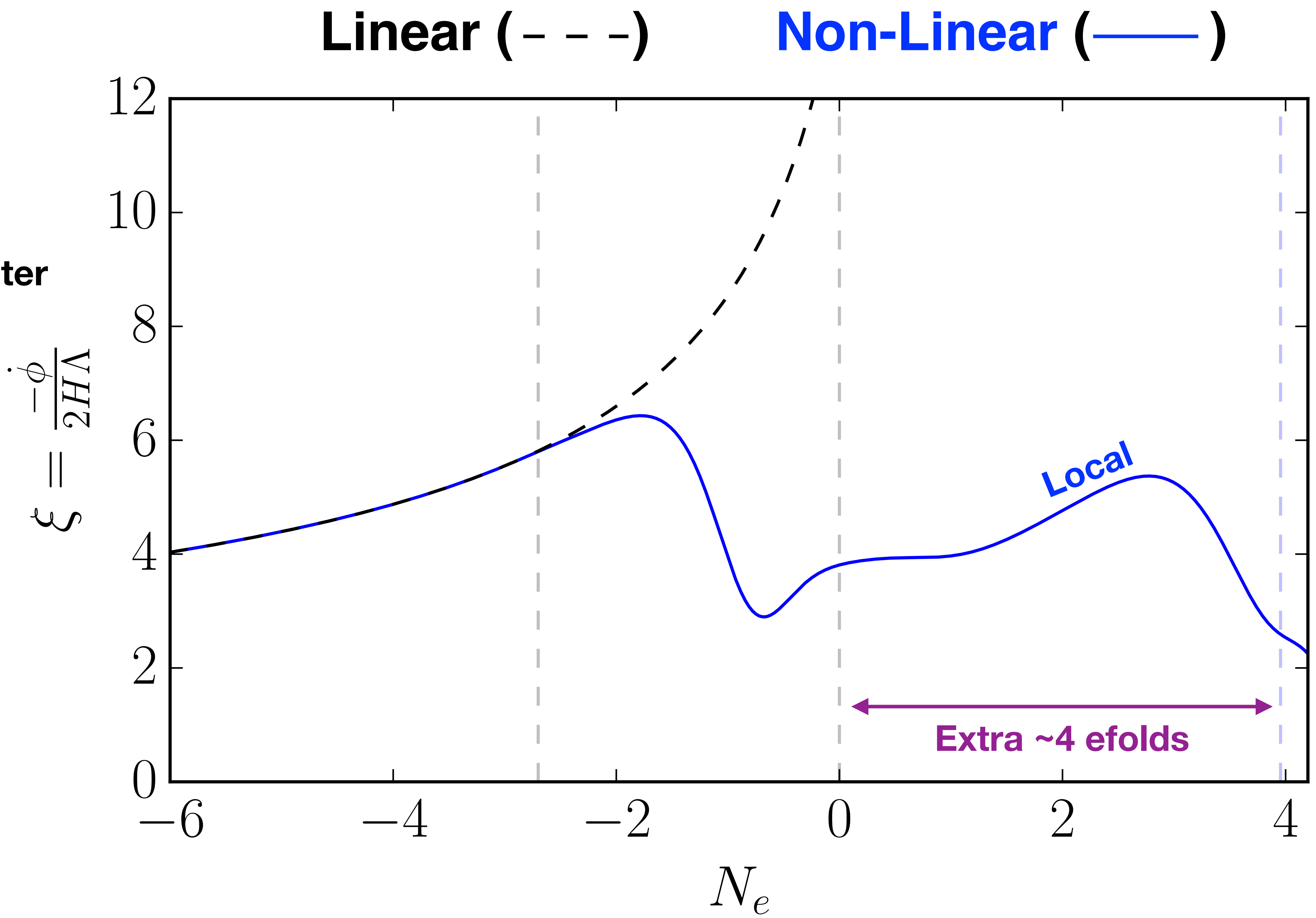
Linear (⋯⋯⋯) **Non-Linear (——)**

Epsilon Parameter



$\boxed{\alpha_\Lambda \equiv 15}$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18$)



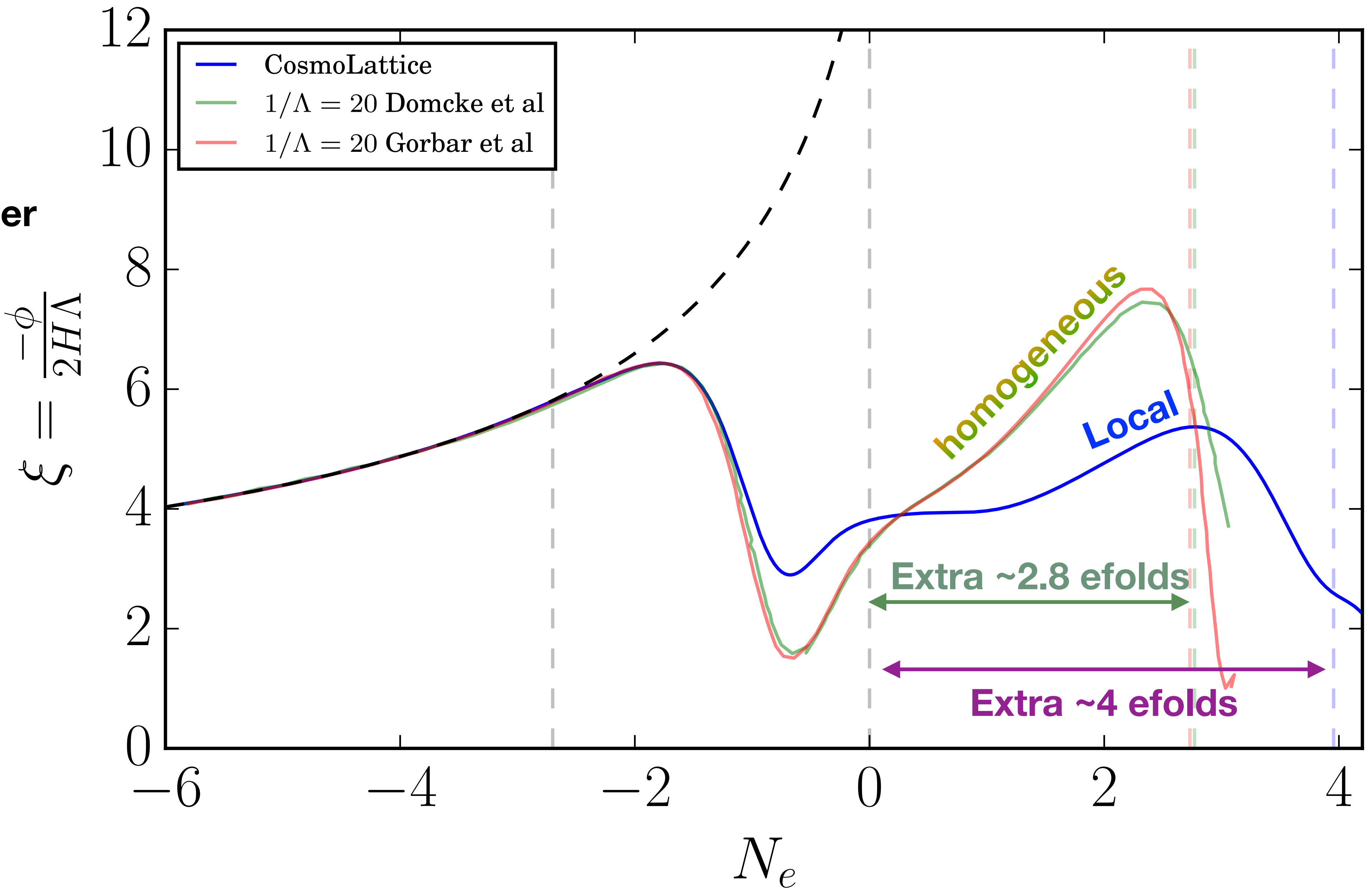
Xi parameter

$\alpha_\Lambda \equiv 18$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

Linear (- - -) **Non-Linear (——)**

Xi parameter

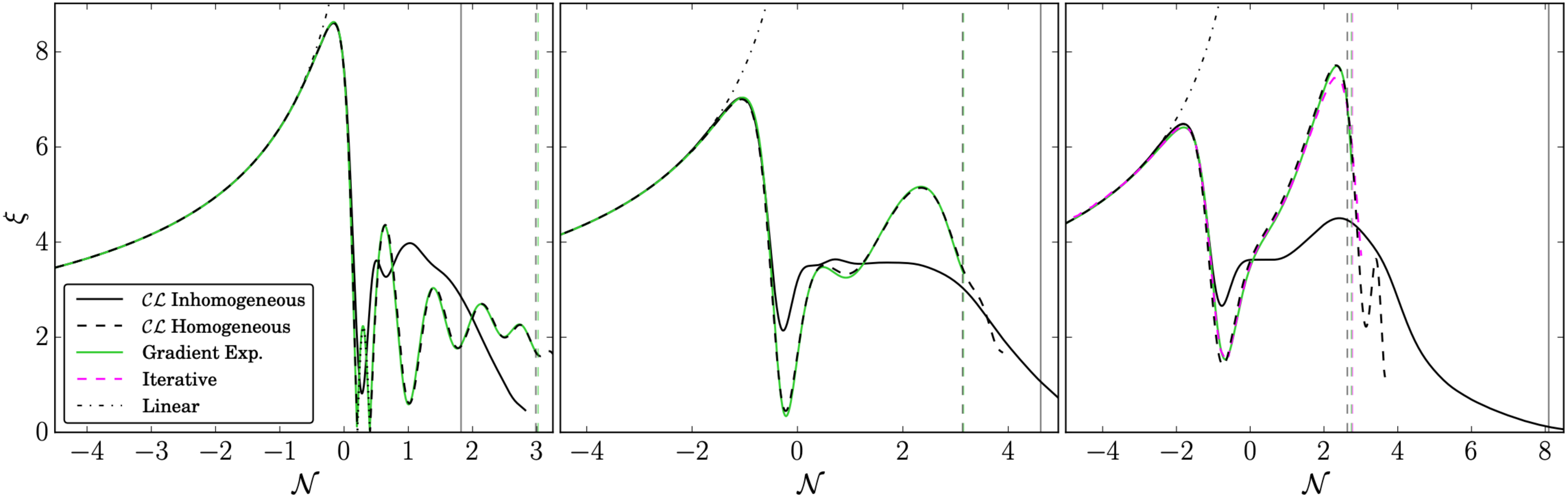


$\alpha_\Lambda \equiv 18$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{\alpha}$)

($\alpha = 15, 18, 20$)

$\alpha_\Lambda = 15$ $\alpha_\Lambda = 18$ $\alpha_\Lambda = 20$



$\Delta\mathcal{N}_{Hom} \simeq 3$ $\Delta\mathcal{N}_{Hom} \simeq 3$ $\Delta\mathcal{N}_{Hom} \simeq 3$

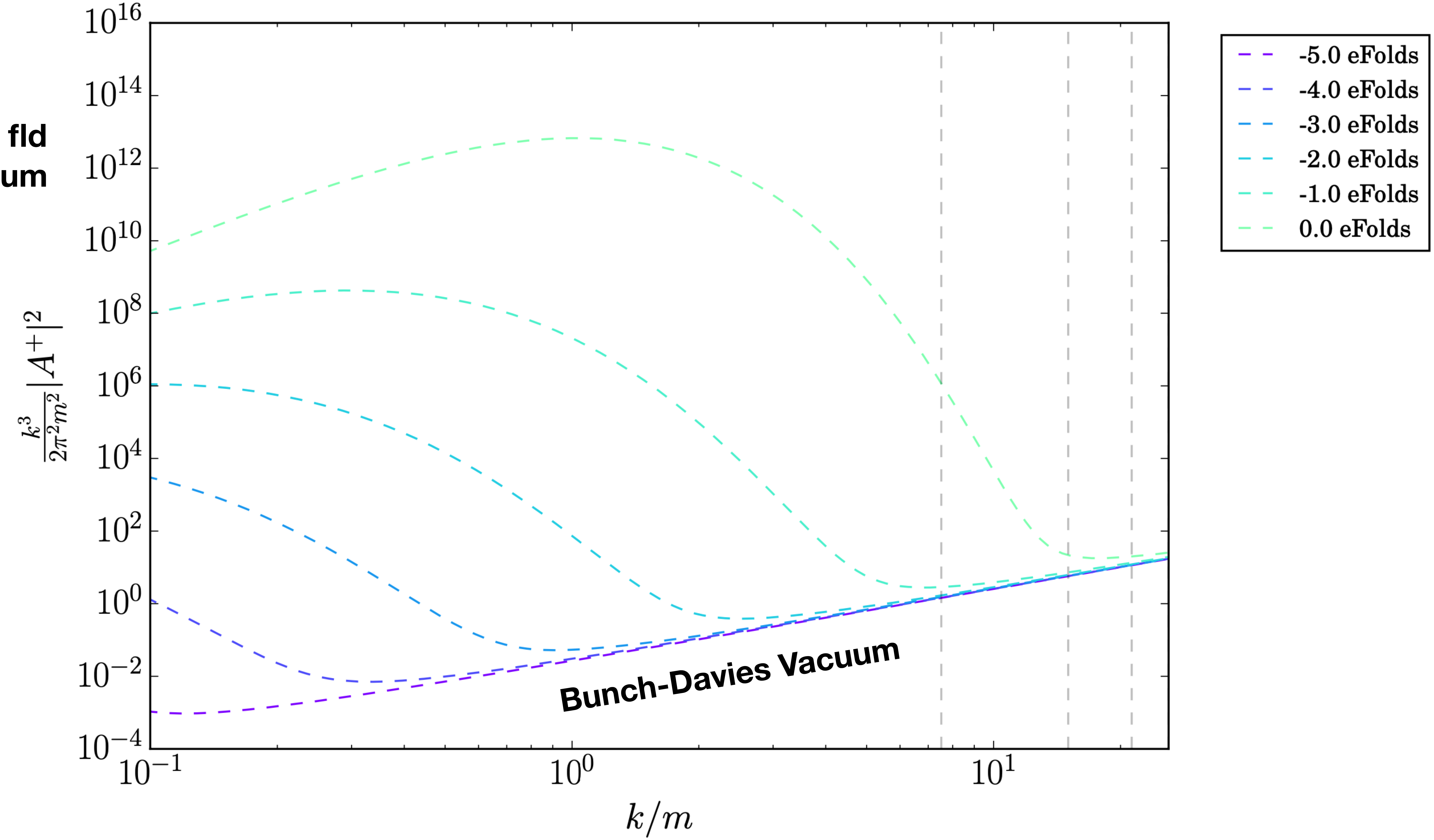
$\Delta\mathcal{N}_{InH} \simeq 1.8$ $\Delta\mathcal{N}_{InH} \simeq 4.5$ $\Delta\mathcal{N}_{InH} \simeq 8$

Gauge Amplification

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 18$)

Linear regime (- - -)

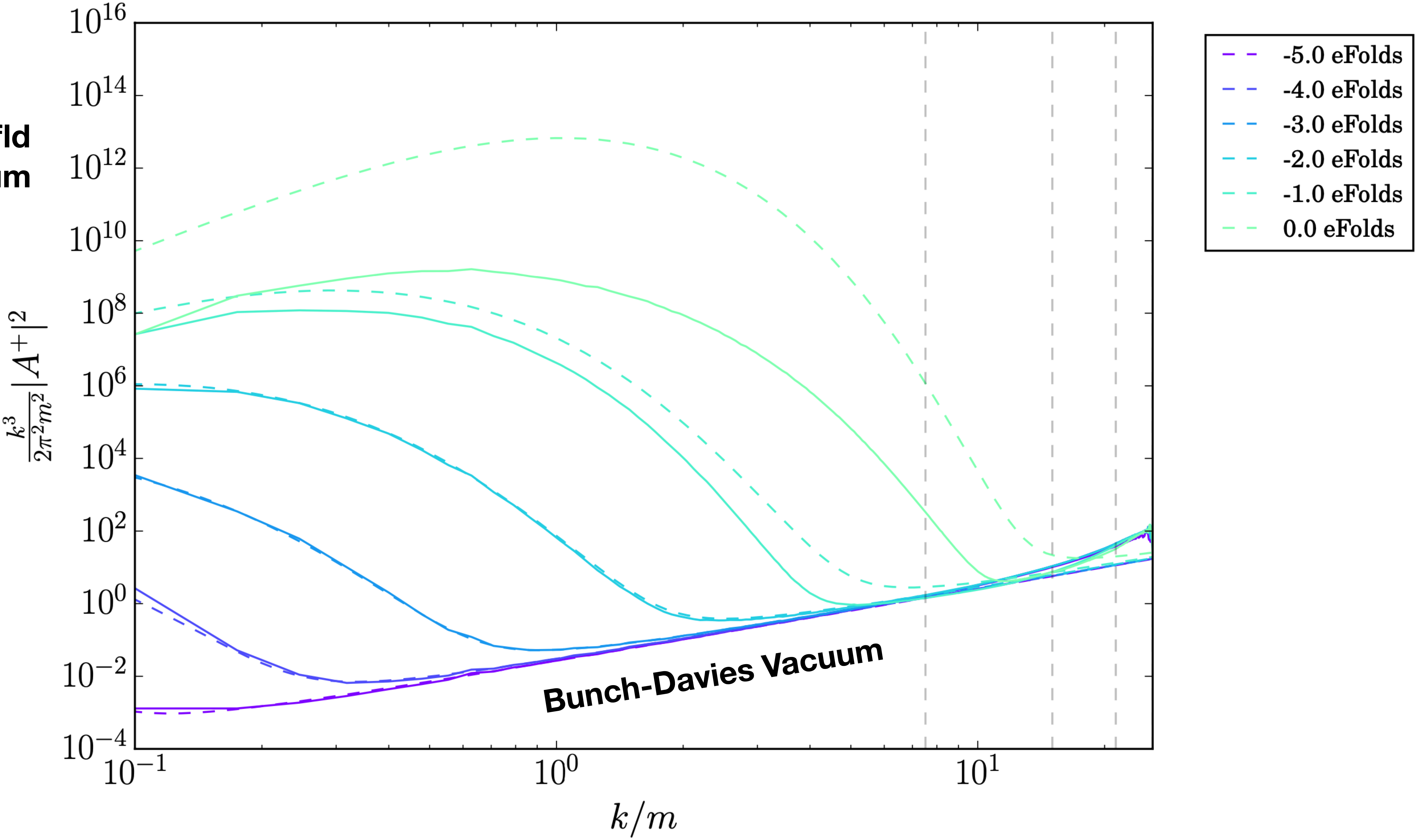
Gauge fld Spectrum



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18$)

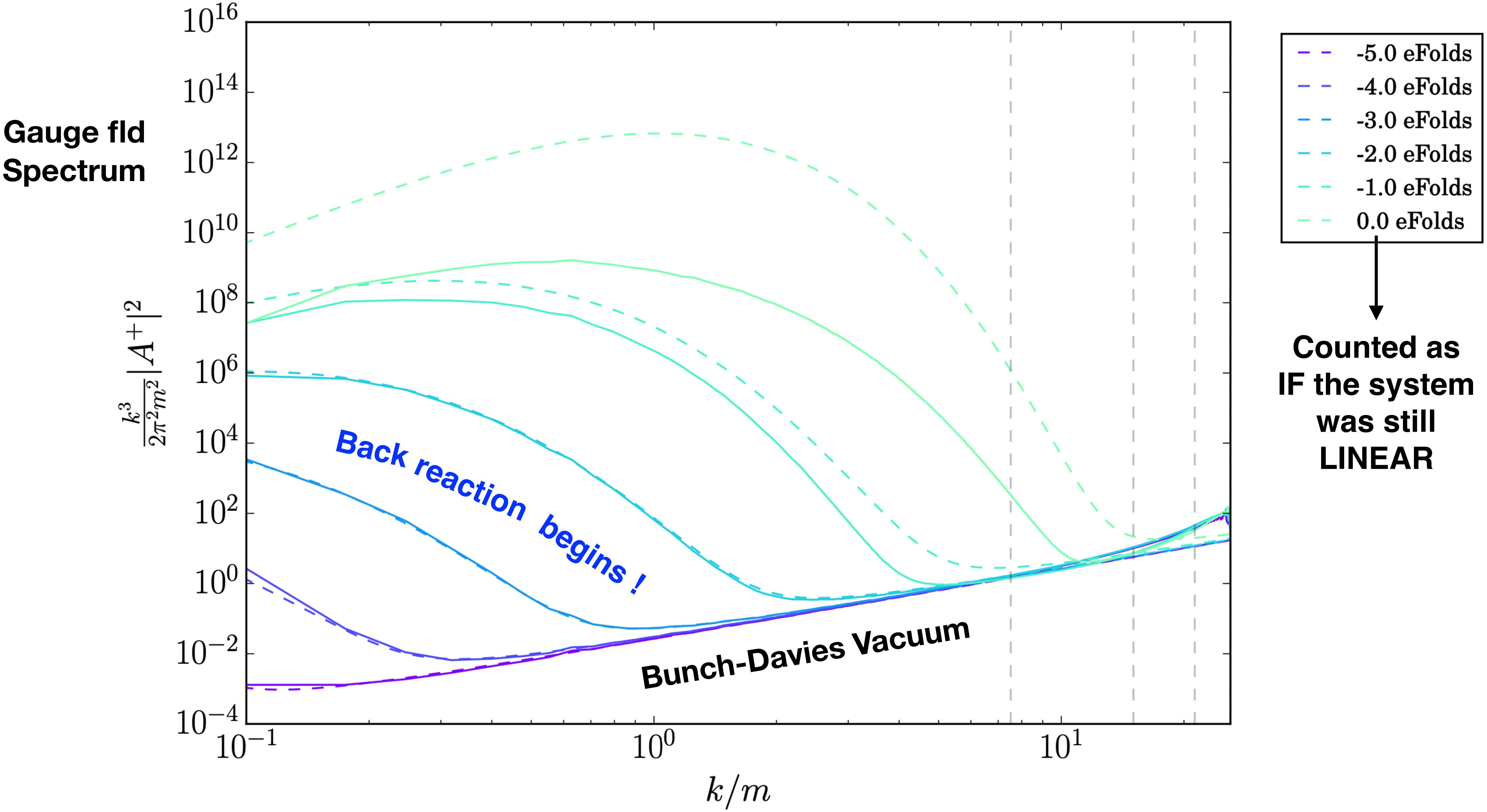
Linear regime (- - -) Non-Linear regime (—)

Gauge fld Spectrum



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

Linear regime (- - -) Non-Linear regime (—)

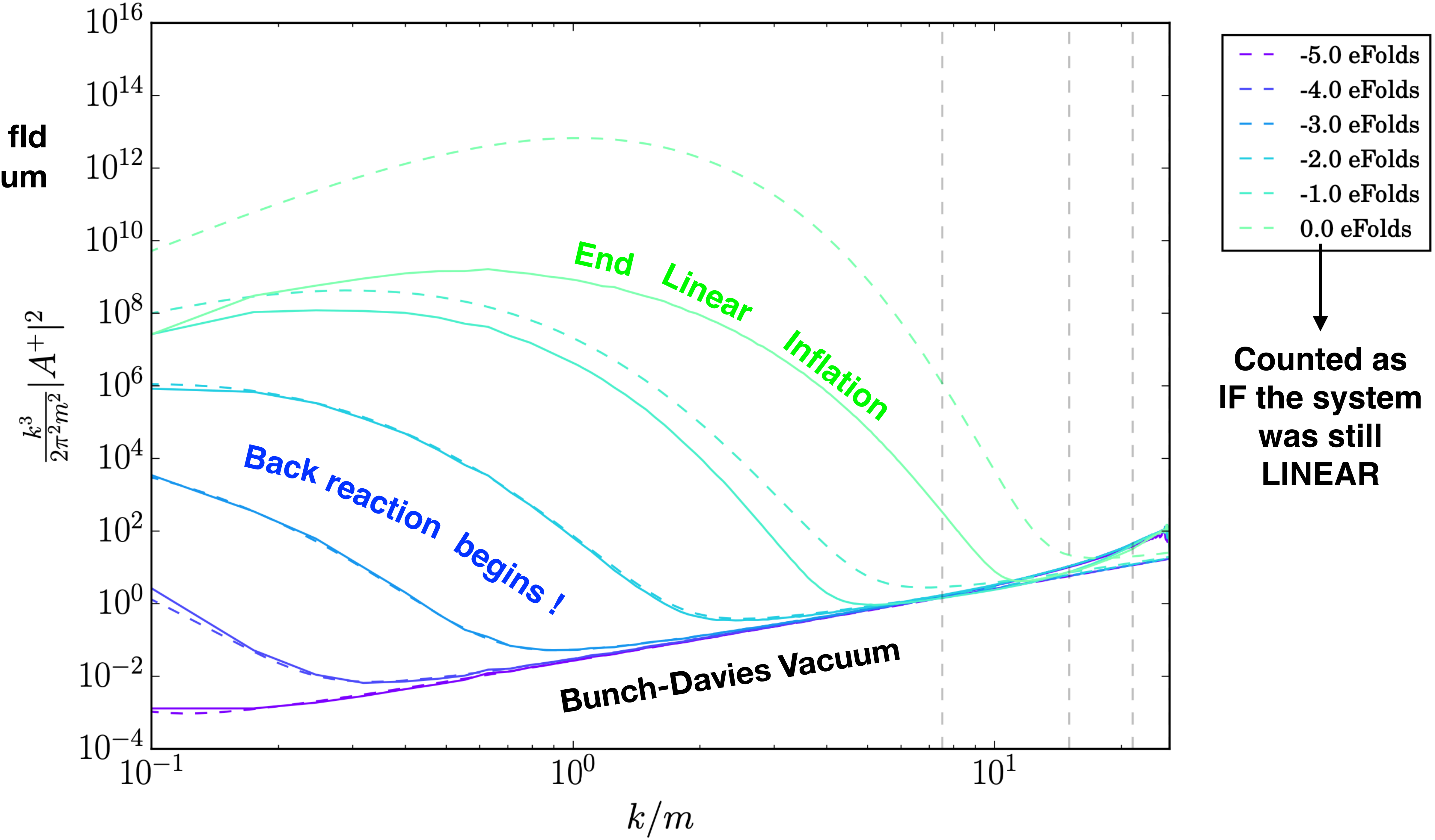


Counted as IF the system was still LINEAR

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

Linear regime (- - -)   Non-Linear regime (—)

Gauge fld Spectrum



End Linear Inflation

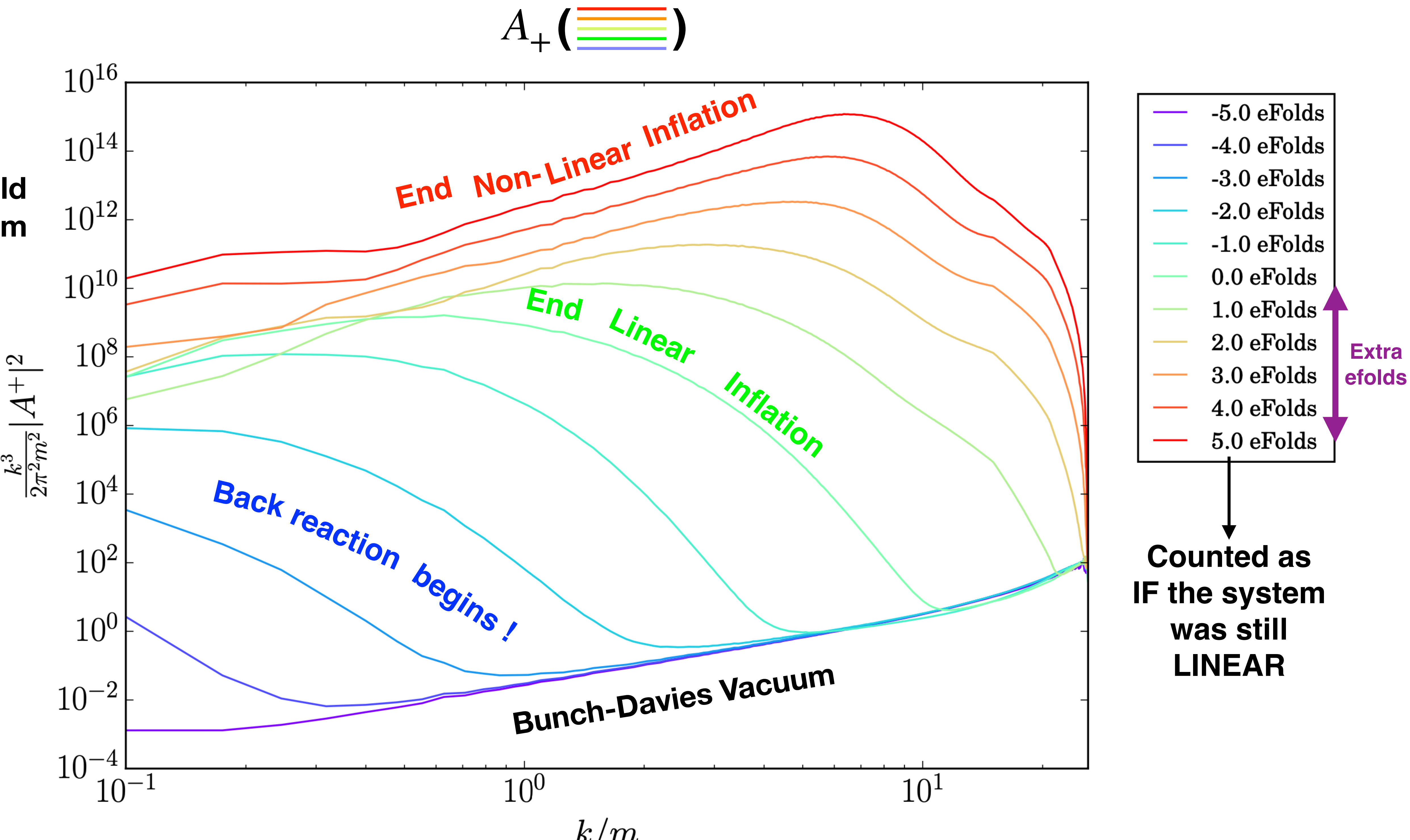
Back reaction begins !

Bunch-Davies Vacuum

Counted as IF the system was still LINEAR

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18$)

Gauge fld Spectrum



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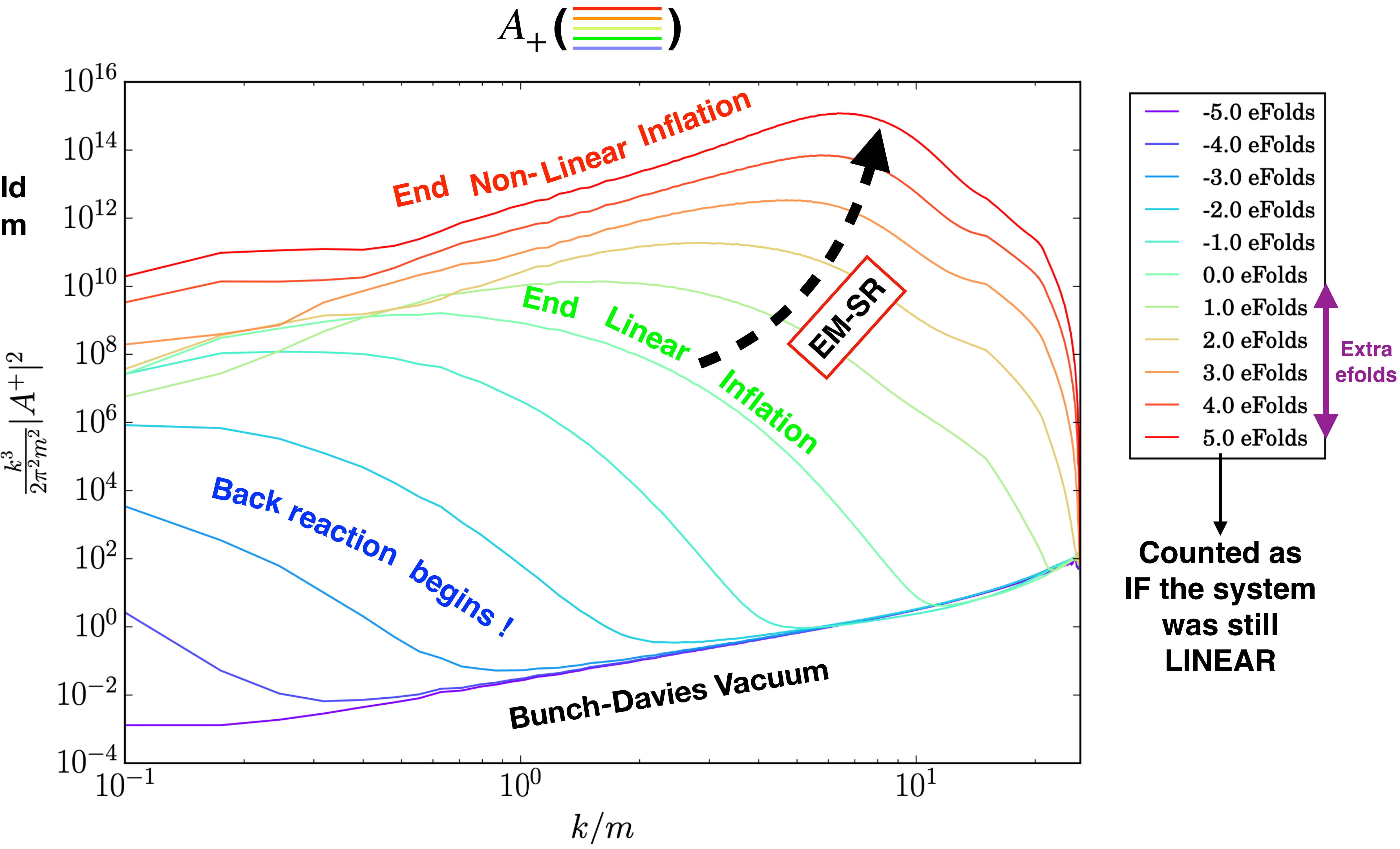
Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda=18\right)$

Gauge fld Spectrum



Counted as IF the system was still LINEAR

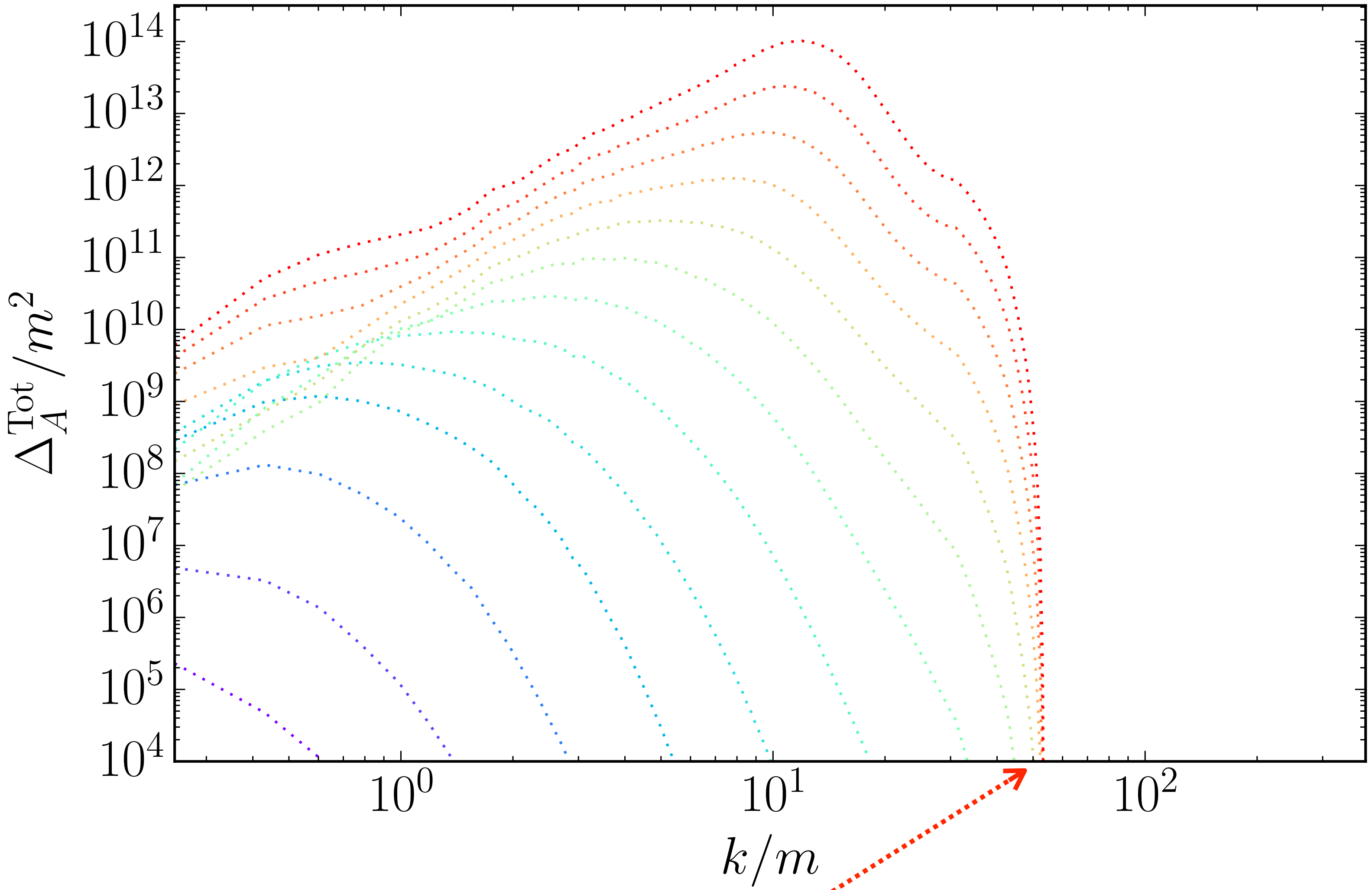
UV sensitivity (convergence)

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \alpha_\Lambda = 18$)

UV sensitivity

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

UV sensitivity

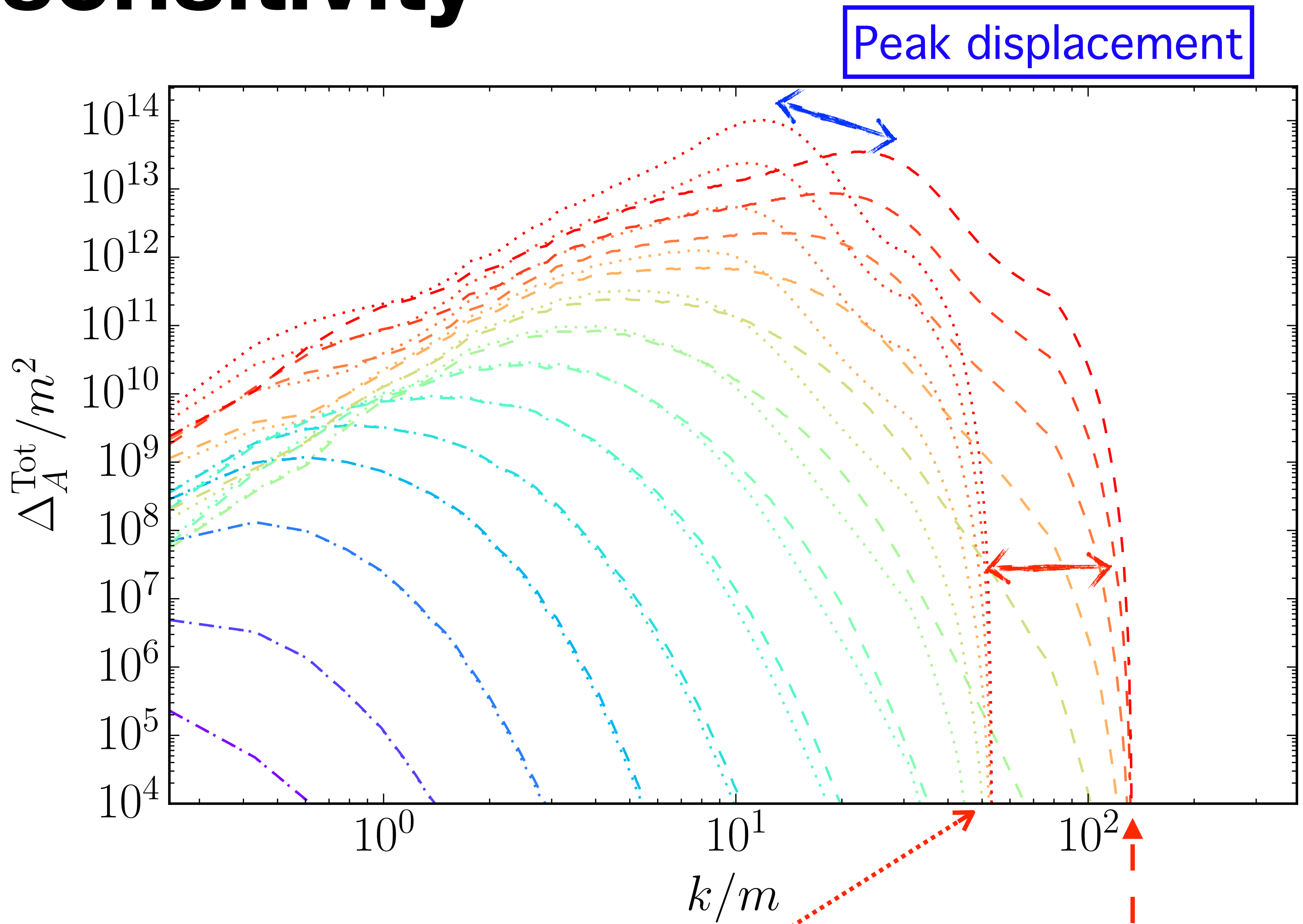


$N = 320$

$k_{\mathrm{UV}}/k_{\mathrm{IR}} \approx 280$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

UV sensitivity



$N = 320$

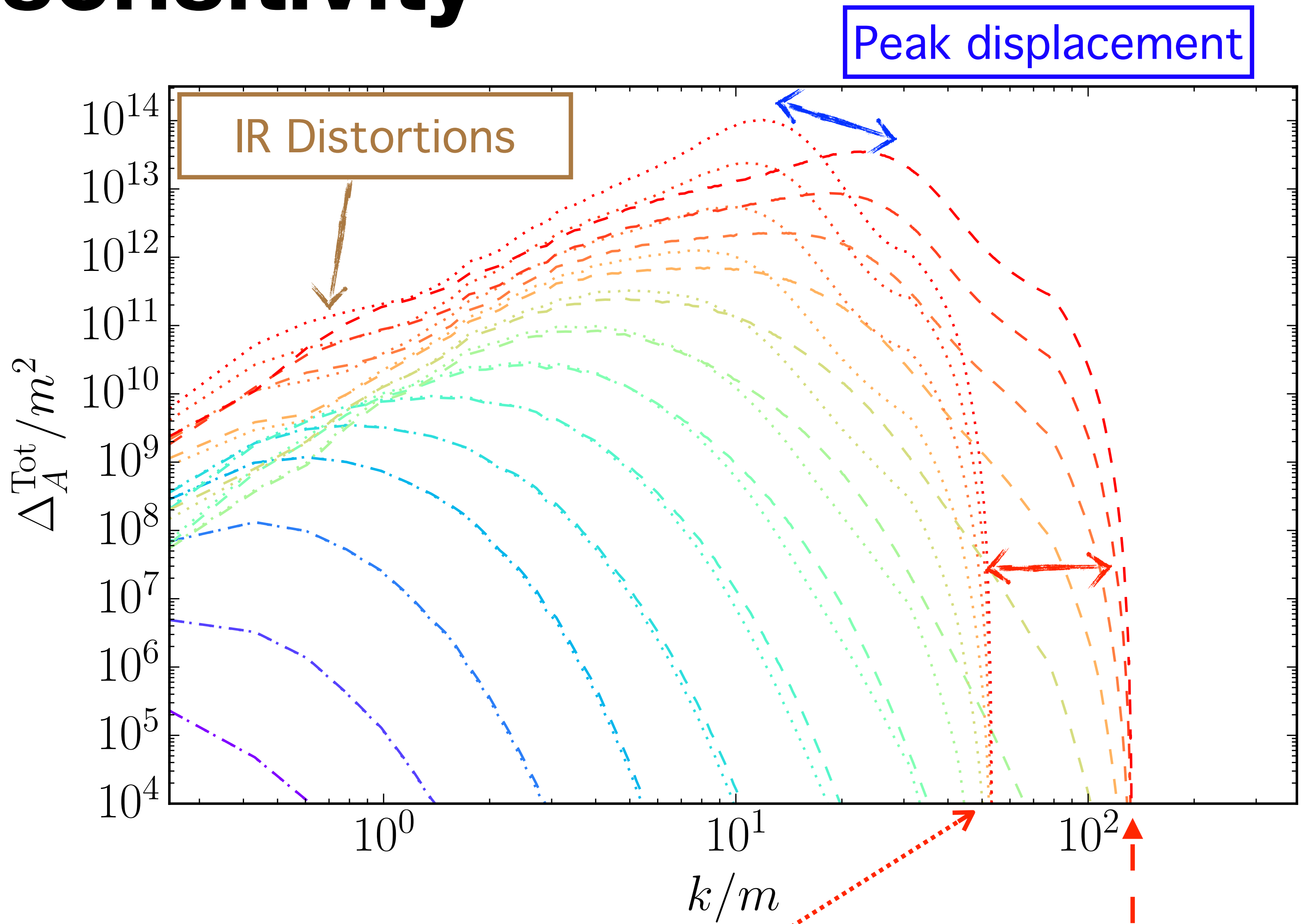
$k_{\rm UV}/k_{\rm IR} \approx 280$

$N = 800$

$k_{\rm UV}/k_{\rm IR} \approx 690$

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 18\right)$

UV sensitivity



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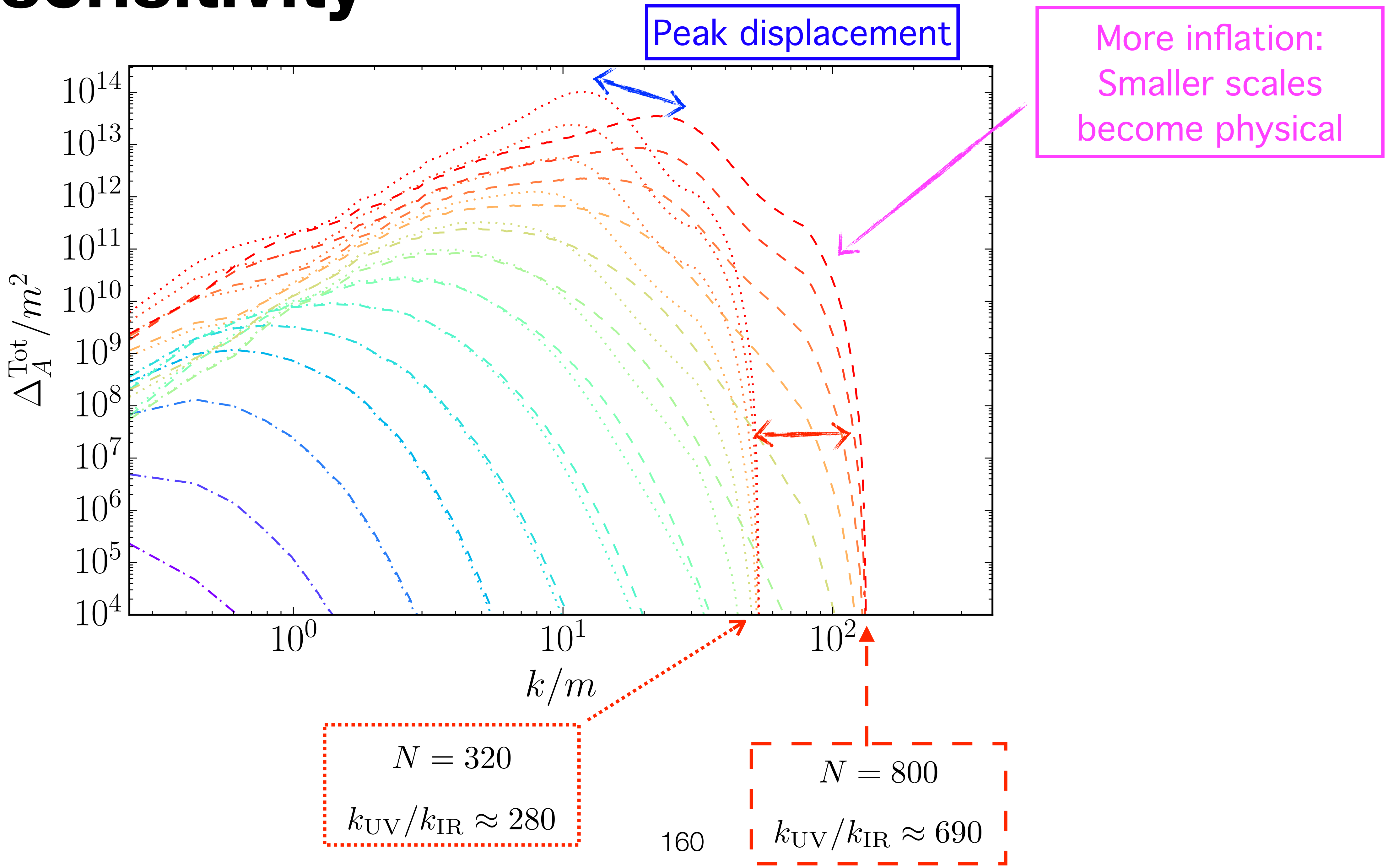
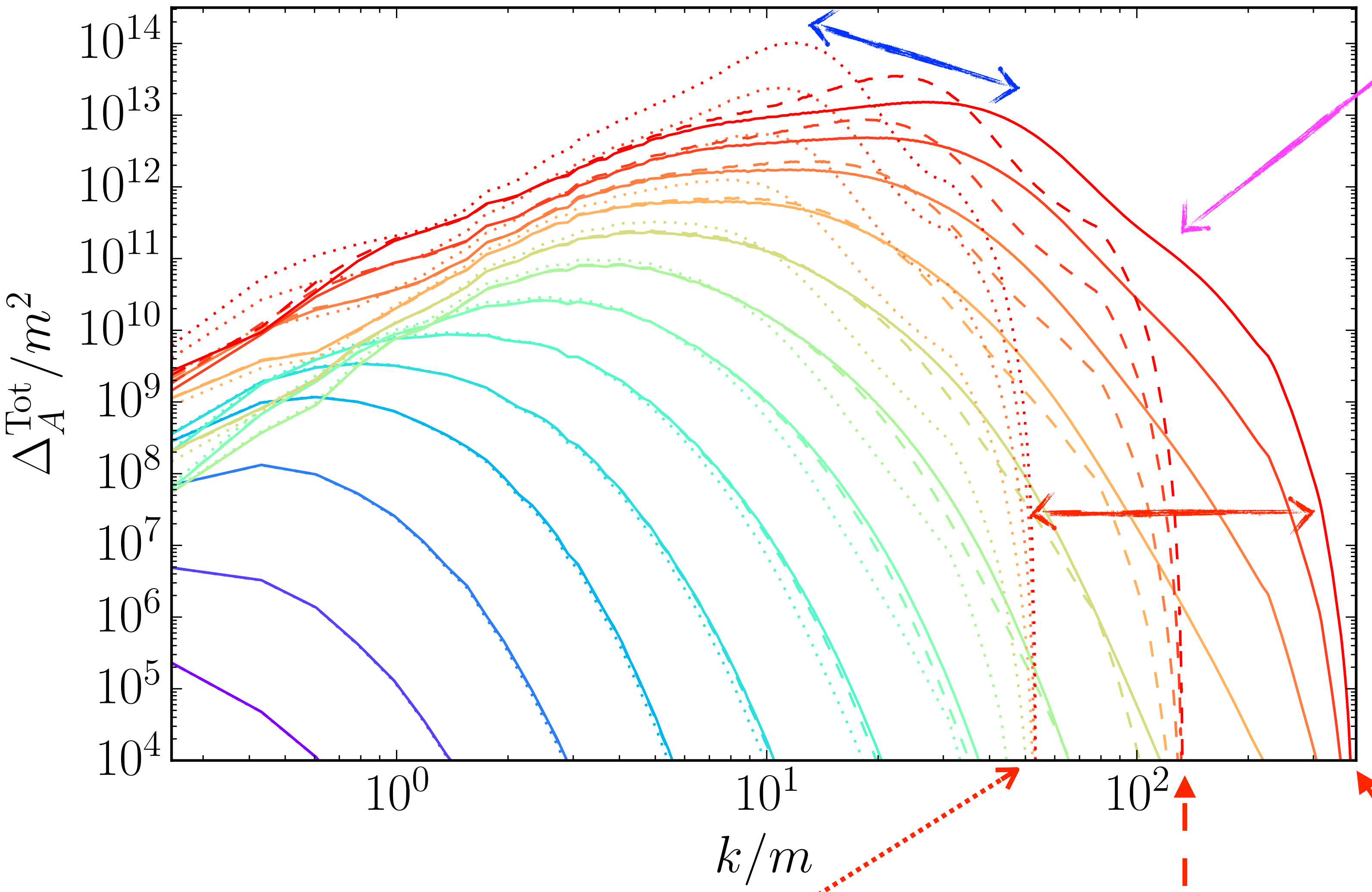
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

UV sensitivity



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

UV sensitivity



Peak displacement

More inflation:
Smaller scales
become physical

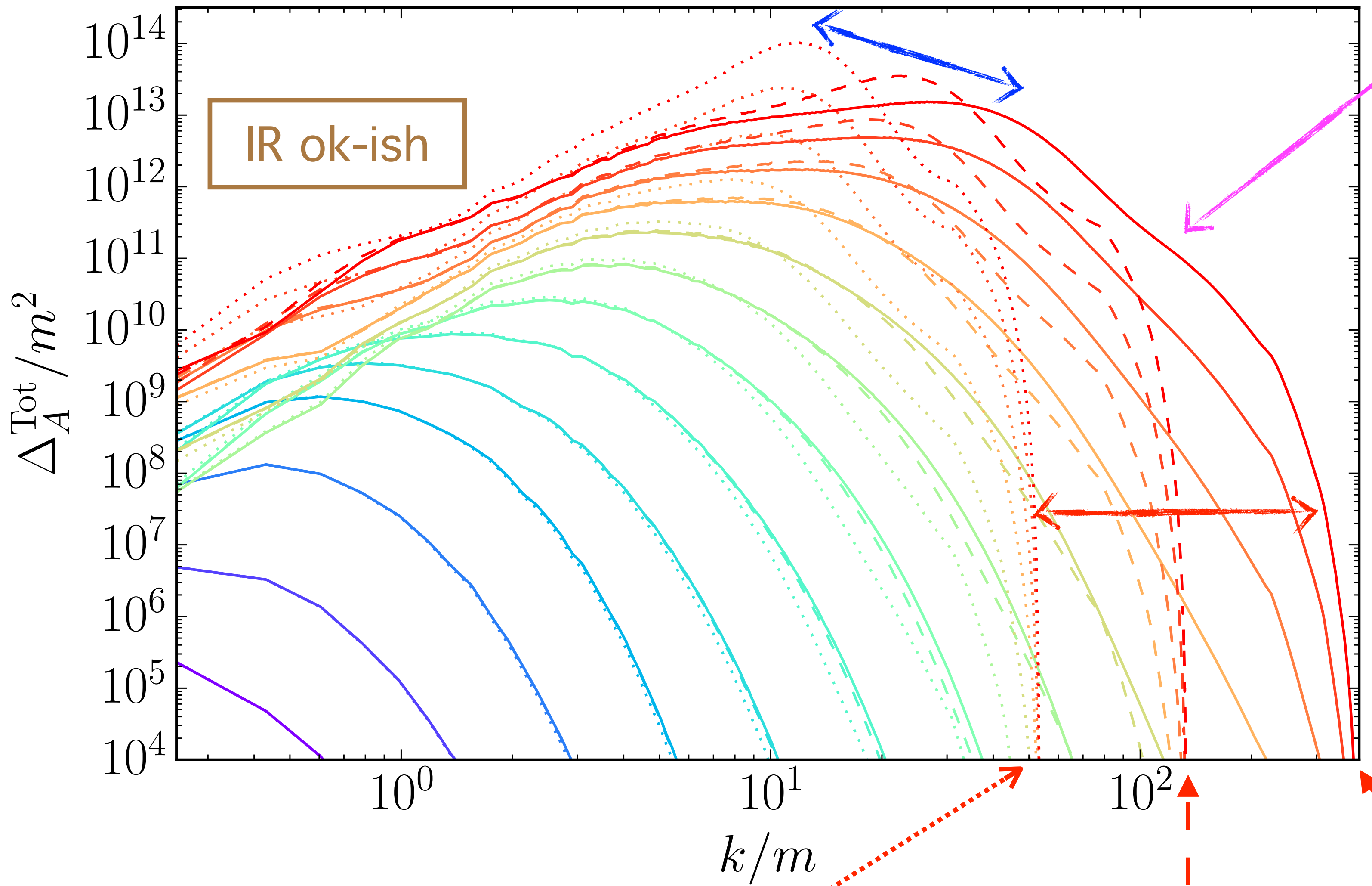
$N = 320$
$k_{\rm UV}/k_{\rm IR} \approx 280$

$N = 800$
$k_{\rm UV}/k_{\rm IR} \approx 690$

$N = 2340$
$k_{\rm UV}/k_{\rm IR} \approx 2000$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 18$)

UV sensitivity



Peak displacement

More inflation:
Smaller scales
become physical

IR ok-ish

$N = 320$

$k_{\rm UV}/k_{\rm IR} \approx 280$

$N = 800$

$k_{\rm UV}/k_{\rm IR} \approx 690$

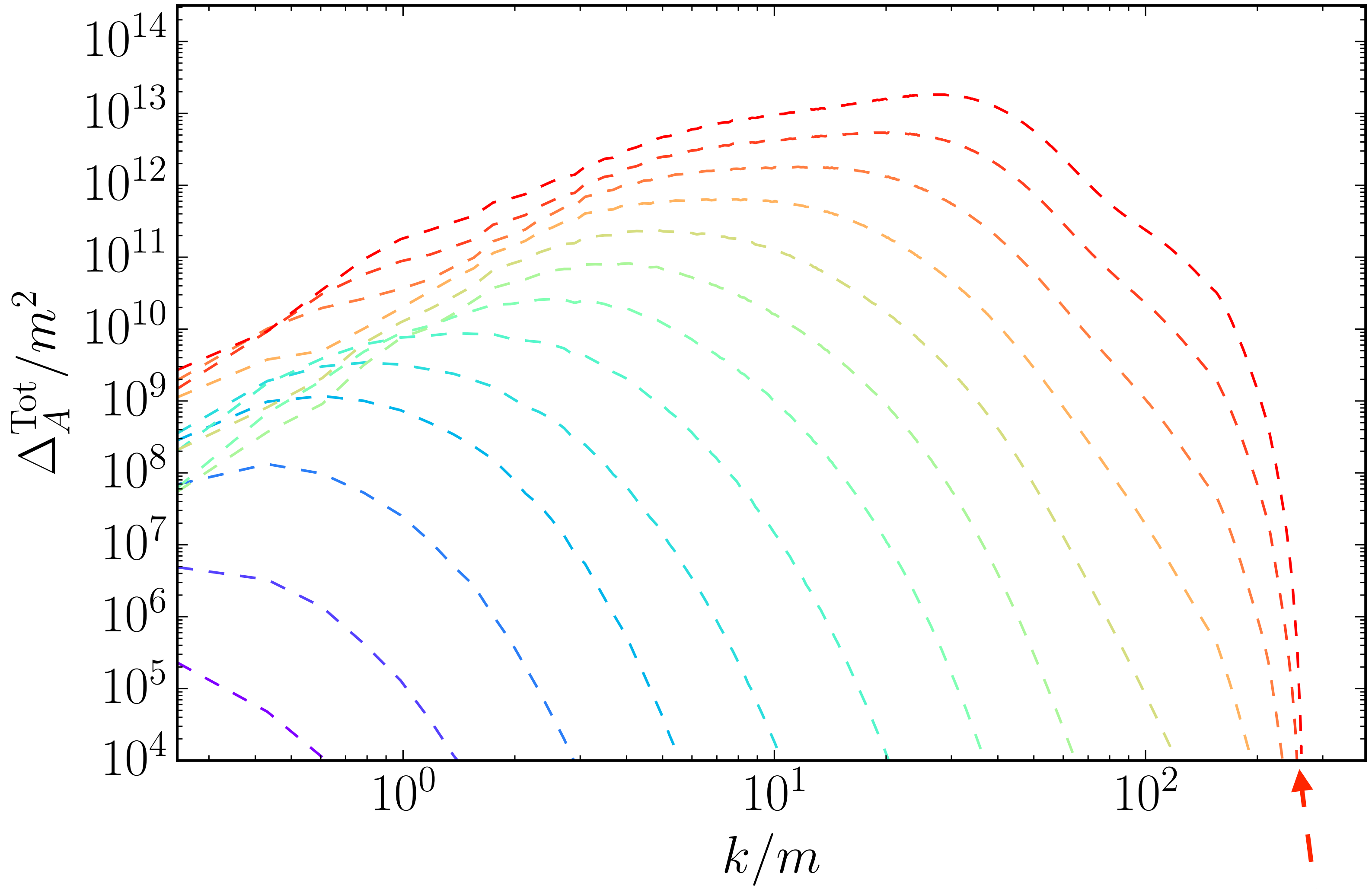
$N = 2340$

$k_{\rm UV}/k_{\rm IR} \approx 2000$

Extreme UV dependence! Simulations in quasi-dS!

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 18$)

UV convergence ?



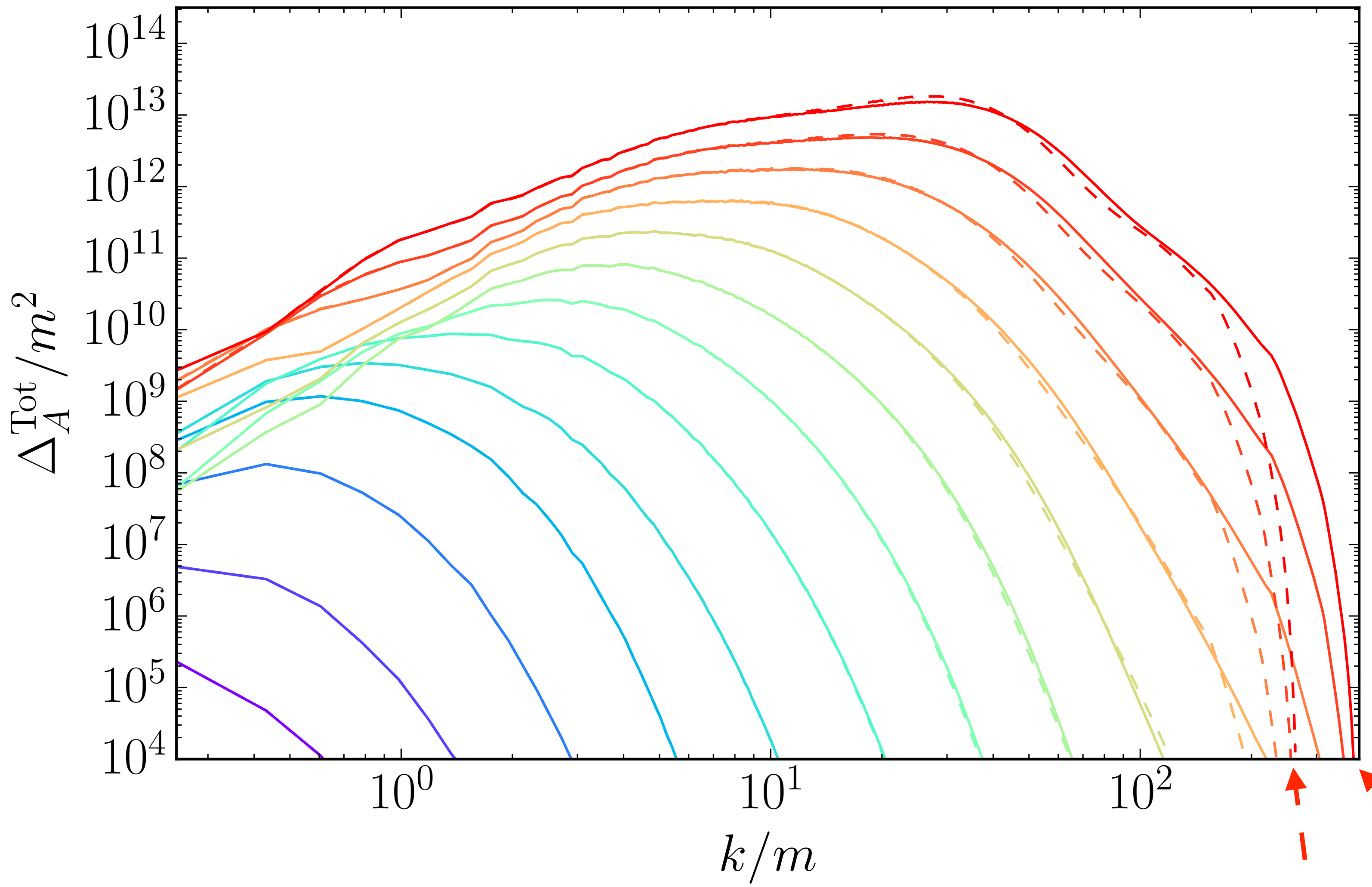
$N = 1600$

$k_{\rm UV}/k_{\rm IR} \approx 1400$

Extreme UV dependence! Simulations in quasi-dS!

Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda=18\right)$

UV convergence ?



IR = IR

Mid $\simeq$ Mid

UV $\neq$ UV

$N = 1600$

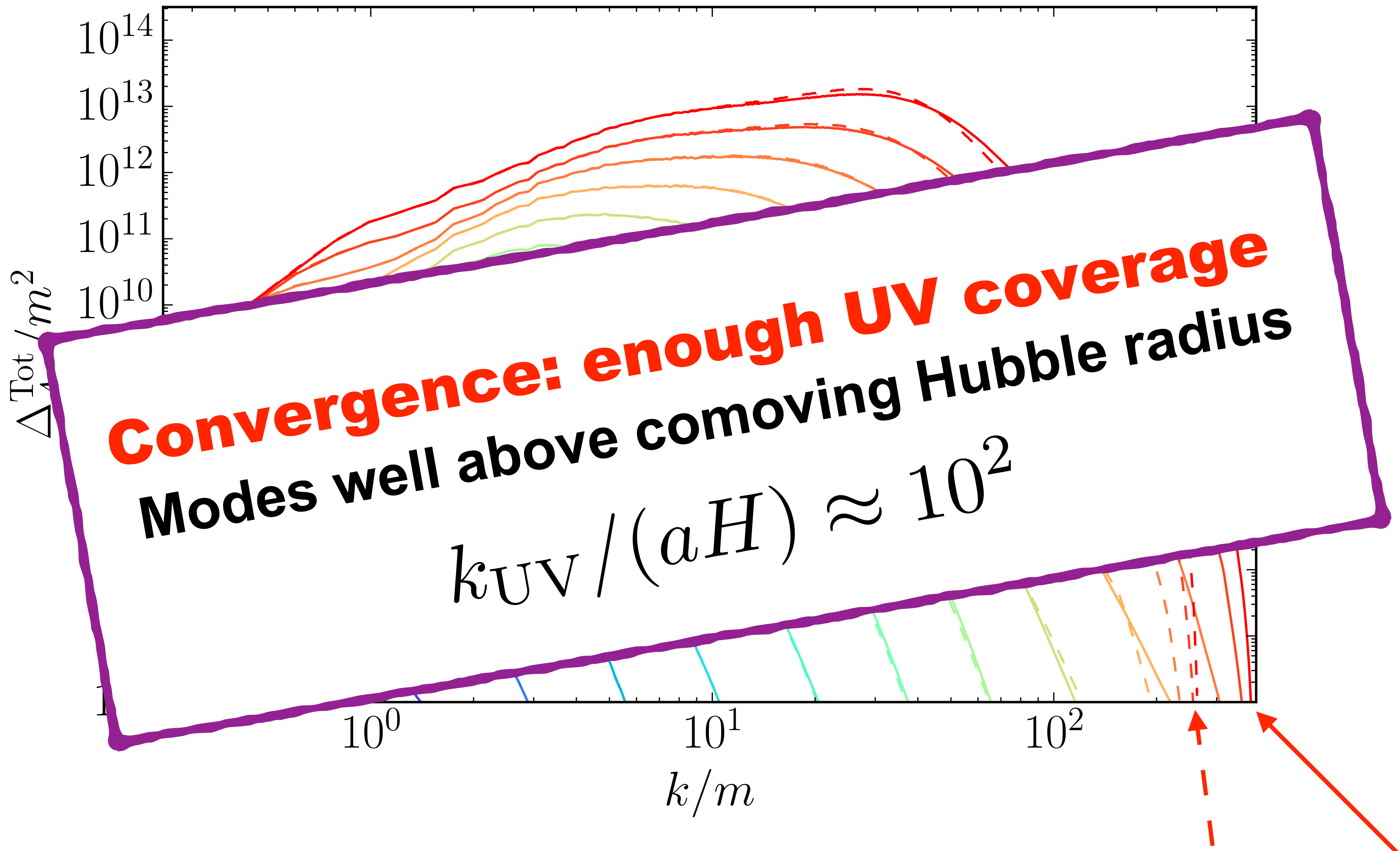
$k_{\rm UV}/k_{\rm IR} \approx 1400$

$N = 2340$

$k_{\rm UV}/k_{\rm IR} \approx 2000$

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \alpha_\Lambda = 18\right)$

UV convergence ?



Convergence: enough UV coverage

Modes well above comoving Hubble radius

$$k_{\rm UV}/(aH) \approx 10^2$$

Gigantic lattices !

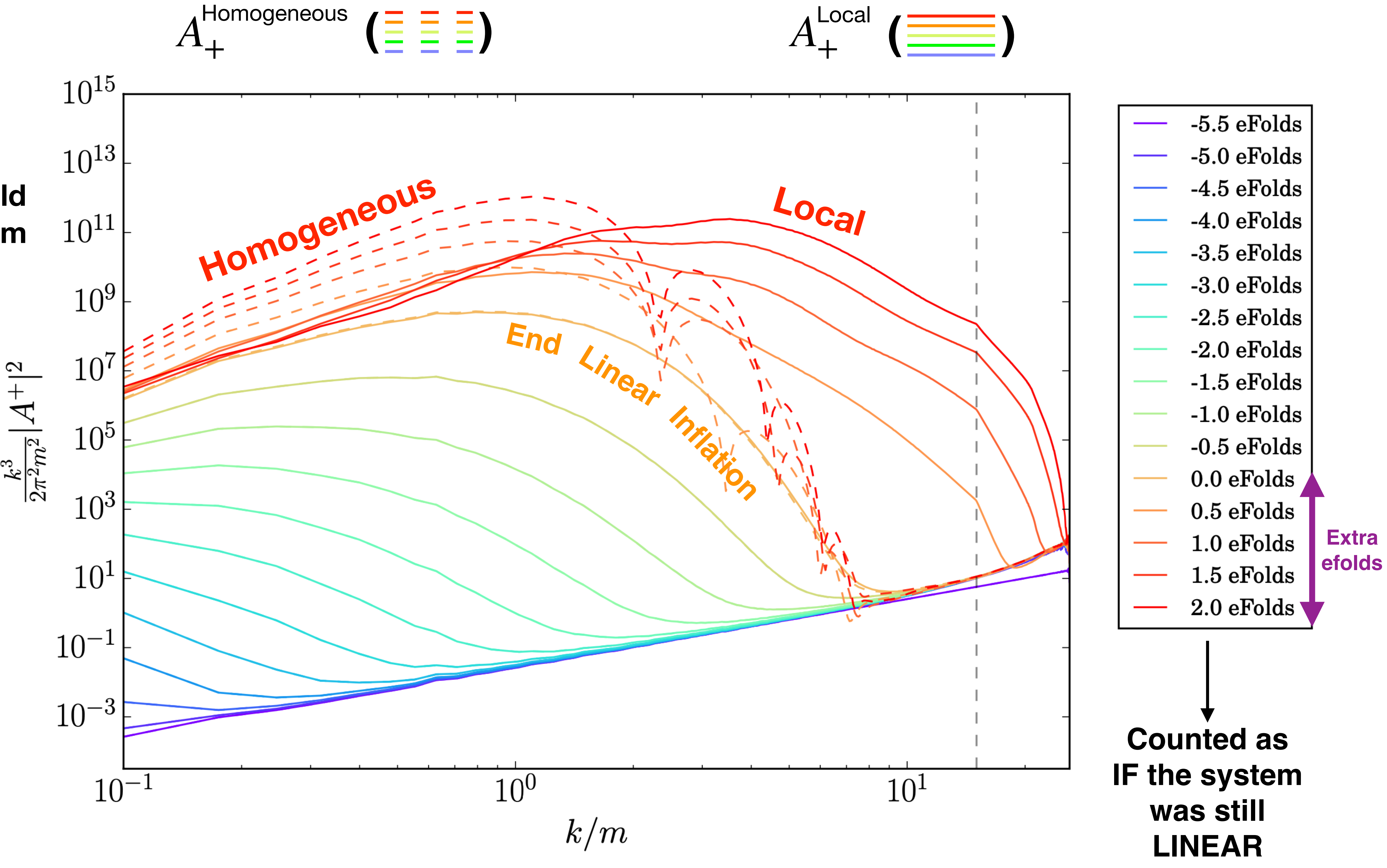
$N = 1600$

$k_{\rm UV}/k_{\rm IR} \approx 1400$

$N = 2340$

$k_{\rm UV}/k_{\rm IR} \approx 2000$

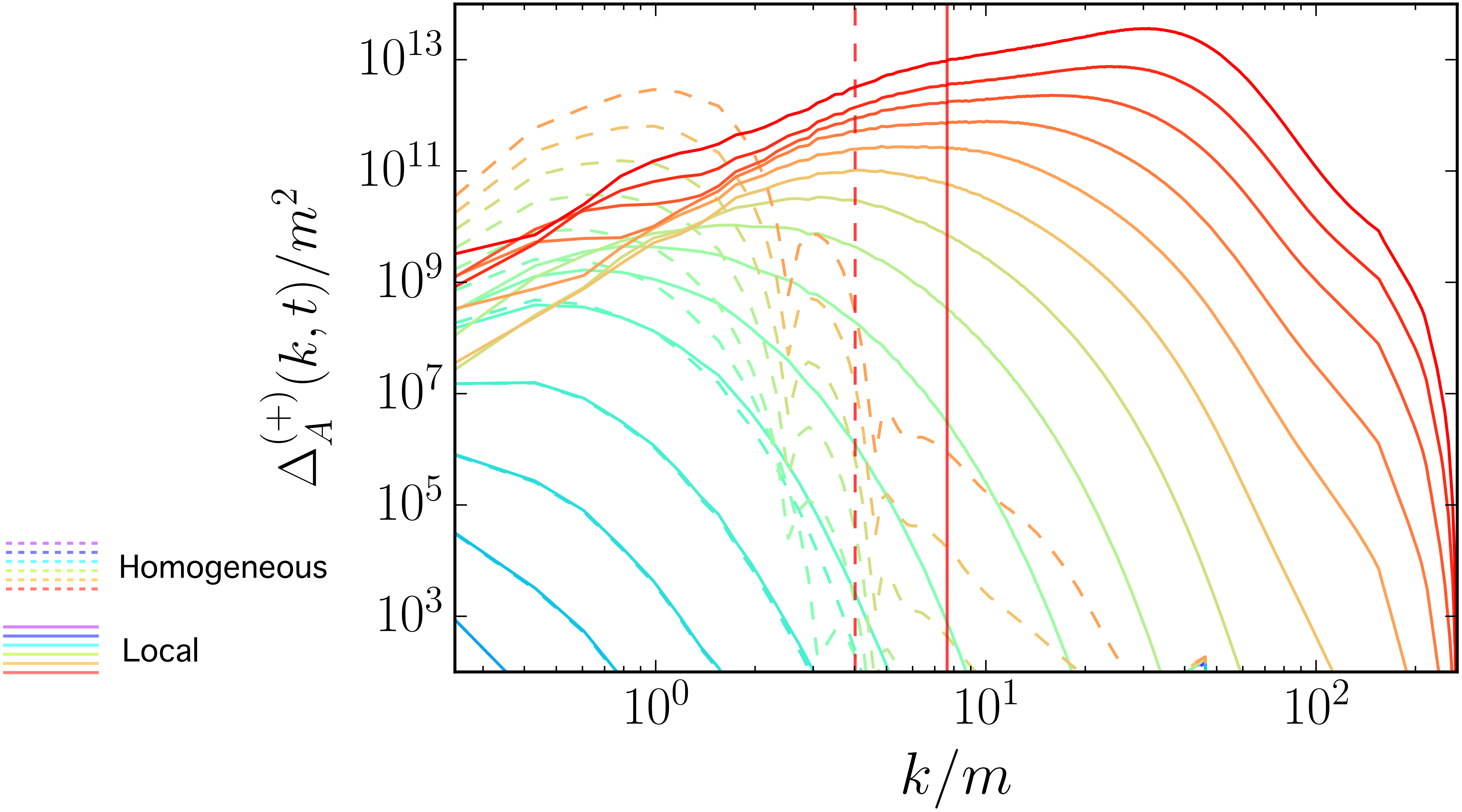
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \alpha_\Lambda = 15 \right)$



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 15$)

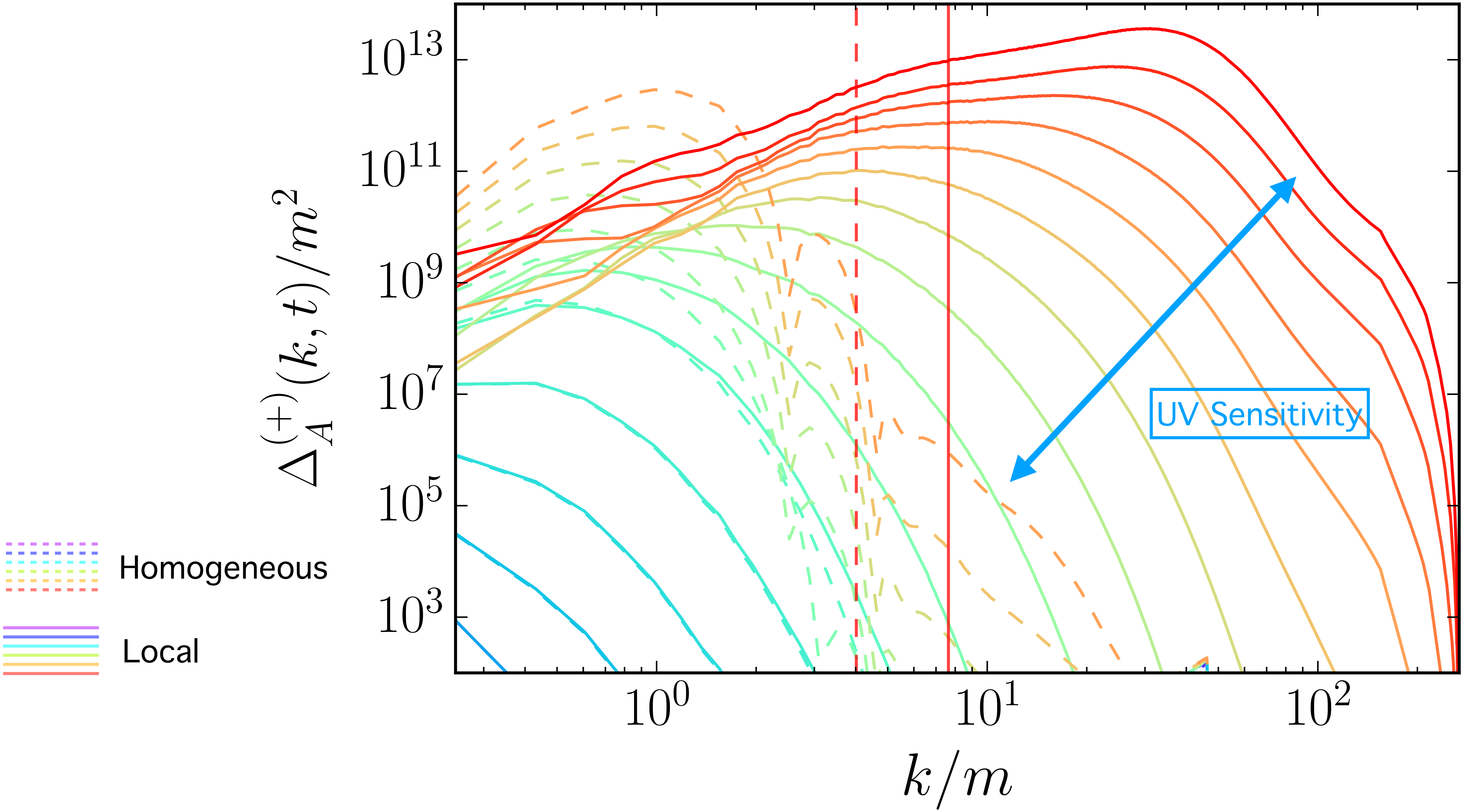
Homogeneous vs Inhomogeneous

ZOOM



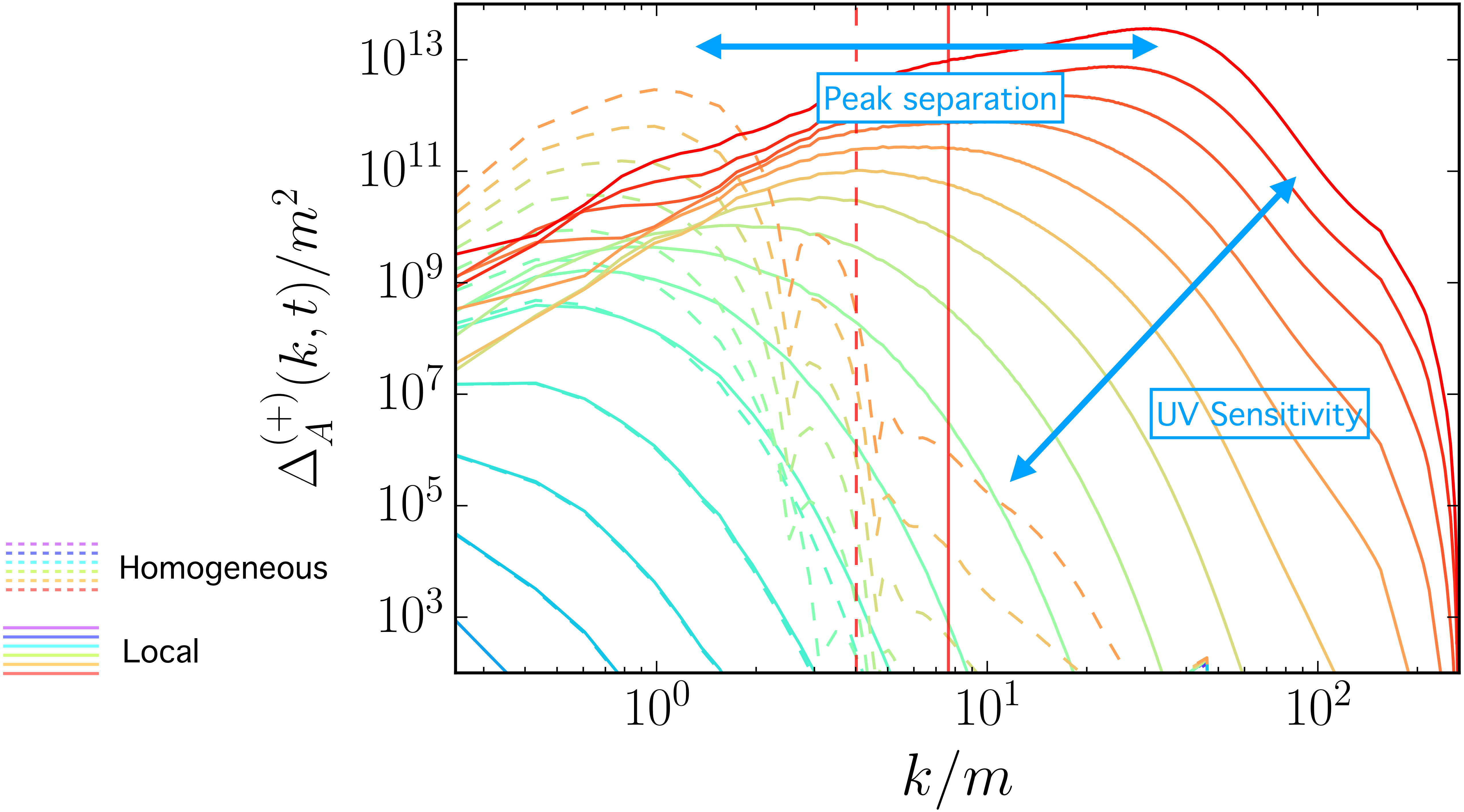
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda = 15\right)$

Homogeneous vs Inhomogeneous



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 15$)

Homogeneous vs Inhomogeneous

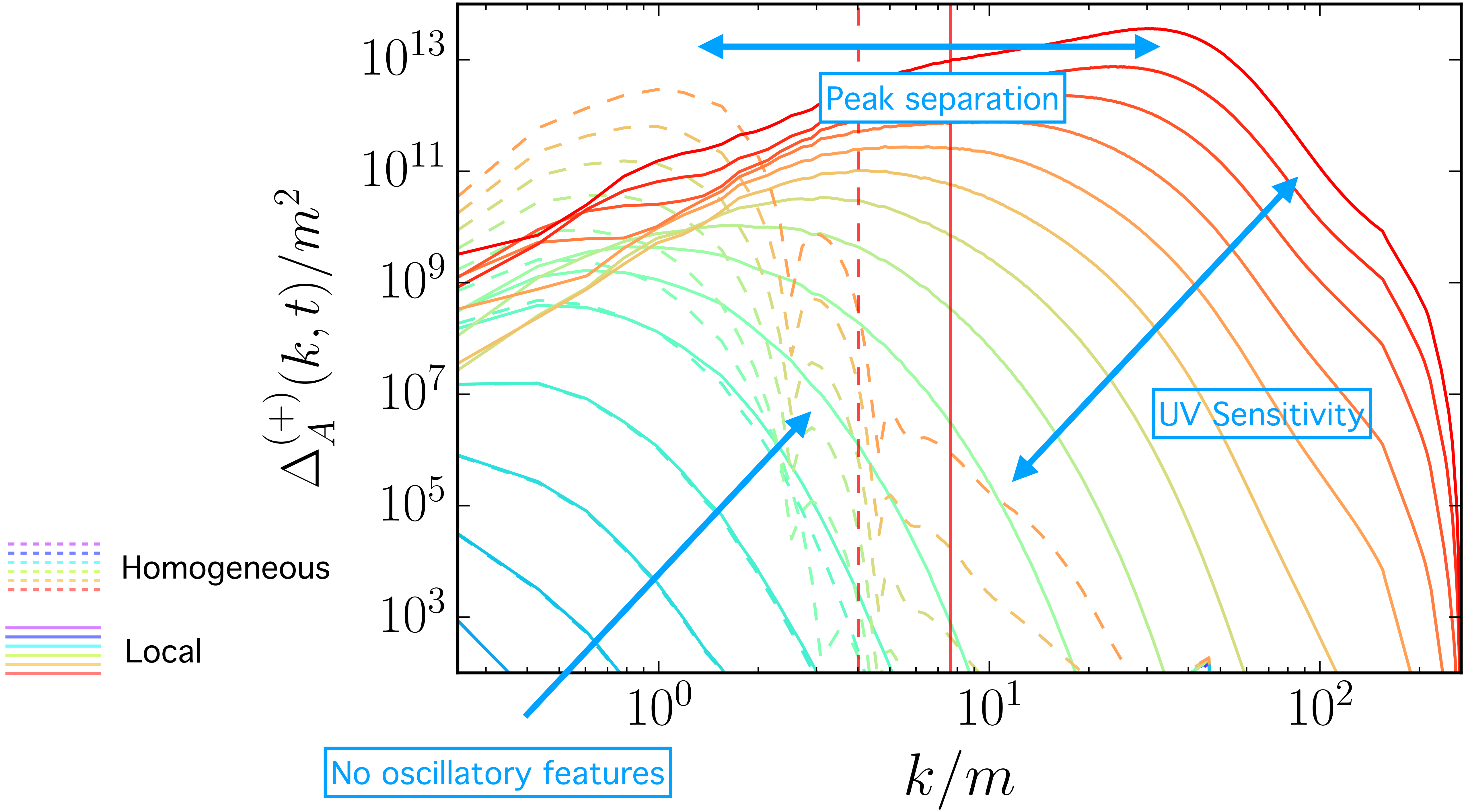


Peak separation

UV Sensitivity

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 15$)

Homogeneous vs Inhomogeneous



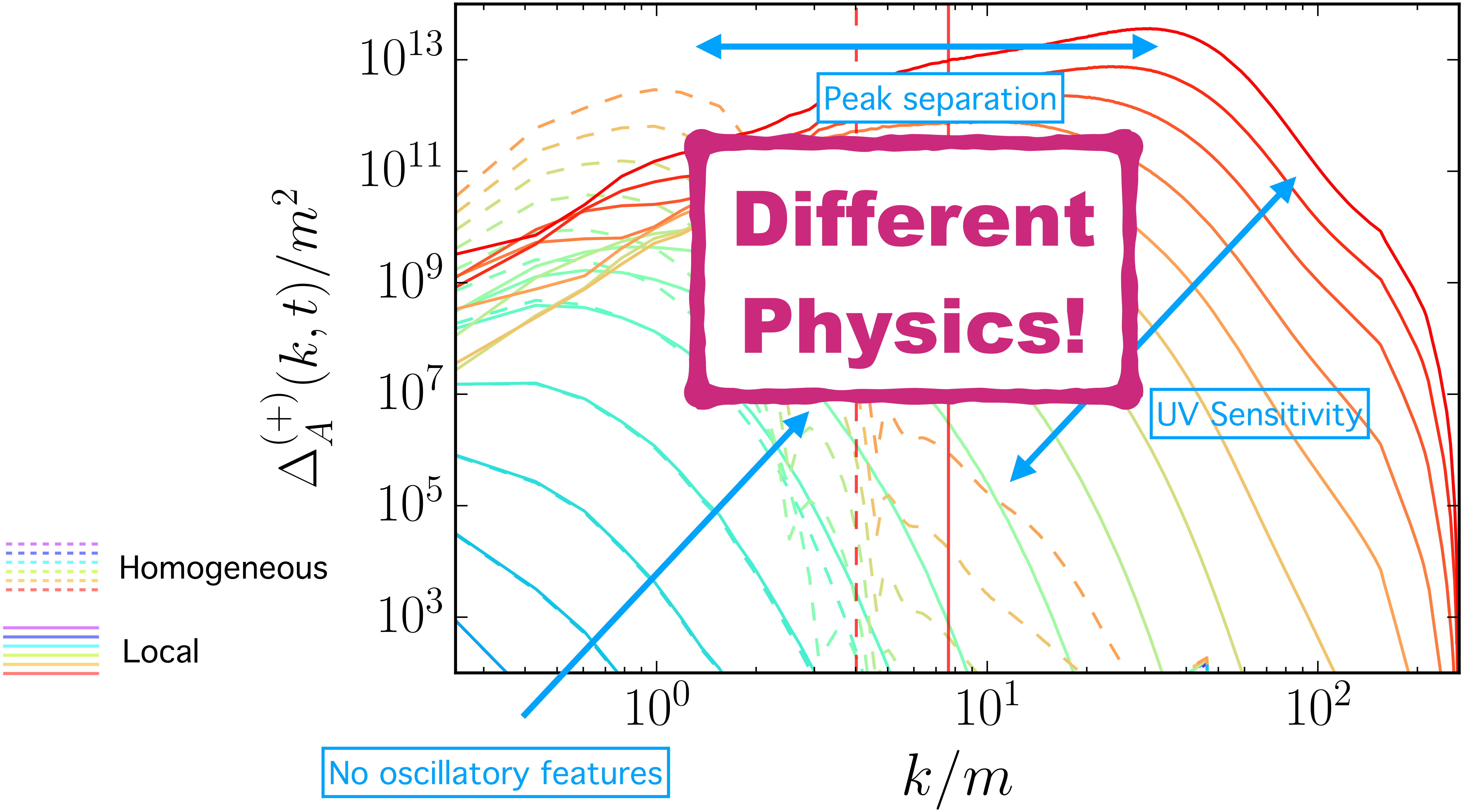
Peak separation

UV Sensitivity

No oscillatory features

Axion-Inflation $\left(V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \alpha_\Lambda = 15\right)$

Homogeneous vs Inhomogeneous



Peak separation

Different Physics!

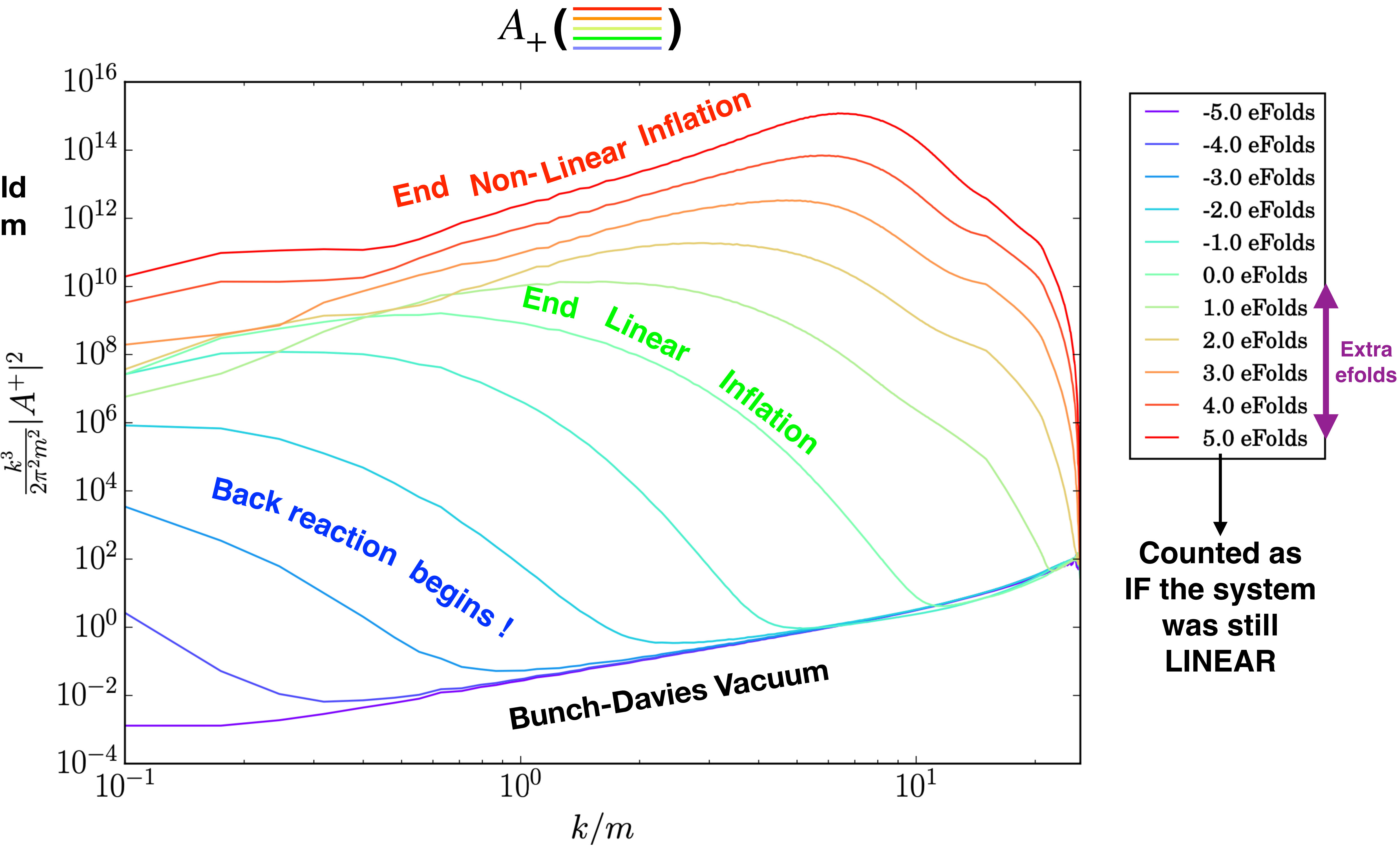
UV Sensitivity

No oscillatory features

Chirality

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \alpha_\Lambda = 18$)

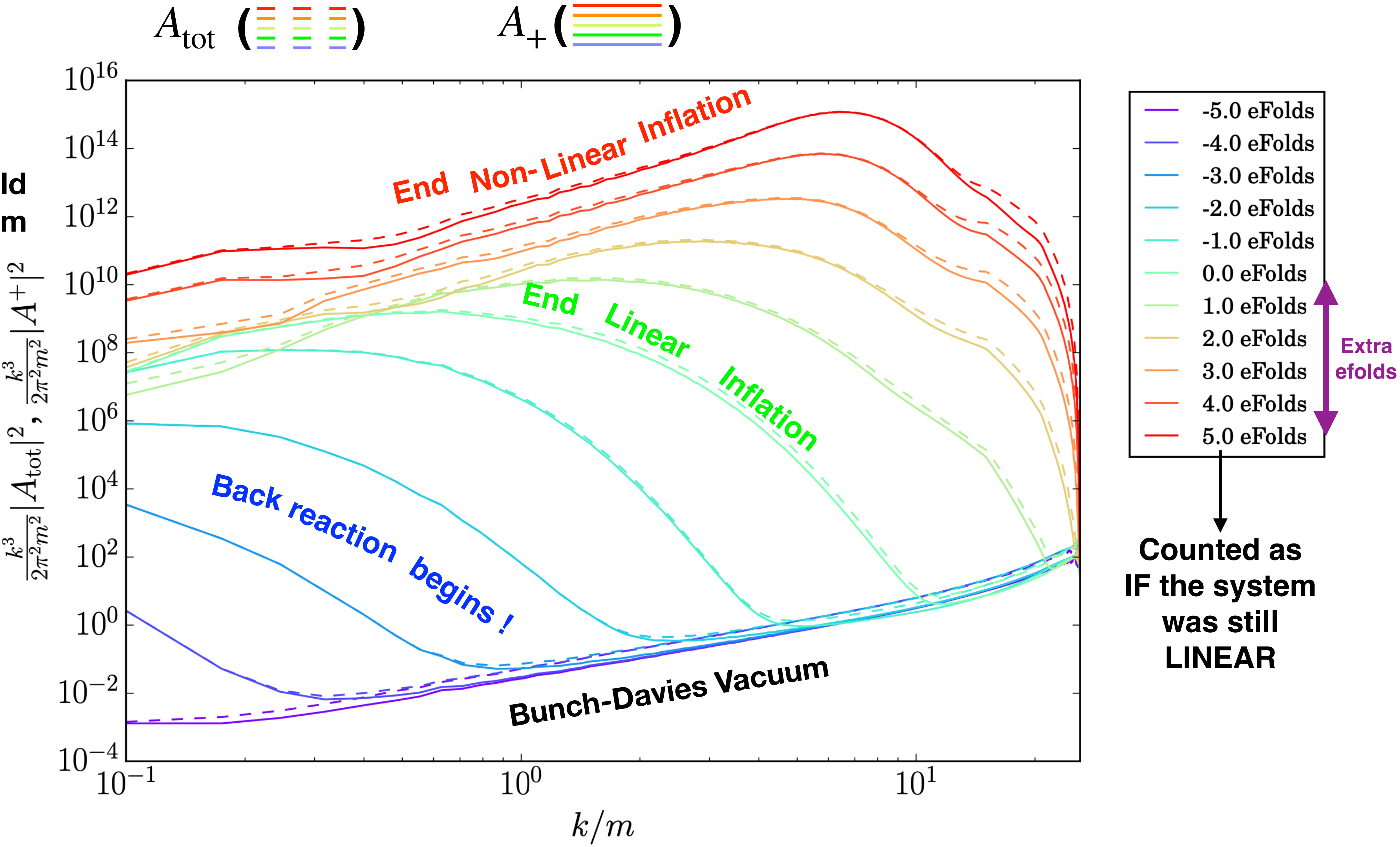
Gauge fld Spectrum



Counted as IF the system was still LINEAR

Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \alpha_\Lambda=18\right)$

Gauge fld Spectrum



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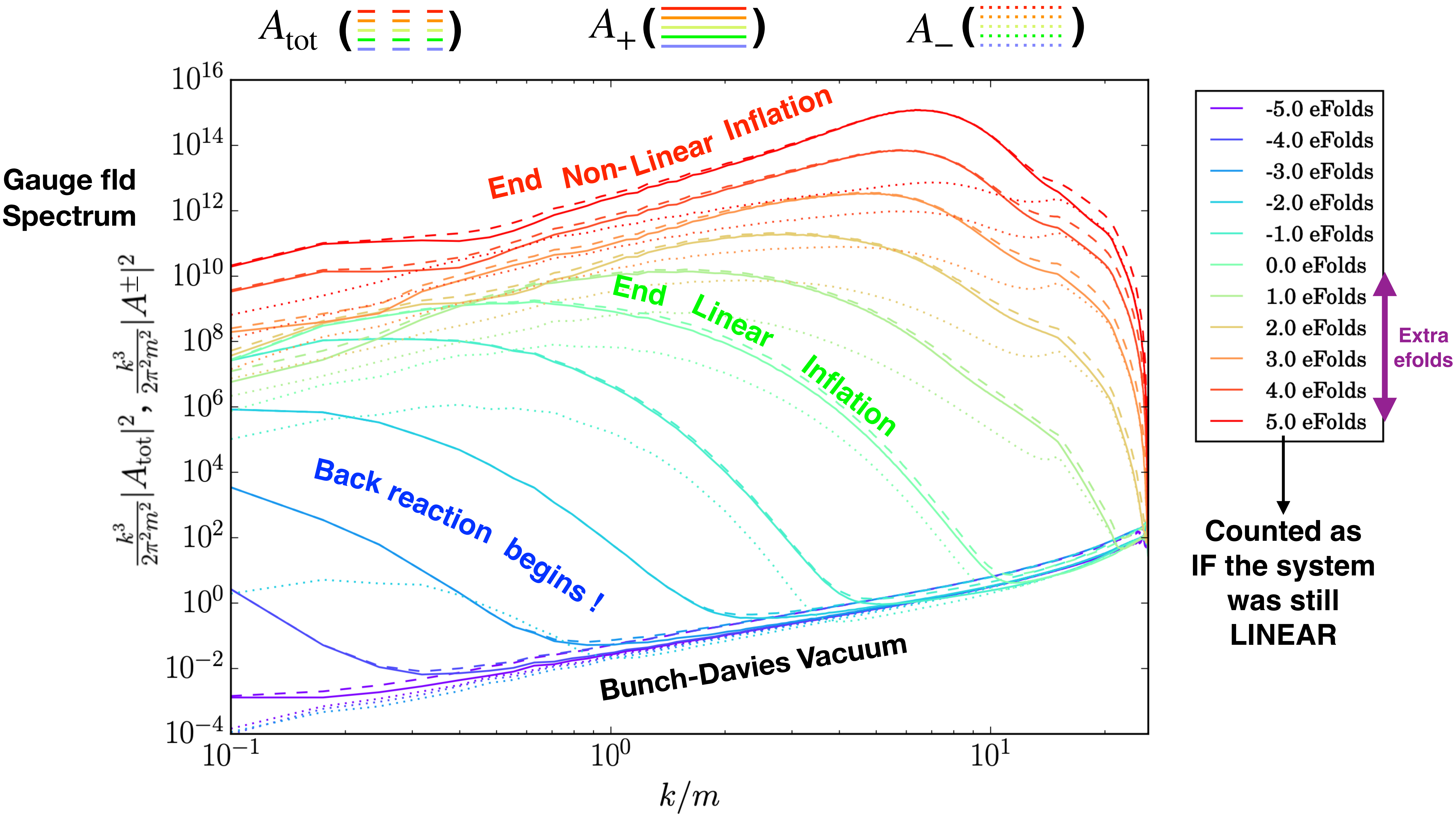
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

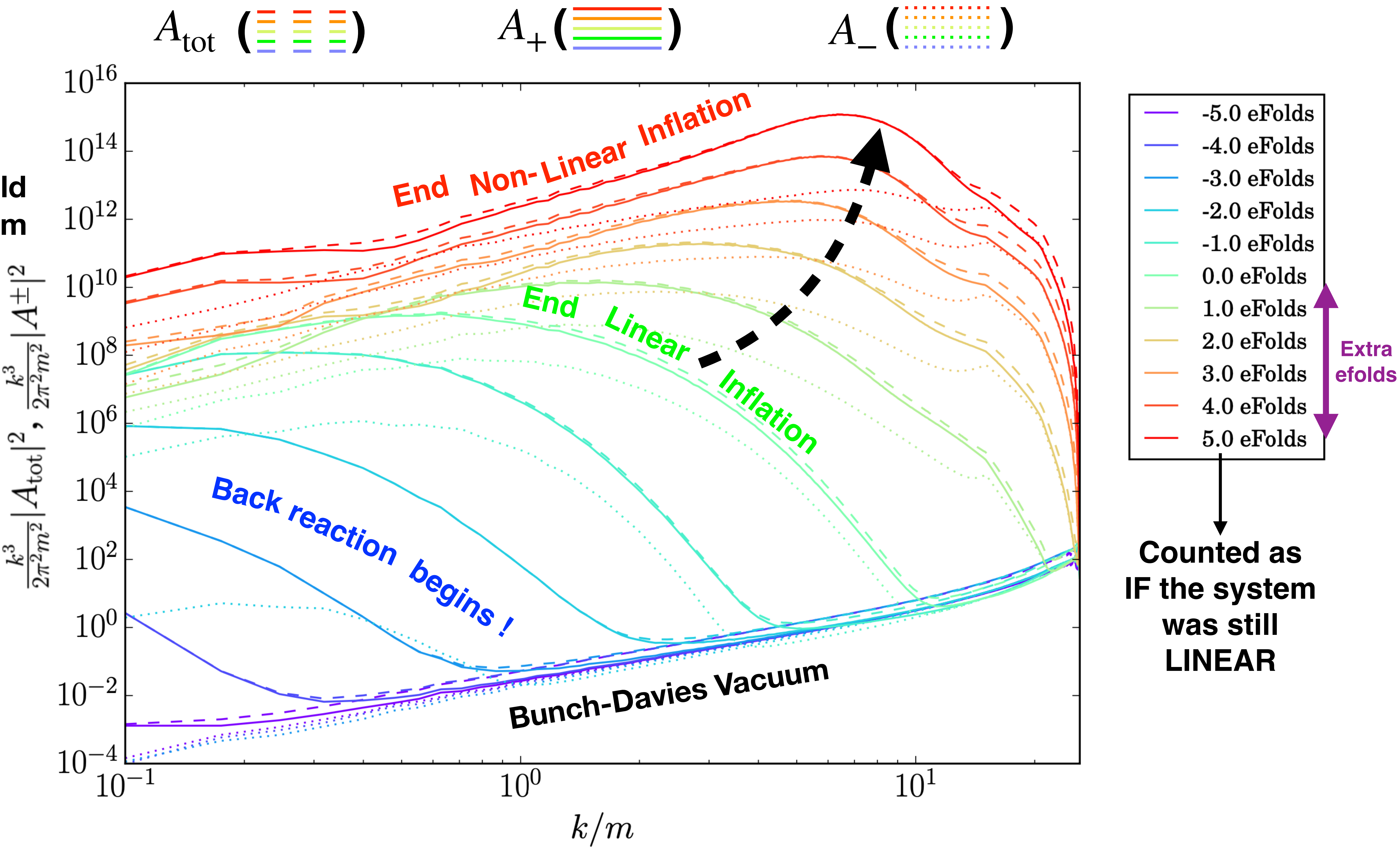


Gauge fld Spectrum

Counted as IF the system was still LINEAR

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

Gauge fld Spectrum



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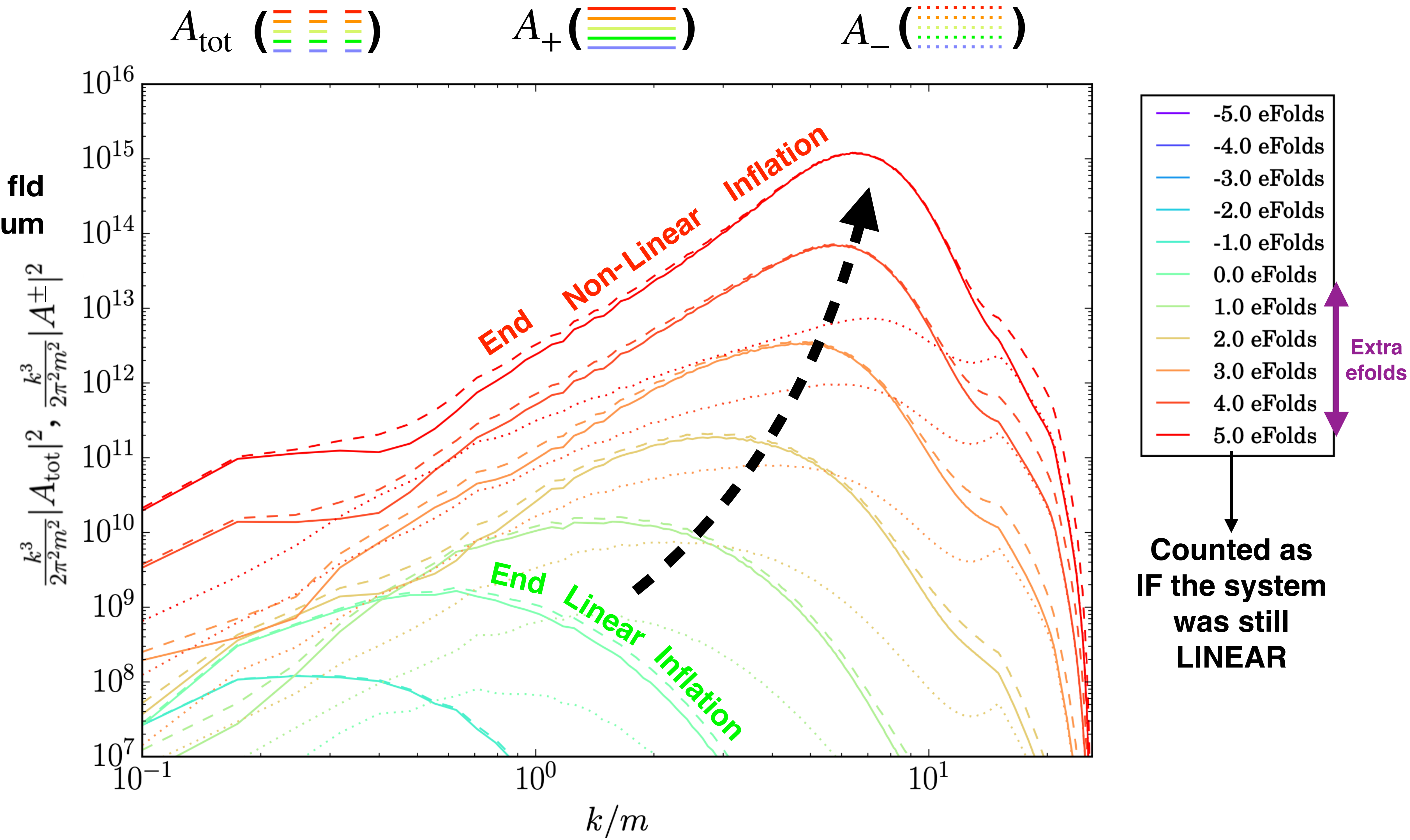
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\alpha_\Lambda = 18$)

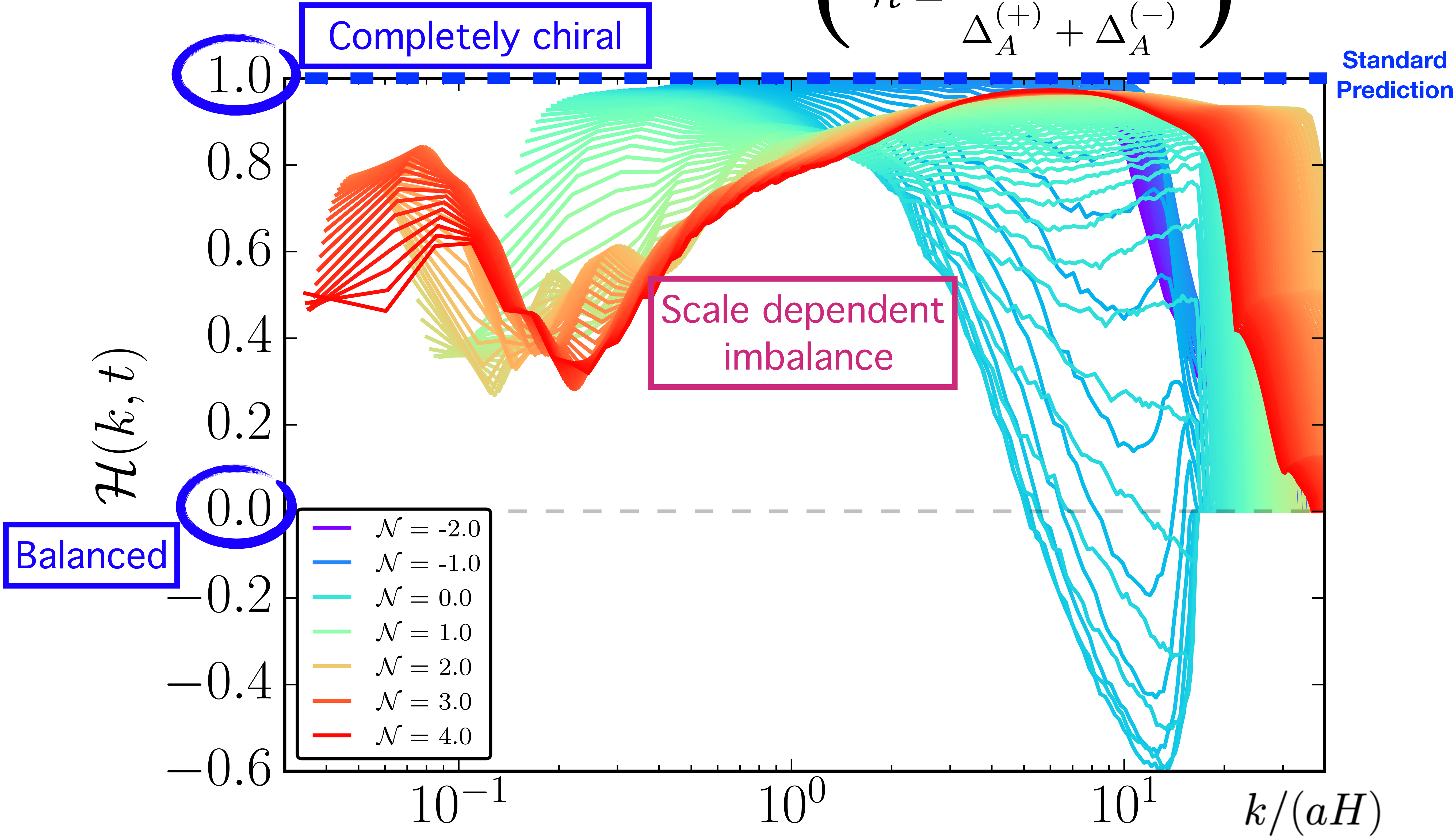
Gauge fld Spectrum



Counted as IF the system was still LINEAR

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

$$\left(\mathcal{H} = \frac{\Delta_A^{(+)} - \Delta_A^{(-)}}{\Delta_A^{(+)} + \Delta_A^{(-)}} \right)$$



Completely chiral

Standard Prediction

Scale dependent imbalance

Balanced

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{X}$)

$(X = 15, 20, 25)$

Summary

* ξ Controls the Gauge field excitation

* Linear change in ξ : exponential response in A_μ

* Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !

* Adding Schwinger pair production easy via $\vec{J} = \sigma \vec{E}$

* Other phenomena: BAU, Magnetogenesis, …

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{X}$)

$(X = 15, 20, 25)$

Summary

Phys. Rev. Lett. *131 (2023) 15, 151003*

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* **Linear change in ξ : exponential response in A_μ**

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* **Other phenomena: BAU, Magnetogenesis, …**

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda = \frac{m_p}{X}\right)$

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Summary

* ξ **Controls the Gauge field excitation**

* **Linear change in** ξ **: exponential response in** A_μ

Work in Progress ...

* **Predictions/constraints (PNG, PBH and GWs) depend crucially on** ξ **: we will re-assess real observability !**

* **Adding Schwinger pair production easy via** $\vec{J} = \sigma\vec{E}$

* **Other phenomena: BAU, Magnetogenesis, …**

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Summary

* ξ Controls the Gauge field excitation

* Linear change in ξ : exponential response in A_μ

* Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !

Future work ...

* Adding Schwinger pair production easy via $\vec{J} = \sigma \vec{E}$

* Other phenomena: BAU, Magnetogenesis, …

Example II

Cosmic String Loops (+ GW emission)

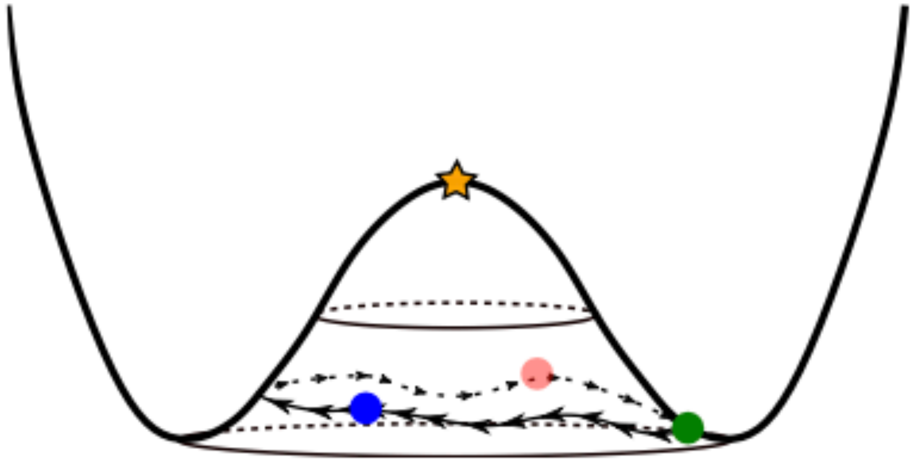
ArXiv:2308.08456 [astro-ph]

(Submitted to Phys. Rev. Lett.)

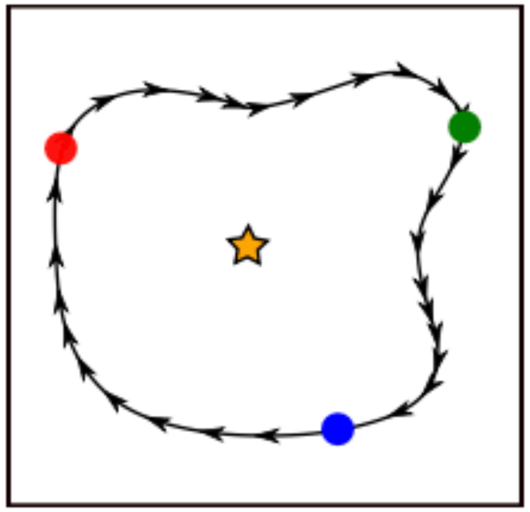
with ***J. Baeza-Ballesteros, E. Copeland, J. Lizarraga***
(PhD student)

Cosmic String Formation

Cosmic strings are **one-dimensional topological defects**



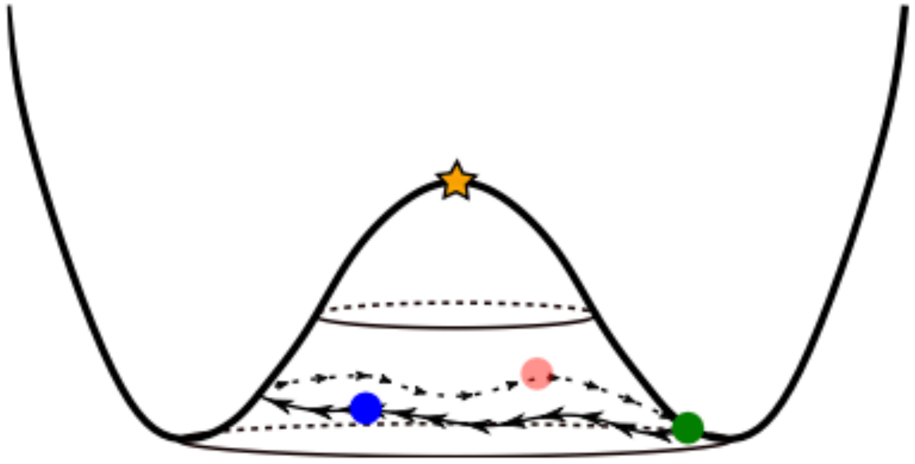
Symm. Breaking Fld ('Higgs')



Different Vacua

Cosmic String Formation

Cosmic strings are **one-dimensional topological defects**



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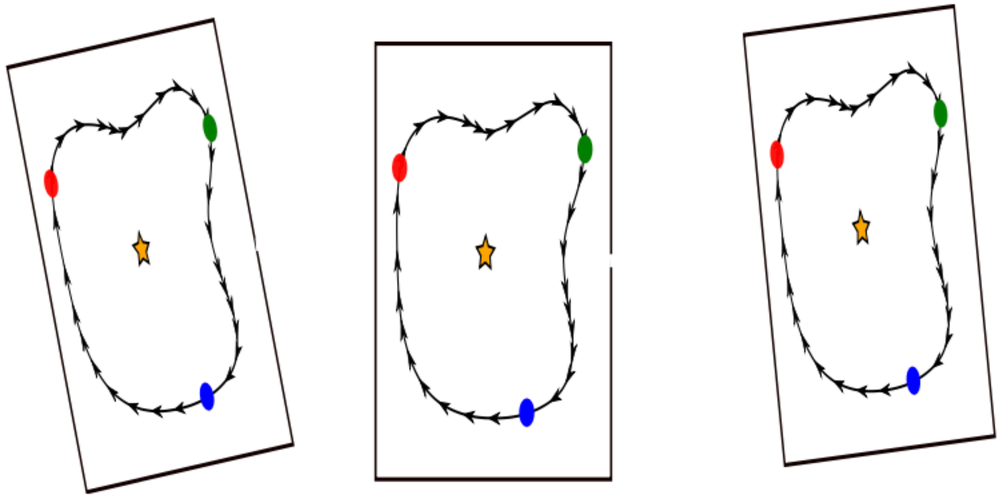
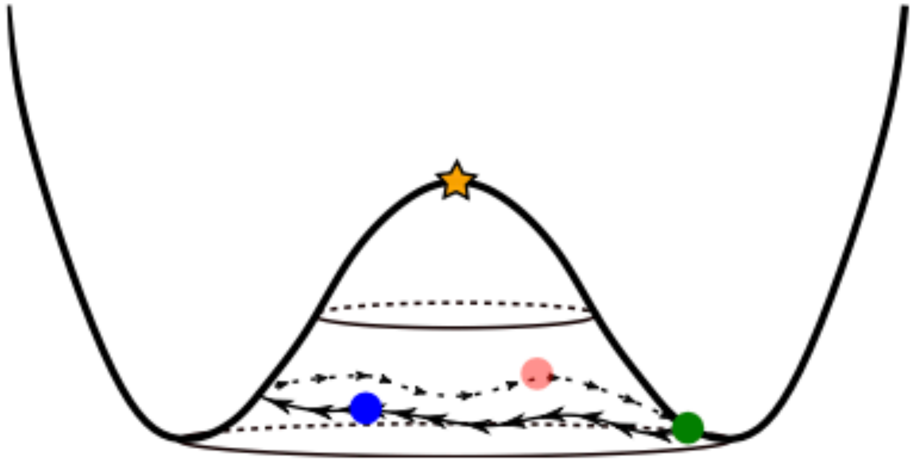


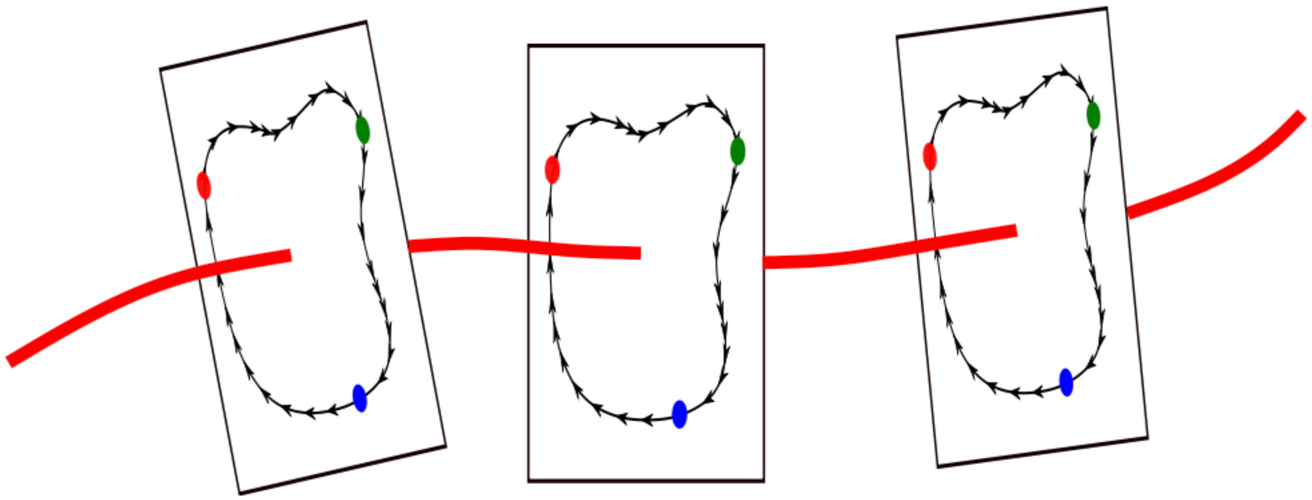
Different Vacua (at different locations)

Cosmic String Formation

Cosmic strings are **one-dimensional topological defects**



Symm. Breaking Fld (‘Higgs')



Different Vacua (at different locations)

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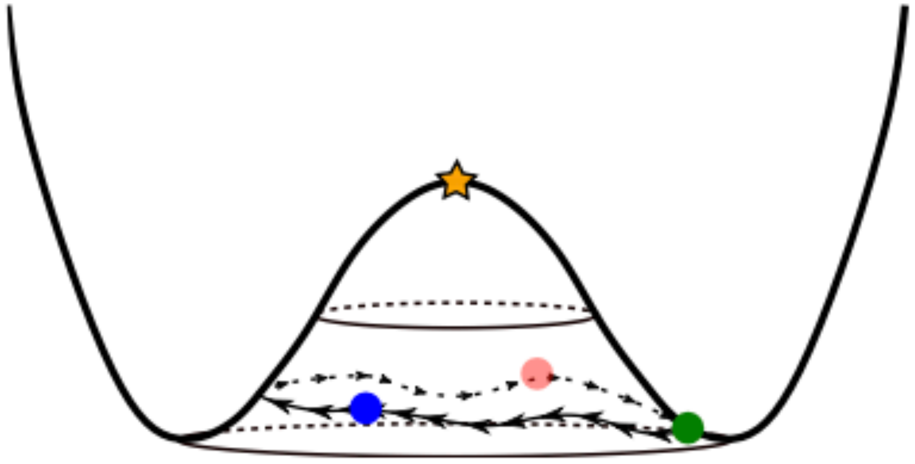

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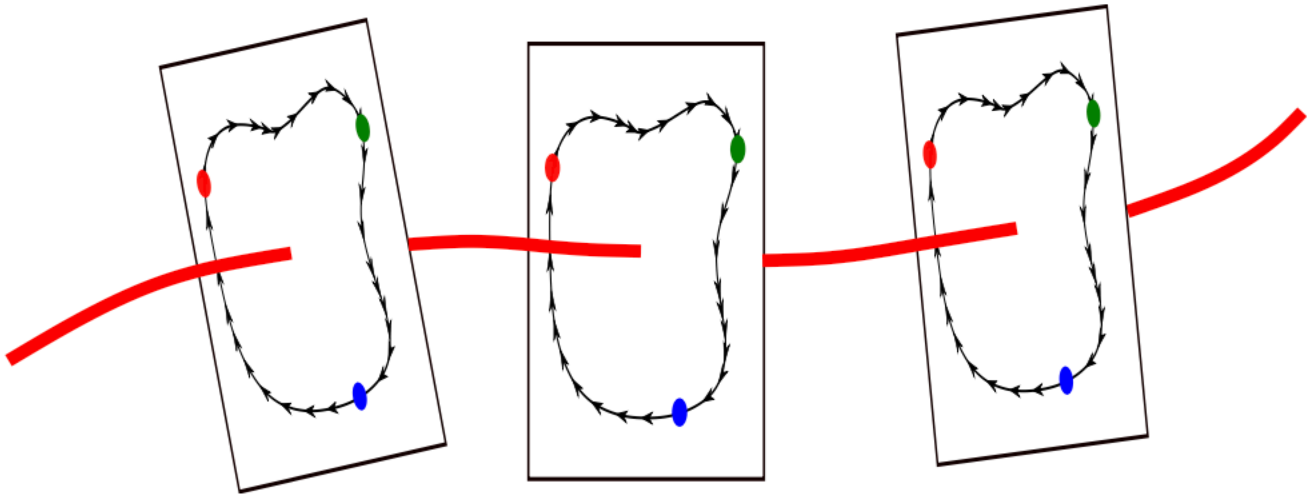


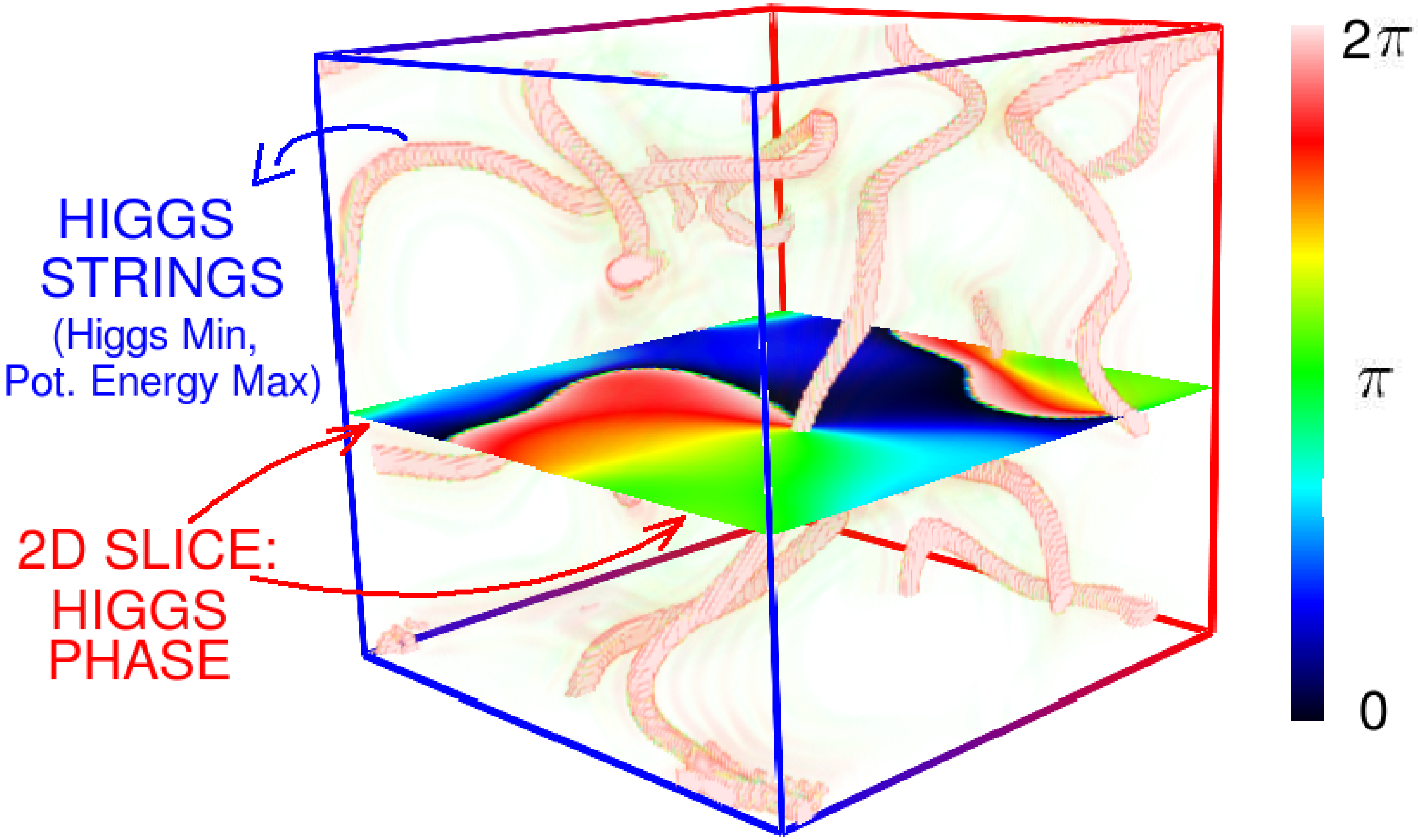
Different Vacua (at different locations)

Global (scalar) or Local (Scalar + Gauge fld.)

Cosmic String Formation

[Cosmic Strings: Global (scalar) or Local (Scalar + Gauge fld.)]

e.g. Global cosmic strings

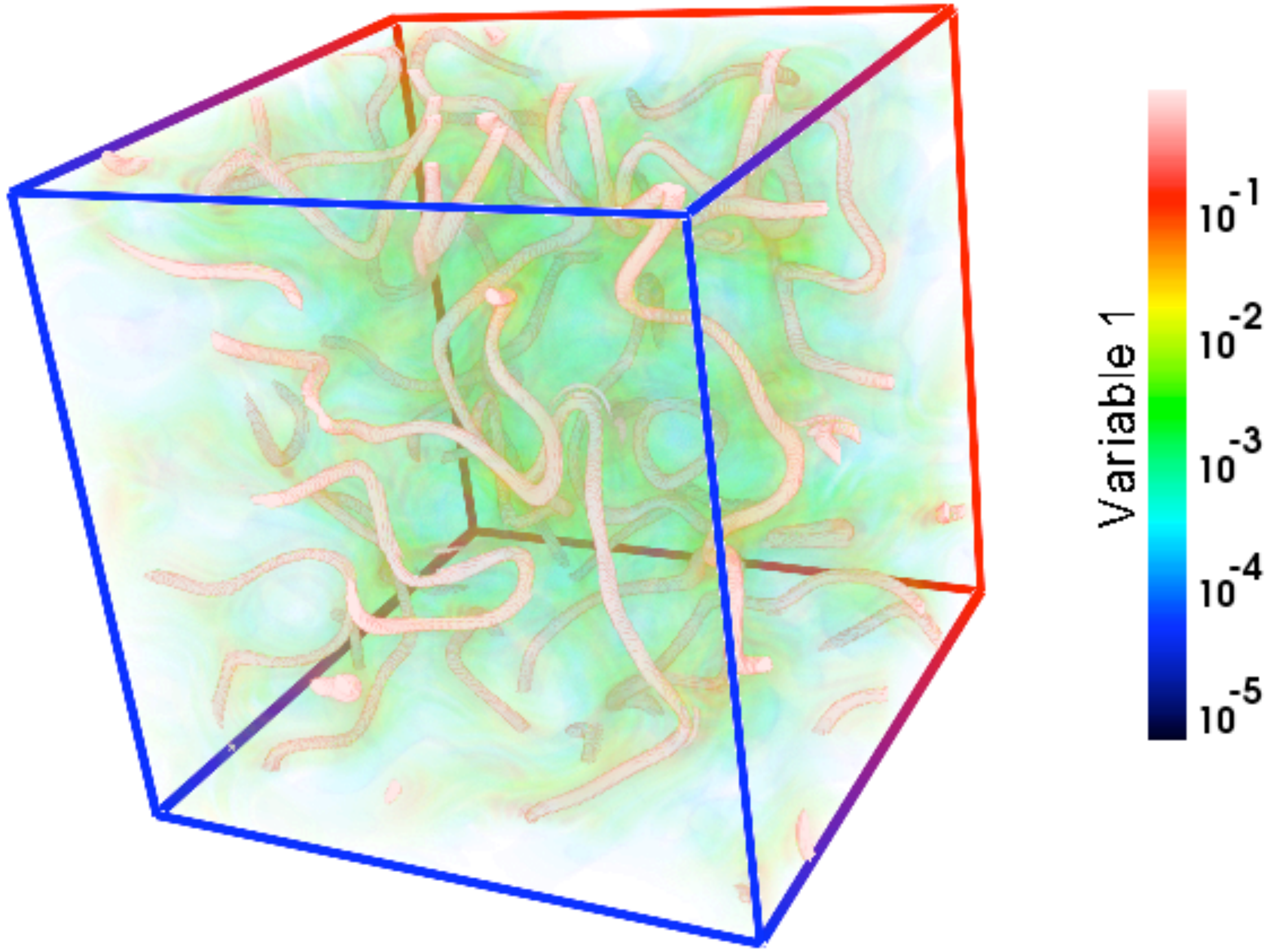


DGF, PhD Thesis 2010

Cosmic String Formation

[Cosmic Strings: Global (scalar) or Local (Scalar + Gauge fld.)]

Intensity of magnetic energy density



DGF, PhD Thesis 2010

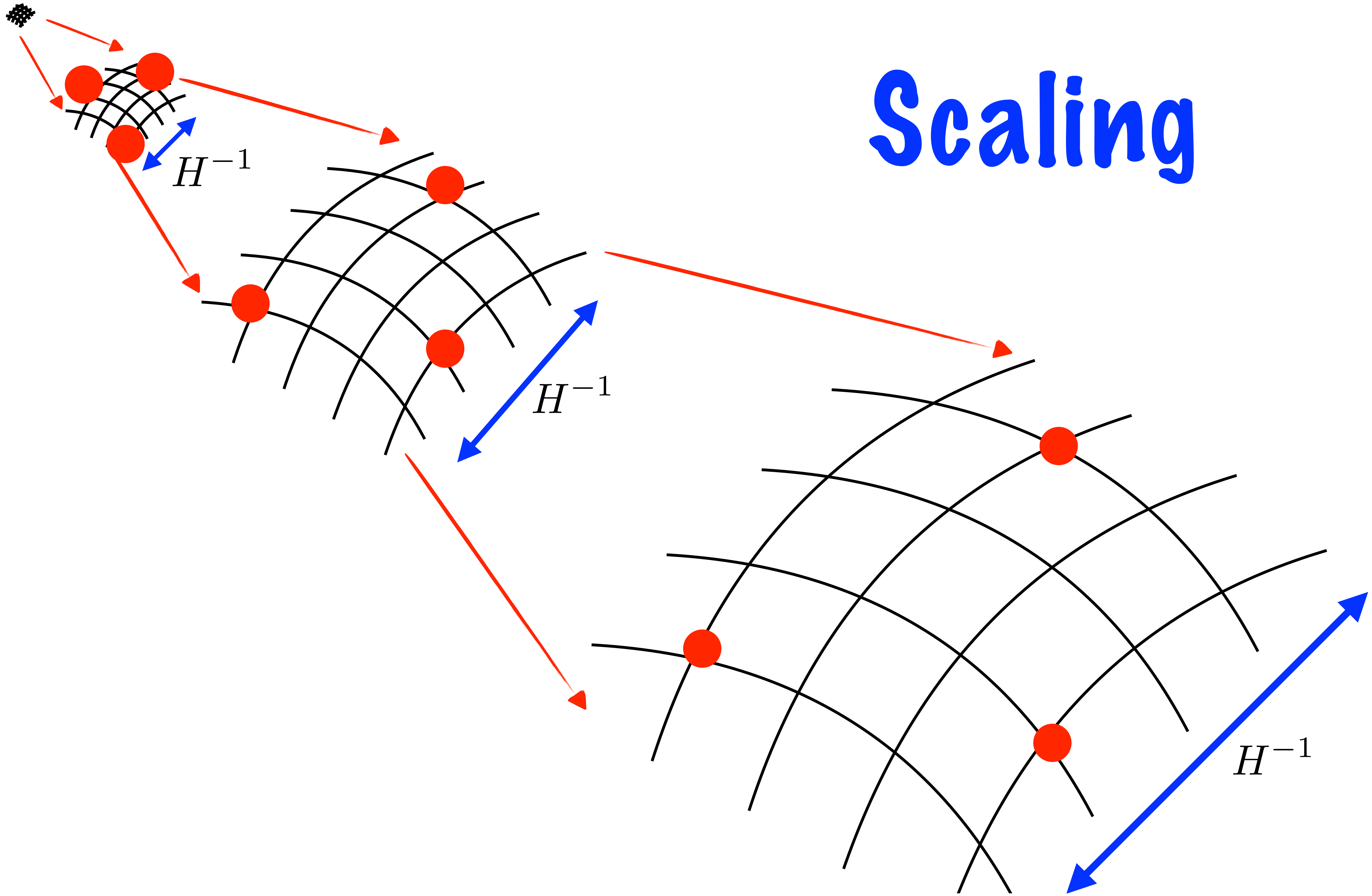
Cosmic String Networks

* Scaling dynamics

* Infinitely thin

* Inter-commutation

Cosmic String Networks



Scaling

Cosmic String Networks

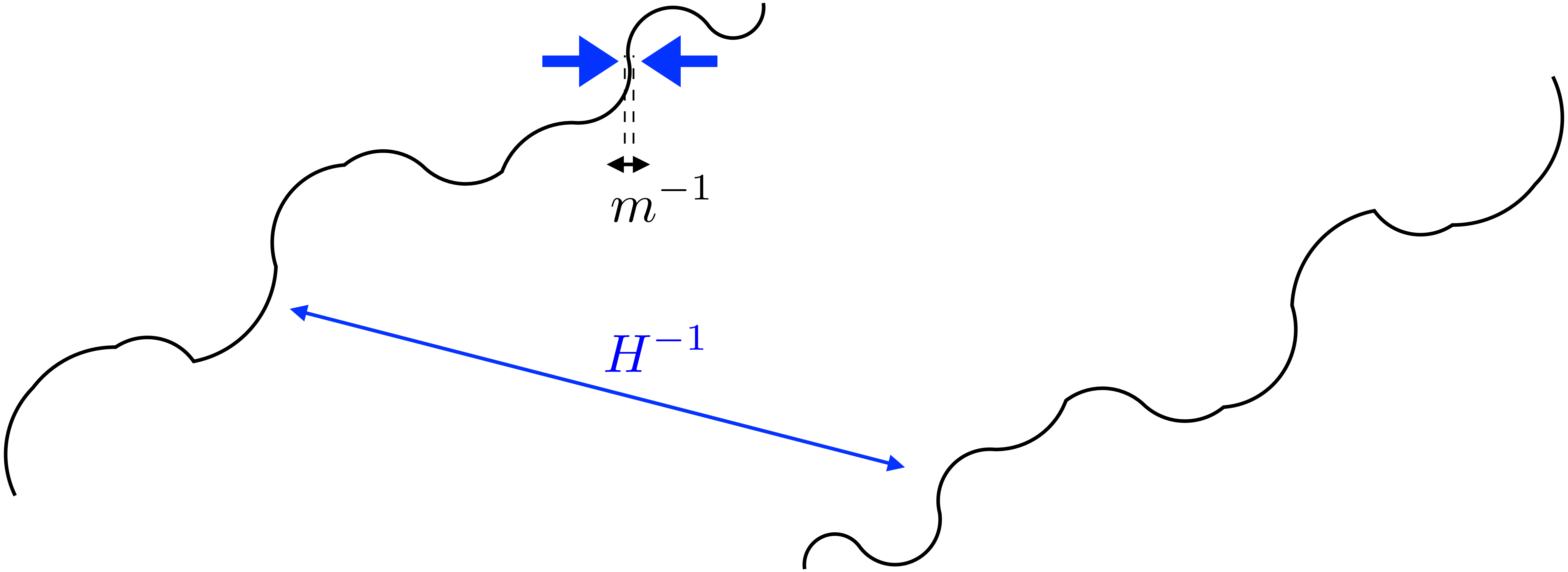
* Scaling dynamics

* Infinitely thin

* Inter-commutation

Cosmic String Networks

Infinitely thin: $H^{-1} \gg m^{-1}$



Nambu-Goto

Cosmic String Networks

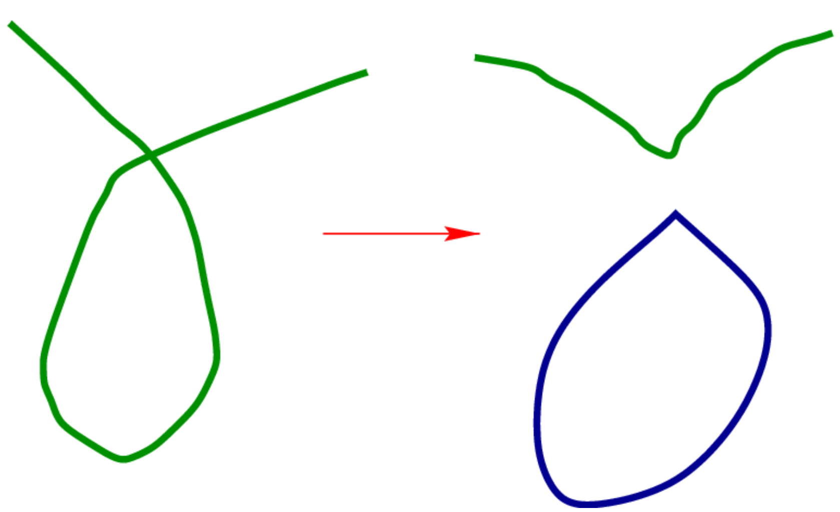
* Scaling dynamics

* Infinitely thin

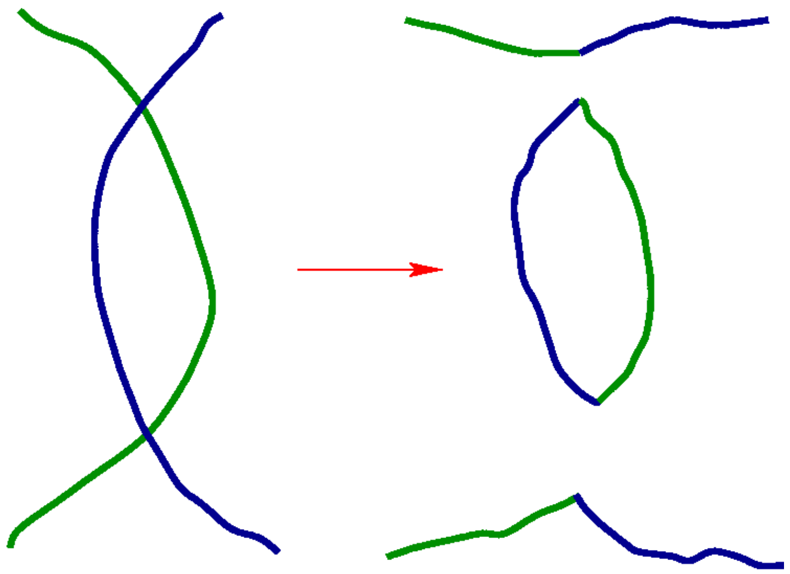
* Inter-commutation

Cosmic String Networks

Intercommutation



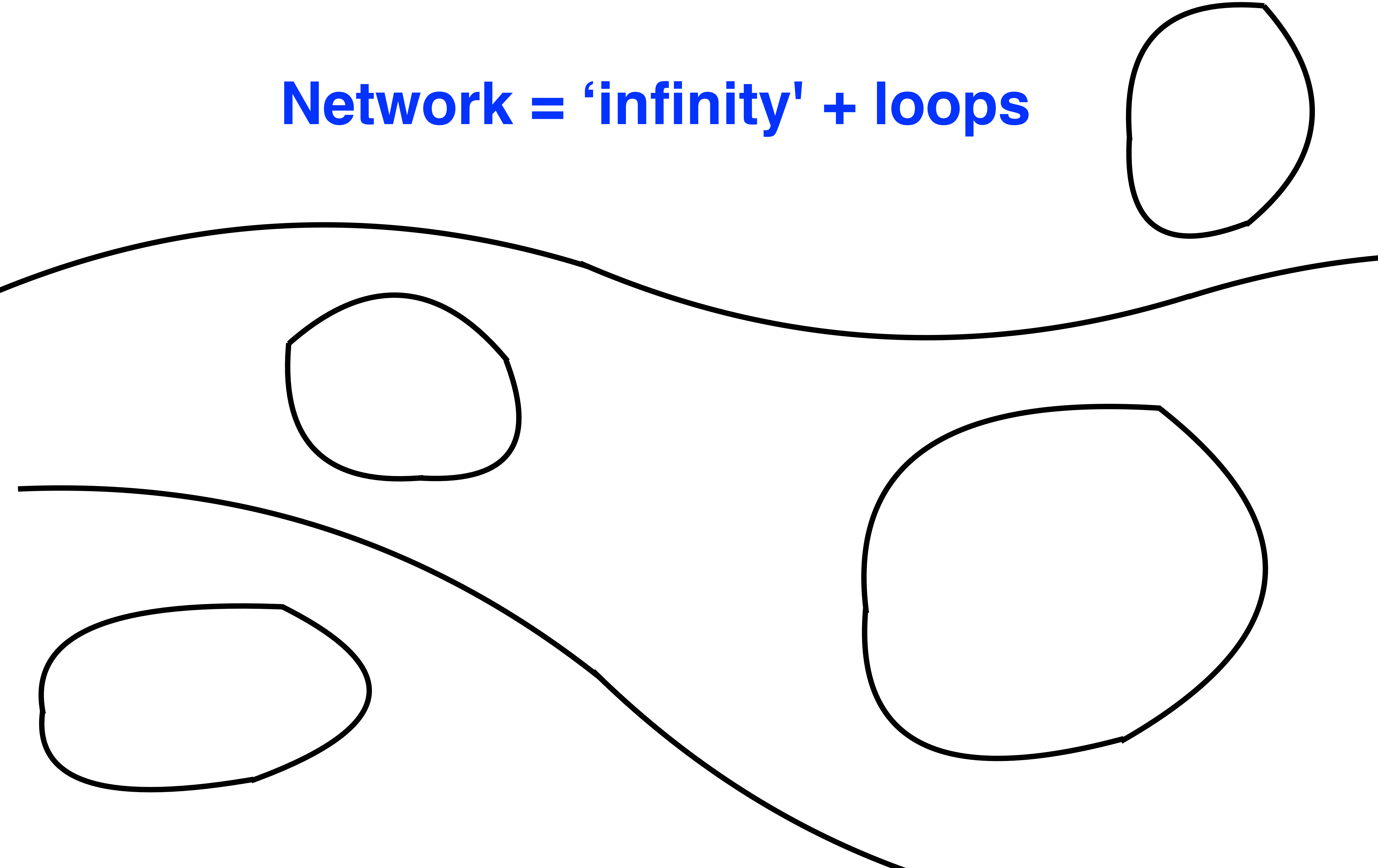
Loops !



Loops !

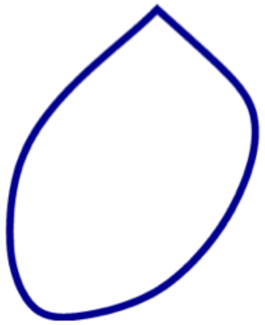
Cosmic String Networks

Network = ‘infinity' + loops



Cosmic String Networks

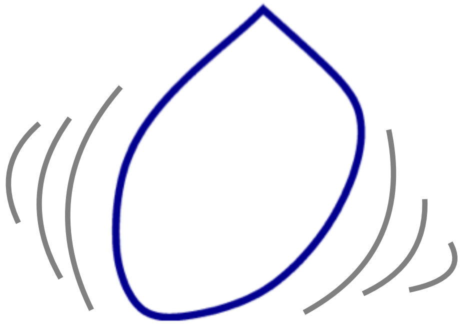
Loops are formed !



Cosmic String Networks

Loops are formed !

Vibrate under their tension !

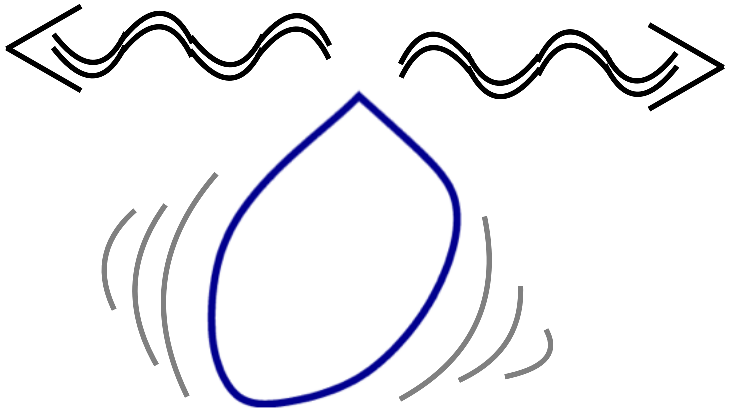


Periodic Oscillations

Cosmic String Networks

Loops are formed !

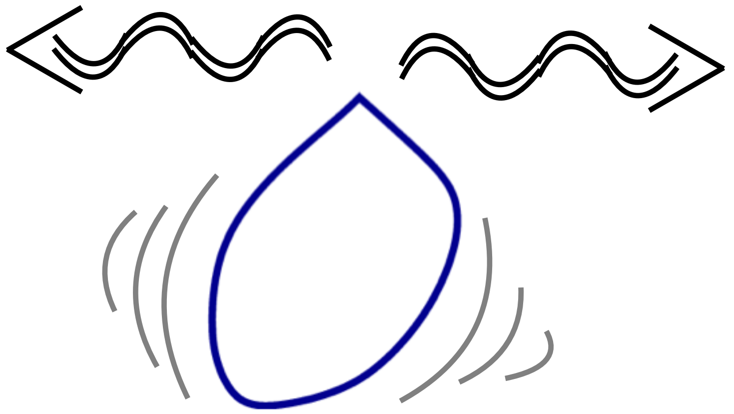
Vibrate under their tension !



Gravitational Waves (GW) are emitted !

Cosmic String Networks

Loops are formed !



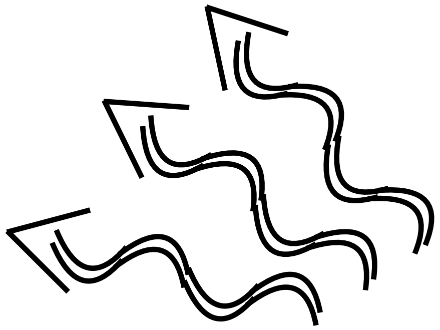
Vibrate under their tension !

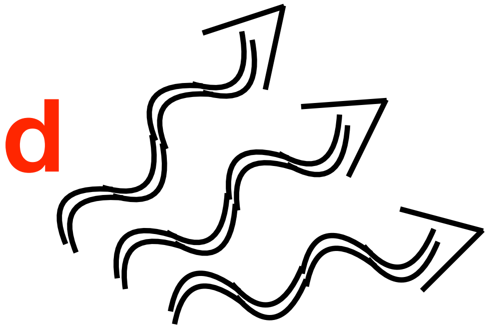
Gravitational Waves (GW) are emitted !

Superposition from many loop signals

=

Gravitational Wave Background





Cosmic String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops

Cosmic String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops

  ↳ 'Decay' to loops

Cosmic String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

→ Decay to GWs **(Vilenkin ’81)**

- String networks = Infinite strings + Loops

→ 'Decay' to loops

Cosmic String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops → Decay to GWs
  - Infinite strings → 'Decay' to loops
- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \quad \longrightarrow \quad P_{\mathrm{GW}} = \dot{E}_{\mathrm{GW}} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

Cosmic String Networks

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But ...

Field-theory strings can also decay via **particle emission**

Cosmic String Networks

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Goal: Particle and GW emission

Cosmic String Networks

Traditional picture → **Nambu-Goto approximation** (zero width)

↱ Decay to GWs

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↳ 'Decay' to loops

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But …

Field-theory strings can also decay via **particle emission**

Goal: Particle and GW emission using lattice simulations

Cosmic String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

→ Decay to GWs

- String networks $=$ Infinite strings $+$ Loops

→ 'Decay' to loops

- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \longrightarrow P_{\mathrm{GW}} = \dot{E}_{\mathrm{GW}} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

But …

Field-theory strings can also decay via **particle emission**

Goal: Particle and GW emission using lattice simulations

First time simultaneously !

String Loop: Particle & GW emission

GOAL

Dynamics of an isolated loop
and its particle & GW emission

String Loop: Particle & GW emission

GOAL

Dynamics of an isolated loop
and its particle & GW emission

Today's focus on
… Global Strings

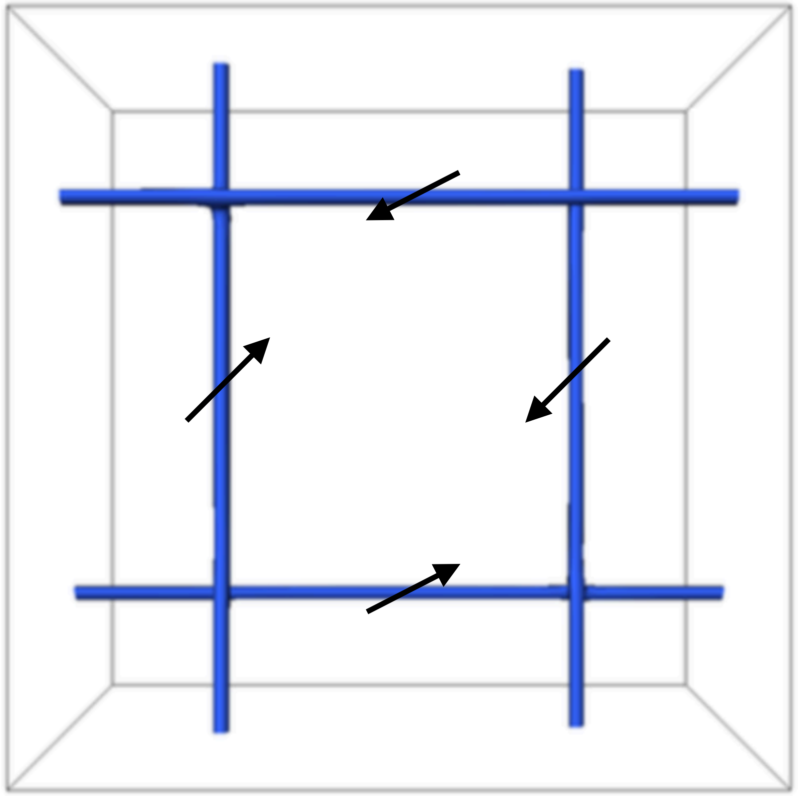
[but Local String
analysis coming !]

String Loop: Particle & GW emission

GOAL

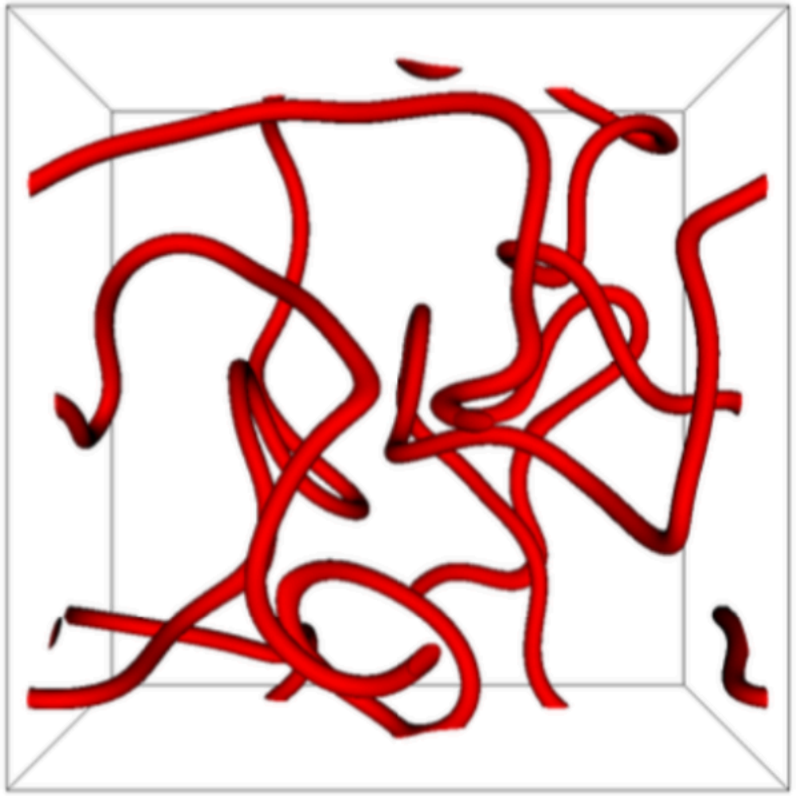
Dynamics of an isolated loop
and its particle & GW emission

Case I : Nielsen-Olesen



(following Vachaspati et al 2020)

Case II : Network



(following Lizarraga et al 2020/21)

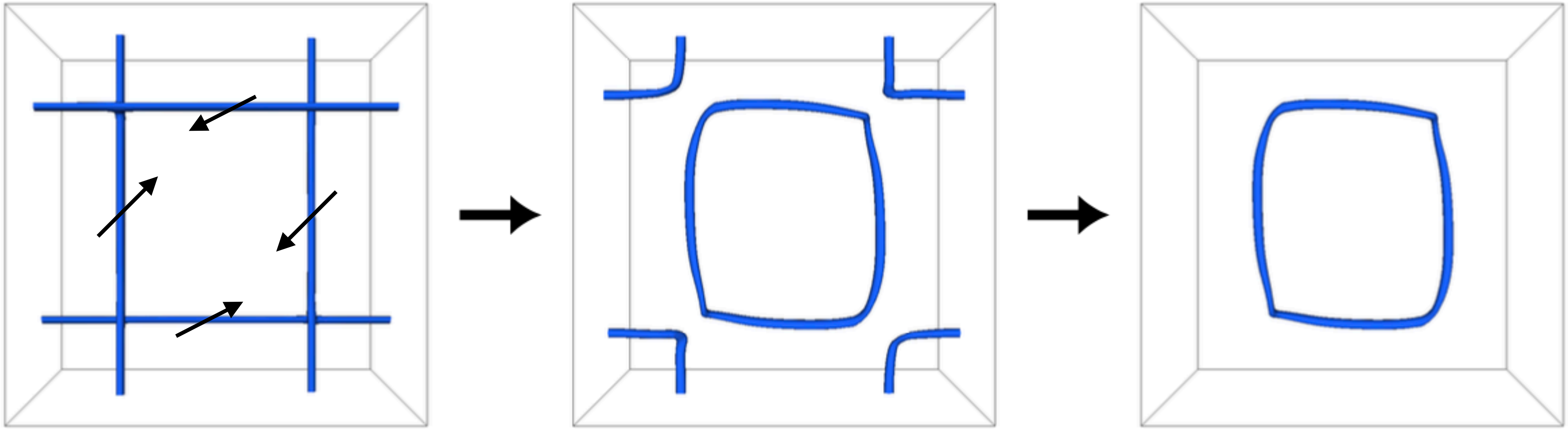
String Loop: Particle & GW emission

GOAL

Dynamics of an isolated loop and its particle & GW emission

Case I: 'Artificial'

— Isolate the inner loop —



Boost 2 string pairs

Intersect —> 2 Loops Inner/Outer

Isolate Inner Loop

String Loop: Particle & GW emission

GOAL

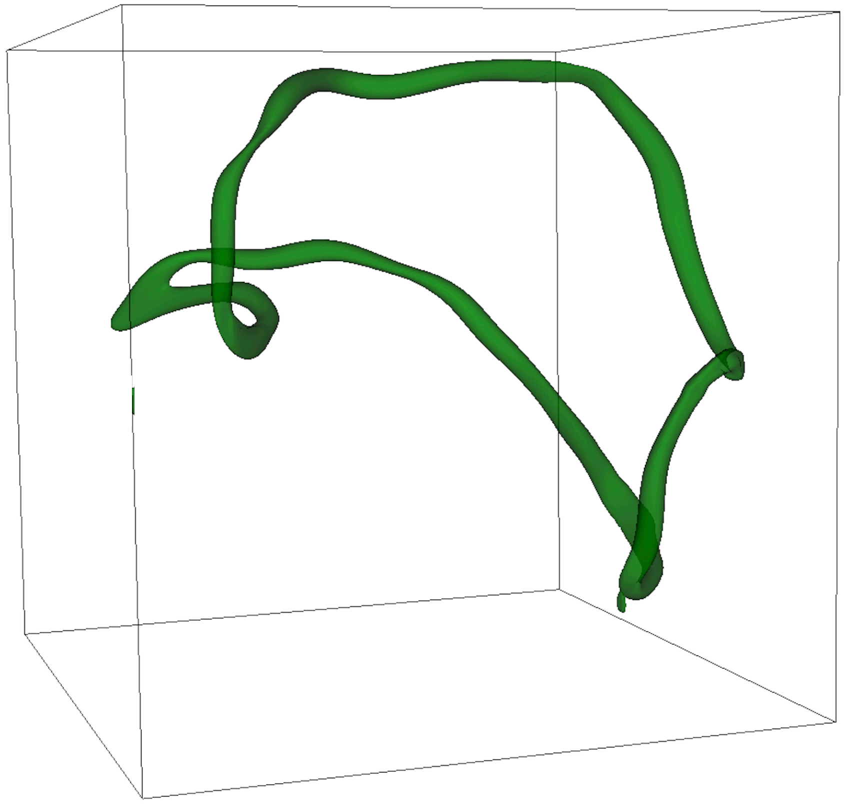
Dynamics of an isolated loop and its particle & GW emission

Case II: Network

— Only one loop remains eventually —

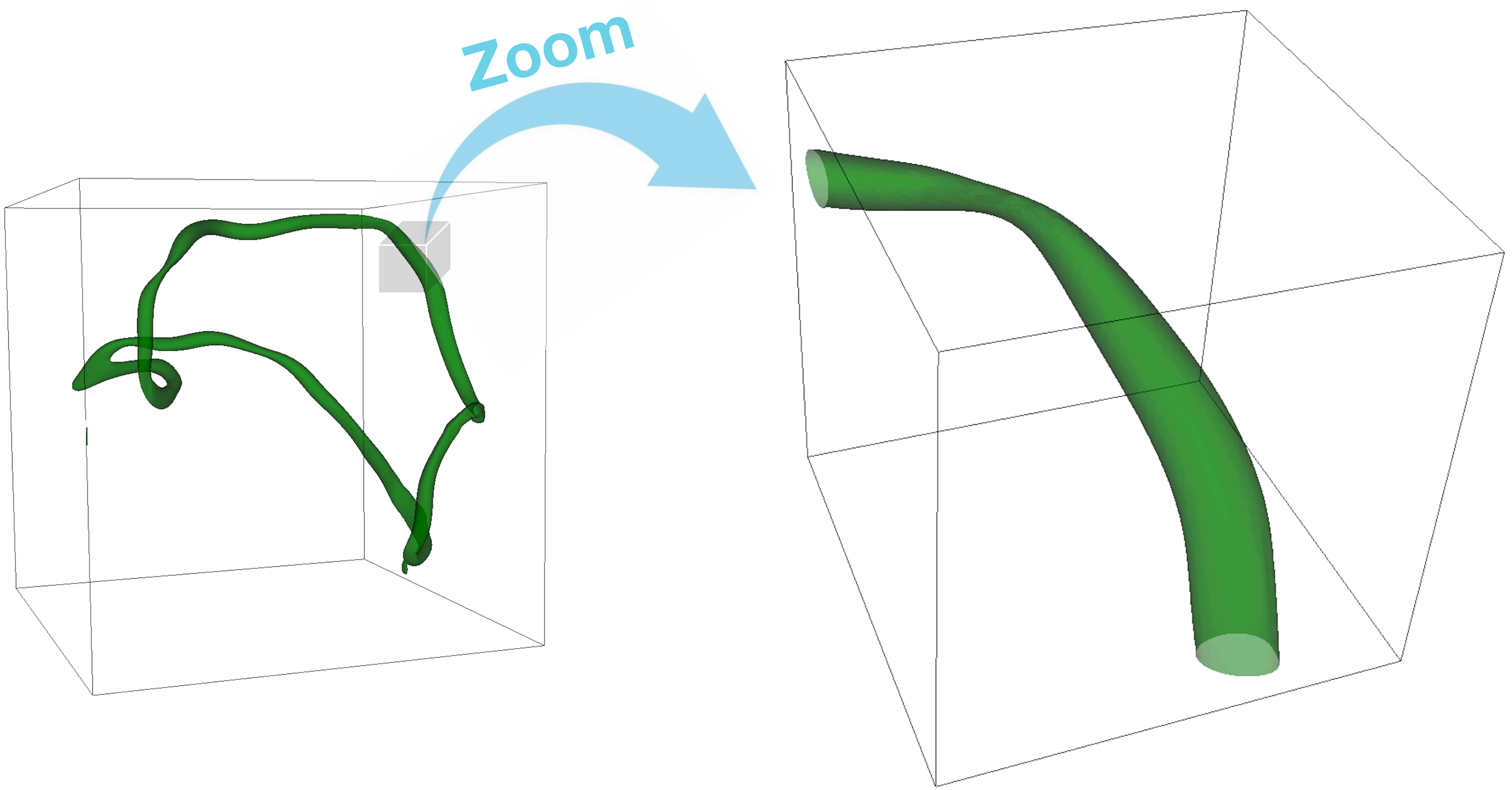
String Loop: Particle & GW emission

Loop Resolution



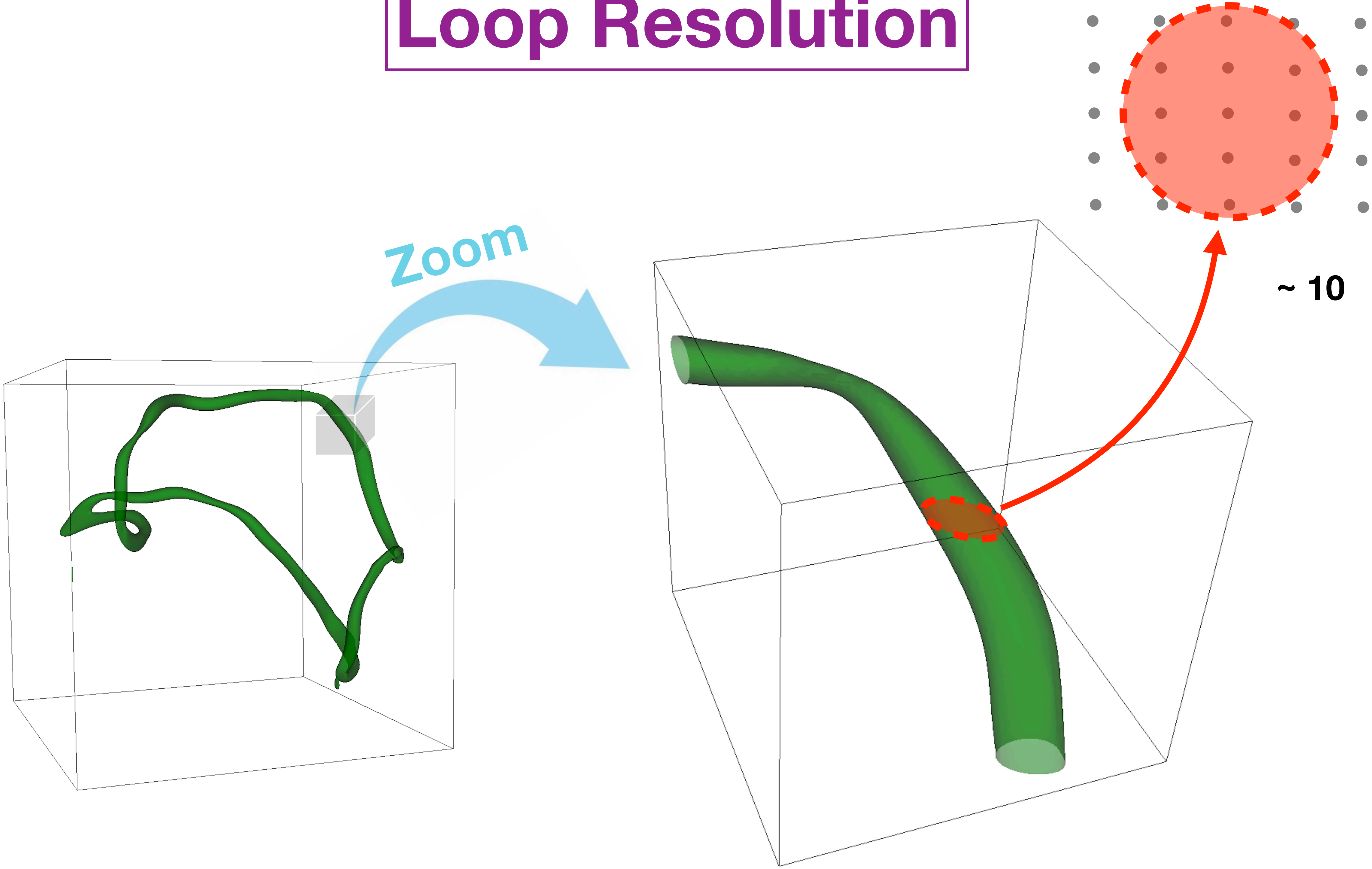
String Loop: Particle & GW emission

Loop Resolution



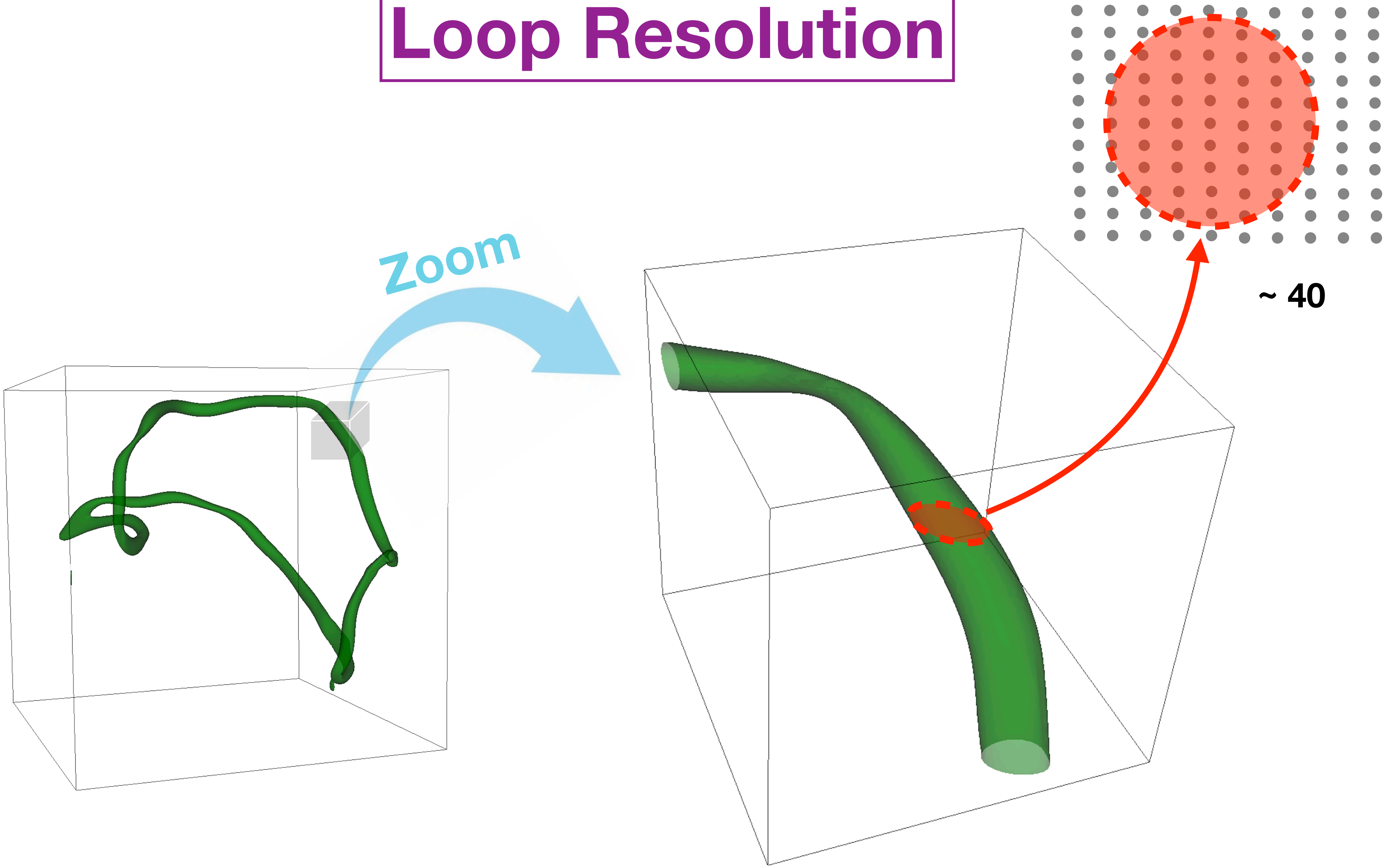
String Loop: Particle & GW emission

Loop Resolution



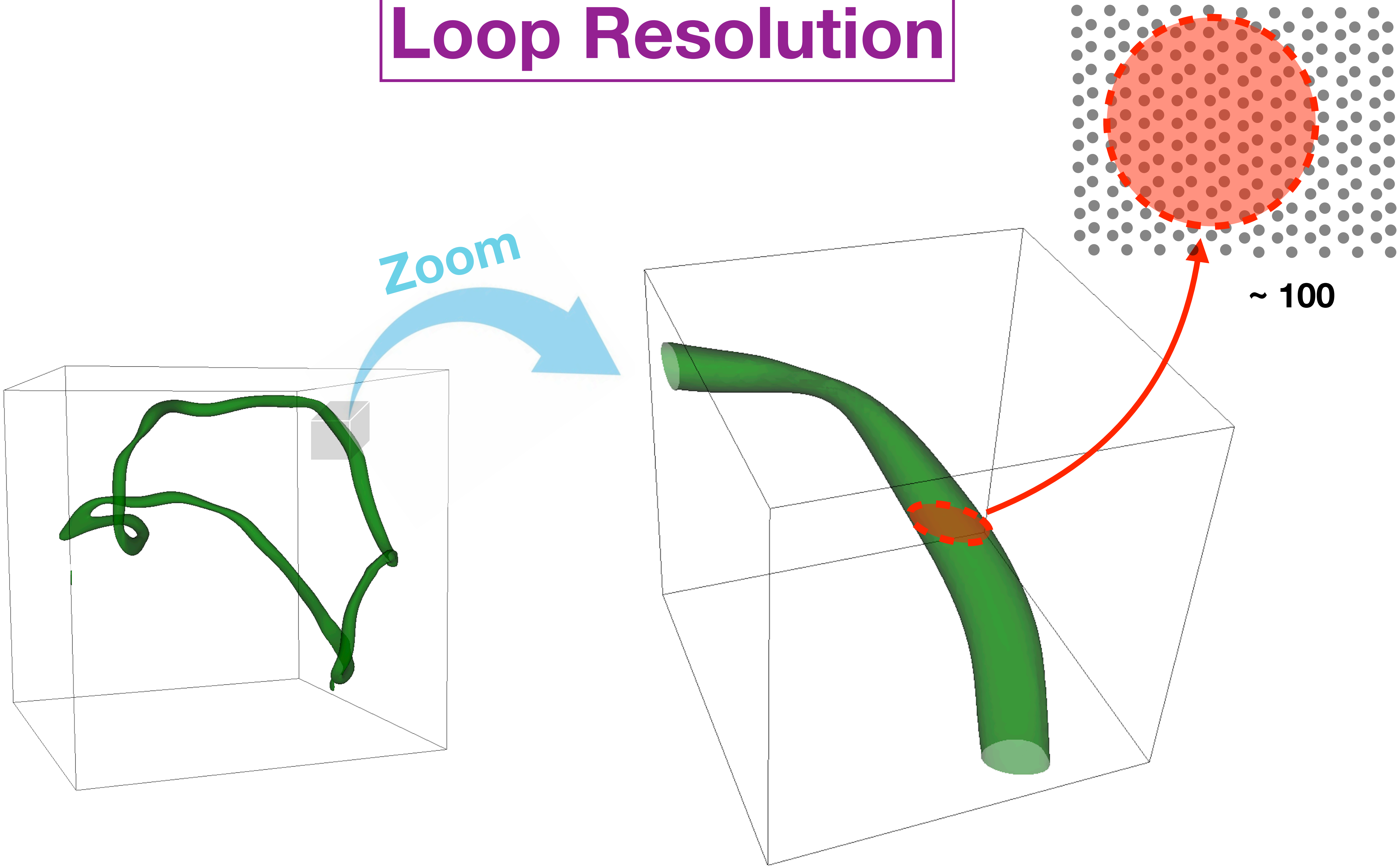
String Loop: Particle & GW emission

Loop Resolution



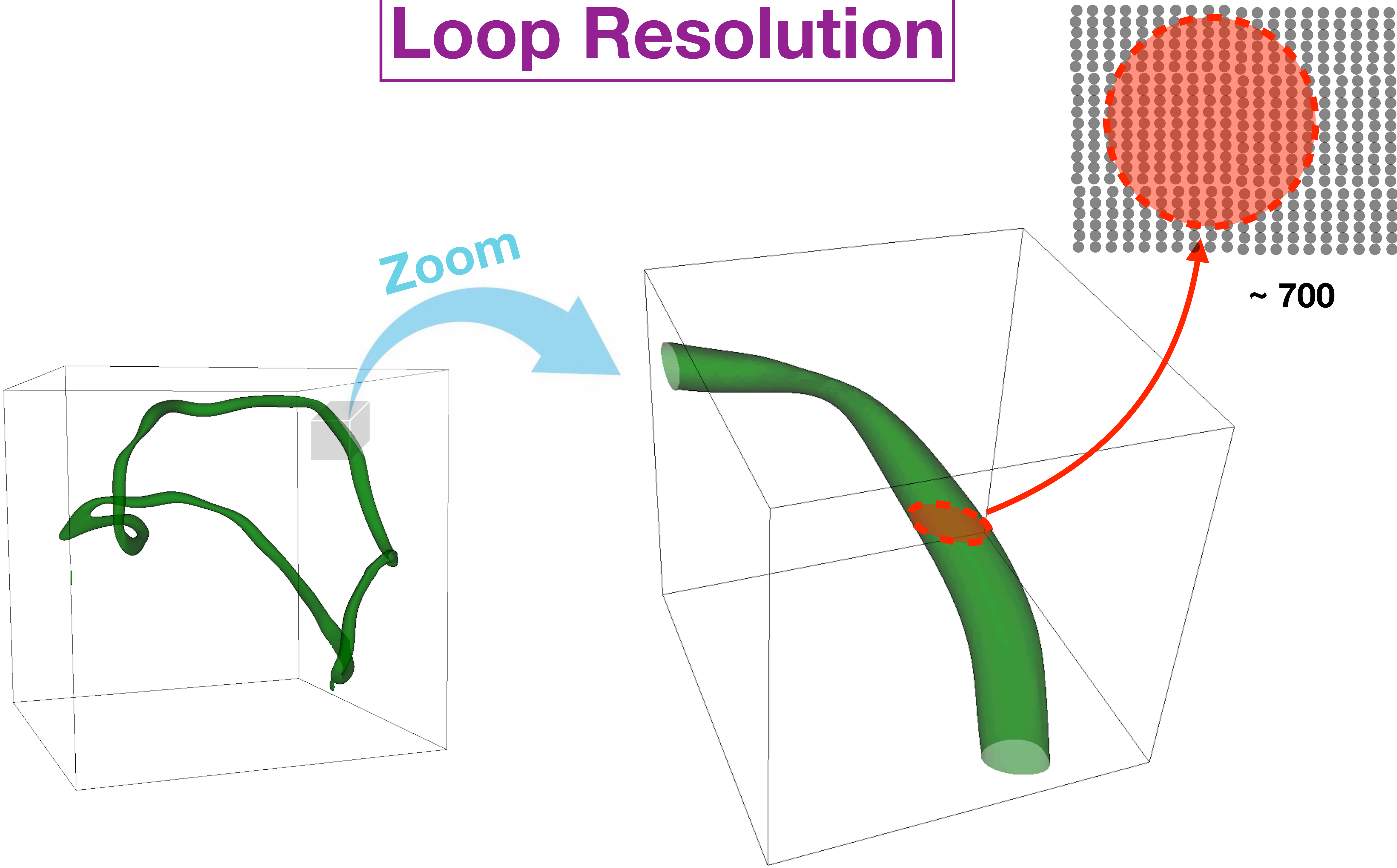
String Loop: Particle & GW emission

Loop Resolution



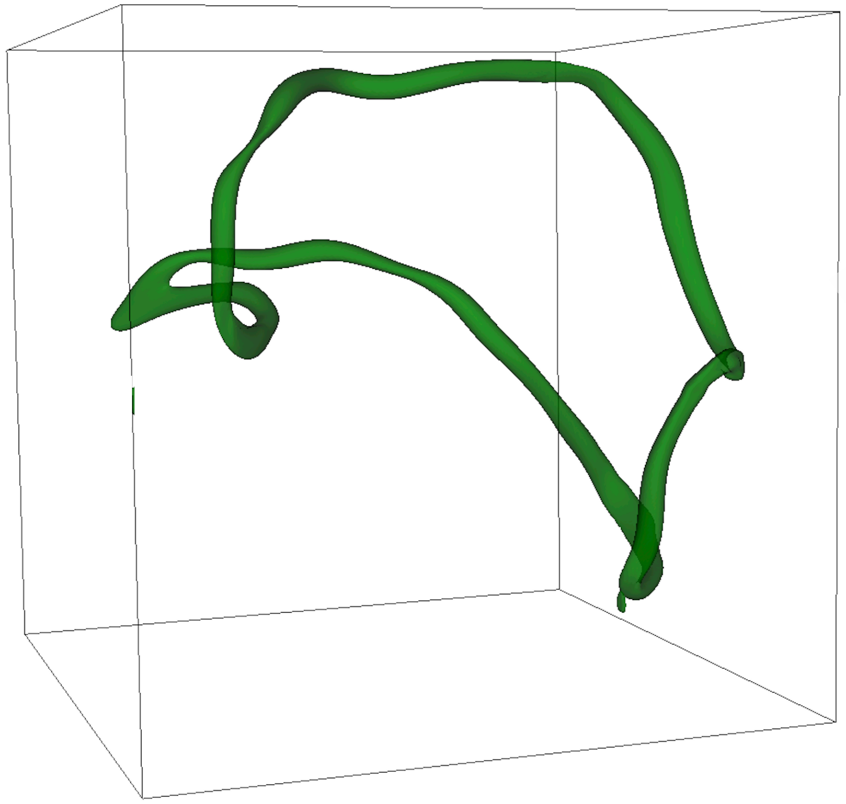
String Loop: Particle & GW emission

Loop Resolution



String Loop: Particle emission

Decay of a Loop



Higgs isosurface

String Core

String Loop: Particle emission

Decay of a Loop

String Loop: Particle emission

Decay of a Loop

String Loop: Particle emission

Decay of a Loop

String Loop: Particle emission

Decay of a Loop

String Loop: Particle emission

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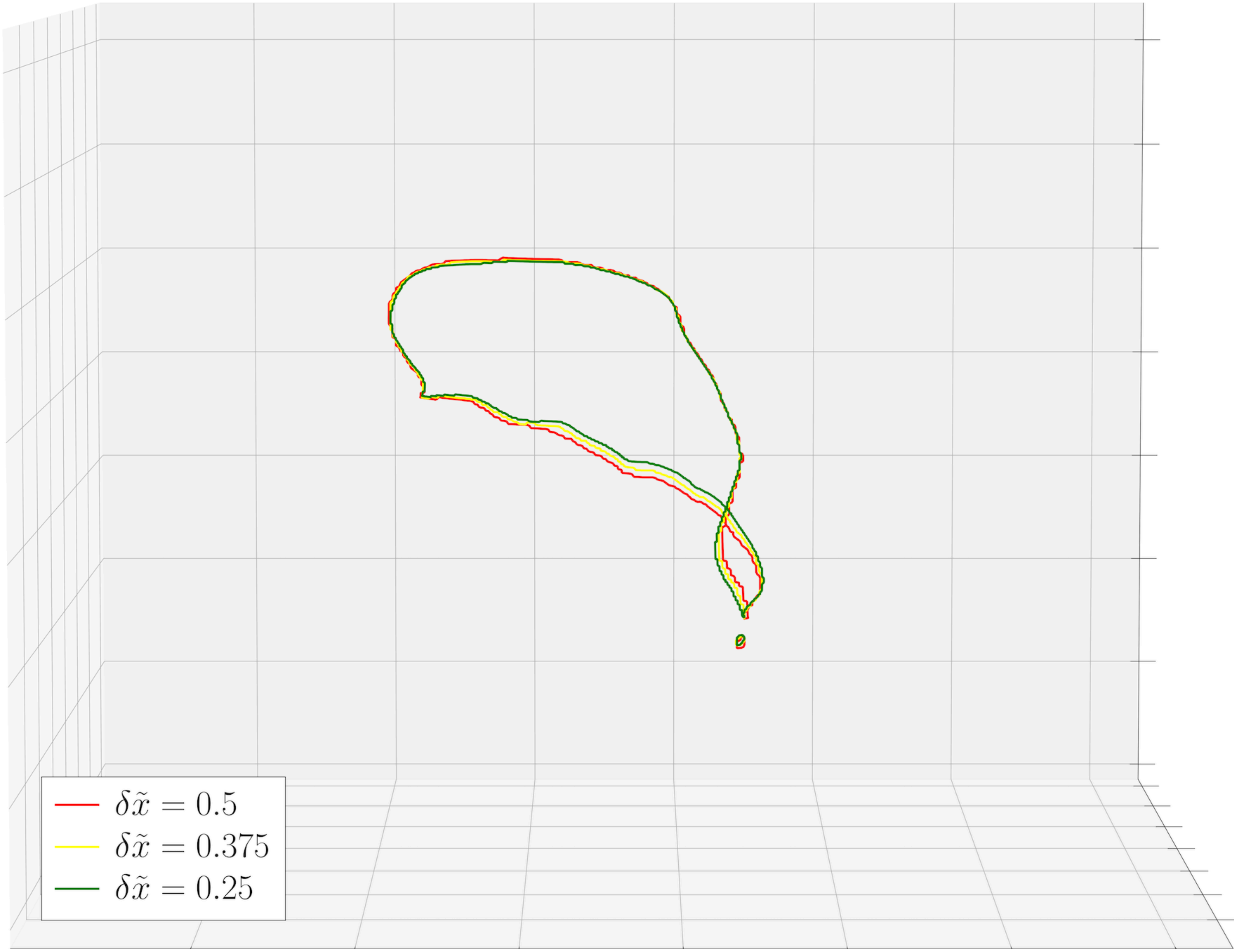
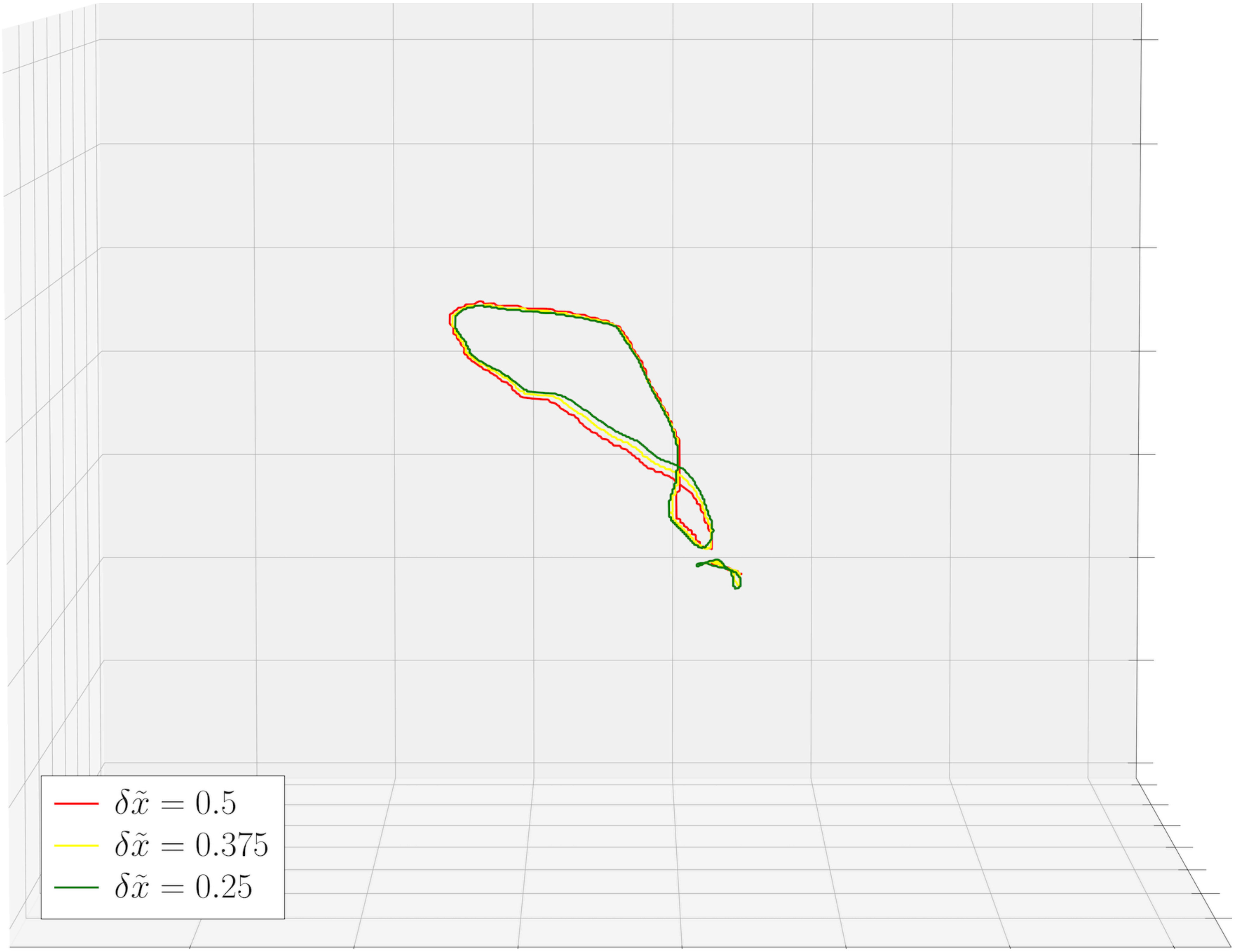
String Loop: Particle emission

Decay of a Loop



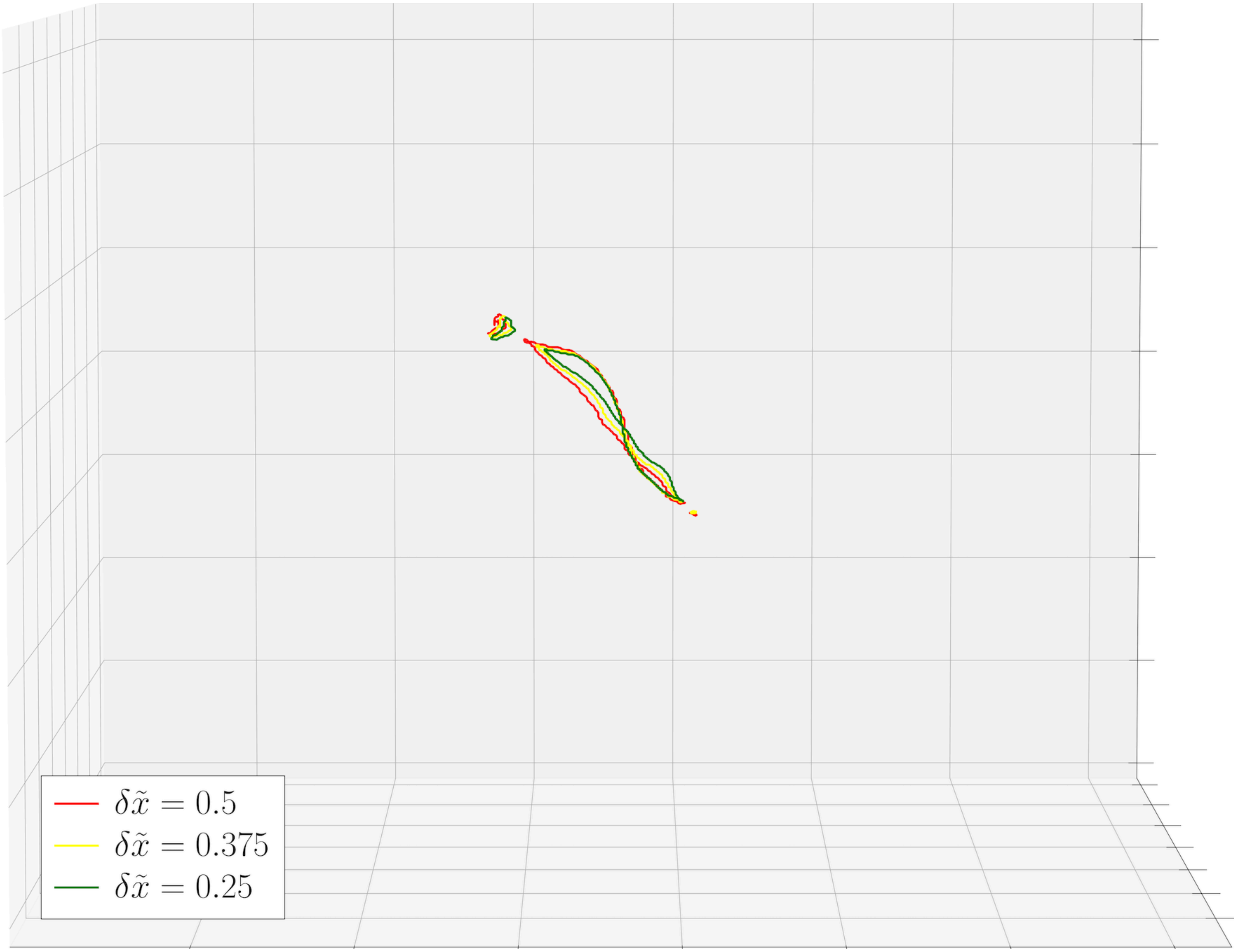
String Loop: Particle emission

Decay of a Loop



String Loop: Particle emission

Decay of a Loop

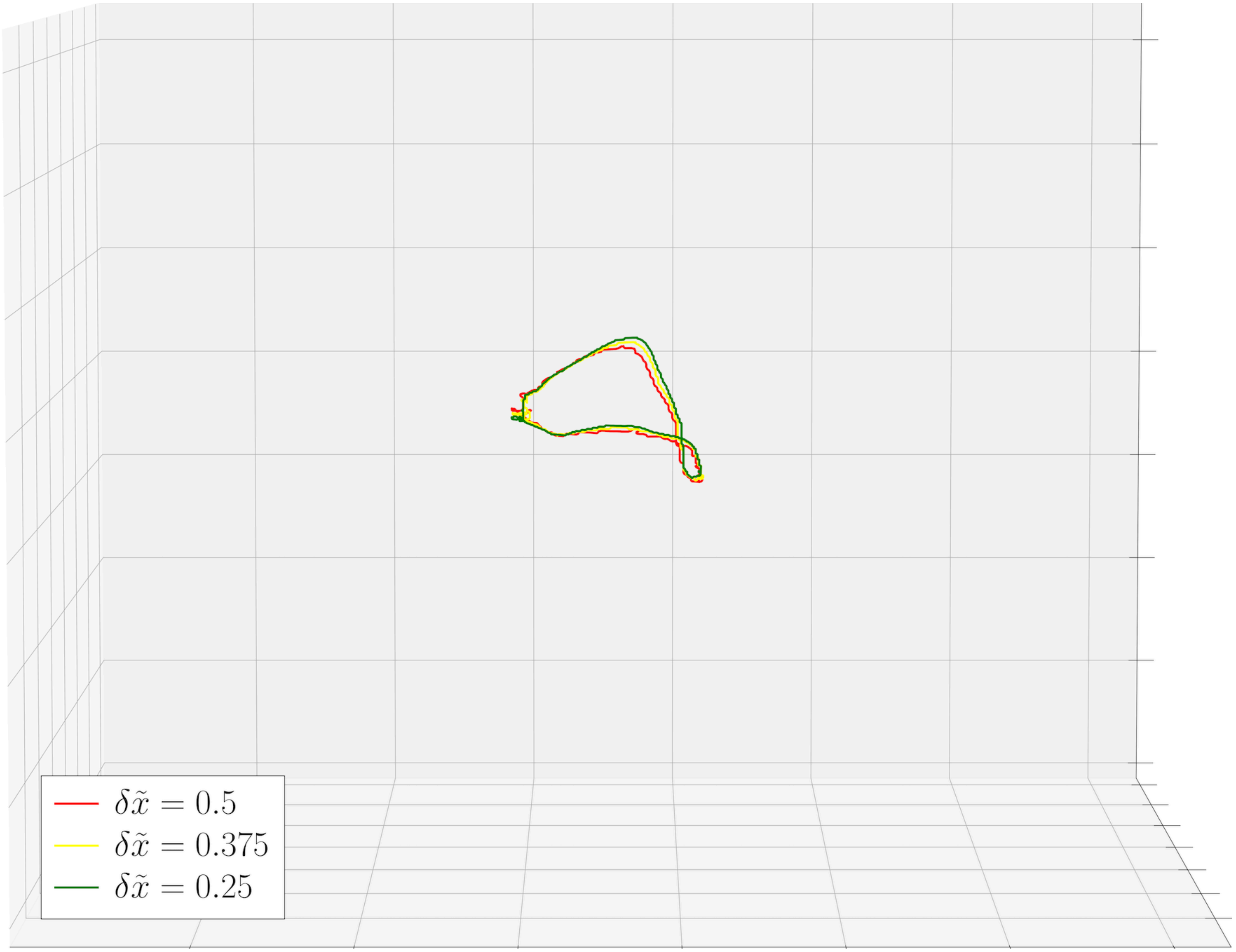


String Loop: Particle emission

Decay of a Loop

String Loop: Particle emission

Decay of a Loop

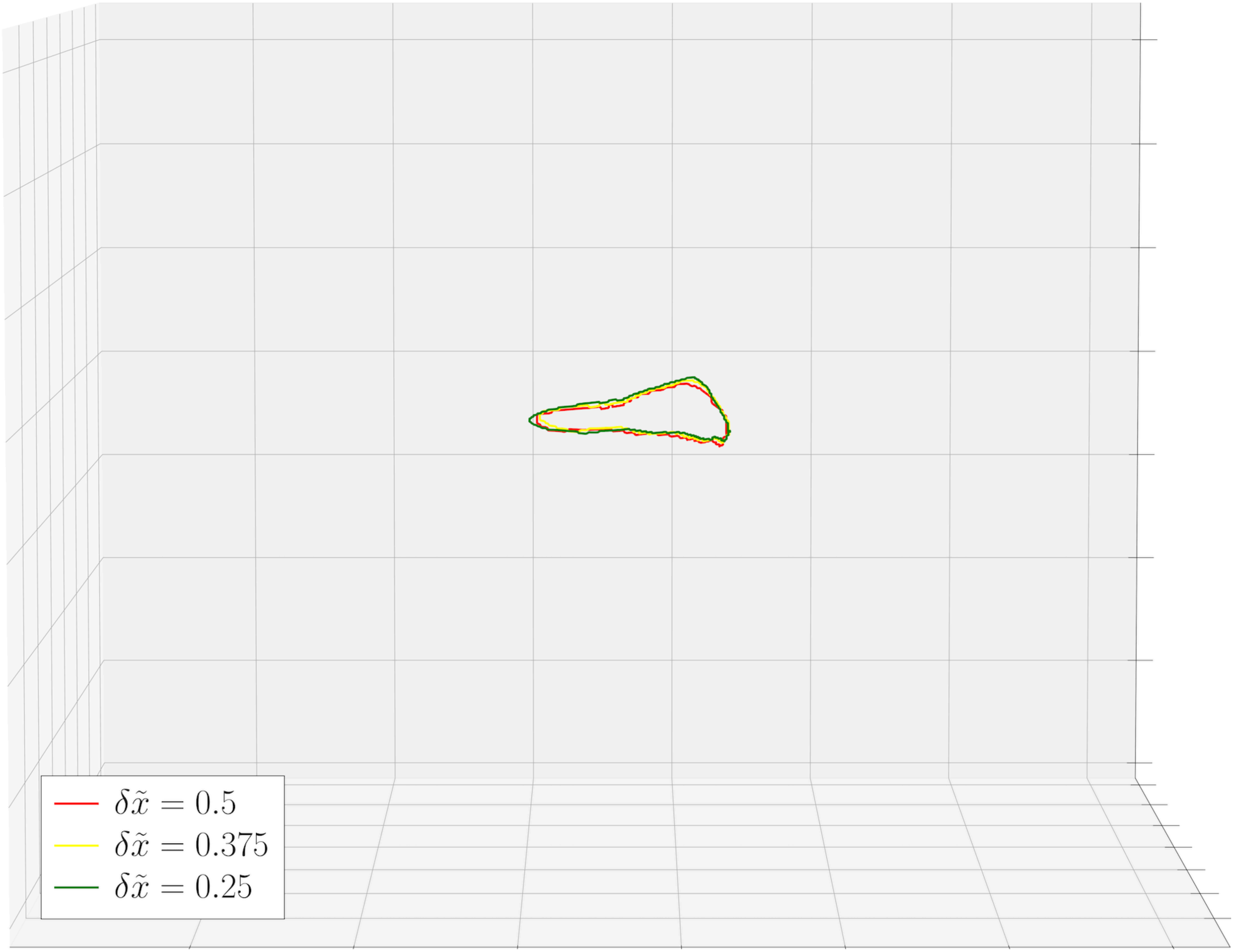


String Loop: Particle emission

Decay of a Loop

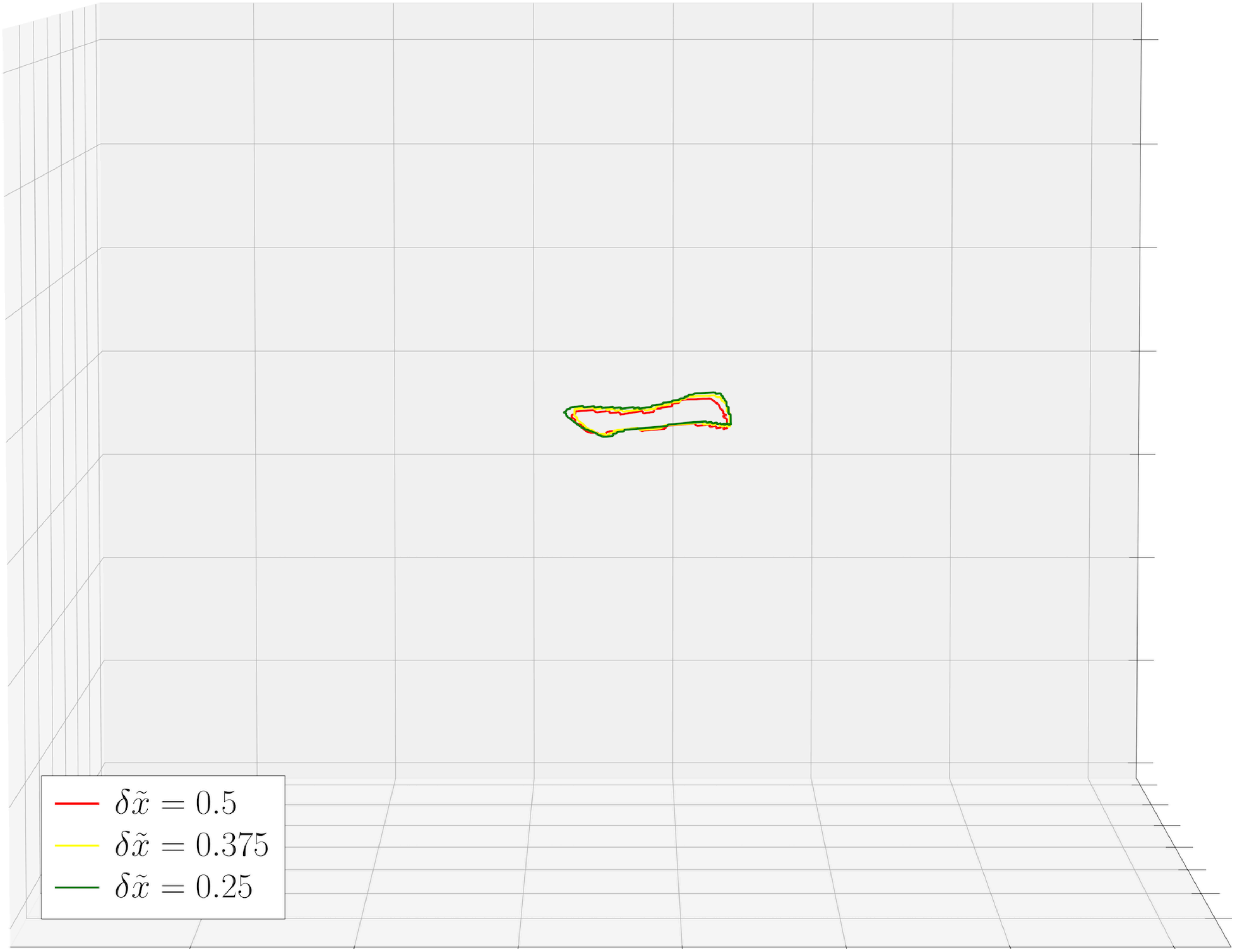
String Loop: Particle emission

Decay of a Loop



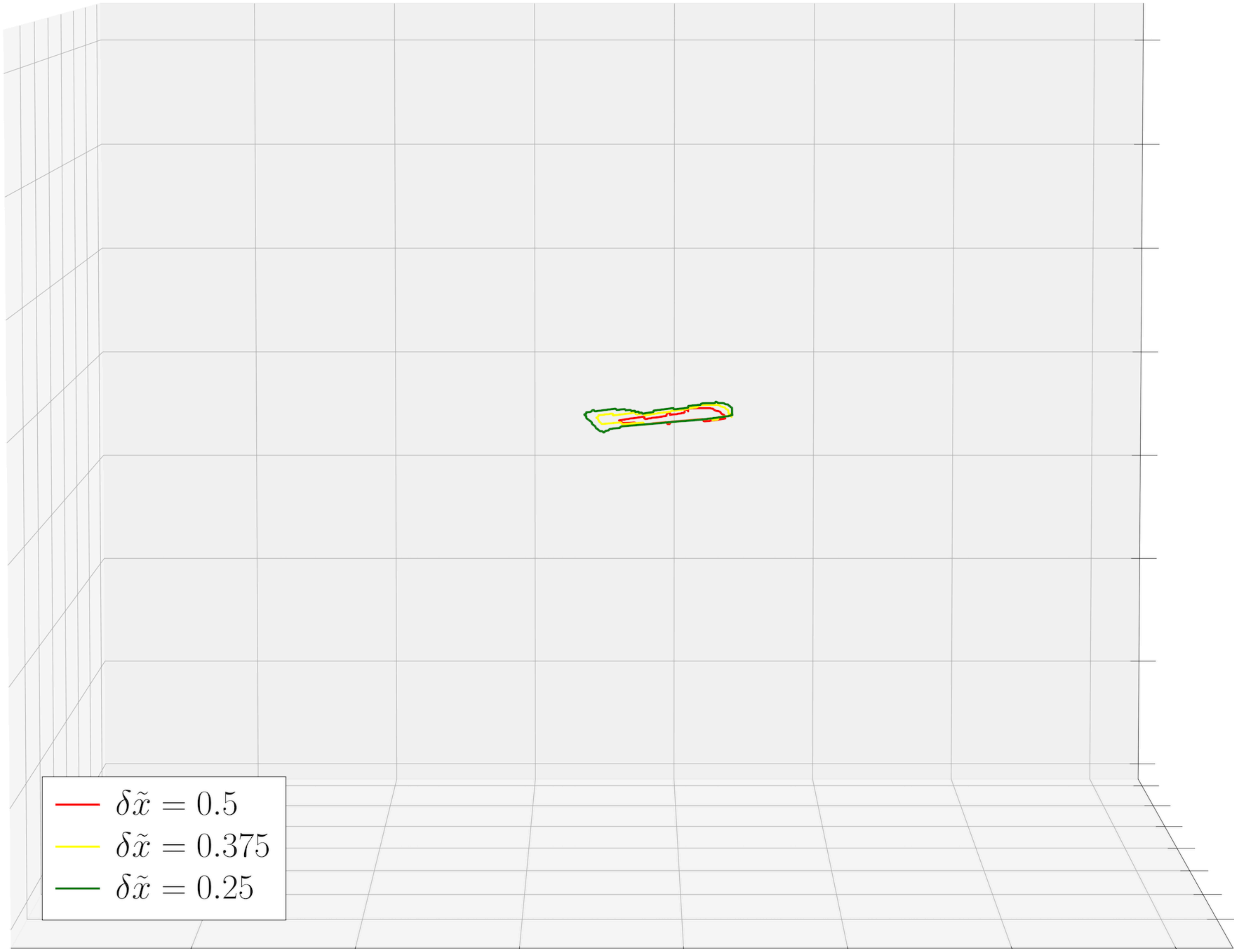
String Loop: Particle emission

Decay of a Loop



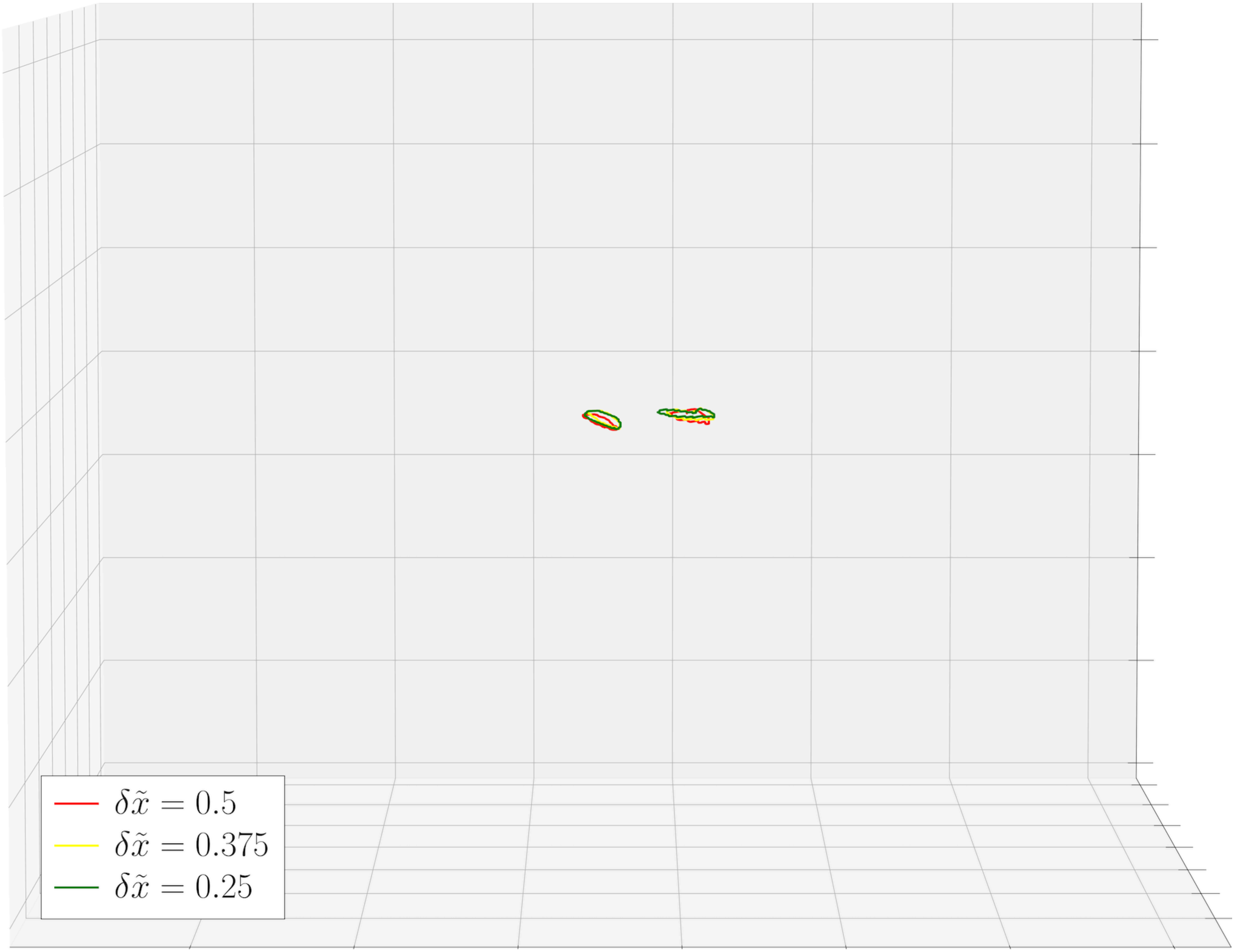
String Loop: Particle emission

Decay of a Loop



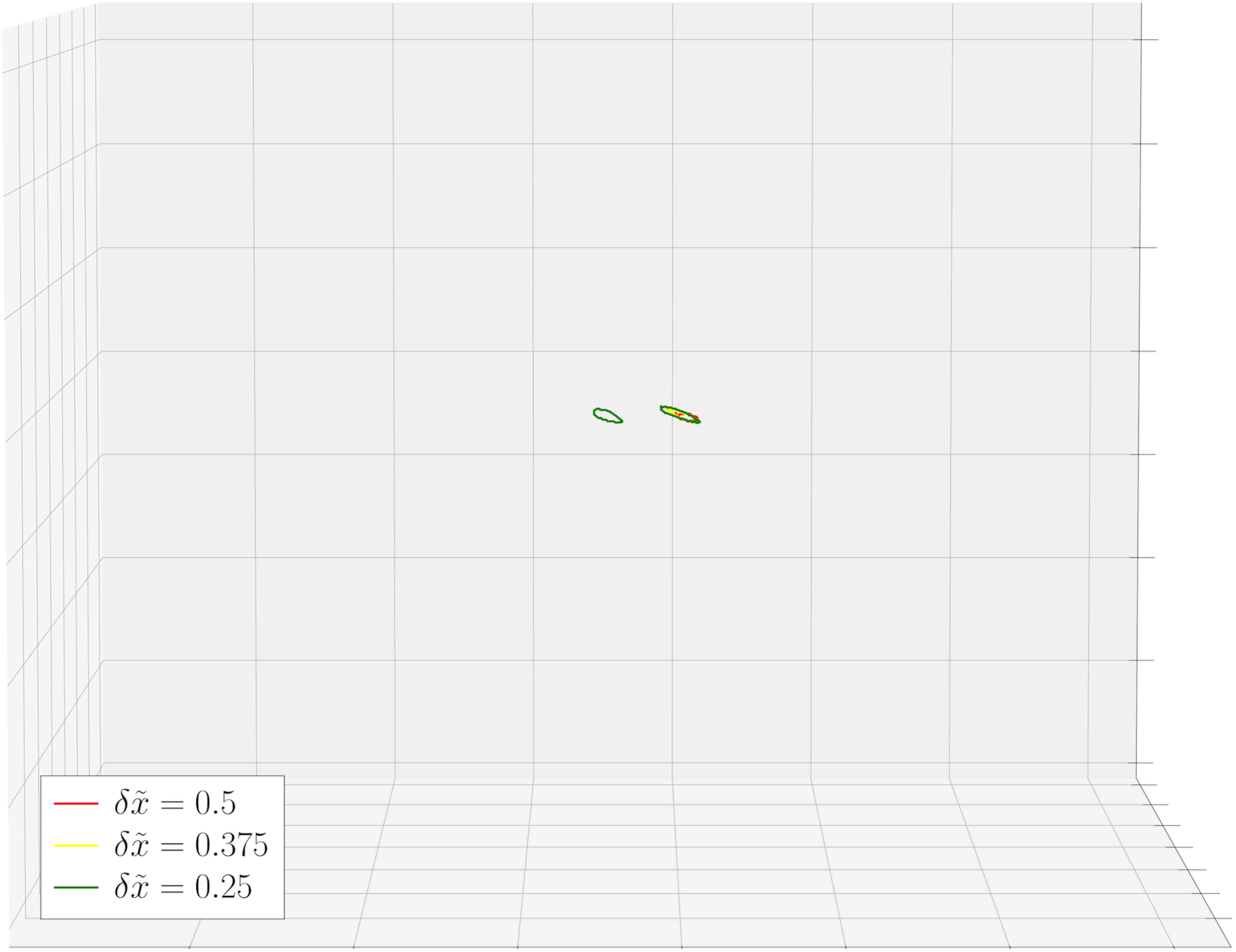
String Loop: Particle emission

Decay of a Loop



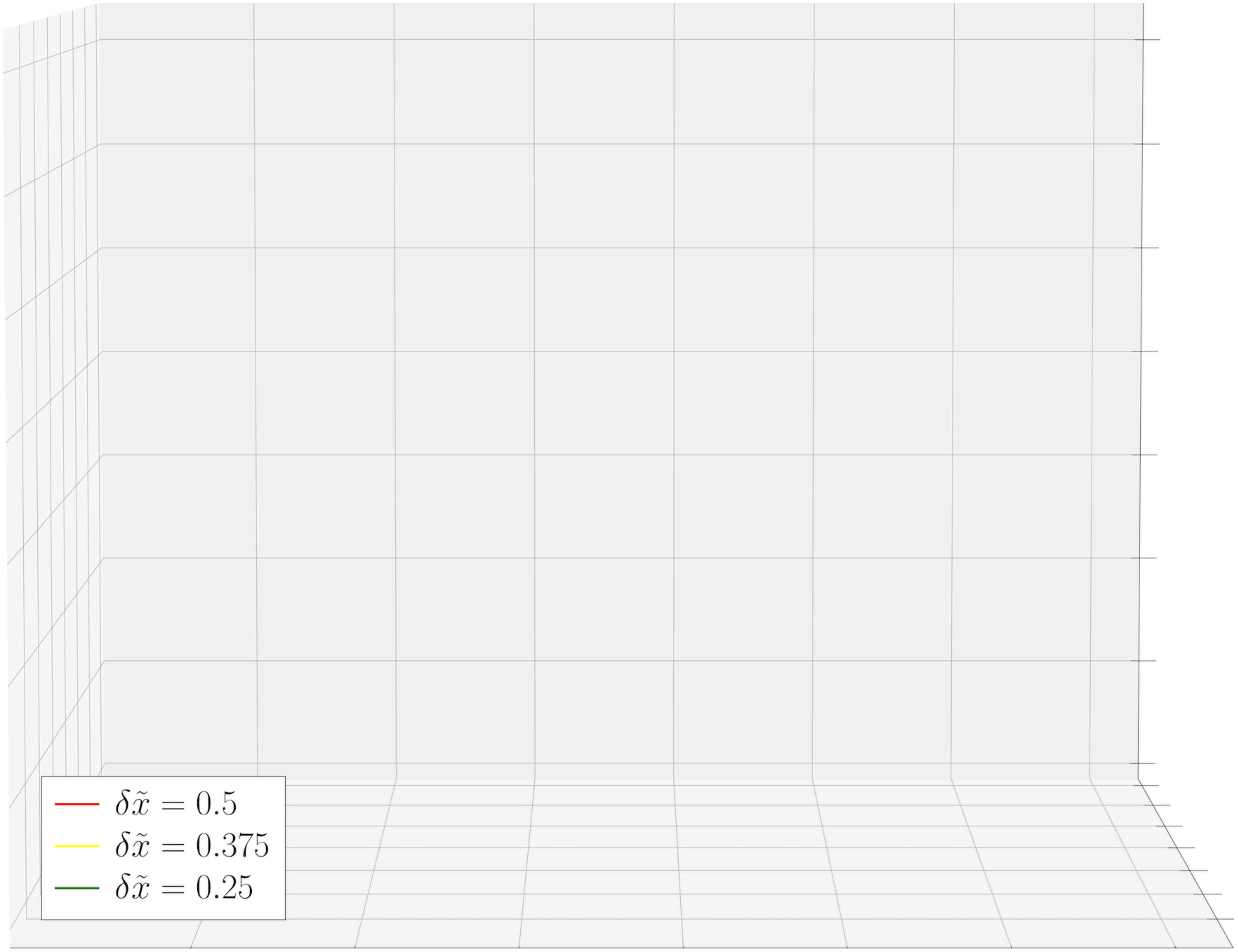
String Loop: Particle emission

Decay of a Loop



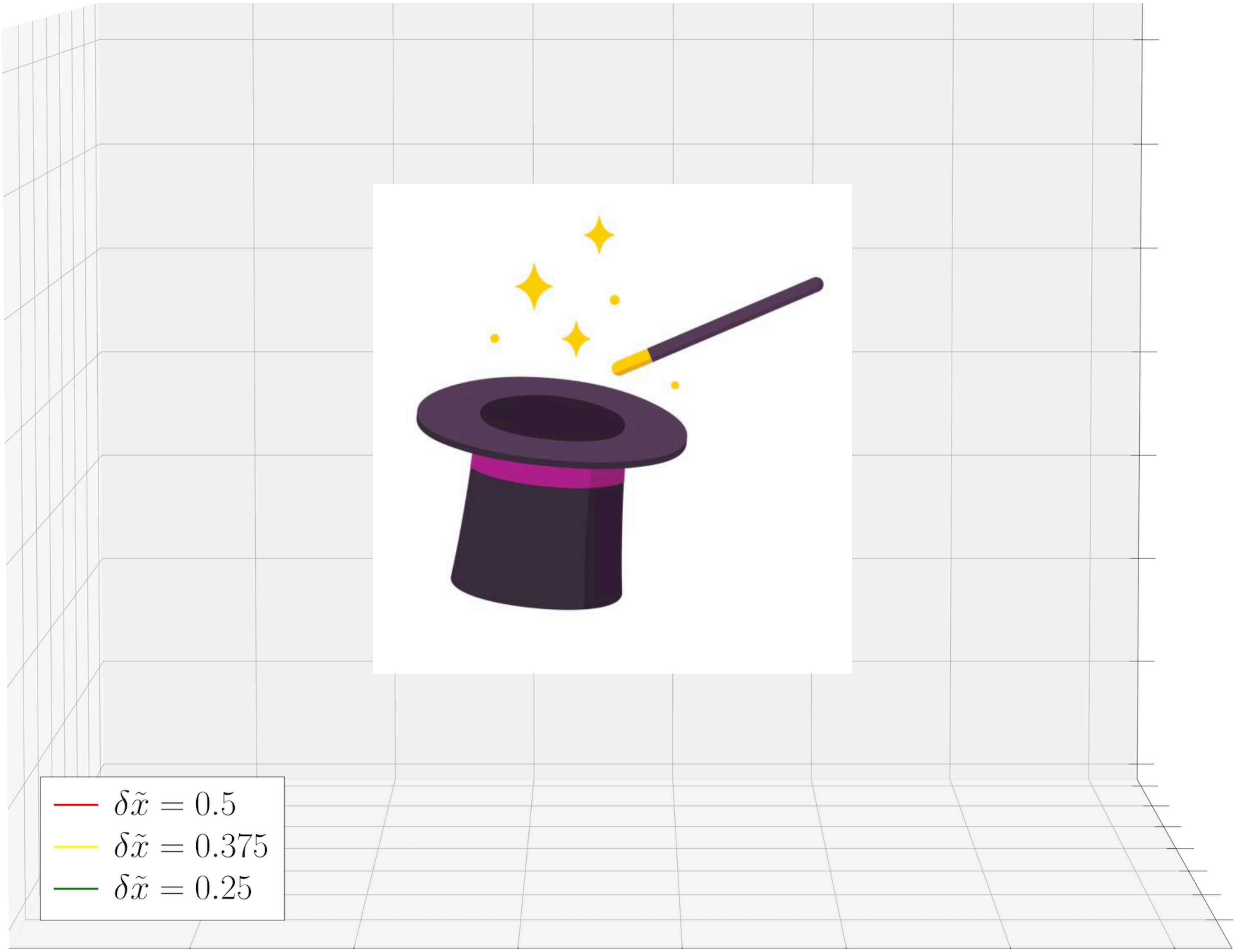
String Loop: Particle emission

Decay of a Loop



String Loop: Particle emission

Decay of a Loop



String Loop: Particle emission

Decay Time

(Due to Particle Emission)

$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$$

$$\boxed{\tilde{L} = \sqrt{\lambda}\, v\, L}$$

String Loop: Particle emission

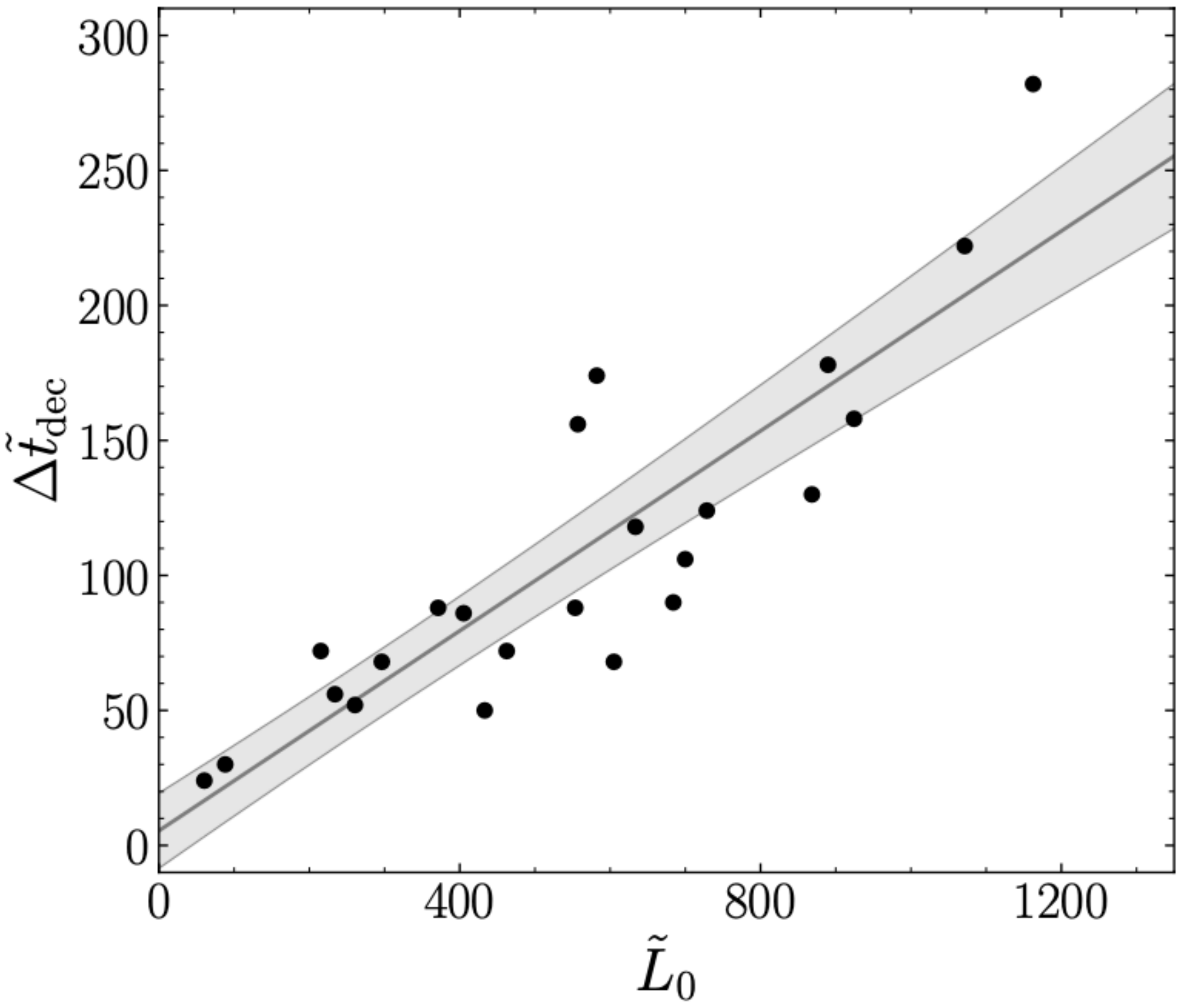
Decay Time

(Due to Particle Emission)

$$\tilde{t} = \sqrt{\lambda}\, v\, t$$

$$\tilde{L} = \sqrt{\lambda}\, v\, L$$

Network



String Loop: Particle emission

Decay Time

(Due to Particle Emission)

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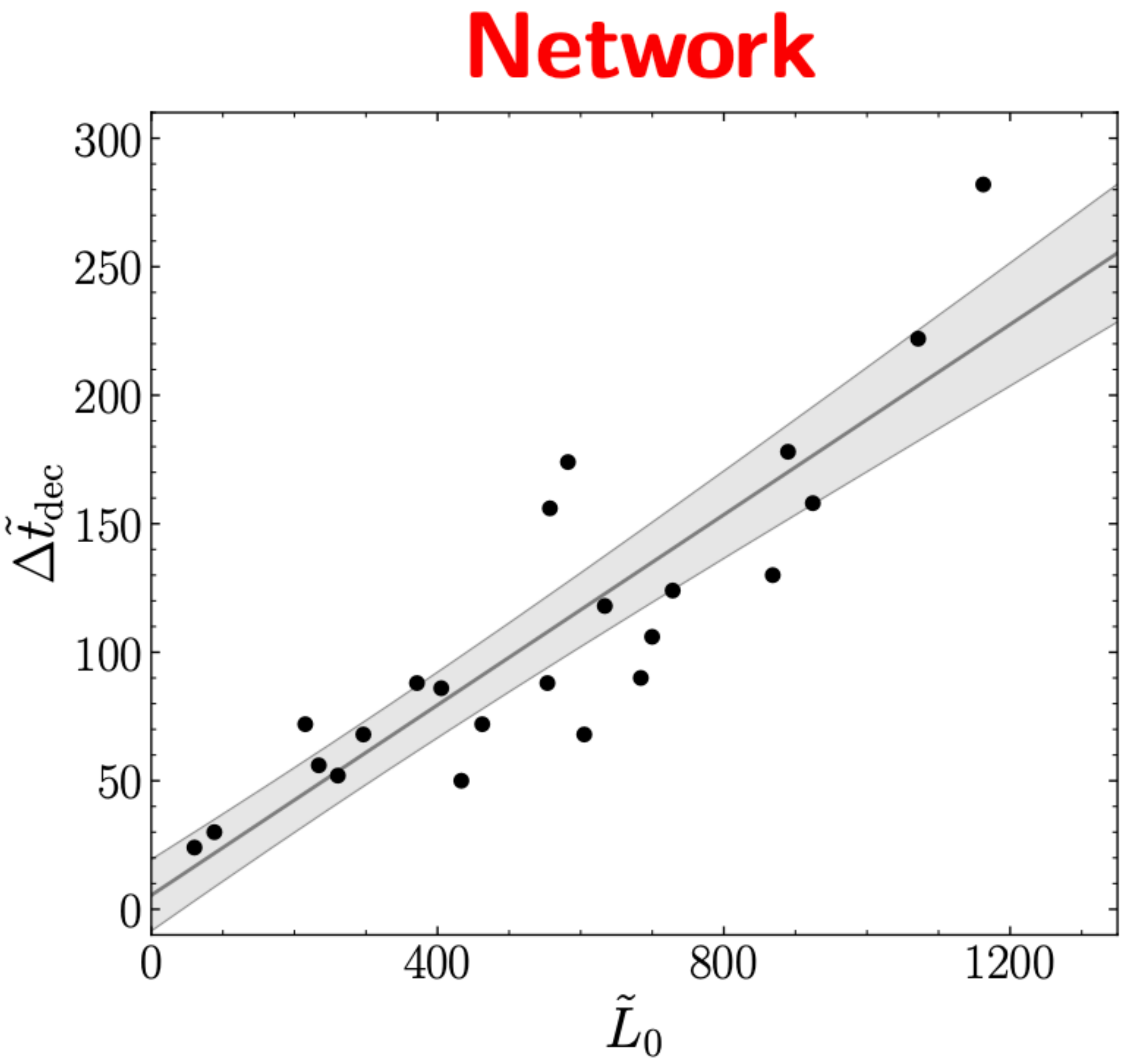
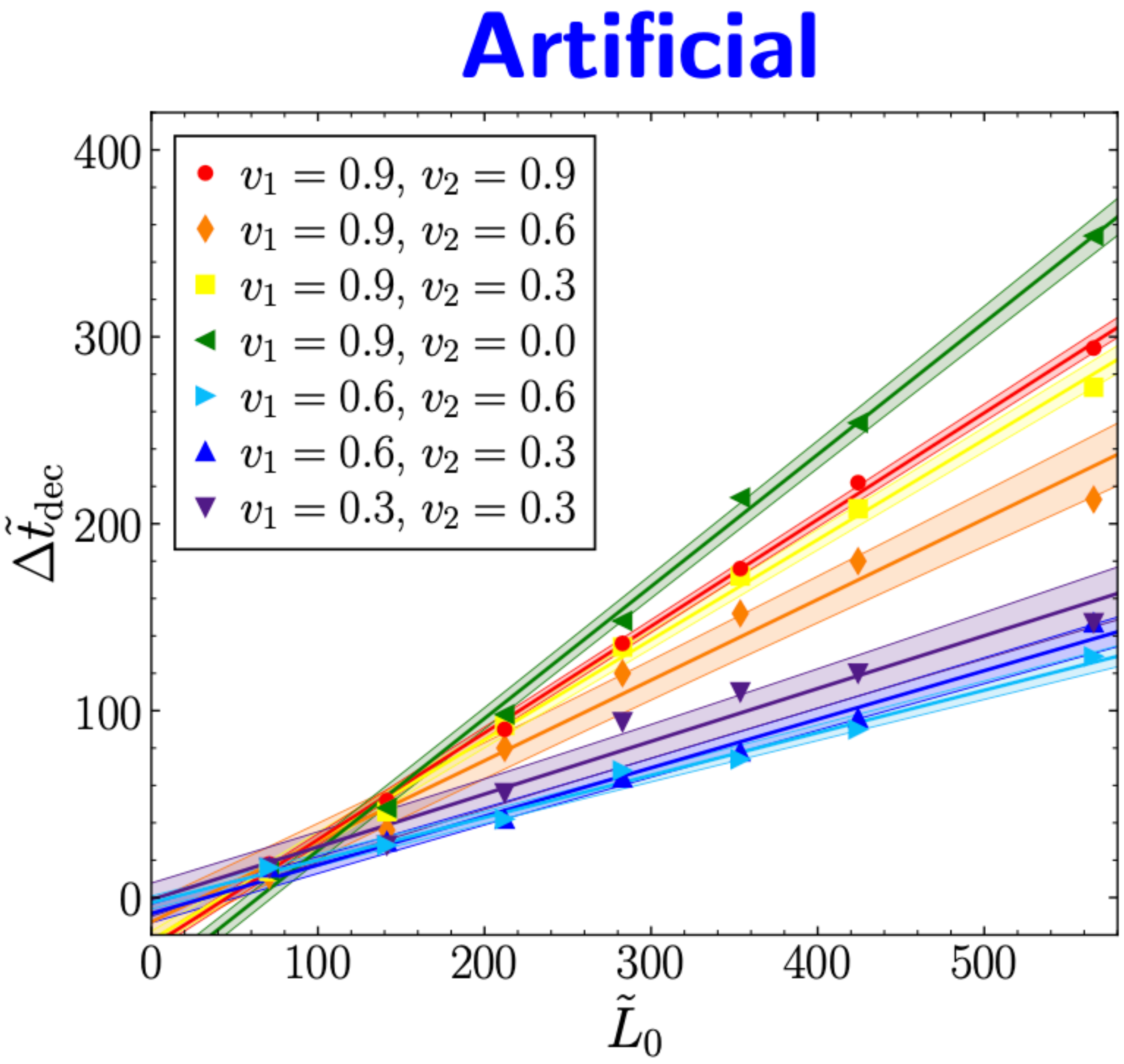
$$\boxed{\tilde{L} = \sqrt{\lambda}\, v\, L}$$





(See also Saurabh, Vachaspati, Pogosian)

String Loop: Particle emission

Decay Time

(Due to Particle Emission)

$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$$

$$\boxed{\tilde{L} = \sqrt{\lambda}\, v\, L}$$

Angular momentum

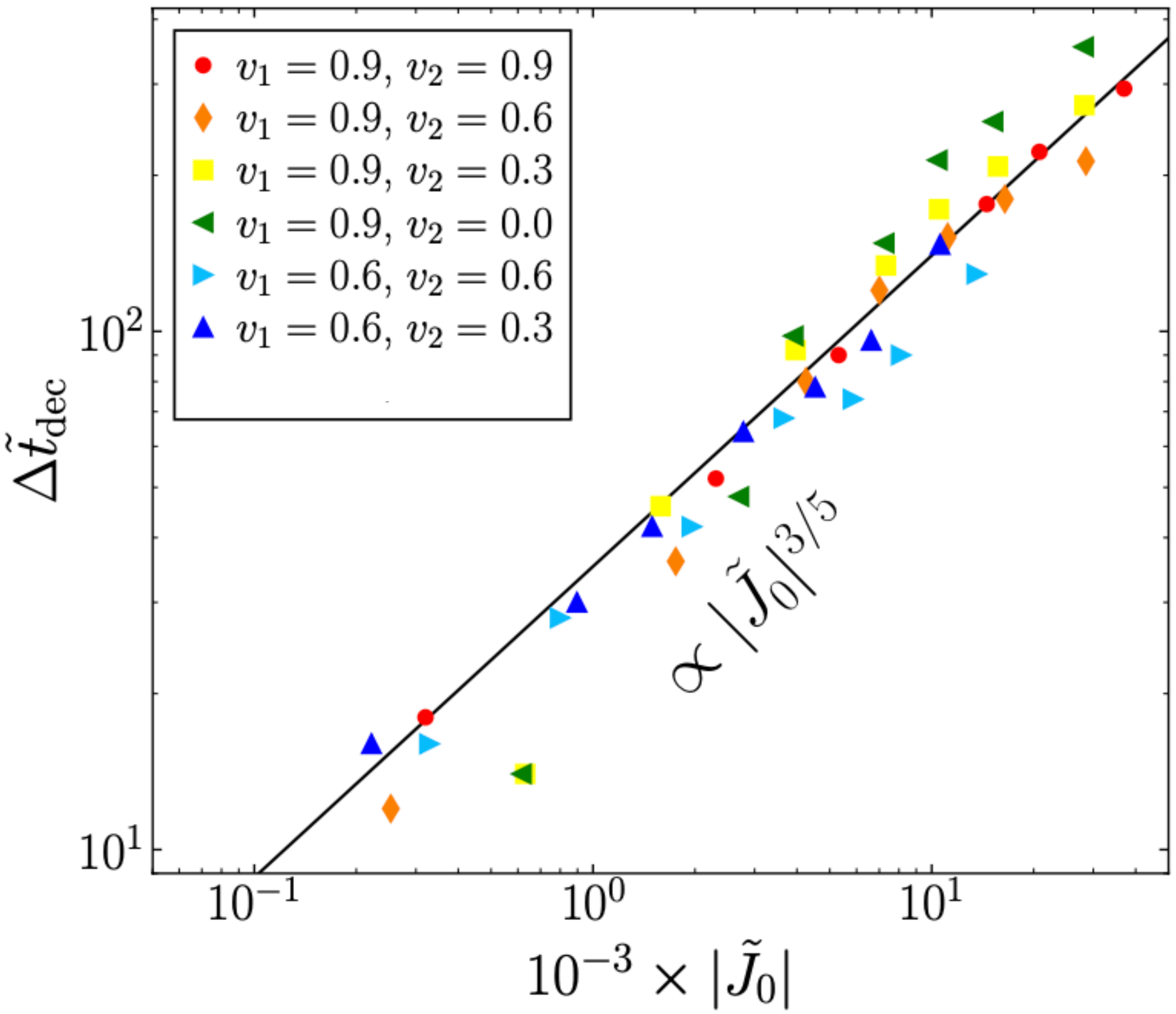
Universal scaling !

$$\vec{J} = -2\int_{\text{str}} \mathrm{d}^3x\, \mathrm{Re}\left[\vec{x} \times \dot{\varphi}\vec{\nabla}\varphi^*\right]$$

$\downarrow$

Distance to loop's geometric center

Artificial



String Loop: Particle emission

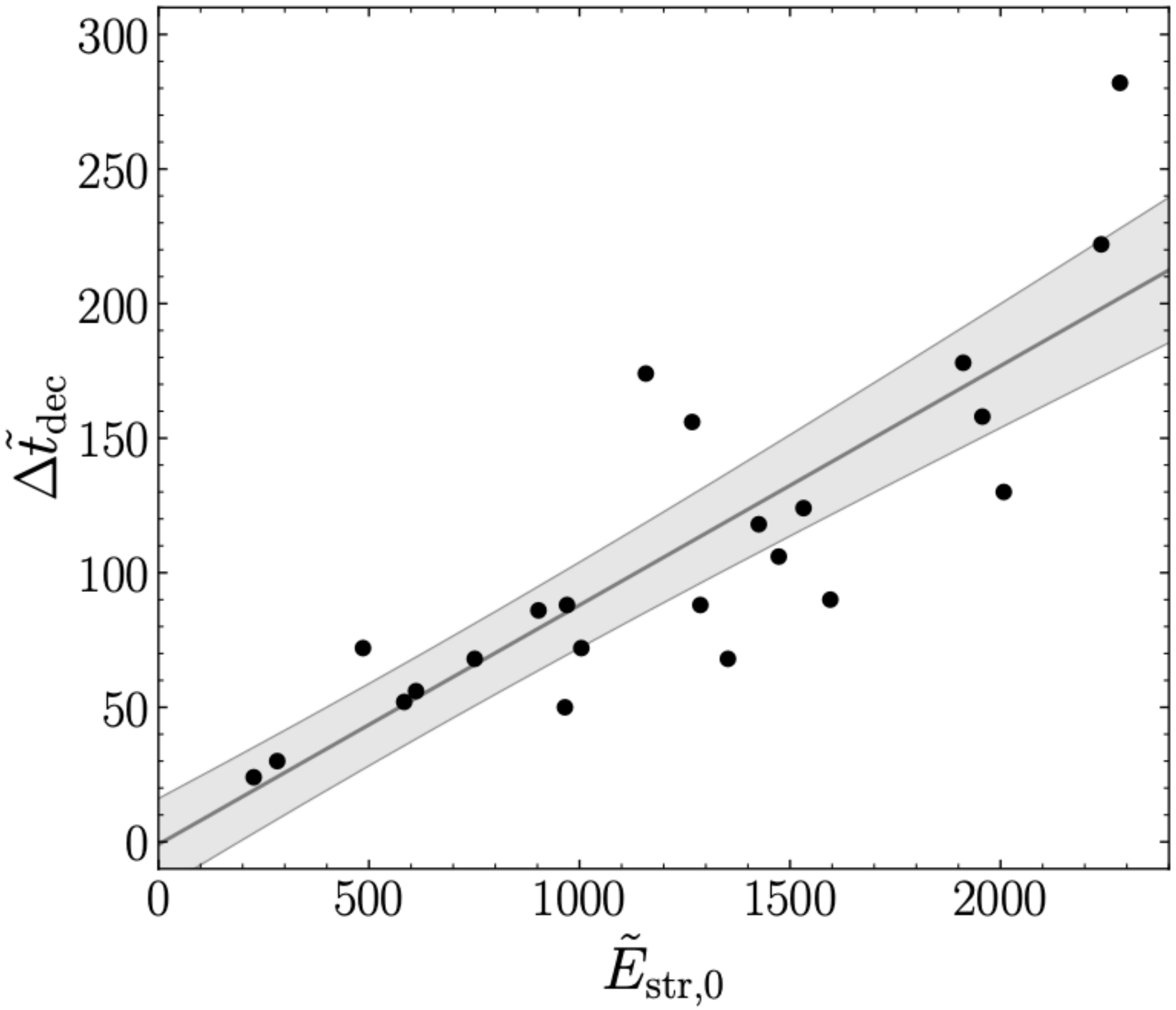
Decay Time

(Due to Particle Emission)

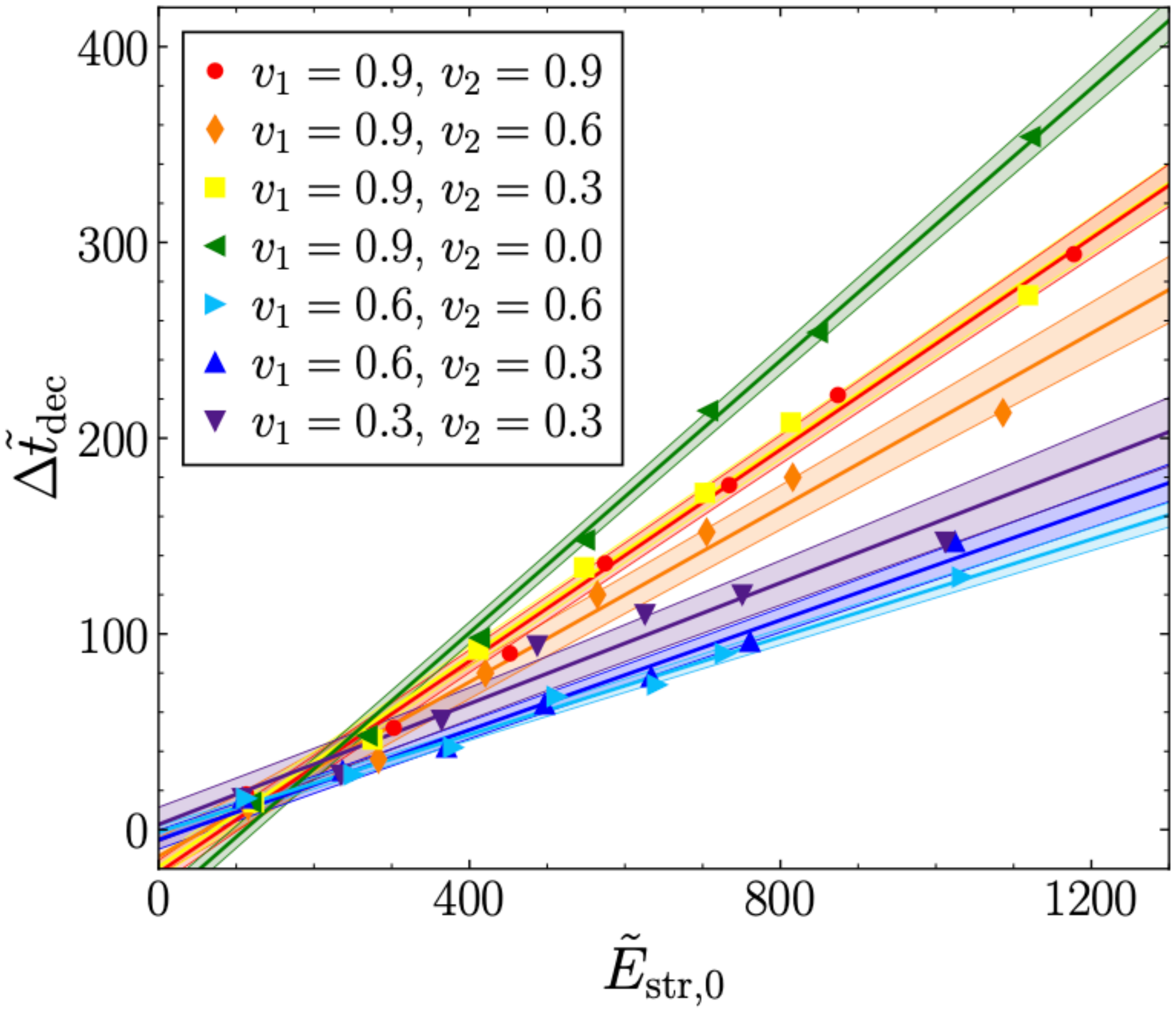
$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$$

$$\boxed{\tilde{L} = \sqrt{\lambda}\, v\, L}$$

Network



Artificial



String Loop: Particle emission

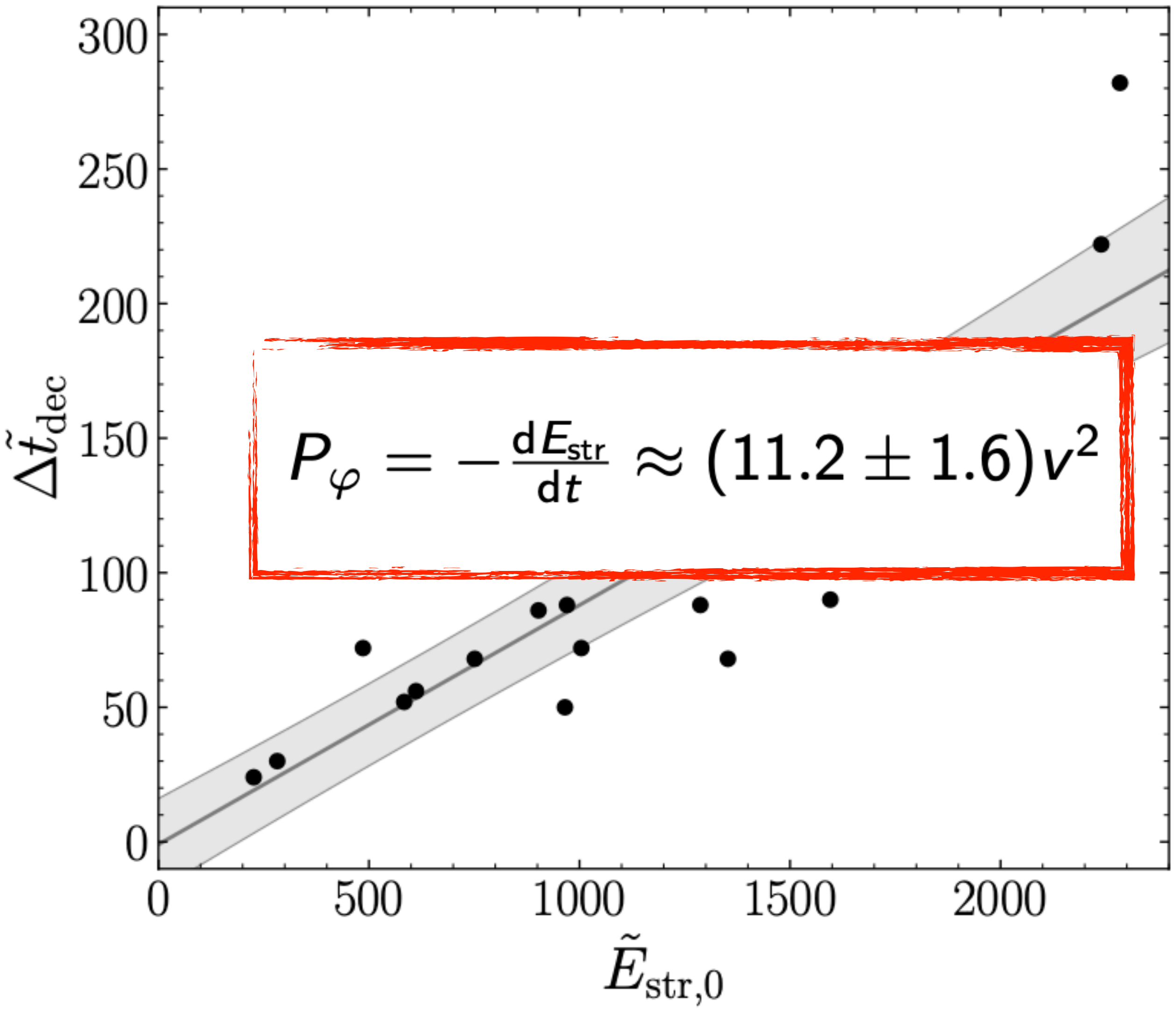
Decay Time

(Due to Particle Emission)

$$\tilde{t} = \sqrt{\lambda}\, v\, t$$

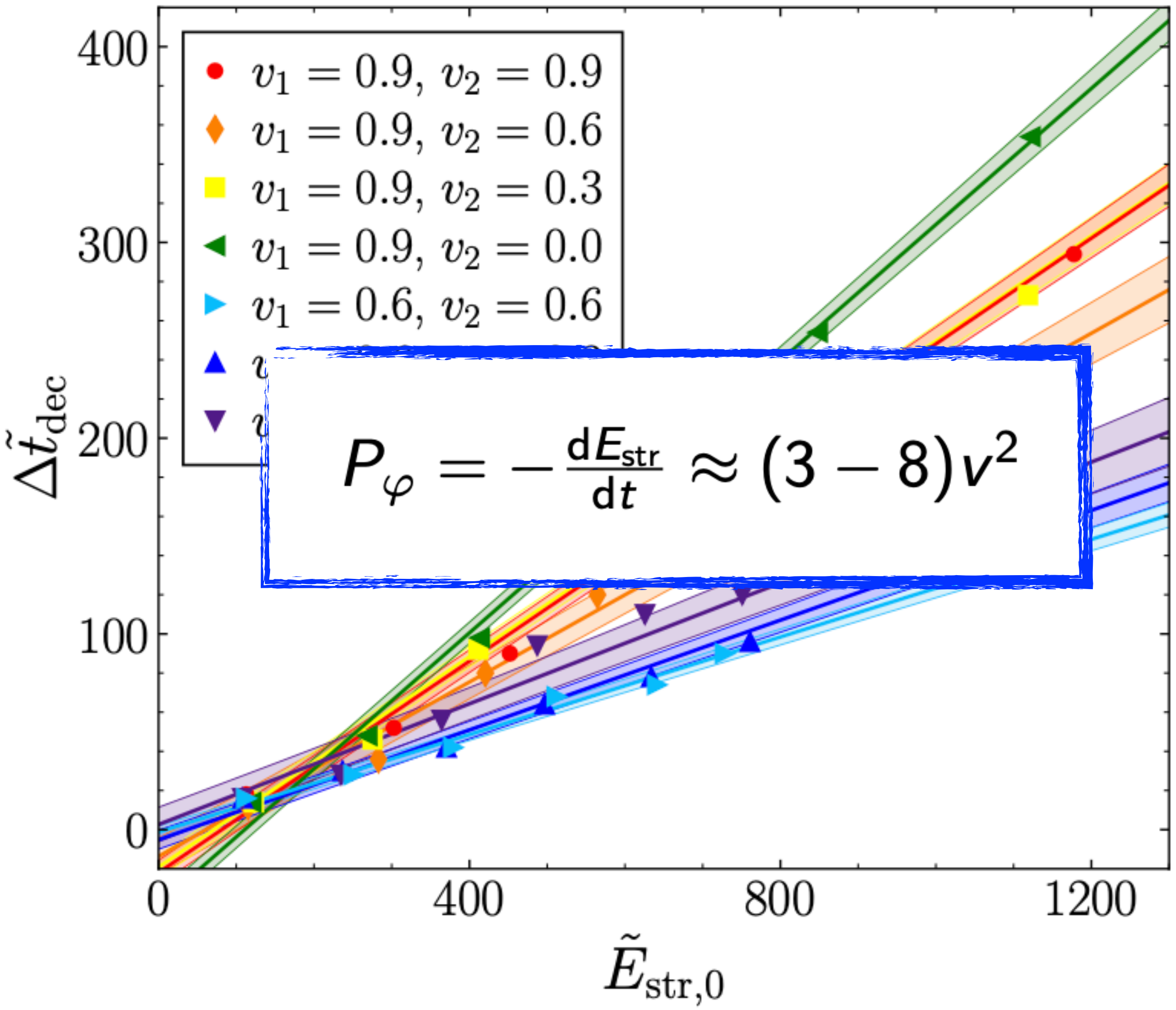
$$\tilde{L} = \sqrt{\lambda}\, v\, L$$

Network



$$P_\varphi = -\frac{\mathrm{d}E_{\rm str}}{\mathrm{d}t} \approx (11.2 \pm 1.6)v^2$$

Artificial



$$P_\varphi = -\frac{\mathrm{d}E_{\rm str}}{\mathrm{d}t} \approx (3 - 8)v^2$$

String Loop: Particle emission

Decay Time

(Due to Particle Emission)

$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$$

$$\boxed{\tilde{L} = \sqrt{\lambda}\, v\, L}$$

Network

$$\boxed{P_\varphi = -\frac{\mathrm{d}E_{\rm str}}{\mathrm{d}t} \approx (11.2 \pm 1.6)v^2}$$

Artificial

$$\boxed{P_\varphi = -\frac{\mathrm{d}E_{\rm str}}{\mathrm{d}t} \approx (3 - 8)v^2}$$

String Loop: Particle emission

Decay Time

(Due to Particle Emission)

$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$$

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Artificial

$$\boxed{P_\varphi = -\frac{\mathrm{d}E_{\mathrm{str}}}{\mathrm{d}t} \approx (3 - 8)v^2}$$

Independent* of Resolution & Length !

[*within the error]

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$

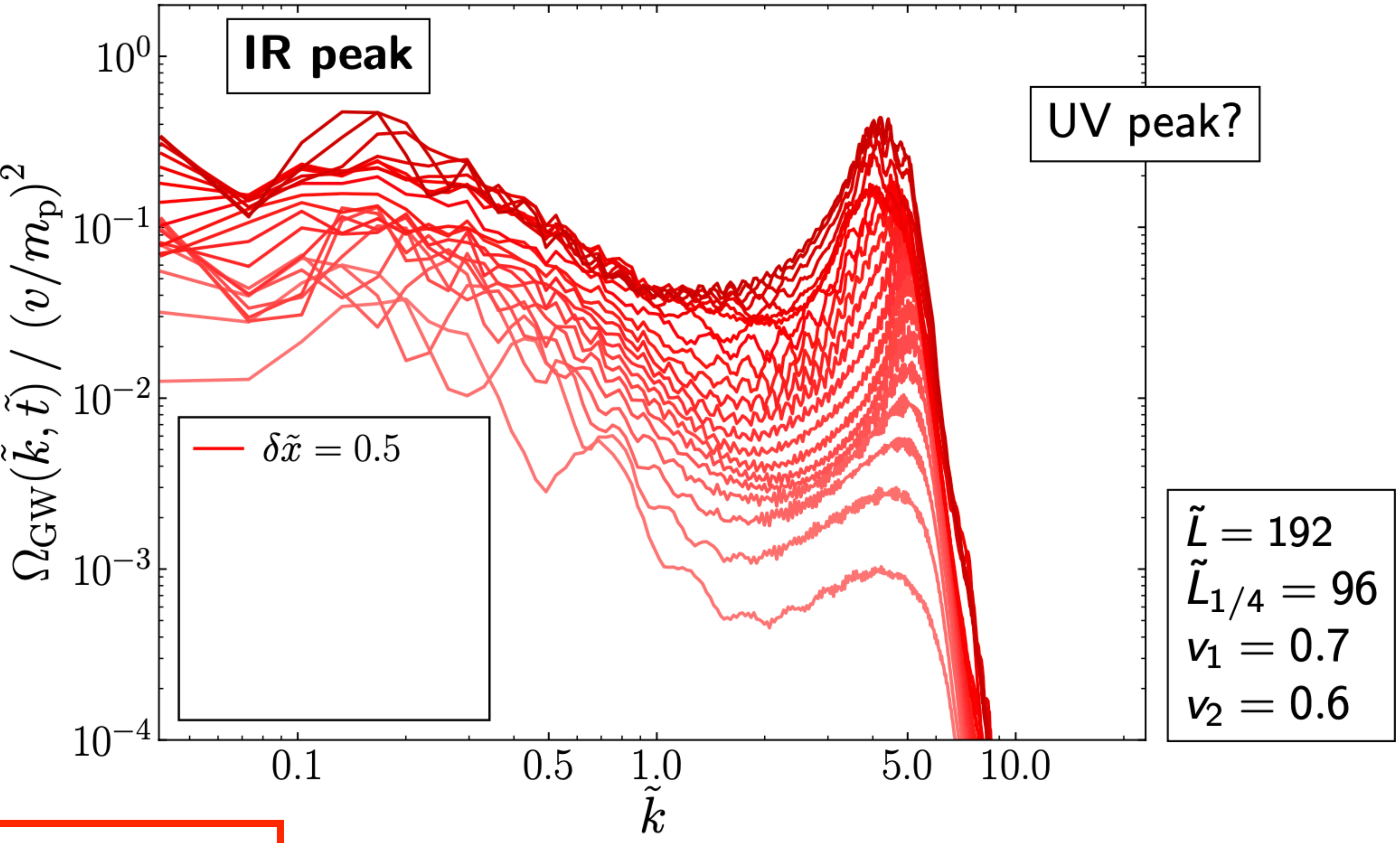
$\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$

$\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)



$\boxed{\delta\tilde{x} = 0.500 \quad \Rightarrow \quad N_* \sim 25}$

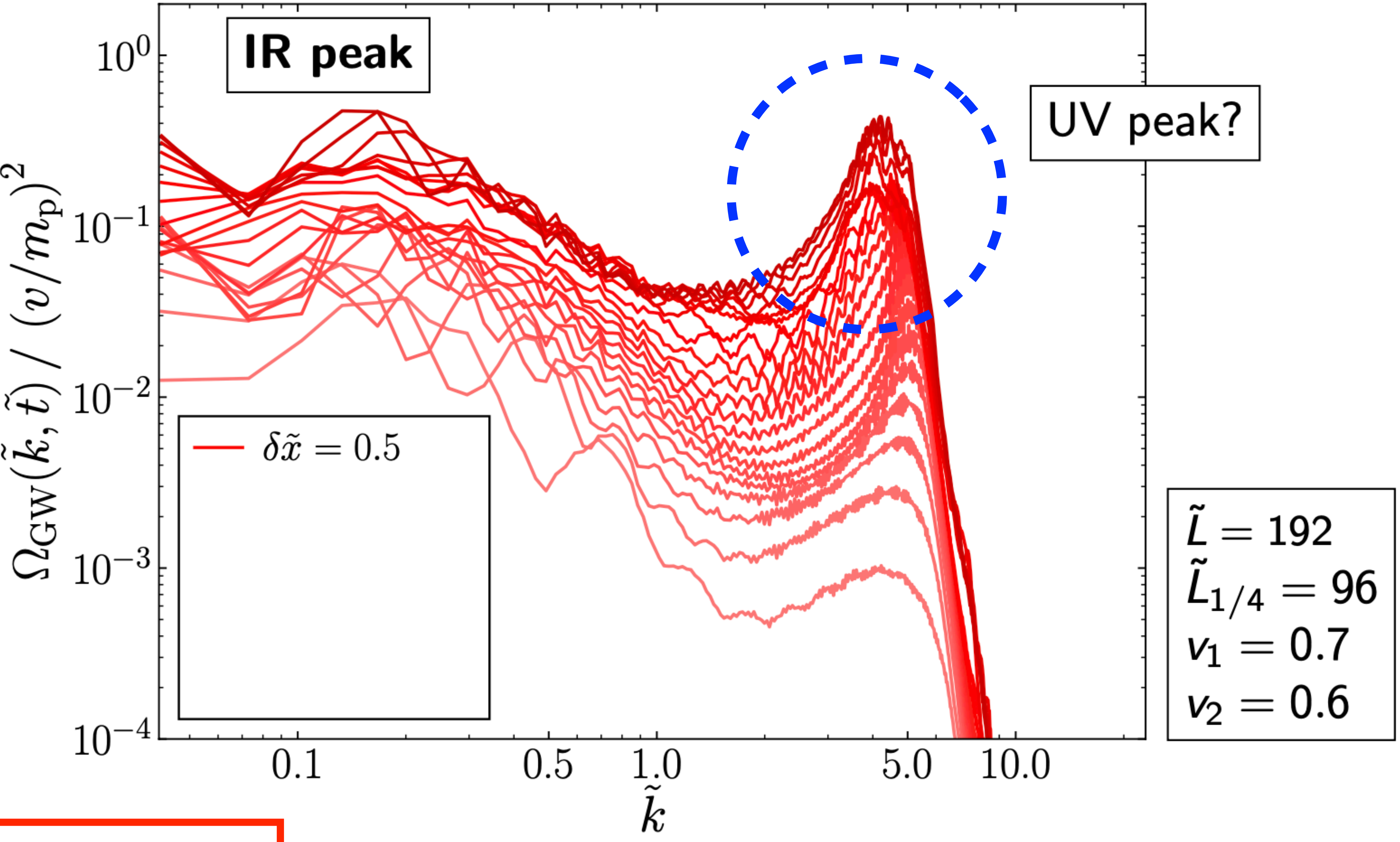
String Loop: GW emission

$\tilde{t} = \sqrt{\lambda}\, v\, t$

$\tilde{l} = \sqrt{\lambda}\, v\, l$

$\tilde{k} = k/\sqrt{\lambda}\, v$

(Network Loop)



$\delta\tilde{x} = 0.500 \quad \Rightarrow \quad N_{*} \sim 25$

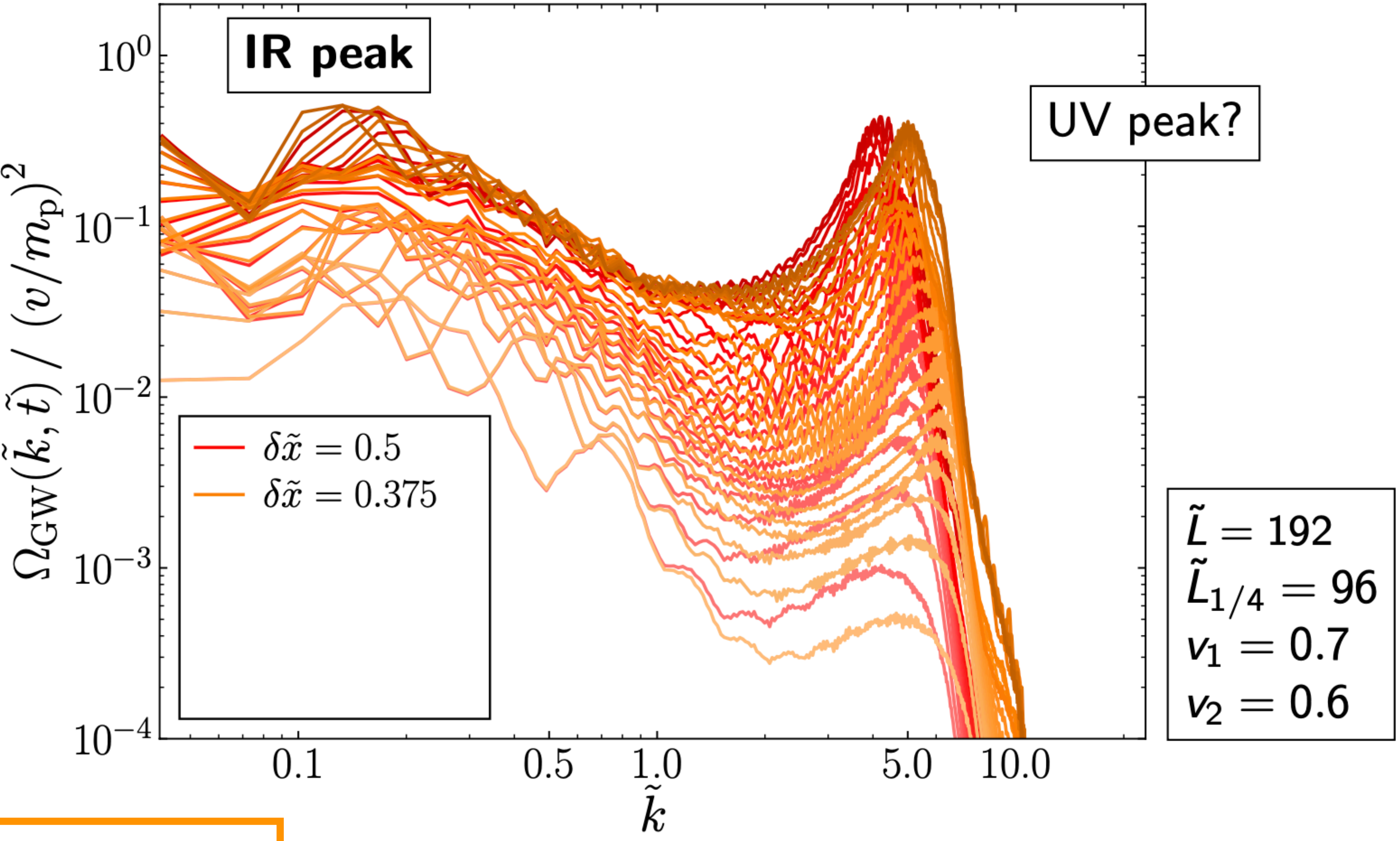
String Loop: GW emission

$\tilde{t} = \sqrt{\lambda}\, v\, t$

$\tilde{l} = \sqrt{\lambda}\, v\, l$

$\tilde{k} = k/\sqrt{\lambda}\, v$

(Network Loop)



$\delta\tilde{x} = 0.375 \quad \Rightarrow \quad N_* \sim 45$

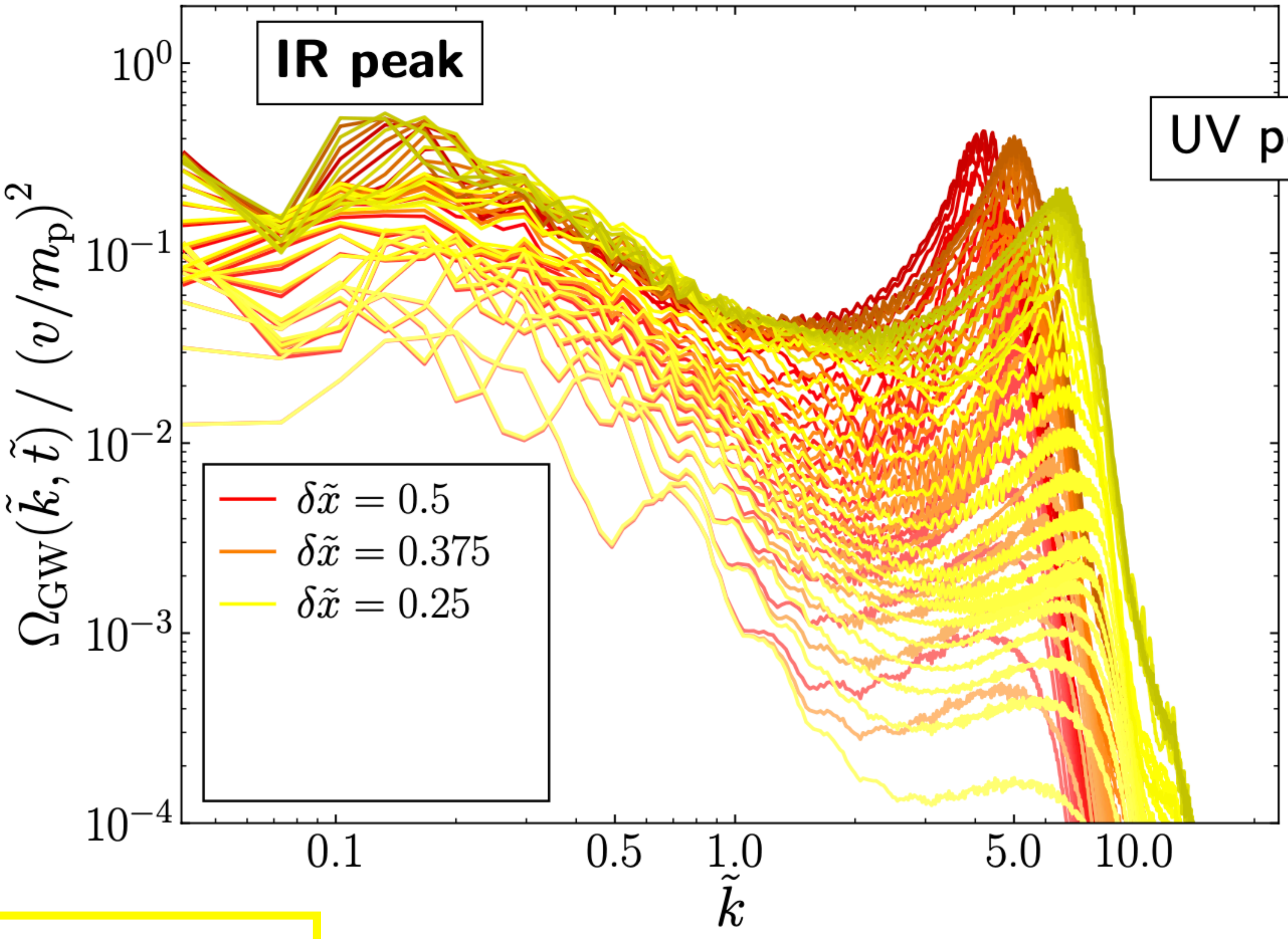
String Loop: GW emission

$\tilde{t} = \sqrt{\lambda}\, v\, t$

$\tilde{l} = \sqrt{\lambda}\, v\, l$

$\tilde{k} = k/\sqrt{\lambda}\, v$

(Network Loop)



IR peak

UV peak?

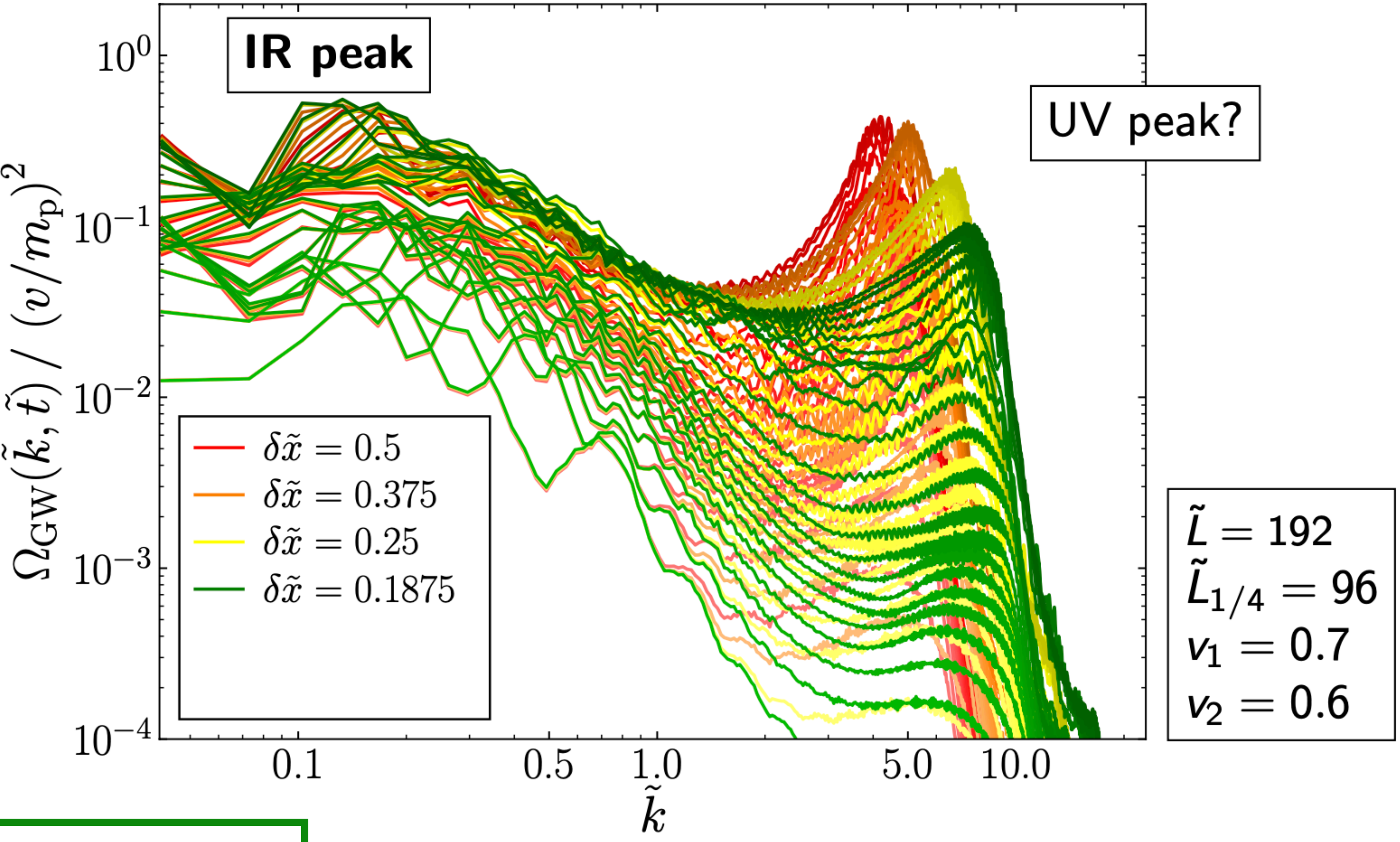
$\tilde{L} = 192$
$\tilde{L}_{1/4} = 96$
$v_1 = 0.7$
$v_2 = 0.6$

$\delta\tilde{x} = 0.250 \quad \Rightarrow \quad N_{*} \sim 100$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)

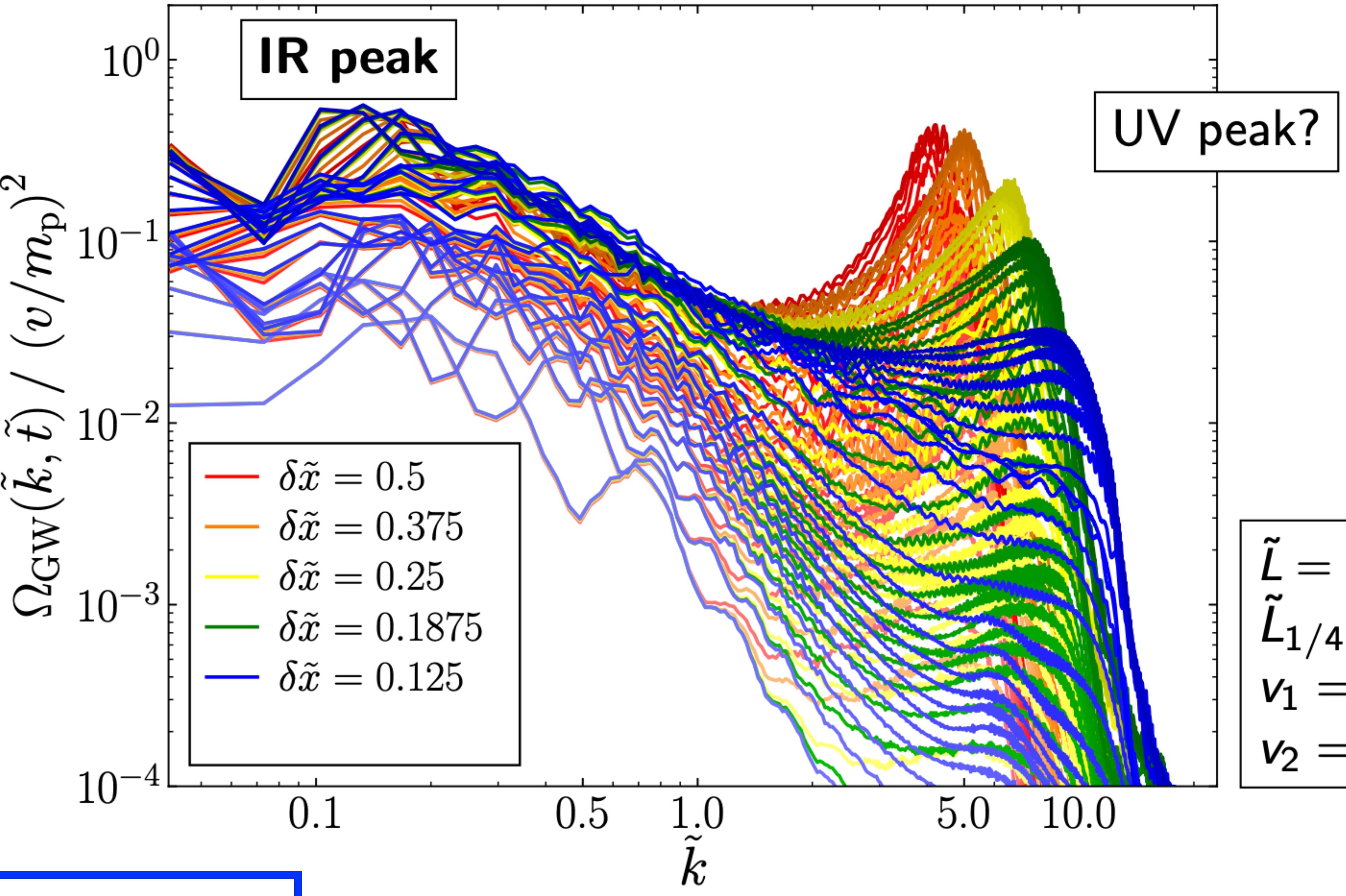


$\boxed{\delta\tilde{x} = 0.1875 \quad \Rightarrow \quad N_* \sim 180}$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)



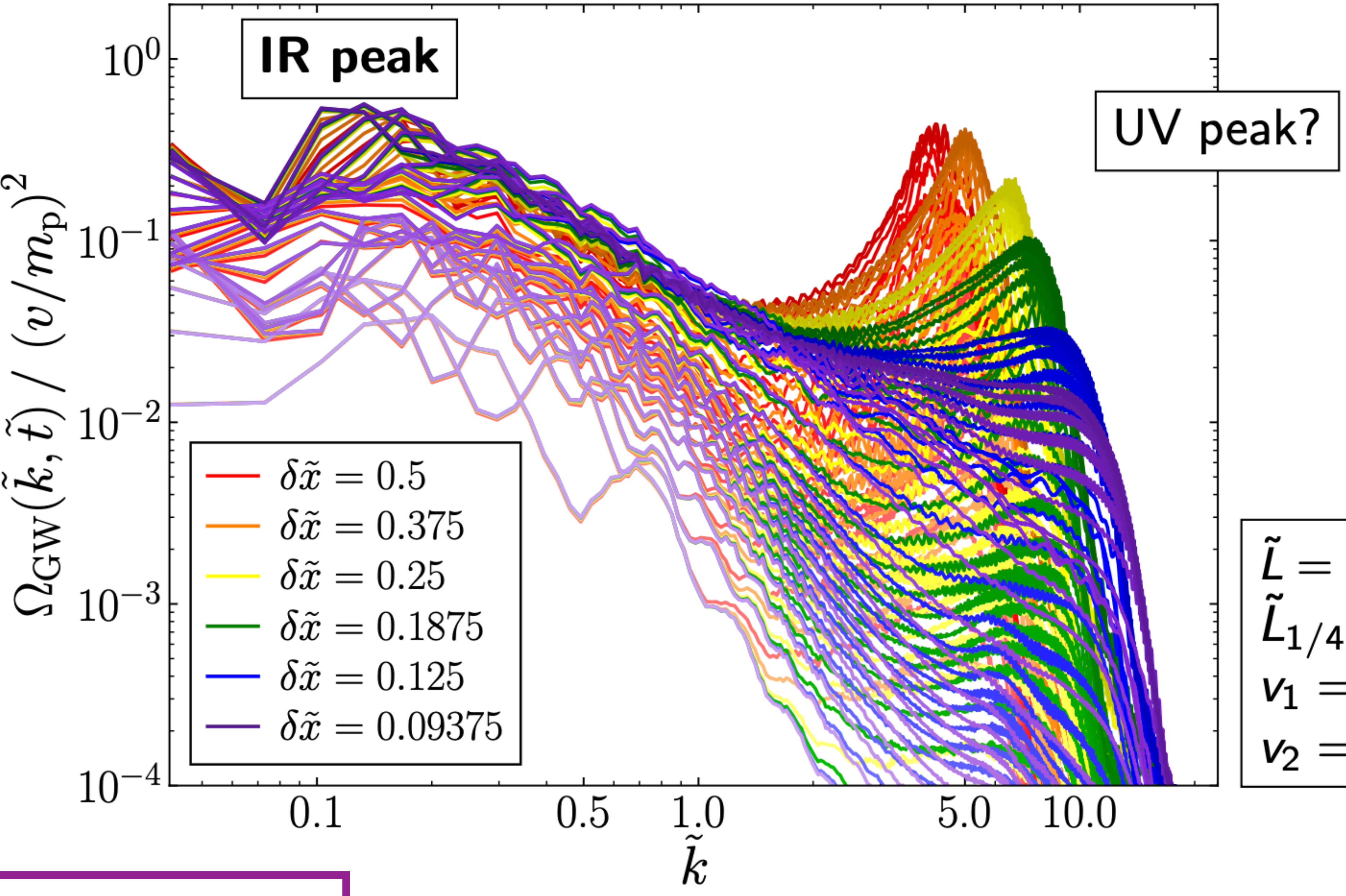
$\tilde{L} = 192$
$\tilde{L}_{1/4} = 96$
$v_1 = 0.7$
$v_2 = 0.6$

$\boxed{\delta\tilde{x} = 0.125 \quad \Rightarrow \quad N_* \sim 400}$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)



IR peak

UV peak?

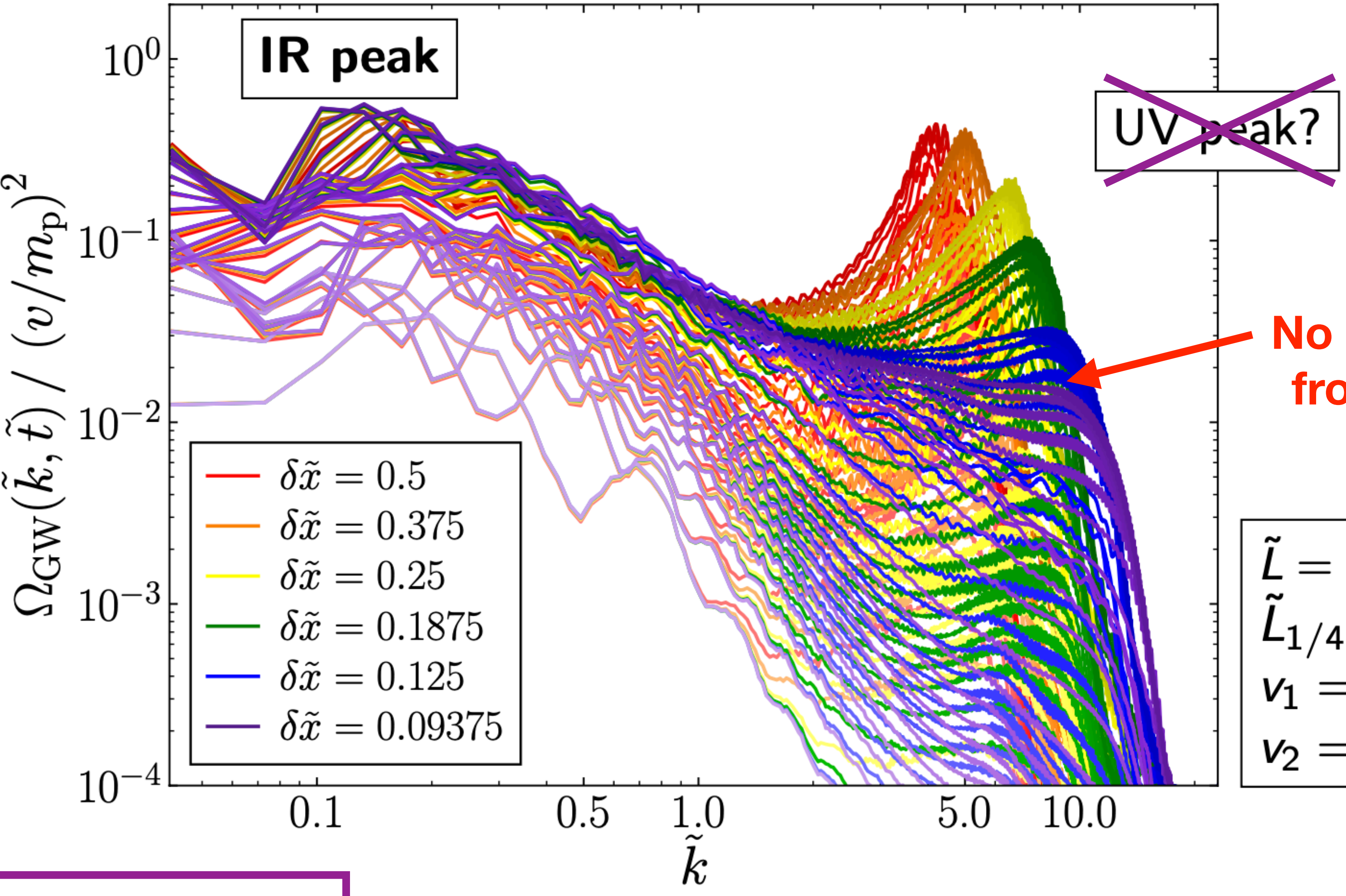
$\tilde{L} = 192$
$\tilde{L}_{1/4} = 96$
$v_1 = 0.7$
$v_2 = 0.6$

$\boxed{\delta\tilde{x} = 0.09375 \quad \Rightarrow \quad N_* \sim 700}$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)



(UV lattice Artifact)

No GW emission from the core !

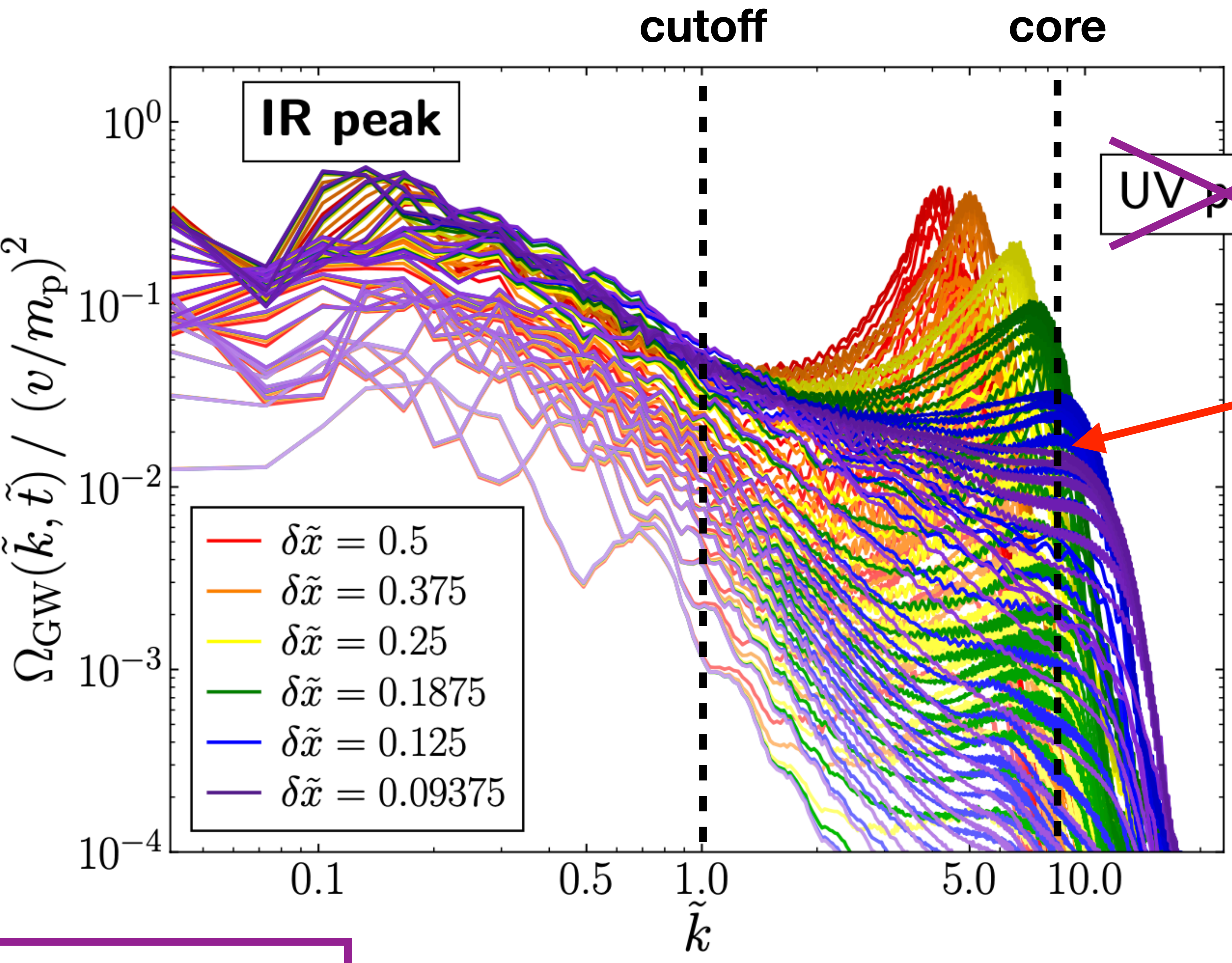
$\tilde{L} = 192$
$\tilde{L}_{1/4} = 96$
$v_1 = 0.7$
$v_2 = 0.6$

$\boxed{\delta\tilde{x} = 0.09375 \quad \Rightarrow \quad N_* \sim 700}$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Network Loop)



~~UV peak?~~

(UV lattice Artifact)

No GW emission from the core !

$\tilde{L} = 192$
$\tilde{L}_{1/4} = 96$
$v_1 = 0.7$
$v_2 = 0.6$

$\boxed{\delta\tilde{x} = 0.09375 \quad \Rightarrow \quad N_* \sim 700}$

String Loop: GW emission

$\tilde{t} = \sqrt{\lambda}\, v\, t$

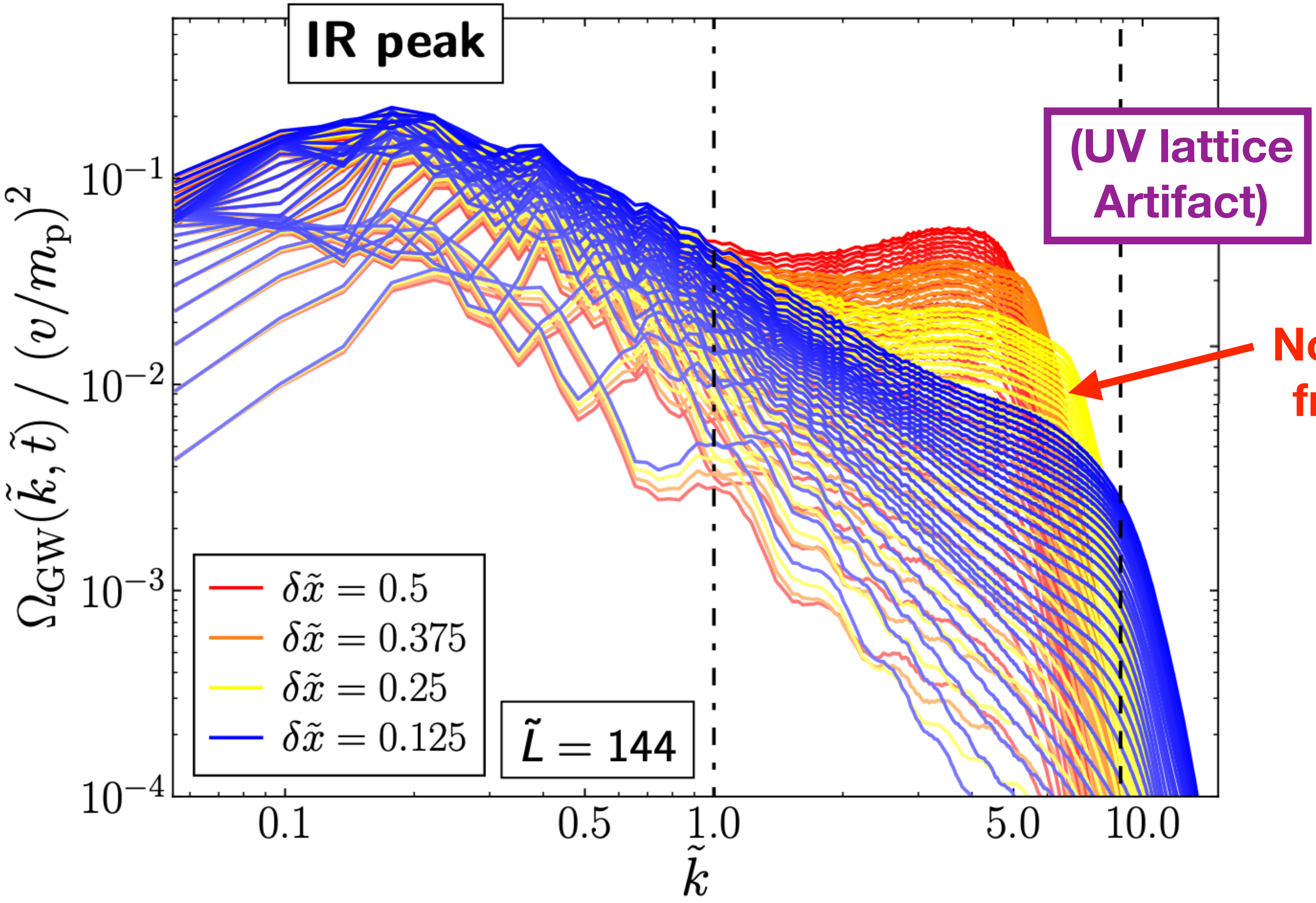
$\tilde{l} = \sqrt{\lambda}\, v\, l$

$\tilde{k} = k/\sqrt{\lambda}\, v$

(Artificial Loop)

cutoff

core



IR peak

(UV lattice Artifact)

Same as Before !

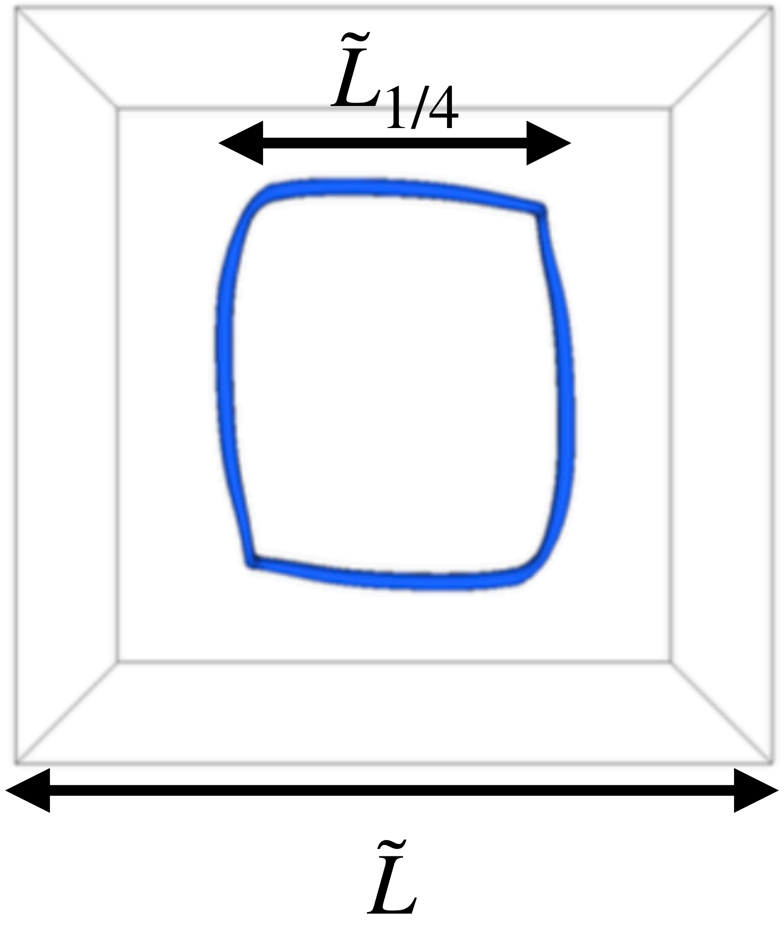
No GW emission from the core !

String Loop: GW emission

$$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t} \qquad \boxed{\tilde{l} = \sqrt{\lambda}\, v\, l} \qquad \boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$$

(Artificial Loop)

IR effects



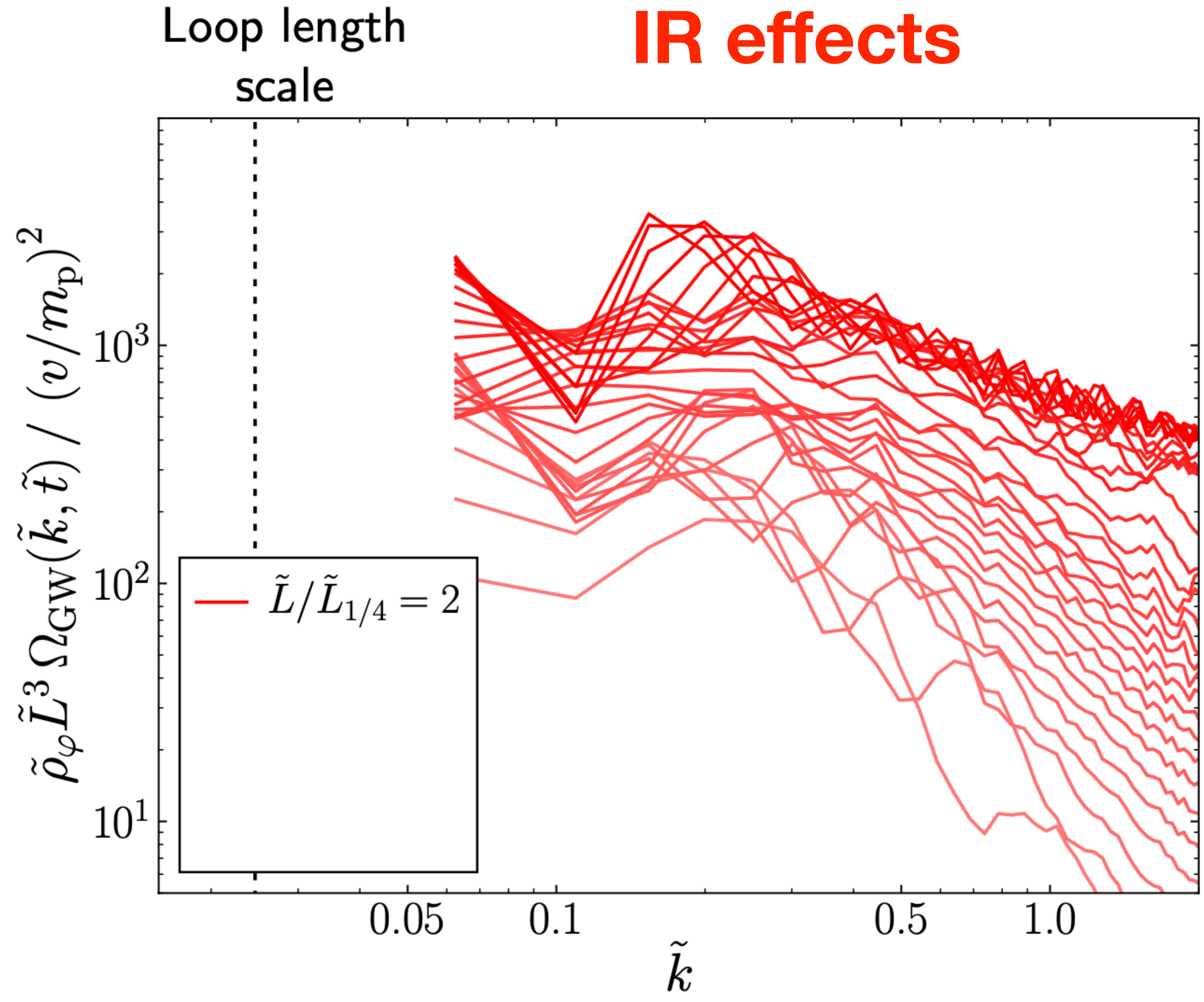
$$\tilde{L}/\tilde{L}_{1/4} = 2,3,4,\ldots,8$$

String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Artificial Loop)

IR effects



$\tilde{L}_{1/4} = 96$
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String Loop: GW emission

$\tilde{t} = \sqrt{\lambda}\, v\, t$

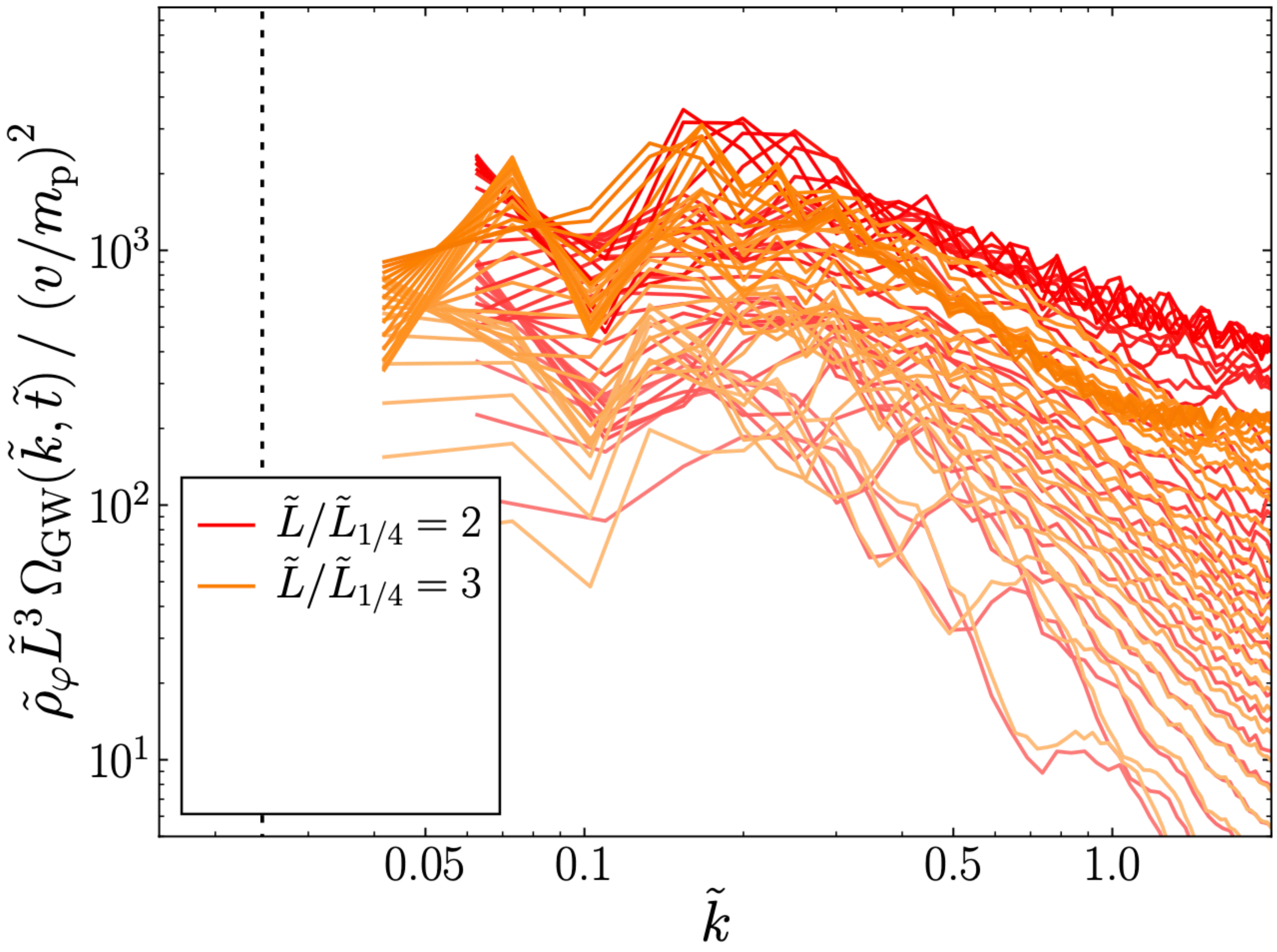
$\tilde{l} = \sqrt{\lambda}\, v\, l$

$\tilde{k} = k/\sqrt{\lambda}\, v$

(Artificial Loop)

Loop length scale

IR effects



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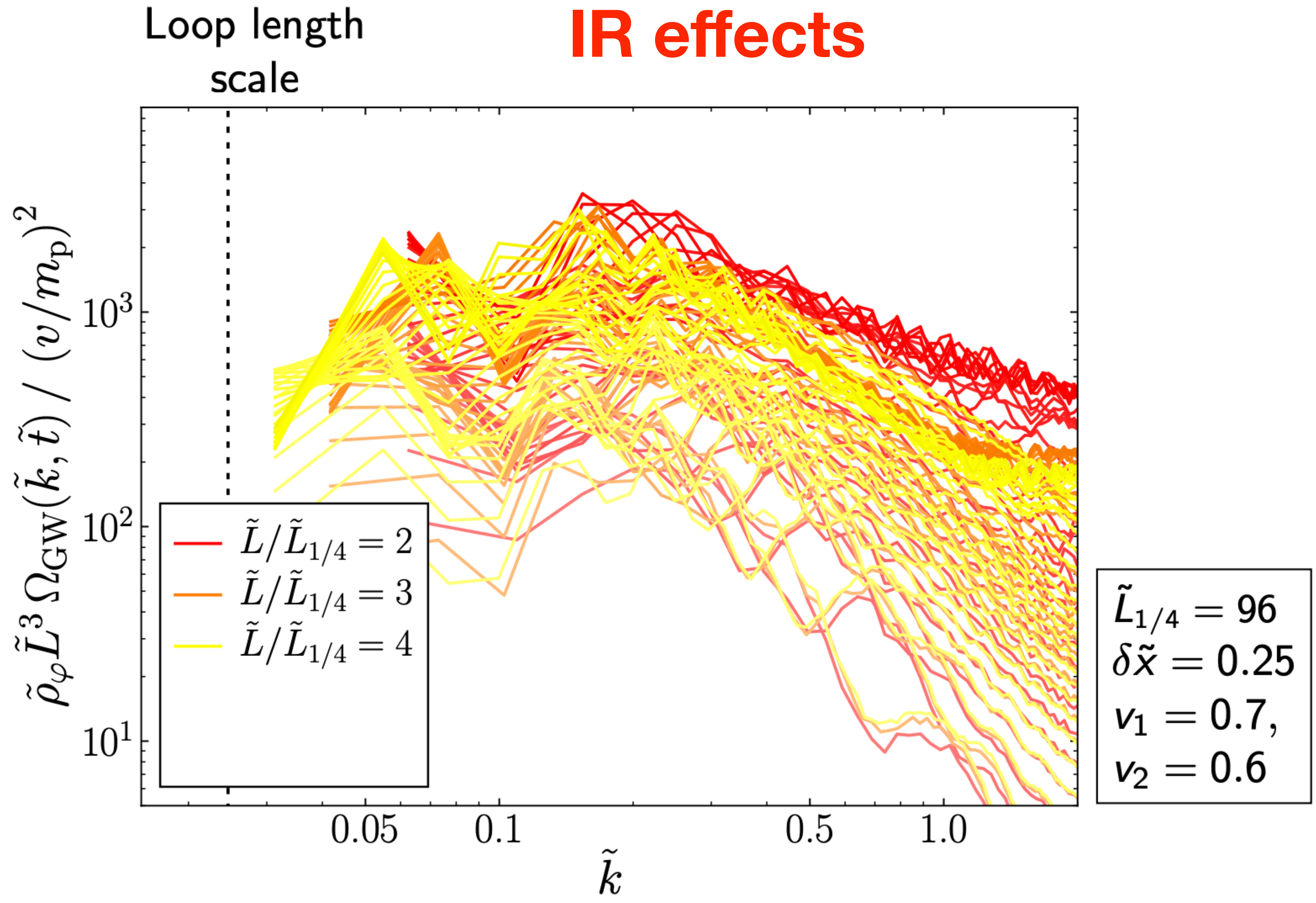

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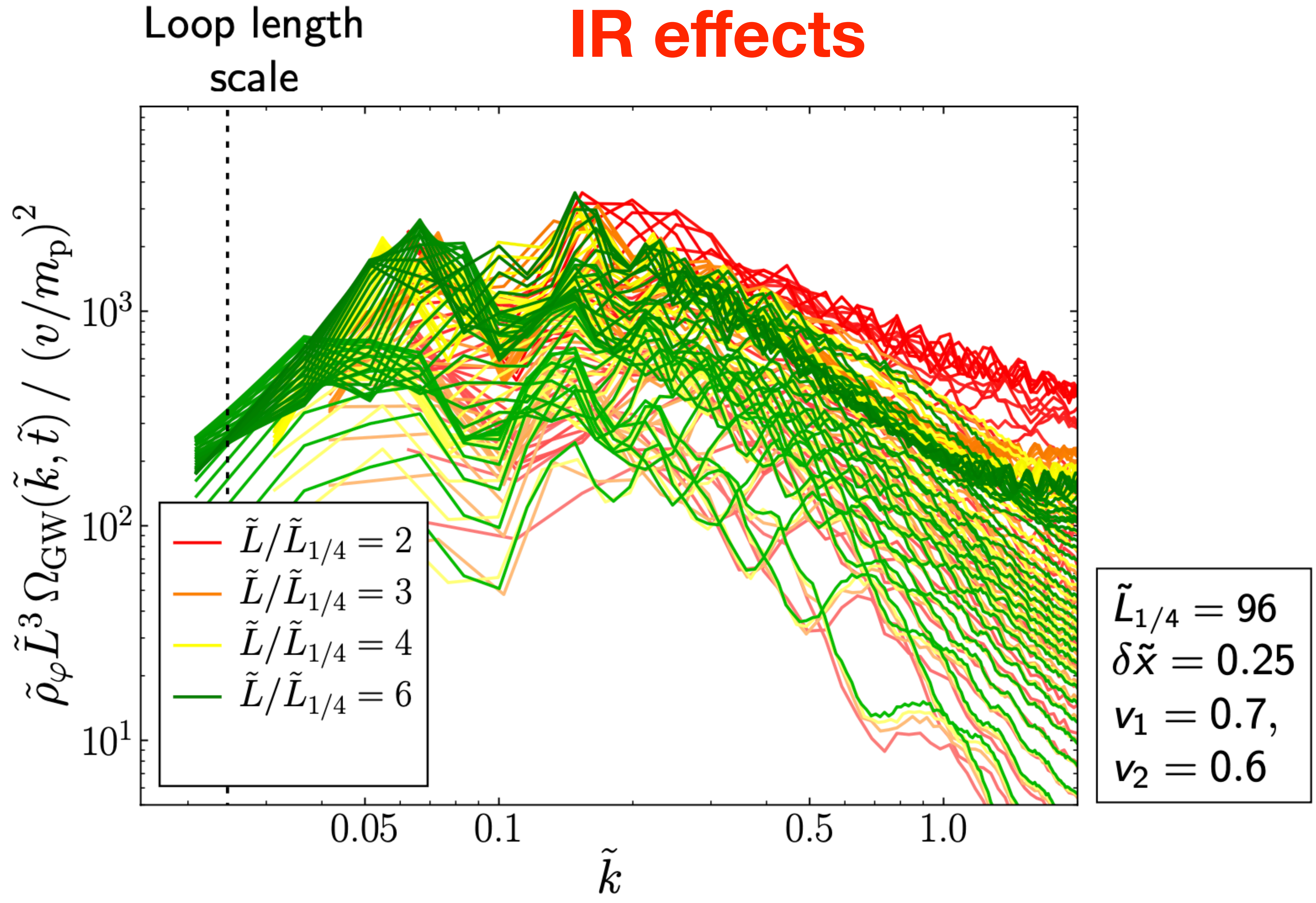
(Artificial Loop)



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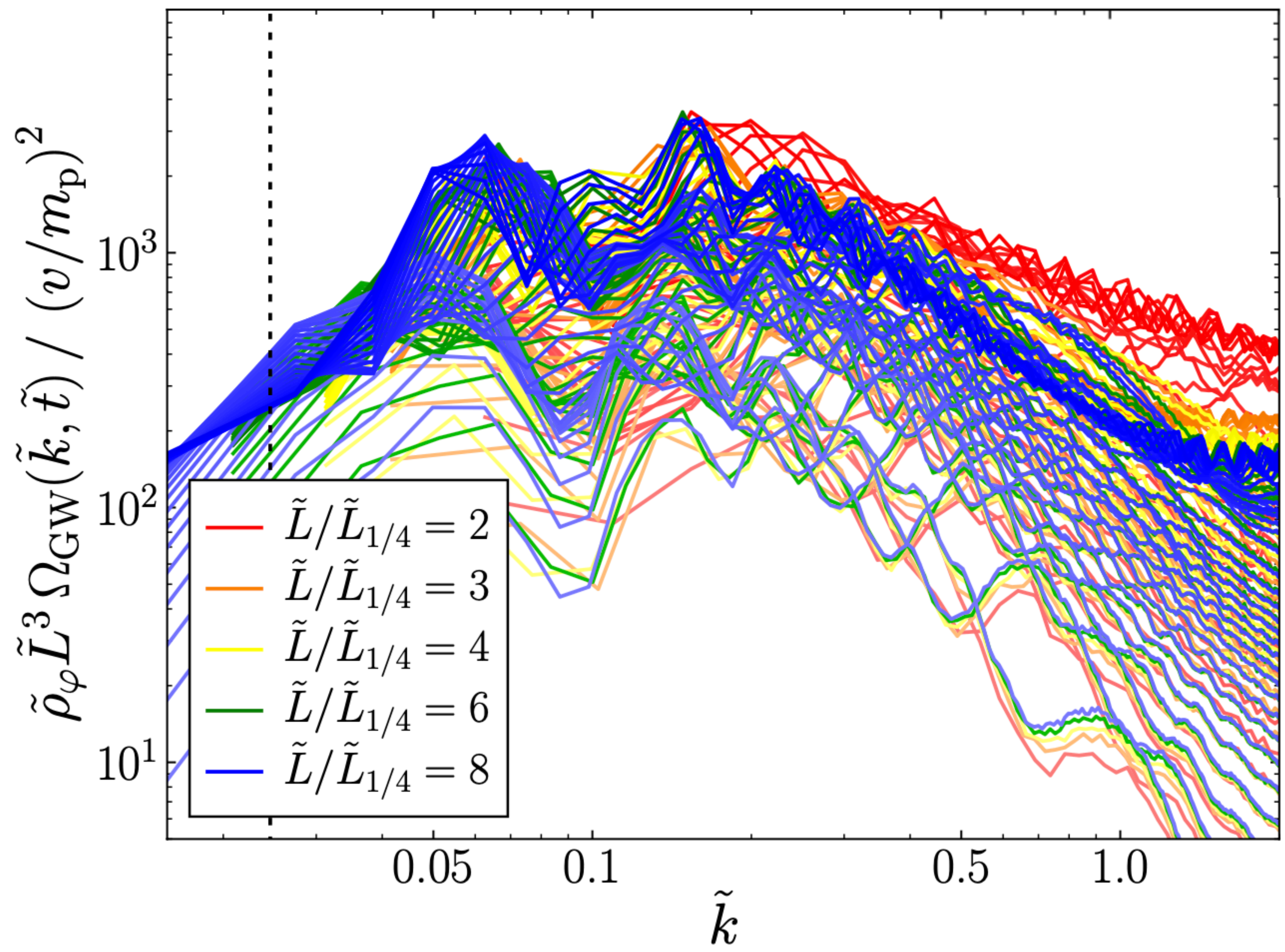

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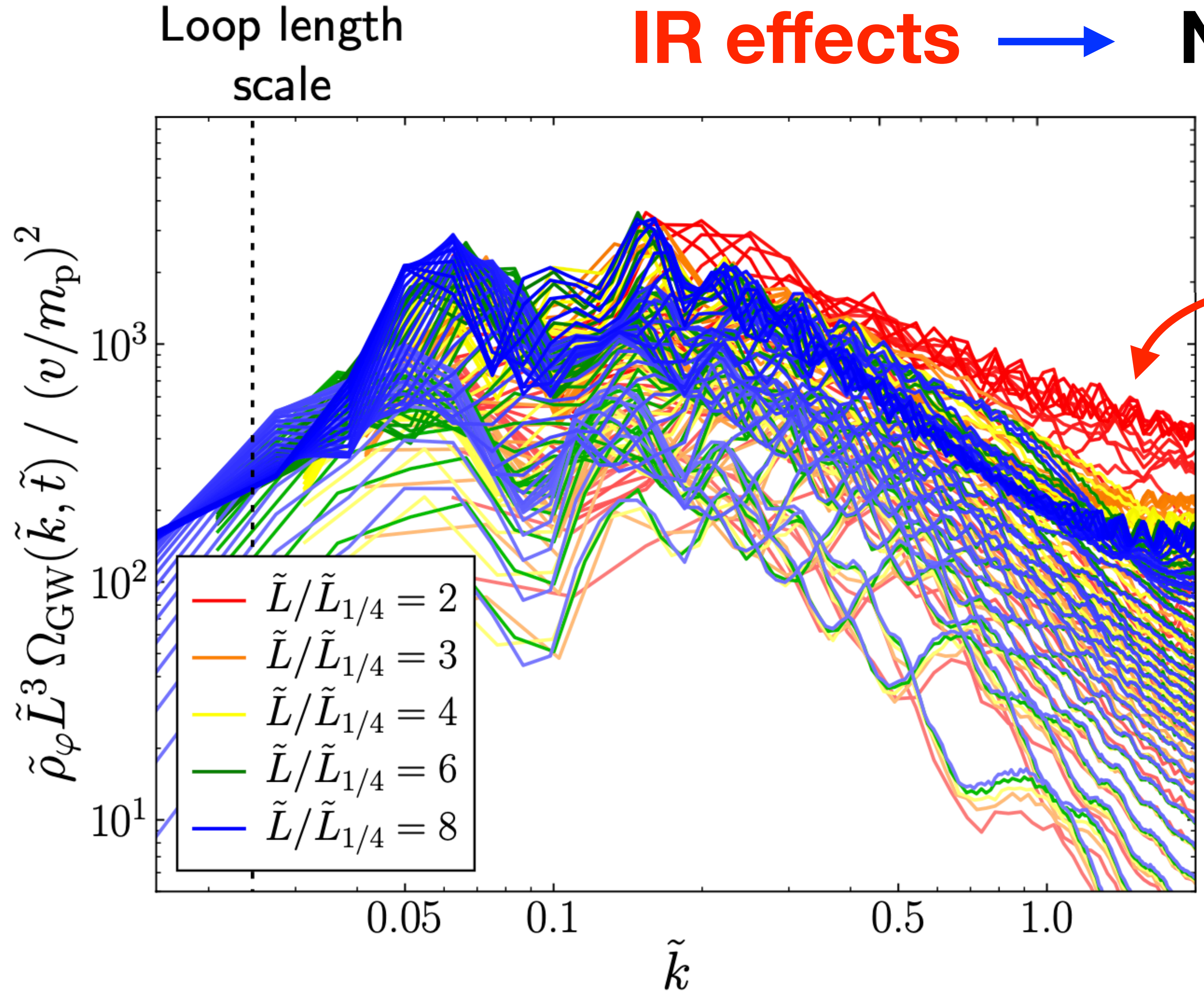
String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Artificial Loop)

IR effects → **Negligible !**



(Except for small $\tilde{L}/\tilde{L}_{1/4}$)
(~2)

$\tilde{L}_{1/4} = 96$
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$v_1 = 0.7,$
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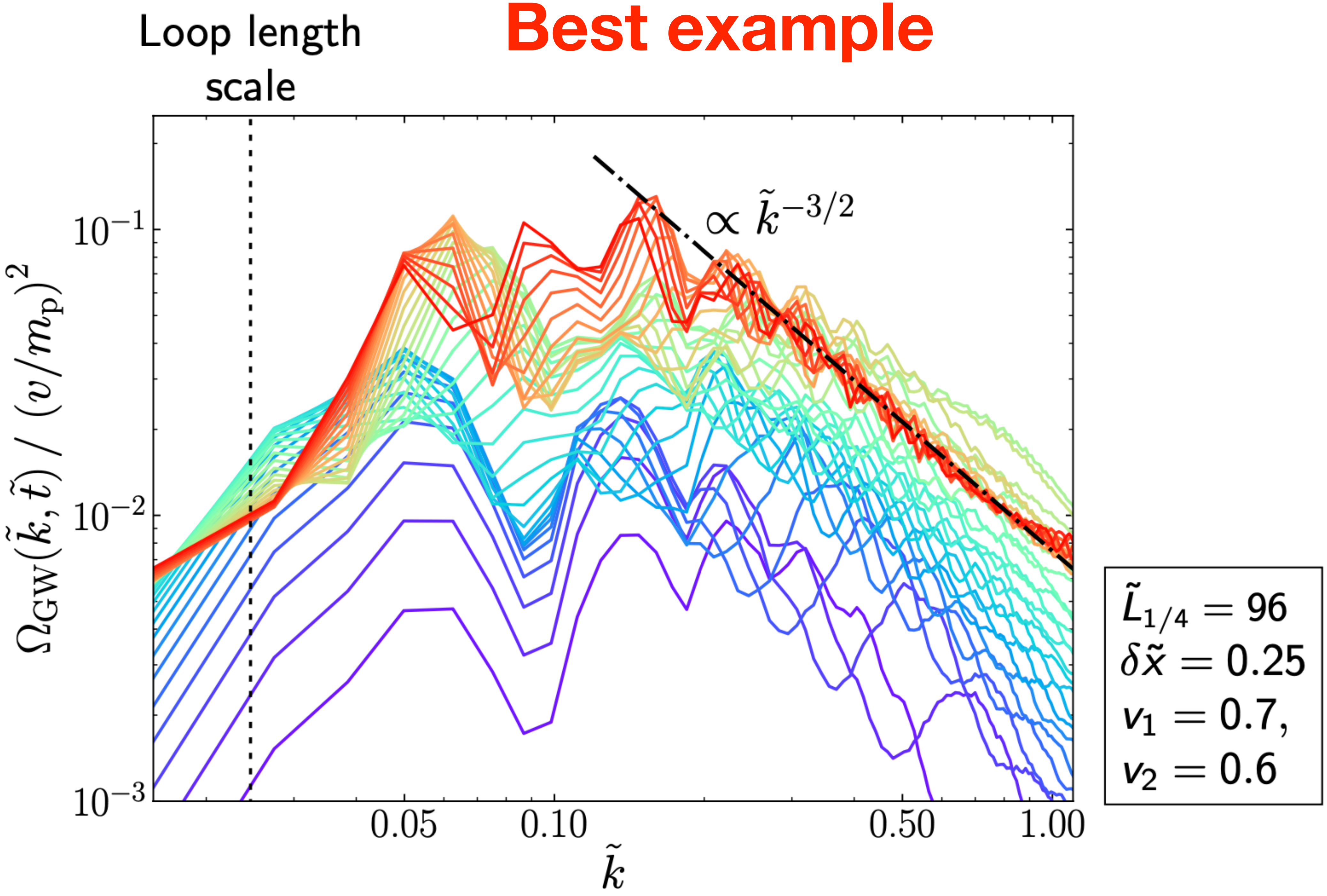
String Loop: GW emission

$\boxed{\tilde{t} = \sqrt{\lambda}\, v\, t}$ $\boxed{\tilde{l} = \sqrt{\lambda}\, v\, l}$ $\boxed{\tilde{k} = k/\sqrt{\lambda}\, v}$

(Artificial Loop)

Best example



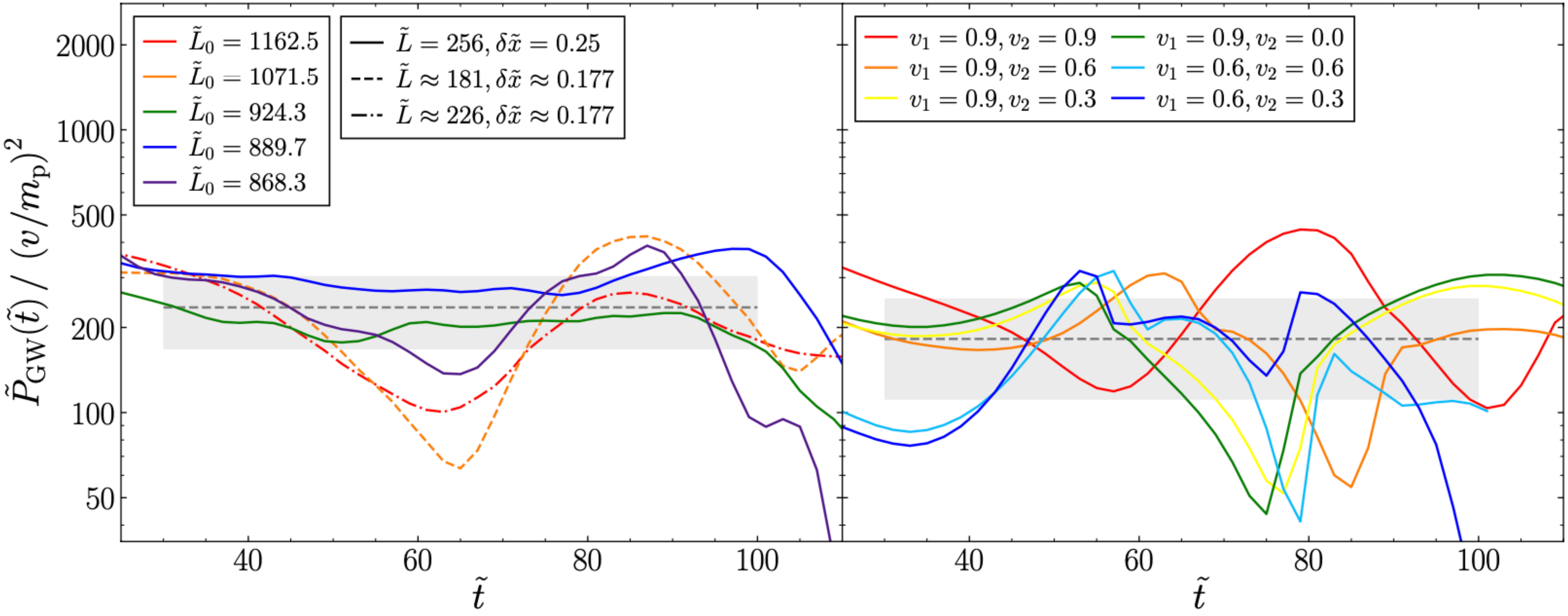
String Loop Dynamics + GW emission

GW Power Emitted

$$P_{\rm GW}(t) = L^3\rho_\varphi \left\langle \frac{d}{dt'}\int_0^{k_c} \Omega_{\rm GW}(k,t')\,{\rm d}\log k \right\rangle_T$$

(Rolling Average)

Network **Artificial**



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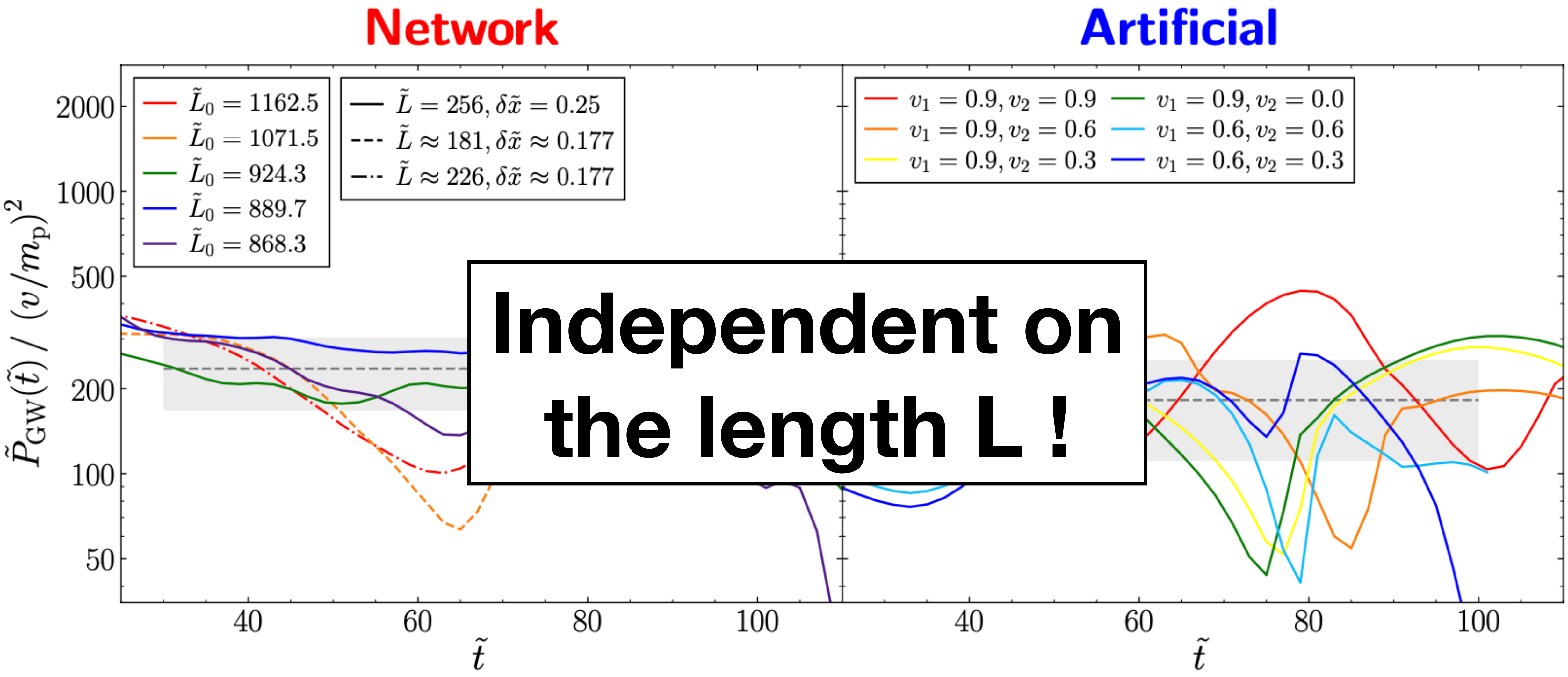

Independent on the length L !

String Loop Dynamics + GW emission

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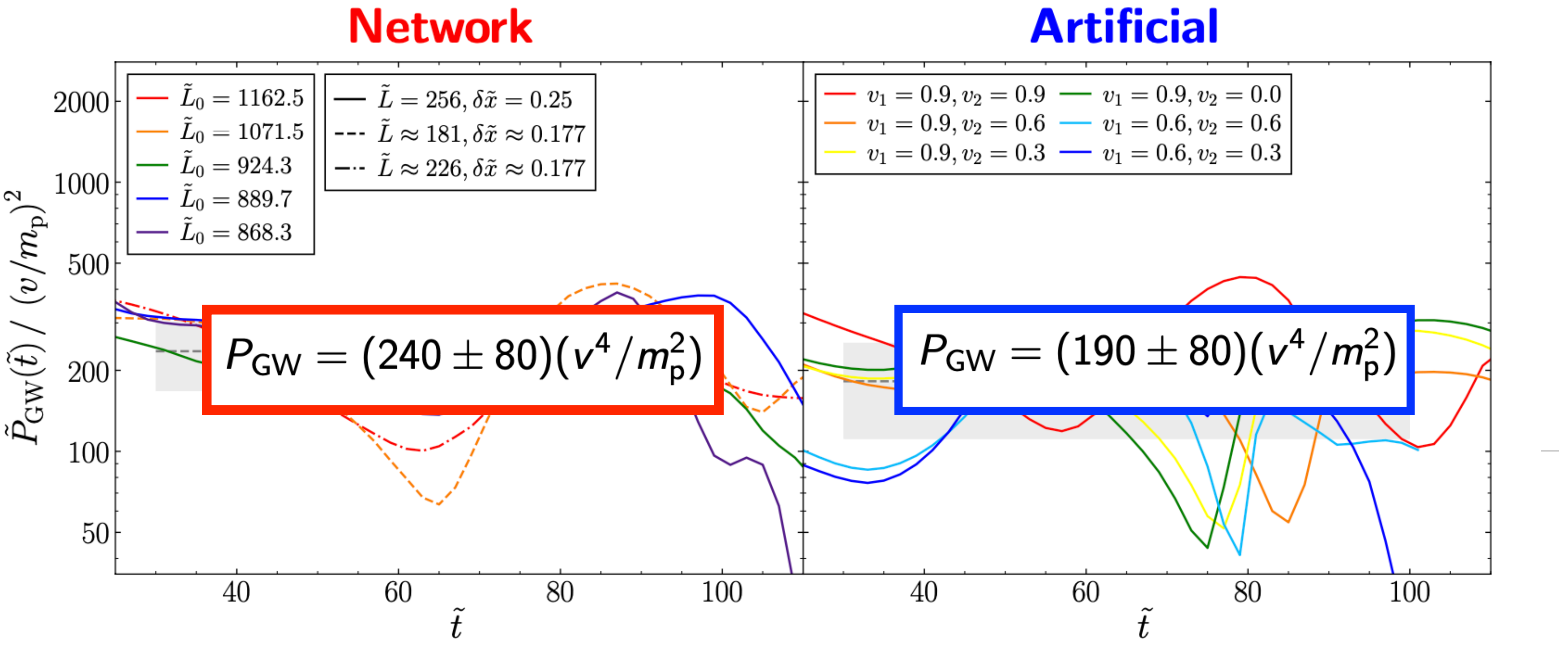

Network

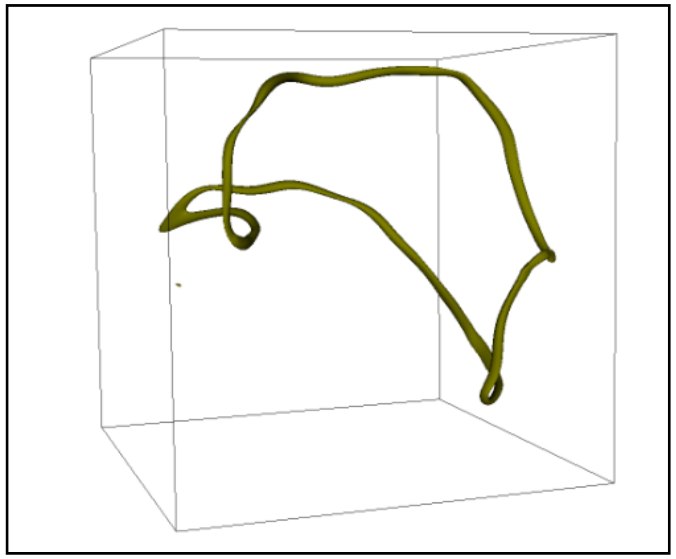
Artificial

$P_{\rm GW} = (240 \pm 80)(v^4/m_{\rm p}^2)$

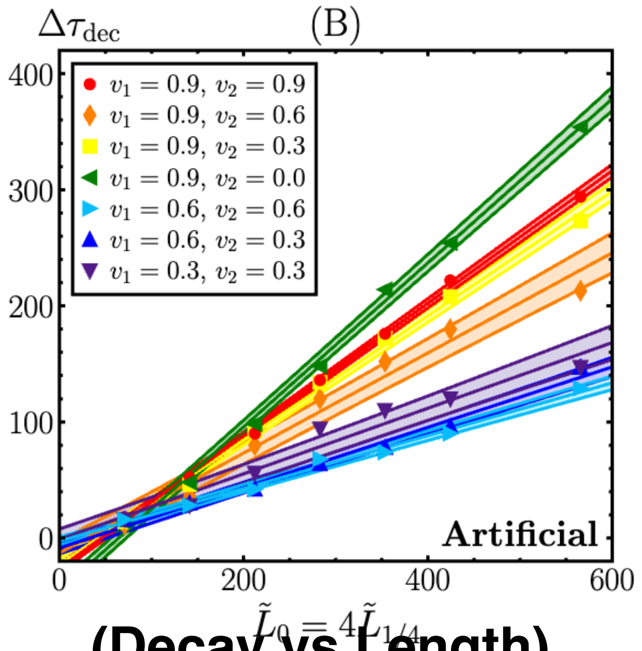
$P_{\rm GW} = (190 \pm 80)(v^4/m_{\rm p}^2)$

String Loop Dynamics + GW emission

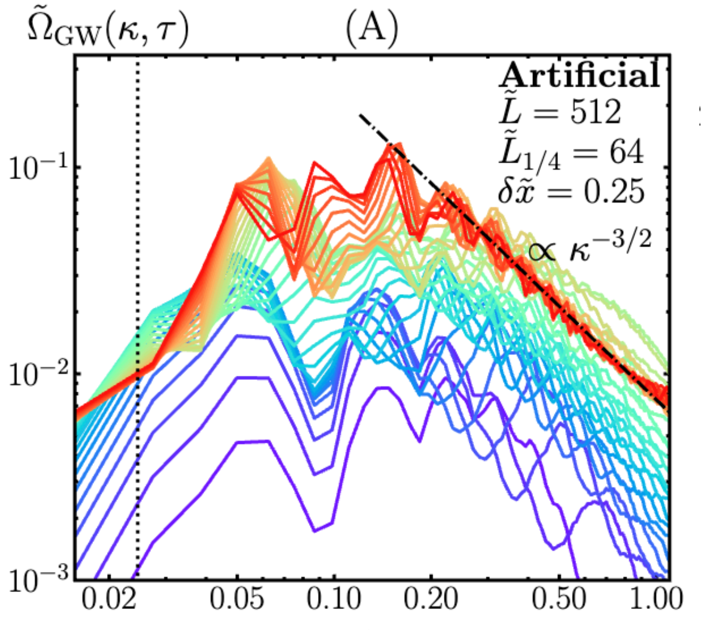
GW Power Emitted



(Loops isolated)



(Decay vs Length)



(GW emission)

String Loop Dynamics + GW emission

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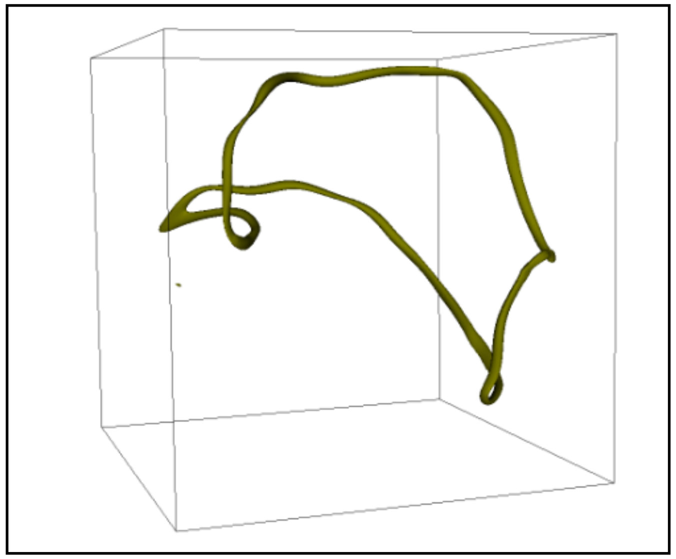

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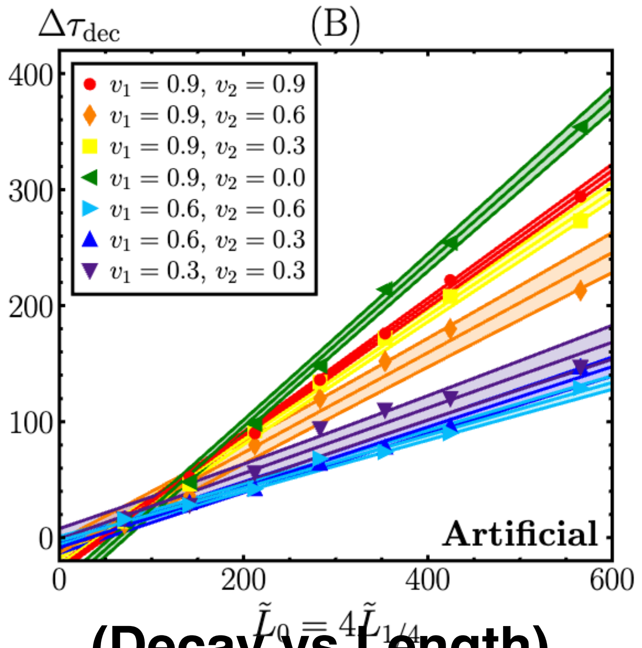

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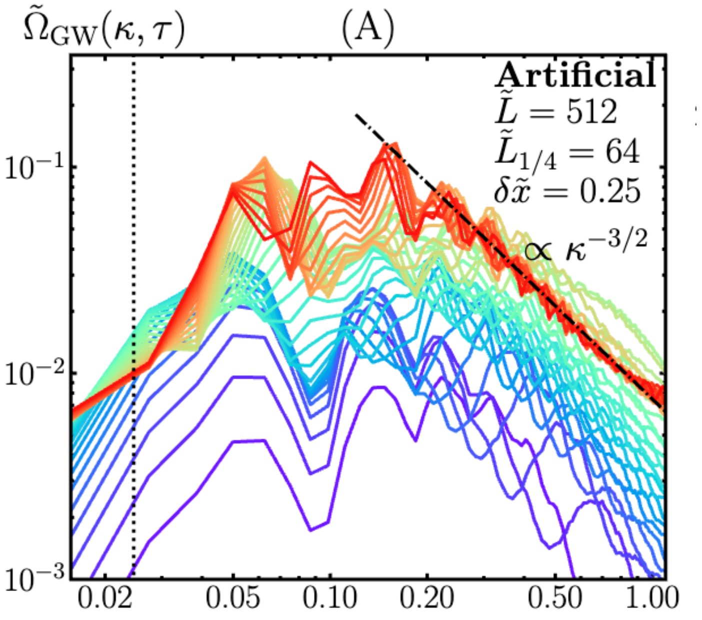


(GW emission)

Baeza-Ballesteros et al, 2023

(Global Strings)

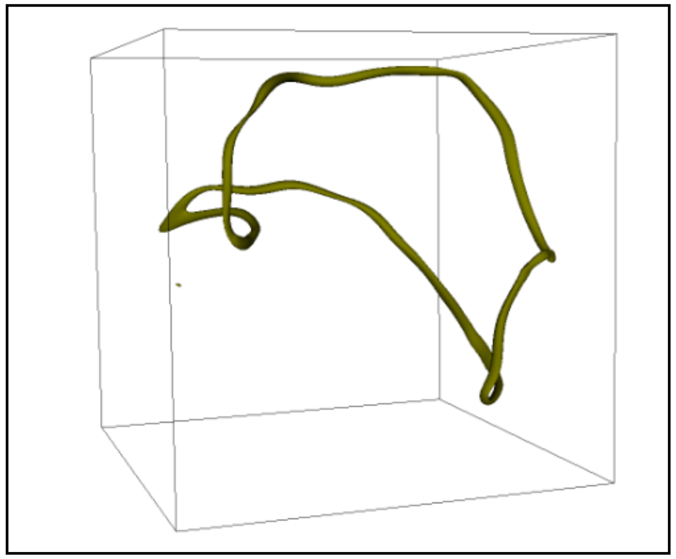
$[\ L/w \simeq 80 - 1700\ ]$

$$\boxed{\frac{P_{\rm GW}}{P_\phi} \simeq \mathcal{O}(10)\left(\frac{v}{m_p}\right)^2} \ll 1$$

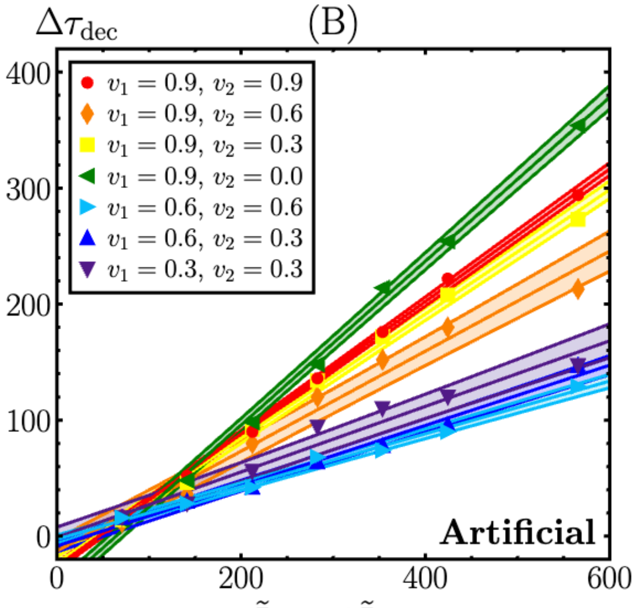
$\Big($ $v^2/m_{\rm p}^2 \lesssim 10^{-6} - 10^{-3}$ [Lopez-Eiguren, et al. (2017), Benabou, et al. (2023)] $\Big)$

String Loop Dynamics + GW emission

GW Power Emitted



(Loops isolated)



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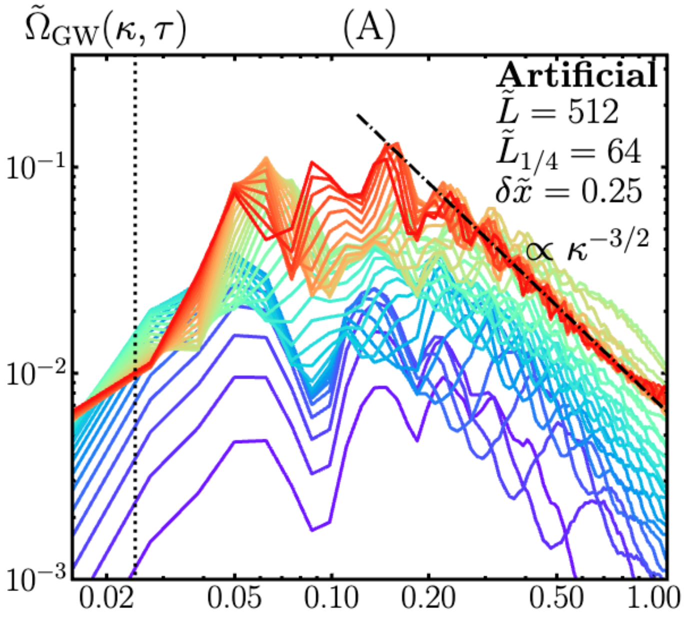

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So what happens with Local Strings ?

String Loop Dynamics + GW emission

Results will impact on

Real evaluation of GW emission

Re-evaluation of PTA constraints

(Pulsar Time Array)

String Loop Dynamics + GW emission

Results will impact on

Real evaluation of GW emission

Re-evaluation of PTA constraints

(Pulsar Time Array)

Implications for

Dark Matter Axion string network

Local (Abelian-Higgs) string network

Comparison with Nambu-Goto

GUT models

….

Non-minimal

Almost …
the End

If you want to learn
how to "latticesize"
your problems …

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CosmoLattice

1st CL School 2022: Sept 5-8

@Valencia:

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2nd CL School 2023: Sept 25-29

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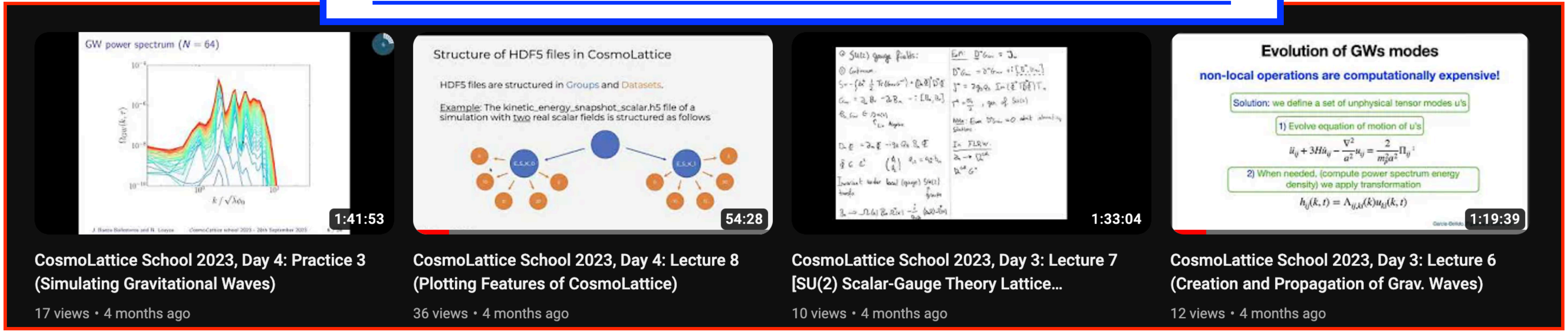
https://www.youtube.com/@CosmoLattice/videos



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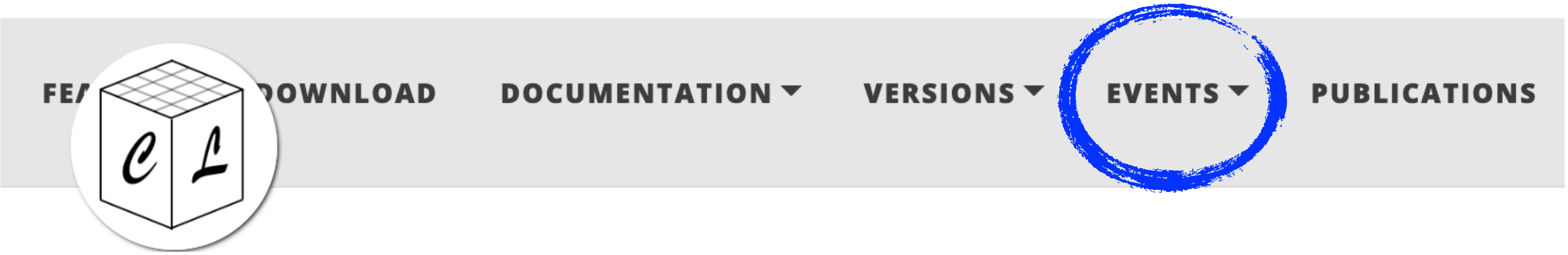
3rd CL School 2024: XXXXX

Details for CL School 2024 TBA at:

https://cosmolattice.net



Thanks for your attention

Merci pour votre attention

Thanks for your attention

Merci pour votre attention

Back Slides

Constraints
Applications
Program Variables
Axion Inflation

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\phi) + \frac{\phi}{4\Lambda} F\tilde{F} \quad \Rightarrow \quad A''_\pm + \left(k^2 \pm \frac{k\dot{\phi}}{\Lambda H \tau}\right) A_\pm = 0$$

L.Sorbo et al 2006-2012

Chiral instability

$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

$(\xi \propto \dot{\phi})$

A_+ exponentially amplified

Only one chirality of gauge field then… chiral GWs !

Example, GW prediction

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

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L.Sorbo et al 2006-2012

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Only
one chirality
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Example, GW prediction

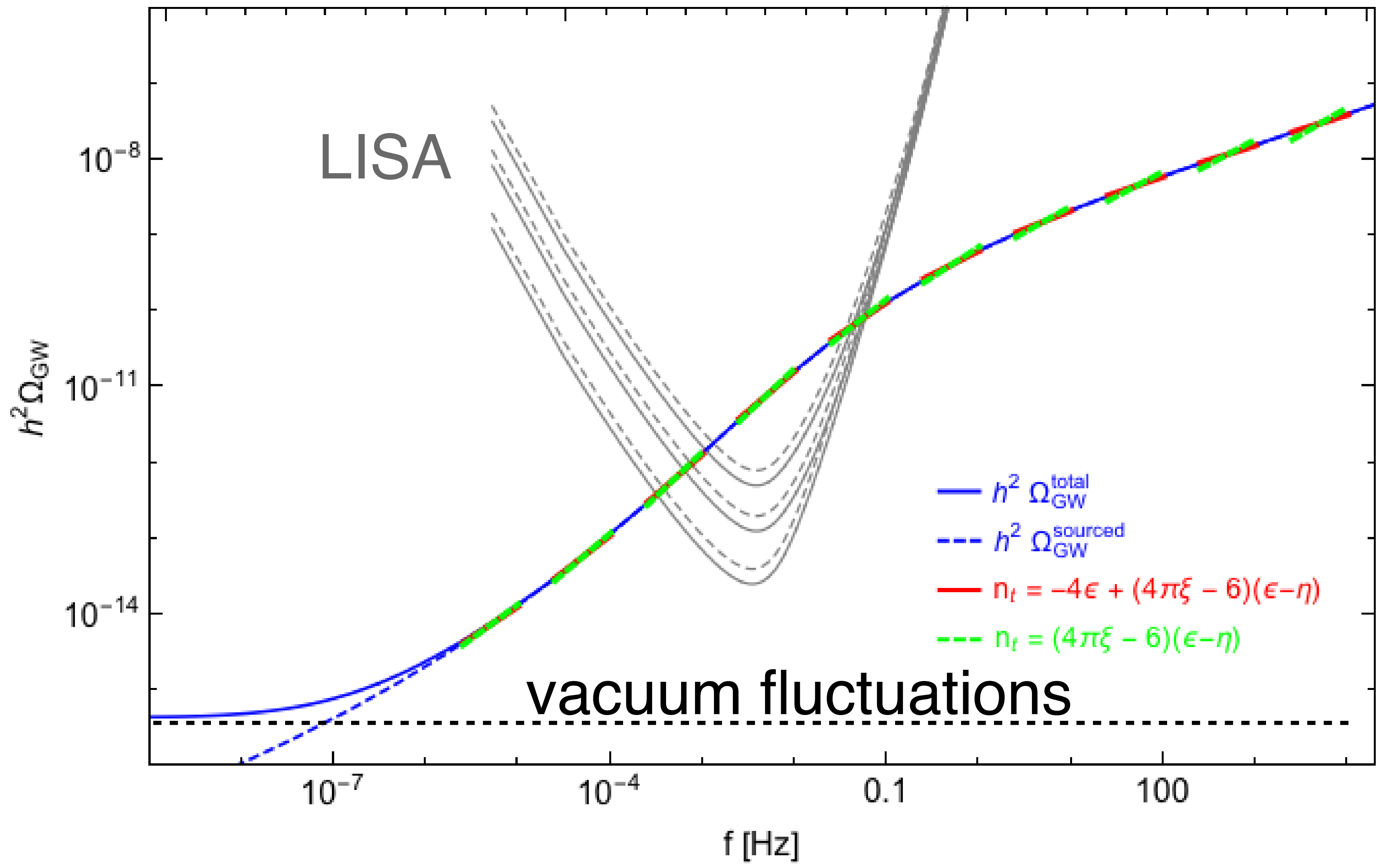
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij} \propto \{E_iE_j + B_iB_j\}^{TT}$$

well calculated ?

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



Blue-Tilted

+ Chiral

+ Non-G

GW background

Bartolo et al ’16, 1610.06481

INFLATIONARY MODELS

Axion-Inflation

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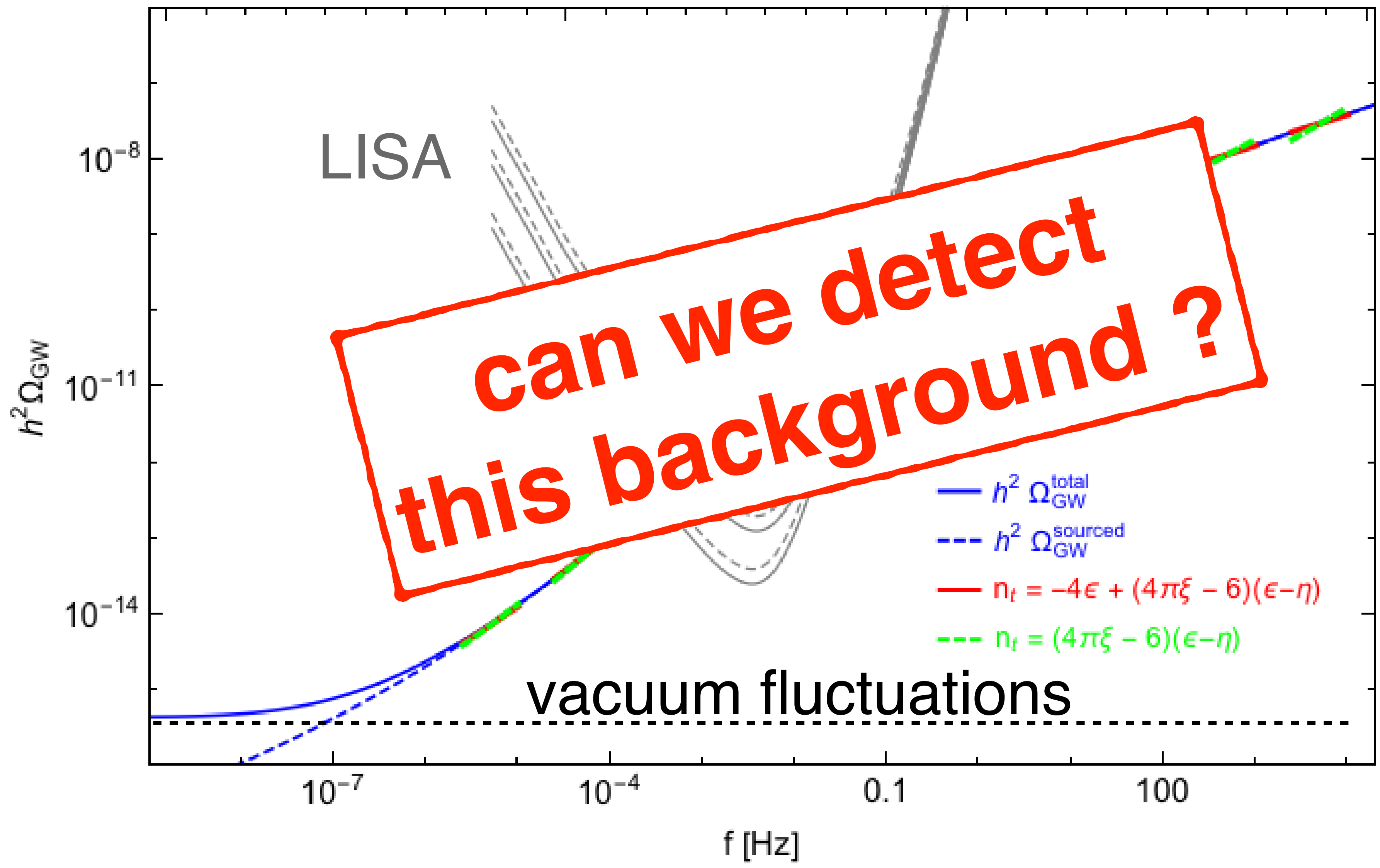


can we detect this background ?

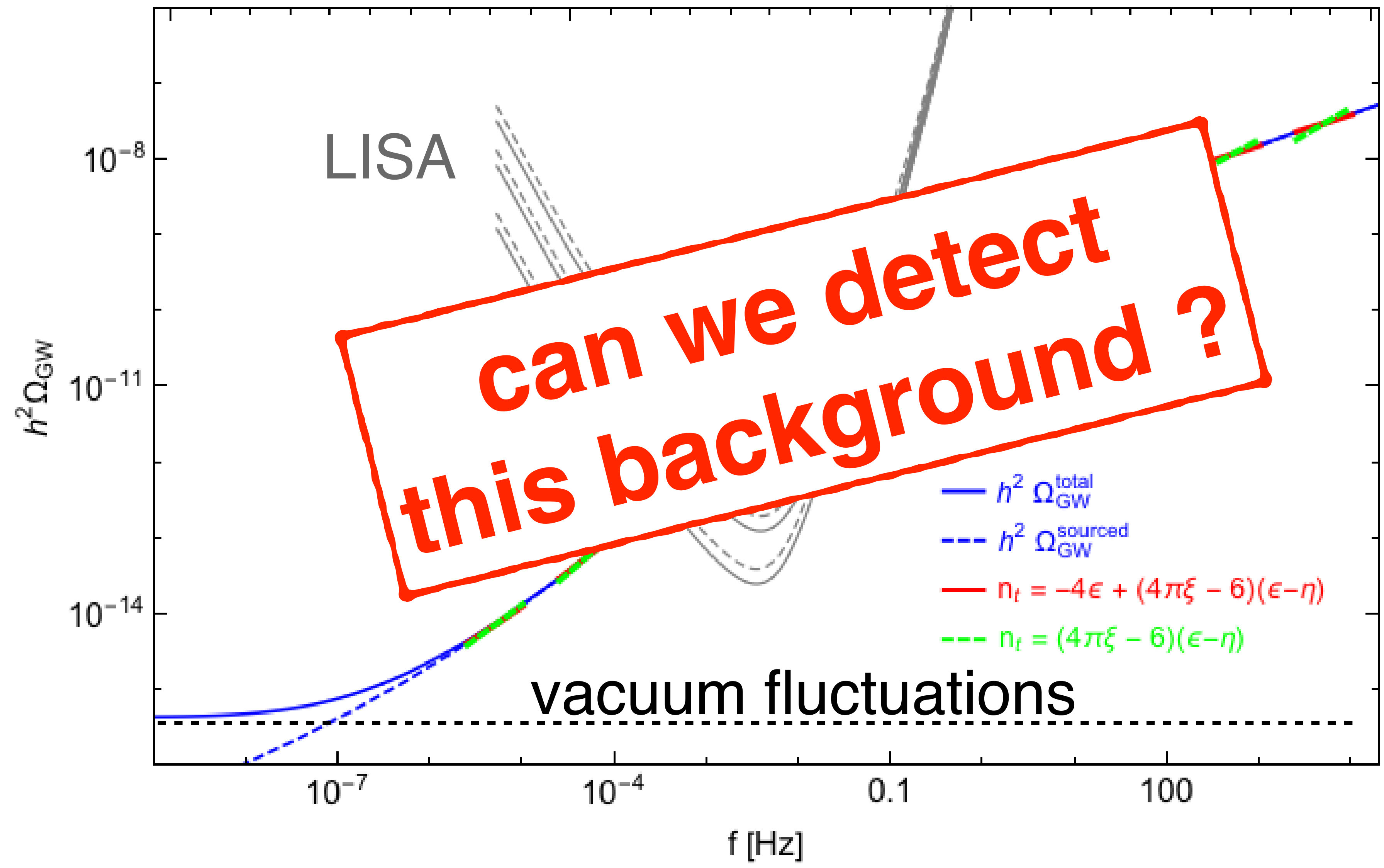
Blue-Tilted
+ Chiral
+ Non-G
GW background

Bartolo et al '16, 1610.06481

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



can we detect this background ?

Blue-Tilted

+ Chiral

+ Non-G

GW background

As $A_+ \propto e^{\dot{\phi}}$, GWs very sensitive to choice of $V(\phi)$ and calculation details

Bartolo et al '16, 1610.06481

Constraints

Energy conservation

- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{\rho}{3m_p^2}}$$

$$\Big\downarrow$$

$$\Delta_e \equiv \frac{\langle \mathrm{LHS} - \mathrm{RHS} \rangle}{\langle \mathrm{LHS} + \mathrm{RHS} \rangle}$$

Transparencies worked out together with Paco Torrentí

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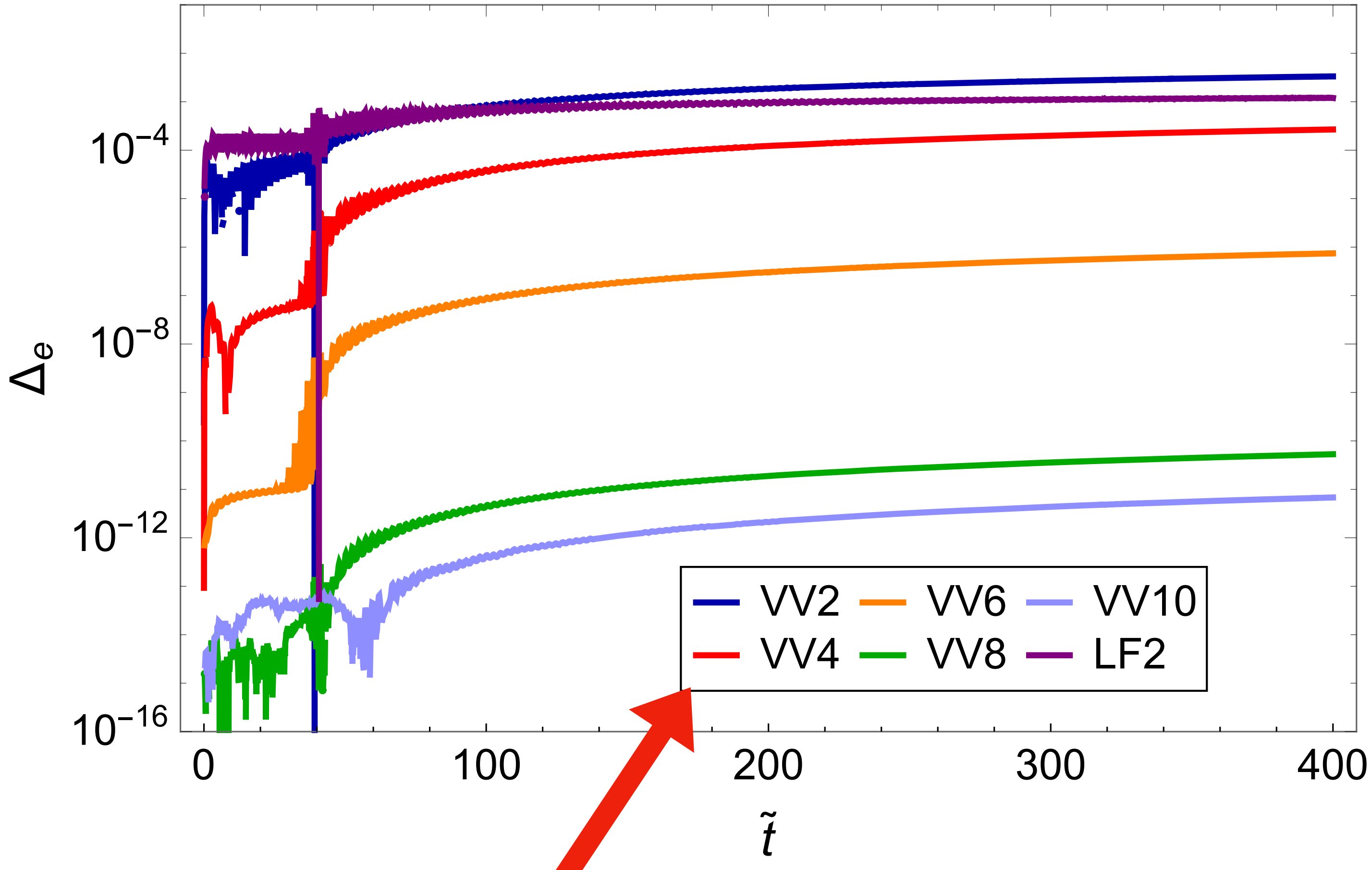

Evolution algorithms:

- **VVn:** Velocity-verlet of accuracy order O(dt^n)
- **LF2:** Staggered leapfrog, accuracy order O(dt^2)

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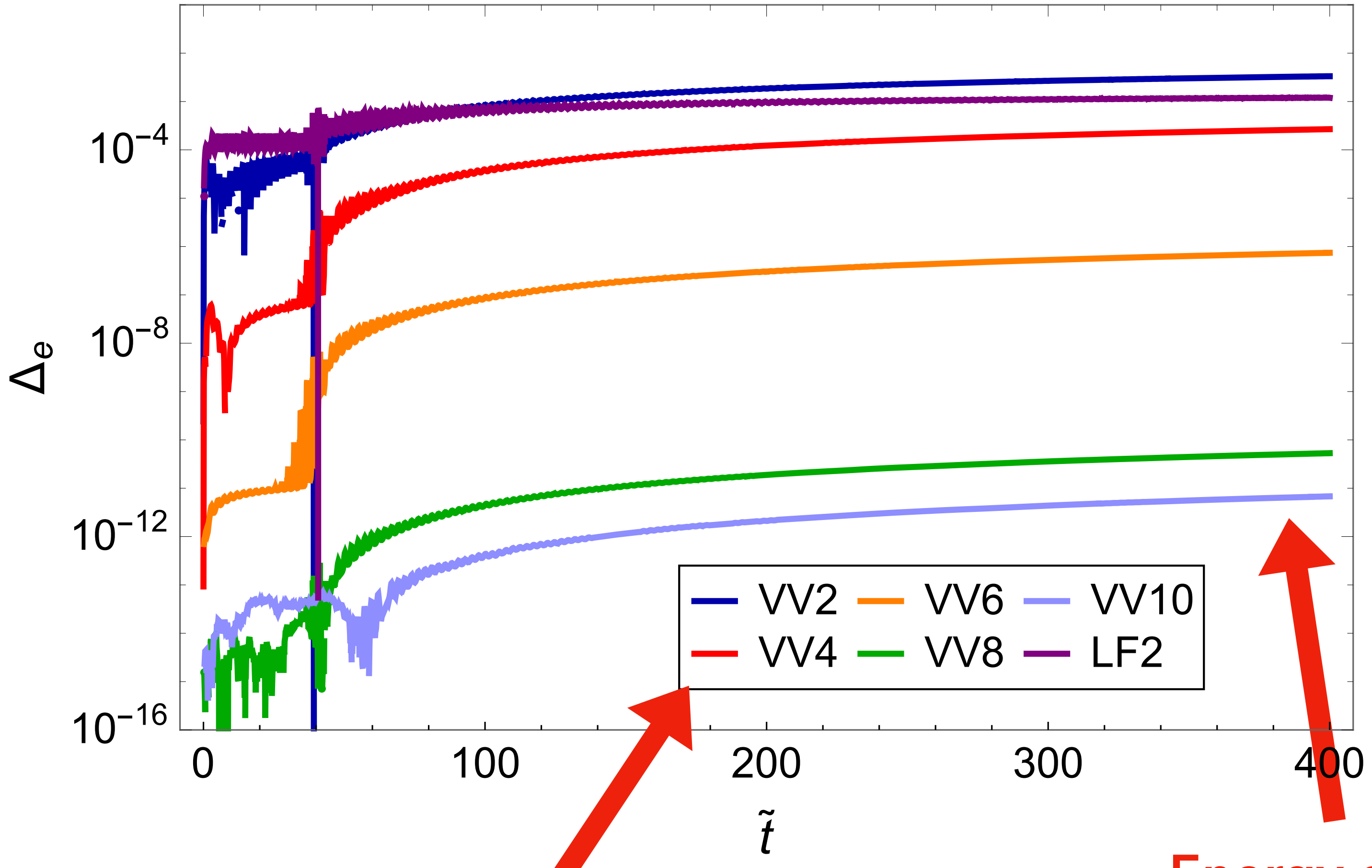

Evolution algorithms:

- **VVn:** Velocity-verlet of accuracy order O(dt^n)
- **LF2:** Staggered leapfrog, accuracy order O(dt^2)

Energy conserved up to machine precision for VV10!

Transparencies worked out together with Paco Torrentí

Gauge theories: Gauss constraint

- Preservation of U(1) & SU(2) **Gauss constraints** (for all integrators!)

$$\boxed{\begin{aligned} \partial_i F_{0i} &= a^2 J_0^A \\ (\mathscr{D}_i)_{ab}(G_{0i})^b &= a^2 (J_0)_a \end{aligned}}$$

Gauge charges

$$\Delta_g \equiv \frac{\langle\sqrt{(\mathrm{LHS}-\mathrm{RHS})^2}\rangle}{\langle\sqrt{(\mathrm{LHS}+\mathrm{RHS})^2}\rangle}$$

Transparencies worked out together with Paco Torrentí

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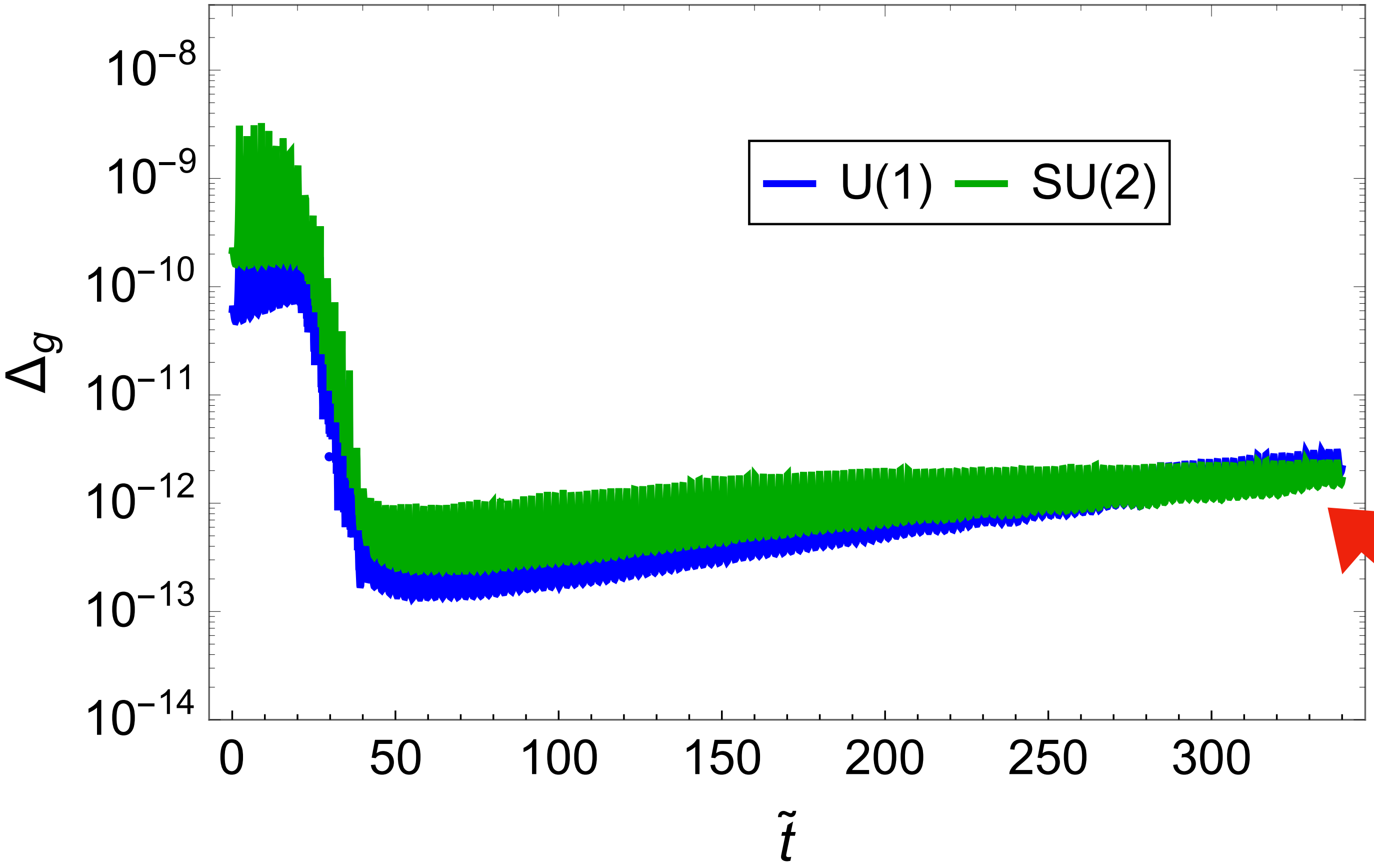
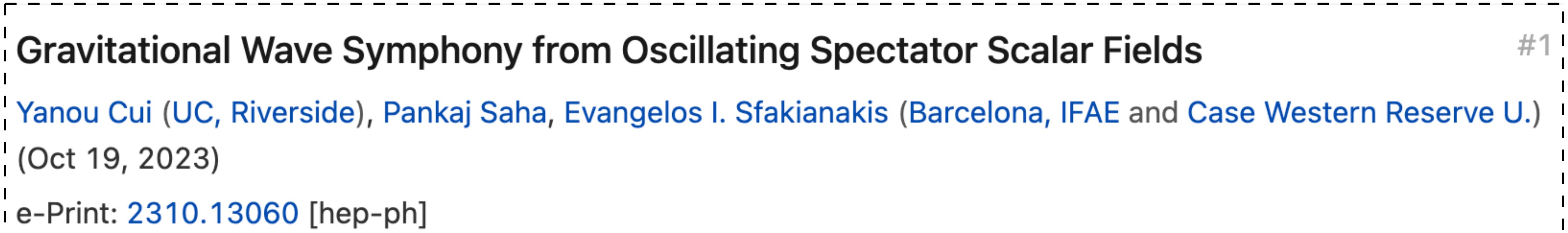


Gauss constraint preserved up to machine precision

Transparencies worked out together with Paco Torrentí

Applications
(papers)

Applications



Gravitational Wave Symphony from Oscillating Spectator Scalar Fields

#1

Yanou Cui (UC, Riverside), Pankaj Saha, Evangelos I. Sfakianakis (Barcelona, IFAE and Case Western Reserve U.) (Oct 19, 2023)

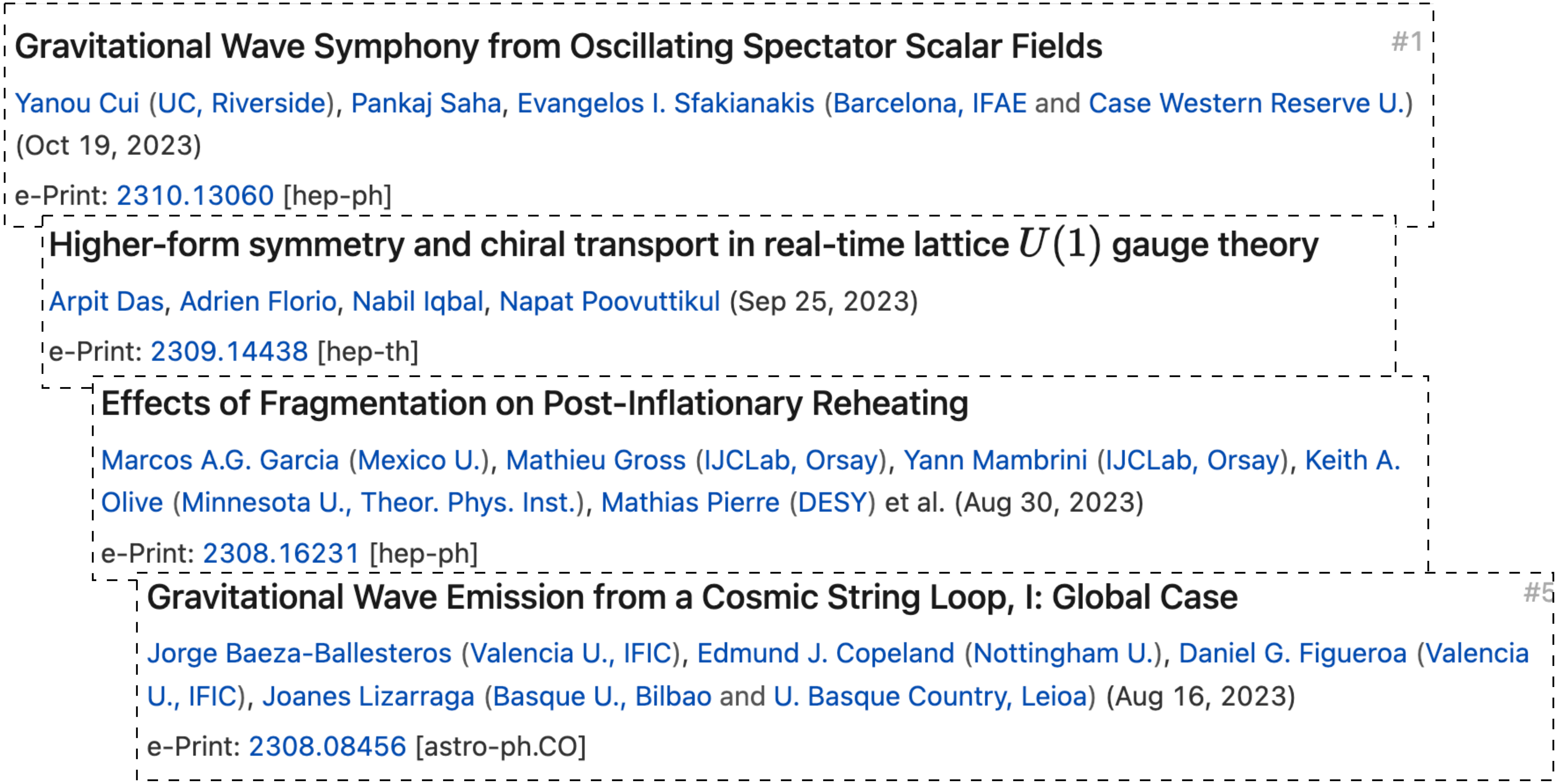
e-Print: 2310.13060 [hep-ph]

Higher-form symmetry and chiral transport in real-time lattice $U(1)$ gauge theory

Arpit Das, Adrien Florio, Nabil Iqbal, Napat Poovuttikul (Sep 25, 2023)

e-Print: 2309.14438 [hep-th]

Applications



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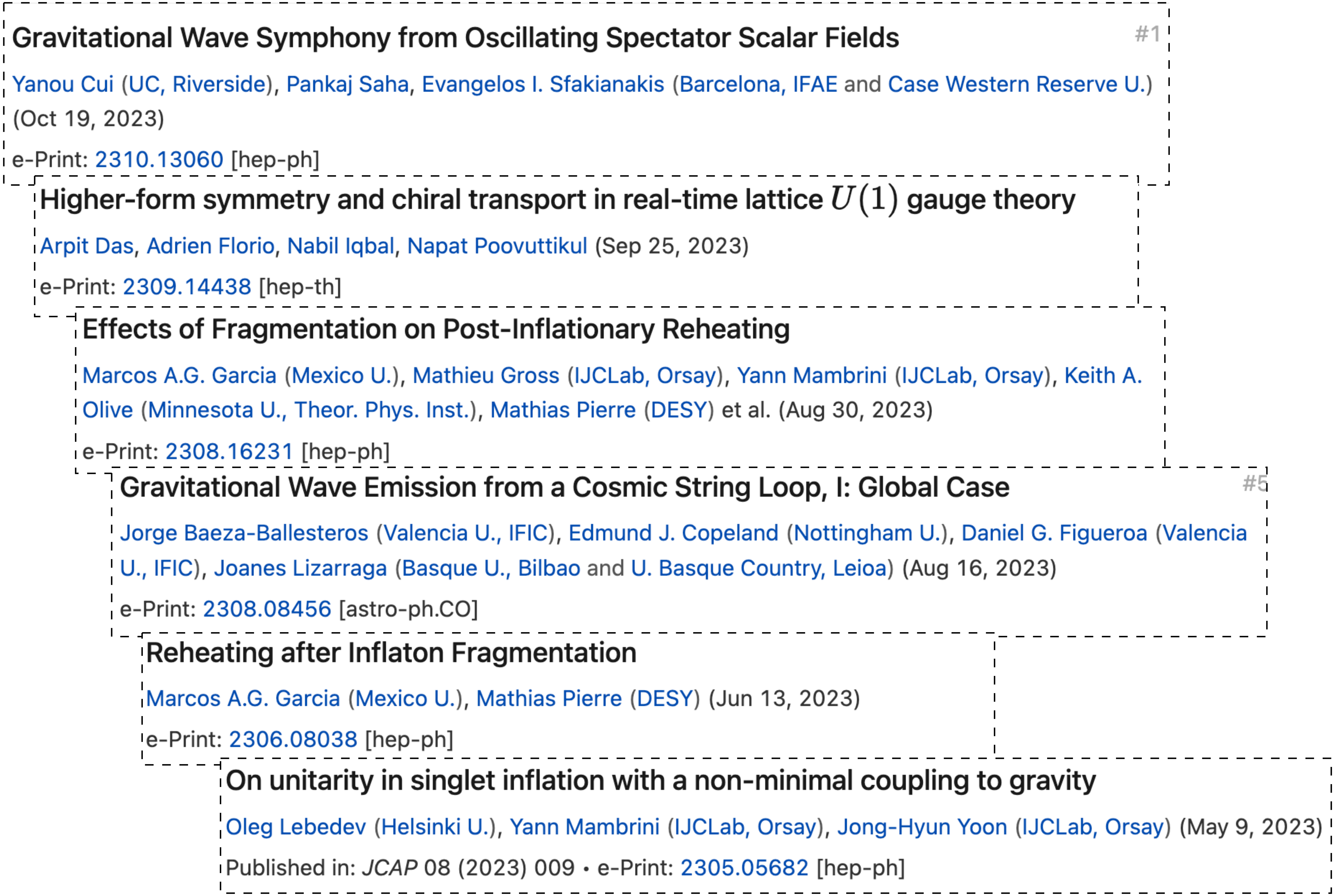
Gravitational Wave Emission from a Cosmic String Loop, I: Global Case #5

Jorge Baeza-Ballesteros (Valencia U., IFIC), Edmund J. Copeland (Nottingham U.), Daniel G. Figueroa (Valencia U., IFIC), Joanes Lizarraga (Basque U., Bilbao and U. Basque Country, Leioa) (Aug 16, 2023)

e-Print: 2308.08456 [astro-ph.CO]

Applications



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e-Print: 2308.08456 [astro-ph.CO]

Reheating after Inflaton Fragmentation

Marcos A.G. Garcia (Mexico U.), Mathias Pierre (DESY) (Jun 13, 2023)

e-Print: 2306.08038 [hep-ph]

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Oleg Lebedev (Helsinki U.), Yann Mambrini (IJCLab, Orsay), Jong-Hyun Yoon (IJCLab, Orsay) (May 9, 2023)

Published in: *JCAP* 08 (2023) 009 • e-Print: 2305.05682 [hep-ph]

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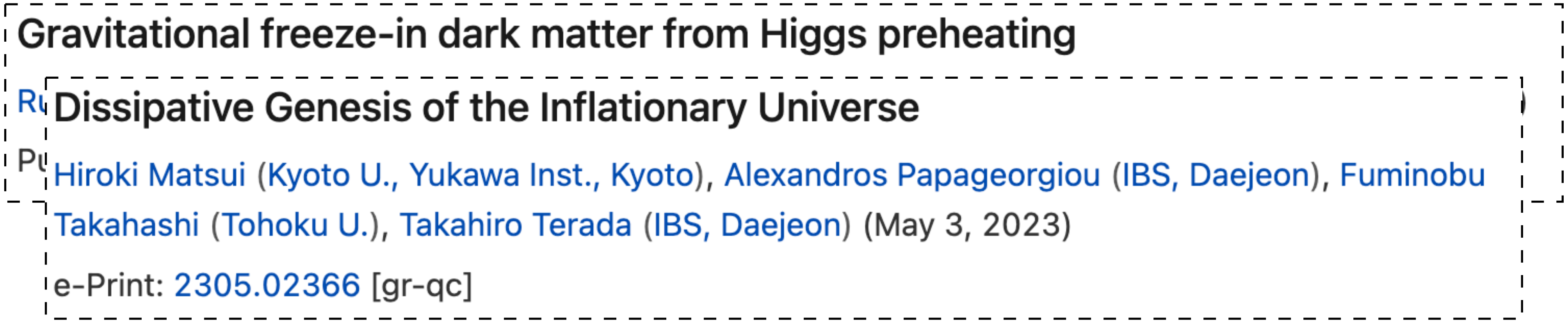
Gravitational freeze-in dark matter from Higgs preheating

Ruopeng Zhang (Chongqing U.), Zixuan Xu (Chongqing U.), Sibo Zheng (Chongqing U.) (May 4, 2023)

Published in: *JCAP* 07 (2023) 048, *JCAP* 07 (2023) 048 • e-Print: 2305.02568 [hep-ph]

Applications



Gravitational freeze-in dark matter from Higgs preheating

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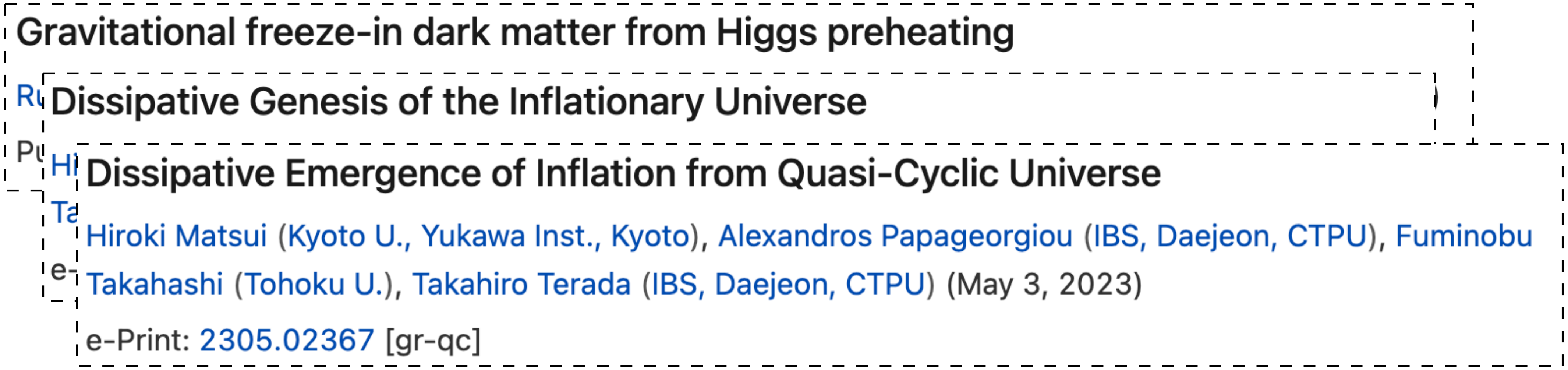
Dissipative Genesis of the Inflationary Universe

Hiroki Matsui (Kyoto U., Yukawa Inst., Kyoto), Alexandros Papageorgiou (IBS, Daejeon), Fuminobu Takahashi (Tohoku U.), Takahiro Terada (IBS, Daejeon) (May 3, 2023)

e-Print: 2305.02366 [gr-qc]

Applications



Gravitational freeze-in dark matter from Higgs preheating

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Dissipative Genesis of the Inflationary Universe

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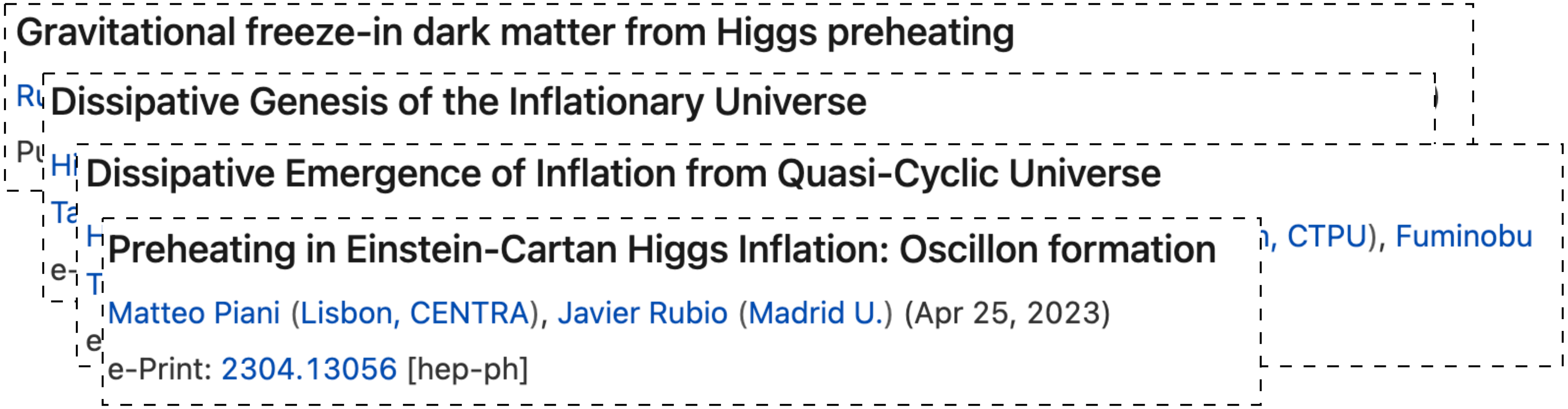
Dissipative Emergence of Inflation from Quasi-Cyclic Universe

Hiroki Matsui (Kyoto U., Yukawa Inst., Kyoto), Alexandros Papageorgiou (IBS, Daejeon, CTPU), Fuminobu Takahashi (Tohoku U.), Takahiro Terada (IBS, Daejeon, CTPU) (May 3, 2023)

e-Print: 2305.02367 [gr-qc]

Applications



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Dissipative Emergence of Inflation from Quasi-Cyclic Universe

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n, CTPU), Fuminobu

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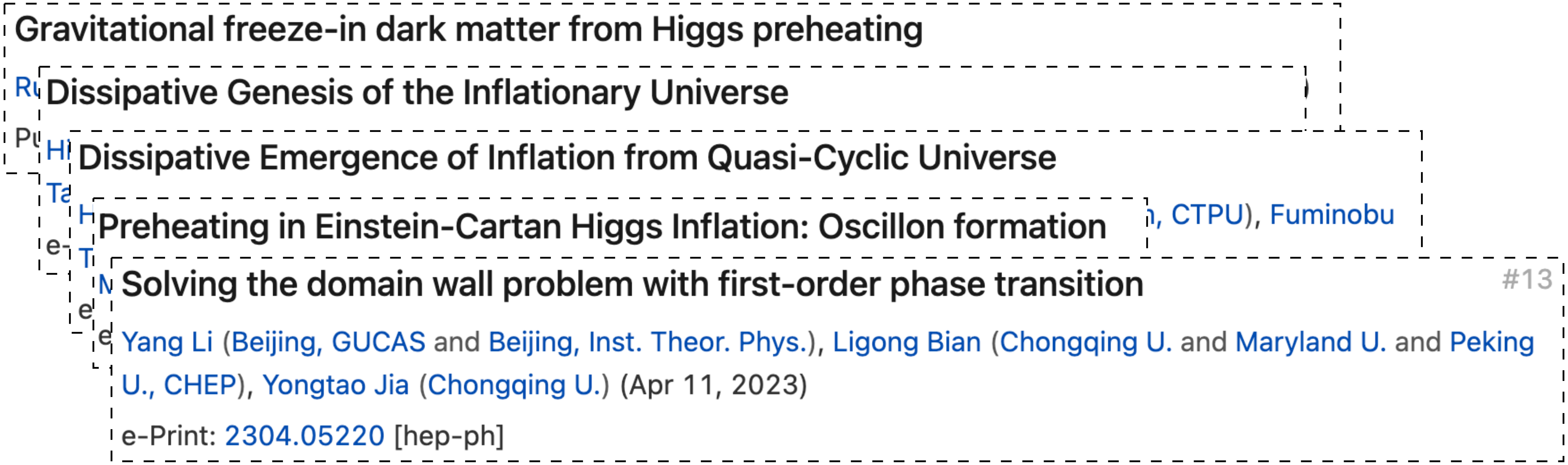
Preheating in Einstein-Cartan Higgs Inflation: Oscillon formation

Matteo Piani (Lisbon, CENTRA), Javier Rubio (Madrid U.) (Apr 25, 2023)

e-Print: 2304.13056 [hep-ph]

Applications



Gravitational freeze-in dark matter from Higgs preheating

Dissipative Genesis of the Inflationary Universe

Dissipative Emergence of Inflation from Quasi-Cyclic Universe

CTPU), Fuminobu

Preheating in Einstein-Cartan Higgs Inflation: Oscillon formation

Solving the domain wall problem with first-order phase transition

#13

Yang Li (Beijing, GUCAS and Beijing, Inst. Theor. Phys.), Ligong Bian (Chongqing U. and Maryland U. and Peking U., CHEP), Yongtao Jia (Chongqing U.) (Apr 11, 2023)

e-Print: 2304.05220 [hep-ph]

Applications

Gravitational freeze-in dark matter from Higgs preheating

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, CTPU), Fuminobu

Preheating in Einstein-Cartan Higgs Inflation: Oscillon formation

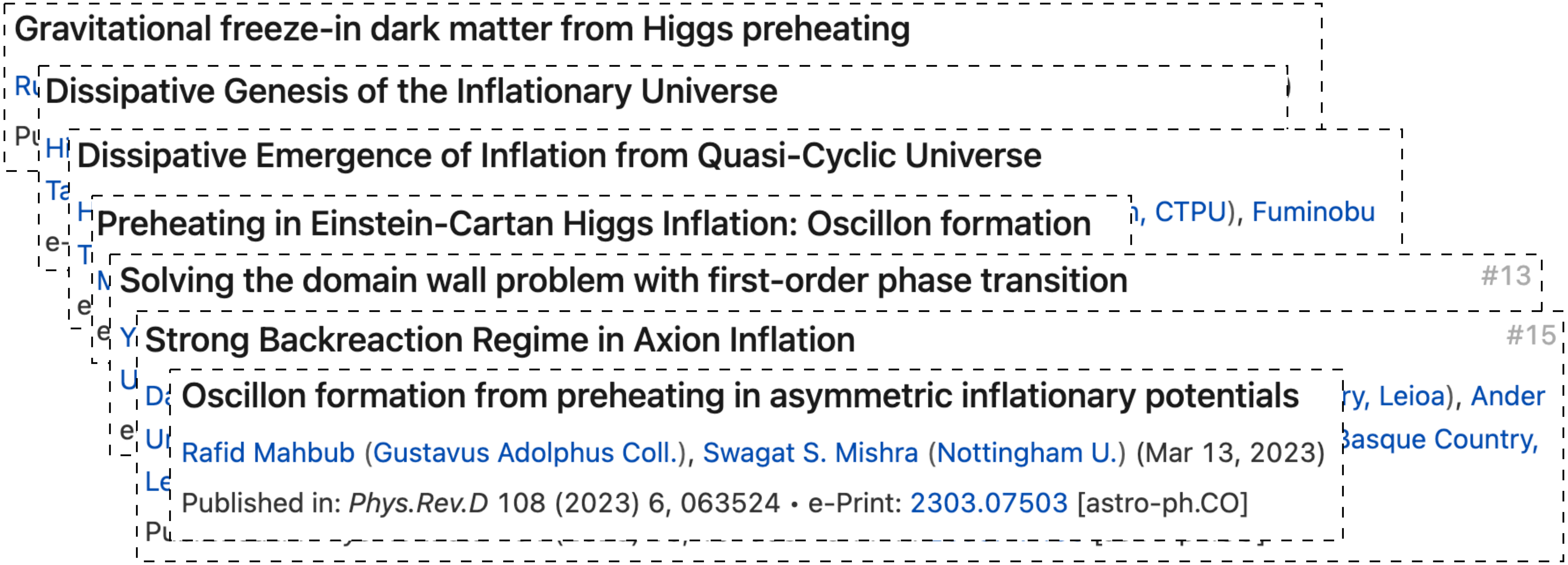
Solving the domain wall problem with first-order phase transition #13

Strong Backreaction Regime in Axion Inflation #15

Daniel G. Figueroa (Valencia U., IFIC), Joanes Lizarraga (Basque U., Bilbao and U. Basque Country, Leioa), Ander Urio (Basque U., Bilbao and U. Basque Country, Leioa), Jon Urrestilla (Basque U., Bilbao and U. Basque Country, Leioa) (Mar 30, 2023)

Published in: *Phys.Rev.Lett.* 131 (2023) 15, 151003 • e-Print: 2303.17436 [astro-ph.CO]

Applications



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ry, Leioa), Ander

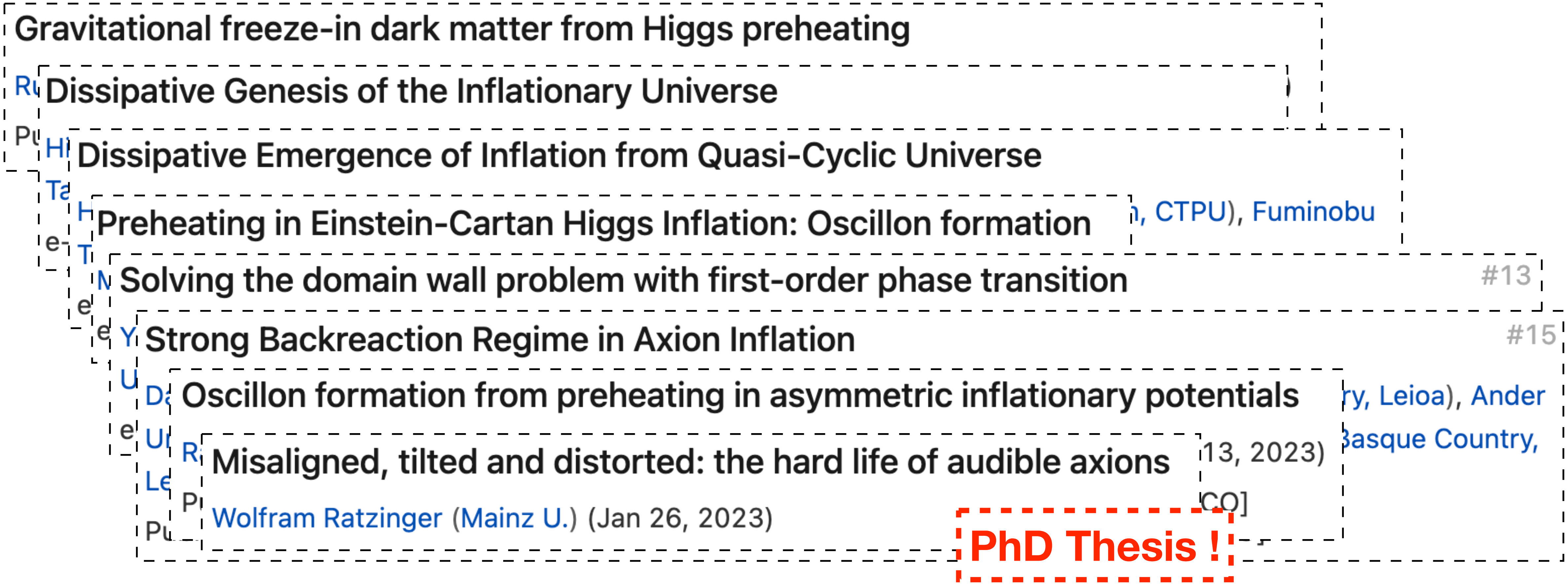
Basque Country,

Oscillon formation from preheating in asymmetric inflationary potentials

Rafid Mahbub (Gustavus Adolphus Coll.), Swagat S. Mishra (Nottingham U.) (Mar 13, 2023)

Published in: *Phys.Rev.D* 108 (2023) 6, 063524 • e-Print: 2303.07503 [astro-ph.CO]

Applications



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Oscillon formation from preheating in asymmetric inflationary potentials

ry, Leioa), Ander

asque Country,

Misaligned, tilted and distorted: the hard life of audible axions

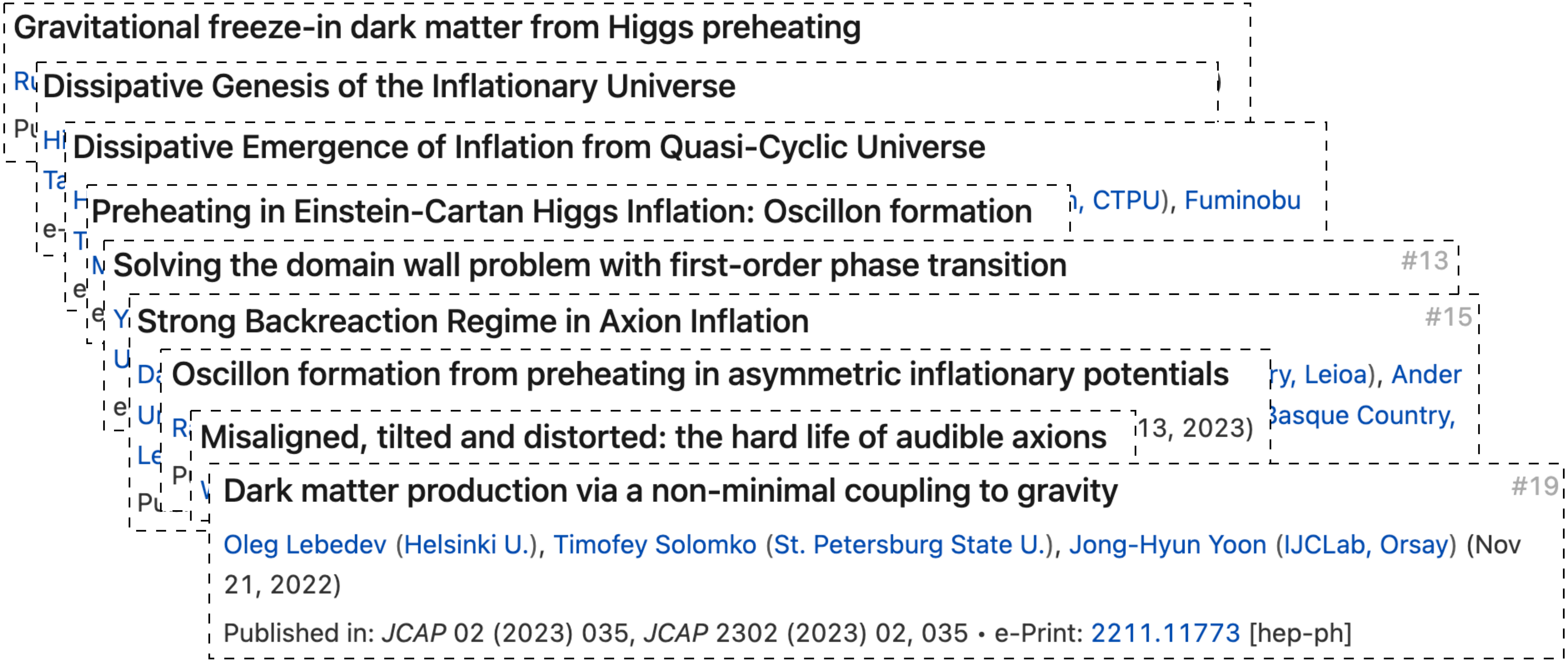
13, 2023)

CO]

Wolfram Ratzinger (Mainz U.) (Jan 26, 2023)

PhD Thesis !

Applications



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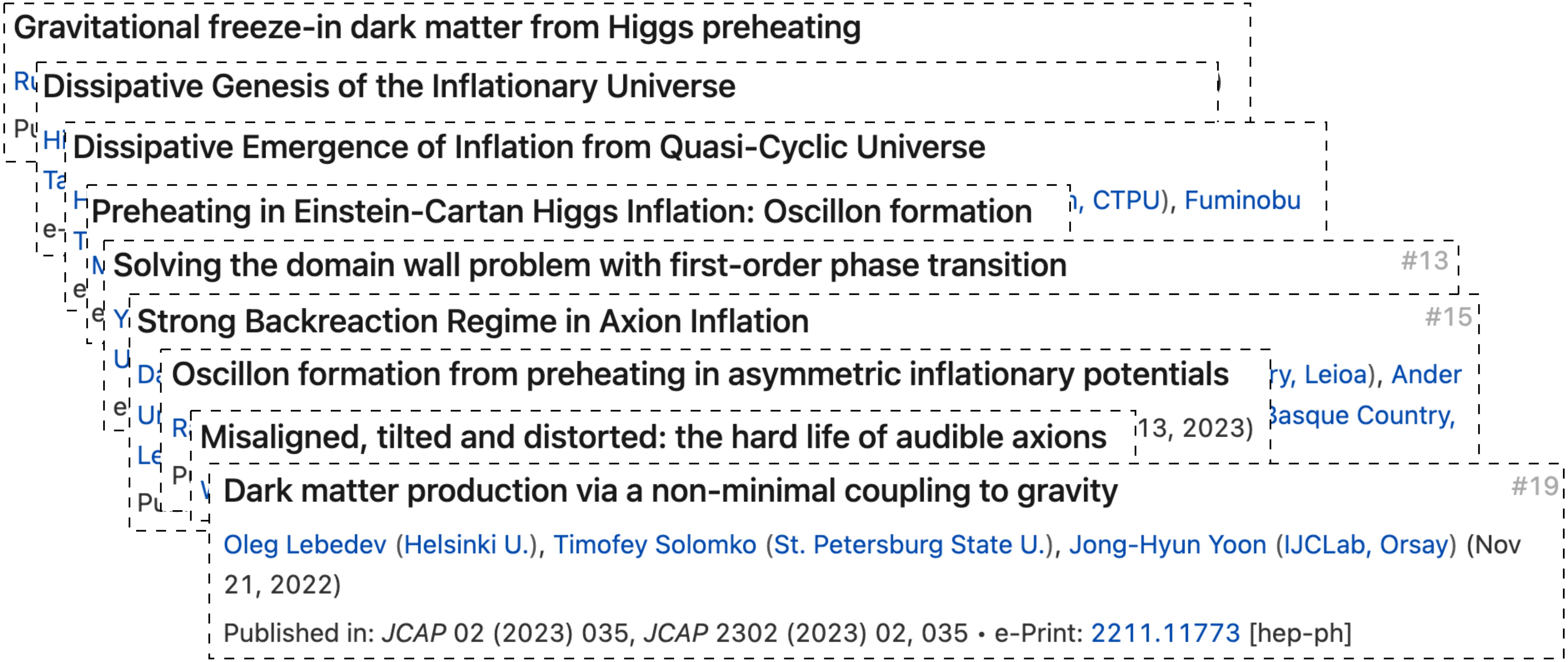
Dark matter production via a non-minimal coupling to gravity

#19

Oleg Lebedev (Helsinki U.), Timofey Solomko (St. Petersburg State U.), Jong-Hyun Yoon (IJCLab, Orsay) (Nov 21, 2022)

Published in: *JCAP* 02 (2023) 035, *JCAP* 2302 (2023) 02, 035 • e-Print: 2211.11773 [hep-ph]

Applications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Published in: *JCAP* 02 (2023) 035, *JCAP* 2302 (2023) 02, 035 • e-Print: 2211.11773 [hep-ph]

Just in 2023 ….

Program Variables

CosmoLattice – Program variables

- Equations solved in (dimensionless) ***program variables*:**

Choose:

$$\{\alpha, \omega_*, f_*\}$$

→

$$d\tilde{\eta} \equiv a^{-\alpha}\omega_* dt$$
$$d\tilde{x}^i \equiv \omega_* dx^i$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \qquad \tilde{\varphi} = \frac{\varphi}{f_*} \qquad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \qquad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}$$

Gauge fields

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Gauge fields

How do I choose them ?

CosmoLattice – Program variables

- Equations solved in (dimensionless) ***program variables***:

Choose:

$$\{\alpha, \omega_*, f_*\} \rightarrow \boxed{\begin{array}{c} d\tilde{\eta} \equiv a^{-\alpha}\omega_* dt \\ d\tilde{x}^i \equiv \omega_* dx^i \end{array}}$$

Space and time

$$\boxed{\tilde{\phi} = \frac{\phi}{f_*} \qquad \tilde{\varphi} = \frac{\varphi}{f_*} \qquad \tilde{\Phi} = \frac{\Phi}{f_*}}$$

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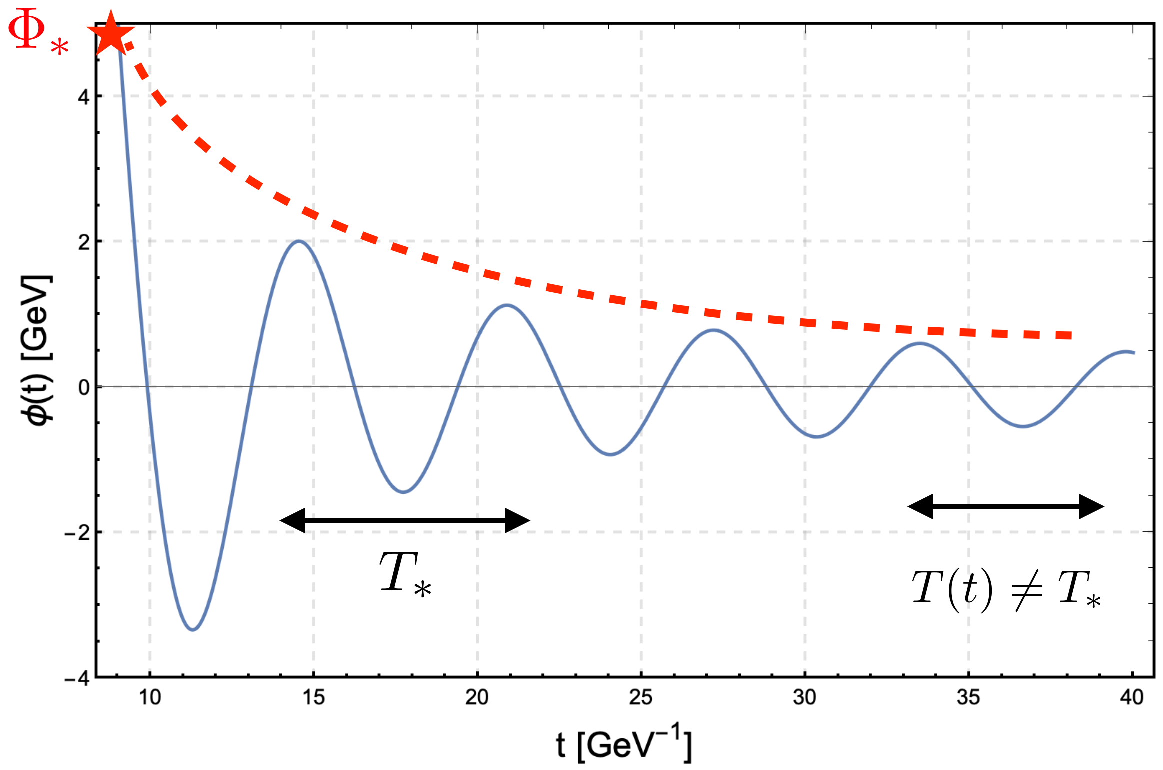
$$\boxed{\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \qquad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}}$$

Gauge fields

Example: $\phi(t) \simeq \Phi_* \times f_{\rm osc}(t)$



CosmoLattice – Program variables

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Choose:

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Space and time

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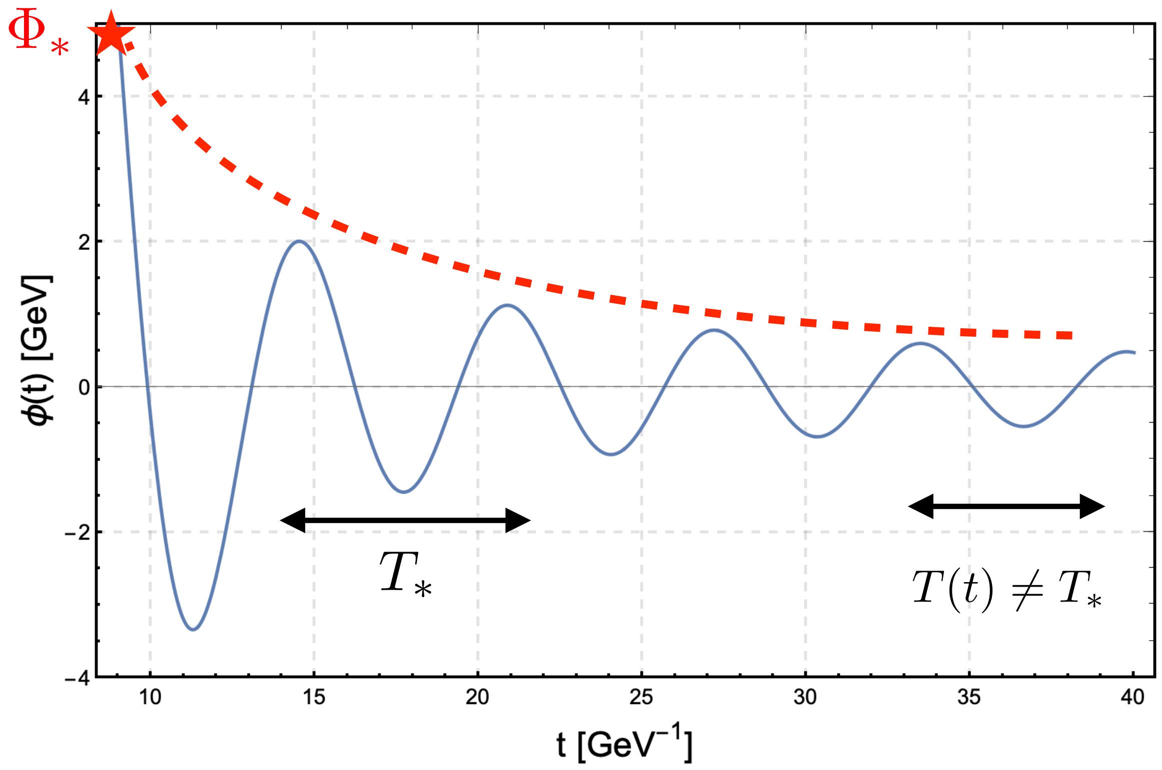

$$\begin{cases} f_* = \Phi_* \\ \omega_* = 1/T_* \\ \alpha \longrightarrow \text{Make period constant in } \tilde{\eta} \end{cases}$$

CosmoLattice – Program variables

- ➤ Equations solved in (dimensionless) ***program variables*:**

Choose:

$$\{\alpha, \omega_*, f_*\} \longrightarrow \boxed{\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha}\omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}}$$

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Scalar fields

$$\boxed{\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \qquad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}}$$

Gauge fields

- ➤ Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

$$\boxed{\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|)} \longrightarrow \frac{\partial\tilde{V}}{\partial\tilde{\phi}}, \frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}, \frac{\partial\tilde{V}}{\partial|\tilde{\Phi}|}, \frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\varphi}|^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\Phi}|^2},$$

CosmoLattice – Program variables

- Equations solved in (dimensionless) ***program variables*:**

Choose:

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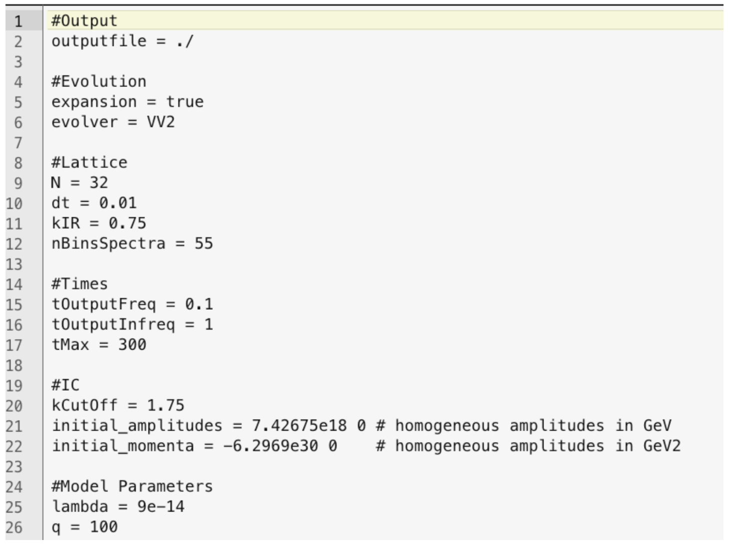
Gauge fields

- Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

$$\boxed{\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|)} \rightarrow \frac{\partial\tilde{V}}{\partial\tilde{\phi}}, \frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}, \frac{\partial\tilde{V}}{\partial|\tilde{\Phi}|}, \frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\varphi}|^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\Phi}|^2},$$

- **Parameters** passed via **one file** (input.txt)

(no need to re-compile !)



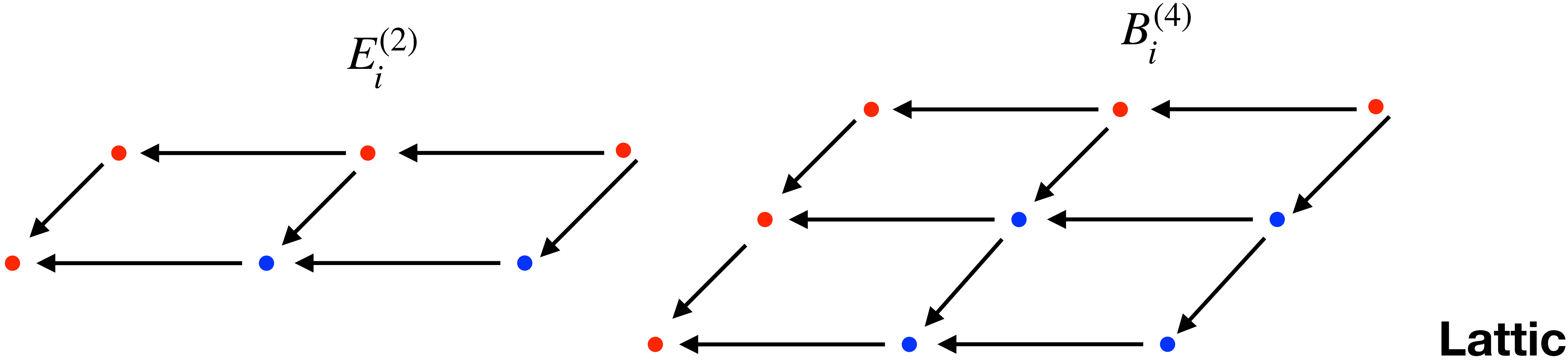
```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e-14
q = 100
```

Axion-inflation extra stuff

Axion-Inflation

PROBLEM: **PNG, GW and PBH** ⟶ **Analytical approximations !**

Let's "latticize" the system of EOM !



Lattice gauge techniques

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2 \phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2} E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

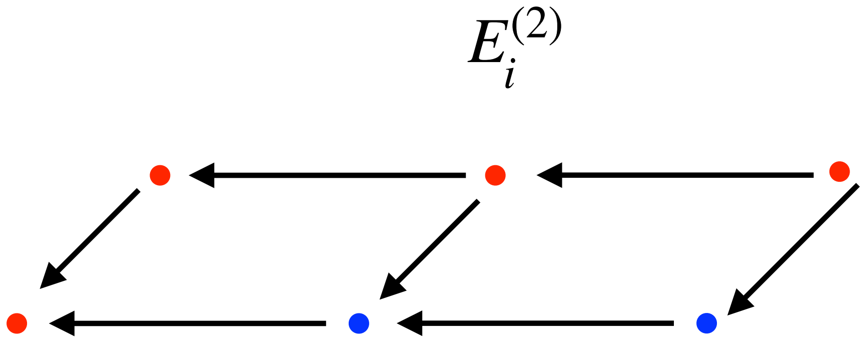
Gauge Fld EoM

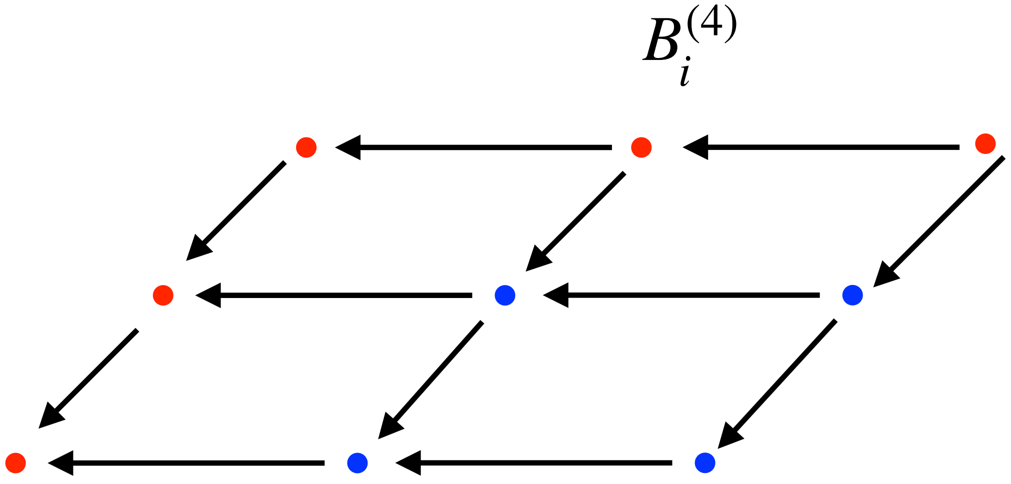
$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}} E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i} B^{(4)}_{i,+i}\right)$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i} \;, \quad \text{(Gauss Law)}$$

DGF, Shaposhnikov 2017
Canivete, DGF 2018





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Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

1. Lattice Gauge Inv: $A_\mu \longrightarrow A_\mu + \Delta_\mu^+\alpha$

2. Cont. Limit to $\mathcal{O}(dx^2)$

3. Lattice Bianchi Identities: $\Delta_i^-(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}) = 0\,, \ldots$

4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L = \Delta_\mu^+ K^\mu$ (**CS current**)
   $[\,F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu\,]$

Exact Shift Sym. on the lattice !

LATTICE FORMULATION of $\phi F\tilde{F}$

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Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i\frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) + \frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i\Delta_i^-E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

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$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \underline{\frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)}$$

Gauge Fld EoM

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$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \underline{\frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B}},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\underline{\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)}$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \underline{\frac{1}{a\Lambda}\pi_\phi\vec{B}} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$
$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\underline{\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i\Delta_i^-E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
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$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\underline{\vec{\nabla}\phi\times\vec{E}}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)\,, \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\,, \quad \text{(Gauss Law)}
\end{aligned}$$

Expansion

$$\begin{aligned}
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 &= \frac{a^2}{3m_{\rm pl}^2}\rho_L\,, \\
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}
\end{aligned}$$

LATTICE FORMULATION of $\phi F \tilde{F}$

Lattice Formulation

EoM

$$\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad +\frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right) \\
&\quad +\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}$$

Expansion

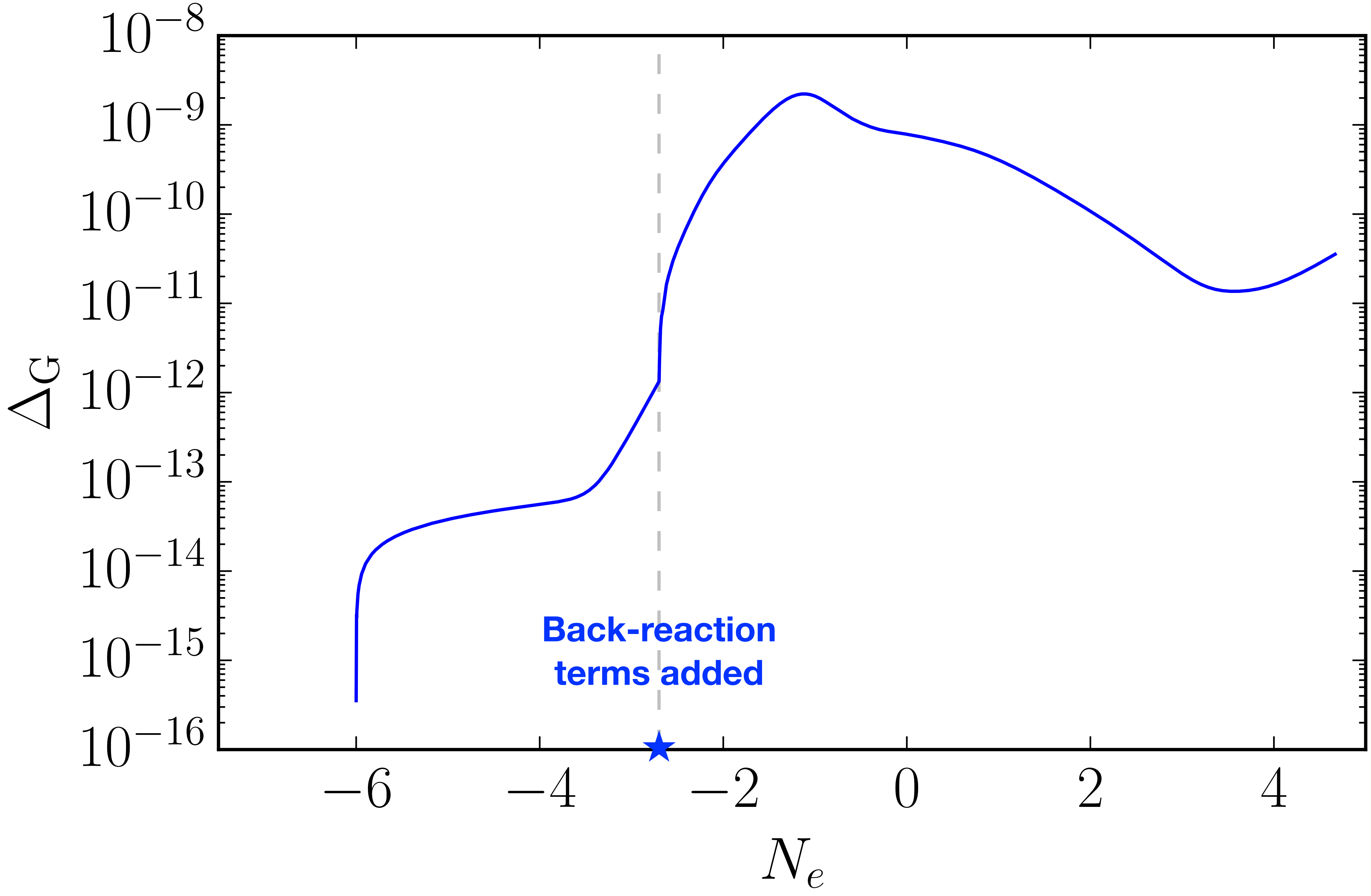
$$\begin{aligned}
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 &= \frac{a^2}{3m_{\rm pl}^2}\rho_L, \\
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}
\end{aligned}$$

$$\begin{aligned}
\rho_L &= \bar{H}^{\rm kin} + \frac{1}{a^2}\frac{1}{2}(\bar{H}^{\rm grad}_{-\hat{0}/2} + \bar{H}^{\rm grad}_{+\hat{0}/2}) + \frac{1}{2}(\bar{H}^{\rm pot}_{-\hat{0}/2} + \bar{H}^{\rm pot}_{+\hat{0}/2}) + \frac{1}{a^2}\frac{1}{2}(\bar{H}^E_{-\hat{0}/2} + \bar{H}^E_{+\hat{0}/2}) + \frac{1}{a^4}\bar{H}^B, \\
(\rho_L + 3p_L)_{+\hat{0}/2} &= 2(\bar{H}^{\rm kin} + \bar{H}^{\rm kin}_{+\hat{0}}) - 2\bar{H}^{\rm pot}_{+\hat{0}/2} + \frac{2}{a^2_{+\hat{0}/2}}\bar{H}^E + \frac{1}{a^4_{+\hat{0}/2}}(\bar{H}^B + \bar{H}^B_{+\hat{0}}),
\end{aligned}$$

$$\left(\begin{aligned}
&\bar{H}^{\rm kin} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{\pi_\phi^2}{2}\right\rangle \qquad \bar{H}^{\rm grad} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}\sum_i(\Delta_i^+\phi_{+\frac{\hat{0}}{2}})^2\right\rangle, \qquad \bar{H}^{\rm pot} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}m^2\phi^2_{+\frac{\hat{0}}{2}}\right\rangle \\
&\bar{H}^E = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_i\frac{1}{2}E^2_{i,+\frac{\hat{0}}{2}}\right\rangle \quad \bar{H}^B = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_{i,j}\frac{1}{4}(\Delta_i^+A_j - \Delta_j^+A_i)^2\right\rangle
\end{aligned}\right)$$

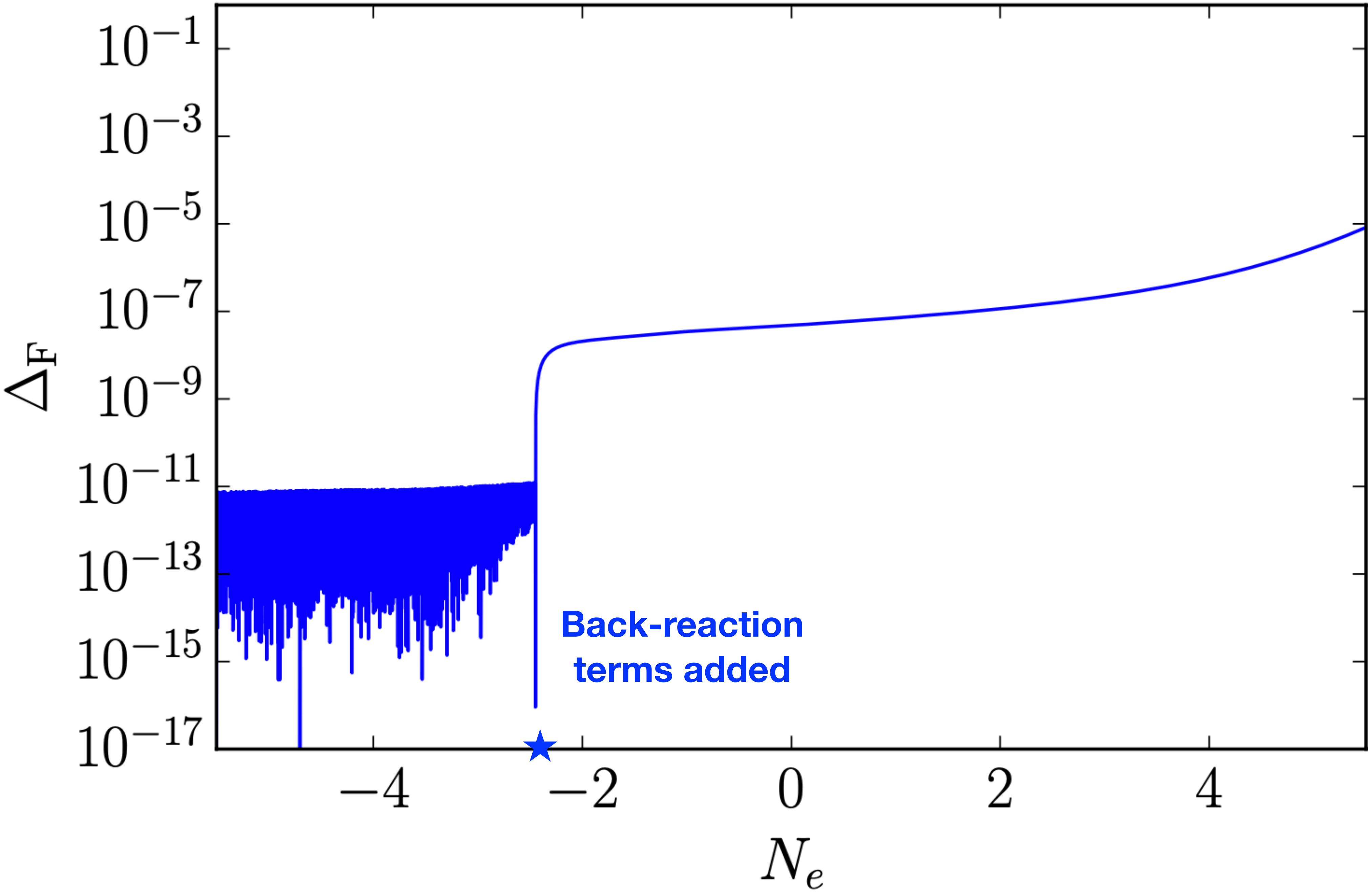
Gauss Constraint

$$\vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla}\phi \cdot \vec{B}$$



Hubble Constraint

$$\pi_a^2 = \frac{a^2}{3m_{pl}^2}(K_\phi + G_\phi + V + K_A + G_A) \quad ; \quad \pi_a \equiv \dot{a}$$



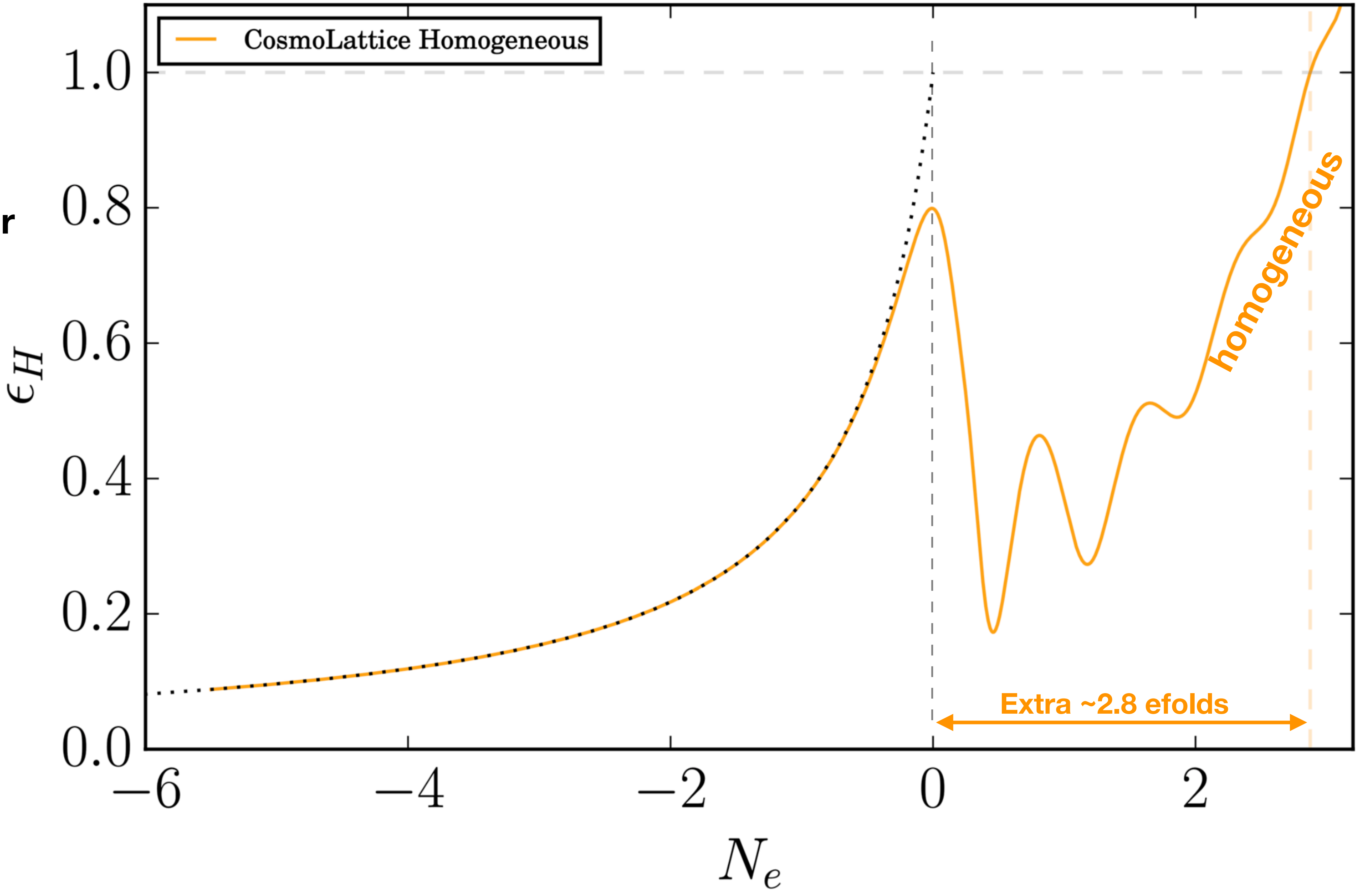
Back-reaction terms added

$$V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \boxed{\Lambda = \frac{m_p}{15}}$$

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \Lambda = \frac{m_p}{15}\right)$

Linear (········) **Non-Linear (——)**

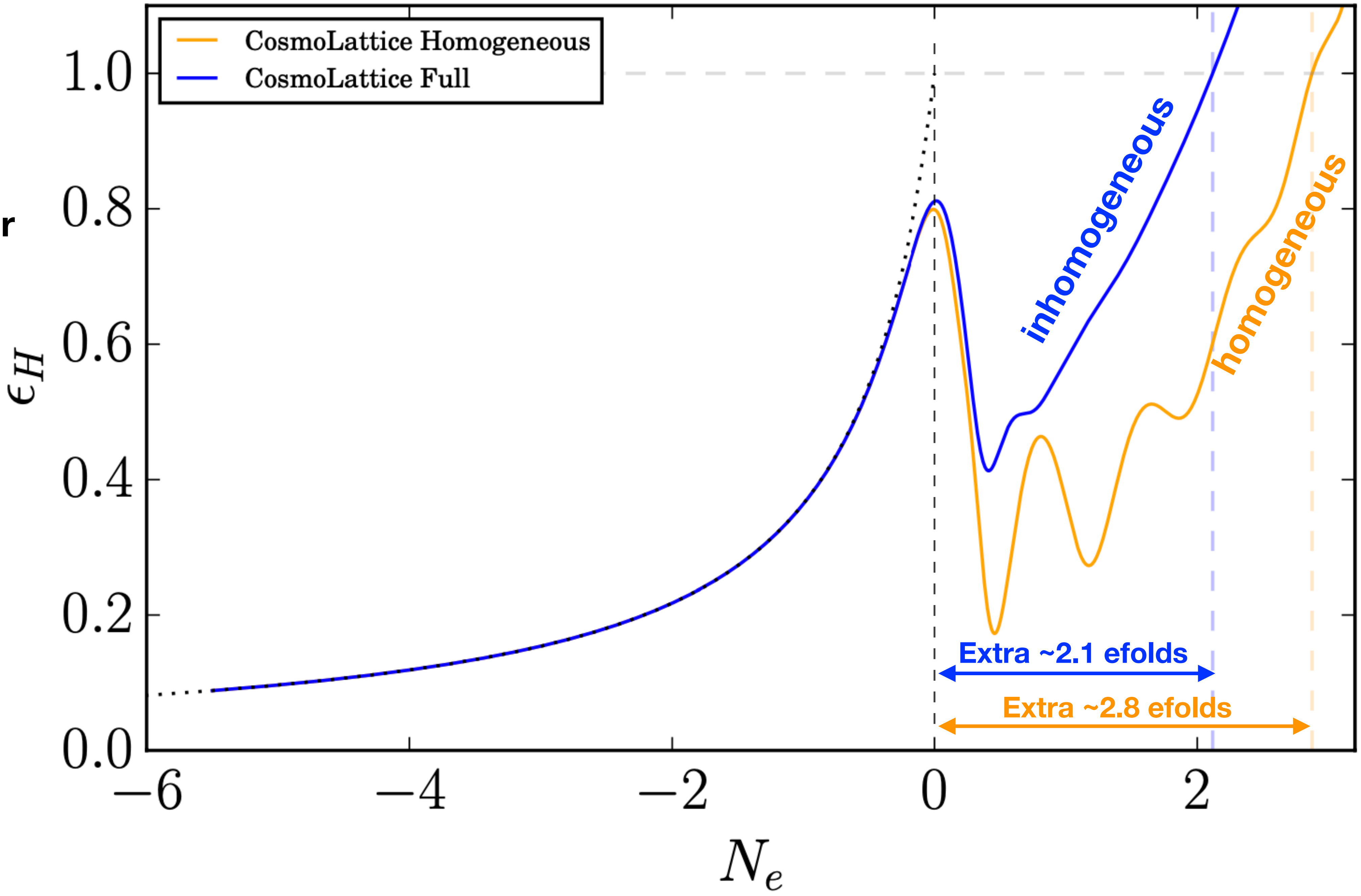
Epsilon Parameter



Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda=\frac{m_p}{15}\right)$

Linear (……..)   **Non-Linear (——)**

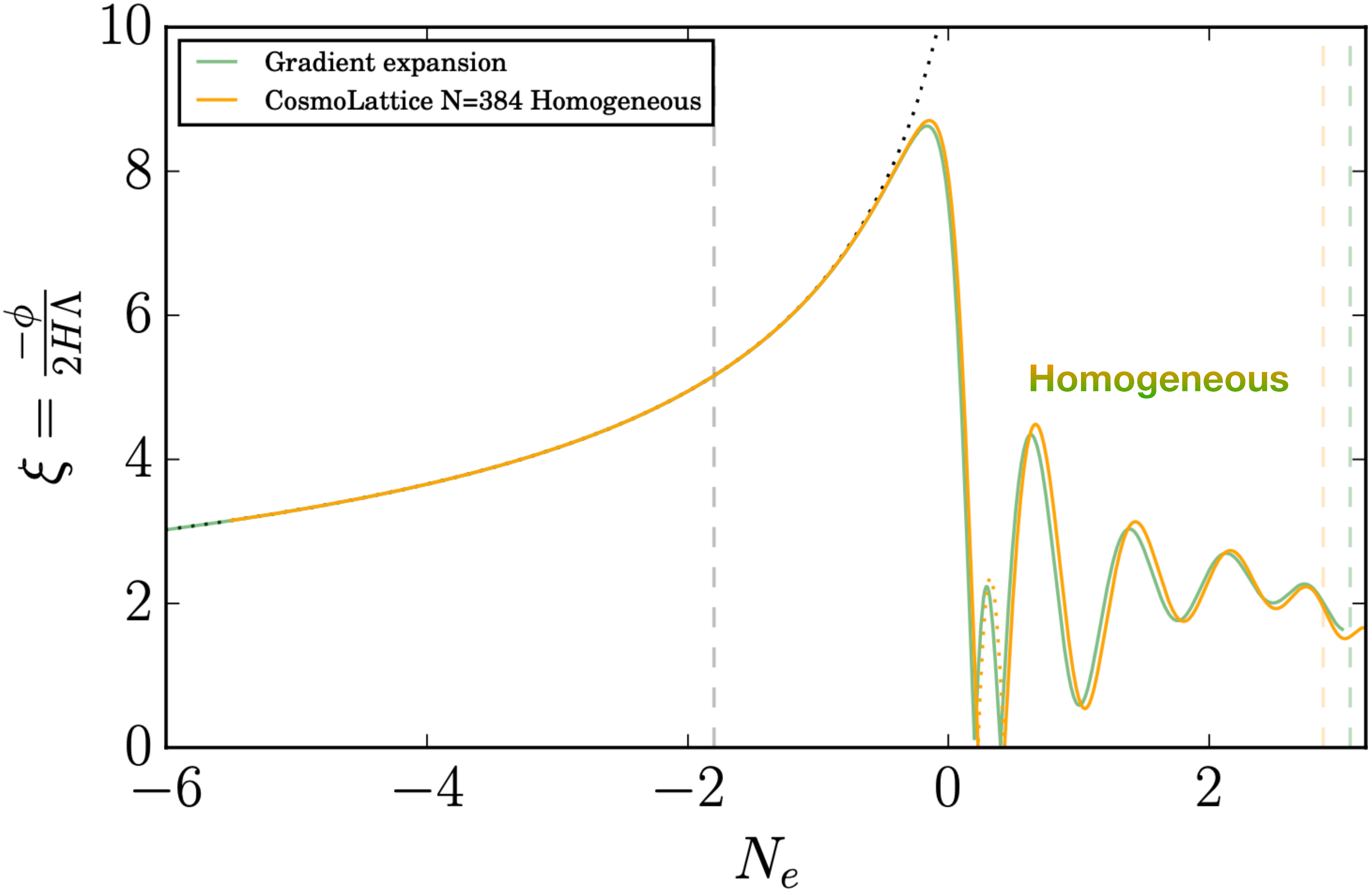
Epsilon Parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{15}$)

Homogeneous (🟧🟩)

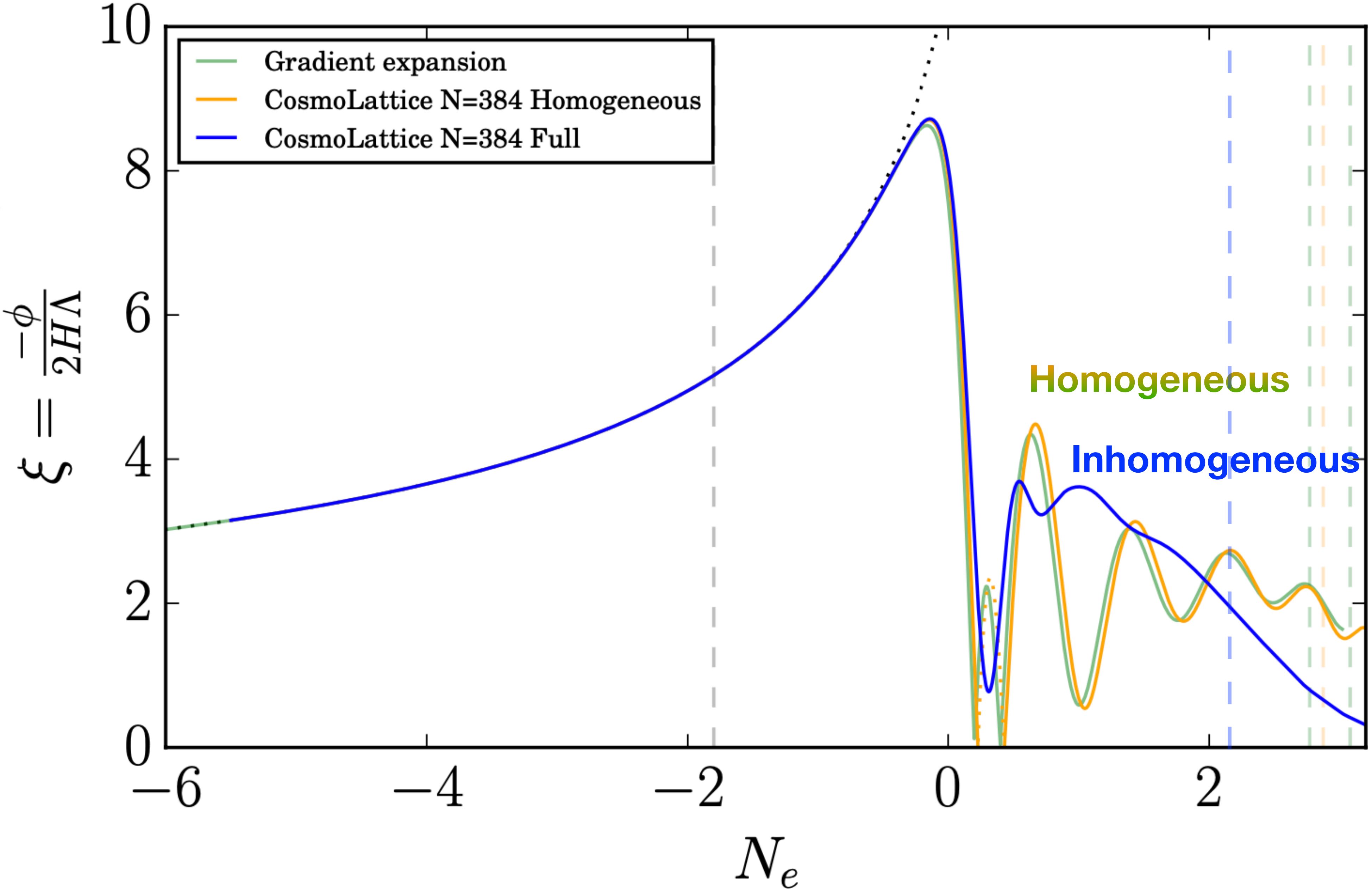
Xi parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{15}$)

Homogeneous (━━) **In-Homogeneous** (━━)

Xi parameter



Example III

Non-minimally coupled Scalar fields in the Jordan Frame

with

A. Florio, T. Opferkuch and B. Stefanek

SciPost, accepted ; 2112.08388 [astro-ph.CO]

End

Non-minimally coupled Scalars

Set-up

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \mathcal{L}_\chi + \mathcal{L}_{\text{inf}} + \tfrac{1}{2}\xi_\phi\phi^2 R\right]$$

or $\tfrac{1}{2}\xi_\chi\chi^2 R$

- Inflaton ϕ

$$\mathcal{L}_{\text{inf}} = -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\text{inf}}(\phi)$$

- Spectator field χ

$$\mathcal{L}_\chi = -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi,\phi)$$

- Non minimal coupling to gravity ξ_ϕ or ξ_χ

- Stay in Jordan frame

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Set-up

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \mathcal{L}_\chi + \mathcal{L}_{\rm inf} + \cancel{\tfrac{1}{2}\xi_\phi\phi^2 R}\right]$$

or $\frac{1}{2}\xi_\chi\chi^2 R$ **Spectator fld non-min. Coupled**

- Inflaton ϕ

$$\mathcal{L}_{\rm inf} = -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\rm inf}(\phi)$$

- Spectator field χ

$$\mathcal{L}_\chi = -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi,\phi)$$

- Non minimal coupling to gravity ξ_ϕ or ξ_χ
- Stay in Jordan frame

End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

EoM

$$\nabla_\mu \nabla^\mu \chi + \frac{\partial V}{\partial \chi} + \xi_\chi \chi R = 0$$

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Non-minimally coupled Scalars

EoM

$$\nabla_\mu \nabla^\mu \chi + \frac{\partial V}{\partial \chi} + \xi_\chi \chi R = 0$$

$$R = F(\chi)\Big((1-6\xi)\,\langle \partial^\mu \chi \partial_\mu \chi \rangle$$
$$+4\left(\langle V \rangle - \tfrac{3\xi}{2}\langle \chi V_{,\chi} \rangle\right) - \langle \rho_m \rangle - 3\langle p_m \rangle \Big)$$

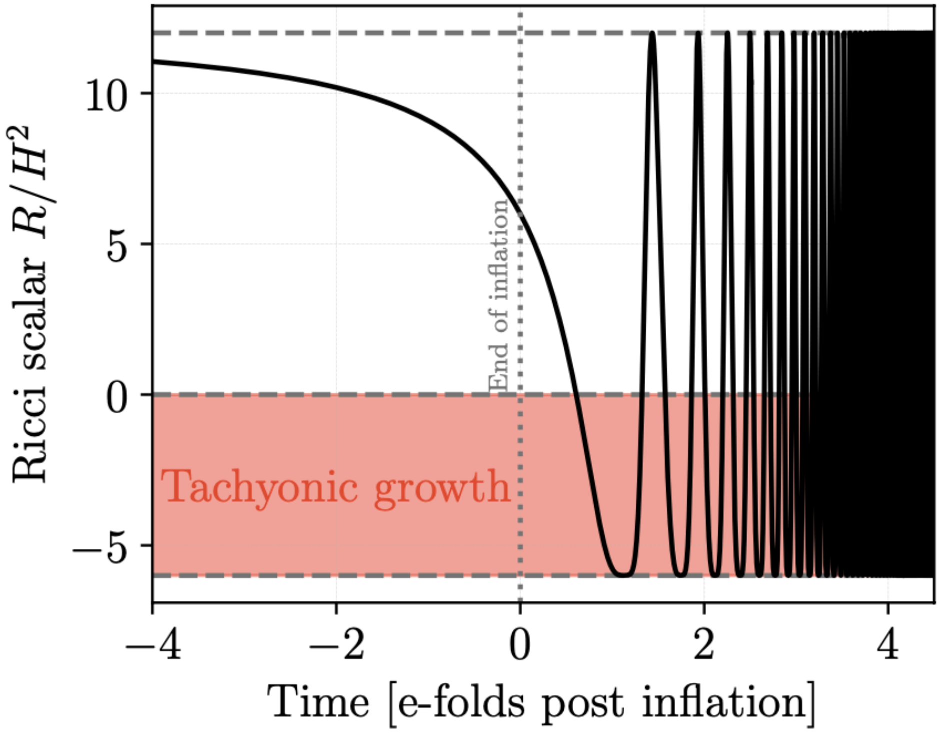
$$F(\chi) = \frac{1}{M_P^2\left[1+(6\xi-1)\xi\langle\chi^2\rangle/M_P^2\right]}$$

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

- Standard inflaton

$$V_{inf} \propto \tanh^4(\tilde{\phi})$$



Curvature Oscillates !

(sourced by Inflaton Oscillations)

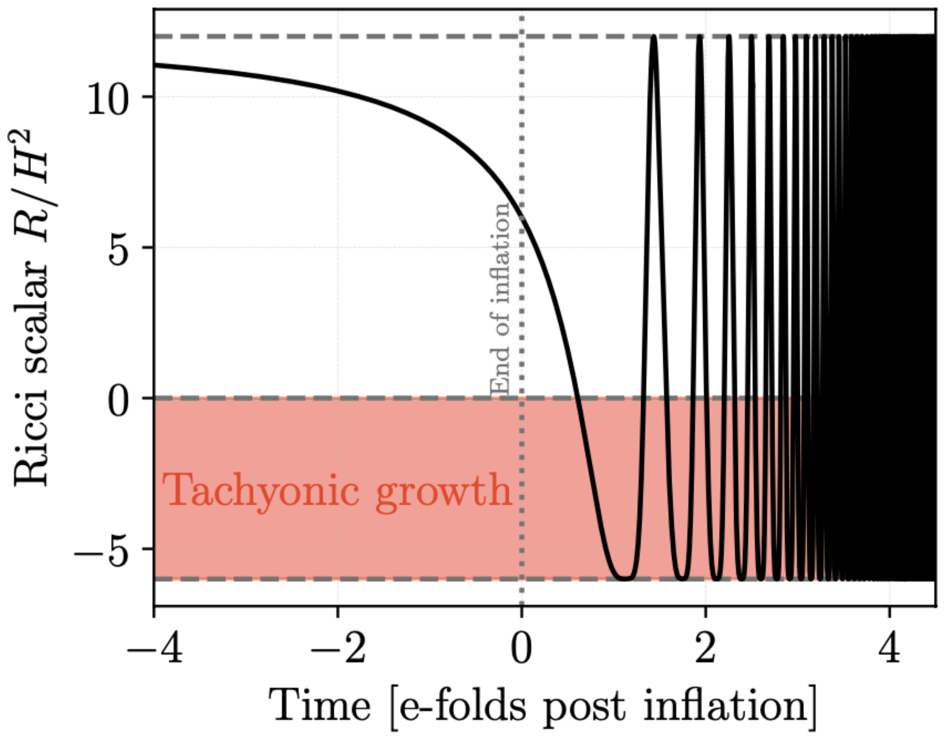
End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

- Standard inflaton

$V_{inf} \propto \tanh^4(\tilde{\phi})$



Geometric Preheating

[Basset & Liberati '99]

$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$

The preheat field is excited exponentially

Curvature Oscillates !

(sourced by Inflaton Oscillations)

Transparency worked out with Adrien Florio

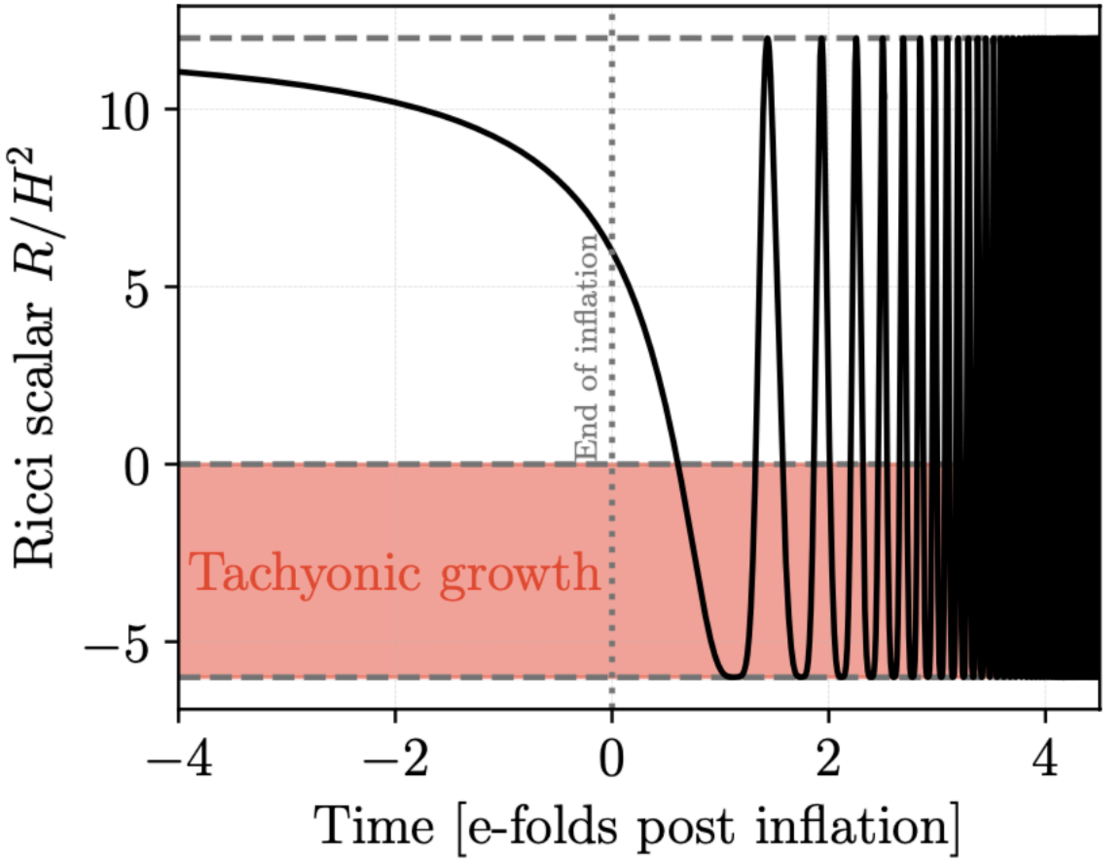
Non-minimally coupled Scalars

Geometric Preheating

[Basset & Liberati '99]

$$\xi\chi^2 R \quad ,\xi = 10{,}50{,}100$$

How is the preheat field excited?



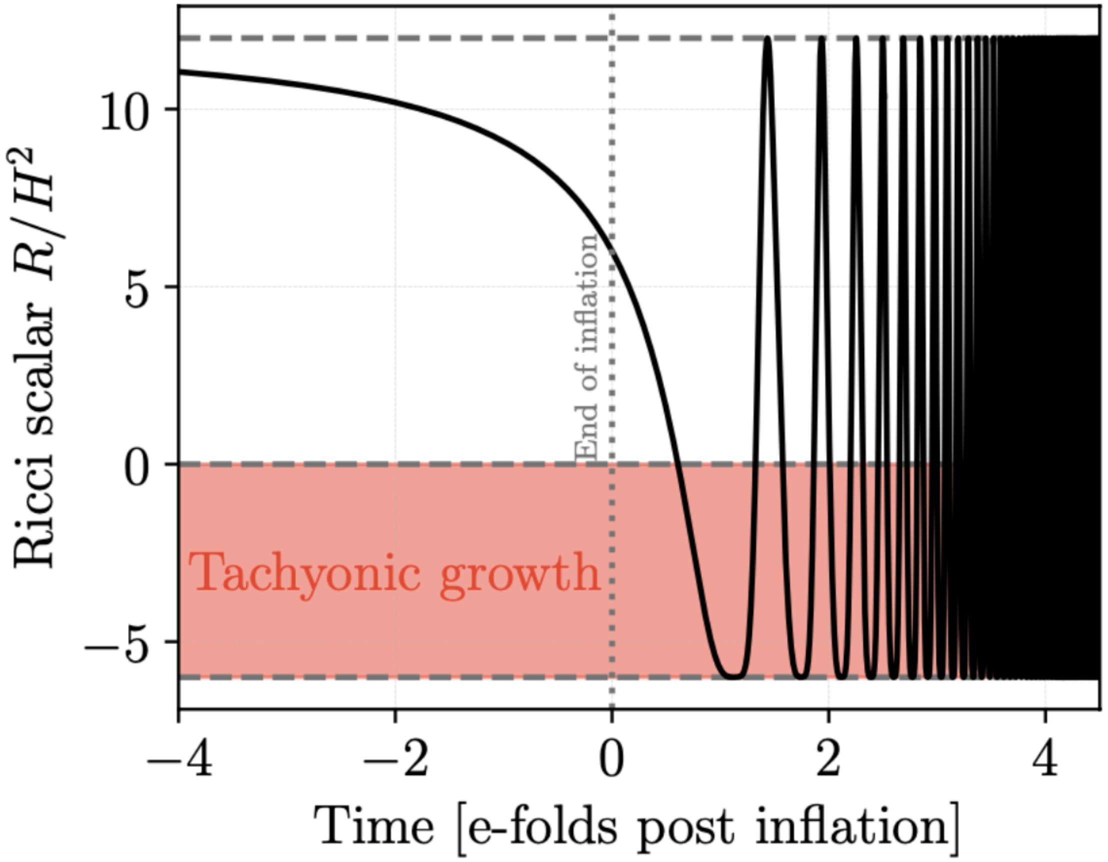
Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating

[Basset & Liberati ’99]

$$\xi \chi^2 R \quad , \xi = 10,50,100$$



How is the preheat field excited?

$$m_\chi^2 \propto \left(\frac{\partial^2 V}{\partial \chi^2} + \xi R \right)$$

Tachyonic term every time R < 0

End

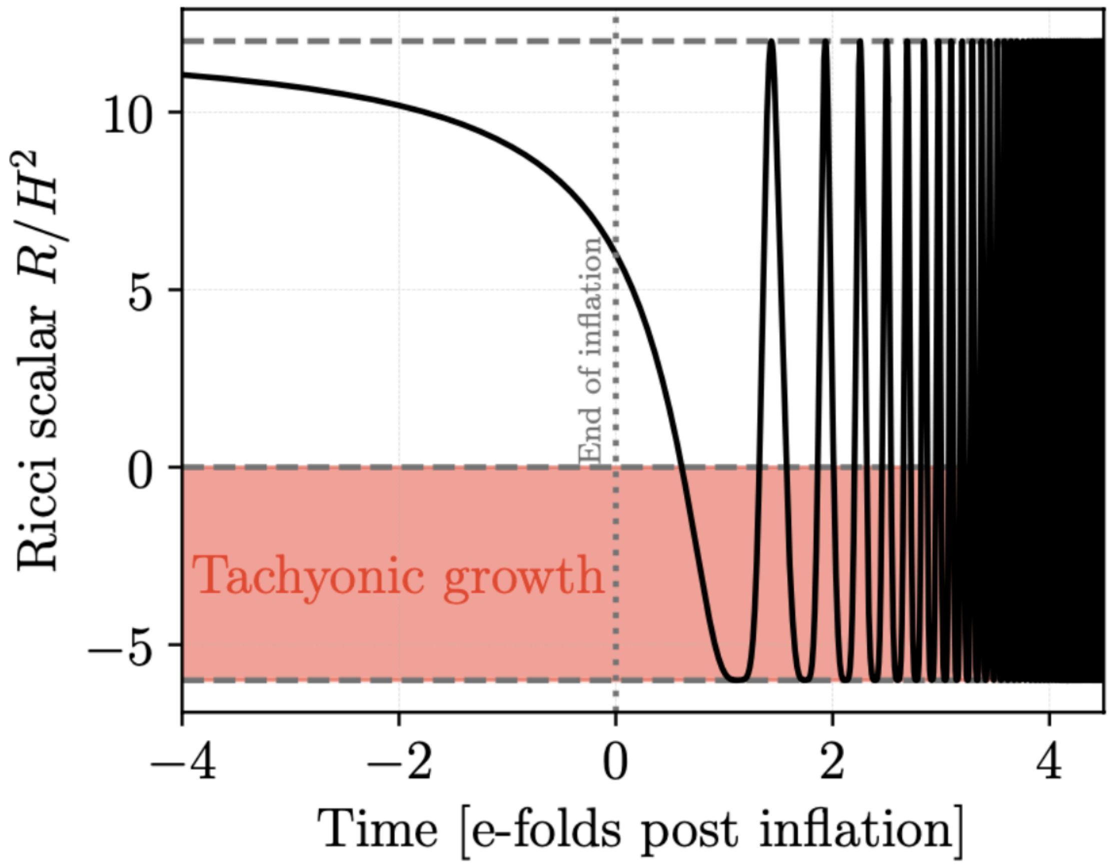
Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating

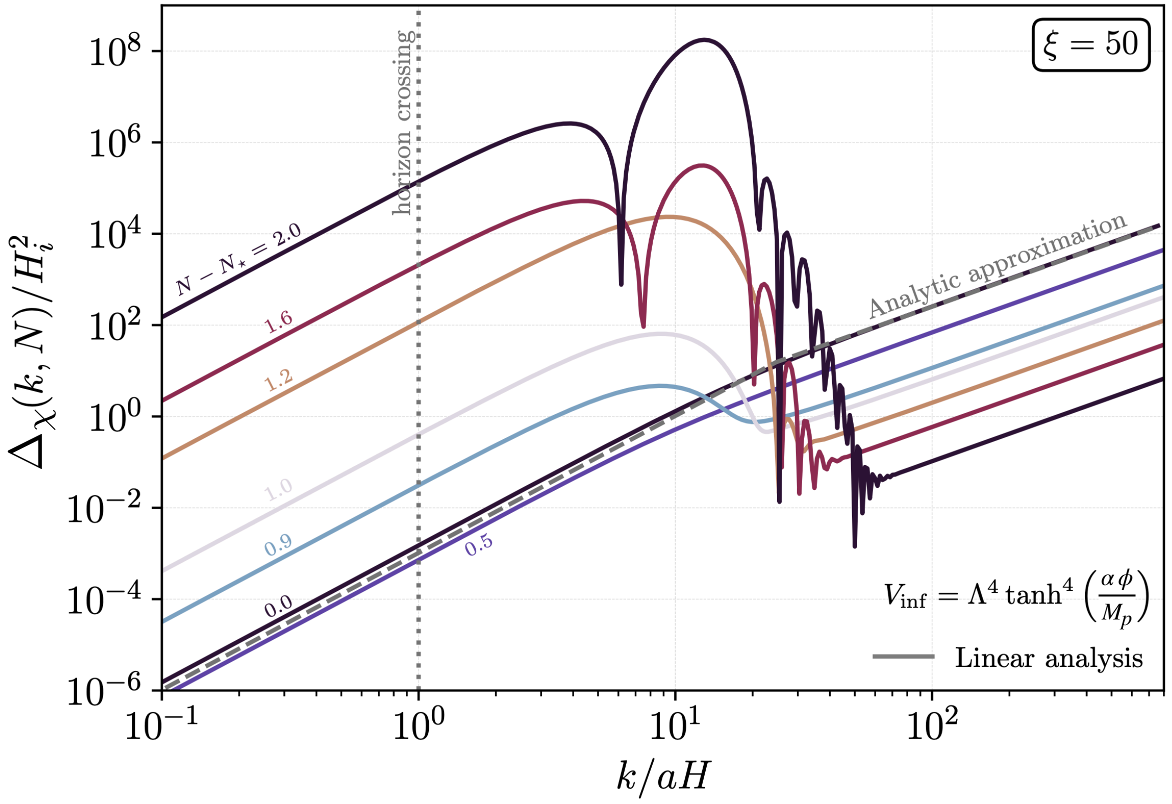
[Basset & Liberati '99]

$$\xi\chi^2 R \quad , \xi = 10{,}50{,}100$$



How is the preheat field excited?

Linear Regime



Transparency worked out with Adrien Florio

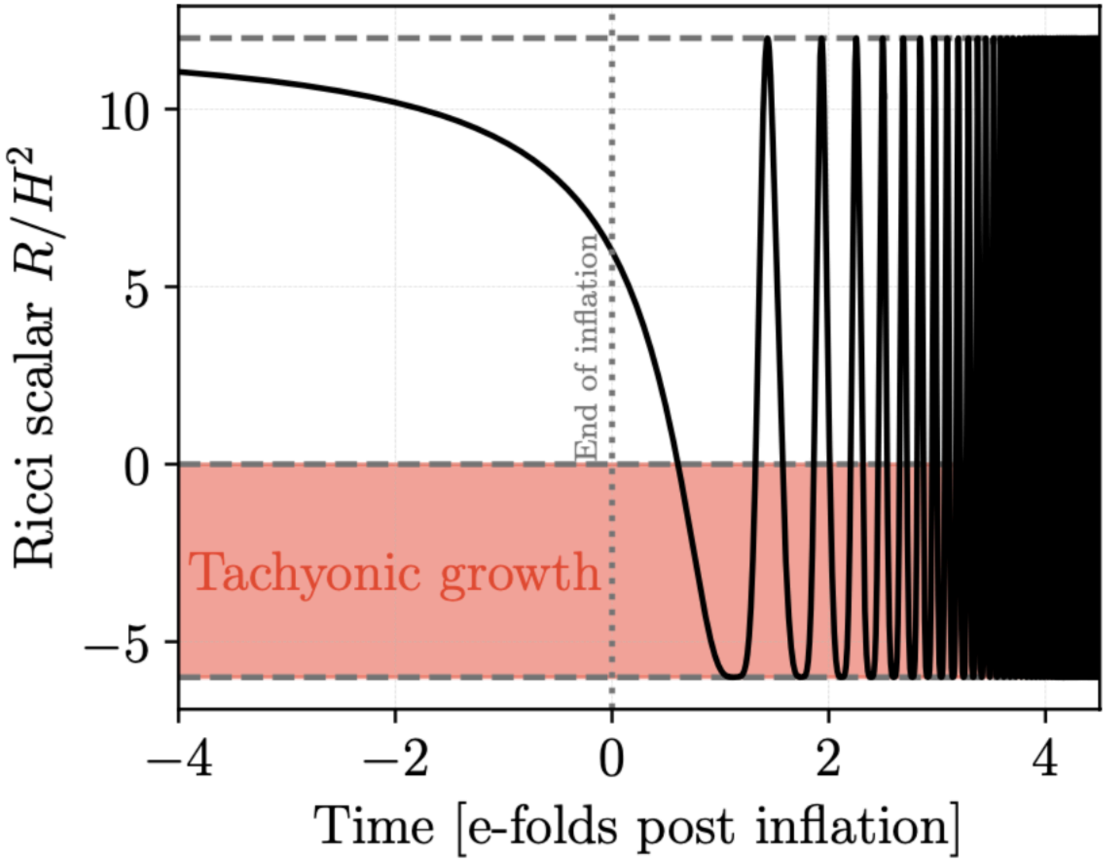
Non-minimally coupled Scalars

Geometric Preheating

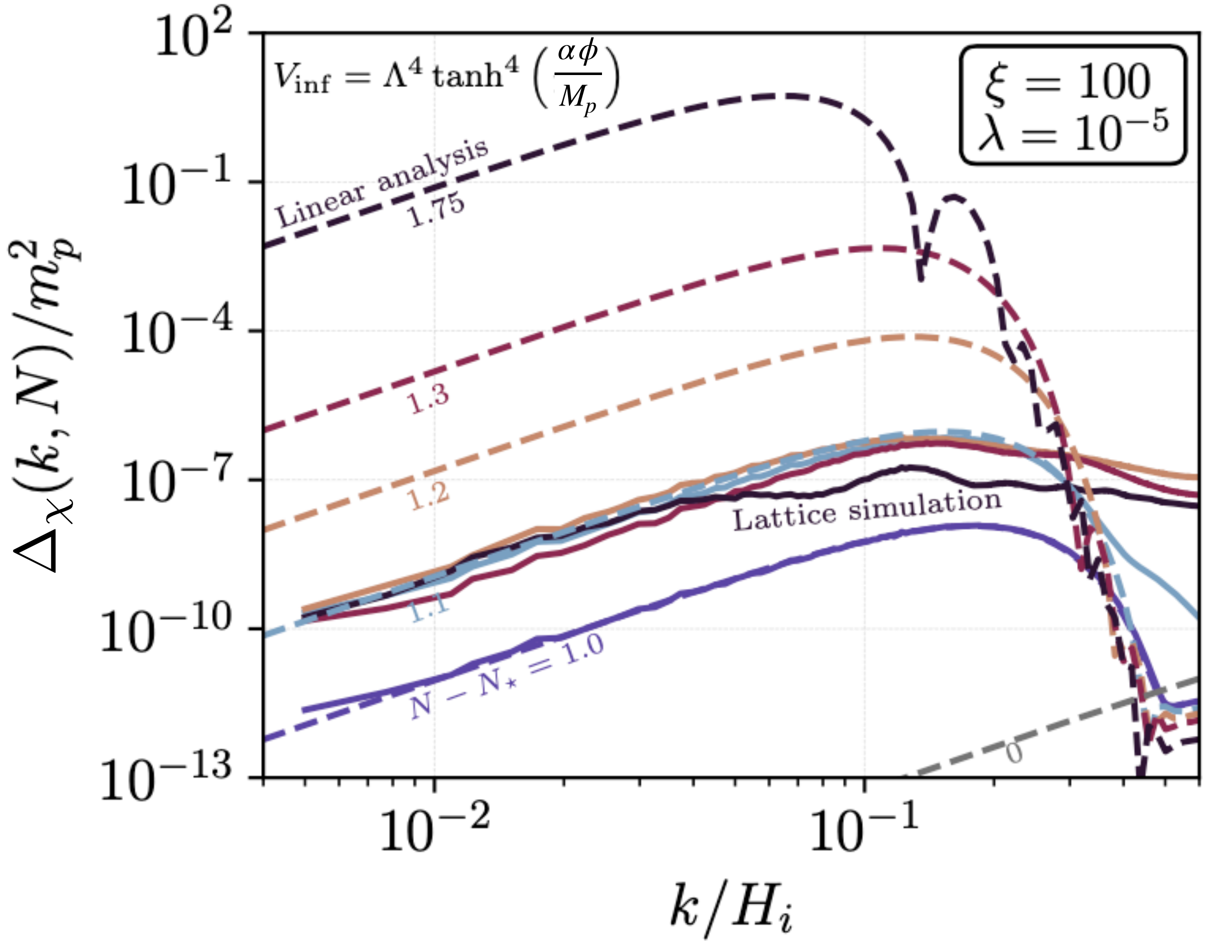
[Basset & Liberati ’99]

$$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$$

How is the preheat field excited?



Non-Linear Regime



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

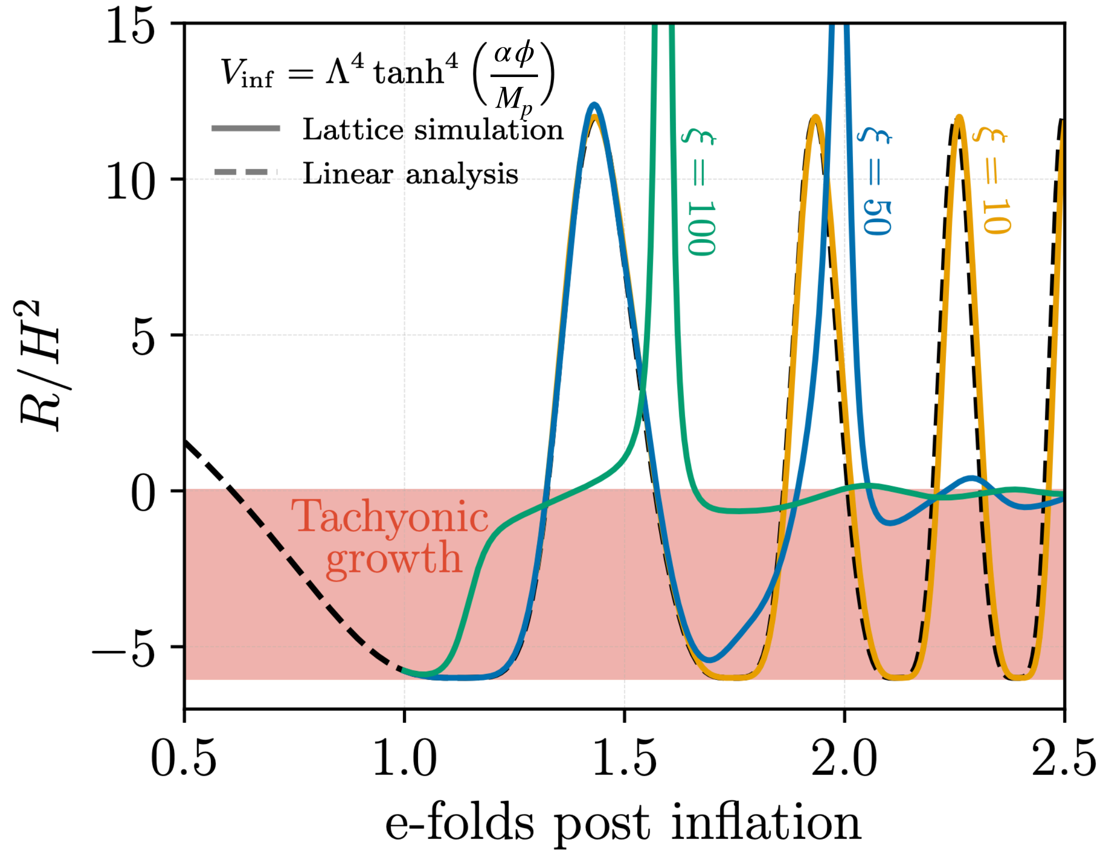
Geometric Preheating

[Basset & Liberati '99]

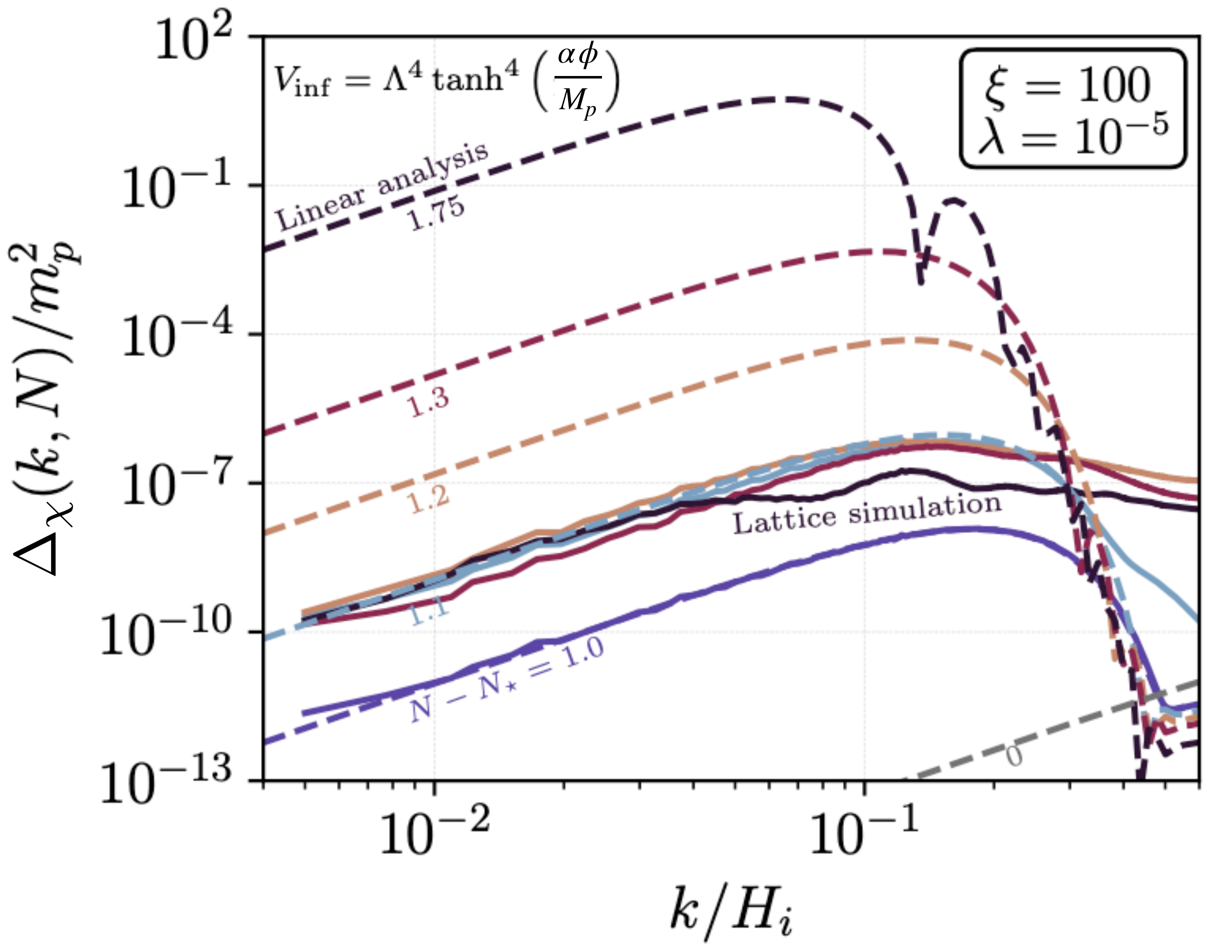
$$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$$

How is the preheat field excited?

Back-reaction



Non-Linear Regime



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

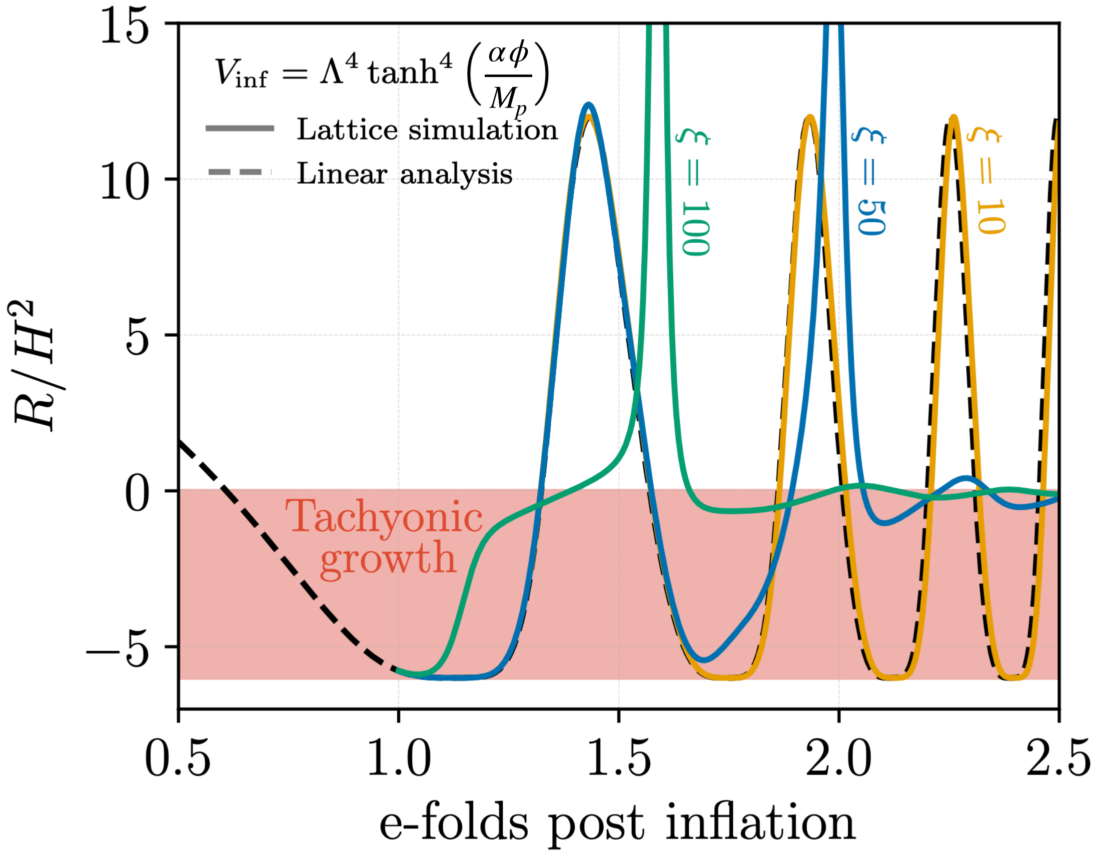
Geometric Preheating

[Basset & Liberati ’99]

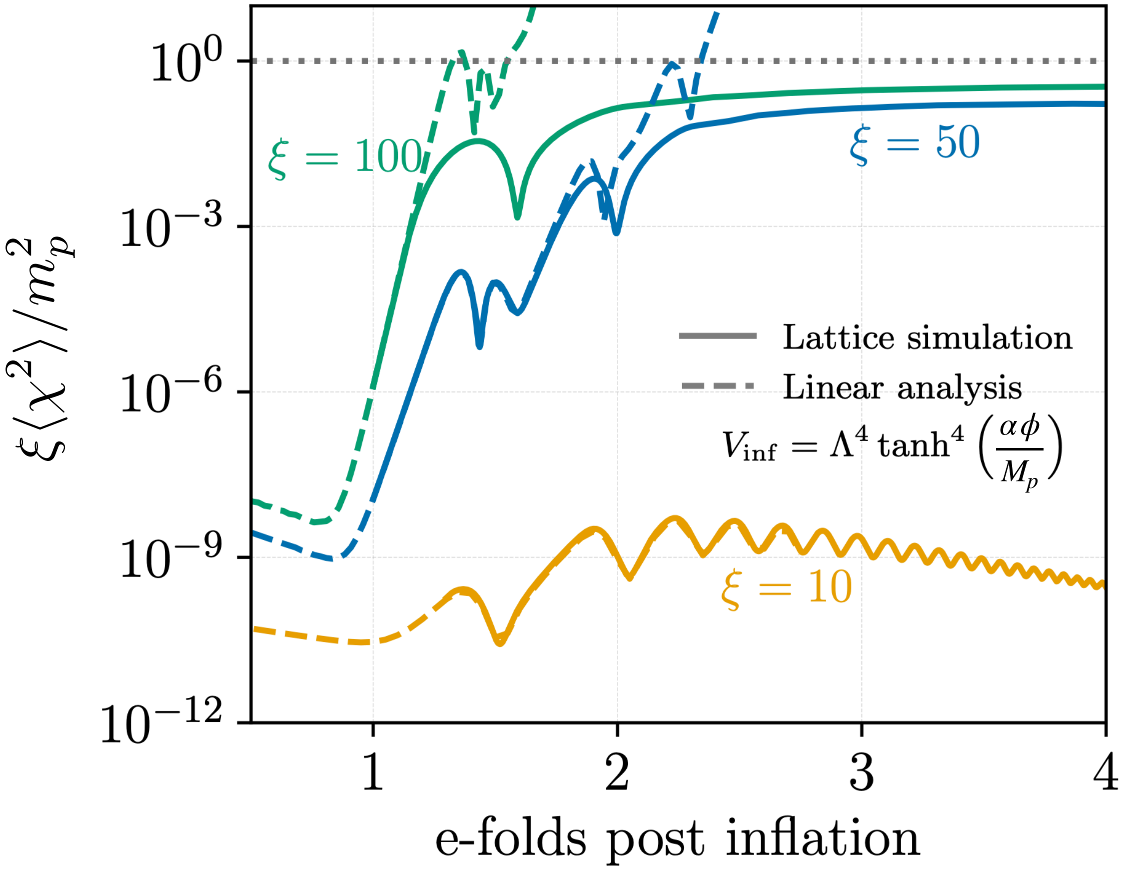
$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$

How is the preheat field excited?

Back-reaction



Non-Linear Regime

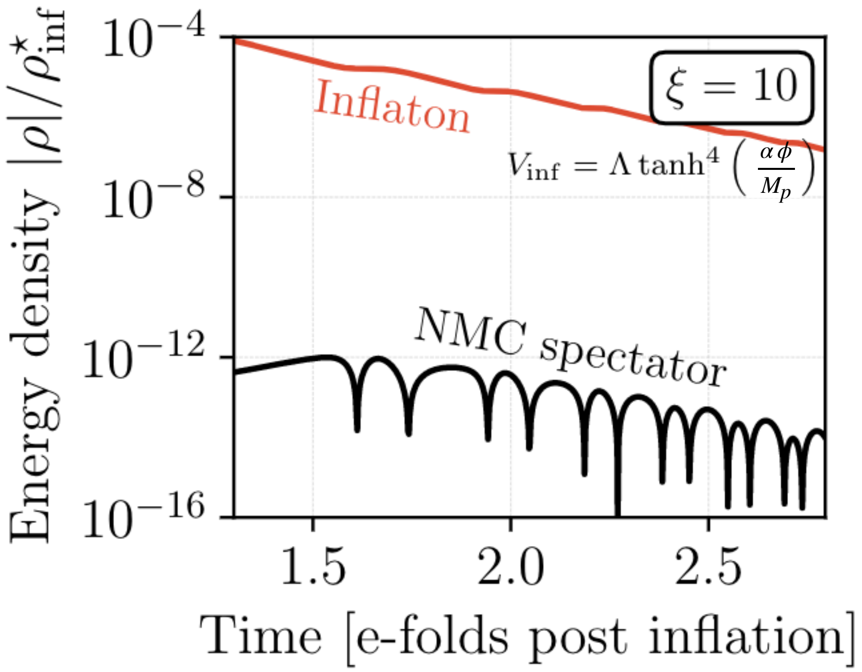


End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

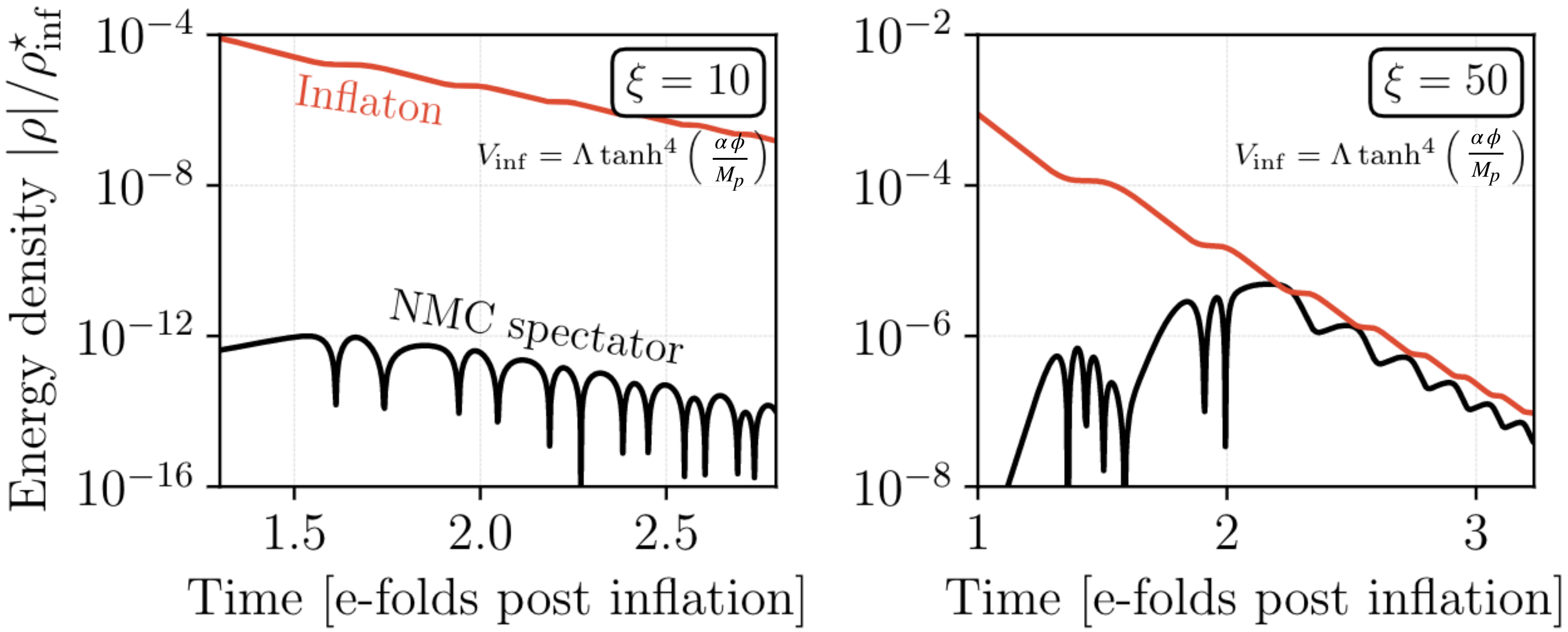
Geometric Preheating excitation (e.g. p = 4)



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating excitation (e.g. p = 4)

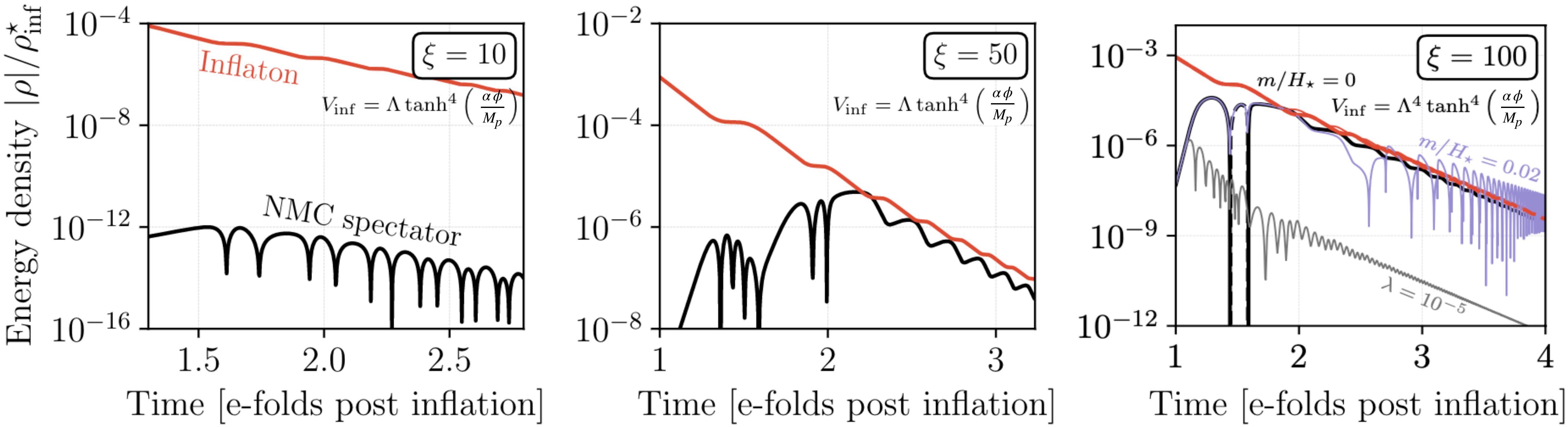


End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

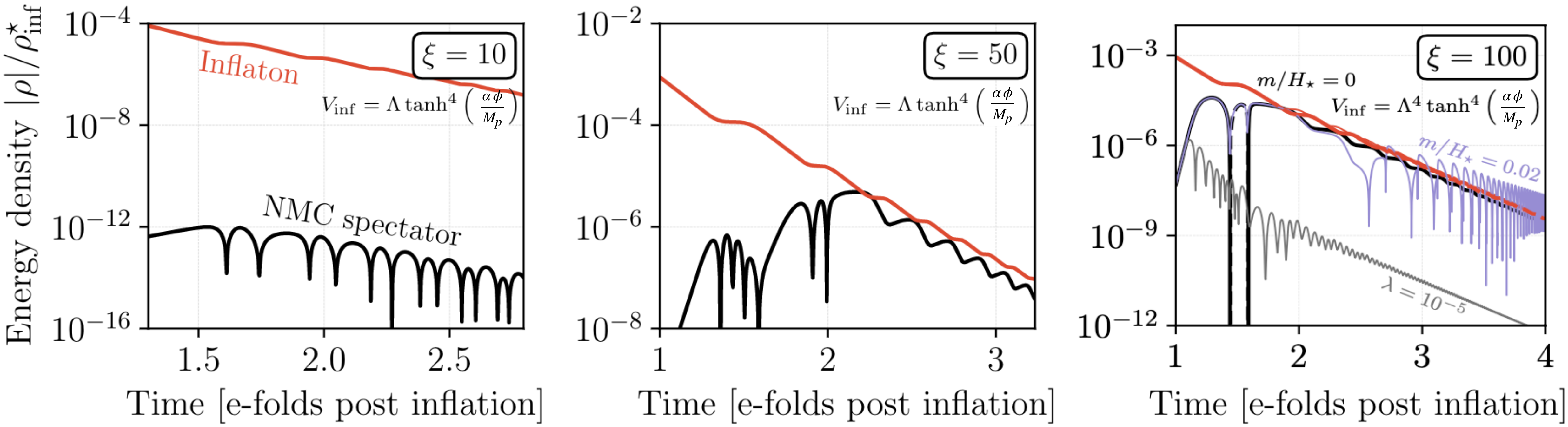
Geometric Preheating excitation (e.g. p = 4)



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating excitation (e.g. p = 4)



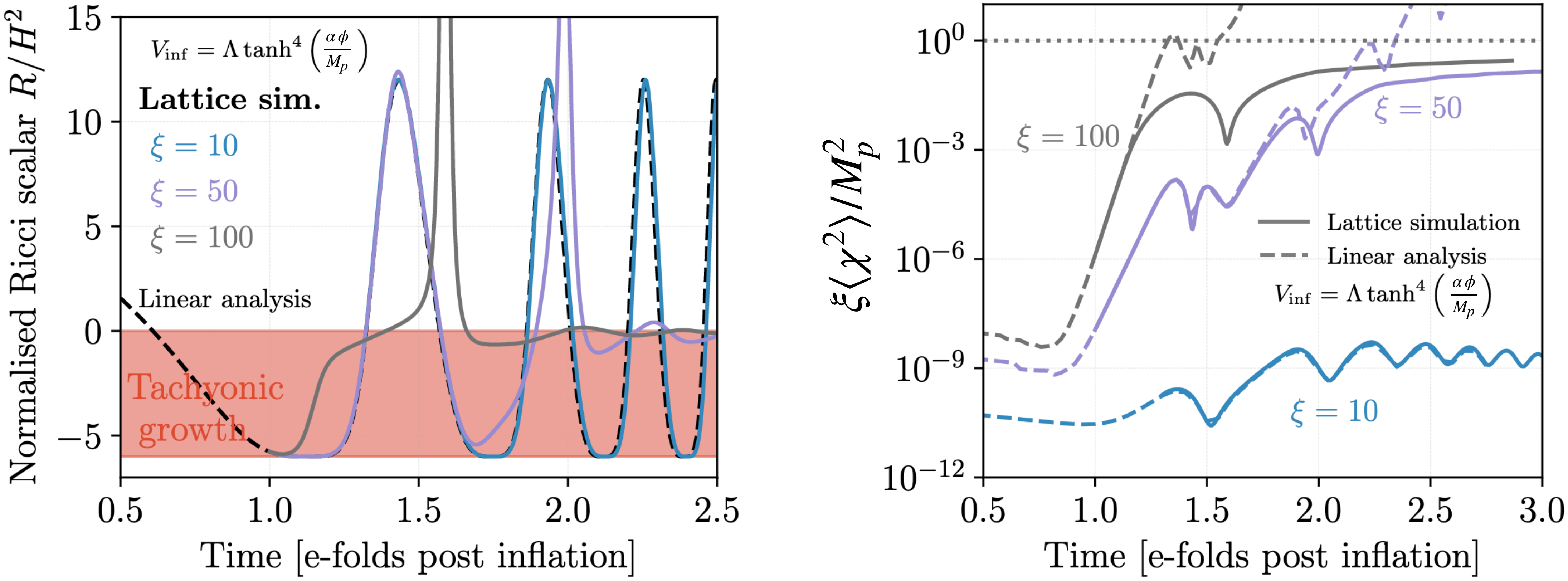
We can do it for any **p**

$V(|\phi|^p) \propto |\phi|^p$

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Full non-linear Geometric Preheating

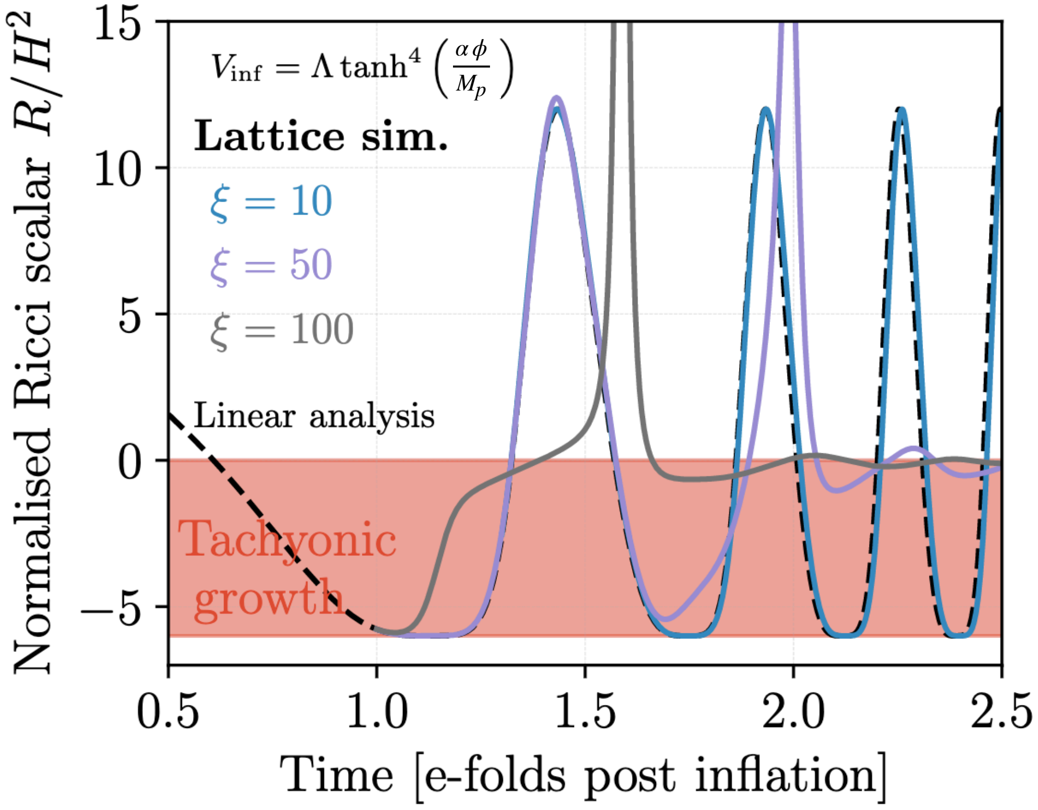


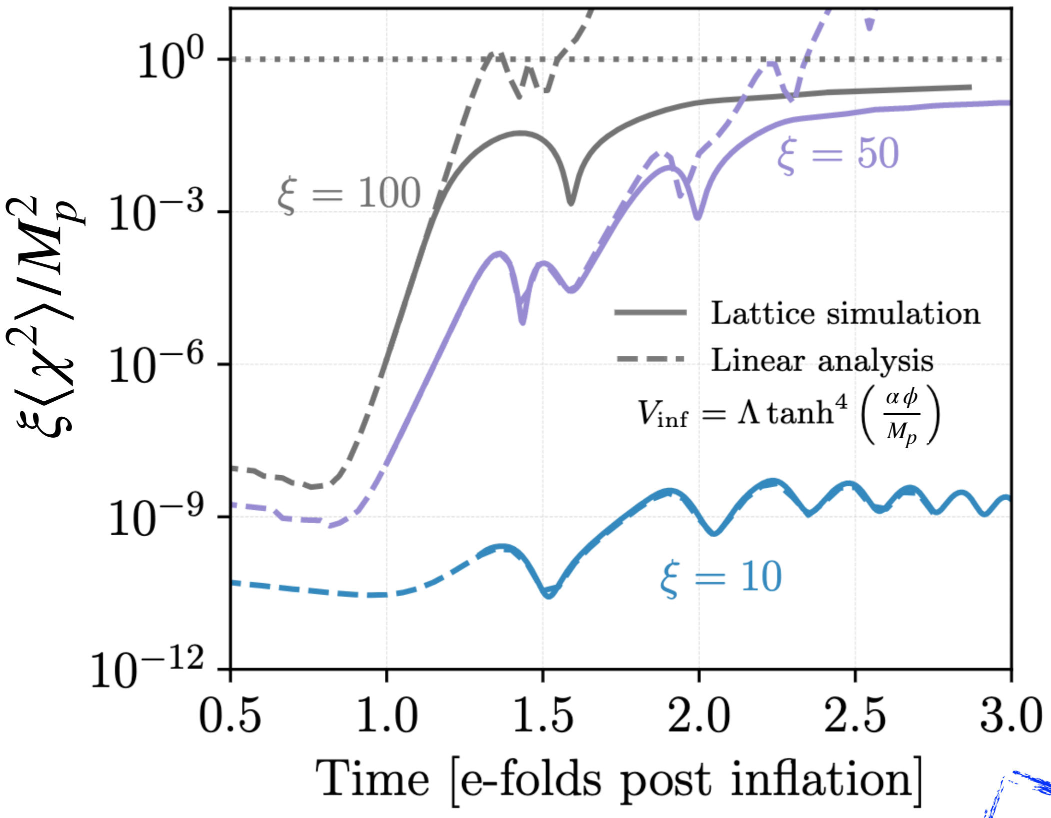
End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Full non-linear Geometric Preheating





We can compare Jordan vs Einstein frames

Work in progress

End

Transparency worked out with Adrien Florio