Large scale magnetic field, vacuum Energy of Universe, and nontrivial topology in QCD

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Generation, evolution, and observations of cosmological magnetic fields

EPFL Bernoulli Center, Lausanne, Switzerland, April 30, 2024

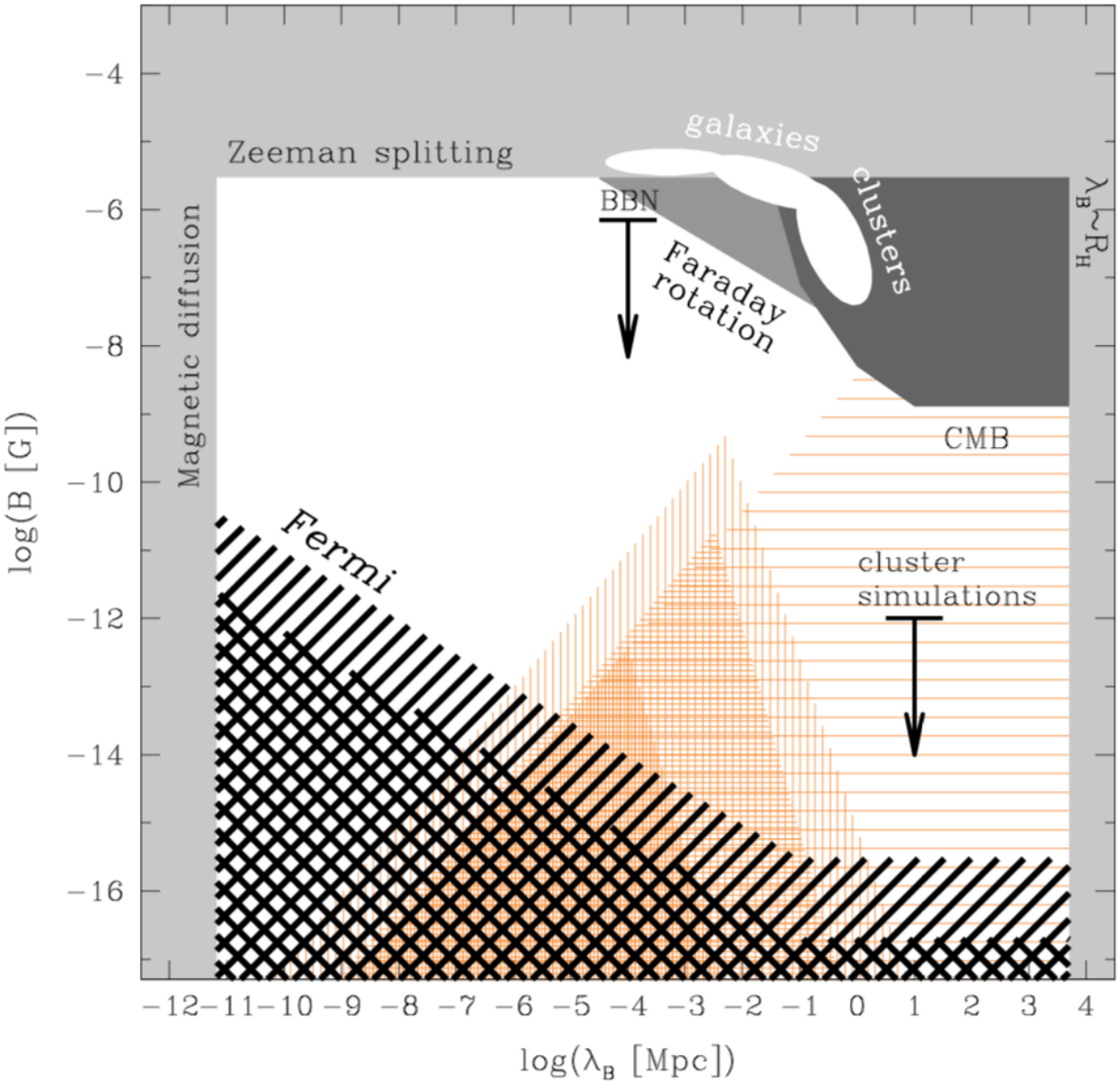
0. Motivation.

- It is known [From Neronov & Vovk, 2010]. The long ranged B-field $10^{-15}G \lesssim B \lesssim 10^{-9}G$ must exist in the Universe. It must be correlated on enormous Gpc scales, see slides below

- It is also known: such enormous Gpc correlation length is hard to achieve within conventional ideas

- We propose that this Gpc correlation length is a result of energy transfer from a specific DE model to magnetic energy through **helical instability**

- The DE in this framework emerges as a result of dynamics of the topologically nontrivial (helical) sectors in QCD. It is amazingly close to the observed DE today: $\rho_{\rm DE} \sim H\Lambda_{QCD}^3 \sim (10^{-3}eV)^4$



Constraints on the B field [From Neronov & Vovk]. The B-field correlated on Gpc scales must exist: $10^{-15}G \lesssim B \lesssim 10^{-9}G$

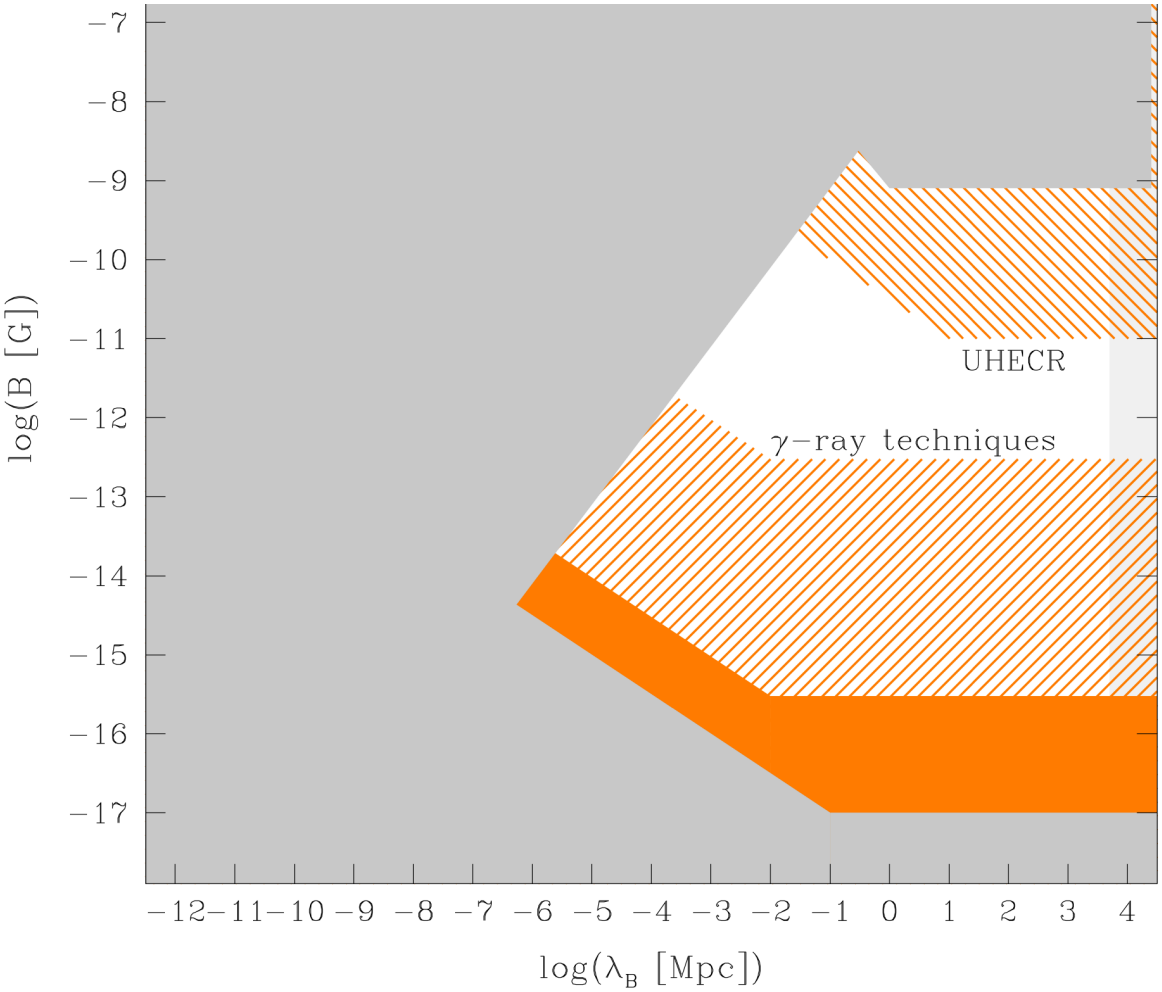


Fig. 14 The range of IGMF parameters accessible by next-generation γ-ray and UHECR telescopes. The orange shaded region shows the range of parameters for which delayed emission following bright flares is detectable at the energies below ~ 1 TeV. The lower orange hatched region shows the range of parameters for which extended emission is detectable below ~ 1 TeV. The upper orange hatched region shows the range of parameters which can be constrained by future UHECR telescopes.

Future experiment constraints (from UHECR and gamma rays telescopes) on the B field [From Durrer&Neronov, 2013].

1. Preliminary: energy definition in QFT

- We want to argue that there is a novel type of energy in strongly coupled QCD. This energy has "non-dispersive" nature, and can not be expressed in terms of conventional scattering amplitudes.

- It explicitly contradicts to the "folk theorem" that the S-matrix contains all the information about all physical observables.

- All these novel effects are due to the nontrivial topological sectors in the gauge systems and tunnelling transitions between them.

- The effect is non-local in nature, and can not be expressed in terms of local curvature in gradient expansion. It is expressed in terms of a non-local characteristics of the system -**the holonomy**.

2. Topological susceptibility

- A convenient way to explain the nature of new type of vacuum energy is to study the topological susceptibility (*it is the key element in the resolution of the so-called U(1) problem in QCD, Witten, Veneziano, 1979*).

$$\chi_{YM} = \int d^4x \, \langle q(x), q(0) \rangle \neq 0 \qquad \frac{\partial^2 E_{\rm vac}(\theta)}{\partial \theta^2} = \chi_{YM}$$

To avoid confusion: This is the Wick's T-product, not Dyson's

- χ_{YM} does not vanish, though $q(x) \sim \partial_\mu K^\mu(x)$. It has ``wrong sign", see below. It can not be related to any physical propagating degrees of freedom. Furthermore, it has a pole in momentum space

$$\lim_{k\to 0} \int d^4x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4}$$

- There is a massless pole (Veneziano ghost), but there are no any physical massless states in the system.

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0|q|n\rangle\langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

- conventional physical degrees of freedom always contribute with sign (-) while one needs sign (+) to satisfy WI and resolve the U(1) problem

$$\chi_{non-dispersive} = \int d^4x \, \langle q(x), q(0)\rangle = \frac{1}{N^2}|E_{vac}| > 0$$

- Conventional terms (related to propagating degrees of freedom) always produce $\exp(-\Lambda_{QCD} L)$ behaviour at large distances.

- Witten simply postulated this term, while Veneziano assumed the unphysical field, the so-called the "Veneziano ghost" to saturate "wrong" sign in χ.

- In some models this contact non-dispersive term with "wrong" sign (+) can be explicitly computed. It is originated from the tunnelling effects between the degenerate topological sectors of the theory.

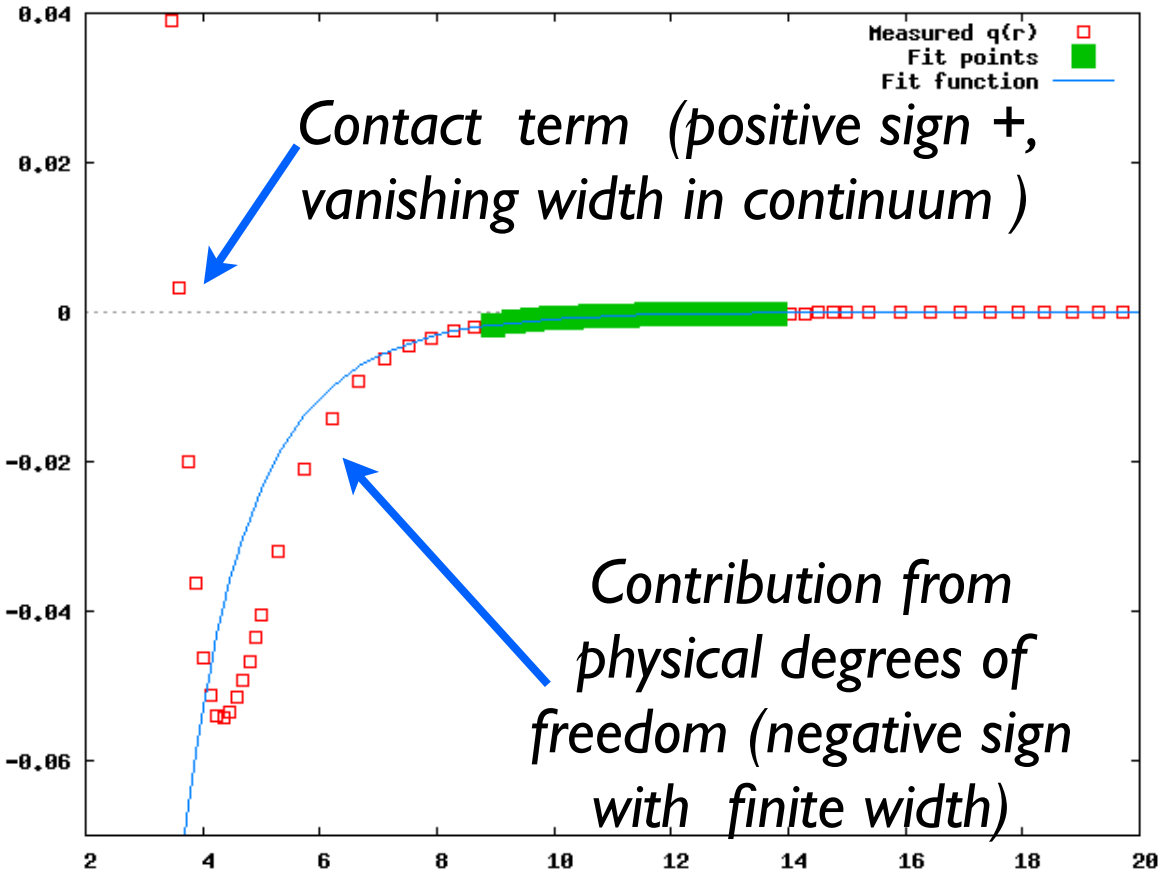
- These contributions can not be described in terms of conventional degrees of freedom (wrong sign);

- They are inherently non-local in nature as they are related to the tunnelling processes which are formulated in terms of the NON-LOCAL large gauge transformation operator and HOLONOMY;

- These terms may exhibit the LONG RANGE features even through QCD has a gap (similar to the CM topological insulators);

- The effects have been explained in terms χ_{YM} . However, the θ -dependent portion of energy $E_{\rm vac}(\theta)$ (relevant for the cosmological applications) has all these unusual features due to the relation

$$\chi_{YM} = \frac{\partial^2 E_{\rm vac}(\theta)}{\partial^2 \theta}|_{\theta=0}$$



The topological susceptibility $\chi(r)$ as a function of r. Wrong sign for χ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r=0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.

3. Definition of the Gravitating Energy

- We assume that the relevant (gravitating) energy which enters the Friedman's equation is the difference $\Delta E = (E_{\mathrm{FLRW}}(H) - E_{\mathrm{Mink}})$ similar to computations of the Casimir energy, when the difference ΔE is observed. This assumption was, in fact, originally formulated by Zeldovich in 1967.

- We can not (by technical reasons) to perform the computations in FLRW background. However, we can proceed with computations in a toy model formulated on hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$ when role of $H \sim 10^{-33}\ eV$ plays parameter $\kappa \to 0$

- We want to argue that a nontrivial holonomy generates a linear correction $\Delta E(\kappa) \sim \kappa$ in contrast with conventional expectation $\Delta E \sim R\,[\mathbb{H}^3_\kappa] \sim \kappa^2$

- Technically, we want to see a linear (rather than very small quadratic κ^2) correction in the ratio

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq 1 + \mathcal{O}\left(\frac{\kappa}{\Lambda_{QCD}}\right).$$

- If the same pattern persists in real FLRW Universe one could estimate

$$\Delta E_{\rm vac} \sim \kappa \sim L^{-1} \sim \Lambda^4_{QCD}\left(\frac{1}{\Lambda_{QCD} L}\right) \sim (10^{-3}{\rm eV})^4$$

- In other words, we interpret the observed Dark Energy as a modification of the QCD vacuum $\sim \kappa$ energy due to a nontrivial topology (not expressible in terms of local curvature $R\,[\mathbb{H}^3_\kappa] \sim \kappa^2$)

- It has the same non-dispersive nature (can not be expressed in terms of propagating dof), it is non-local in nature (not expressible in terms of the local curvature), and it has a positive sign.

- Historical comments: many people from different fields had advocated (after *Zeldovich, 1967*) a similar idea on the RHS for the Friedman's equation

$$\Delta E(L) = [E(L) - E_{\rm Mink}]$$

- $$E(L) \equiv -(\beta V)^{-1} \ln \mathcal{Z}$$

- *James Bjorken (particle physics), 2001,*
  Ralf Schuetzhold (GR), PRL, 2002;
  Grisha Volovik (CM physics), 2008 +many more

- I personally adopted this idea in 2009, mostly due to the intense (and never ending) discussions with Grisha Volovik in the relation with his COSLAB (Cosmology in a Laboratory) activities.

4. Holonomy and the linear correction $\kappa \sim 1/\mathcal{T}$ in hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$

- Normally it is expected that all corrections due to the time-dependent (curved) background are proportional to the local curvature $R\left[\mathbb{H}^3_\kappa\right] \sim \kappa^2$

- We want to test these ideas in gauge theories with nontrivial holonomy. In this case corrections are not reduced to the local observables. The IR regularization plays key role in all computations.

- Specifically, we compute the ratio which explicitly shows the linear correction $\sim \kappa$

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq \left(1 - \frac{\nu\bar{\nu}}{2} \cdot \frac{\kappa}{\Lambda_{QCD}}\right). \quad E_{\rm vac} \equiv -\frac{1}{\beta V} \ln \mathcal{Z}$$

- The key feature of the configurations which generate the vacuum energy is that the fields are helical with quantum numbers of $q(x) \sim \partial_\mu K^\mu(x)$

- Precisely these P-odd colour- singlet topologically protected configurations can be sensitive to arbitrary large distances (there is no cutoff at the QCD scale as naively one could expect, see below)

$$\lim_{k\to 0}\int d^4x e^{ikx}\langle K_\mu(x), K_\nu(0)\rangle \sim \frac{k_\mu k_\nu}{k^4} \qquad K_0 \propto \mathbf{A}^a \cdot \mathbf{B}^a$$

- The generation of the large scale magnetic field is based on specific features of the topologically induced vacuum DE which itself is generated by the helical configurations.

- This mechanism can be thought as generation by the axion which auxiliary (not dynamical) field as the axion couples to topological density $\sim a \cdot \partial_\mu K^\mu$

- *Q: How a system with a gap could be ever sensitive to arbitrary large distances?*

- A1: The long range order in gapped QCD is similar to Aharonov -Casher effect. If one inserts an external charge into superconductor when electric field is screened $\exp(-r/\lambda)$ a neutral magnetic fluxon will be still sensitive to external charge at arbitrary large distances.

- A2: Long range order in the system emerges because the large gauge transformation operator and holonomy are non-local operators sensitive to far IR-physics, similar to "modular operator" in Aharonov -Casher effect.

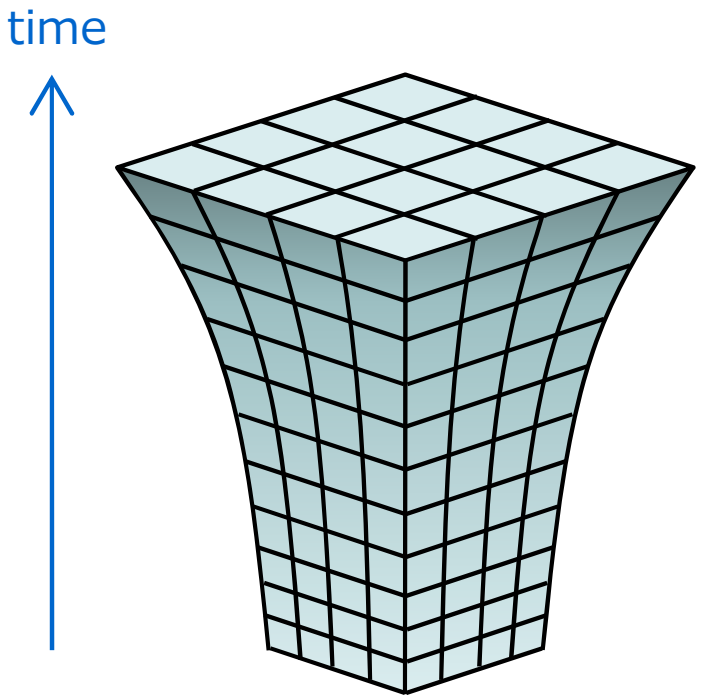
- Are there other hints on a linear dependence on cosmological scale $L \sim H^{-1}$ in a gapped system? (locality suggests quadratic behaviour as $R \sim H^2$)

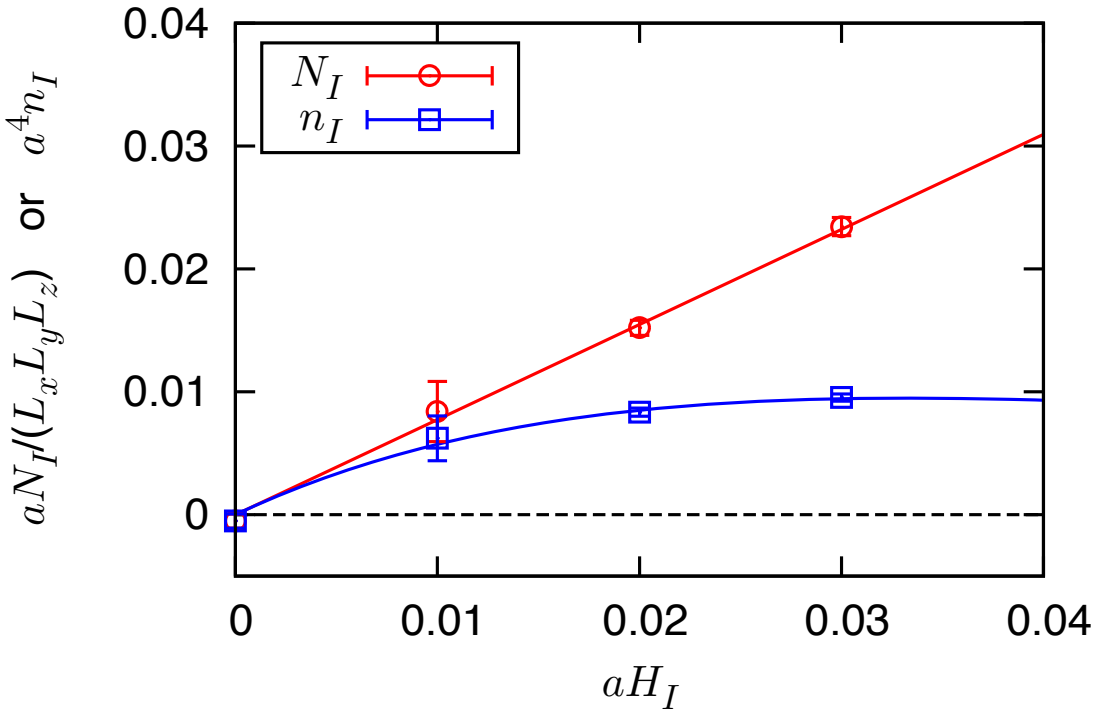
- 1. A number of analytical computations in some simplified models (e.g. deformed QCD).

- 2a. Lattice numerical simulations. In this case the computations of a real part of the energy -momentum tensor $Re\langle T_{\mu\nu}\rangle$ is a hard problem.

- 2b. However, the <u>imaginary (absorptive)</u> portion of the energy-momentum tensor $Im\langle T_{\mu\nu}\rangle$ due to particle production, can be computed, see plot below.

- 2c. Analyticity suggests that the dependence on H must be the same in $Re\langle T_{\mu\nu}\rangle$ and $Im\langle T_{\mu\nu}\rangle$





The plots from A. Yamamoto, arxiv 1405.6665.

1. The expansion in Euclidean space-time was parametrized by the "imaginary" Hubble constant when the lattice action is positively defined;
2. Red curve ——— describes the particle production rate per unit volume per unit time in the background H_I ;
3. The linear dependence on H_I has been observed, $Im[\langle T_{\mu\nu}\rangle] \sim H_I$. It strongly supports our arguments.

5. Applications to the Dark Energy

- Lesson 1: There is a fundamentally new type of the vacuum energy which can not be expressed in terms of any local field (dynamical inflaton field or dynamical DE field)

- Lesson 2: It emerges as a result of tunnelling processes between degenerate topological sectors, and formulated in terms of the "non-dispersive" contact terms and nonlocal holonomy.

- Lesson 3: We identify this new type of energy with cosmological vacuum dark energy (DE).

- The relevant parameters are amazingly close to the observed DE values:

$$\mathcal{T}^{-1} \sim H \sim \frac{\Lambda_{QCD}^3}{M_{PL}^2} \sim 10^{-33} eV, \quad \rho_{\rm DE} \sim H\Lambda_{QCD}^3 \sim (10^{-3} eV)^4, \quad \mathcal{T} \sim H^{-1} \sim \frac{M_{PL}^2}{\Lambda_{QCD}^3} \sim 10\,{\rm Gyr},$$

- It explicitly shows that the vacuum QCD contribution to the Hubble constant and DE is expressed in terms of the $\Lambda_{\rm QCD}$

- This energy will be eventually transferred to the Maxwell EM fields (so-called helical instability) on the time scale of $\alpha^{-2} H^{-1} \sim \alpha^{-2} 10^{10}$ years

- Result: the large scale magnetic field (with correlation length of entire visible Universe) will be generated during the evolution of the Universe when DE starts to dominate the dynamics.

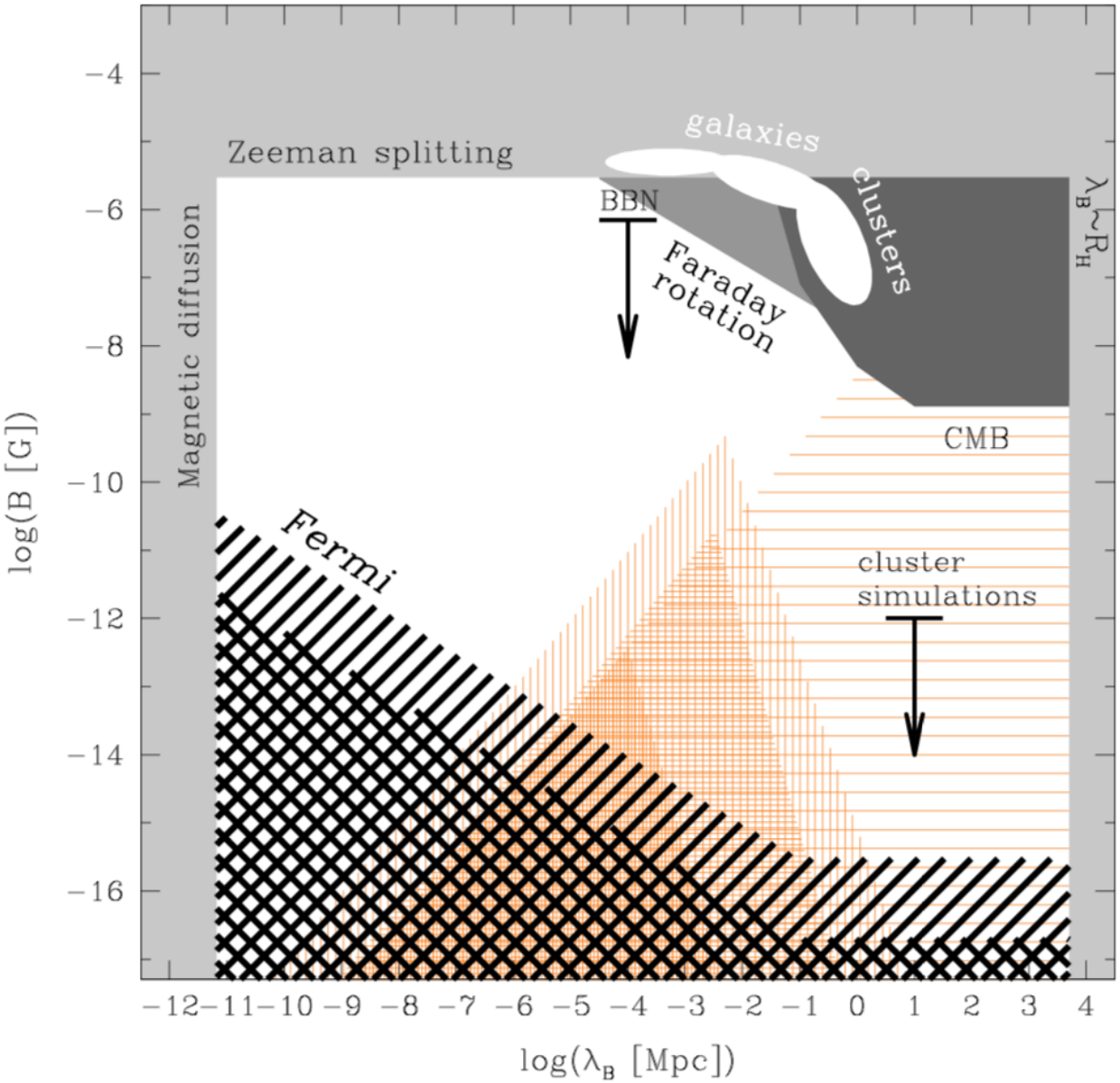
6. Large Scale Magnetic field

- There are many ideas how the observed cosmological magnetic field is generated including primordial mechanisms (many people sitting in the room here were pioneers in this field).

- The B field correlated on enormous (Gpc) scale must exist. What is the origin of such correlation?

- We advocate unorthodox mechanism which is dramatically different from all previous approaches: the B- field is generated with enormous coherence scale from topologically induced DE

- No need for any amplification mechanisms as it is characterized by the largest possible scale at the moment of formation



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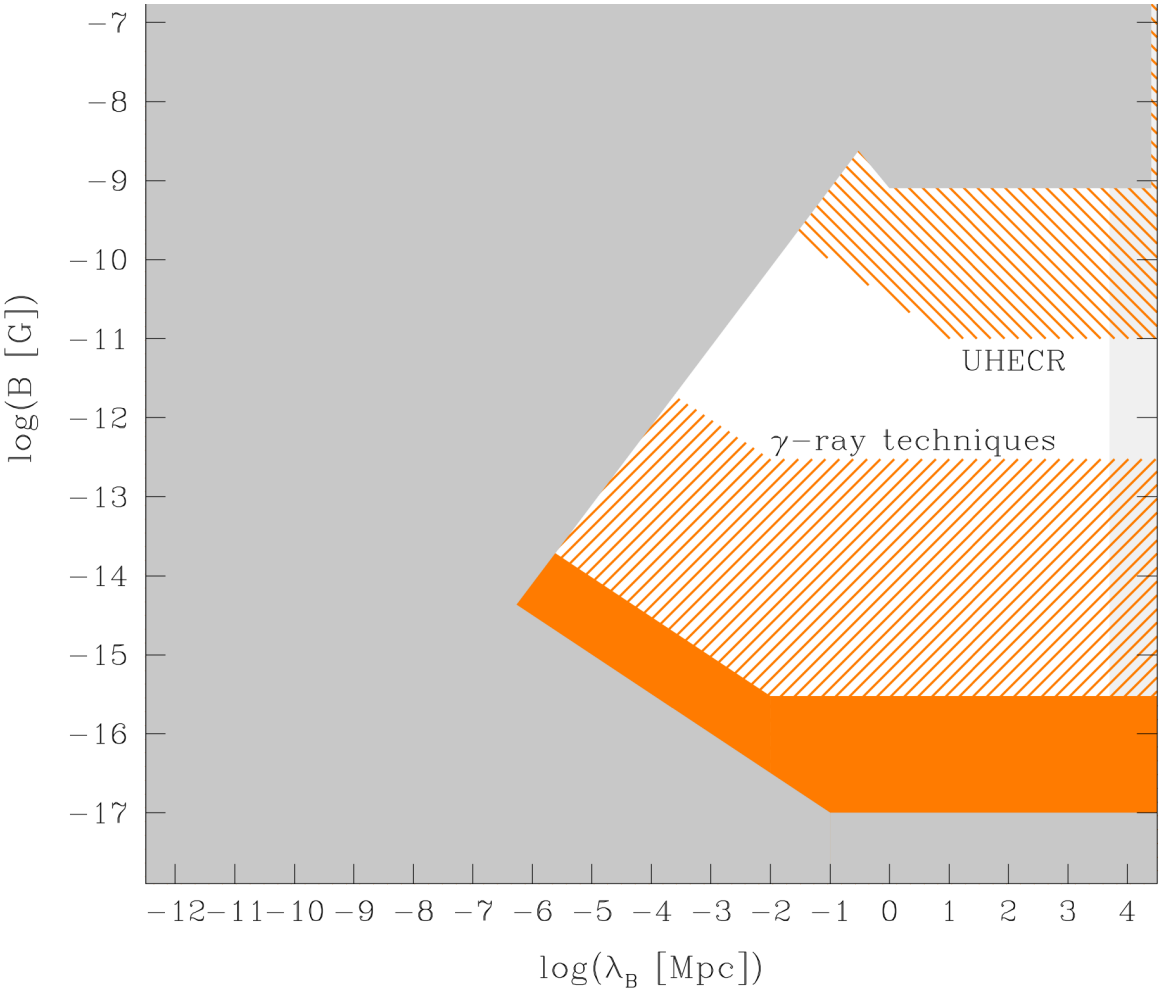

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Future experiment constraints (from UHECR and gamma rays telescopes) on the B field [From Durrer&Neronov, 2013].

- The starting point is the conventional Effective Lagrangian in terms of the auxiliary field (it can be exactly derived from topological configurations (KvBLL calorons with nontrivial holonomy)

$$\mathcal{L}_{b\gamma\gamma}(x) = \frac{\alpha}{4\pi} N \frac{\sum_i Q_i^2}{N_f} \left[\theta + b(x,H)\right] \cdot F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$$

- It generates well known extra term with $\mu_5 \sim H_0$

$$\vec{\nabla} \times \vec{B} = \sigma \vec{E} + \frac{\alpha}{2\pi} N \frac{\sum_i Q_i^2}{N_f} \cdot \left(\mu_5 \vec{B}\right), \quad \mu_5 \equiv \langle \dot{b}(x,H) \rangle$$

- Similar equations have been studied before (e.g. dynamical axion field). The difference here is that the μ_5 does not satisfy any equation of motion as there is no canonical kinetic term for auxiliary field $b(x,H)$. The μ_5 is non-propagating field

- The P-odd long ranged field $b(x,H)$ was introduced as the Lagrange multiplier to account for tunnelling events, generating the DE in this framework. It has the axion quantum numbers, and it is sensitive to arbitrary large distances

- It is known that the presence of the μ_5 term leads to the helical instability. In the present context it implies the generation of the magnetic field on the huge scales where μ_5 is correlated.

- The instability develops for large wavelengths:

$$B(t) = B_0 \exp(\gamma t), \quad k < \frac{\alpha}{\pi}\mu_5, \quad \mu_5 \sim H$$

- This effect leads to the generation of the magnetic field correlated on the enormous scales.

- The order of magnitude estimates suggest (present time),

$$B \sim 10^{-10} G$$

7. Few technical details

- We illustrate the idea with oversimplified ansatz [Boyarrsky,Frohlich & Ruchayskiy, 2015].

$$\vec{B} = B(t)\big[\sin(kz),\ \cos(kz),\ 0\big],\quad \vec{E} = -\tfrac{1}{k}\dot{B}(t)\big[\sin(kz),\ \cos(kz),\ 0\big] = -\tfrac{1}{k}\dot{\vec{B}}.$$

- It leads to the following equation for $\mathrm{B(t)}$

$$kB(t) = -\tfrac{\sigma}{k}\dot{B}(t) + \tfrac{\alpha}{2\pi}N\tfrac{\sum_i Q_i^2}{N_f}\cdot\left(\mu_5\vec{B}\right),\quad \mu_5 \equiv \langle\dot{b}(x,H)\rangle \propto H$$

- Crucial point here is that $\mu_5 \equiv \langle\dot{b}(x,H)\rangle \propto H$ is not a dynamical field (like axion). It's dynamics entirely determined by the dynamics of the topological sectors of QCD in time dependent background $\mathrm{H(t)}$

- This equation has a solution

$$B(t) = B_0 \exp(\gamma t), \quad \gamma = \frac{k}{\sigma}\left[\frac{\alpha}{\pi}\bar{c}H - k\right], \quad \bar{c} \propto \frac{N\sum_i Q_i^2}{2N_f}$$

- This implies development of the helical instability

$$\gamma > 0 \quad \Rightarrow \quad k < \frac{\alpha}{\pi}\bar{c}H.$$

- This is well known phenomenon and discussed previously in applications to the cosmology and heavy-ion collisions [Carroll, Field & Jackiw; Joyce & Shaposhnikov....]

- Generically, we expect the field to be twisted as it is highly helical. For simple configuration we expect magnetic helicity directly related to energy density

$$\mathcal{H} \equiv \int \vec{A}\cdot\vec{B}d^3x, \qquad \frac{\mathcal{H}(t)}{V} \approx \frac{B^2(t)}{k}.$$

Concluding comments on Dark Energy & B field

- QCD vacuum energy is different for different background (Minkowski vs deSitter). This difference generates correct order of magnitude for the observed DE today

$$\rho_{\rm DE} \equiv \Delta E \equiv (E_{\rm deSitter} - E_{\rm Mink})$$

$$H \sim \frac{\Lambda^3_{\rm QCD}}{M^2_{\rm PL}} \sim 10^{-33}{\rm eV}, \qquad \rho_{\rm DE} \sim H\Lambda^3_{\rm QCD} \sim (10^{-3}{\rm eV})^4$$

- There are no any new fields associated with $\rho_{\rm DE}(t)$

- The long range magnetic field is generated as a result of variation of the QCD tunnelling transition rate in time dependent background determined by the Hubble $\rm H(t)$.

- The source of the magnetic energy in this framework is the DE, i.e. $B^2 \propto \rho_{\rm DE}$

- The energy-transfer time scale is $\alpha^{-2} H^{-1} \text{years}$ (similar ideas applied to reheating after inflation predict number e-folds $N_{\rm e-folds} \sim \alpha_s^{-2}(H_0) \sim 10^2$

- It is expected a number of P-odd phenomena on scales of the entire visible Universe

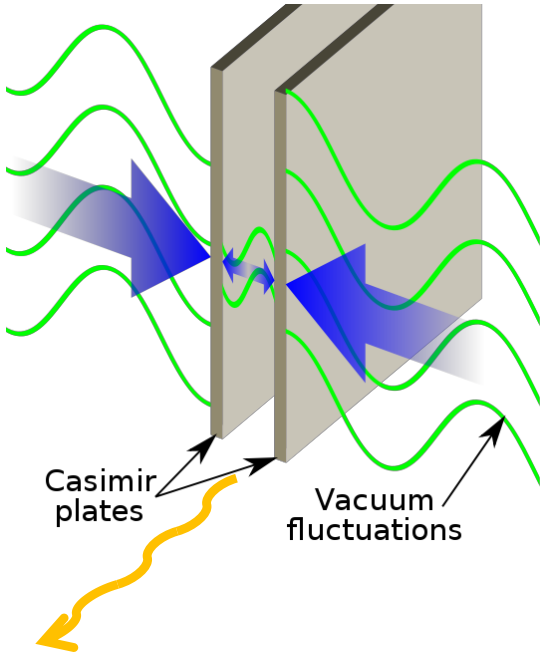
- This effect in many respects is similar to the Dynamical Casimir Effect, see slides below

- The difference is that instead of vacuum's virtual photons one should consider the vacuum P-odd configurations describing the tunnelling transitions between different topological sectors

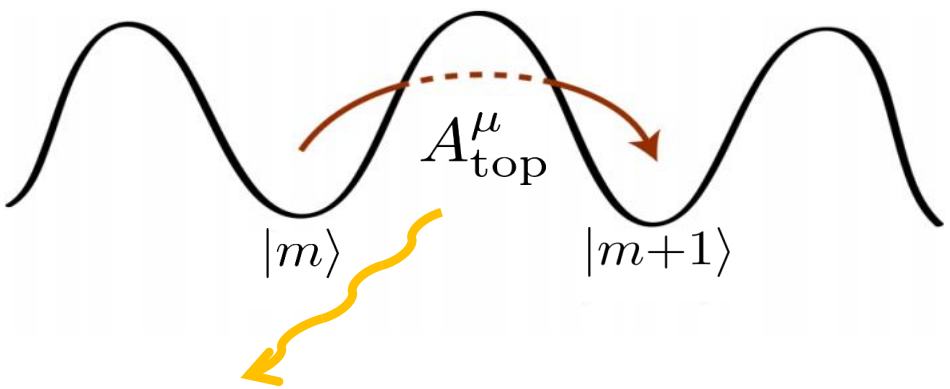
Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study this new physics phenomenon:

- When the Maxwell system is formulated on four-torus there will be an <u>extra contribution</u> to the Casimir pressure, not related to the physical propagating photons with two transverse polarizations (4-torus has nontrivial holonomy).

- The Maxwell system on the 4-torus shows all signs (degeneracy, etc) which are normally attributed to the topologically ordered CM systems.



This picture illustrates a new physics phenomenon



The emission of real physical photons as a result of tunnelling transitions in time dependent background is precisely the effect discussed in this talk: the cosmological magnetic field is generated as a result of variation of the vacuum tunnelling transitions

Extra Slides

1. on physical meaning of $\mathbb{S}^1$ in computations
2. applications to Inflation and reheating

Physical meaning of $\mathbb{S}^1$ (the size of $\mathcal{T}$)

(appendix A3 from paper with Barvinsky)

- Originally $\mathcal{T}$ was introduced in Euclidean space for computations in the weak coupling regime with given holonomy.

$$L = \mathcal{P} \exp\left(i \int_0^{\mathcal{T}} dx_4 A_4(x_4, |\mathbf{x}| \to \infty) \right) .$$

- It should not be confused with real size in 4D in Minkowski space.

- In weakly coupled gauge theories (such as deformed QCD) all computations can be carried out explicitly with fixed $\mathcal{T}$. One can see explicitly confinement, fractionally 1/N charged monopoles (instanton quarks), generation of the vacuum energy expressed in terms of the auxiliary field $b(x, H)$, etc

- In strongly coupled real QCD (when you start from the very begging from $\mathcal{T} \to \infty$) such computations cannot be done as calorons with nontrivial holonomy cannot be constructed

- How do we know about anything about holonomy defined on $\mathbb{S}^1$ if it was not a part of construction to begin with?

- It turns out that the holonomy can be dynamically generated (emerging) in strongly coupled regime.

- The well known example is $2d\ CP^{N-1}$ model defined on $\mathbb{R}^2$ when the only integer values instantons with trivial holonomy were introduced into the system

- However, when all the instants are taken into account (grand canonical ensemble) the fractional topological charge 1/N dynamically emerges. It looks exactly as we were started with nontrivial holonomy defined on $\mathbb{S}^1$. However, the semiclassical description is not justified.

- Therefore, in this "emergent" case defined on $\mathbb{R}^2$ the effective size $\mathbb{S}^1$ is generated dynamically.

- The main lesson in the present context: the effective size $\mathbb{S}^1$ can be unlinked from the so-called bootstrap equations and should be treated as free parameter to be fixed from observations: $\mathcal{T} \sim H^{-1}$

- Sufficiently large size of $\mathbb{S}^1$ is consistent with presently available CMB observations for $\mathcal{T} \gtrsim H^{-1}$

Cosmological application: inflation

- This proposal represents a synthesis of two previous, and naively unrelated, ideas:

- 1. self-consistent generation of the de Sitter behaviour based on the Euclidean $\mathbb{S}^3 \times \mathbb{S}^1$ gravitational instantons, garlands. They can be thought as the thermal (so is the $\mathbb{S}^1$) version of Hartle-Hawking instantons *(Barvinsky and Co)*,

- Inflation stage starts after "nucleation" of the system from $\mathbb{S}^3 \times \mathbb{S}^1$ instanton. Analytical continuation of the scale factor leads to the de Sitter behaviour *(Barvinsky and Co)*,

How the inflation ends. The Reheating

- In a simplified case (no SM particles) the Hubble constant H and the energy density remain constant after the "nucleation" from the gravitational instanton. The system assumes conventional Lorentzian signature.

- The solution after nucleation corresponds to inflationary de-Sitter behaviour

$$w = \frac{\Delta P}{\Delta \rho} = -1, \quad \mathrm{a}(t) \sim \exp(Ht), \quad H \sim \mathcal{T}^{-1},$$

$$\Delta\rho \equiv \rho_{\mathrm{vac}}[\mathbb{S}^3 \times \mathbb{S}^1] - \rho_{\mathrm{vac}}[\mathbb{R}^4], \quad \Delta p \equiv p_{\mathrm{vac}}[\mathbb{S}^3 \times \mathbb{S}^1] - p_{\mathrm{vac}}[\mathbb{R}^4],$$

- This would be the final destination of the Universe if the interaction with SM particles is switched off

- When the interaction with SM particles is switched back on the inflation ends as a result of this interaction with SM particles.

- The computational procedure is well defined in principle (profoundly complicated in practise):

- 1.One should describe the relevant Euclidean configurations satisfying proper boundary conditions (similar to calorons with nontrivial holonomy defined on $\mathbb{R}^3 \times \mathbb{S}^1$)

- 2. One should compute the corresponding path integral accounting for the tunnelling transitions;

- 3.One should compute (ρ, p) in the presence of all massless SM gauge fields (γ, W, Z, g)

- 4. One should subtract the corresponding expressions $\rho(\mathbb{R}^4), p(\mathbb{R}^4)$ to derive $(\Delta\rho,\ \Delta p)$

- While these steps are well defined in principle, it is not feasible now to perform the computations

- There is analogy with the Dynamical Casimir Effect (DCE) when photons are radiated from time-dependent background.

- the difference is: in our case photons are emitted not from conventional quantum vacuum fluctuations, but from configurations describing the vacuum tunnelling processes.

- This is a hard technical problem: tunnelling is described in Euclidean path integral while the emission represents inherent Minkowski process

- Fortunately, the key features can be understood using alternative technique formulated in terms of the auxiliary topological non-propagating field, similar to the Veneziano ghost field in QCD.

- The end of inflation is described by such effective auxiliary field. It is fixed by triangle anomaly:

$$\mathcal{L}_{b\gamma\gamma} = \frac{\alpha(H_0)}{8\pi} N Q^2 \left[\theta - b(x)\right] \cdot F_{\mu\nu}\tilde{F}^{\mu\nu} \,,$$

- where $b(x)$ is topological non-propagating field (Lagrange multiplier). This technique has been explicitly tested in simplified solvable systems

- In Condensed Matter this technique is well known when summation over topological sectors is replaced by auxiliary (non-propagating) field, e.g. in topologically ordered systems

- In QCD context $\dot{b} = \mu_5$ implies that the helical instability will be developed (studies were done in relation with the chiral magnetic effect in QCD where μ_5 is the chiral chemical potential).

- In cosmology context ($\mu_5 \sim H_0$) the typical time scale for development of the helical instability is

$$\tau_{\rm instability} \sim \frac{1}{\alpha_s^2 \mu_5} \quad \rightarrow \quad \tau_{\rm inflation} \sim \frac{1}{\alpha_s^2(H_0) H_0} \quad \rightarrow \quad N_{\rm e-folds} \sim \alpha_s^{-2}(H_0) \sim 10^2$$

- We identify $\tau_{\rm instability}$ with $\tau_{\rm inflaton}$

- The number of e-folds $N_{\rm e-folds} \sim \alpha_s^{-2}(H_0) \sim 10^2$

- The deviation from pure De-Sitter is related to the coupling with SM (we know and love) and not to some ad-hoc local inflaton potential $V(\Phi)$