Fluid and Magnetic Spectra in First-Order Phase Transitions

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Tanmay Vachaspati, Teerthal Patel…

«Exploring the Early Universe with
Gravitational Waves and Primordial Magnetic Fields»

Cosmological Magnetic Fields

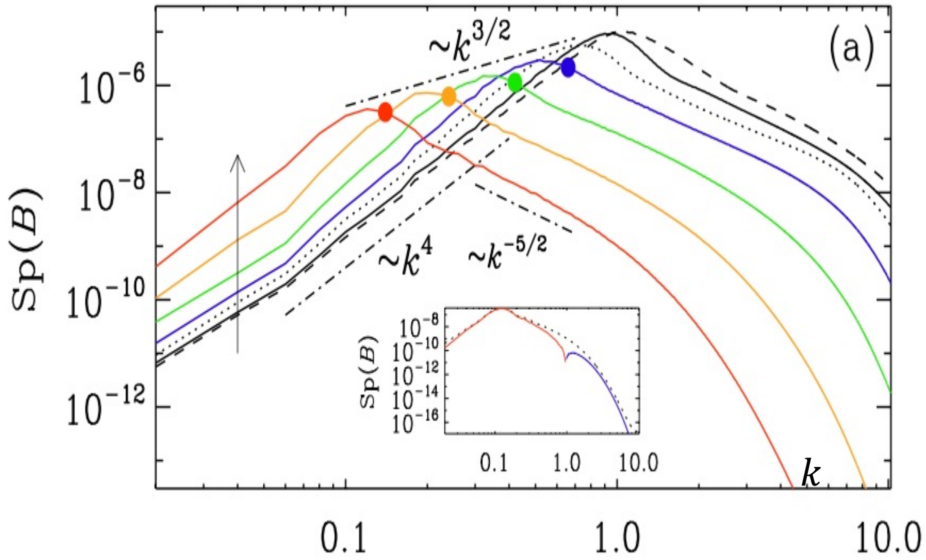
- Different bounds on different scales → e.g. on Mpc scales $10^{-16}G < B < 10^{-9}G$ (lower bounds from blazars and upper from CMB)

- Magnetohydrodynamics (MHD):

$$\partial_\mu T^{\mu\nu}_{fluid} = -J_\rho F^{\nu\rho}, \quad \partial_\mu F^{\mu\nu} = J^\nu$$

[1409.3723] $J_i - v_i \, \rho_e = \sigma \, (E + v \times B)_i \; with \; \sigma \propto T$

- $Re_M \gg 1$ → MHD Turbulence → Inverse Cascade
  At which scale should it stop?



[Brandenburg, Kamada, Schober - 2302.00512]

In radiation domination → $T^{\mu\nu}_{fluid}$ traceless → MHD conformally invariant
(no Hubble constant in the rescaled equations in conformal time)

Even if for the evolution of magnetic fields to large scales we can stick to pure MHD, in order to produce the initial magnetic field we may need other fields (scalars, pseudoscalars…)

Pure MHD $B(t=0)=0 \rightarrow B(t)=0 \quad \forall\, t$

→ Need for a magnetogenesis mechanism

EW Magnetogenesis

- EWSSB → $|\Phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$
- Higgs inhomogeneities in causally disconnected zones

  → Vacuum Manifold $S^2 \times S^1$
- Monopoles and Strings $\vec{\nabla} \cdot \vec{B} \neq 0 \rightarrow$ $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g}(\partial_\mu \hat{\Phi}^\dagger \partial_\nu \hat{\Phi} - \partial_\nu \hat{\Phi}^\dagger \partial_\mu \hat{\Phi})$
- Annihilation of monopole-antimonopole pairs

  with residual $\vec{B} \neq 0$ (see Vachaspati and Patel talks)

[Vachaspati - 2010.10525]
[Patel, Vachaspati - 2108.05357]
['t Hooft - Nucl. Phys. B 79 (1974) 276]

(E)MHD + scalar

FLRW in α time → $ds^2 = -a(\eta)^{2\alpha}\, d\eta^2 + a(\eta)^2\, d\vec{x}^2$ [Figueroa et al. - 2006.15122]

Adding a real scalar → $\partial_t^2\phi - a^{-2(1-\alpha)}\nabla^2\phi + (3-\alpha)H\,\partial_t\phi = -a^{2\alpha}\frac{\partial V}{\partial\phi}$

$T^{\mu\nu}_{tot} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{EM} + T^{\mu\nu}_{\phi}$ $\qquad$ $\nabla_\mu T^{\mu\nu}_{tot} = 0$

$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2(x,t)g_{\mu\nu}$

$T^{\mu\nu} \to \tilde{T}^{\mu\nu} = \Omega^{-6}(x,t)T^{\mu\nu}$ $\longrightarrow$ $\tilde{\nabla}_\mu\tilde{T}^{\mu\nu} + \tilde{T}\,\partial^\nu \ln\Omega = 0$

Conformal transformation which makes MHD flat in radiation domination

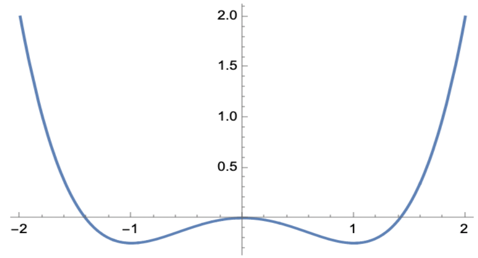
$\Omega(x,t) = a^{-1}(t) \to \nabla_\mu T^{\mu 0} - T\,H = 0$ where $T \neq 0$ $\quad$ ($T^{\mu\nu}_{\phi}$ has non-zero trace)

→ Need to consider the expansion of the universe also in numerical simulations

Electroweak Cosmological Phase Transition

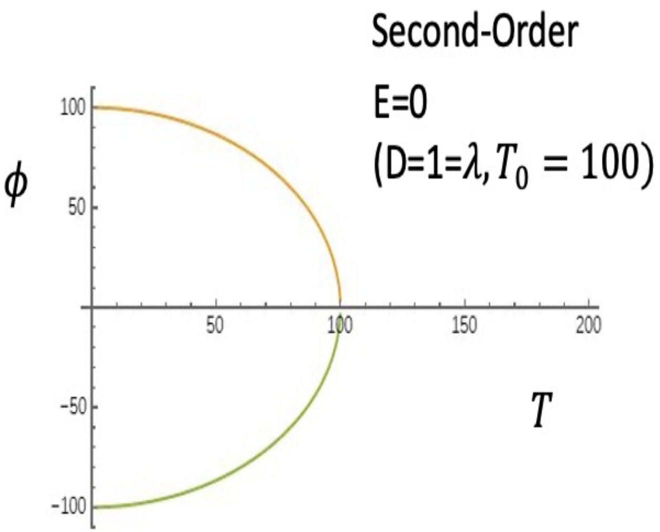
Tree Level Renormalizable Higgs Potential → $V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \rightarrow \phi^2_{vacuum} = \frac{\mu^2}{\lambda}$

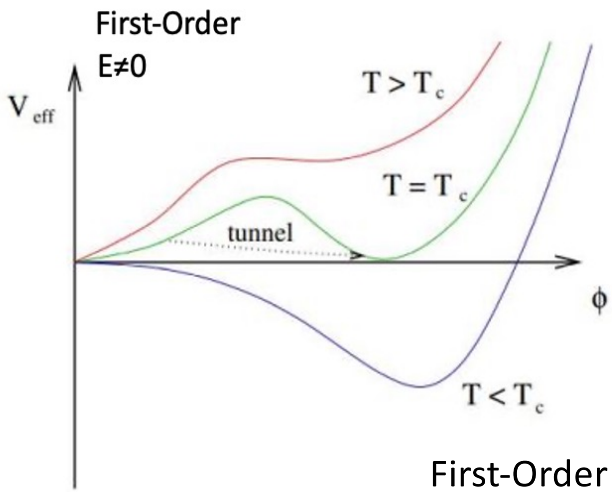
$SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \rightarrow SU(3)_C \bigotimes U(1)_{em}$

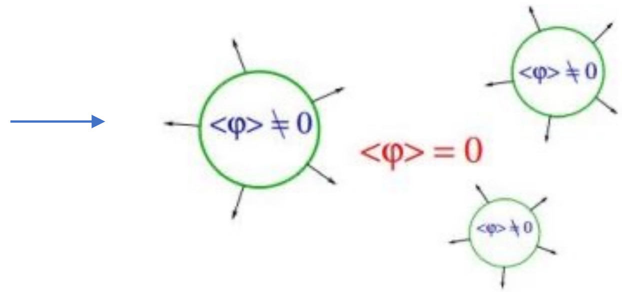


1-Loop Thermal Effective Potential (high-T) → $V^{eff}_{1-Loop}\,(\phi, T) \approx D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$

[Dine et al. – arXiv:hep–ph/9203203]







First-Order requires BSM → [Shaposhnikov - 1992 (Ph. L. B 277)]

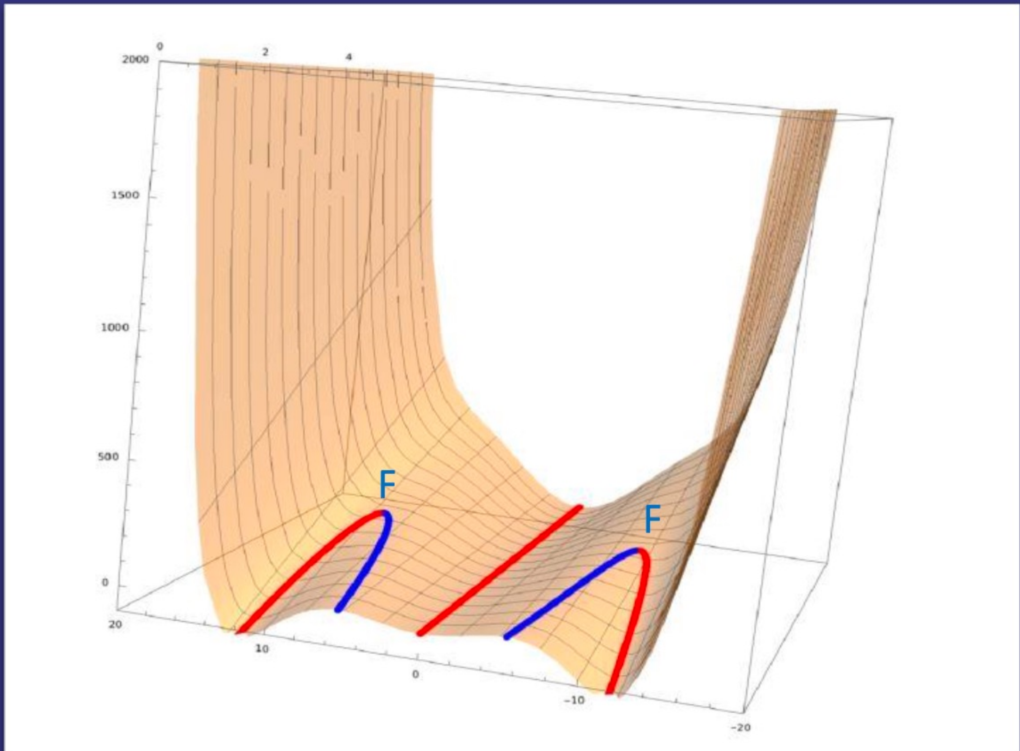
First-Order vs Second-Order

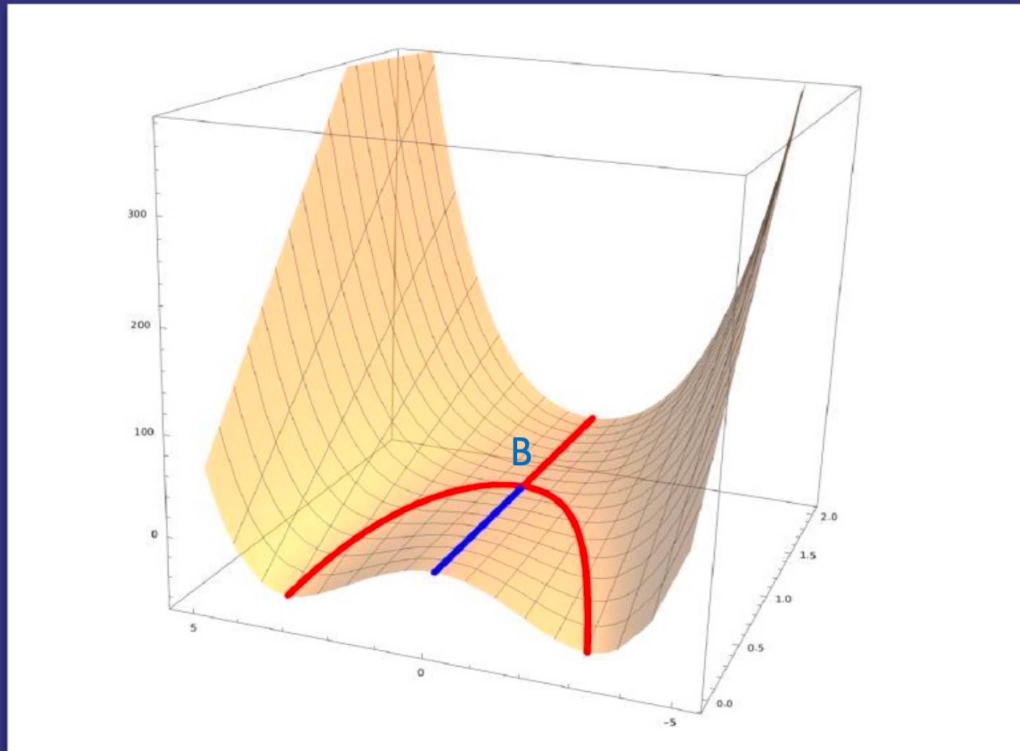
First-Order Phase Transition

$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$

Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$



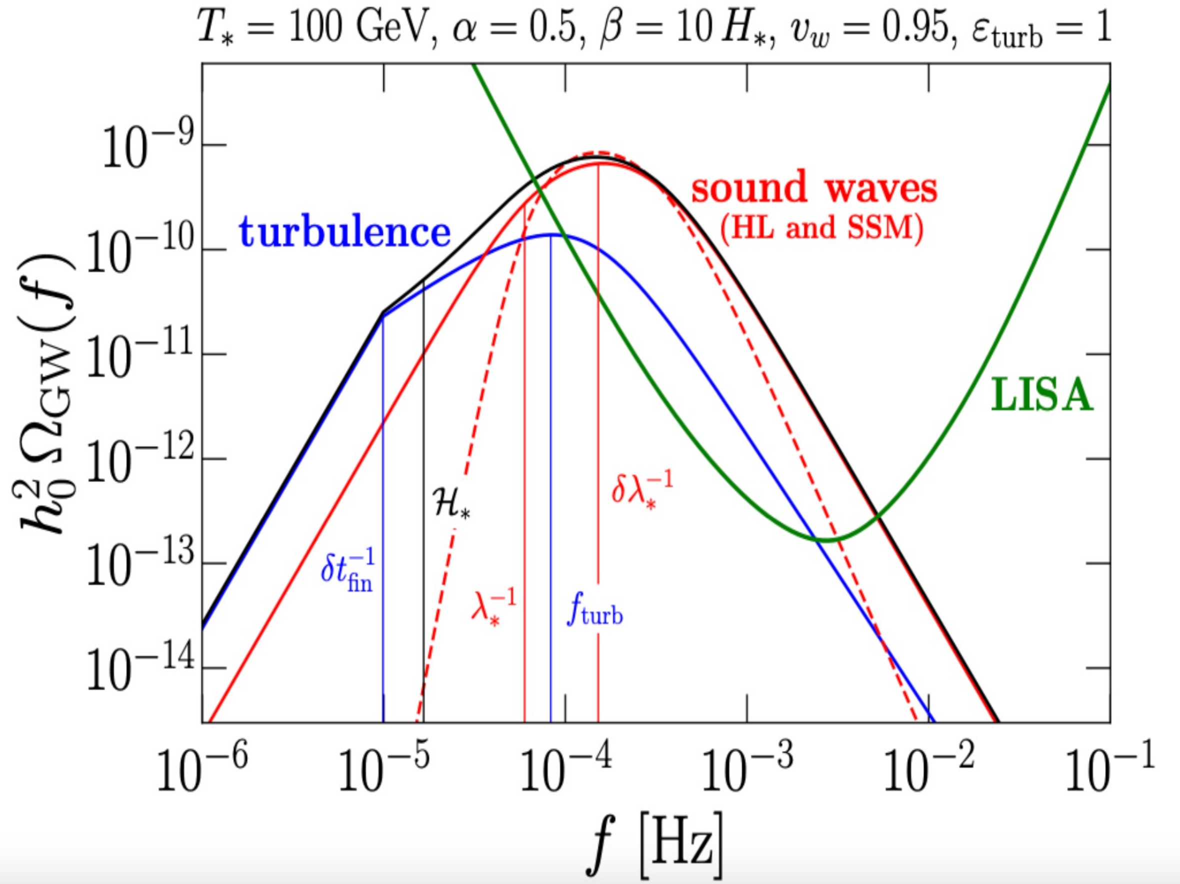


Gravitational Waves from First-Order Phase Transitions

SGWB from First-Order Phase Transitions

$$T_{ij} \supset \omega\, \gamma^2 v_i v_j + B_i B_j + \partial_i \phi\, \partial_j \phi$$

- Bubble Collisions → from breaking spherical symmetry of the expanding bubbles
- Sound Waves → from fluid compressional modes induced by bubble collisions
- MHD Turbulence → from vortical modes produced by scalar-gauge-fluid dynamics



Predictions for the Electroweak Phase Transition in the sensitivity range of LISA

[Roper Pol et al. - 2201.05630, 2307.10744 (above plot), 2308.12943]
[Jinno et al. - 2209.04369]

GWs and unequal-time correlator of the source

- Radiation dominated FLRW → $(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2)\, h_{ij} = 16\pi G \rho_c a^2(\eta)\, \Pi_{ij}(\vec{k},\eta)$

  with $\Pi_{ij}(\vec{k},\eta) = \Lambda_{ijlm} T_{lm}(\vec{k},\eta) = \left(P_{il}P_{jm} - \frac{1}{2} P_{ij}P_{lm}\right) T_{lm}(\vec{k},\eta)$ $\qquad P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$

- Gravitational wave energy density fraction at present time

$$\Omega_{GW}(\eta_0, k) = 3\, \mathcal{T}_{GW}\, k \int \frac{d\eta_1}{\eta_1} \frac{d\eta_2}{\eta_2} \cos k(\eta_0 - \eta_1) \cos k(\eta_0 - \eta_2)\, \mathrm{E}_\Pi(\eta_1, \eta_2, k)$$

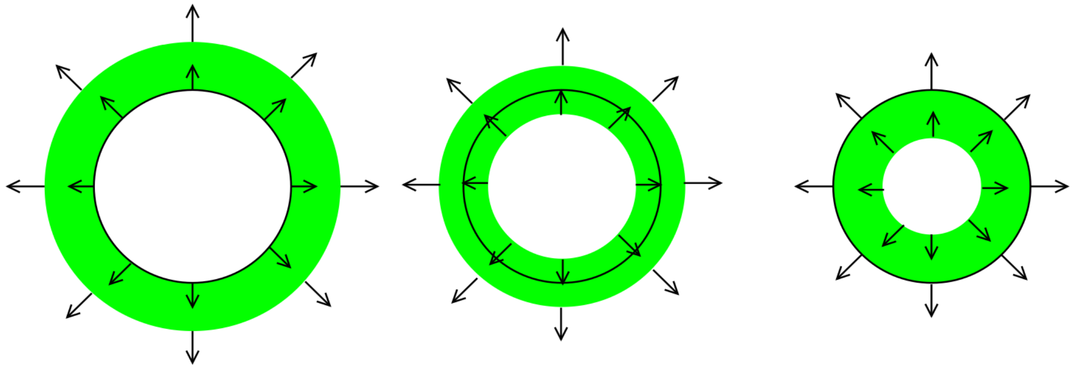
- Unequal-time correlator of the source $\left\langle \Pi_{ij}(\eta_1, \vec{k}) \Pi_{ij}^*(\eta_2, \overrightarrow{k'}) \right\rangle = (2\pi)^6 \frac{E_\Pi(\eta_1,\eta_2,k)}{4\pi k^2}\ \delta^3(\vec{k} - \overrightarrow{k'})$

[Roper Pol, Procacci, Caprini - 2308.12943]

Unequal time correlator of the source: before collisions

- Perfect fluid description for the plasma → $T^{\mu\nu}_{fluid} = (\rho + p) u^\mu u^\nu + p\, \eta^{\mu\nu}$
- Scalar-fluid friction leads to stationary profiles → $\vec{v} = \sum_i \hat{r}_i\, v_{ip}(\xi)$, $\quad \xi = \frac{r_i}{t}$, $\quad i = 1, \dots, N_B$
- $v_{ip}(\xi)$ from spherical symmetric solutions of $\partial_\mu T^{\mu\nu}_{fluid} = 0$
  (depending on phase transition strenght,
  bubble wall velocity and
  fluid equation of state)

$$T_{ij}(\vec{k}) \propto \hat{k}_i\, \hat{k}_j \rightarrow \Pi_{ij}(\vec{k}, \eta) = 0$$
$$\rightarrow E_\Pi(\eta_1, \eta_2, k) = 0$$



[Espinosa et al. - 1004.4187 (above picture)]

→ We should have no GWs before collisions
(maybe at high frequencies from spherical symmetry breaking quantum fluctuations [2403.20164])

Fluid velocity spectrum: before collisions

- [Caprini et al. - 0711.0593] → Non-zero vorticity and GWs production *before collisions* (after averaging over nucleation locations)

- [Roper Pol, Caprini, Procacci, Midiri - 24**.*****]
  Analytical study of irrotational fluid perturbations in First-Order Phase Transitions

- Equal time two-point correlator computed by averaging over bubble nucleation locations and nucleation times $B_{ij}(\vec{x},\vec{y}) = \langle v_i(\vec{x}) v_j(\vec{y}) \rangle_{x_0,t_0} = B_{ij}(\vec{x}-\vec{y})$
  $< v_i(\vec{k}) v_j^*(\vec{k'}) > = (2\pi)^3 \delta^3(\vec{k}-\vec{k'}) F_{ij}(\vec{k})$

- $B_{ij}(\vec{r}) = \hat{r}_i \hat{r}_j B_L(r) + (\delta_{ij} - \hat{r}_i \hat{r}_j) B_N(r) \ \rightarrow B_L(r) = B_N(r) + r\frac{d}{dr} B_N(r)$

- $F_{ij}(\vec{k}) = \hat{k}_i \hat{k}_j F_L(k) + (\delta_{ij} - \hat{k}_i \hat{k}_j) F_N(k) \ \rightarrow \ F_N(k) = 0$ (compatible with $\vec{\nabla} \times \vec{v} = 0$)

- No vorticity produced by averaging over nucleation locations, in contrast with previous studies

Gravitational Waves: before collisions

- Two-point fourth moment of velocities (homogeneous and isotropic fields)

$$< T_{ij}(\vec{x})T_{lm}(\vec{y}) > \approx < v_i(\vec{x})v_j(\vec{x})v_l(\vec{y})v_m(\vec{y}) > = B_{ij,lm}(\vec{r})$$

$$T_{ij}(\vec{k}) \approx \frac{1}{(2\pi)^3}\int v_i(\vec{k}-\vec{q})v_j(\vec{q})d^3\vec{q}$$

$$< T_{ij}(\vec{k})T^*_{lm}(\vec{k'}) > = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k'})F_{ij,lm}(\vec{k})$$

$$\begin{aligned} F_{ij,lm}(\vec{k}) = {} & F_1(k)\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m + F_2(k)\left(\delta_{ij}\hat{k}_l\hat{k}_m + \delta_{lm}\hat{k}_i\hat{k}_j\right) \\ & + F_3(k)\left(\delta_{il}\hat{k}_j\hat{k}_m + \delta_{im}\hat{k}_j\hat{k}_l + \delta_{jm}\hat{k}_i\hat{k}_l + \delta_{jl}\hat{k}_i\hat{k}_m\right) + F_4(k)\left(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}\right) \\ & + F_5(k)(\delta_{ij}\delta_{lm}) \end{aligned}$$

(See Monin and Yaglom, Mechanics of Turbulence Vol. 2)

$$\longrightarrow < \Pi_{ij}(\vec{k})\Pi^*_{ij}(\vec{k}') > = \Lambda_{ijlm} < T_{ij}(\vec{k})T^*_{lm}(\vec{k}') > = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')\ 4\ F_4(k)$$

- Before collisions → $F_{ij,lm}(\vec{k}) = F_1(k)\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m$ → No Gravitational Waves!

On the applicability of Wick's Theorem

- In [0711.0593] Wick's theorem applied to 2-point correlator of fluid energy momentum tensor (EMT) indicating GW production even before collisions
- Before collisions bubble velocity distribution actually far from gaussian

  → Violation of Wick's theorem's hypothesis

  → Need direct computation of 2-point fourth moment of fluid velocities

$$
\begin{aligned}
< \Pi_{ij}(k)\Pi^*_{ij}(k') > \propto\ & < v_i(k)v_j(k)v_i^*(k')v_j^*(k') > \\
\neq\ & < v_i(k)v_j(k) > < v_i(k')v_j(k') > \\
& + < v_i(k)v_i^*(k') > < v_j(k)v_j^*(k') > \\
& + < v_i(k)v_j^*(k') > < v_j(k)v_i^*(k') >
\end{aligned}
$$

- What about after collisions?
- Assuming restoration of gaussianity → GWs spectrum computed from scalar gradients, fluid velocity and magnetic field spectra, otherwise need to compute 2-point correlator of EMT

[Jinno, Takimoto - 1605.01403v2]

Unequal-time correlator of the source: after collisions

- After bubble collisions assuming fluid linear perturbations (weak FOPTs) → sound waves
- For sound waves, considering only the fluid contribution to the Π_{ij} and assuming gaussianity (and Wick's theorem)

$$E_\Pi(\eta_1, \eta_2, k) \propto k^2 \int dp \int d\tilde{p}\, f(p, \tilde{p}, k)\, E_{kin}(\eta_1, \eta_2, p) E_{kin}(\eta_1, \eta_2, \tilde{p})$$

where $E_{kin}(\eta_1, \eta_2, p)$ is the unequal-time correlator of fluid velocities

[Roper Pol, Procacci, Caprini - 2308.12943]

→ Simulations of weak FOPTs showing sound waves as the main source of GWs
[Hindmarsh et al. - 1504.03291]

- If non-linearities develop → Given $Re \gg 1$, $Re_M \gg 1$ → HD or MHD (if $B \neq 0$) turbulence
[Arnold et al. - arXiv:hep-ph/0010177v1]

→ Simulations of strong FOPTs showing vorticity production after bubble collisions
[Cutting et al. - 1906.00480]

Fluid Velocity vs Magnetic Field Spectra

- $F_{ij}(\vec{k}) = \hat{k}_i\hat{k}_j F_L(k) + (\delta_{ij} - \hat{k}_i\hat{k}_j)F_N(k)$
- Irrotational fluid velocities (e.g. bubble expansion or sound waves)

$\vec{\nabla} \times \vec{v} = 0 \ \rightarrow \ F_N(k) = 0$

- Purely vortical fields (e.g. incompressible fluid or *monopole-free* magnetic fields)

$\vec{\nabla} \cdot \vec{B} = 0 \ \rightarrow \ F_L(k) = 0$

$$F_{ij}(k) = \frac{1}{2\pi^2} \int_0^\infty \frac{\sin(kr)}{kr} B_{ij}(r)\, r^2\, dr$$

If $B_{ij}(r)$ go to zero for large r faster than any power law $\rightarrow$ existence of small k expansion

$F_N(k) = F_N^{(0)} + F_N^{(2)}\,k^2 + F_N^{(4)}\,k^4 + \ \dots;\ F_L(k) = F_L^{(0)} + F_L^{(2)}\,k^2 + F_L^{(4)}\,k^4 + \ \dots;\ F_N^{(0)} = F_L^{(0)}$

Both sound waves (L) and magnetic fields (N) should respect $F_{L\backslash \mathrm{N}} \approx F_{L\backslash \mathrm{N}}^{(2)}\,k^2$ for small k $\rightarrow$

$E(k) = 2(4)\pi k^2 F_{L(N)} \approx 2(4)\pi F_{L(\mathrm{N})}^{(2)}\,k^4$

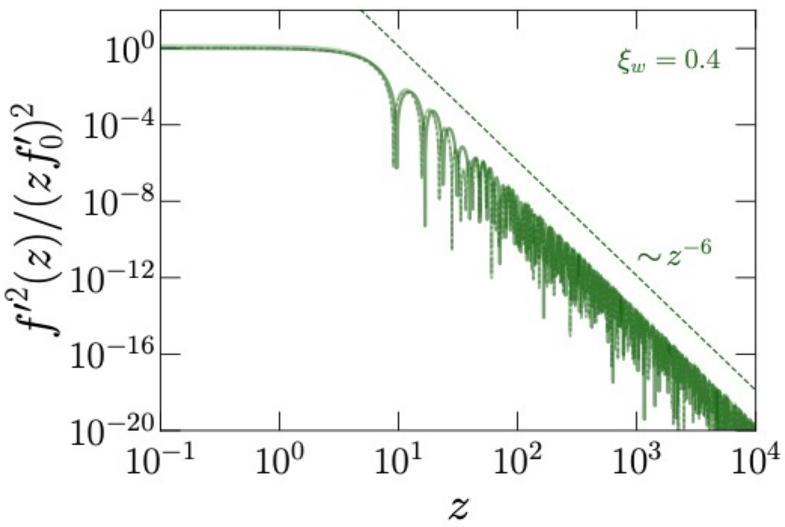
(See Monin and Yaglom, Mechanics of Turbulence Vol. 2)

Large scales behavior of causal spectra ($k \ll 1$)

- If we think that from causality we should have a scale at which $B_{ij}(r)$ is identically zero or at least decays exponentially to zero
- As a consequence → for purely irrotational or vortical fields $F(k) \propto k^2$
- While for mixed cases (also for $\vec{\nabla} \cdot \vec{B} \neq 0$) $F(k) \propto k^0$

Causality condition valid for the bubble expansion phase (large scales scaling k^2, while small scales scaling k^{-4} from discontinuities in the second derivative of $B_L(r)$ related to the discontinuities in the velocity profiles)

[Roper Pol, Caprini, Procacci, Midiri 24**.*****]



→As a consequence of causality we also expect (and find) the same k^3 scaling in the large scale GW spectrum for both sound waves and MHD turbulence

Causality violating spectra from strings and monopoles?

- Spectra scaling as k for arbitrary small k
  (see Vachaspati and Patel talks)

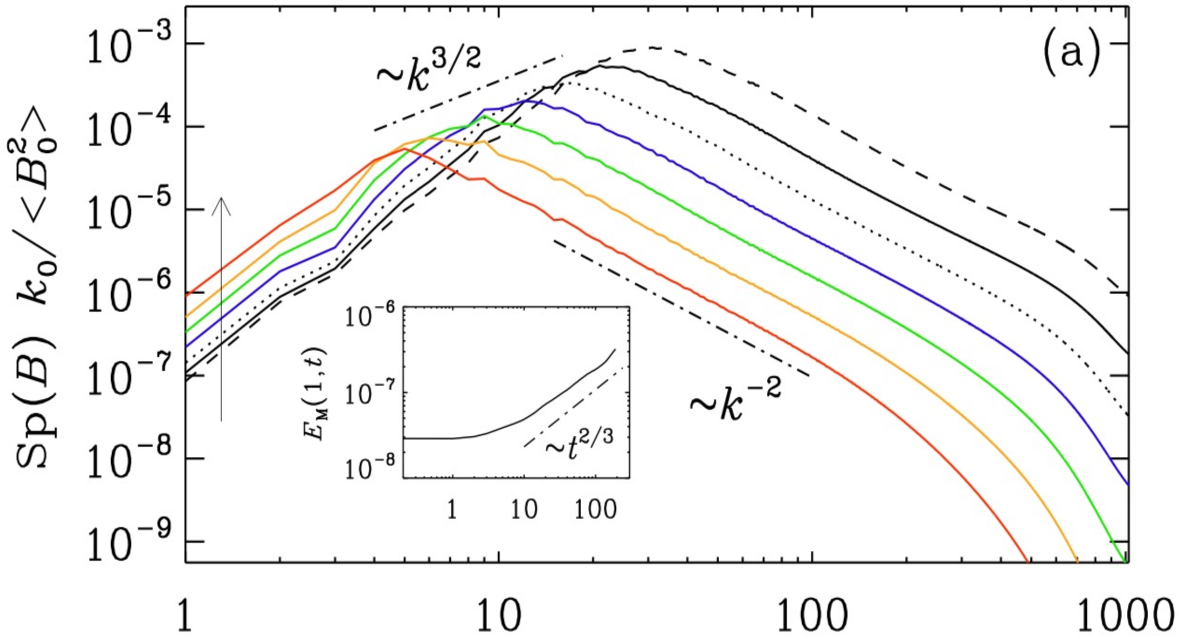
can be obtained only with «non-causal» real space correlation → for example (*after monopole/anti-monopole annihilation*) from $\vec{\nabla} \cdot \vec{B} = 0$ and with

$$B_L(r) = \frac{const}{1+\left(\frac{r}{r_*}\right)^4}$$

$$B_N(r) = \frac{1}{2r}\frac{d}{dr}(r^2 B_L)$$

$$\rightarrow \quad F_N(k) \propto k \qquad (E(k) \propto k^3)$$

(k^0 or k^2 at Hubble scale?)



[Brandenburg, Sharma, Vachaspati - 2307.04602]

Summary

- We need lattice simulations of scalar-gauge-fluid dynamics (*also considering the expansion of the universe*) in order to understand GW and Magnetic Field production from First-Order Phase Transitions (FOPTs)
- 2-point unequal time correlator (UETC) of the Energy Momentum Tensor of scalar fields, fluid velocities and magnetic fields (or 2-point UETC of the fields when gaussianity is a valid hypothesis) is required for the analytical derivation of the GW spectrum from FOPTs
- Analytical study of fluid and magnetic spectra can help in the interpretation of the magnetic fields and GWs produced

Thanks for your attention!