Dynamos in SWIFT

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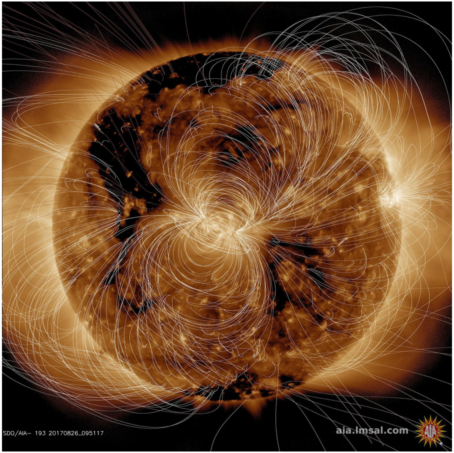
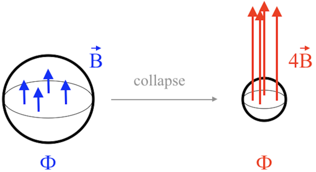
Magnetic fields in space

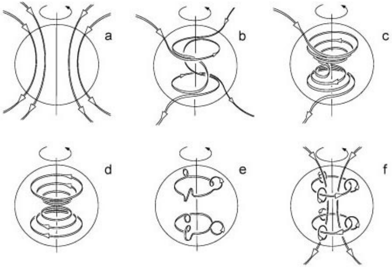
Many astrophysical objects in the Universe host magnetic field of various magnitude:

- **Neutron stars**: $\sim 10^{12} - 10^{15}$ G
- **Stars**: $\sim 1 - 10^{3}$ G
- **Planets**: ~ 1 G
- **Galaxies**: $\sim 10^{-5} - 10^{-6}$ G
- **Galaxy clusters**: $\sim 10^{-6} - 10^{-7}$ G



Cosmic magnetic field evolution





Sources of astrophysical magnetic fields growth:

1. Adiabatic contraction of gas during gravitational collapse

$$B \sim \rho^{\frac{2}{3}} \quad (1)$$

   where ρ - gas density

2. In collapsed structures **dynamo** converts turbulent motion energy into magnetic field

$$B \sim e^{\lambda t} \quad (2)$$

- Growth cannot happen indefinetely, magnetic field saturates when $E_{mag} \sim E_{turb}$

Dynamo - instability in induction equation

Evolution equations for magnetic field and fluid velocity

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = \frac{1}{\rho}[\vec{J} \times \vec{B}] + \vec{f}_{other} \tag{3}$$

$$\partial_t \vec{B} = \text{curl}[\vec{\mathcal{E}}_{emf}] + \eta \Delta \vec{B}, \quad \vec{\mathcal{E}}_{emf} = [\vec{v} \times \vec{B}] \tag{4}$$

- $\text{curl}[\vec{\mathcal{E}}_{emf}]$ is responsible for MF growth
- if $\vec{\mathcal{E}}_{emf} \sim B_j, \partial_j B_k$, **dynamo instability** can occur leading to exponential growth
- magnetic field grows until Lorentz force $[\vec{J} \times \vec{B}]$ becomes large enough to alter $\vec{\mathcal{E}}_{emf}$, **MF saturation happens**

Mean field dynamo I

If separation of scales is possible, $\lambda_{turb} < \lambda_{\text{large scale}}, \quad \tau_{turb} < \tau_{\text{large scale}}$:

$$\vec{B} = \langle\vec{B}\rangle + \delta\vec{B}, \qquad \vec{v} = \langle\vec{v}\rangle + \delta\vec{v} \tag{5}$$

The equations for mean and fluctuations of magnetic fields become:

$$\partial_t\langle\vec{B}\rangle = \text{curl}[\langle\delta\vec{v}\times\delta\vec{B}\rangle] + \eta\Delta\langle\vec{B}\rangle \tag{6}$$

$$\partial_t\delta\vec{B} = \text{curl}[\delta\vec{v}\times\langle\vec{B}\rangle] + \eta\Delta\delta\vec{B} \tag{7}$$

$$\partial_t\delta\vec{v} = \frac{1}{\rho}\left[[\vec{\nabla}\times\langle\vec{B}\rangle]\times\delta\vec{B} + [\vec{\nabla}\times\delta\vec{B}]\times\langle\vec{B}\rangle\right] + \delta\vec{f}_{other} \tag{8}$$

Mean field dynamo II

Fluctuations can be separated to background part and the one that depends on the mean field:

$$\delta\vec{v} = \delta\vec{v}_0 + \delta\vec{v}_1[\langle\vec{B}\rangle] + ..., \qquad \delta\vec{B} = \delta\vec{B}_0 + \delta\vec{B}_1[\langle\vec{B}\rangle] + ... \tag{9}$$

Evolution equations for $\delta\vec{v}_1$ and $\delta\vec{B}_1$:

$$\partial_t \delta\vec{B}_1 = \text{curl}[\delta\vec{v}_0 \times \langle\vec{B}\rangle] + \eta\Delta\delta\vec{B}_1 \tag{10}$$

$$\partial_t \delta\vec{v}_1 = \frac{1}{\rho}\left[[\vec{\nabla} \times \langle\vec{B}\rangle] \times \delta\vec{B}_0 + [\vec{\nabla} \times \delta\vec{B}_0] \times \langle\vec{B}\rangle\right] + \delta\vec{f}_{other} \tag{11}$$

Electromotive force (neglecting quadratic corrections and assuming $\delta\vec{v}_0$ and $\delta\vec{B}_0$ are uncorrelated):

$$\vec{\mathcal{E}}_{emf} = \langle\delta\vec{v} \times \delta\vec{B}\rangle \simeq \langle\delta\vec{v}_0 \times \delta\vec{B}_1\rangle + \langle\delta\vec{v}_1 \times \delta\vec{B}_0\rangle \tag{12}$$

Mean field dynamo III

Using integrated equations of motion for $\delta\vec{v}_1$ and $\delta\vec{B}_1$ electromotive force:

$$\boldsymbol{\mathcal{E}}_i = \int_0^\infty d\tau \int_V d^3\boldsymbol{\xi} \left[\mathcal{A}_{ij}(t,\, \boldsymbol{x};\, \tau,\, \boldsymbol{\xi})\langle\vec{B}\rangle_j(t-\tau,\, \boldsymbol{x}+\boldsymbol{\xi}) + \right.$$
$$\left. + \mathcal{B}_{ijk}(t,\, \boldsymbol{x};\, \tau,\, \boldsymbol{\xi})\frac{\partial\langle\vec{B}\rangle_j(t-\tau,\, \boldsymbol{x}+\boldsymbol{\xi})}{\partial x_k} + \dots\right].$$

$\langle\vec{B}\rangle$ is smooth over $\xi \sim \lambda_{turb}$ and $\tau \sim \tau_{turb}$:

$$\boldsymbol{\mathcal{E}}_i = \alpha_{ij}\langle\vec{B}\rangle_j(t,\, \boldsymbol{x}) + \beta_{ijk}\partial_j\langle\vec{B}\rangle_k(t,\, \boldsymbol{x}) + \dots$$

Mean field dynamo IV

If turbulence is homogeneous and isotropic:

$$\vec{\mathcal{E}}^{emf} = \alpha\langle\vec{B}\rangle - \beta \text{curl}\langle\vec{B}\rangle \tag{13}$$

Equations for mean magnetic field induce instability due to α term - **alpha effect**:

$$\partial_t\langle\vec{B}\rangle = \alpha\cdot\text{curl}\langle\vec{B}\rangle + (\eta+\beta)\Delta\langle\vec{B}\rangle \tag{14}$$

It can be shown that:

$$\alpha = c_1\cdot\langle\delta\vec{v}_0\cdot[\vec{\nabla}\times\delta\vec{v}_0]\rangle - c_2\cdot\langle\delta\vec{B}_0\cdot[\vec{\nabla}\times\delta\vec{B}_0]\rangle \tag{15}$$

$$\beta = c_3\cdot\langle\delta\vec{v}_0\cdot\delta\vec{v}_0\rangle \tag{16}$$

For the alpha effect to occur one of the correlators $\langle\delta\vec{v}_0\cdot[\vec{\nabla}\times\delta\vec{v}_0]\rangle$ or $\langle\delta\vec{B}_0\cdot[\vec{\nabla}\times\delta\vec{B}_0]\rangle$ should be non-zero

Nessesary conditions for any dynamos

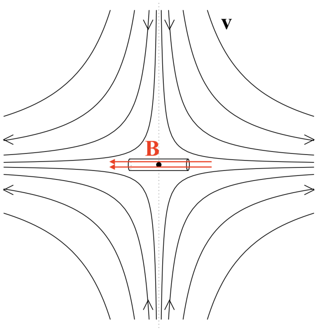
Induction equation

$$\partial_t \vec{B} + (\vec{\nabla}\vec{v})\vec{B} = (\vec{B}\cdot\vec{\nabla})\vec{v} - \vec{B}\cdot \mathrm{div}\vec{v} + \eta\Delta\vec{B} \quad (17)$$

where terms on RHS describe influence of:

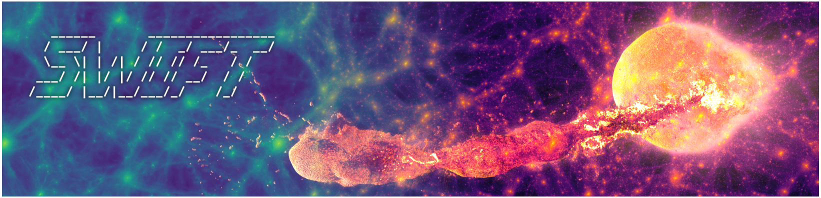
1. flow velocity shear
2. matter compression
3. ohmic diffusion

- Existance of shear zones
- $\eta \neq 0$
- Flow can't be 2D (Zeldovich anti dynamo theorem)



Stagnation point as example of shear zone

SWIFT with MHD I



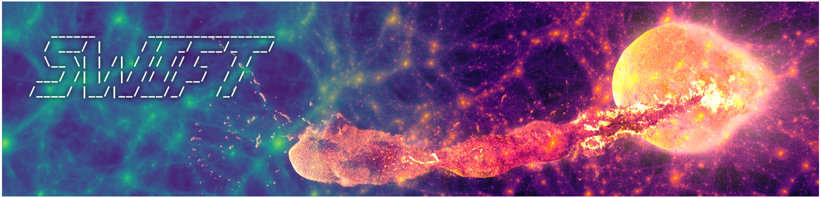
Dynamos can be studied with numerical simulations such as SWIFT code

- SPH based code - solves evolution equations (including MHD) at particle positions
- This is equivalent to MHD equations in a frame that moves with fluid

Induction equation (Moving frame)

$$\frac{d\vec{B}}{dt} = (\vec{B}\cdot\vec{\nabla})\vec{v} - \vec{B}\cdot \mathrm{div}\vec{v} + \eta\Delta\vec{B} \quad (18)$$

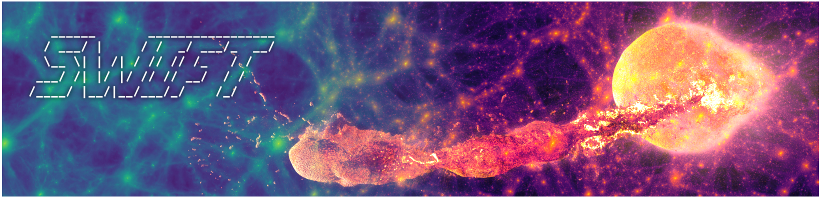
SWIFT with MHD II



MHD implementations in SWIFT:

- FDI: direct-induction, evolves $\vec{B}$, additional scalar field cleans $\mathrm{div}\vec{B}$ (Dedner cleaning)
- ODI: similar to FDI but evolves $\frac{\vec{B}}{\rho}$, employs more sophisticated $\mathrm{div}\vec{B}$ cleaning and shock capturing
- VP: evolves vector potential $\vec{A}$, $\vec{B} = \mathrm{curl}\vec{A}$, divergence is zero by construction

SWIFT with MHD III



Numerical evolution of astrophysical MFs is complicated:

- the apperent magnetic field might be caused by numerical instabilities as well as physical processes
- in the case of large simulations some regions might become under-resovled, leading to incorrect evolution of MFs

The dynamo implementation in SWIFT should be checked with a set of well studied dynamo tests

Roberts flow 1 in SWIFT I

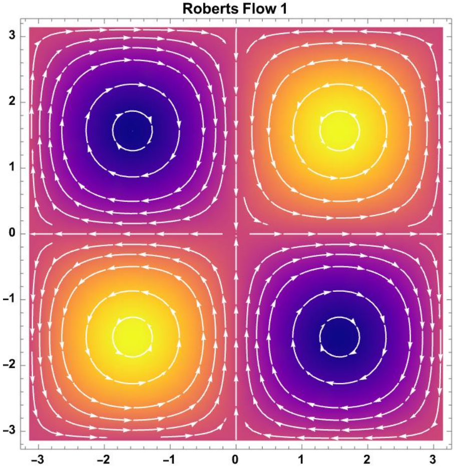
Roberts found 4 simple dynamo capable flows.

G.O.Roberts, 1972

Roberts flow 1:

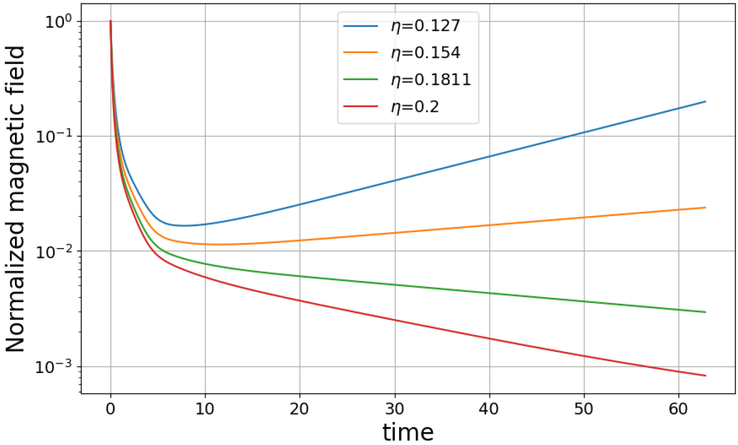
$$v_x = v_0 \sin k_0 x \cos k_0 y$$
$$v_y = -v_0 \cos k_0 x \sin k_0 y$$
$$v_z = \omega_0 \sin k_0 x \sin k_0 y$$

- shows α effect
- is a large scale dynamo
- is a slow dynamo



Roberts flow 1. Density plot of $v_z(x, y)$ and velocity stream lines in xy plane

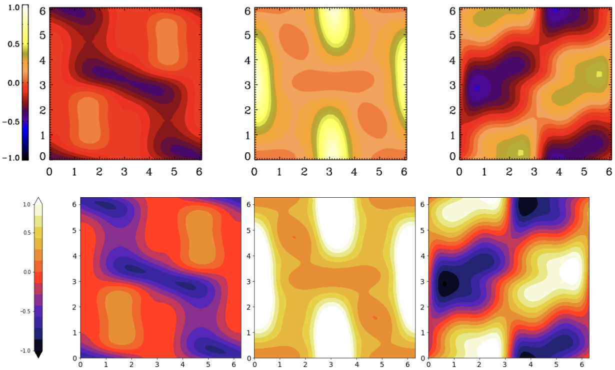
Roberts flow 1 in SWIFT II



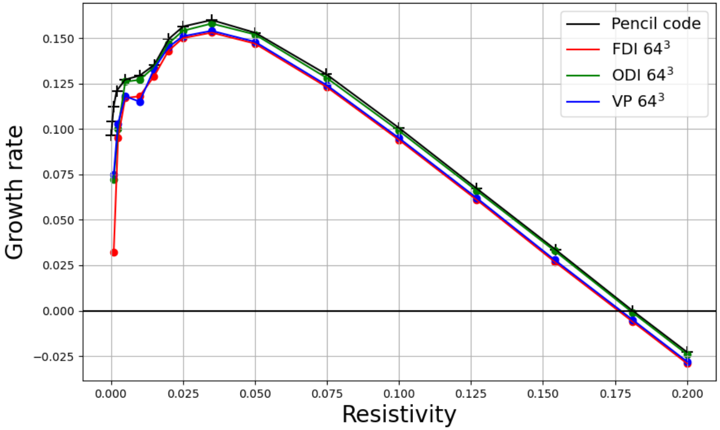
All 3 schemes show existance of exponential growth and decay solutions

Roberts flow 1 in SWIFT III

Initially random MFs start growing and develop a steady pattern

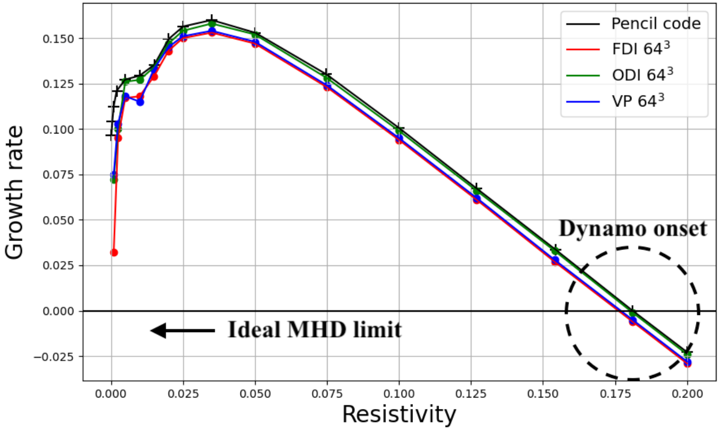


Magnetic field components ($B_x/B_{rms}, B_y/B_{rms}, B_z/B_{rms}$) for Pencil code (top, provided by A.Brandenburg) and for SWIFT (bottom)



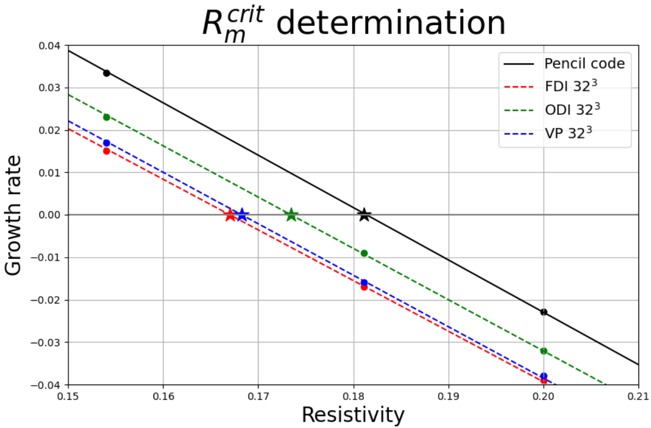
Growth rate vs resistivity for Pencil and SWIFT codes

Roberts flow 1 in SWIFT V



Growth rate vs resistivity for Pencil and SWIFT codes

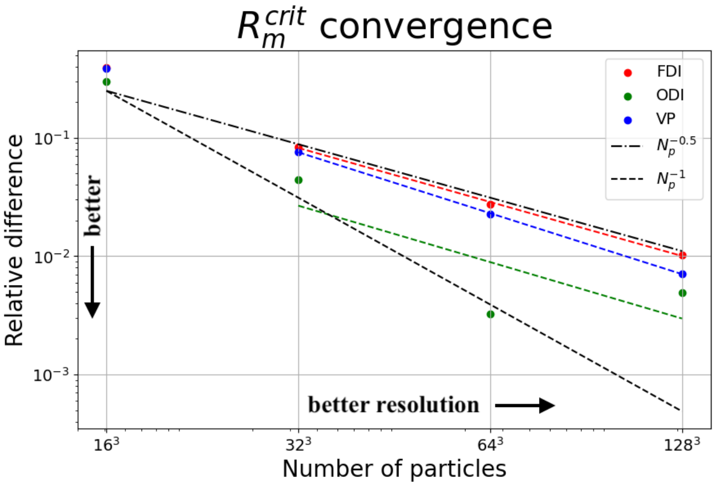
Roberts flow 1 in SWIFT VI



Growth rate vs resistivity for Pencil and SWIFT codes

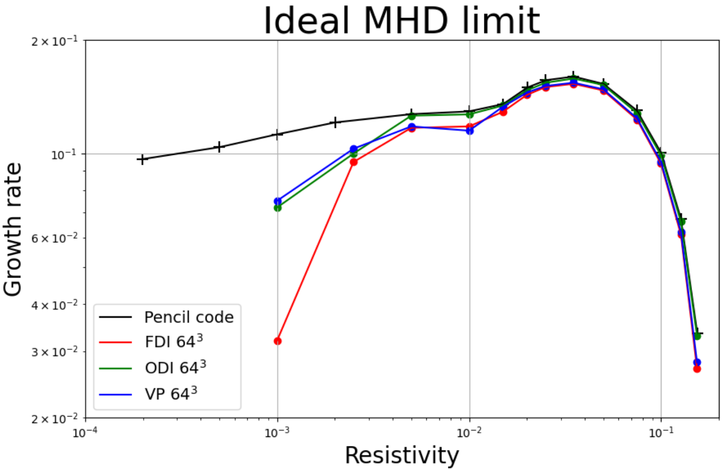
We determine **critical resistivity** and R_m^{crit} with linear fit of growth rate vs resistivity dependence. Black curve corresponds to $R_m^{crit} = 5.51$

Roberts flow 1 in SWIFT VII



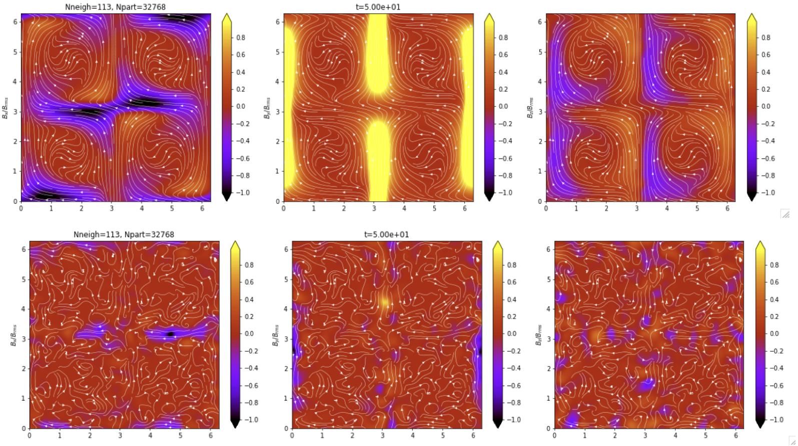
All 3 MHD implementations in SWIFT converge to critical magnetic reynolds number $R_m^{crit} = 5.51$ as $N_p^{-0.5}$

Roberts flow 1 in SWIFT VIII



For smaller resistivity growth rates deviate more from Pencil code. For 64^3 runs at $\eta_{min} \sim 10^{-3}$ dynamo stops

Roberts flow 1 in SWIFT IX



Magnetic field pattern for $\eta > \eta_{min}$ (top) and for $\eta < \eta_{min}$ (bottom). At $\eta \sim \eta_{min}$ the pattern gets destroyed due to lack of resolution

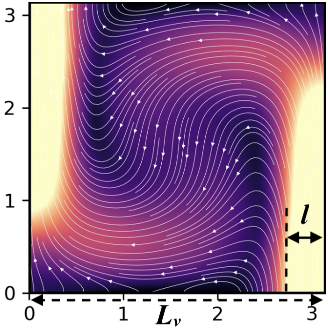
Resolution limit estimate I

- Fluid vortices wind the magnetic fields, while diffusion reconnects the MF lines
- Typical winding and diffusion times:

$$t_w \sim \frac{L_v}{v_{rms}} \qquad t_{diff} \sim \frac{l^2}{\eta} \qquad (19)$$

- Eventually, diffusion balances winding, forming steady magnetic field pattern with typical thickness l:

$$t_w \sim t_{diff} \;\rightarrow\; l \sim \frac{L_v}{\sqrt{R_m}} \qquad (20)$$



$|B|/B_{rms}$ and magnetic field lines inside one vortex

Resolution limit estimate II

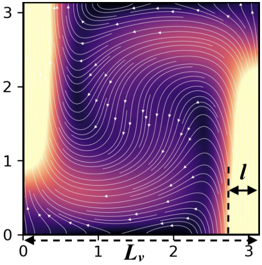
- The pattern becomes unresolved when the MF features are simulated by a few particles

$$l \sim d_{part} \qquad (21)$$

- Upper bound on achivable magnetic reynolds number:

$$R_m \lesssim \frac{L_v^2}{d_{part}^2} \qquad (22)$$

- In our 64^3 runs magnetic pattern breakdown should happen at $\eta \sim 10^{-3}$



Note, $L_v = L_{box}/2$

Resolution	R_m^{max}
16^3	60
32^3	250
64^3	1000

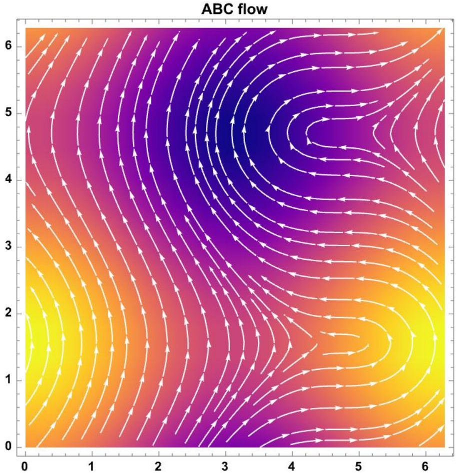
Table: R_m^{max} for RobertsFlows.

ABC flow in SWIFT I

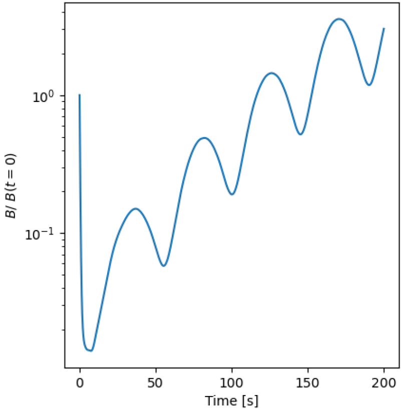
- oscillating solution
- fast dynamo

Velocity profile

$$v_x = A \cdot \sin z - C \cdot \cos y$$
$$v_y = B \cdot \sin x - A \cdot \cos z$$
$$v_z = C \cdot \sin y - B \cdot \cos z$$

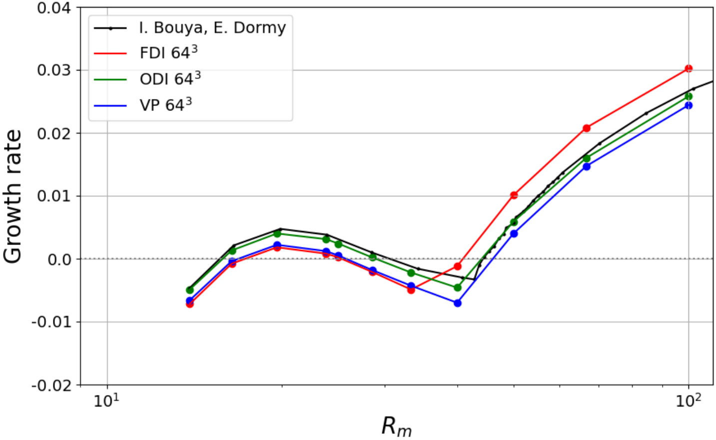


ABC flow in SWIFT II



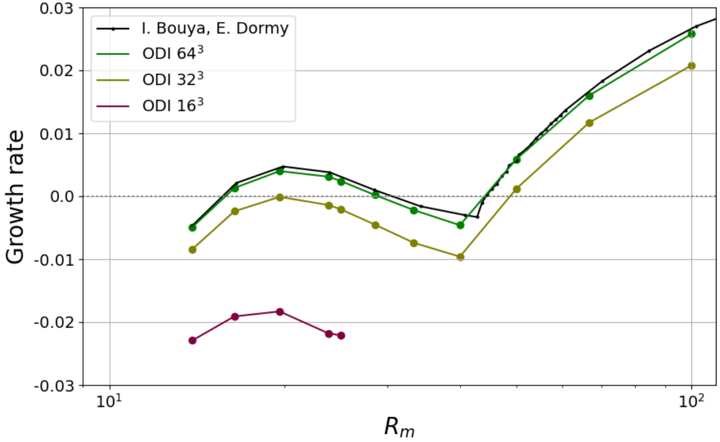
All schemes show growing and decaying oscillating solutions

ABC flow in SWIFT III

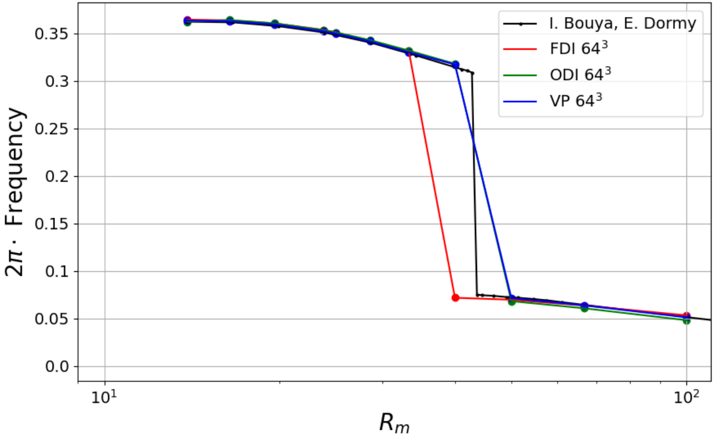


Growth rates vs magnetic reynolds number for SWIFT compared to I. Bouya and E. Dormy article. FDI overestimates growth rates for $R_m > 30$

ABC flow in SWIFT IV

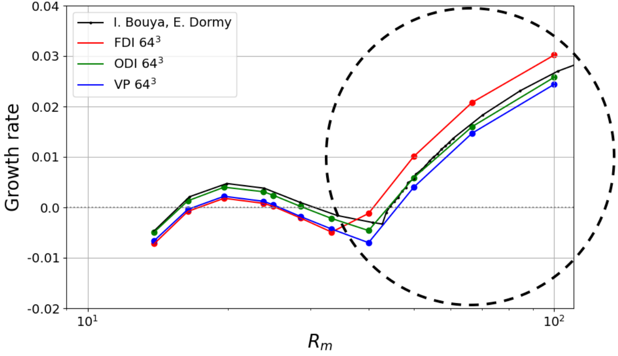


Convergence of growth rates with resolution increase for ODI. Growth for 16^3 stops around $R_m \simeq 30$. VP behaves in a similar way



MF oscillation frequency vs R_m for SWIFT compared to I. Bouya and E. Dormy article. For FDI mode transition happens at smaller R_m.

ABC flow in SWIFT VI



All schemes converge to correct growth rates and frequencies, but FDI now overestimates growth rates at high R_m

Influence of divB on Roberts flow runs I

Dynamo could be influenced by non-zero MF divergence

- ideally evolution of $\text{div}\vec{B}$ follows from induction equation

$$\partial_t \text{div}\vec{B} = \text{div} \cdot \text{curl}[\vec{v} \times \vec{B}] + \eta \Delta \text{div}\vec{B} = \eta \Delta \text{div}\vec{B} \quad (23)$$

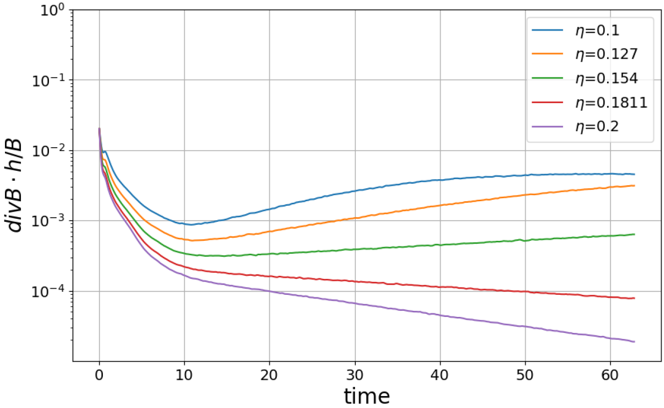
- but in codes curl operator is not exact, $\text{div} \cdot \text{curl}_{num}[\vec{v} \times \vec{B}] \neq 0$ leading to some divergence growth
- $\text{div}\vec{B}$ can source physical fields through electromotive force

$$\vec{\mathcal{E}}_{emf} = [\vec{v}_{phys} \times \vec{B}_{mon}] + [\vec{v}_{unphys} \times \vec{B}_{phys}] \quad (24)$$

where $\vec{B}_{mon}$ - monopole part of magnetic field, $\vec{v}_{unphys}$ - motions due to $\vec{B}_{mon}$

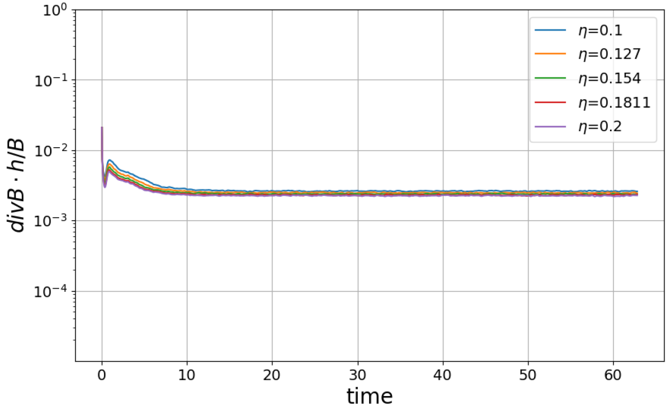
To track this effects we monitor the ratio of $\text{div}\vec{B}$ to largest resolvable magnetic field gradient in SPH, $\frac{|\vec{B}|}{h}$

Influence of divB on Roberts flow runs II



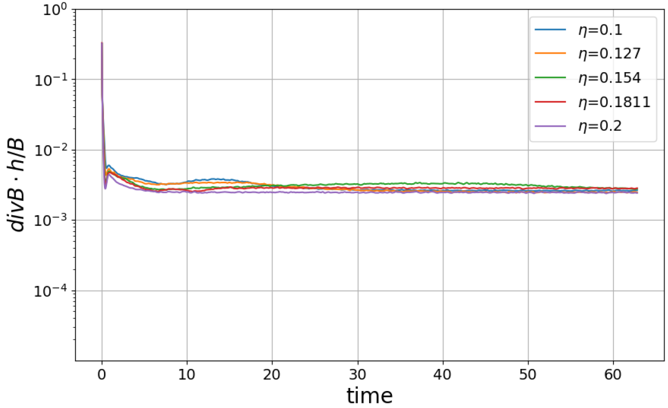
Divergence error remains small for FDI

Influence of divB on Roberts flow runs III



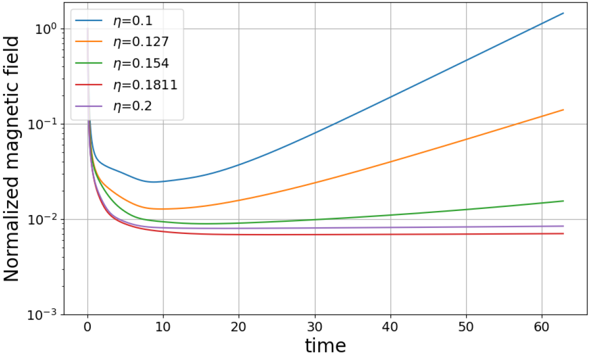
ODI involves more complicated divB cleaning and corrections leading to even less divergence error

Influence of divB on Roberts flow runs IV



Evolution of magnetic field and divB error for ODI if there is large initial divB.

Influence of divB on Roberts flow runs V



- growing modes grow same as before but saturate slower
- no decaying solutions for ODI seen

div$\vec{B}$ cleaning creates nearly homogeneous MFs that hide decaying modes

Conclusions

- All 3 MHD implementations reproduce features of ABC and Roberts flow 1 kinematic dynamos
- The codes demonstrate convergence of growth rate with resolution.
- For Roberts Flow 1 we found that critical R_m converges as $N_p^{-0.5}$
- If magnetic Reynolds number is too high, thickness of magnetic field features becomes smaller than resolution scale, field pattern gets destroyed and dynamo stops.
- For direct induction schemes divergence cleaning keeps divB low during the runs, but it can produce slowly decaying physical fields

Thank you!