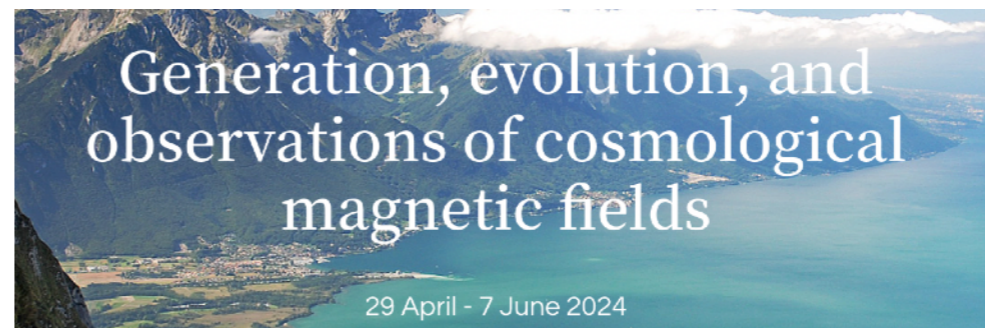
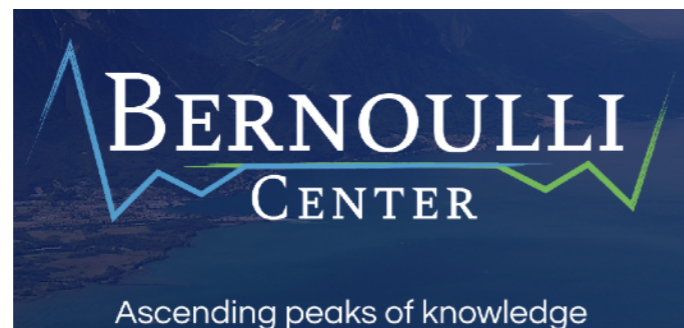


# *Imprints of primordial curvature perturbations on inflationary magnetic fields*

**Rajeev Kumar Jain**

Theoretical Cosmology group  
Department of Physics, IISc Bangalore



# *Outline of the talk*

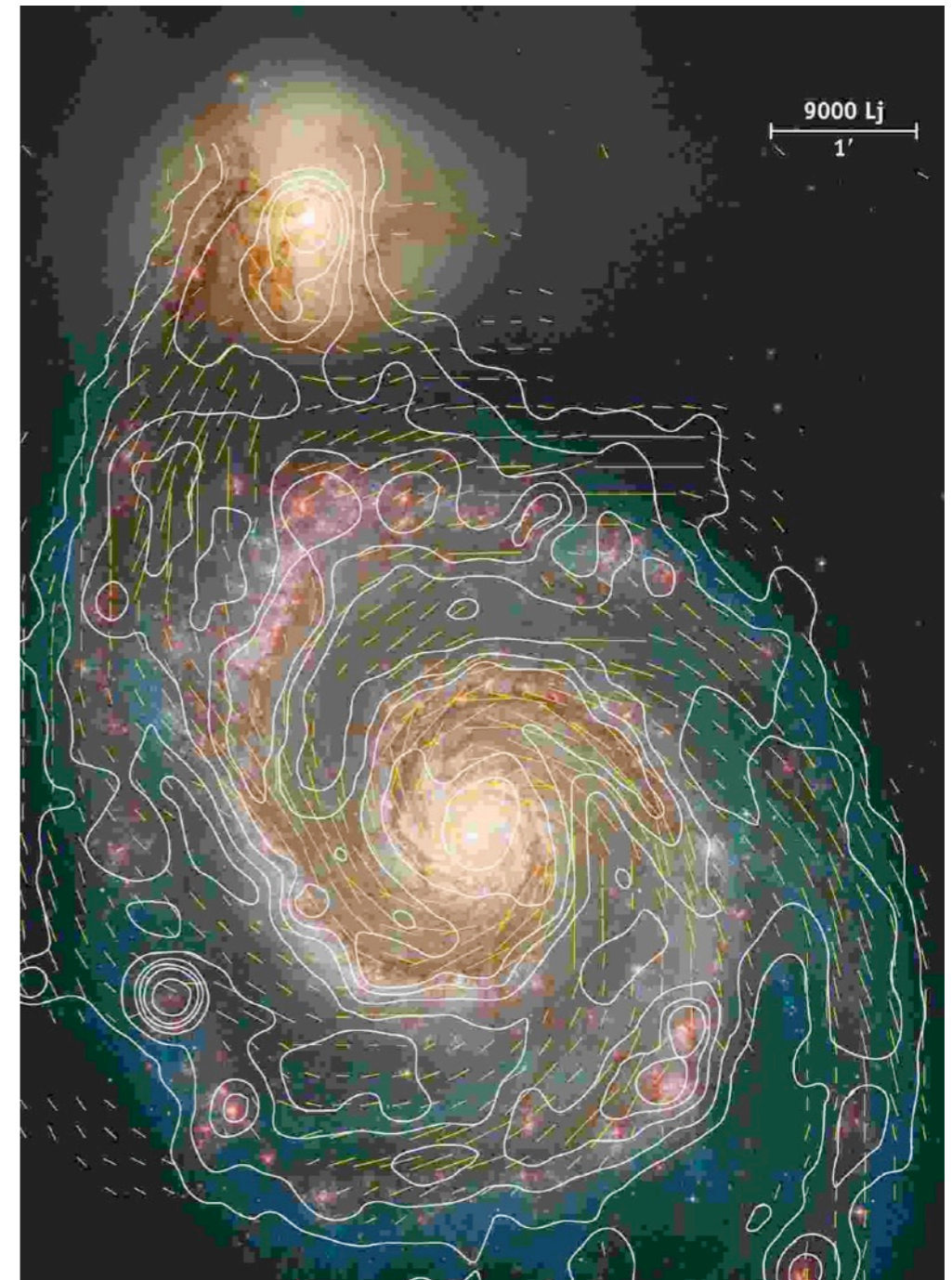
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- *Observational evidence for magnetic fields*
- *Inflationary magnetogenesis — brief overview*
- *IMF from single field inflationary models*
  - *Mild and strong deviations from slow roll*
  - *Imprints on the magnetic field power spectra*
- *IMF from two field inflationary models*
  - *Imprints of PMF on the CMB anisotropies*
- *Cross-correlation of curvature/tensor perturbations with magnetic fields — soft theorems — full results — observational imprints*
- *Conclusions*

# Cosmological magnetic fields

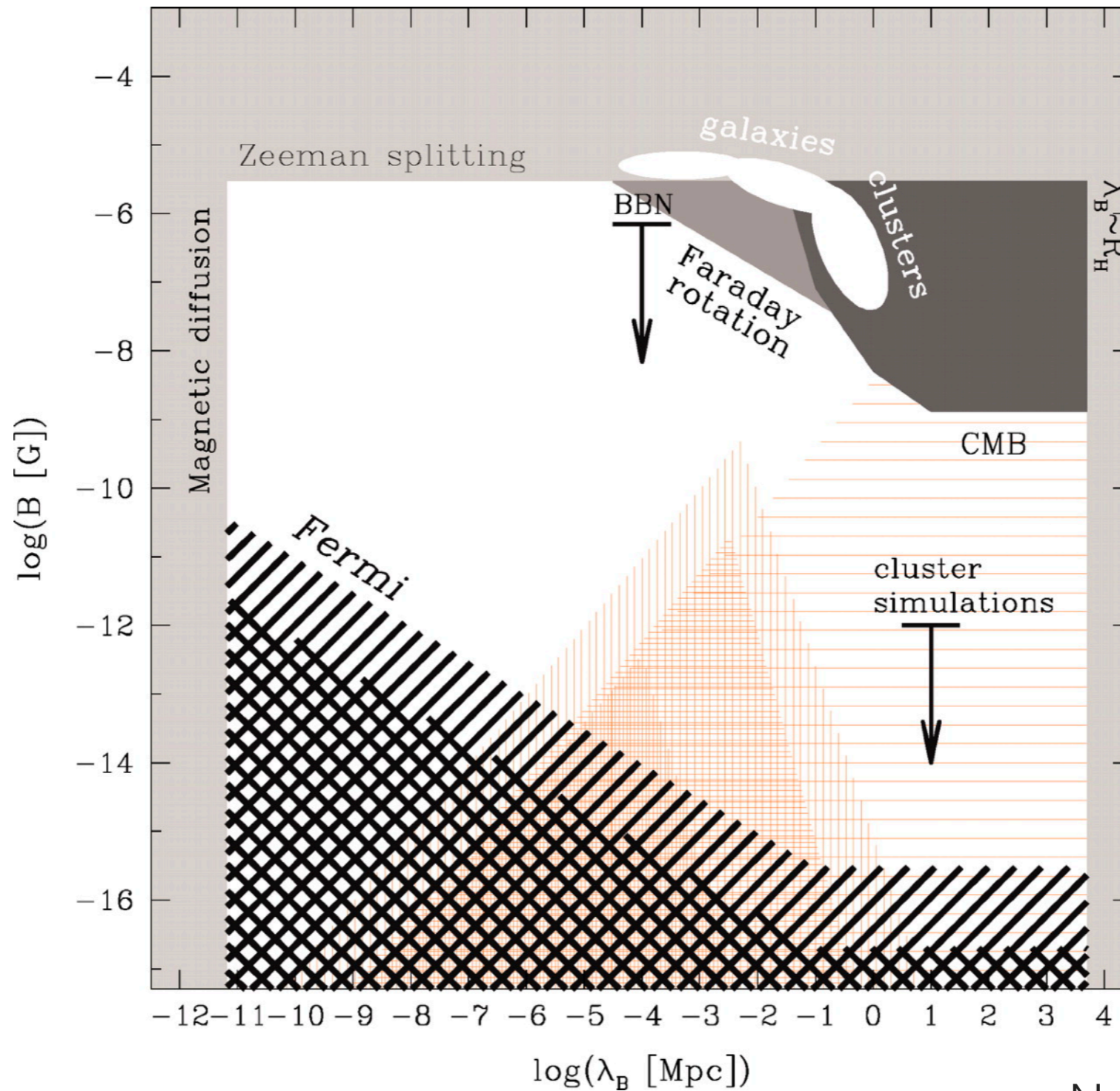
- Our observed universe is magnetized on all scales.
- All the bound structures — stars, galaxies and clusters carry magnetic fields, also present in the intergalactic medium.
- **Stars:**  $B \sim 0.1 - \text{few G}$ .
- **Galaxies:**  $B \sim 1 - 10 \mu\text{G}$  with coherence length as large as 10 kpc.
- **Clusters:**  $B \sim 0.1 - 1 \mu\text{G}$ , coherent on scales up to 100 kpc.
- **Intergalactic medium:**  $B \gtrsim 3 \times 10^{-16} \text{ G}$  on scales of  $\sim 1 \text{ Mpc}$ .

Neronov & Vovk, 2010





# Constraints on cosmic magnetic fields



Neronov & Vovk, 2010



# Primordial magnetic fields from inflation

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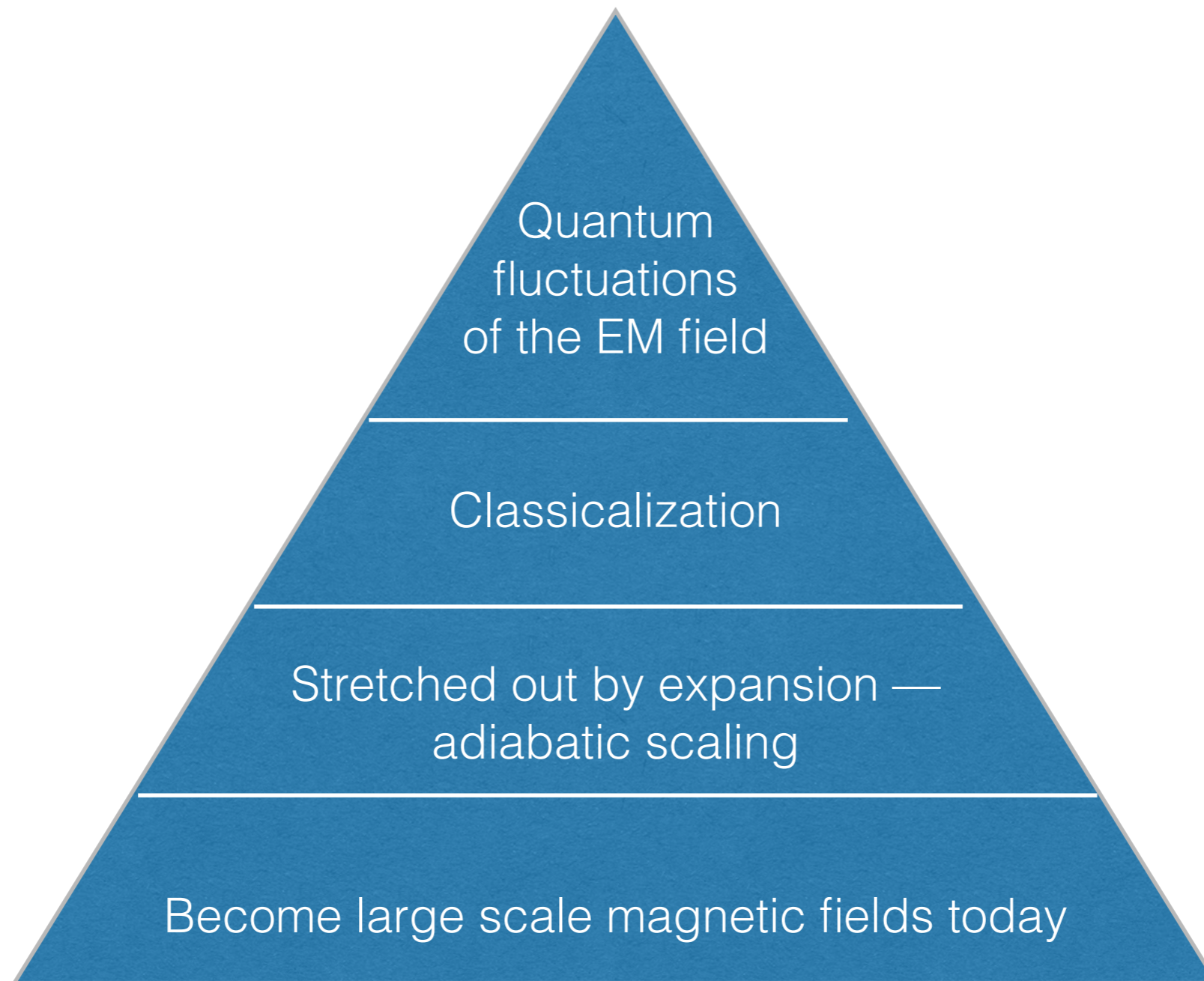
- Inflationary mechanisms — most interesting due to the very nature of inflation — large scale correlations
- Standard Maxwell action is conformally invariant — the electromagnetic fluctuations do not grow in any conformally flat background like FRW.
- A necessary condition — break conformal invariance of the Maxwell theory. (Turner & Widrow, 1988, Ratra, 1992)
- Various possible couplings:

- Kinetic coupling:  $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$

- Axial coupling:  $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$

# *Inflationary magnetogenesis*

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# Inflationary magnetogenesis — basic formalism

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The parity violating term is introduced to the action as

$$S[A^\mu] = -\frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ J^2(\phi) F_{\mu\nu} F^{\mu\nu} - \frac{\gamma}{2} J^2(\phi) F_{\mu\nu} \tilde{F}^{\mu\nu} \right],$$

where  $\tilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta} / \sqrt{-g}) F_{\alpha\beta}$ .

The equation of motion has the form

$$\mathcal{A}_k^\sigma{}'' + \left( k^2 + \frac{2\sigma\gamma k J'}{J} - \frac{J''}{J} \right) \mathcal{A}_k^\sigma = 0,$$

where  $\sigma = \pm 1$  represents positive and negative helicity.

The power spectra of the helical magnetic and electric fields are given by

$$\begin{aligned} \mathcal{P}_B(k) &= \frac{k^5}{4\pi^2 a^4} \left[ |\mathcal{A}_k^+|^2 + |\mathcal{A}_k^-|^2 \right], \\ \mathcal{P}_E(k) &= \frac{k^3}{4\pi^2 a^4} \left[ \left| \mathcal{A}_k^{+'} - \frac{J'}{J} \mathcal{A}_k^+ \right|^2 + \left| \mathcal{A}_k^{-'} - \frac{J'}{J} \mathcal{A}_k^- \right|^2 \right]. \end{aligned}$$

# Inflationary magnetogenesis with kinetic coupling

Power law coupling  $J(\eta) = \left[ \frac{a(\eta)}{a(\eta_e)} \right]^n = \left( \frac{\eta}{\eta_e} \right)^{-n},$

$$\mathcal{P}_B(k) = \frac{H_I^4}{8\pi} \mathcal{F}(m) (-k\eta_e)^{2m+6}, \quad m = \begin{cases} n, & \text{for } n < -\frac{1}{2} \\ -n - 1, & \text{for } n > -\frac{1}{2} \end{cases}$$

$$\mathcal{P}_E(k) = \frac{H_I^4}{8\pi} \mathcal{G}(m) (-k\eta_e)^{2m+4}, \quad m = \begin{cases} n, & \text{for } n < \frac{1}{2} \\ 1 - n, & \text{for } n > \frac{1}{2} \end{cases}$$

$$\mathcal{F}(m) = \frac{1}{2^{2m+1} \cos^2(m\pi) \Gamma^2(m + 3/2)}, \quad n_B = \begin{cases} 2n + 6, & \text{for } n < -\frac{1}{2} \\ 4 - 2n, & \text{for } n > -\frac{1}{2} \end{cases}$$

$$\mathcal{G}(m) = \frac{1}{2^{2m-1} \cos^2(m\pi) \Gamma^2(m + 1/2)}, \quad n_E = \begin{cases} 2n + 4, & \text{for } n < \frac{1}{2} \\ 6 - 2n, & \text{for } n > \frac{1}{2} \end{cases}$$



# Inflationary magnetogenesis — kinetic + helical

---

Power law coupling  $J(\eta) = \left[ \frac{a(\eta)}{a(\eta_e)} \right]^n = \left( \frac{\eta}{\eta_e} \right)^{-n},$

$$\mathcal{P}_B(k) = \frac{H_I^4}{8\pi^2} \frac{\Gamma^2(|2n+1|)}{|\Gamma(\frac{1}{2} + in\gamma + |n + \frac{1}{2}|)|^2} \times \frac{\cosh(n\pi\gamma)}{2^{|2n+1|-2}} (-k\eta_e)^{5-|2n+1|}.$$

$$\mathcal{P}_E(k) = \frac{H_I^4}{4\pi^2} \frac{\Gamma^2(2|n|)}{|\Gamma(|n| + in\gamma)|^2} \frac{\gamma^2}{1 + \gamma^2} \frac{\cosh(n\pi\gamma)}{2^{2|n|-2}} (-k\eta_e)^{4-2|n|}$$

$$n_B = 5 - |2n + 1|, \quad n_E = 4 - 2|n|.$$

# *Constraints for successful magnetogenesis*

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- Background
  - Strong coupling problem
  - Backreaction issue
- Perturbations
  - Power spectrum constraints
  - Induced bispectrum etc..
- Energy scale of inflation (from tensor modes)
- Schwinger effect — strong E field induces charged particle production



# EM fields power spectra – single field slow roll

In terms of e-folds, the coupling function is expected to be

$$J(N) = \exp [n (N - N_e)].$$

The Klein-Gordon equation for inflaton field is

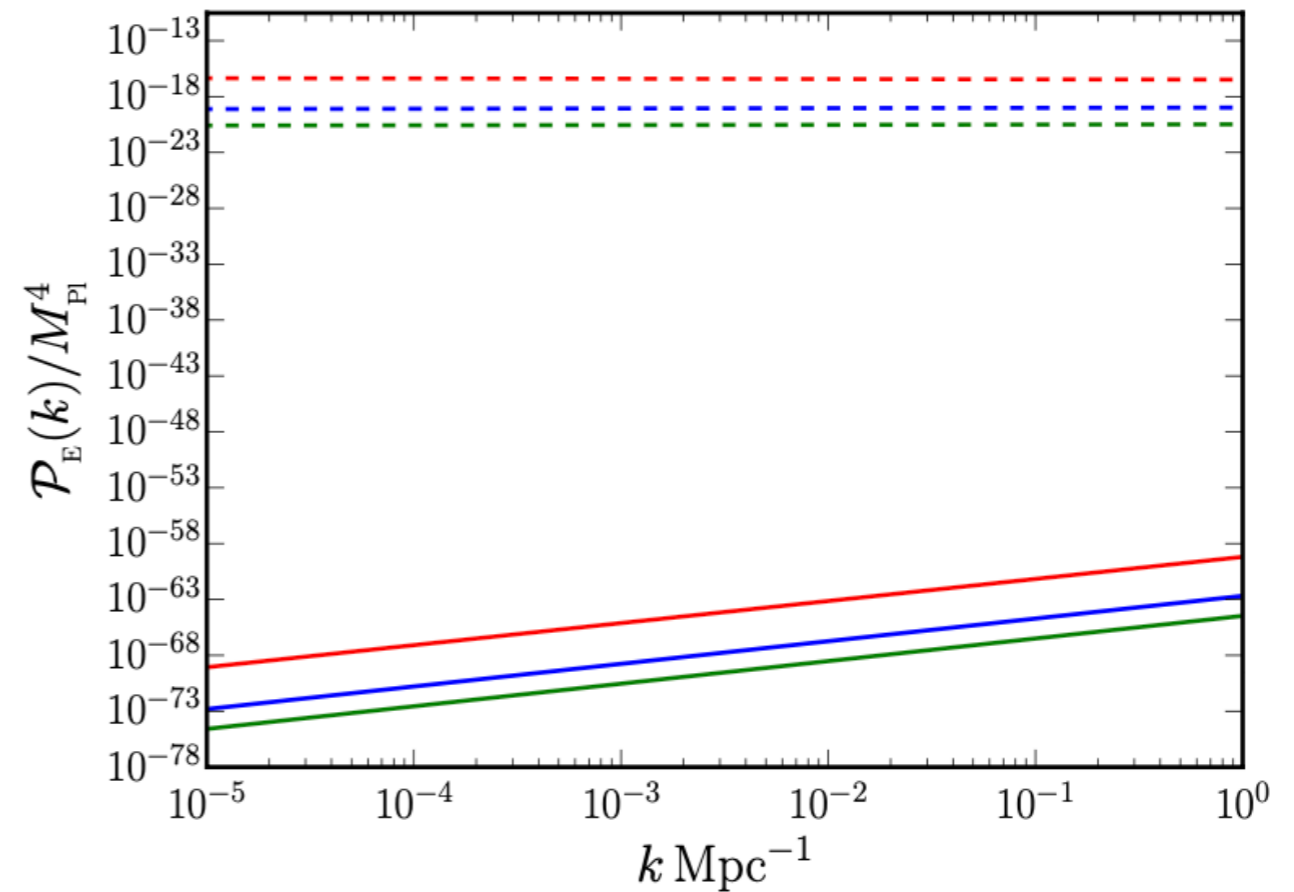
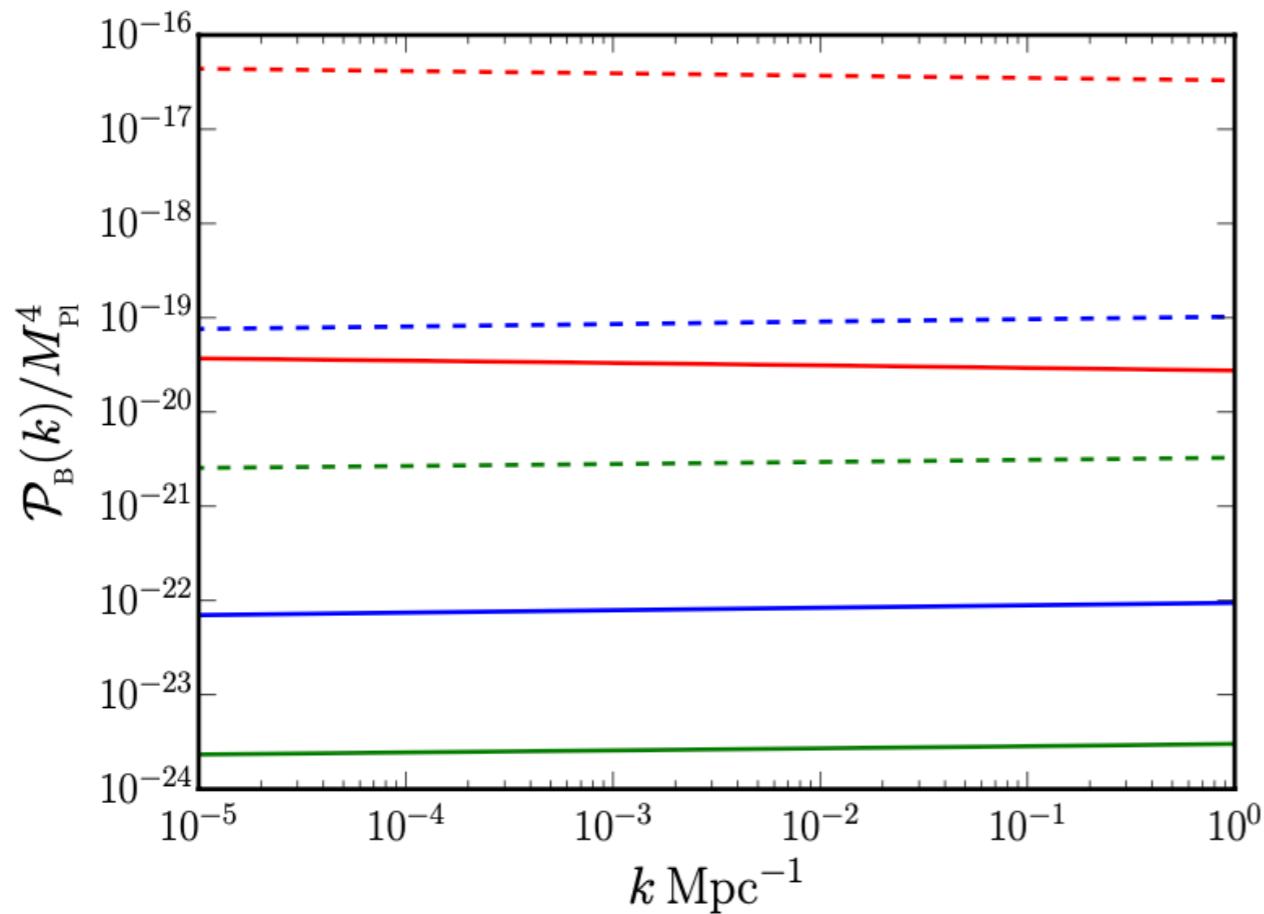
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0.$$

SR Model	Potential	Coupling function $[J(\phi)]$
Quadratic potential (QP)	$\frac{m^2}{2} \phi^2$	$\exp \left[ -\frac{n}{4M_{\text{Pl}}^2} (\phi^2 - \phi_e^2) \right]$
Small field model (SFM)	$V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^q \right]$	$\left( \frac{\phi}{\phi_e} \right)^{n\mu^2/2M_{\text{Pl}}^2} \exp \left[ -\frac{n}{4M_{\text{Pl}}^2} (\phi^2 - \phi_e^2) \right]$
First Starobinsky model (FSM)	$V_0 \left[ 1 - \exp \left( -\sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{Pl}}} \right) \right]^2$	$\exp \left\{ -\frac{3n}{4} \left[ \exp \left( \sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{Pl}}} \right) - \exp \left( \sqrt{\frac{2}{3}} \frac{\phi_e}{M_{\text{Pl}}} \right) - \sqrt{\frac{2}{3}} \left( \frac{\phi}{M_{\text{Pl}}} - \frac{\phi_e}{M_{\text{Pl}}} \right) \right] \right\}$

# EM fields power spectra in slow roll

$$\mathcal{P}_B(k) = \frac{k^5 J^2}{2\pi^2 a^4} |\bar{A}_k|^2 = \frac{k^5}{2\pi^2 a^4} |\mathcal{A}_k|^2,$$

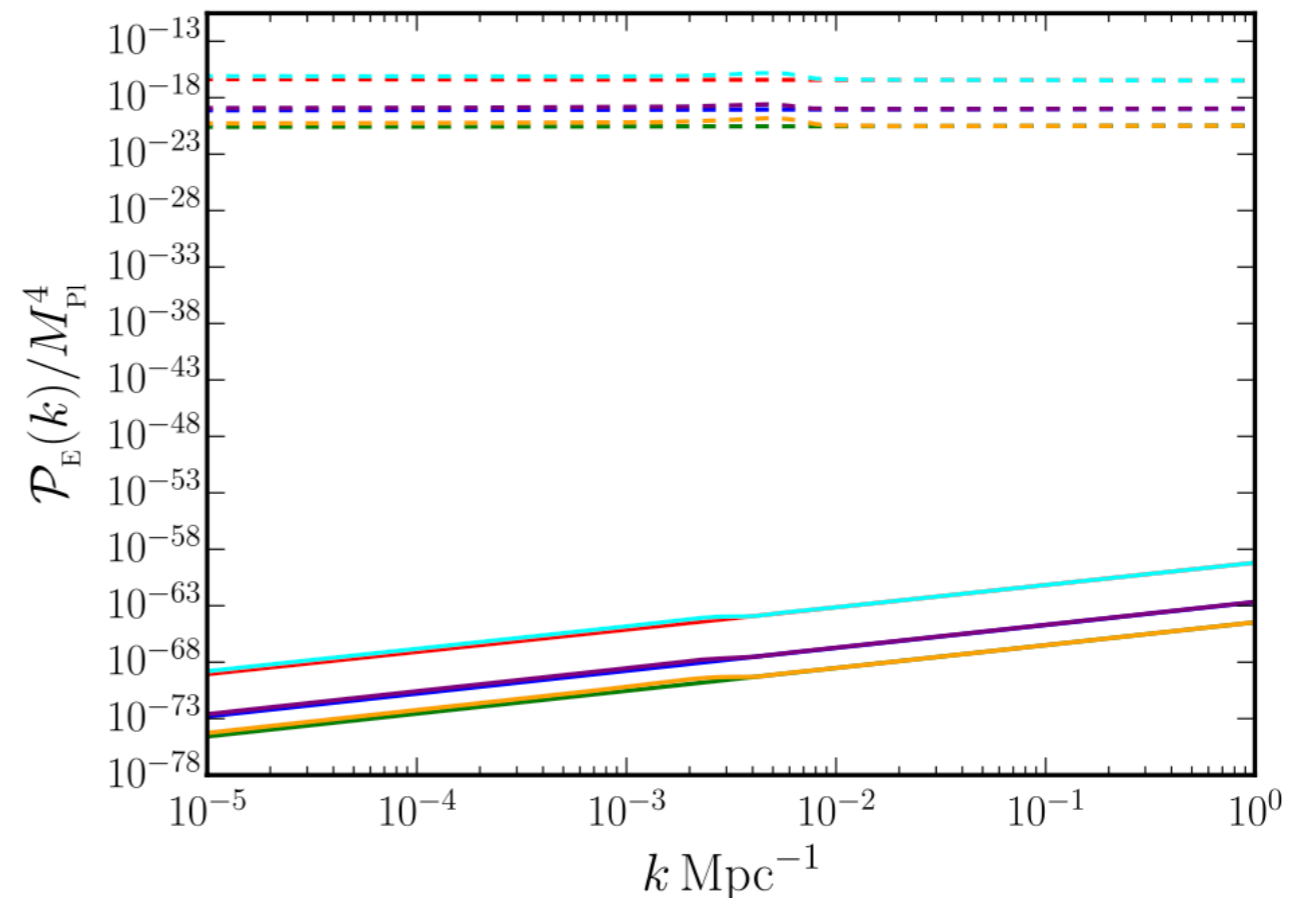
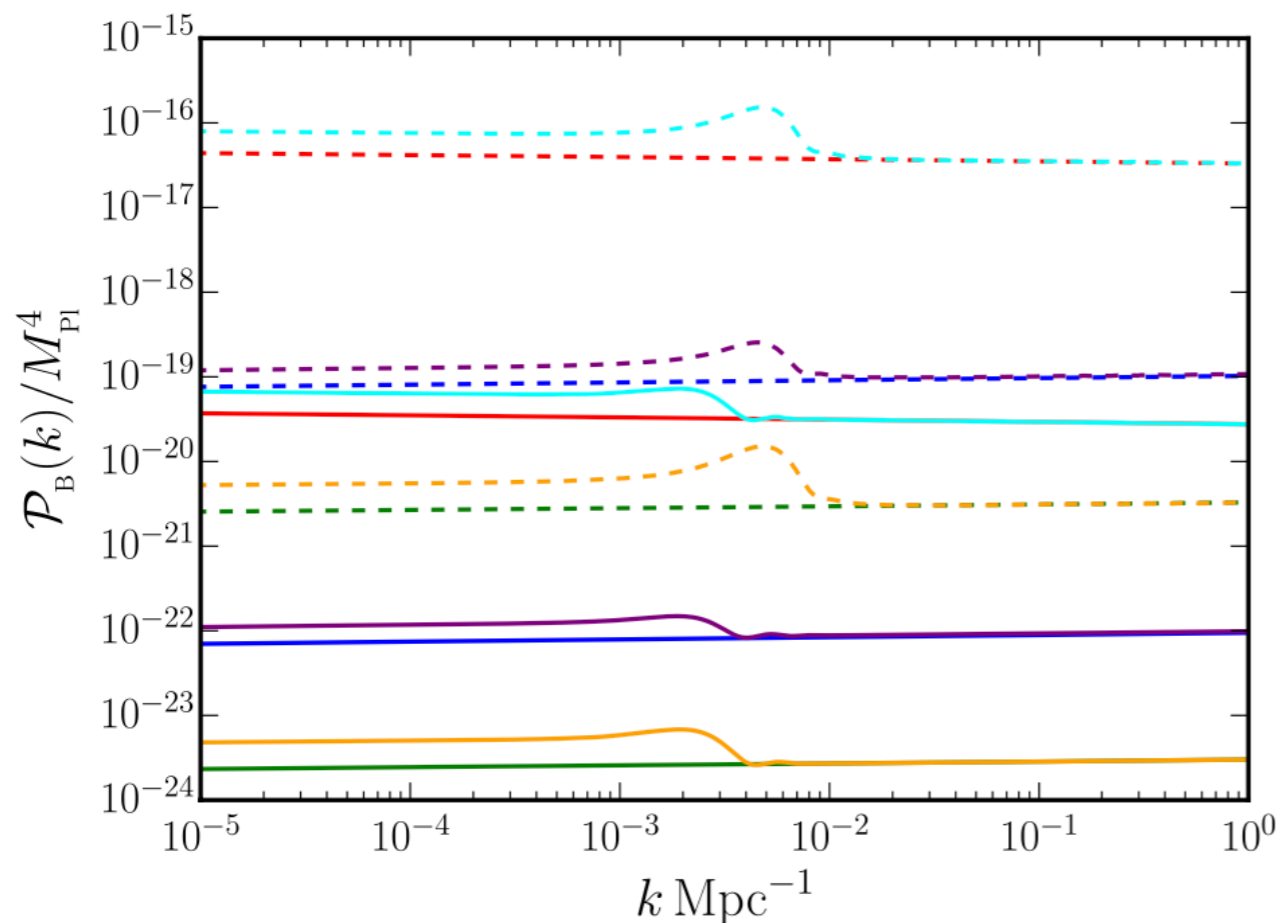
$$\mathcal{P}_E(k) = \frac{k^3 J^2}{2\pi^2 a^4} |\bar{A}'_k|^2 = \frac{k^3}{2\pi^2 a^4} \left| \mathcal{A}'_k - \frac{J'}{J} \mathcal{A}_k \right|^2.$$



# EM spectra — mild deviations from slow roll

Features in the inflaton potential — features in the scalar perturbation spectrum — possible explanation of CMB anomalies

$$V_{\text{step}}(\phi) = V(\phi) \left[ 1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right) \right],$$

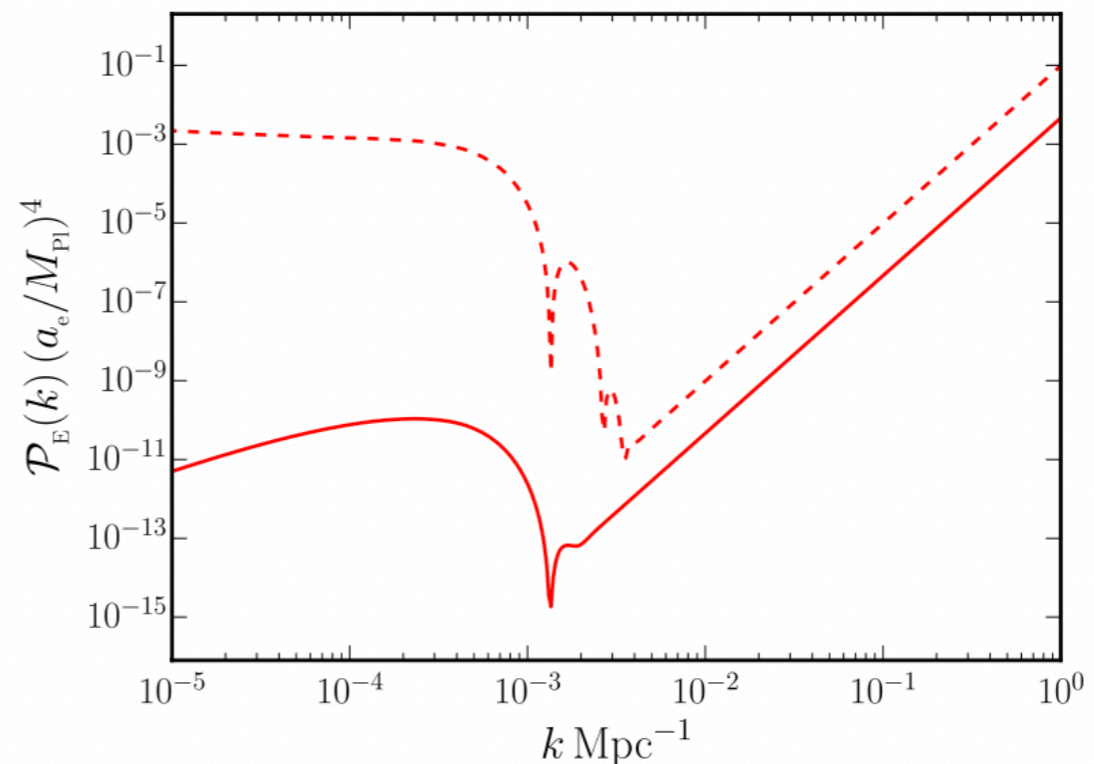
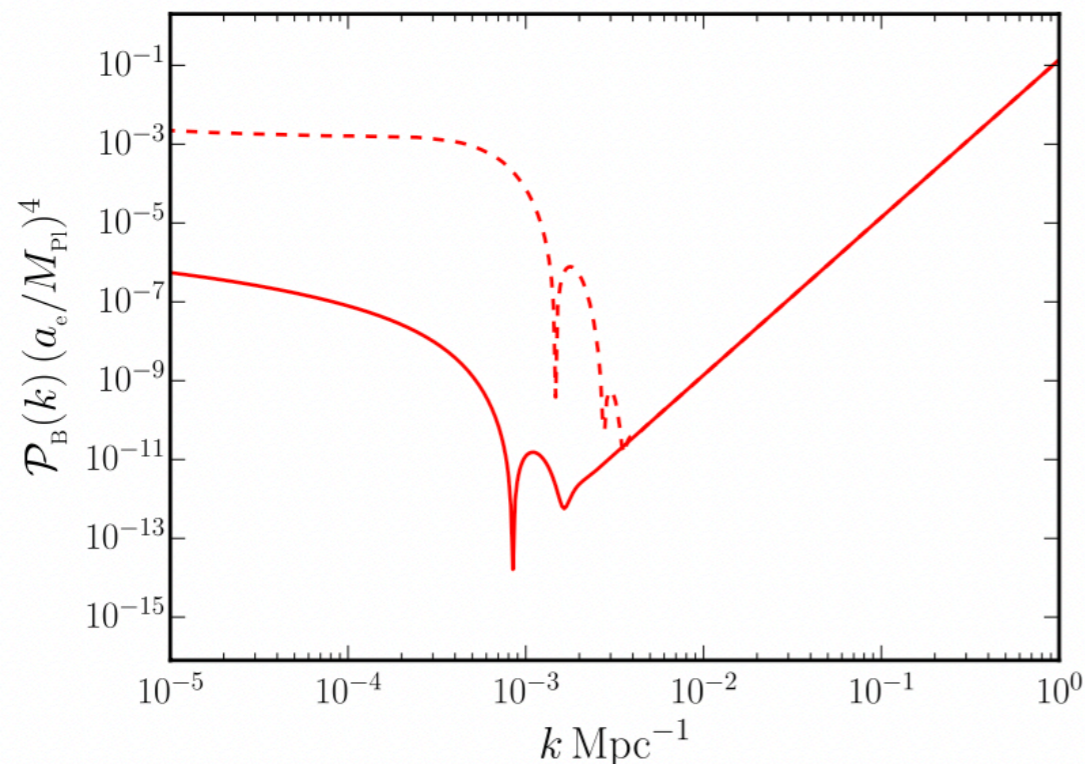
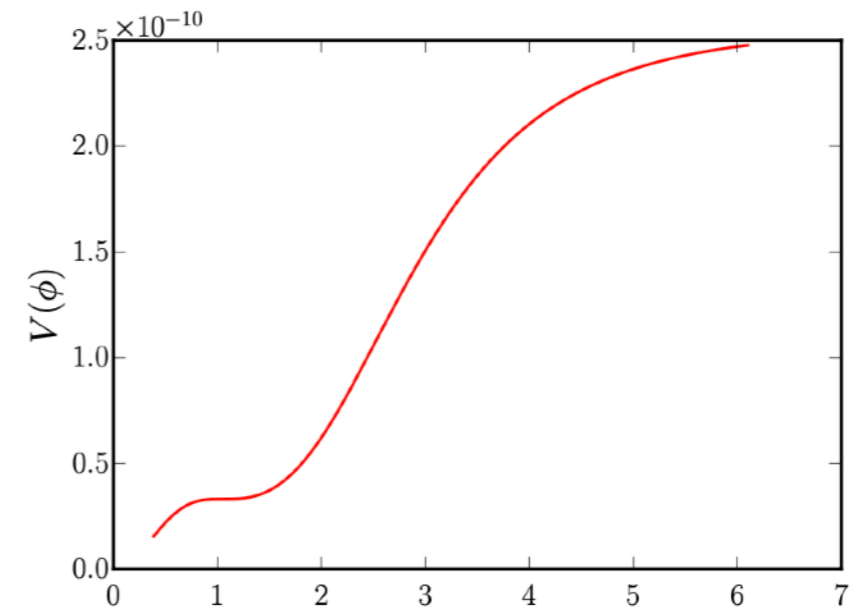


Tripathy, Chowdhury, **RKJ** & Sriramkumar, PRD 105, 063519 (2022)

# EM spectra — ultra slow roll

$$V(\phi) = \frac{m^2}{2} \phi^2 - \frac{2m^2}{3\phi_0} \phi^3 + \frac{m^2}{4\phi_0^2} \phi^4.$$

Polynomial potential — allows USR



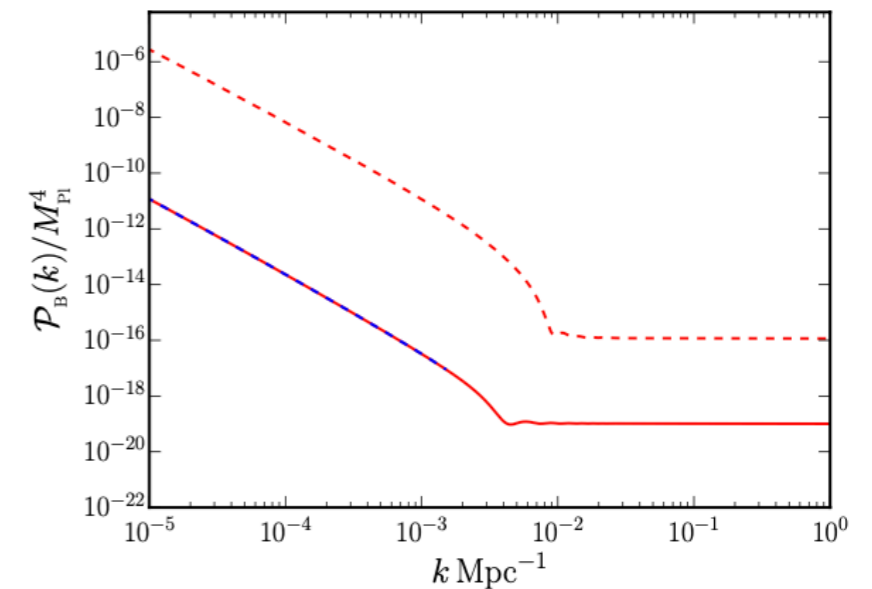
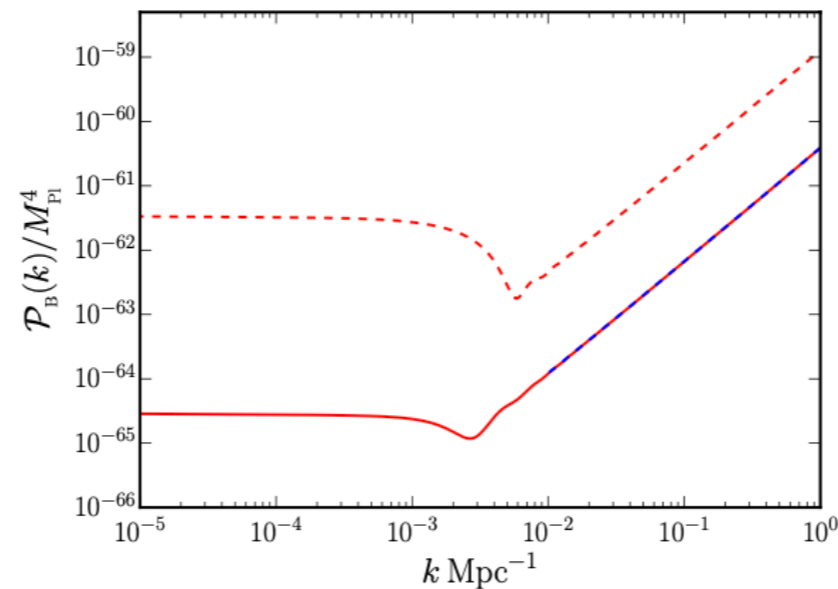
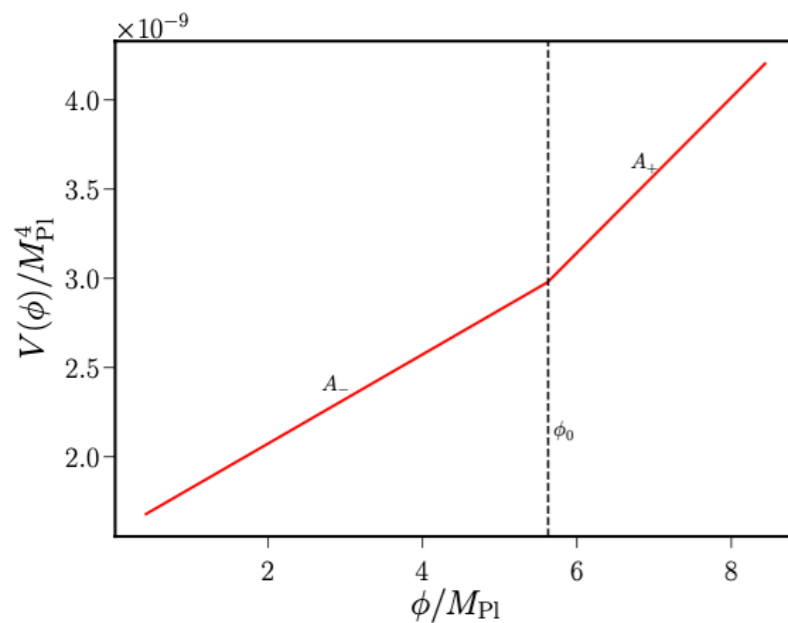
Tripathy, Chowdhury, **RKJ** & Sriramkumar, PRD 105, 063519 (2022)



# EM spectra — strong deviations from slow roll

Starobinsky model is described by the potential

$$V(\phi) = \begin{cases} V_0 + A_+ (\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_- (\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$

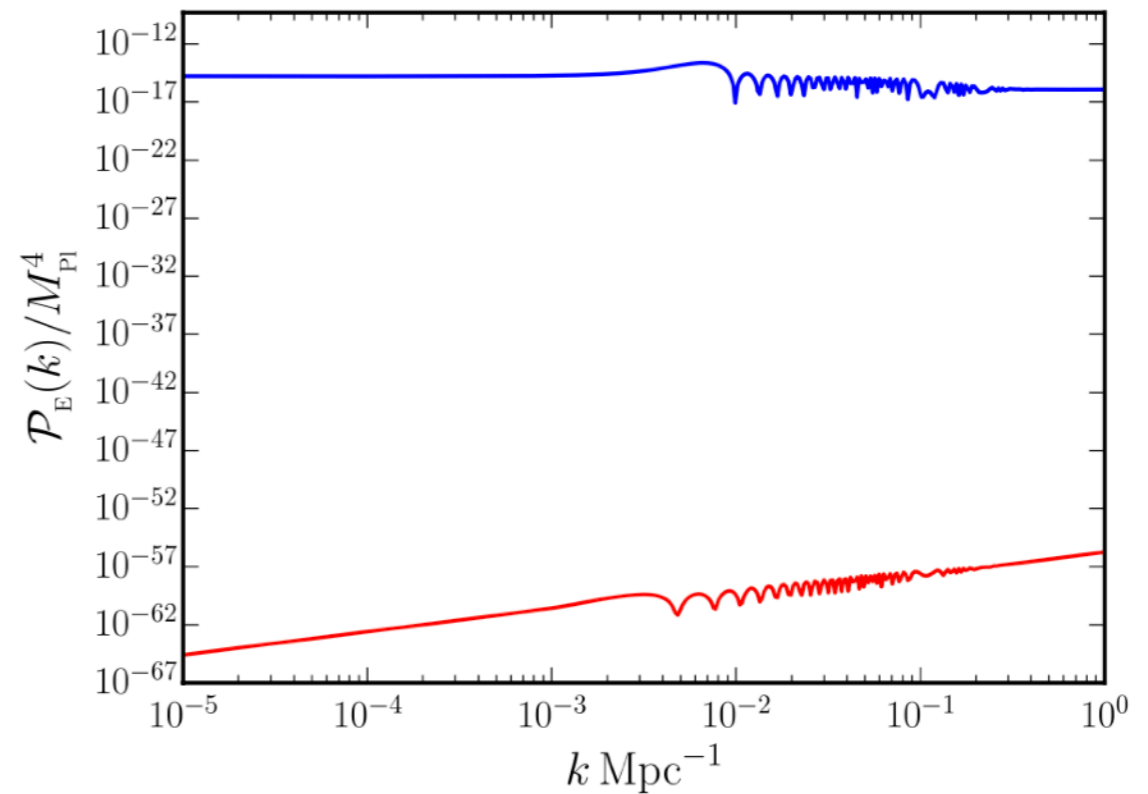
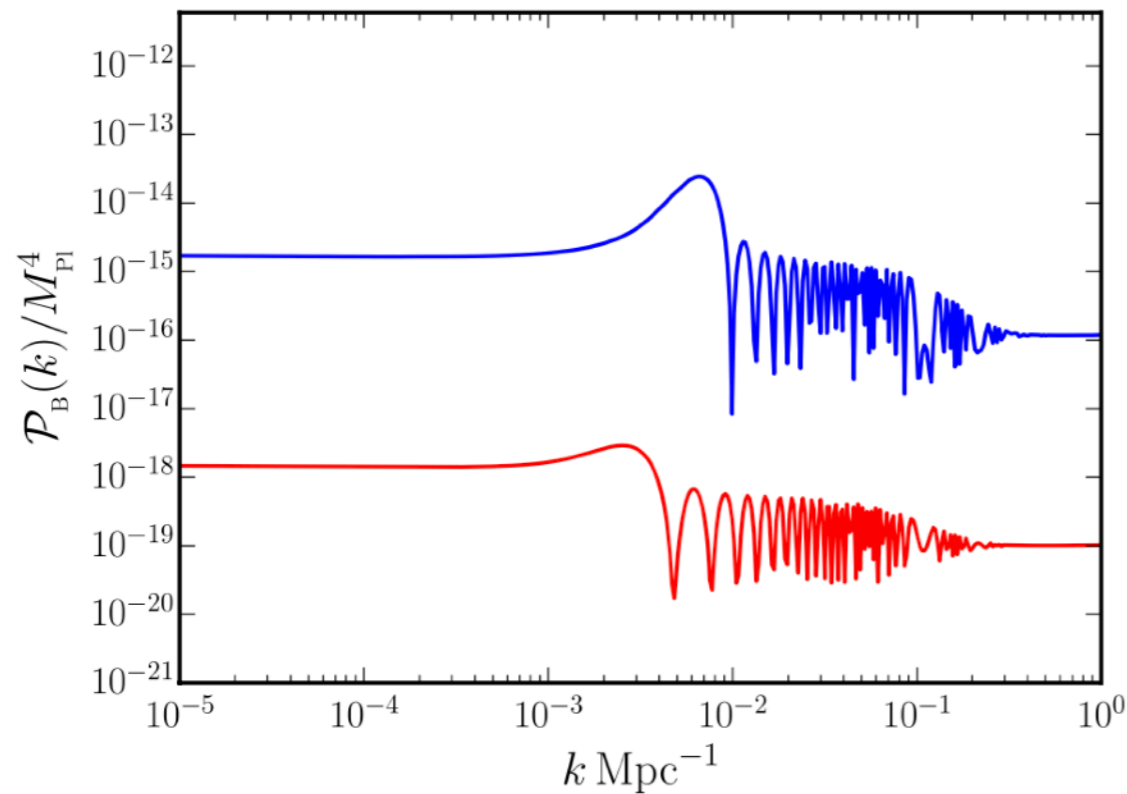


Tripathy, Chowdhury, **RKJ** & Sriramkumar, PRD 105, 063519 (2022)

# EM spectra — strong deviations from slow roll

$$V(\phi) = V_0 + \frac{1}{2}(A_+ + A_-)(\phi - \phi_0) + \frac{1}{2}(A_+ - A_-)(\phi - \phi_0) \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right),$$

$$J(\phi) = \frac{J_1}{2J_{0+}} \left[ 1 + \tanh\left(\frac{\phi - \phi_0}{\Delta\phi_1}\right) \right] J_+(\phi) + \frac{J_1}{2J_{0-}} \left[ 1 - \tanh\left(\frac{\phi - \phi_0}{\Delta\phi_1}\right) \right] J_-(\phi),$$



# PMF from two field models

Action for two field models

$$S[\phi, \chi] = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{f(\phi)}{2} \partial_\nu \chi \partial^\nu \chi - V(\phi, \chi) \right]$$

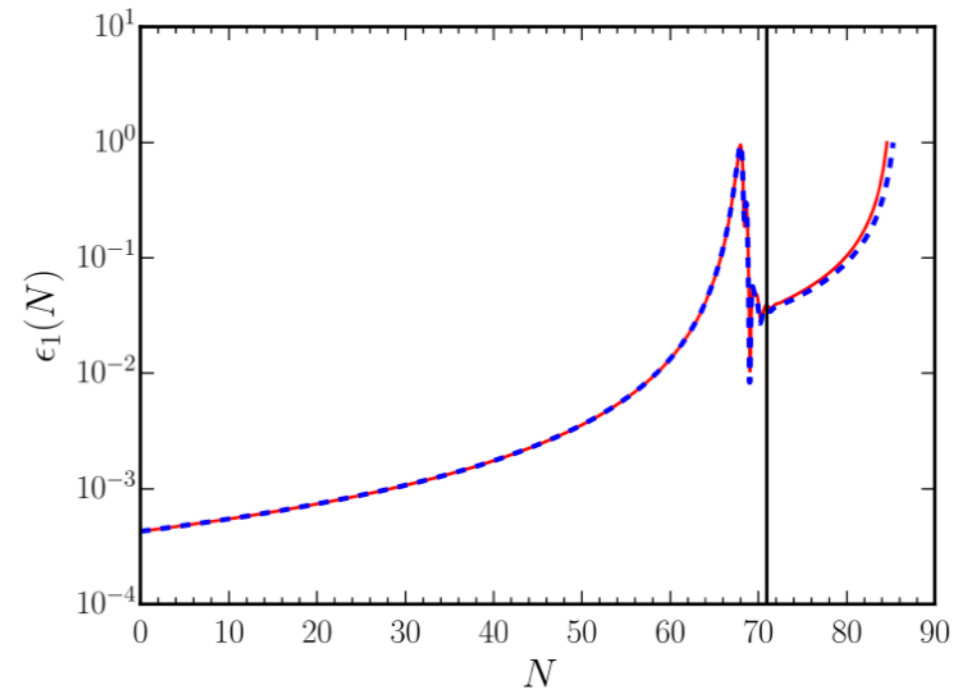
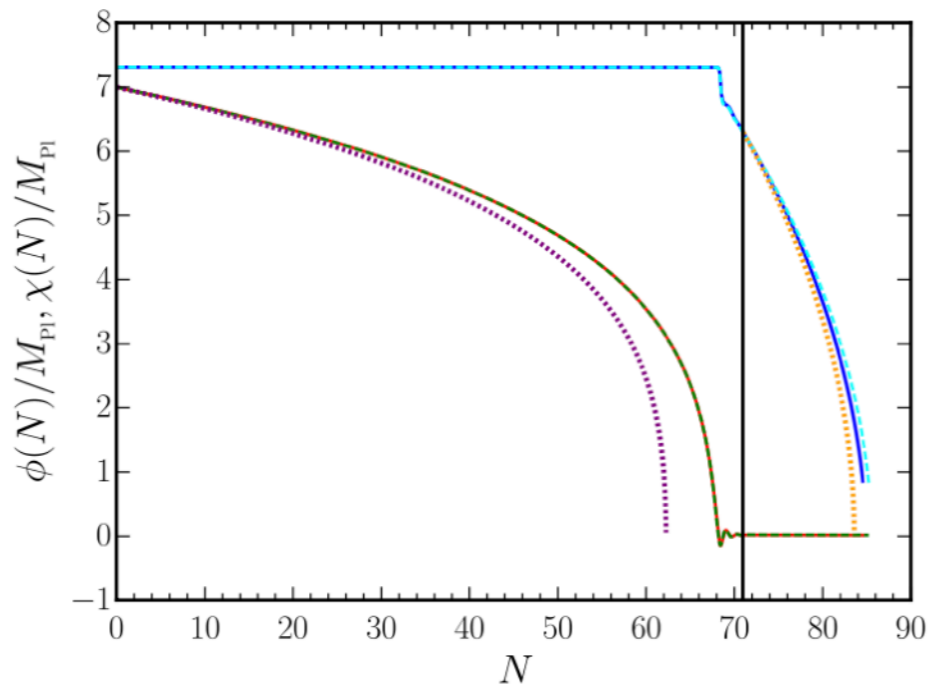
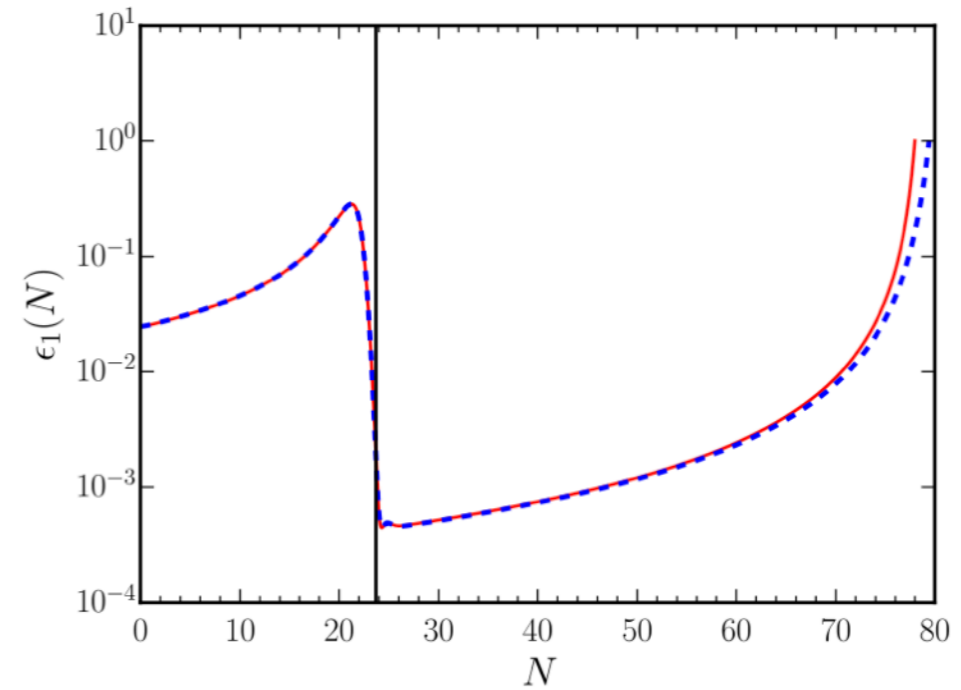
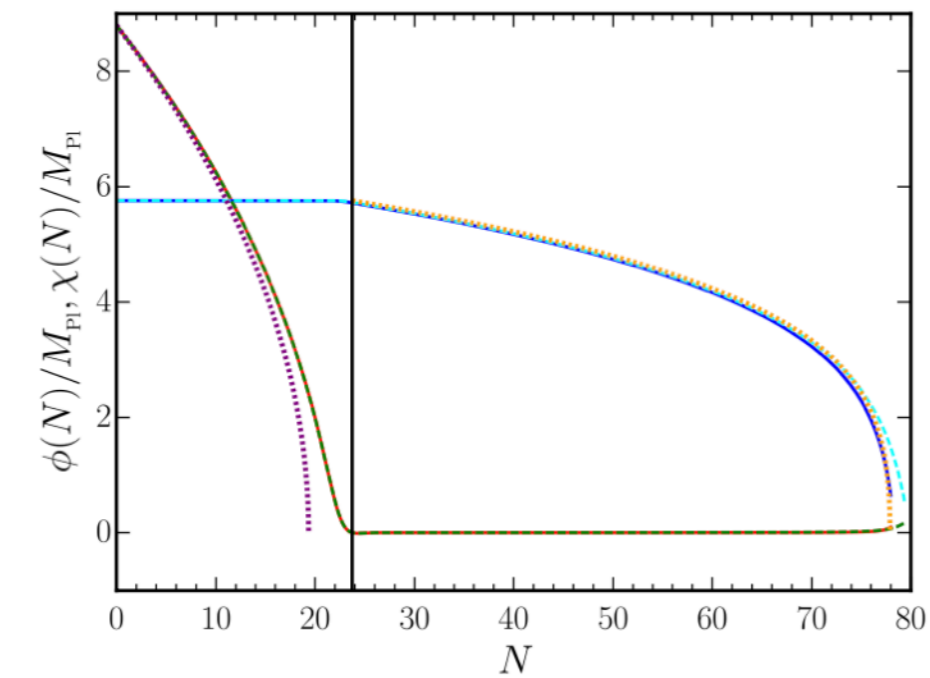
Equations of motion

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V_\phi &= b_\phi e^{2b\dot{\chi}^2}, & H^2 &= \frac{1}{3M_{\text{Pl}}^2} \left( \frac{\dot{\phi}^2}{2} + e^{2b} \frac{\dot{\chi}^2}{2} + V \right), \\ \ddot{\chi} + (3H + 2b_\phi \dot{\phi})\dot{\chi} + e^{-2b} V_\chi &= 0, & \dot{H} &= -\frac{1}{2M_{\text{Pl}}^2} (\dot{\phi}^2 + e^{2b} \dot{\chi}^2). \end{aligned}$$

Two different representative potentials

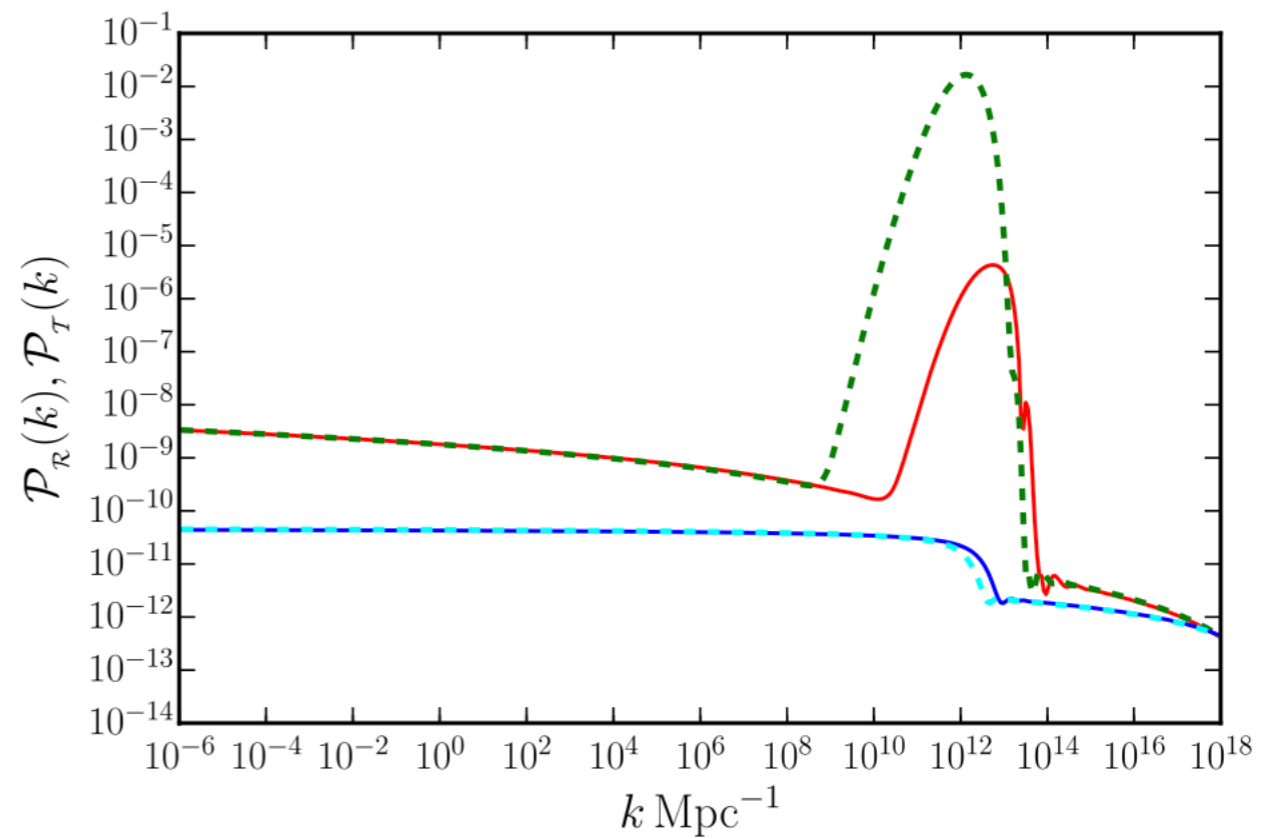
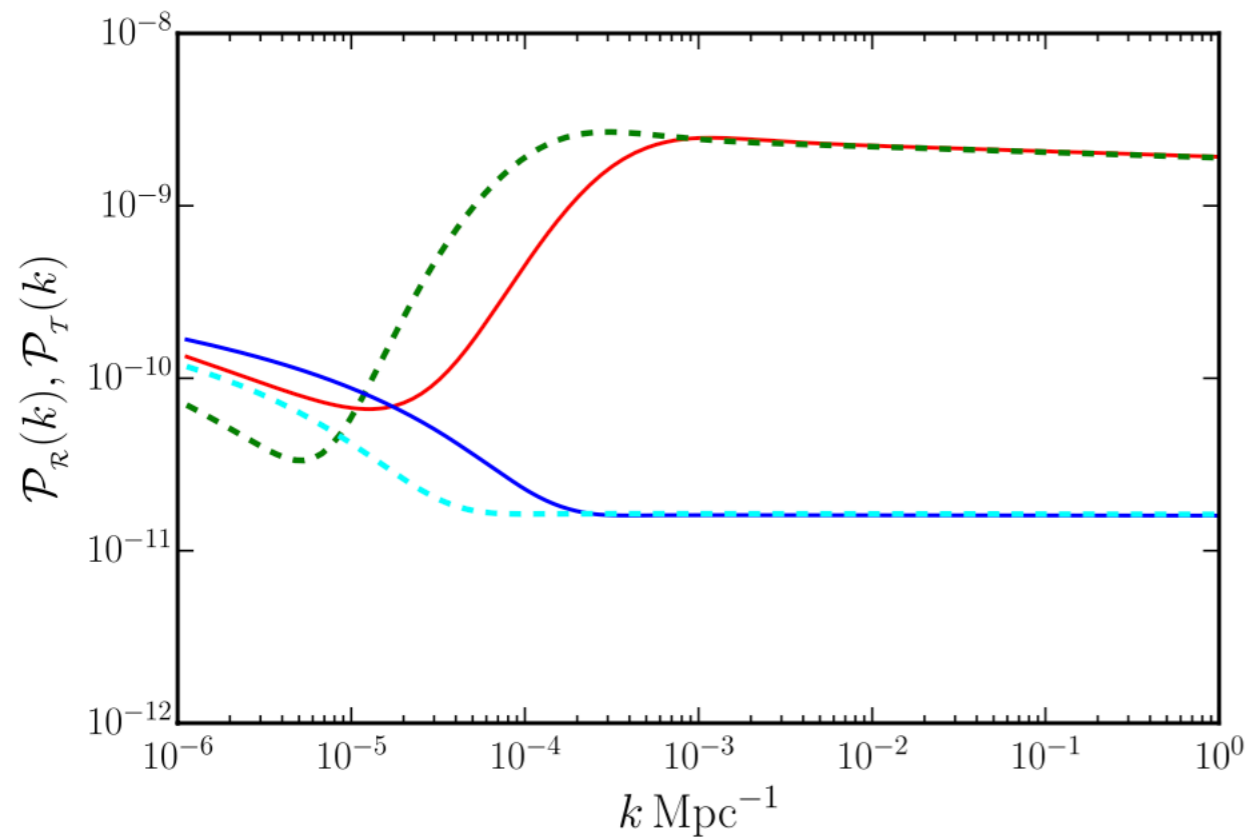
$$V(\phi, \chi) = \frac{m_\phi^2}{2} \phi^2 + V_0 \frac{\chi^2}{\chi_0^2 + \chi^2}, \quad V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2} \chi^2.$$

# Background dynamics — two field models



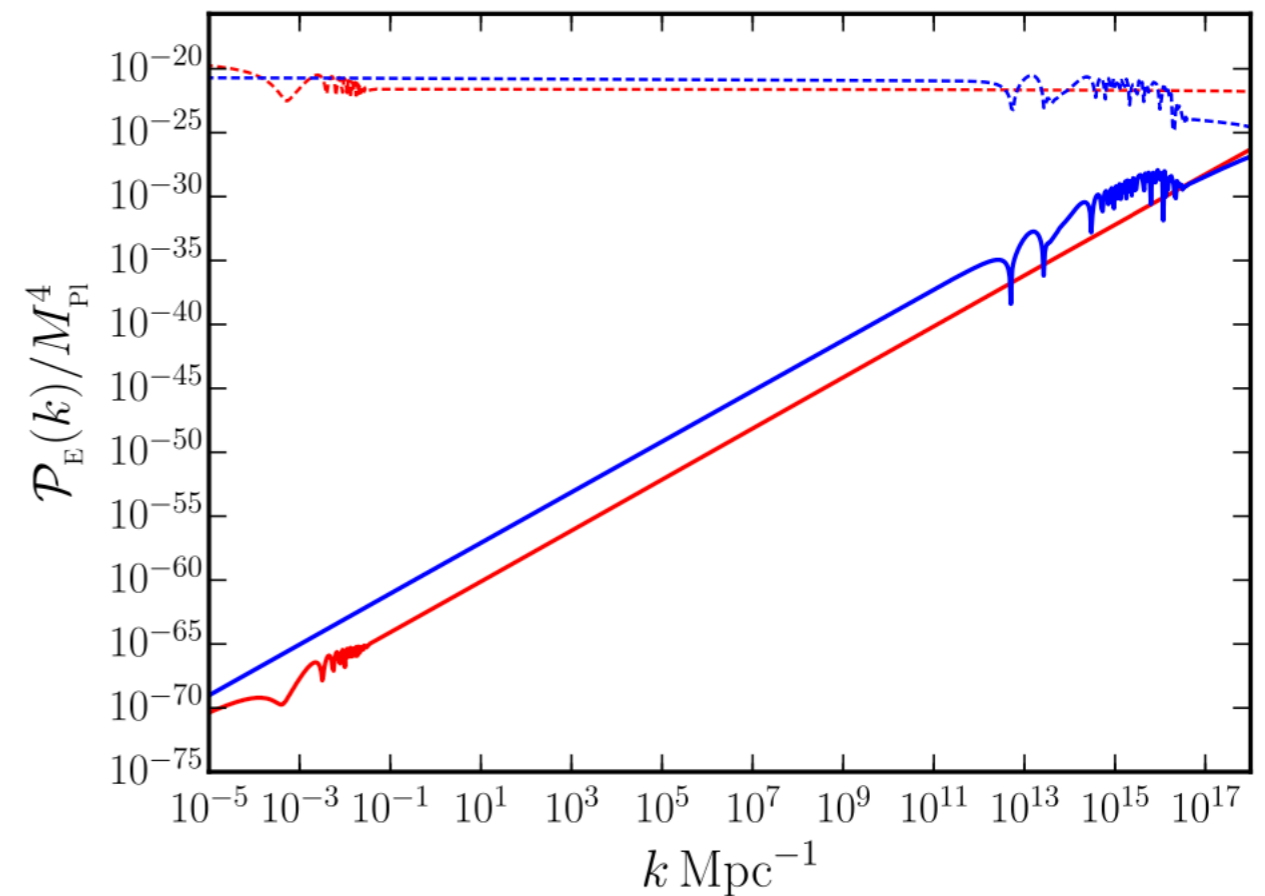
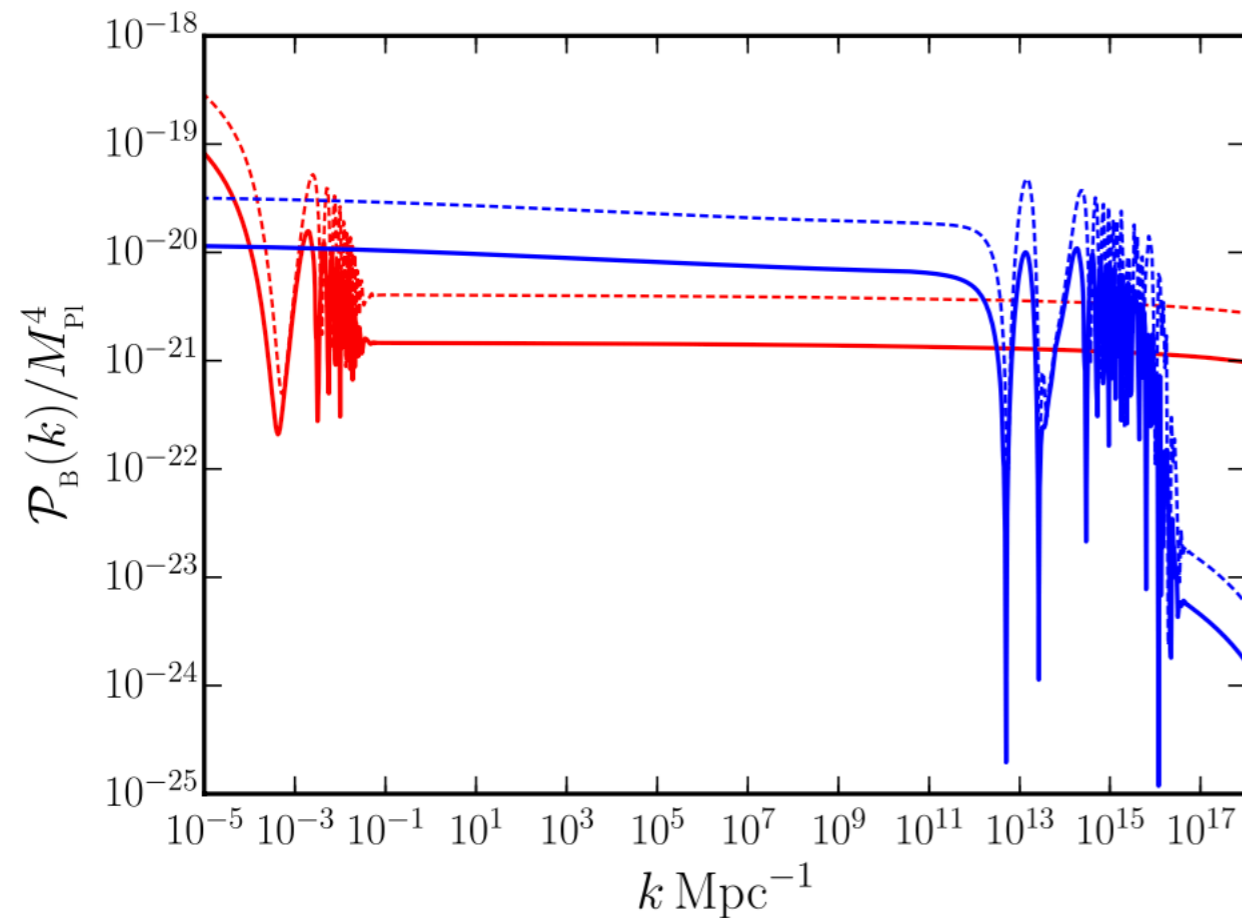


# Primordial scalar and tensor power spectra



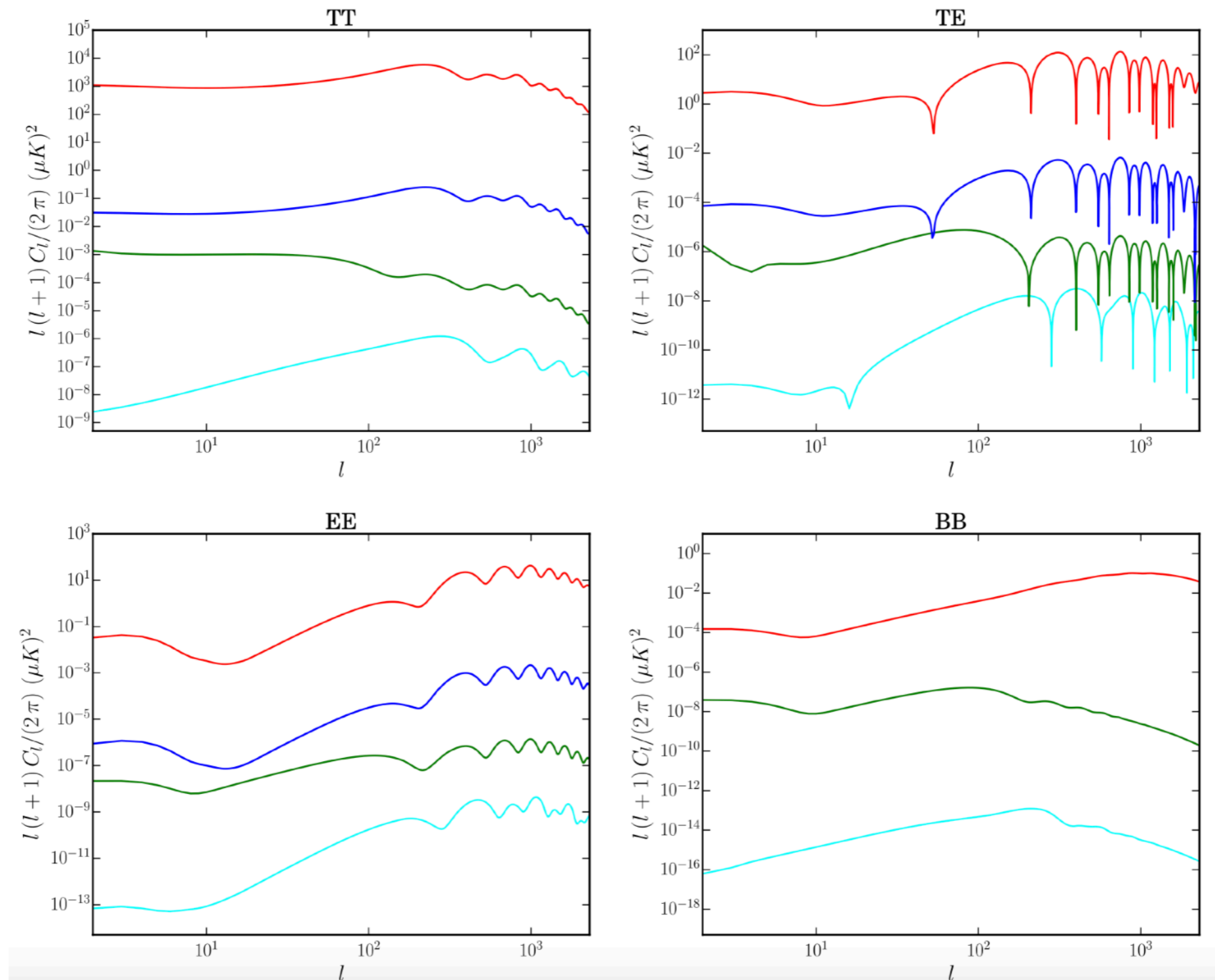
Tripathy, Chowdhury, Ragavendra, **RKJ** & Sriramkumar, PRD 107, 043501 (2023)

# *EM power spectra for two field models*



Tripathy, Chowdhury, Ragavendra, **RKJ** & Sriramkumar, PRD 107, 043501 (2023)

# Imprints of PMF on CMB



Tripathy, Chowdhury, Ragavendra, **RKJ** & Sriramkumar, PRD 107, 043501 (2023)

*Non-Gaussian imprints of  
inflationary magnetic fields*



# *Non-Gaussian imprints of inflationary magnetic fields*

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- Inflationary magnetogenesis — excitation of gauge fields during inflation — a non-trivial cross-correlation of primordial curvature perturbation with magnetic fields.
- Cross-correlations are non-Gaussian in nature — important to understand their strength in a specific scenario.
- A model-independent calculation can not be done as these correlations depend on the coupling function.

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(k_3) \rangle$$

# Primordial non-Gaussianities from inflation

- The primordial perturbations are encoded in the two-point function or the power spectrum

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1)$$

- A non-vanishing three-point function is a signal of primordial non-Gaussianities

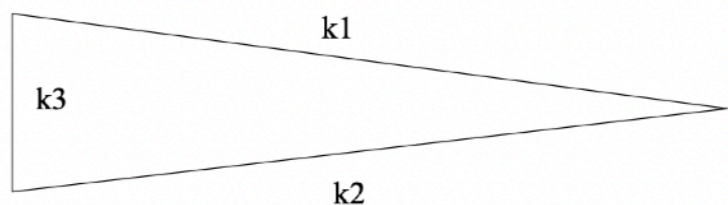
$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$$

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1$$

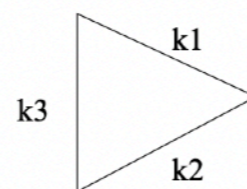
- Introduce  $f_{\text{NL}}$  as a measure of primordial non-Gaussianities

$$f_{\text{NL}} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) + \text{perm.}$$

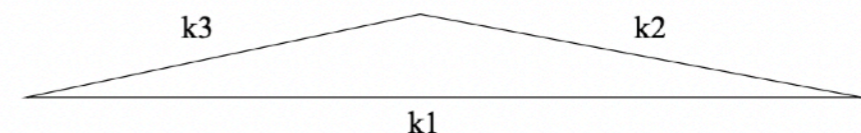
(a) Squeezed



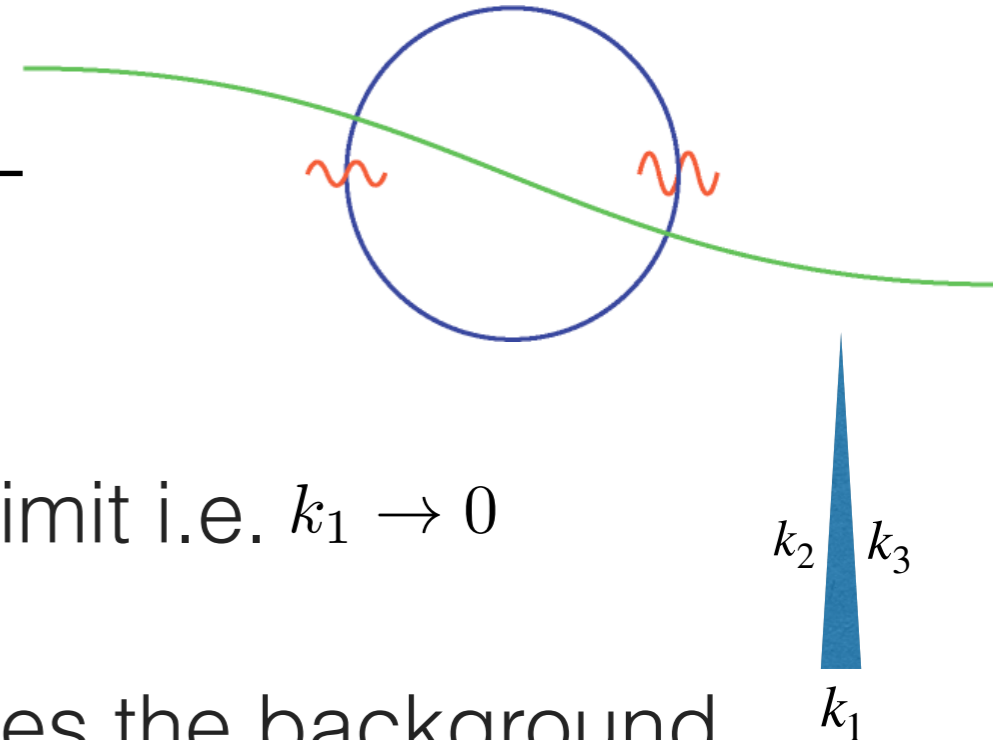
(b) Equilateral



(c) Flattened/Folded



# Semi-classical estimate in the squeezed limit



- Squeezed limit:  $k_1 \ll k_2 \sim k_3$
- Consider  $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$  in the squeezed limit i.e.  $k_1 \rightarrow 0$
- The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t) e^{2\zeta(t, \mathbf{x})} d\mathbf{x}^2$$

- Taylor expand in the rescaled background

$$\langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} = \langle \zeta_{k_2} \zeta_{k_3} \rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle + \dots$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1} \zeta_{k_1} \rangle k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$f_{NL}^{\text{local}} = -(n_s - 1)$$

Maldacena, JHEP 0305, 013 (2002)

# Non-Gaussian cross-correlation with magnetic fields

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- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle \zeta(\mathbf{k}_1) \mathbf{B}(\mathbf{k}_2) \cdot \mathbf{B}(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\zeta BB}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

$$B_{\zeta BB}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \equiv b_{NL} P_{\zeta}(k_1) P_B(k_2)$$

- *Local* resemblance between  $f_{NL}$  and  $b_{NL}$

$$\zeta = \zeta^{(G)} + \frac{3}{5} f_{NL}^{local} \left( \zeta^{(G)} \right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2} b_{NL}^{local} \zeta^{(G)} \mathbf{B}^{(G)}$$

**RKJ** & Sloth, Phys. Rev. D 86, 123528 (2012)



# A novel magnetic consistency relation

- For a kinetic coupling  $\lambda(\phi)F_{\mu\nu}F^{\mu\nu}$ , using our semi-classical approach, the cross-correlation becomes

$$\begin{aligned} & \langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle \\ &= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2) \end{aligned}$$

- With  $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$ , we obtain  $b_{NL} = n_B - 4$

In the squeezed limit  $k_1 \ll k_2, k_3 = k$

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k}_3) \rangle = (n_B - 4) (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)$$

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1) (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)$$

**RKJ** & Sloth, Phys. Rev. D 86, 123528 (2012)

# Full in-in calculation

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- One has to cross-check the consistency relation by doing a complete in-in calculation.

- The final result is

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) |\zeta_{k_1}^{(0)}(\tau_I)|^2 |A_{k_2}^{(0)}(\tau_I)| |A_{k_3}^{(0)}(\tau_I)| \\ \times \left[ \left( \mathbf{k}_2 \cdot \mathbf{k}_3 + \frac{(\mathbf{k}_2 \cdot \mathbf{k}_3)^3}{k_2^2 k_3^2} \right) k_2 k_3 \tilde{\mathcal{I}}_n^{(1)} + 2(\mathbf{k}_2 \cdot \mathbf{k}_3)^2 \tilde{\mathcal{I}}_n^{(2)} \right].$$

- The two integrals can be solved exactly for different values of  $n$ .

# Full in-in calculation

- **The flattened shape:** In this limit,  $k_1 = 2k_2 = 2k_3$ , the cross-correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle \simeq 96 \ln(-k_t \tau_I) P_\zeta(k_1) P_B(k_2)$$

- For the largest observable scale today,  $\ln(-k_t \tau_I) \sim -60$ ,

$$\left| b_{NL}^{flat} \right| \sim 5760$$

- **The squeezed limit:** In this limit,  $k_1 \rightarrow 0$  and

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

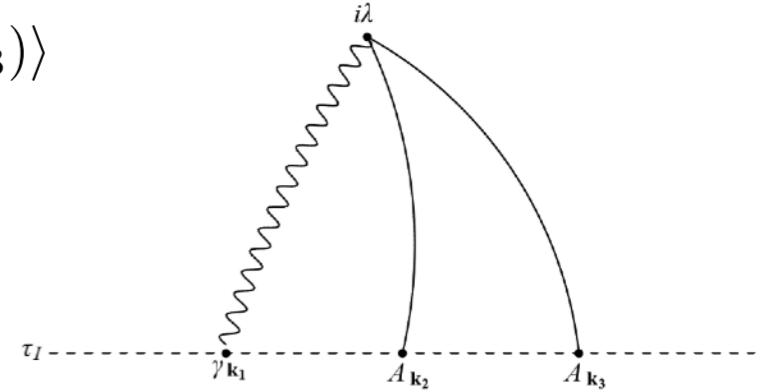
with  $b_{NL} = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$  in agreement with the consistency relation.

**RKJ** & Sloth, JCAP 1302, 003 (2013)

# Cross-correlations with gravitons

$$\langle \gamma(\mathbf{k}_1) \mathbf{A}(\mathbf{k}_2) \cdot \mathbf{A}(\mathbf{k}_3) \rangle, \quad \langle \gamma(\mathbf{k}_1) \mathbf{B}(\mathbf{k}_2) \cdot \mathbf{B}(\mathbf{k}_3) \rangle, \quad \langle \gamma(\mathbf{k}_1) \mathbf{E}(\mathbf{k}_2) \cdot \mathbf{E}(\mathbf{k}_3) \rangle$$

$$ds^2 = -dt^2 + a^2(t) [e^\gamma]_{ij} dx^i dx^j \approx -dt^2 + a^2(t) [\delta_{ij} + \gamma_{ij}] dx^i dx^j$$



In the squeezed (soft) limit

$$\lim_{k_1 \rightarrow 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \left( n - \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2)$$

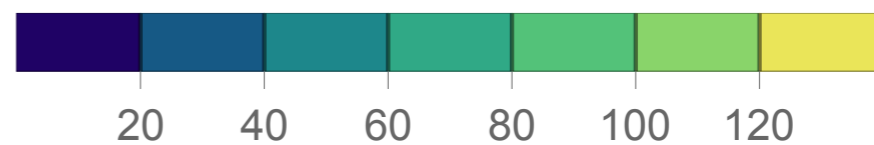
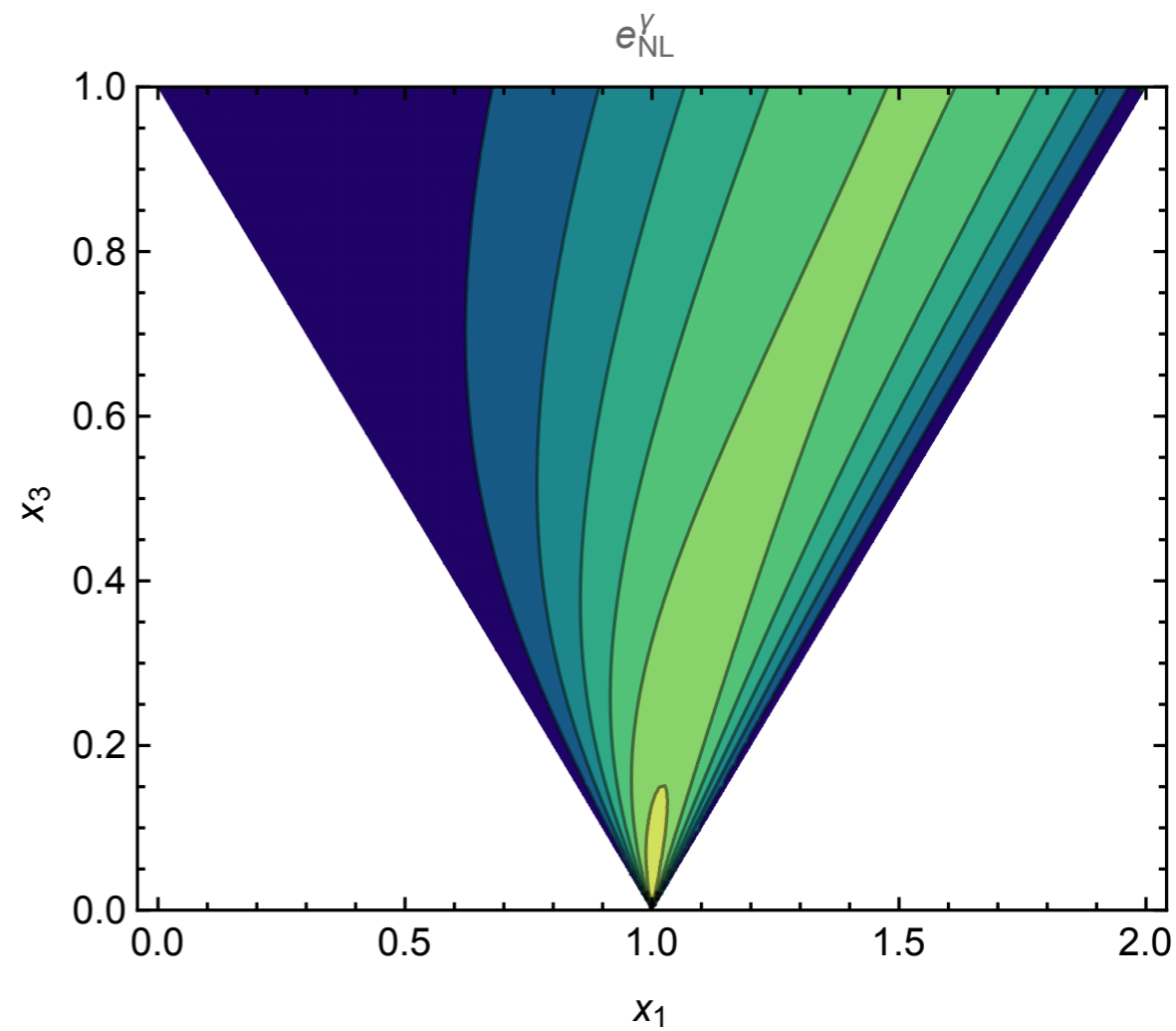
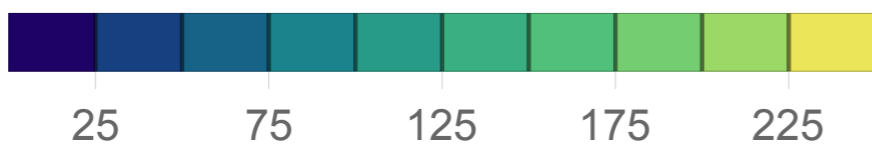
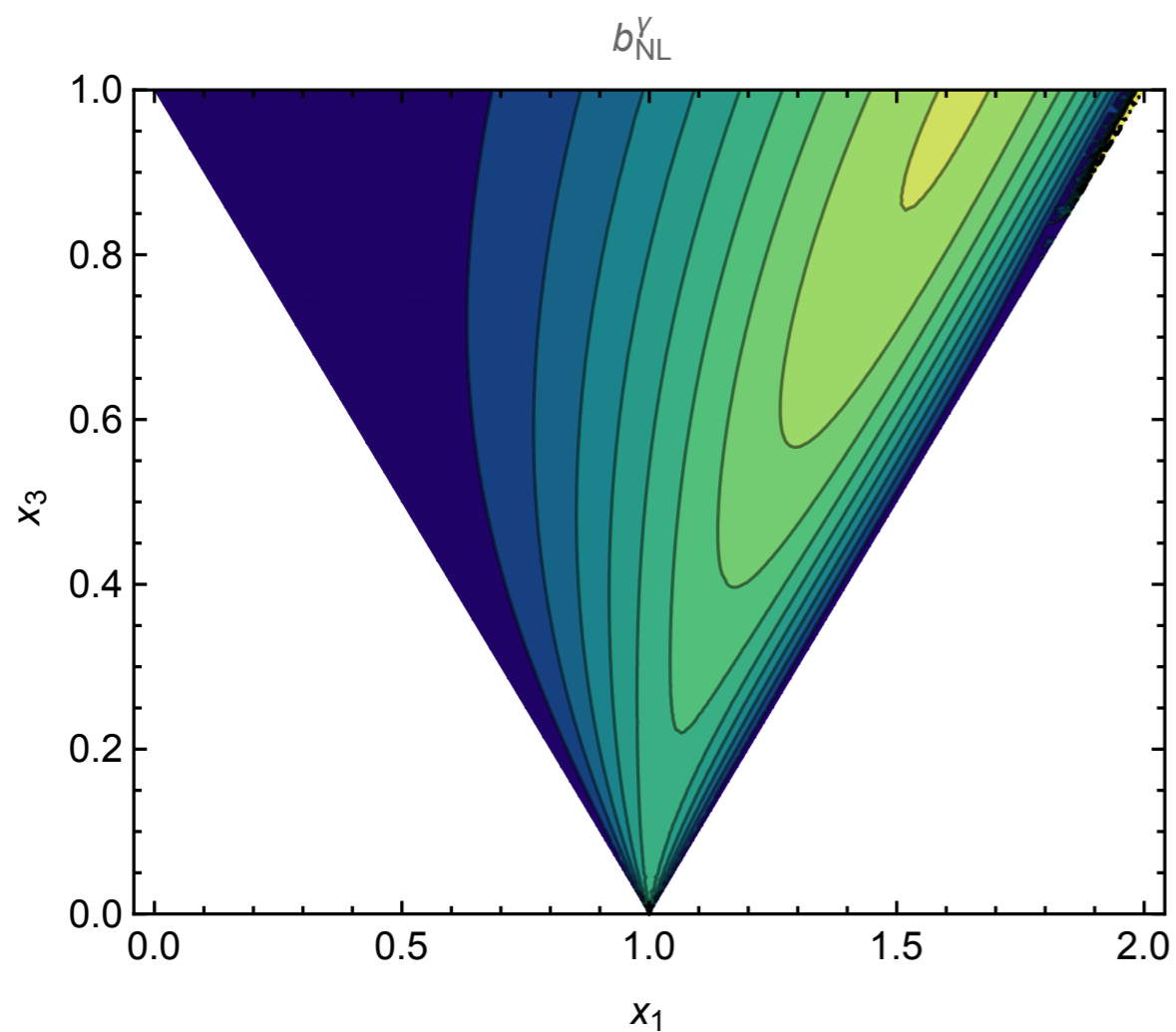
$$\lim_{k_1 \rightarrow 0} \langle \gamma(\tau_I, \mathbf{k}_1) E_\mu(\tau_I, \mathbf{k}_2) E^\mu(\tau_I, \mathbf{k}_3) \rangle = -(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \left( n + \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2} P_\gamma(k_1) P_E(k_2)$$

$$b_{NL}^\gamma = \left( n - \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2}, \quad n > -1/2$$

$$e_{NL}^\gamma = - \left( n + \frac{1}{2} \right) \epsilon_{ij} \frac{k_{2i} k_{2j}}{k_2^2}, \quad n < 1/2$$

**RKJ**, Sai & Sloth, JCAP 03, 054 (2022)

# Cross-correlations with gravitons



RKJ, Sai & Sloth, JCAP 03, 054 (2022)

# Conclusions

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- Inflationary magnetogenesis is promising — problems to get strong enough fields — strong constraints
- Non-trivial imprints of slow roll violations on inflationary magnetic fields — a generic feature in both single and two field models
- CMB imprints are still much smaller — than the CMB TT anisotropies
- Novel cross-correlations of curvature perturbations with magnetic fields — soft theorems — imprints on CMB bispectrum — non-trivial  $\langle \mu T \rangle$  correlations — constraints from observations.



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*Thank you for your attention!*