Structure Formation with Primordial Magnetic Fields: from initial conditions to baryon fraction in halos

based on arxiv:2402.14079 (accepted in JCAP) with Pranjal Ralegankar and Matteo Viel

presented by Mak Pavičević

within the program ***Generation, evolution, and observations of cosmological magnetic fields***
at the **Bernoulli Center, EPFL, Lausanne**

May 14th, 2024

Outline of the talk

1. Introduction on structure formation and power spectra
2. Some previous, related works
3. Our new contribution and main results
   1. Modification of initial conditions for cosmological simulations
   2. Impact of PMFs on baryon fraction in halos
   3. Theoretical ambiguities; non-gaussianity
4. Summary and conclusions
5. Outlook on future projects

So far…

Early Universe physicist: I have a beautiful model of magnetogenesis,
and I can provide a power spectrum for the magnetic field.

Axel: Well, we must take care of the turbulence and inverse cascade.
And then swim in the lake. Turbulently, of course.

Ruth and Chiara: Roses are red, violets are blue.
Causality screams four, inflation knocks on the door.

Kandu: Transition from radiation-dominated epoch to matter-dominated epoch is an elephant in the room.

* Pranjal enters the room *

* the elephant leaves the room *

Tina and Salome: Let us probe the scales much larger than the size of the elephant.

Structure formationist: To ***B*** or not to ***B***, , *that is the question*

* another Pranjal enters the room *

The main message of this talk

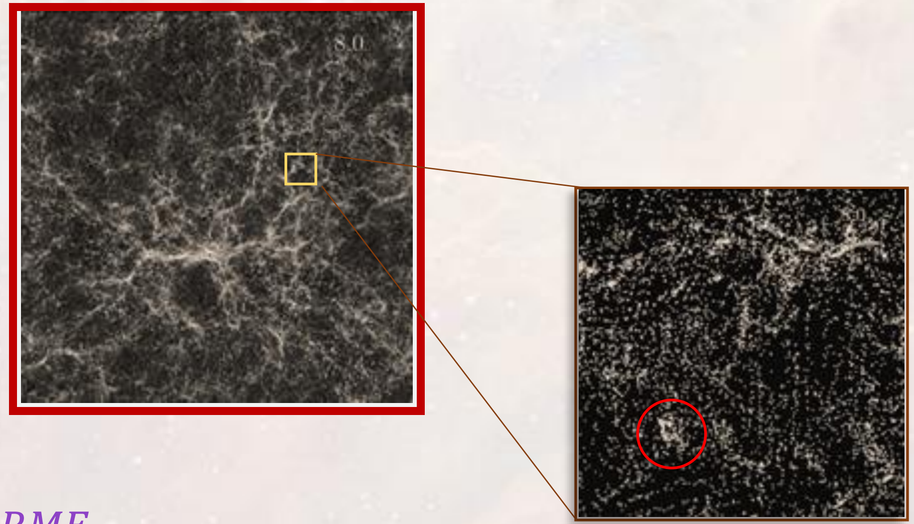
If one consistently sets perturbations of matter (baryons and DM) induced by the Lorentz force of stochastic PMFs in the initial conditions (ICs) of a cosmological simulation, then one should find halos at very early redshifts ($z \gtrsim 8$) with a significant gas component.

$(\delta_b)_{\Lambda CDM}$
$(\delta_{DM})_{\Lambda CDM}$
$(\delta_b)_{PMF}$
$(\delta_{DM})_{PMF}$

ICs (e.g., $z = 100$)

cosmological evolution

formation of structures



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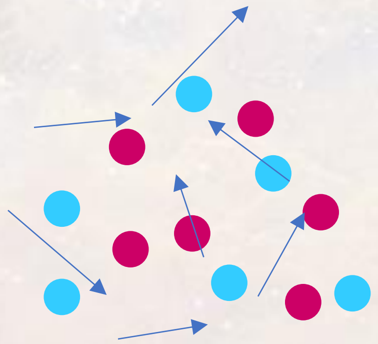
gravity

$(\delta_{DM})_{PMF}$

ICs (e.g., $z = 100$)

* $\vec{B}$ stochastic, gaussian



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b

DM

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Introduction - Structure formation

Insert linear matter power spectrum P(k)

$$\sigma_{\mathrm{R}}^2 \equiv \langle \delta^2(x, z; \mathrm{R}) \rangle = \int_0^\infty \left[\frac{k^3 P(k,z)}{2\pi^2} \right] \tilde{W}^2(k\mathrm{R}) \frac{\mathrm{d}k}{k}.$$

Compute Halo Mass Function

$$\frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}M}(M, z) = \frac{\bar{\rho}_{\mathrm{m}}}{M} f(\sigma_{\mathrm{R}}) \frac{\mathrm{d}\sigma_{\mathrm{R}}^{-1}}{\mathrm{d}M},$$

How are ICs actually implemented in cosmological simulations

- Zeldovich approximation

$$\vec{x} = \vec{x}_\mathrm{i} - \underbrace{\frac{D(a)}{4\pi G \bar{\rho}_m a^3} \nabla \Phi_\mathrm{i}(\vec{x}_\mathrm{i})}_{\propto \frac{\vec{v}}{\dot{D}}} \quad \leftrightarrow \quad \frac{\partial \delta}{\partial t} + \frac{\nabla \cdot \vec{v}}{a} = 0$$

$$\vec{v}(\vec{x}, t) = \sum_{\boldsymbol{k}} v_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \quad \vec{v}_{\boldsymbol{k}} = Haf(\Omega) \frac{i\vec{k}}{k^2} \delta_{\boldsymbol{k}}$$

- Gaussian fields

$$\delta_{\boldsymbol{k}} = A_{\boldsymbol{k}} + iB_{\boldsymbol{k}} = |\delta_{\boldsymbol{k}}| e^{i\varphi_{\boldsymbol{k}}}$$

$$\mathcal{P}(A_{\boldsymbol{k}})\, dA_{\boldsymbol{k}} = \frac{1}{[\pi V^{-1} P(k)]^{1/2}} \exp\left(-\frac{A_{\boldsymbol{k}}^2}{V^{-1} P(k)}\right) dA_{\boldsymbol{k}}$$

Introduction – Power spectra

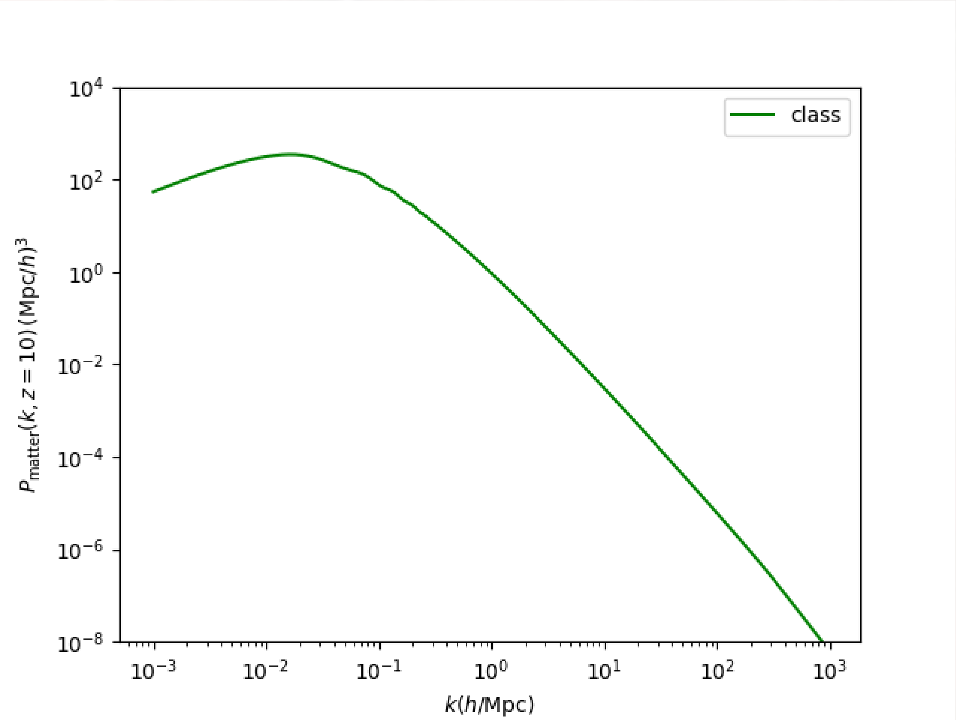
❑ Given a (non-helical) PMF power spectrum post-rec. with 2 parameters

$$\langle B_i(k) B_j^*(k') \rangle = (2\pi)^3 \delta^3(k-k') \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{P_B(k)}{2}$$

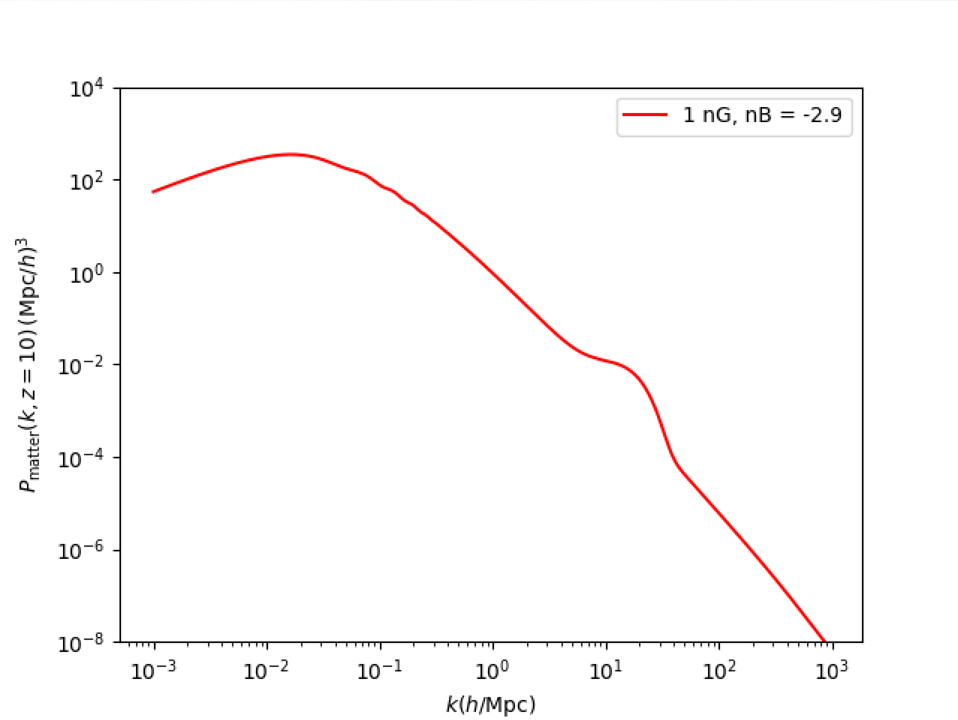
$$P_B(k) \propto B_{1\mathrm{Mpc}}^2 k^{n_B}$$

'averaged over 1 Mpc'

the linear matter power spectrum gains a 'bump' on small scales:



$$P_{\mathrm{matter}}^{\Lambda CDM}(k) + P_{\mathrm{matter}}^{\mathrm{PMF}}(k)$$



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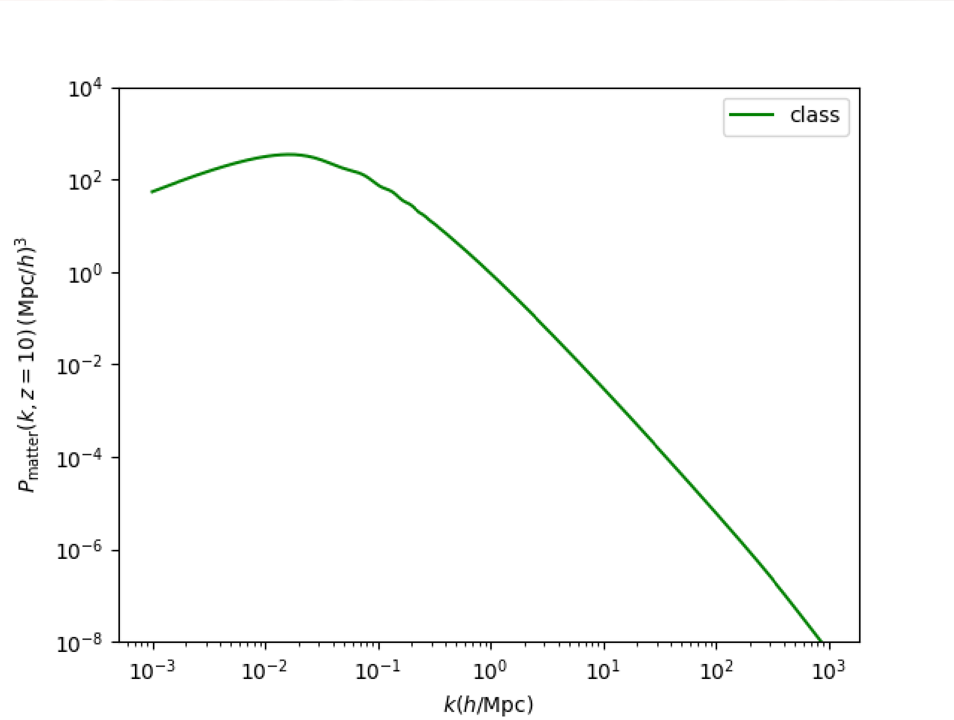
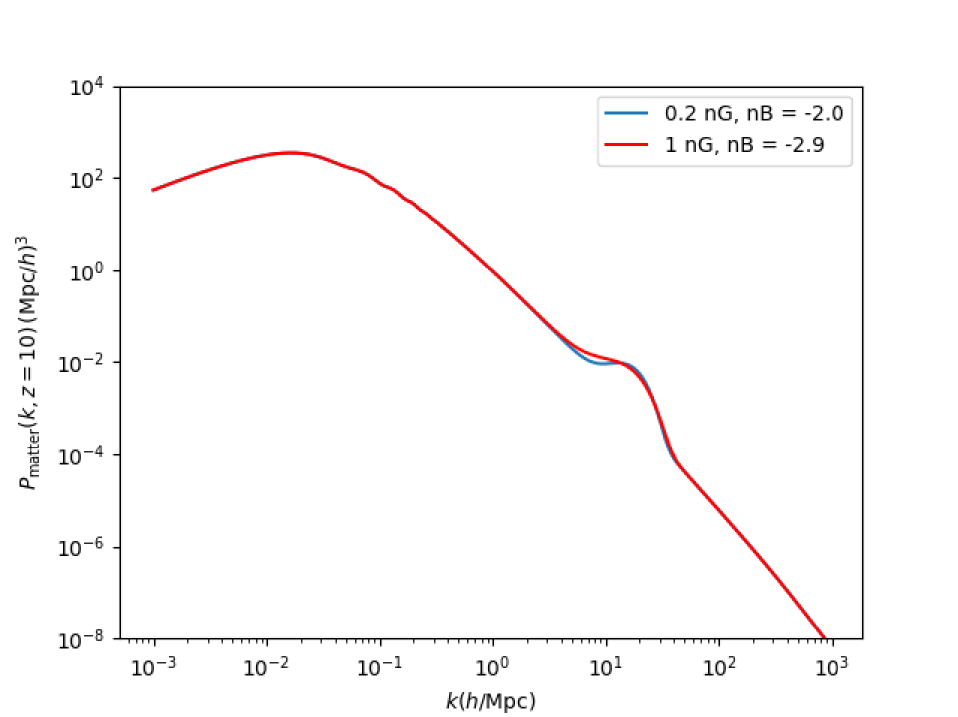


$P^{\Lambda CDM}_{\mathrm{matter}}(k) + P^{\mathrm{PMF}}_{\mathrm{matter}}(k)$



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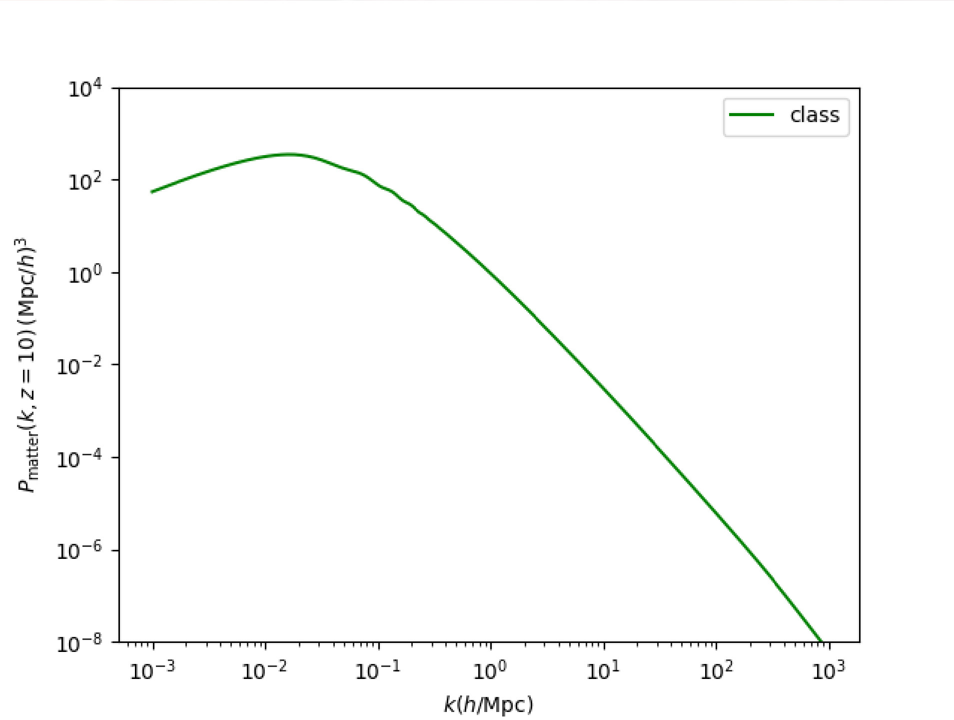

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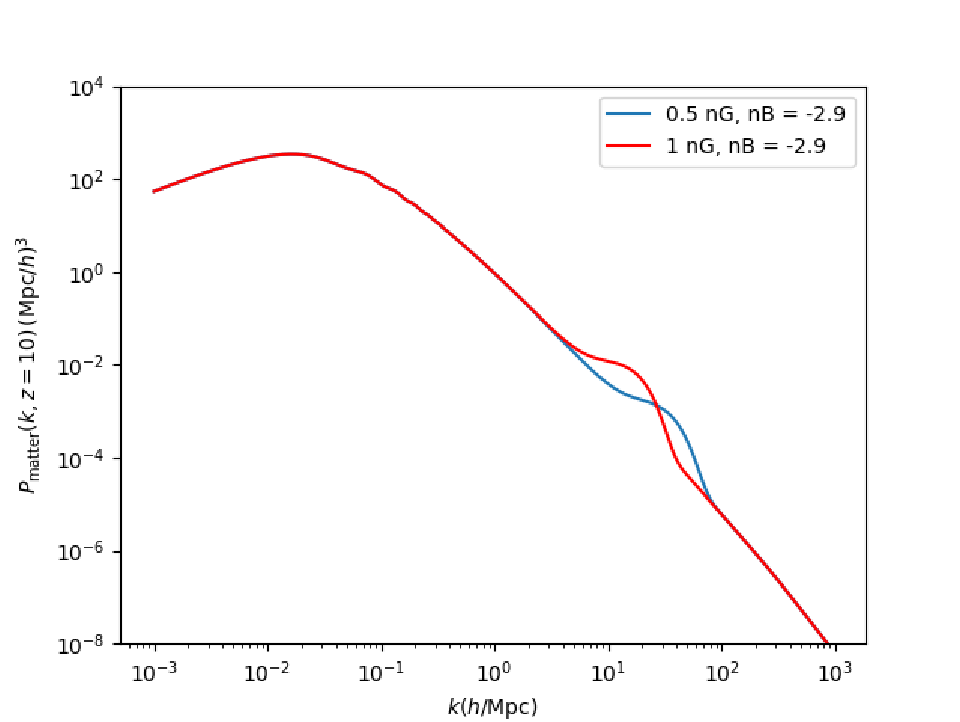
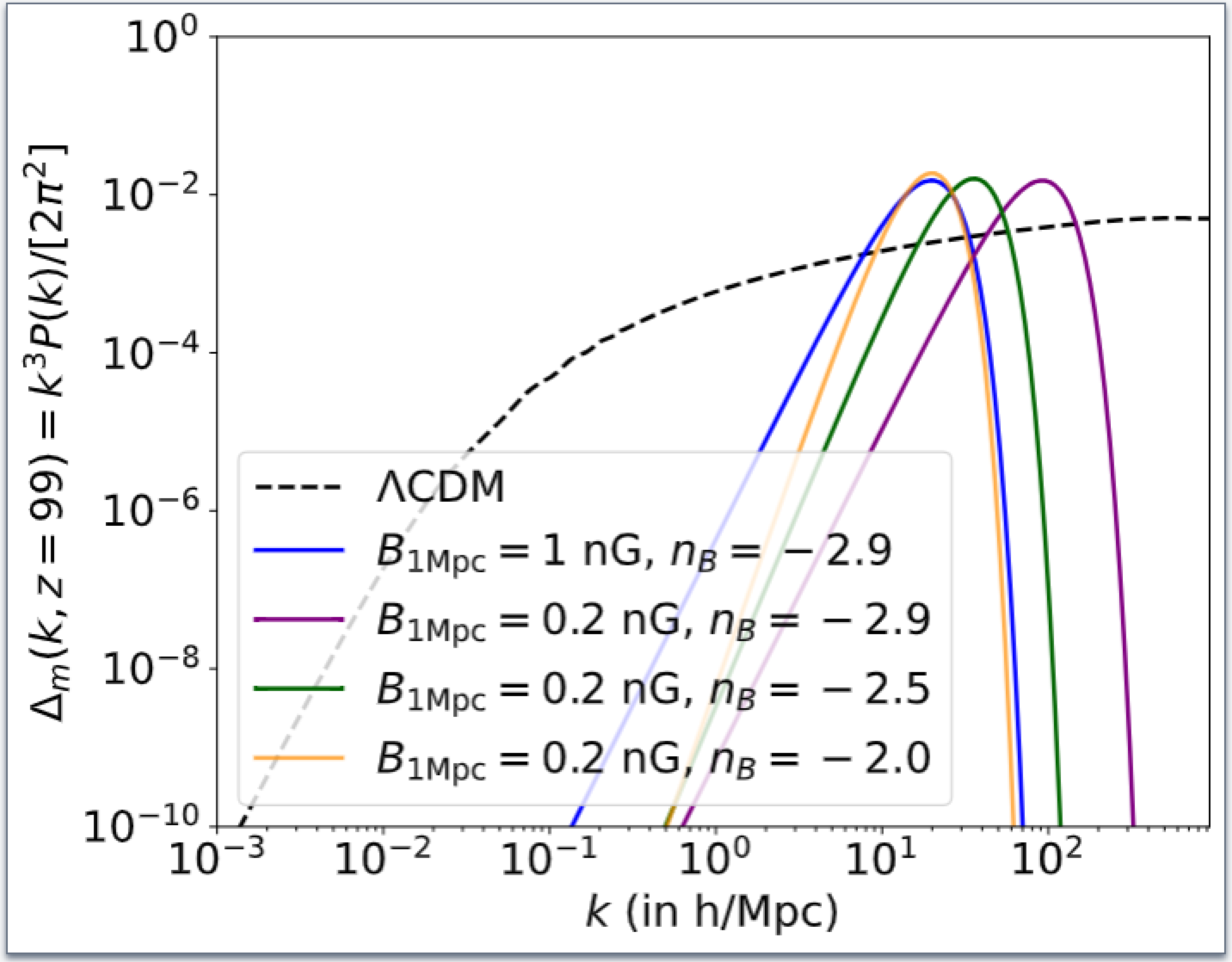


Introduction – Power spectra

- Impact of PMFs on the dimensionless matter power spectrum



$$P_{\mathrm{B}}(k) \propto B_{1\mathrm{Mpc}}^2 k^{n_B}$$

Increase $B_{1\mathrm{Mpc}}$: bump moves to the left (decrease -| |- right)

Bring n_B closer to scale-invariant: bump moves to the right

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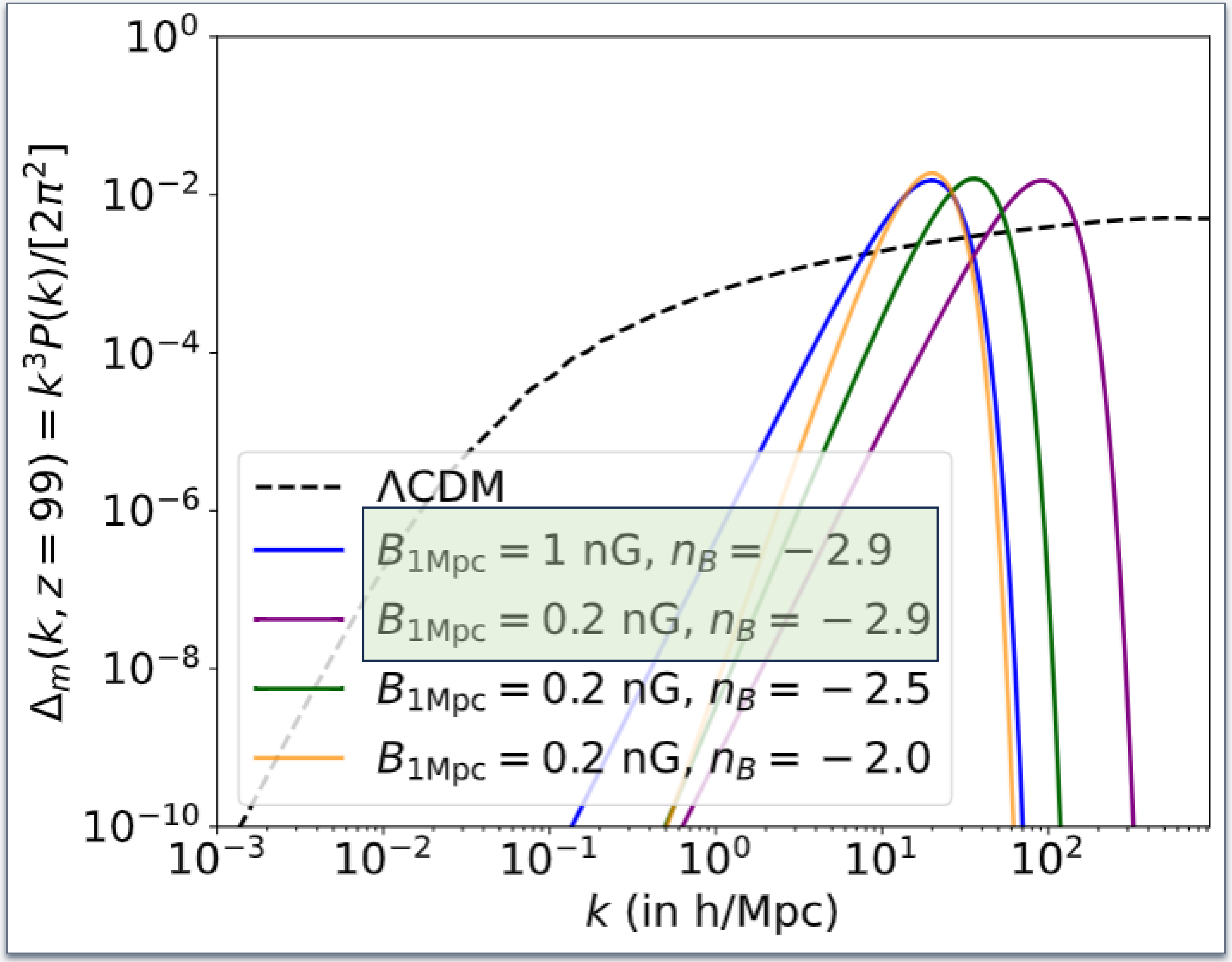

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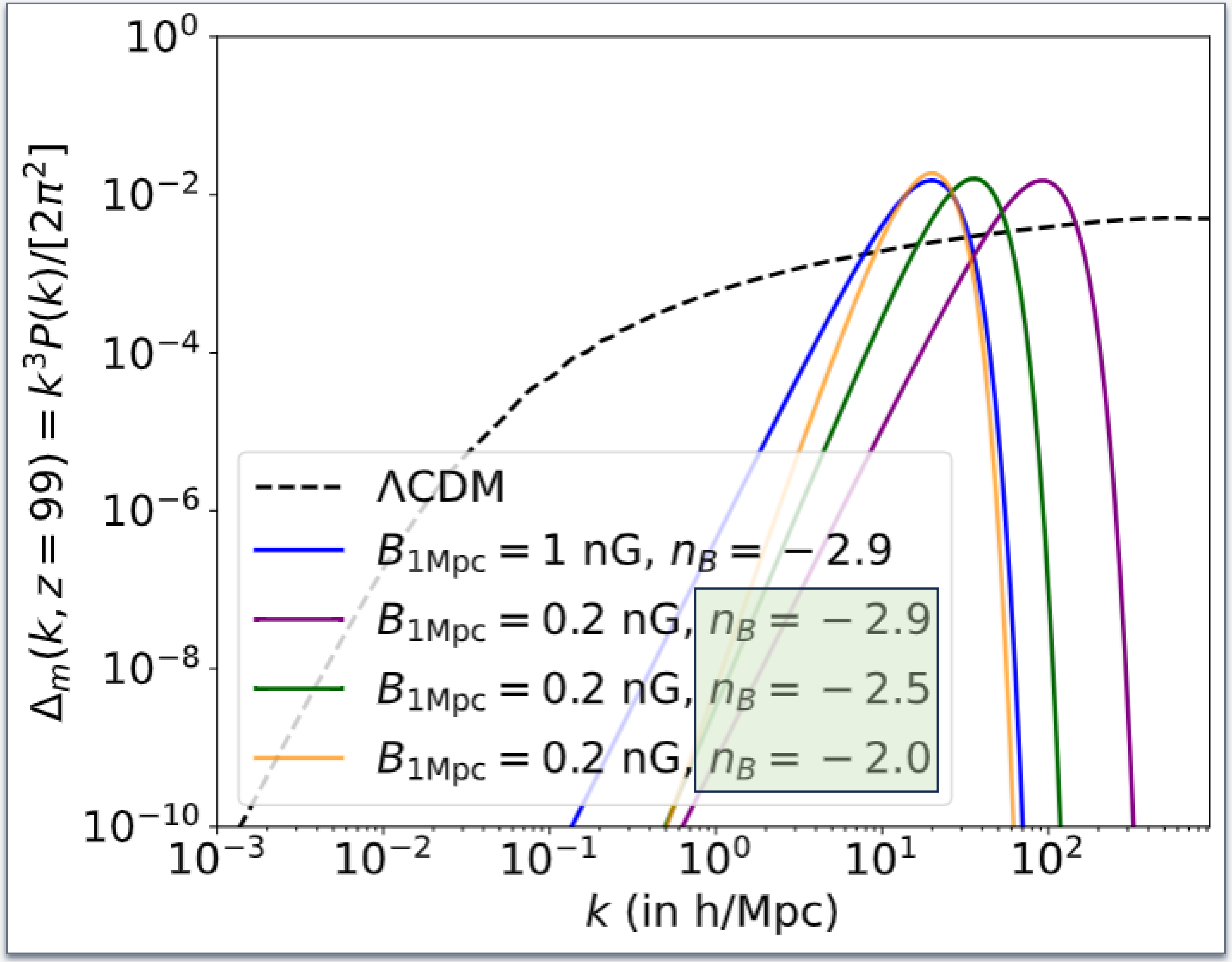


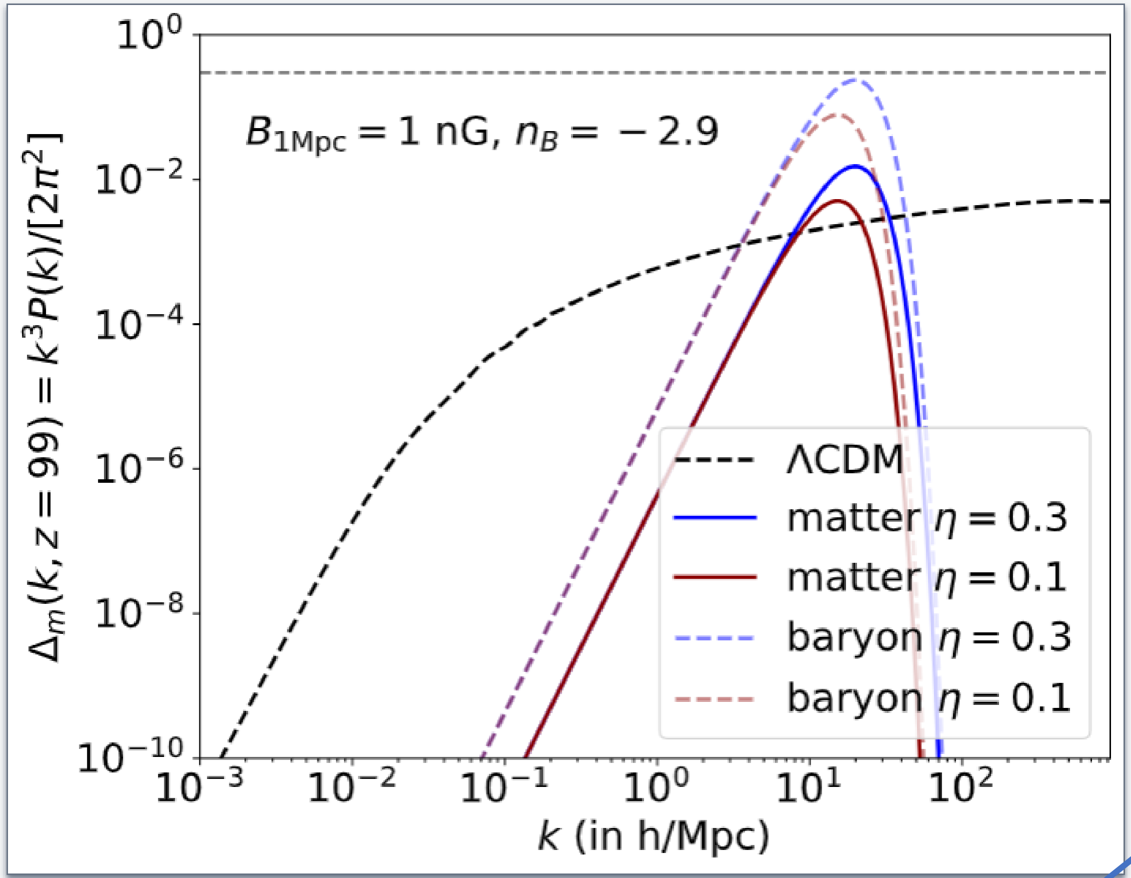
$$P_\mathrm{B}(k) \propto B^2_{1\mathrm{Mpc}} k^{n_B}$$

Increase $B_{1\mathrm{Mpc}}$: bump moves to the left
(decrease -| |- right)

Bring n_B closer to scale-invariant:
bump moves to the right
(only small change in the peak height)

Introduction – Power spectra

- Impact of PMFs on the baryon power spectrum



Lorentz force term

$$S_0 = \frac{\nabla \cdot [(\nabla \times \vec{B}) \times \vec{B}]}{4\pi a^3 \rho_{\rm b}}$$

$$a^2 \frac{\partial^2 \delta_{\rm b}}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_{\rm b}}{\partial a} = -\frac{S_0}{a^3 H^2} + \frac{\nabla^2 \phi}{(aH)^2}$$

$$(\delta_{\rm b})_{\rm PMF} = -\xi_{\rm b}(a) \frac{S_0}{a^3 H^2}$$

damping scale

$$\Delta_{\rm b}^{\rm PMF}(k) \equiv \frac{k^3 P_{\rm b}^{\rm PMF}(k)}{2\pi^2} = 10^{-4} \xi_{\rm b}^2(a) \left(\frac{k}{\rm Mpc^{-1}}\right)^{2n_{\rm B}+10} \left(\frac{B_{1\rm Mpc}}{\rm nG}\right)^4 G_{n_{\rm B}} e^{-2k^2 \lambda_{\rm D}^2}$$

Structure formation with PMFs – some previous works

- ❑ Katz et al. (MNRAS, 2021)
  “Introducing SPHINX-MHD: the impact of primordial magnetic fields on the first galaxies, reionization, and the global 21-cm signal”

- ❑ Sanati et al. (A&A, 2020), [see also recent 2024]
  “Constraining the primordial magnetic field with dwarf galaxy simulations”

- ❑ Sethi & Subramanian (MNRAS, 2005),
  “Primordial magnetic fields in the post-recombination era and early reionization”

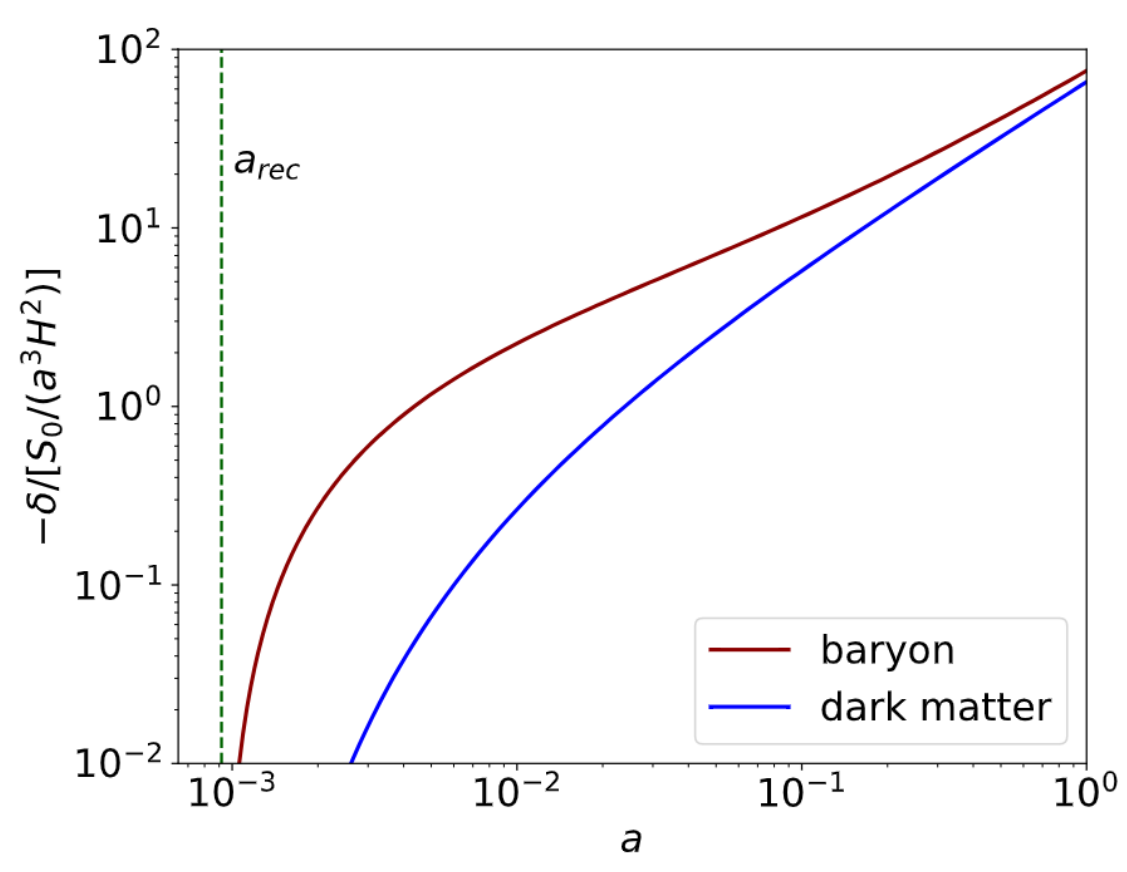
- ❑ Kahniashvili et al. (ApJ, 2012)
  "Constraining primordial magnetic fields through large scale structure"

- ❑ Mtchedlidze et al. (ApJ, 2022) "Evolution of Primordial Magnetic Fields during Large-scale Structure Formation"

Now to our recent work

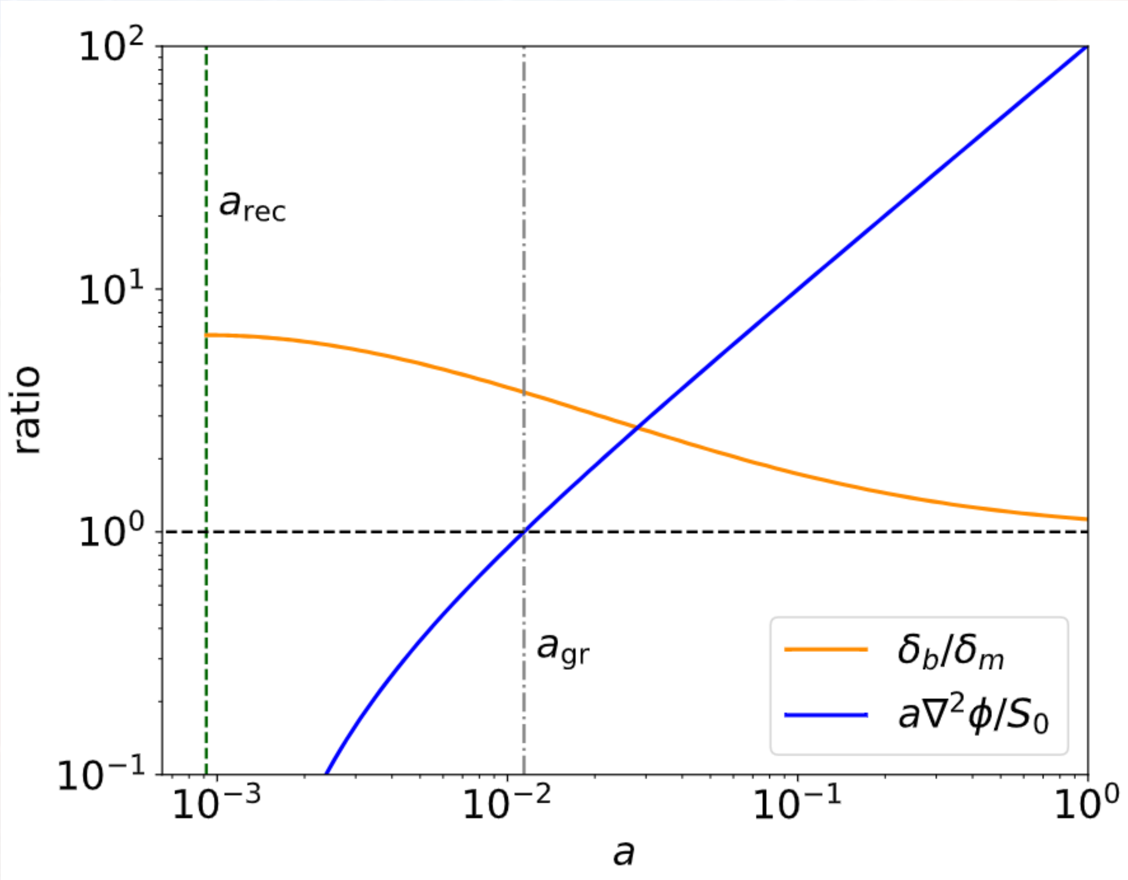
Main features: 1) theory

- PMFs impact baryons and dark matter in a different way



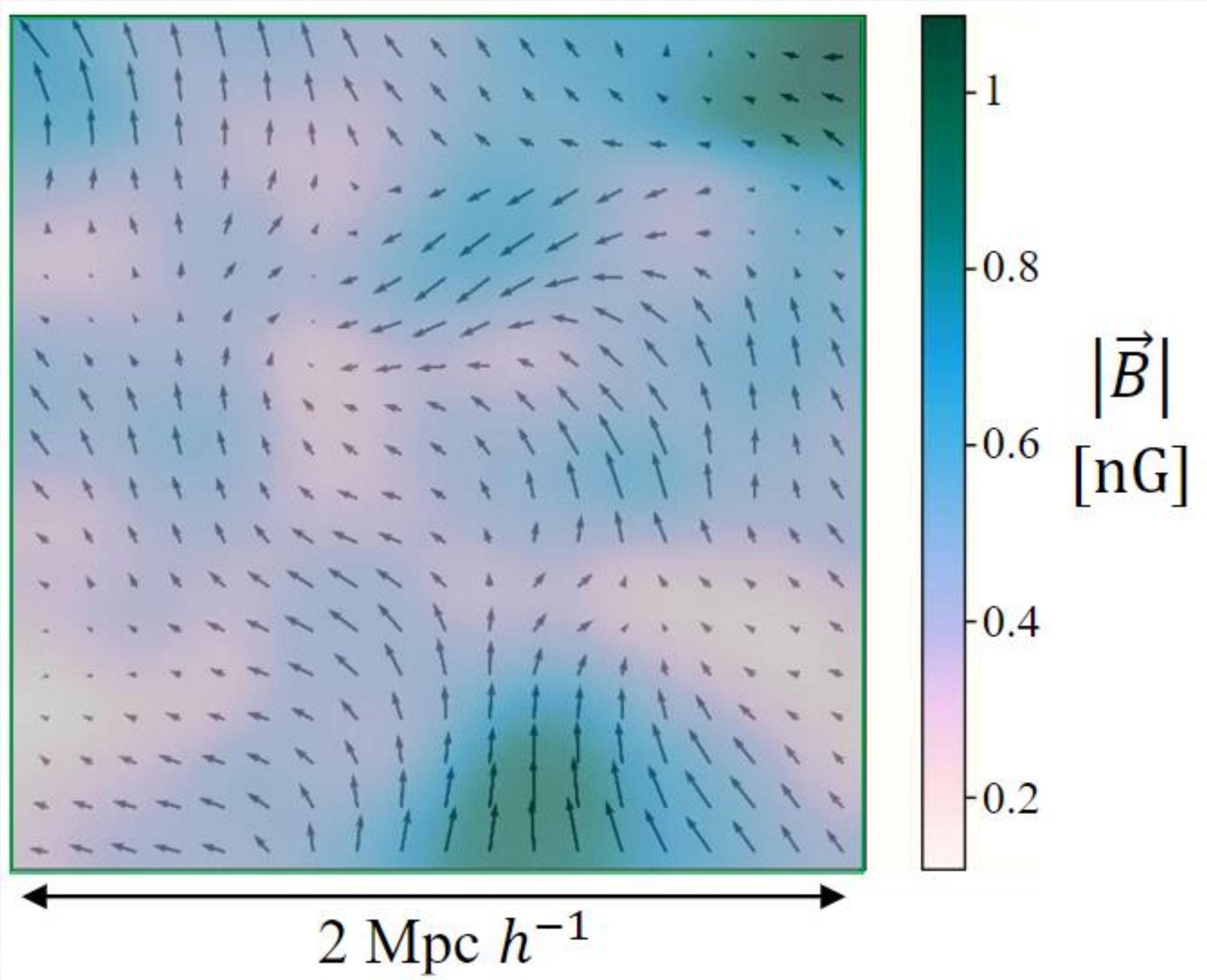
- Baryon fraction

$$f_{\rm b} = \frac{\delta\rho_{\rm b}}{\delta\rho_{\rm DM} + \delta\rho_{\rm b}} = \bar{f}_{\rm b}\frac{\delta_{\rm b}}{\bar{f}_{\rm DM}\delta_{\rm DM} + \bar{f}_{\rm b}\delta_{\rm b}} = \bar{f}_{\rm b}\frac{\delta_{\rm b}}{\delta_{\rm m}}$$



Main features: 2) simulations

- Improved initial conditions code (N-GenIC)



- Displacement of baryons explicitly includes Lorentz force

$$\vec{x}_{\rm dis} = [\vec{x}_{\rm dis}]_{\Lambda\rm CDM} + \xi^{\rm num} \times \frac{(\nabla \times \vec{B}) \times \vec{B}}{(4\pi\rho_b a^3)a^3 H^2}$$

- *We do not* have an MHD simulation. However, this should not alter significantly the structure formation at early redshifts that we are interested in.

Equations and evolution of perturbations

$$\frac{\partial \vec{v}_\mathrm{b}}{\partial t} + H\vec{v}_\mathrm{b} + \frac{(\vec{v}_\mathrm{b}\cdot\nabla)\vec{v}_\mathrm{b}}{a} + \frac{c_\mathrm{b}^2}{a}\nabla\delta_\mathrm{b} = \frac{(\nabla\times\vec{B})\times\vec{B}}{4\pi a^5\rho_\mathrm{b}} - \frac{\nabla\phi}{a}$$

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{a}\nabla\times(\vec{v}_\mathrm{b}\times\vec{B})$$

$$\frac{\partial\delta_b}{\partial t} + \frac{\nabla\cdot\vec{v}_b}{a} + \frac{\nabla\cdot(\delta_b\vec{v}_b)}{a} = 0,$$

$$\nabla^2\phi = \frac{1}{2M_\mathrm{pl}^2}a^2(\rho_\mathrm{b}\delta_\mathrm{b} + \rho_\mathrm{DM}\delta_\mathrm{DM}).$$

Both LCDM and PMF part

Equations and evolution of perturbations

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$$a^2\frac{\partial^2\delta_{\rm b}}{\partial a^2} + a\frac{3}{2}\frac{\partial\delta_{\rm b}}{\partial a} - \frac{3}{2}\frac{\Omega_{\rm b}}{\Omega_{\rm m}(1+a_{\rm eq}/a)}\delta_{\rm b} = -\frac{S_0}{a^3H^2} + \frac{3}{2}\frac{\Omega_{\rm DM}}{\Omega_{\rm m}(1+a_{\rm eq}/a)}\delta_{\rm DM}$$

$$a^2\frac{\partial^2\delta_{\rm DM}}{\partial a^2} + a\frac{3}{2}\frac{\partial\delta_{\rm DM}}{\partial a} - \frac{3}{2}\frac{\Omega_{\rm DM}}{\Omega_{\rm m}(1+a_{\rm eq}/a)}\delta_{\rm DM} = \frac{3}{2}\frac{\Omega_{\rm b}}{\Omega_{\rm m}(1+a_{\rm eq}/a)}\delta_{\rm b}.$$

Equations and evolution of perturbations

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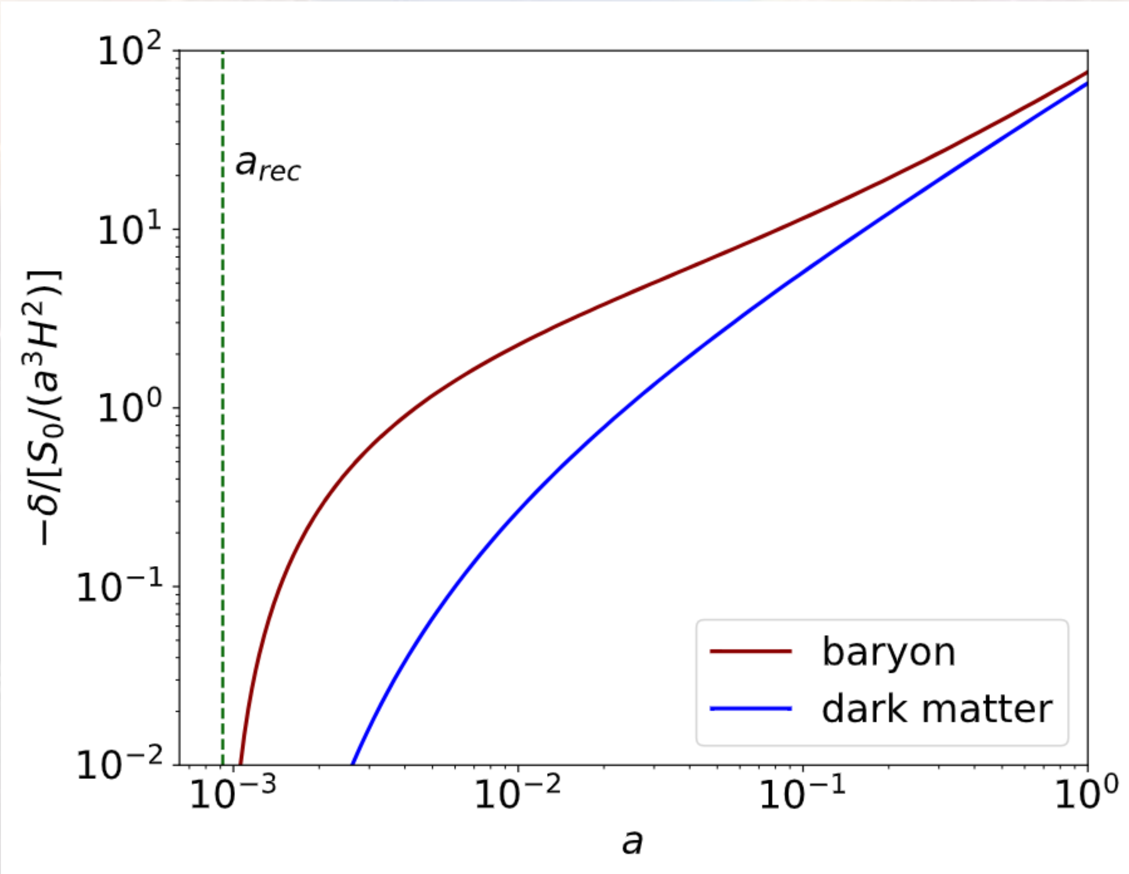
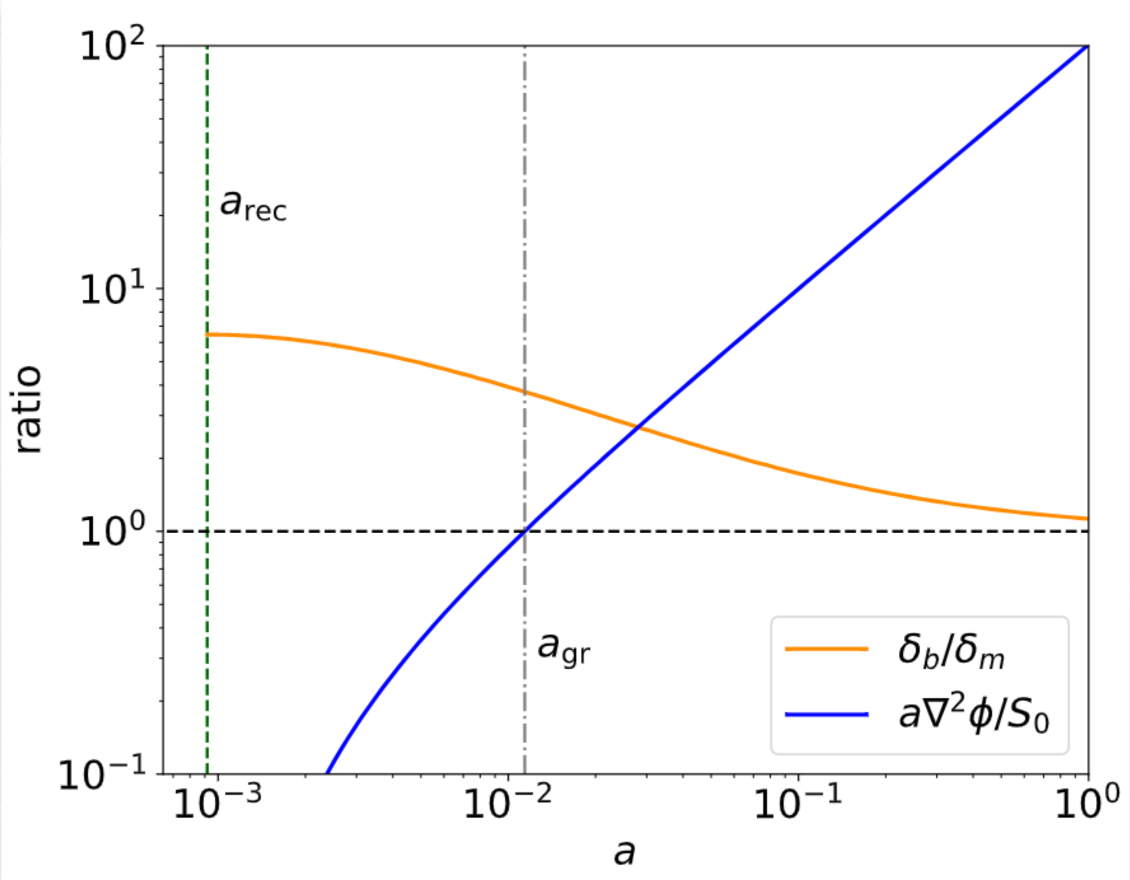
$$\delta_{\rm b}^{\rm PMF}=-\xi_{\rm b}(a)\frac{S_0}{a^3H^2} \qquad \delta_{\rm DM}^{\rm PMF}=-\xi_{\rm DM}(a)\frac{S_0}{a^3H^2}.$$

Equations and evolution of perturbations

$$\delta_{\rm b}^{\rm PMF} = -\xi_{\rm b}(a)\frac{S_0}{a^3H^2} \qquad \delta_{\rm DM}^{\rm PMF} = -\xi_{\rm DM}(a)\frac{S_0}{a^3H^2}.$$





Comment on damping scale and bump height

$$\Delta_{\rm b}^{\rm PMF}(k) \equiv \frac{k^3 P_{\rm b}^{\rm PMF}(k)}{2\pi^2} = 10^{-4}\xi_{\rm b}^2(a)\left(\frac{k}{\rm Mpc^{-1}}\right)^{2n_{\rm B}+10}\left(\frac{B_{\rm 1Mpc}}{\rm nG}\right)^4 G_{n_{\rm B}} e^{-2k^2\lambda_{\rm D}^2}$$

$$\lambda_{\rm D} = 0.1\kappa_{n_{\rm B}} {\rm Mpc}\left(\frac{B}{\rm nG}\right),$$

$$\Delta_{\rm b}^{\rm PMF}(k=\lambda_{\rm D}^{-1}) = \xi_{\rm b}^2(a)\kappa_{n_{\rm B}}^{-4} G_{n_{\rm B}} e^{-2}.$$

$$\Delta_{\rm b}^{\rm PMF}(k=\lambda_{\rm D}^{-1}, a=0.01) = \eta.$$

$$\kappa_{n_{\rm B}} = \left(\frac{G_{n_{\rm B}}}{1.14\eta}\right)^{1/4}.$$

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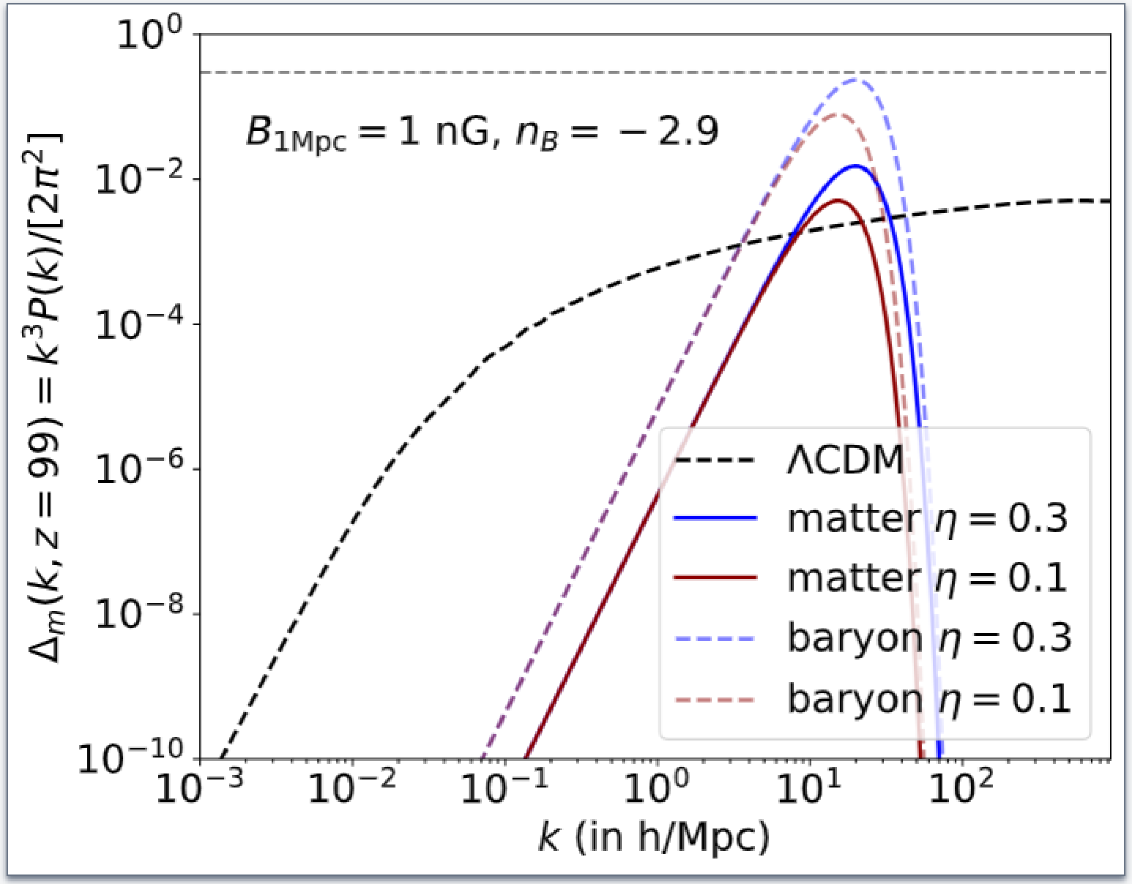

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$$\kappa_{n_{\rm B}} = \left(\frac{G_{n_{\rm B}}}{1.14\eta}\right)^{1/4}.$$

$$\Delta_{\rm b}^{\rm PMF}(k=\lambda_{\rm D}^{-1}, a=0.01) = \eta.$$

Structure formation (ΛCDM + PMFs)

- Standard ICs displacements:

$$\vec{x}_{dis}(k) = \frac{\vec{k}}{k^2}\delta(k).$$

$$\delta = \mathrm{Amp}e^{i\phi}$$

$$\mathrm{Amp} = \mathrm{norm} \times \sqrt{-P(k) \times \log(\mathrm{rand}_1)} \qquad \phi = 2\pi \times \mathrm{rand}_2,$$

Naïve: P(k) is LCDM+PMF in the square root

Less naïve: Two deltas, two amplitudes, two phases

Not-naïve: Displacement of baryons explicitly includes Lorentz force

$$\vec{x}_{\rm dis} = [\vec{x}_{\rm dis}]_{\Lambda\rm CDM} + \xi^{\rm num} \times \frac{(\nabla \times \vec{B}) \times \vec{B}}{(4\pi\rho_b a^3)a^3 H^2}$$

Simulations

- N-body/SPH for DM/gas + star formation, code P-Gadget-3 [see Springel 2005, Springel et al. 2021]
- 2×512^3 particles

Simulation	$B_{1\mathrm{Mpc}}$ [nG]	n_B	η	L_{box} [Mpc/h]	z_{end}	particle displacement method with PMFs
A	1	-2.9	0.3	55	0	Lorentz force
B	1	-2.9	0.3	55	4	$P(k)$ without isocurvature
C	1	-2.9	0.3	55	4	$P(k)$ with isocurvature
D	1	-2.9	0.1	100	0	Lorentz force
E	0.2	-2.0	0.3	55	4	Lorentz force
F	0.5	-2.9	0.3	30	4	Lorentz force

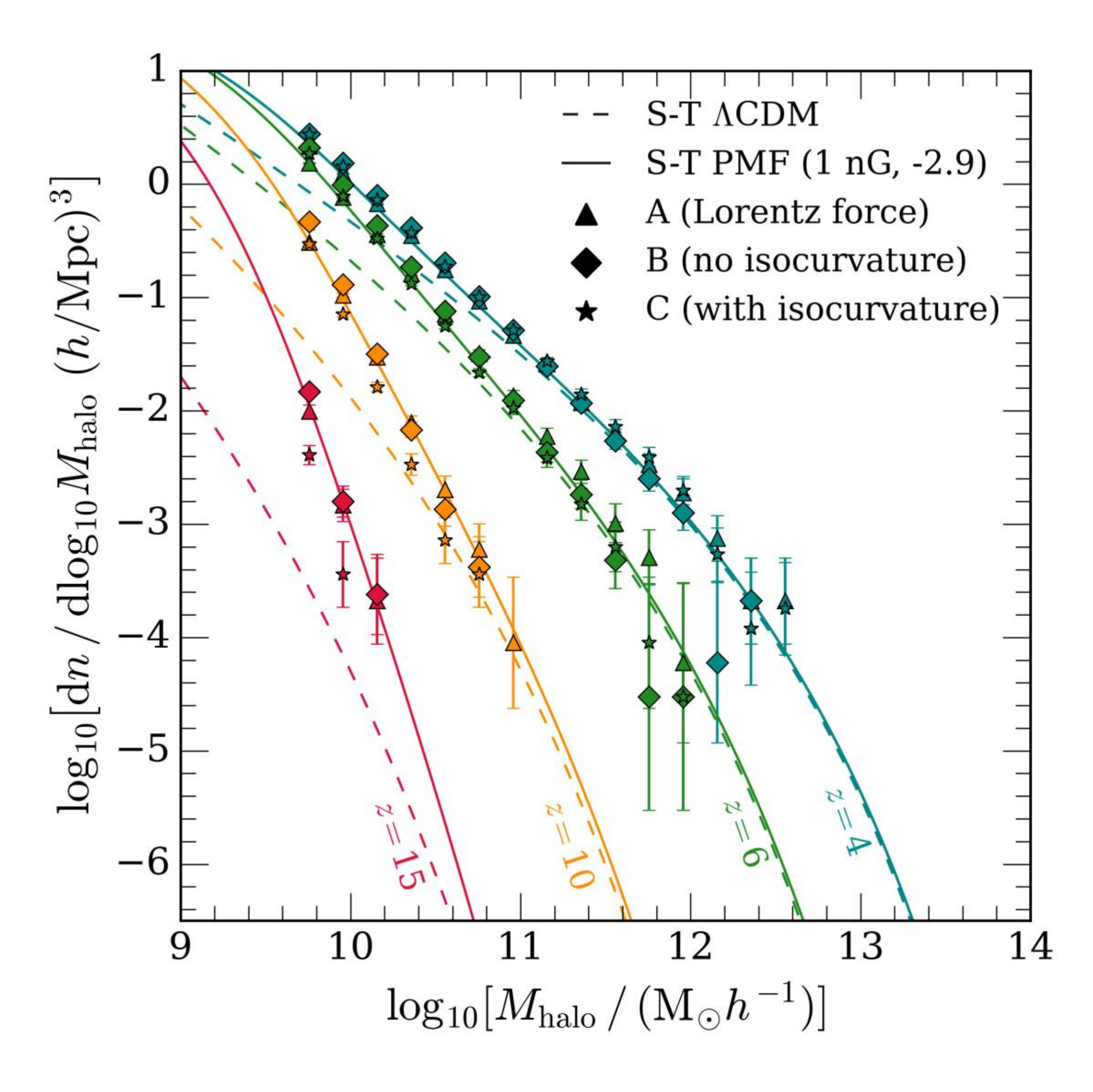
Derived parameters

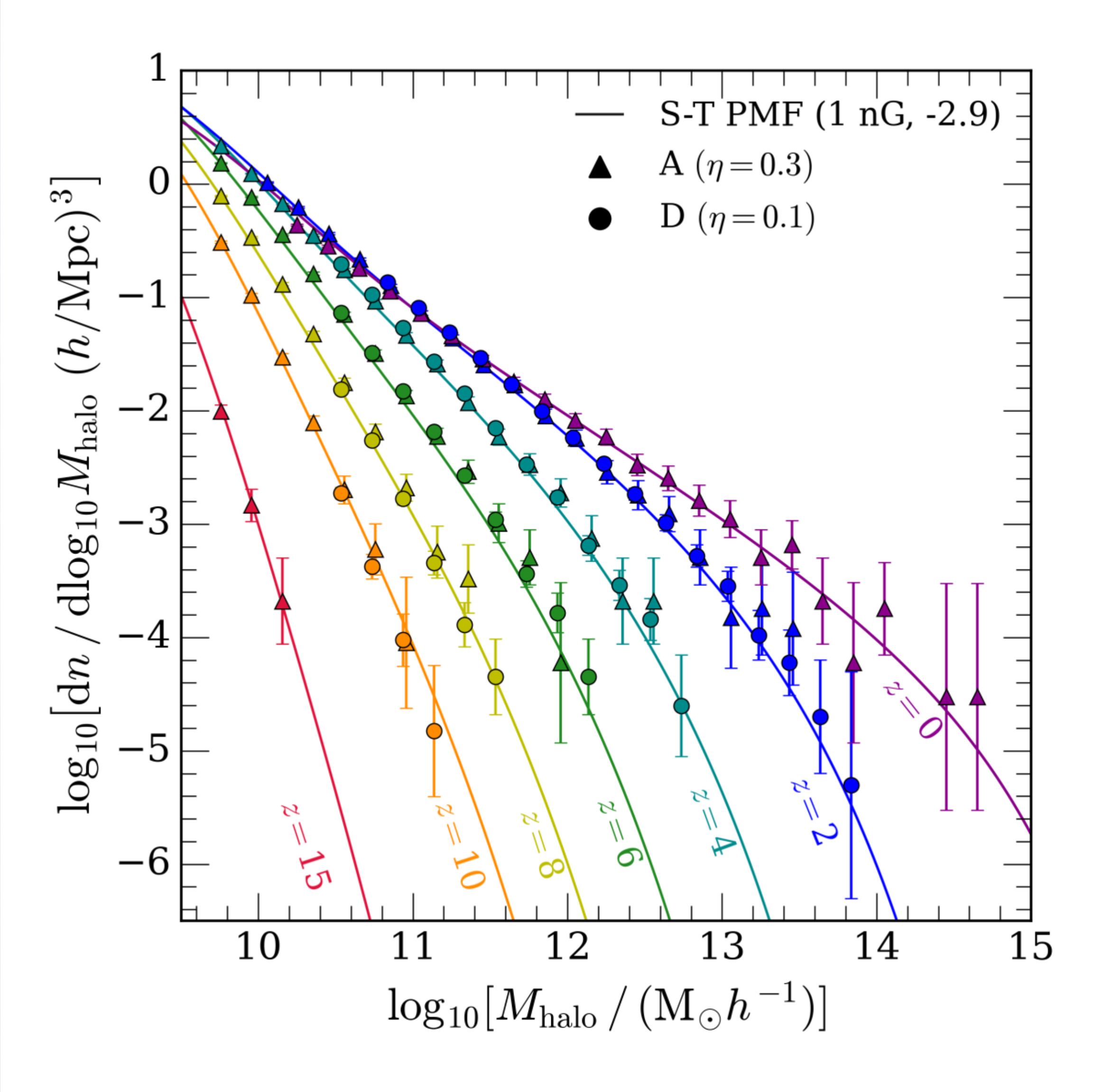
Simulation	l_{soft} [kpc/h]	k_{Nyq} [h/Mpc]	$k_{\mathrm{D}} = 1/\lambda_{\mathrm{D}}$ [h/Mpc]	m_{DM} [$\mathrm{M}_{\odot}/h$]	m_{gas} [$\mathrm{M}_{\odot}/h$]
{A, B, C}, E	4.30	29.25	{19.38}, 16.29	8.94×10^7	1.66×10^7
D	7.81	16.08	14.93	5.37×10^8	9.97×10^7
F	2.34	53.62	37.52	1.45×10^7	2.69×10^6

Cosmological parameters (Planck 2015 data [64])

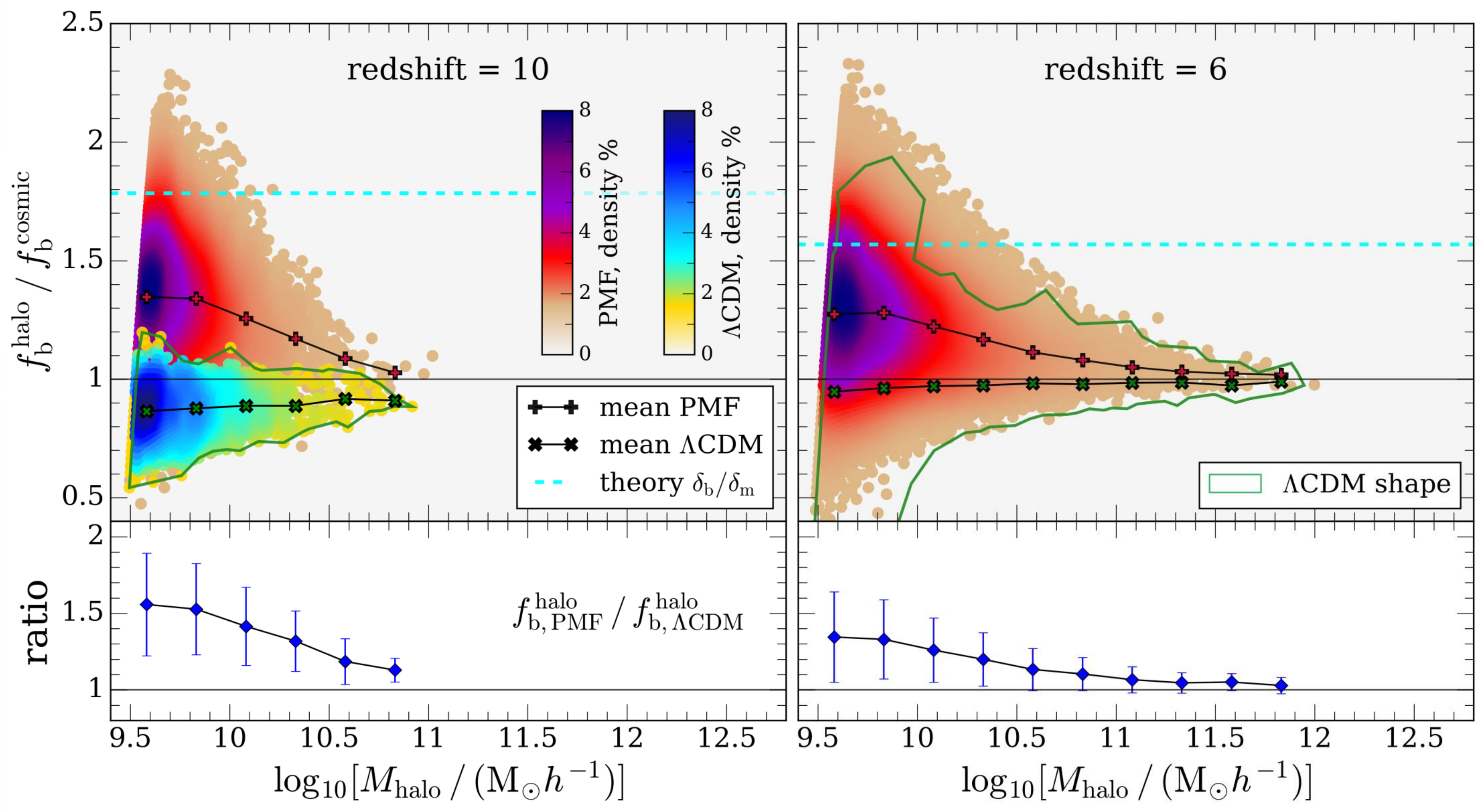
$\Omega_{\mathrm{m}} = 1 - \Omega_{\Lambda}$	Ω_{b}	h
0.308	4.82×10^{-2}	0.678

Structure formation: Halo mass function

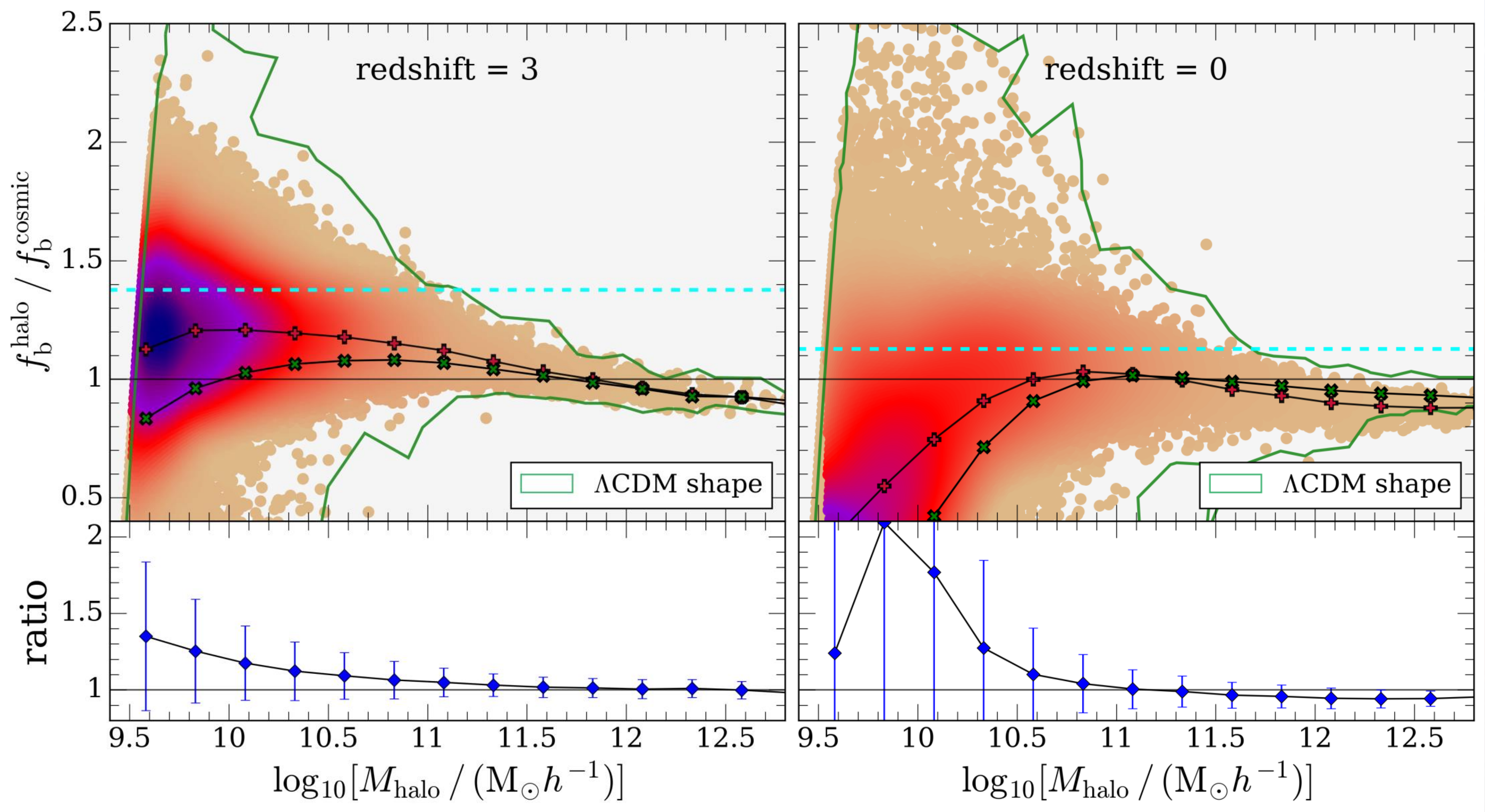




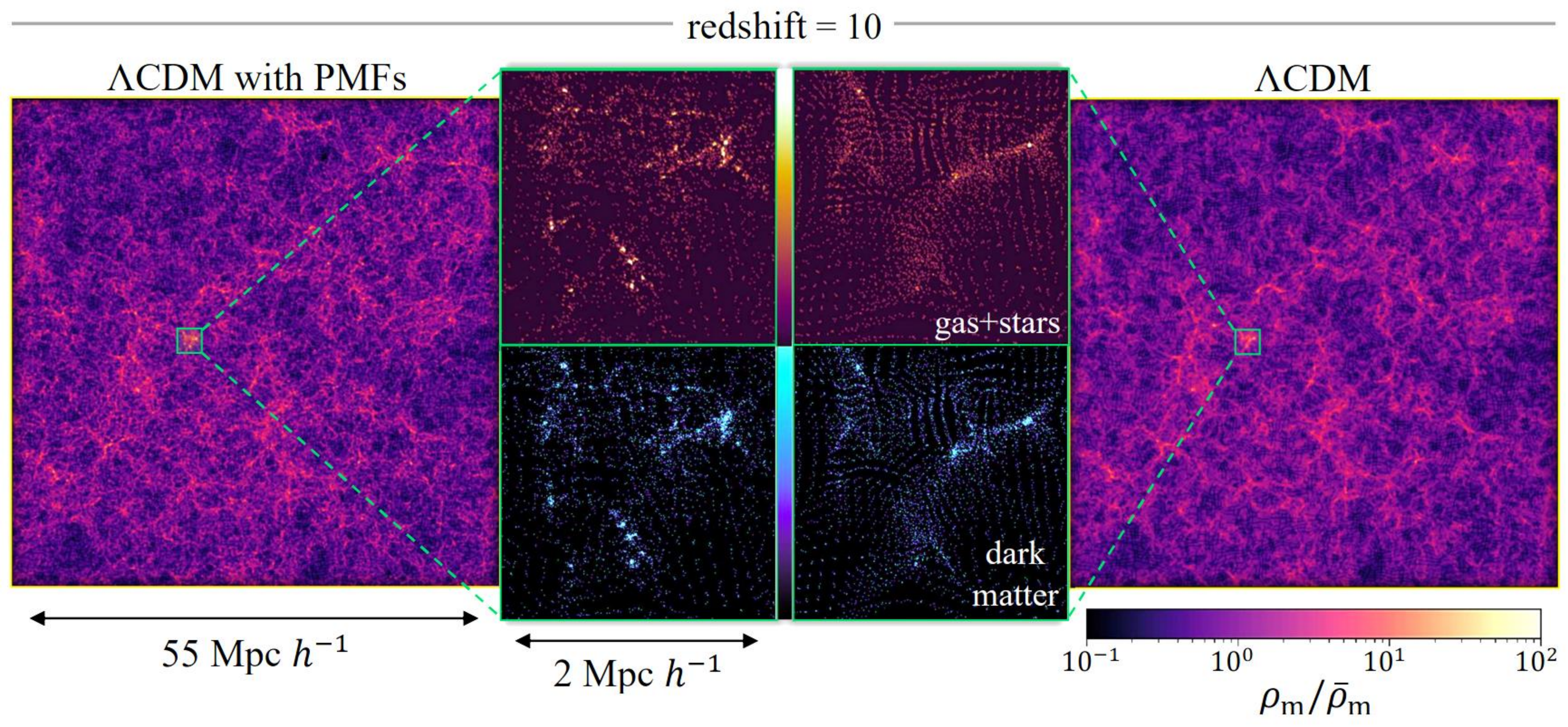
Structure formation: halo baryon fraction



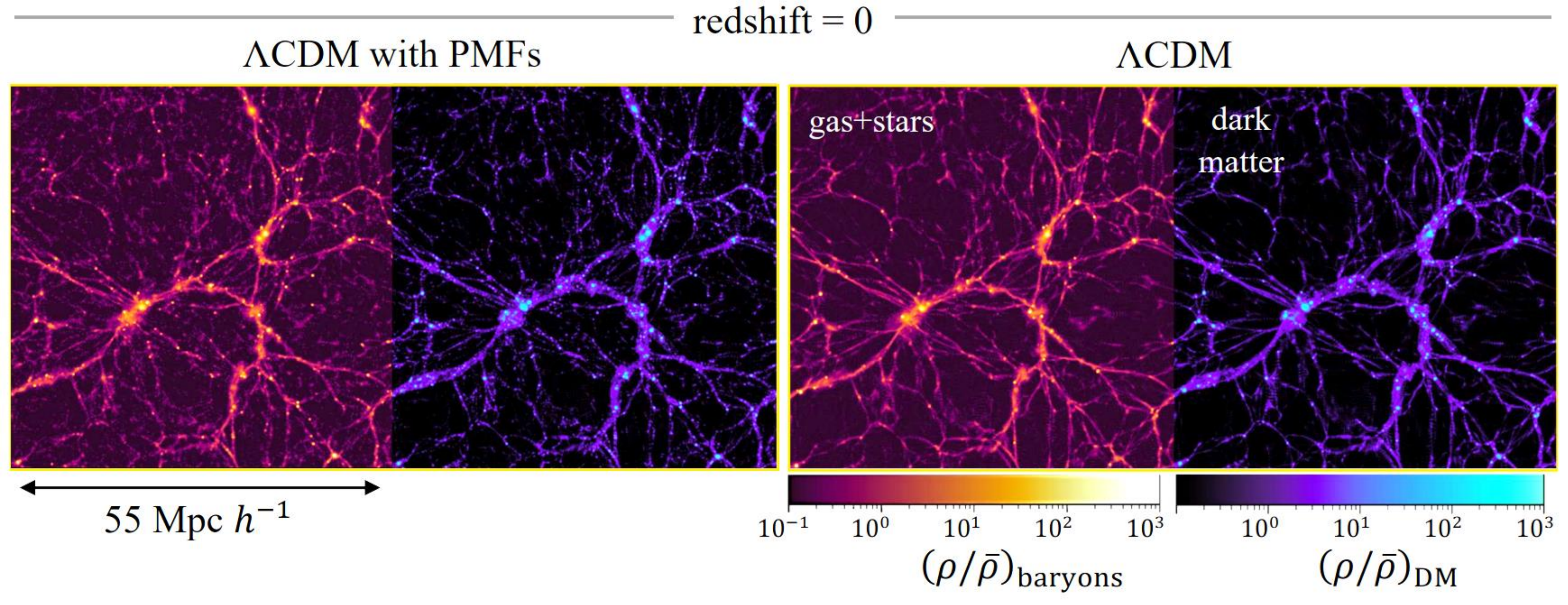
Structure formation: halo baryon fraction



Formation of halos at early redshifts

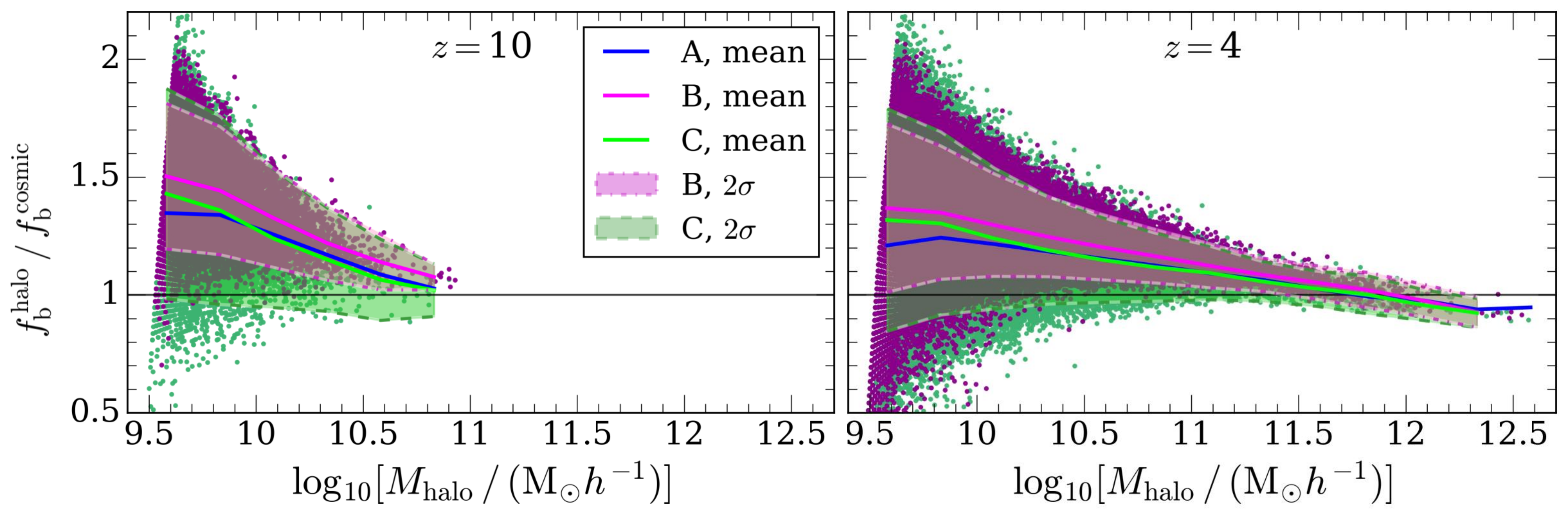


Low redshift structures

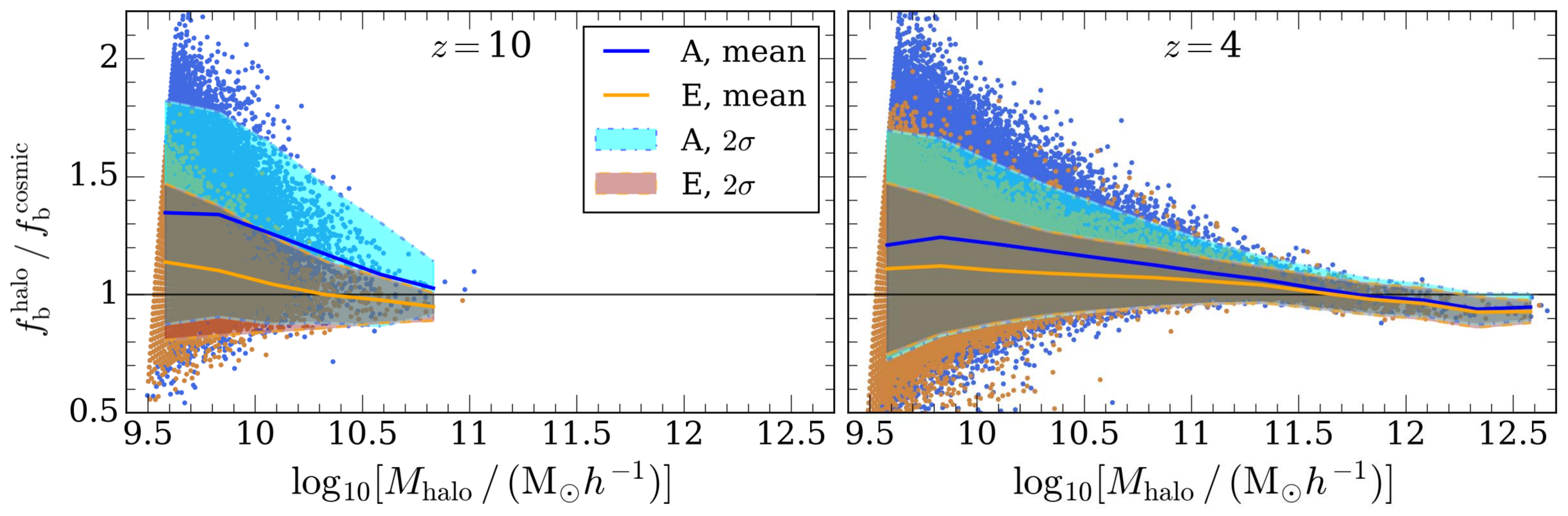


Comparison of different simulations

1 nG, $n_B = -2.9$
A = Lorentz force
B = P(k) uncorrelated
C = P(k) correlated



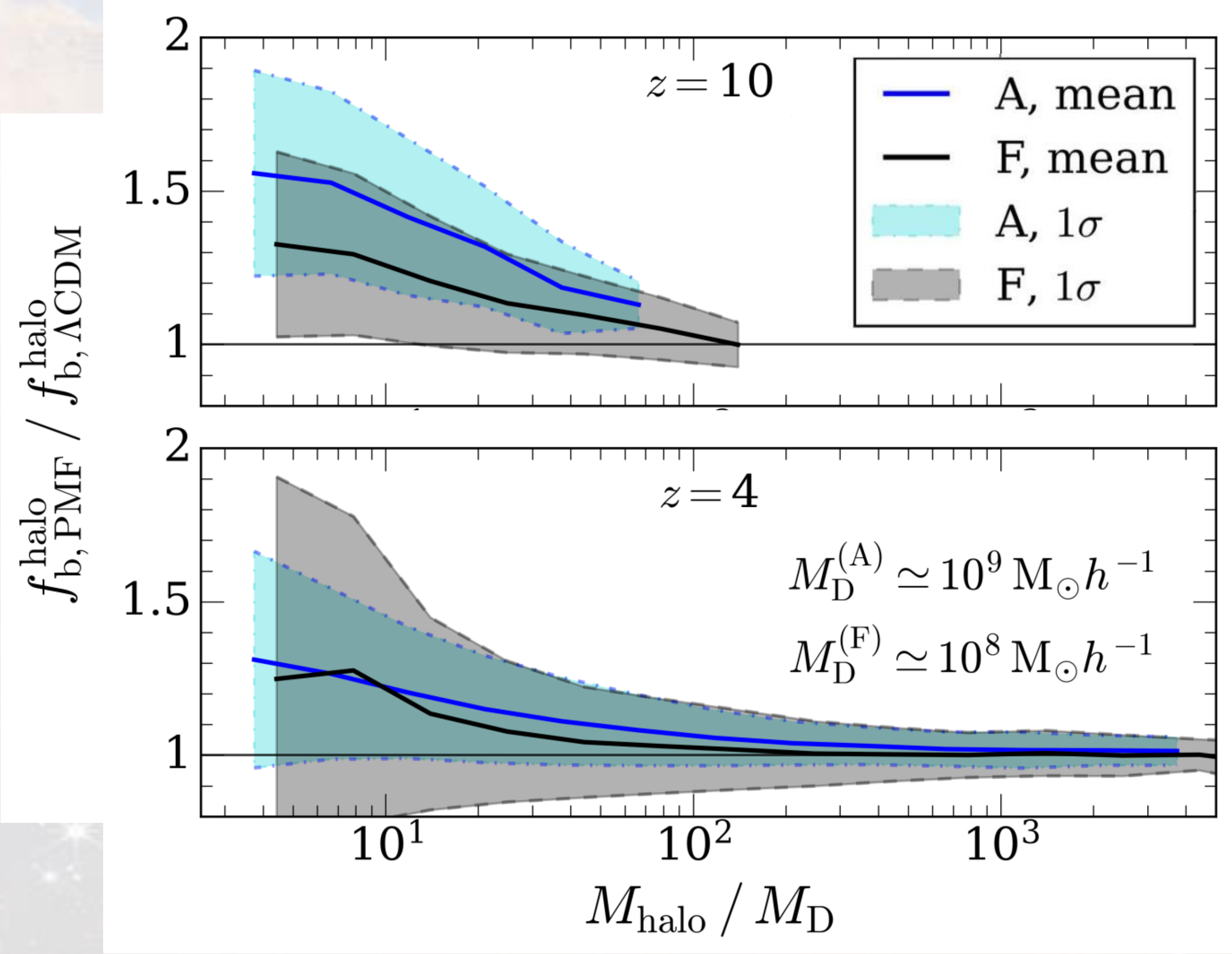
Comparison of different simulations

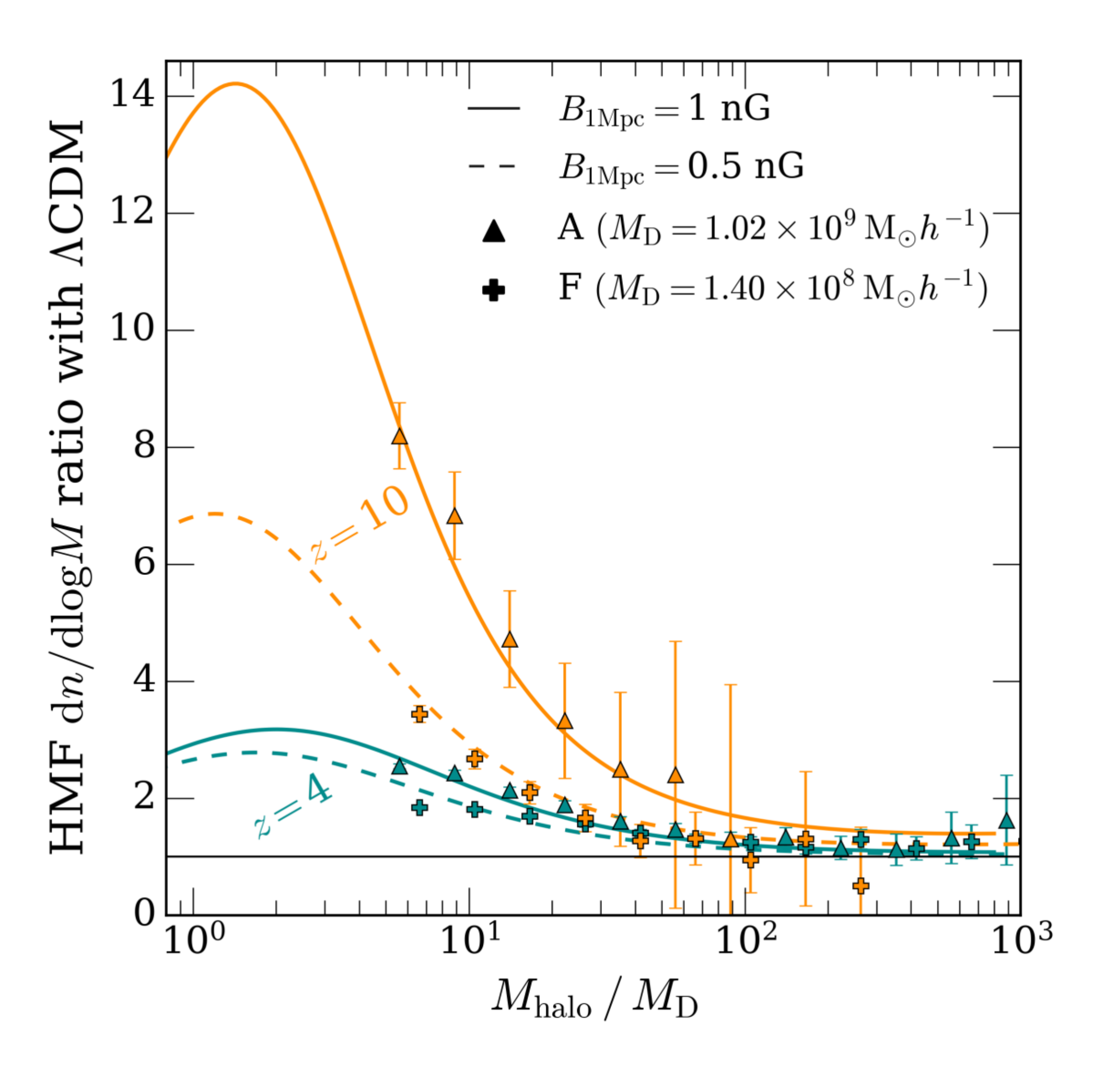


$1\ \mathrm{nG}, n_B = -2.9 \quad vs. \quad 0.2\ \mathrm{nG}, n_B = -2.0$

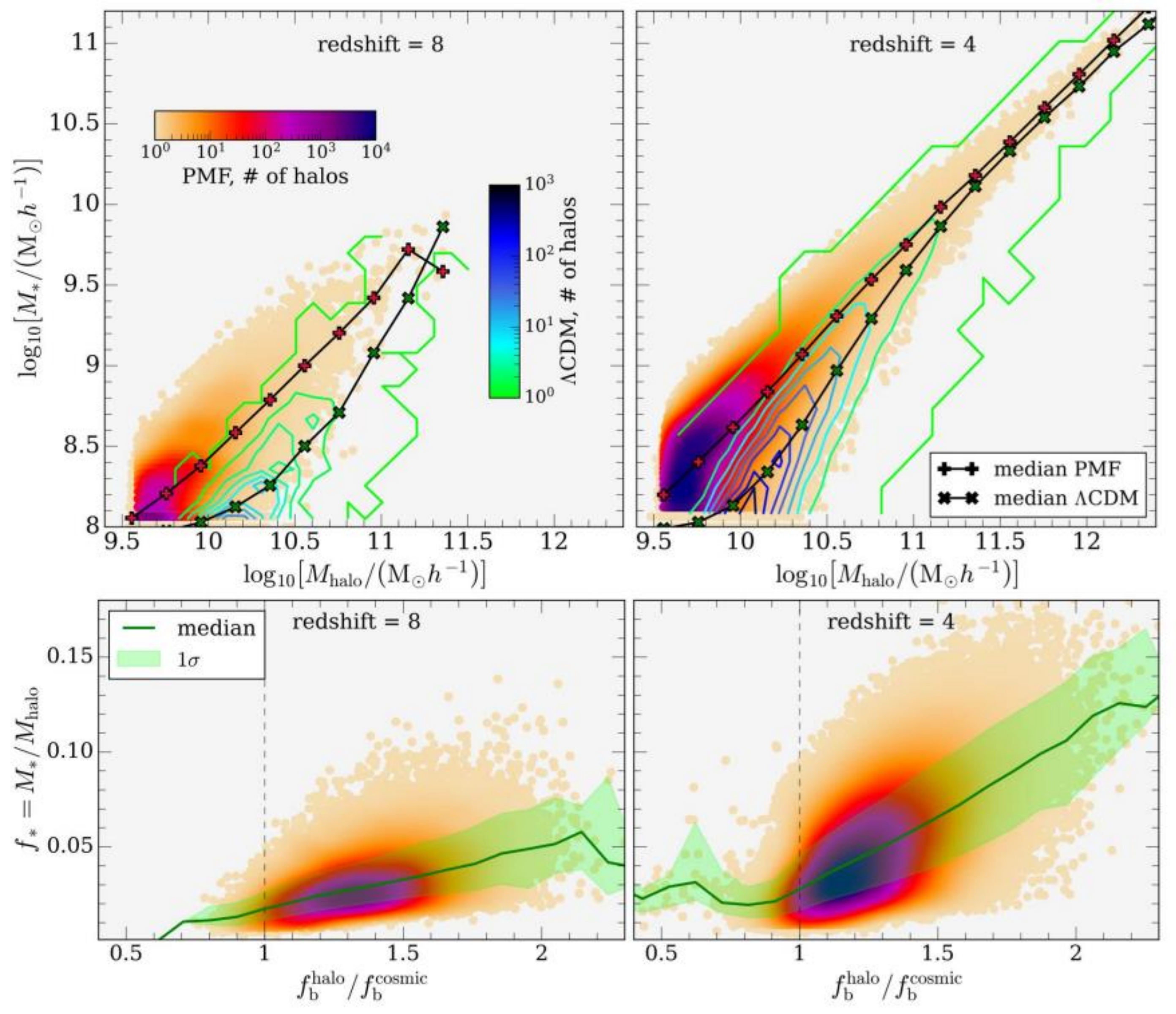
[similar matter power spectra]

Comparison of different simulations

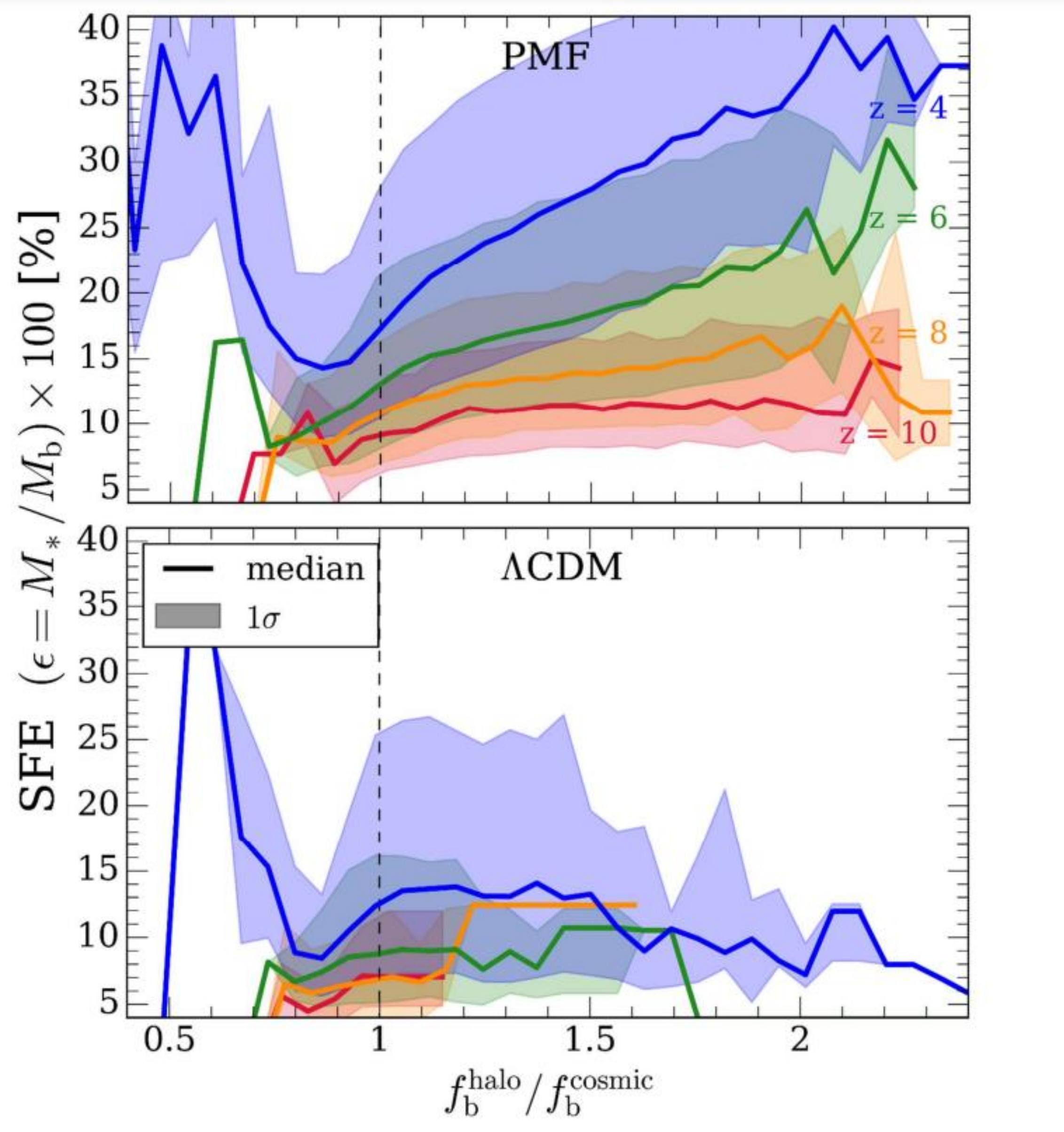


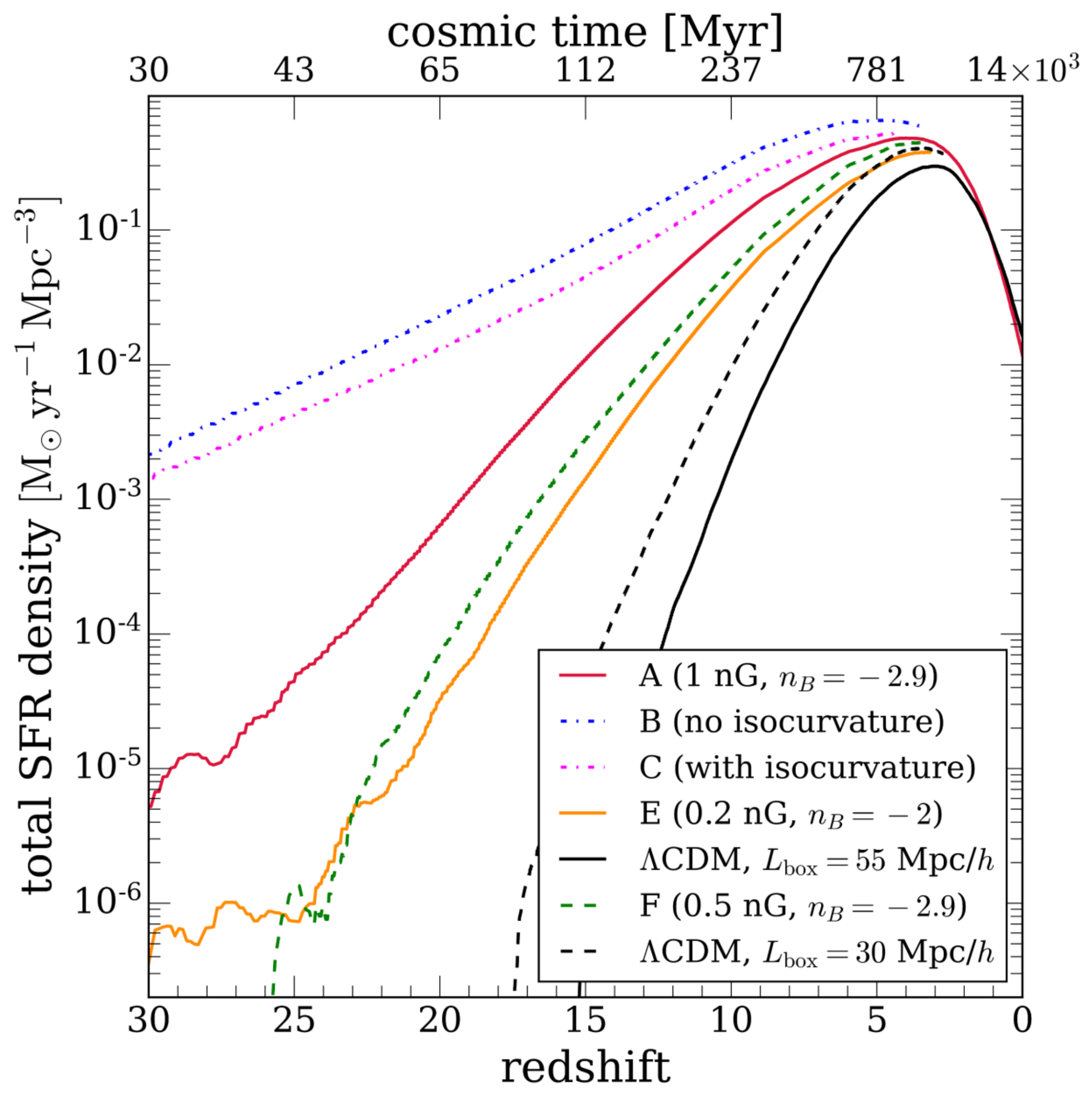


Star formation



Star formation





Summary and conclusions

- Since the PMFs affect directly only baryons we can obtain halos with baryon fraction 1.5 - 2 times larger than in the pure ΛCDM

- Overall, the power spectrum has an enhancement at lower scales, which produces higher abundance of low-mass halos at very early redshifts

- More gas available in halos may give rise to more stars

- Non-gaussianities are negligible*

Outlook on future developments and projects

- Do the full MHD (with AREPO, ENZO, RAMSES), relate parameters to specific magnetogenesis models
- More realistic star formation and galactic properties
- vorticity evolution
- FILAMENTS

*Non-gaussianities