



# Top-Bottom Interference Contribution to Fully-Inclusive Higgs Production

Felix Eschment

with M. Czakon, M. Niggetiedt, R. Poncelet, T. Schellenberger

based on 2312.09896 (accepted by PRL)

May 20, Loopfest 2024



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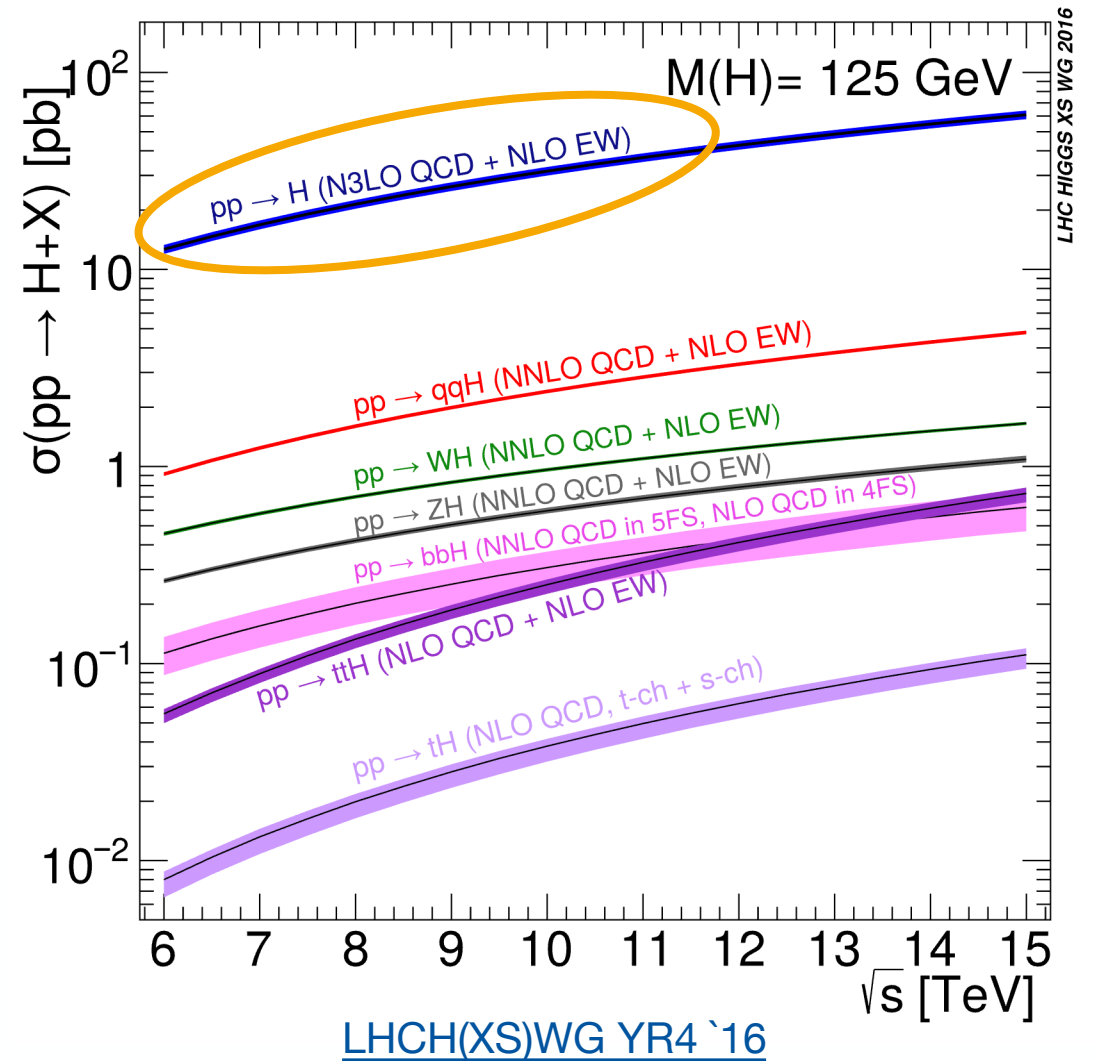
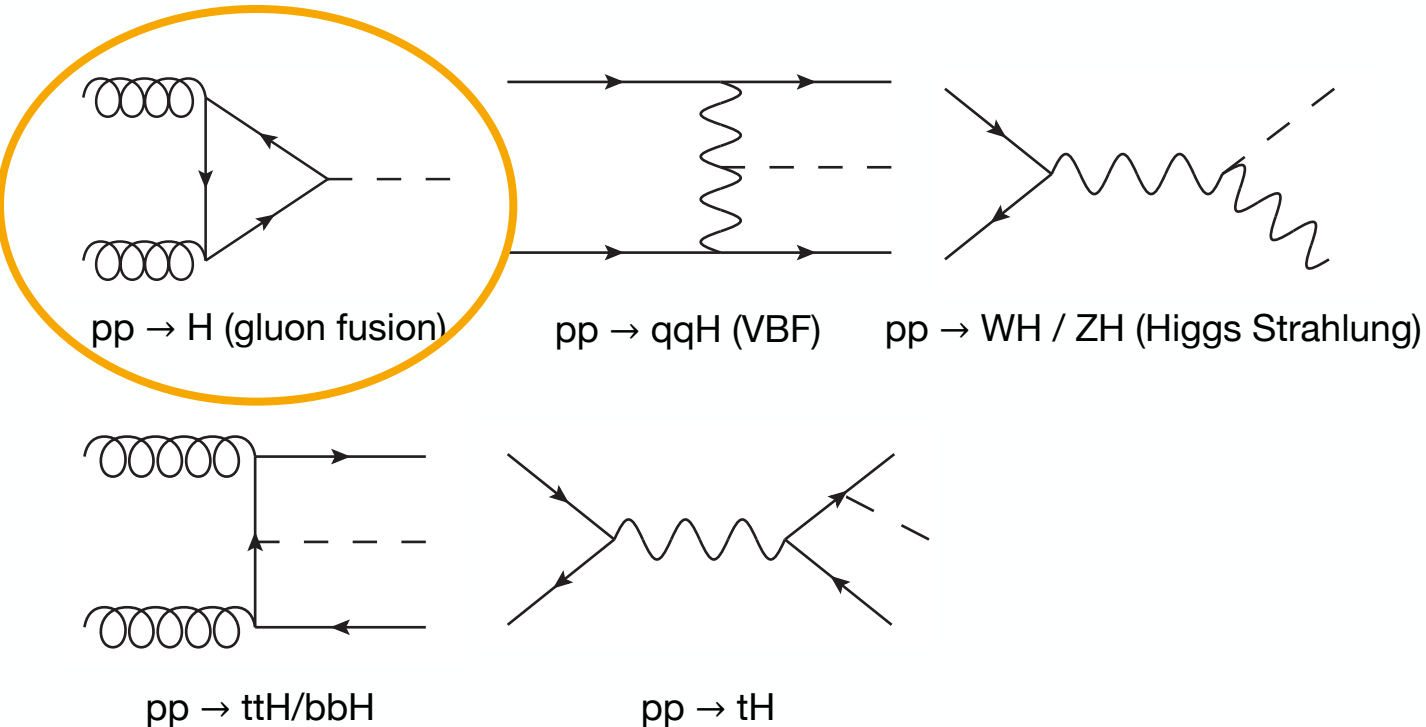
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and 240x.xxxxx

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# Motivation

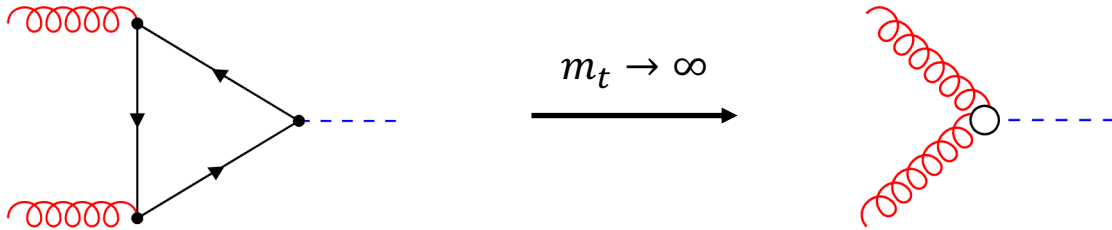
- Higgs production cross section central observable in Higgs physics
- HL-LHC anticipates  $\mathcal{O}(2\%)$  uncertainty



→ Crucial to reduce theory uncertainty on gluon-fusion cross section as much as possible

# Gluon fusion

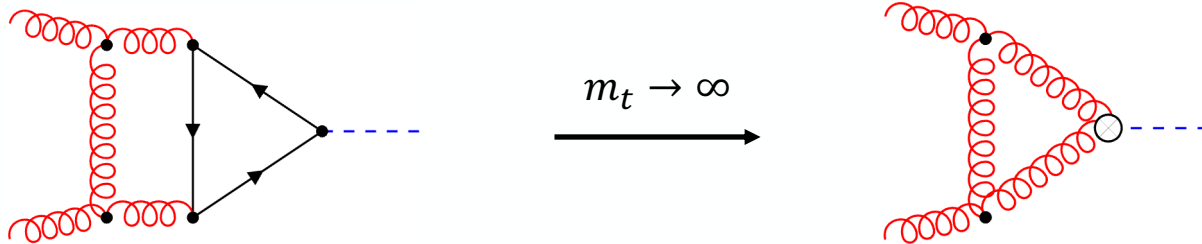
- Loop induced process, LO known for almost 50 years



[Georgi, Glashow, Machacek, Nanopoulos `78](#)

Heavy-top limit (HTL): one fewer loop, one fewer scale  
 → Effective field theory (EFT)

- NLO:



[Graudenz, Spira, Zerwas `93](#)

[Dawson `91](#)  
[Djouadi, Spira, Zerwas `91](#)

Better agreement with **rescaling**:  $\sigma_{\text{HEFT}} = r \sigma_{\text{HTL}}$

$$r = \left| \frac{\text{NLO diagram}}{\text{HTL diagram}} \right|^2 \approx 1.065$$

$m_H = 125 \text{ GeV}$   
 $m_t = 173.055 \text{ GeV}$

→ rEFT, HEFT (Higgs effective field theory)

# Gluon fusion (YR4 `16)

- Yellow report from 2016 ([LHCH\(XS\)WG YR4 `16](#)), following [Anastasiou, Duhr, Dulat, et al. `16](#)

$$\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb} (+4.56\%)}_{-3.27 \text{ pb} (-6.72\%)} \text{ (theory)} \pm 1.56 \text{ pb} (3.20\%) \text{ (PDF} + \alpha_s \text{)} .$$

$\sqrt{S}$	13 TeV
$m_h$	125 GeV
PDF	PDF4LHC15_nnlo_100
$\alpha_s(m_Z)$	0.118
$m_t(m_t)$	162.7 GeV ( $\overline{MS}$ )
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$\mu = \mu_R = \mu_F$	62.5 GeV (= $m_H/2$ )

48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)	<a href="#">Georgi, Glashow, Machacek, Nanopoulos `78</a>
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)	<a href="#">Dawson `91; Djouadi, Spira, Zerwas `91</a>
	- 2.05 pb	(-4.2%)	((t, b, c), exact NLO)	<a href="#">Graudenz, Spira, Zerwas `93</a>
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)	<a href="#">Ravindran, Smith, Van Neerven, `02; Harlander, Kilgore `02; Anastasiou, Melnikov `02</a>
	+ 0.34 pb	(+0.7%)	(NNLO, 1/ $m_t$ )	<a href="#">Harlander, Ozeren, `09; Pak, Rogal, Steinhauser `10; Harlander, Mantler, Marzani, Ozeren, `10</a>
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)	<a href="#">Aglietti, Bonciani, Degrandi, Vicini, `04; Actis, Passarino, Sturm, Uccirati, `08; Anastasiou, Boughezal, Petriello, `09</a>
	+ 1.49 pb	(+3.1%)	(N <sup>3</sup> LO, rEFT)	<a href="#">Anastasiou, Duhr, Dulat, Herzog, Mistlberger `15</a>

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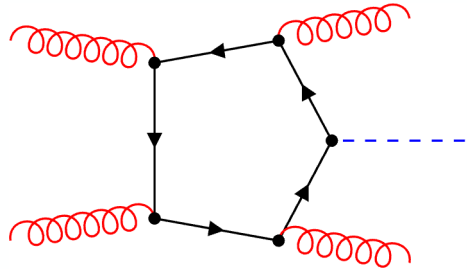
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**Our Goal**

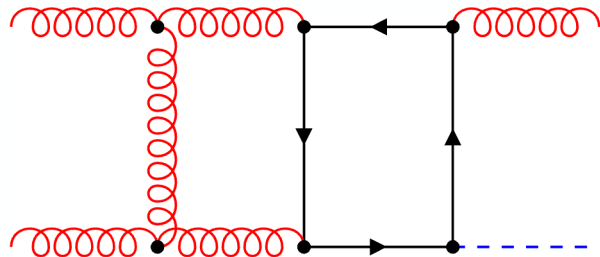
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# Computation

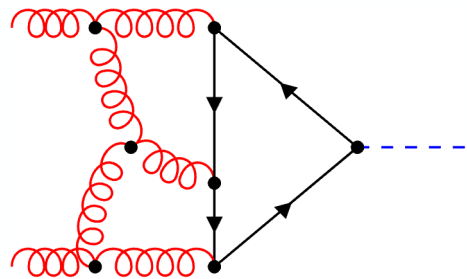
# Ingredients for NNLO



double-real



real-virtual



double-virtual

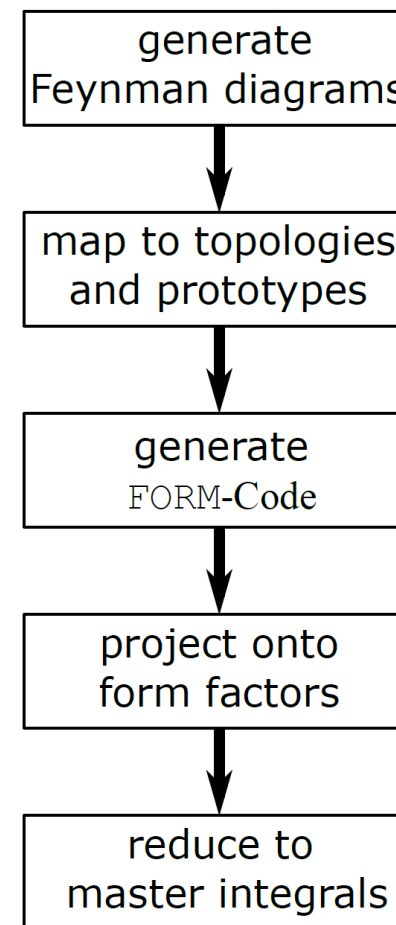
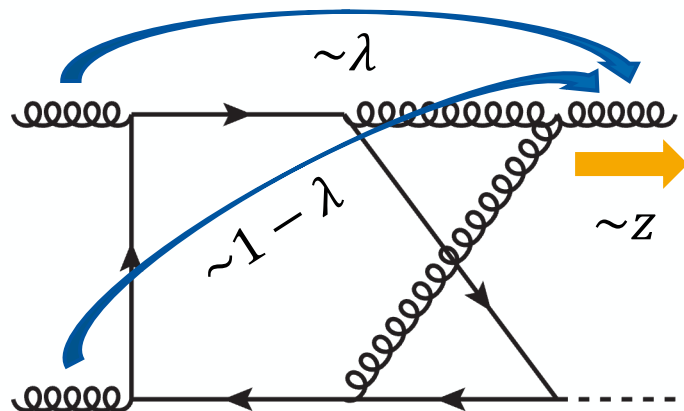
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QCDDLoop ([Carrazza, Ellis, Zanderighi `16](#))

# Real-virtual corrections

- Variables:  $\hat{s}, \hat{t}, \hat{u}, m_H^2, m_q^2$
- Introduce dimensionless variables and fix ratio  $m_q^2 / m_H^2$ 
  - $z$  parametrizes **soft** limit
  - $\lambda$  parametrizes **collinear** limit

$$\begin{array}{l} \hat{t}/\hat{s} = z \lambda \\ \hat{u}/\hat{s} = z (1-\lambda) \end{array} \quad \begin{array}{c} \longrightarrow \\ \longleftarrow \end{array} \quad \begin{array}{l} z = 1 - m_H^2/\hat{s} \\ \lambda = \hat{t}/(\hat{t} + \hat{u}) \end{array}$$



**DiaGen**  
Czakon (private)

**FORM**  
[Vermaseren '00](#)

**Kira⊕FireFly**  
[Klappert, Lange, et al. '20](#)

→ Solve master integrals with differential equations in  $m_q^2/m_H^2, z,$  and  $\lambda$

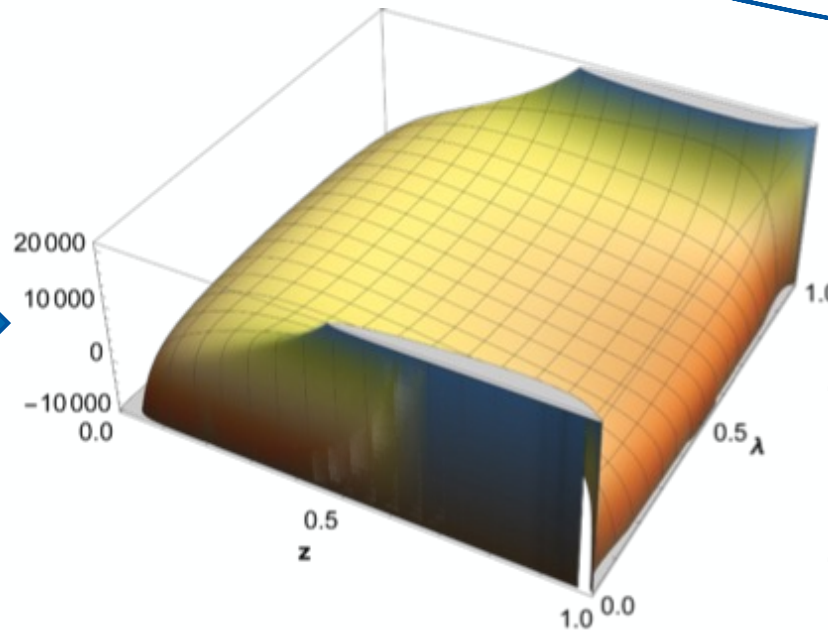
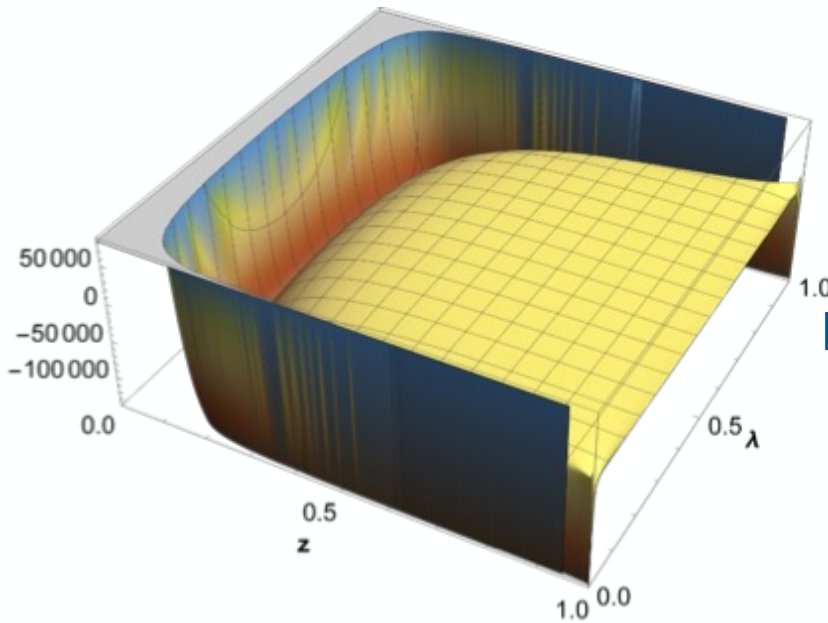
→ Boundary conditions:  $\frac{m_q^2}{m_H^2} \rightarrow \infty$  with large-mass expansion

# Real-virtual corrections

- Create grid with numerical values of squared amplitude
- Subtract IR singularities:

$$\langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle_{\text{regulated}} \equiv \langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle - \left[ \langle M_{\text{HEFT}}^{(1)} | M_{\text{HEFT}}^{(2)} \rangle + \frac{8\pi\alpha_s}{\hat{t}} \left\langle P_{gg}^{(0)} \left( \frac{\hat{s}}{\hat{s} + \hat{u}} \right) \right\rangle \langle F^{(1)} | (F_{\text{exact}}^{(2)} - F_{\text{HEFT}}^{(2)}) \rangle \right]$$

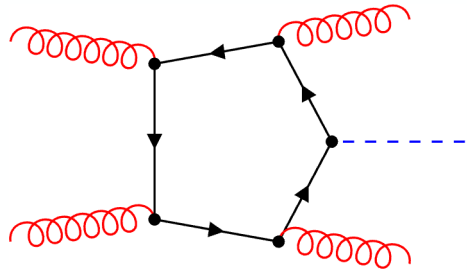
$$r = \frac{\sigma_{t \times b}^{\text{LO}}}{\sigma_{\text{HTL}}^{\text{LO}}} \approx -0.129$$



- Interpolate to any phase-space point with cubic splines
- Add back subtracted terms using analytical expression

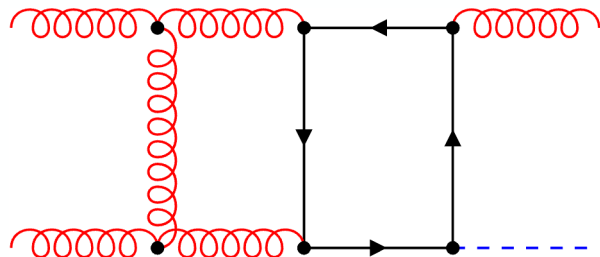


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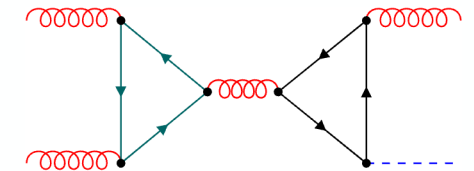
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real-virtual

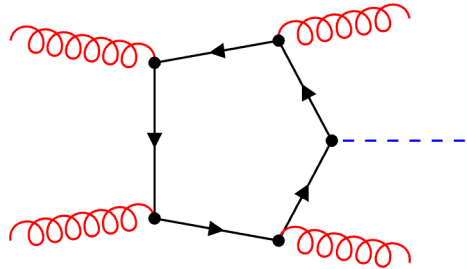
Single massive flavor: Interpolation of [numerical grid](#) of regulated  
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Two massive loops with different masses:  
 → Always factorized into one-loop  
 contributions



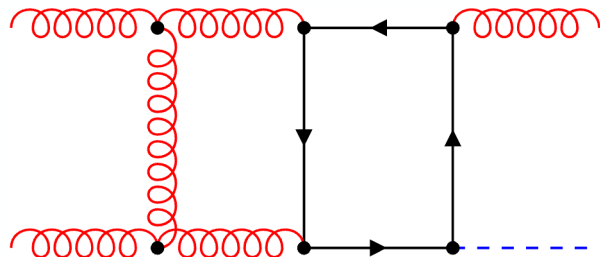
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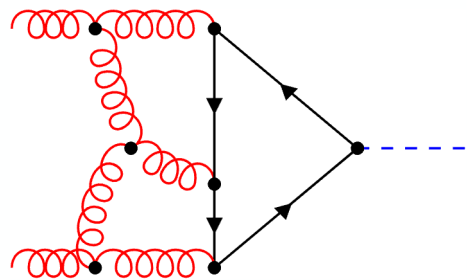
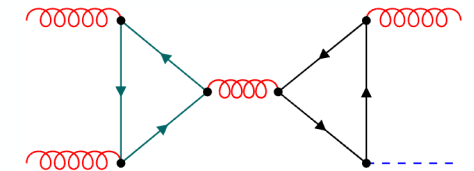
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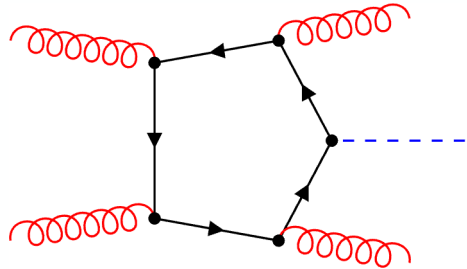


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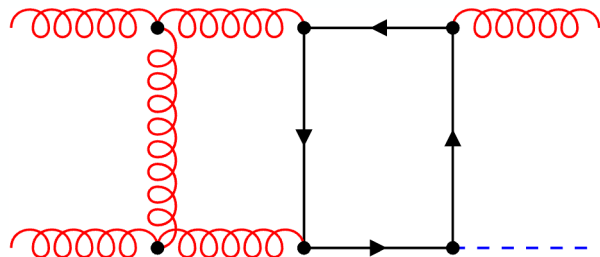
Deep asymptotic expansion in  $m_H^2/m_t^2$ ,  $m_b^2/m_H^2$ , calculated with  
 methods similar to real-virtual:

- Single massive quark flavor: [Czakon, Niggetiedt `20](#)
- Two massive quark flavors: [Niggetiedt, Usovitsch `23](#)

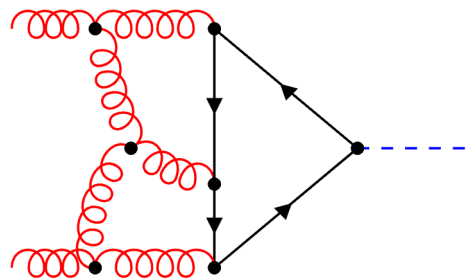
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real-virtual



double-virtual

Calculated in [Del Duca, Kilgore, Oleari, et al. '01](#)  
 We use calculation from [Budge, Campbell, De Laurentis, et al. '20](#)  
 as implemented in MCFM ([Campbell, Ellis '99](#)), scalar integrals with  
[QCDLoop \(Carrazza, Ellis, Zanderighi '16\)](#)

Single massive quark flavor: [Czakon '10](#)  
 amplitude: Analytical expression for IR counter-term  
 Two massive quark flavors: [Czakon '10](#) as implemented in C++  
 code **Stripper**  
 → Always find the appropriate one-loop  
 contributions



Deep asymptotic expansion in  $m_H^2/m_t^2$ ,  $m_b^2/m_H^2$ , calculated with  
 methods similar to real-virtual:

- Single massive quark flavor: [Czakon, Niggetiedt '20](#)
- Two massive quark flavors: [Niggetiedt, Usovitsch '23](#)

# Results: Part I

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon-fusion at the LHC
  - PDF set: **NNPDF31\_nnlo\_as\_0118** [NNPDF Collaboration `17](#)
  - $\mu_R = \mu_F = m_H/2$  (central scale), uncertainties with seven-point variation
  - $m_H = 125$  GeV  $\Rightarrow m_t \approx 173.055$  GeV and  $m_b \approx 4.779$  GeV (both in OS-scheme)
  - HEFT values obtained with SusHi [Harlander, Liebler, Mantler `16](#)

Order	$\sigma_{\text{HEFT}}$ [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b} / \sigma_{\text{HEFT}}$ [%]
$\sqrt{s} = 13$ TeV				
$\mathcal{O}(\alpha_s^2)$	+16.30	–	–1.975	
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	–12
$\mathcal{O}(\alpha_s^3)$	+21.14	–0.303	–0.446(1)	
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-6.5^{+0.9}_{-0.8}$
$\mathcal{O}(\alpha_s^4)$	+9.72	+0.147(1)	+0.434(8)	
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-4.2^{+0.9}_{-0.8}$

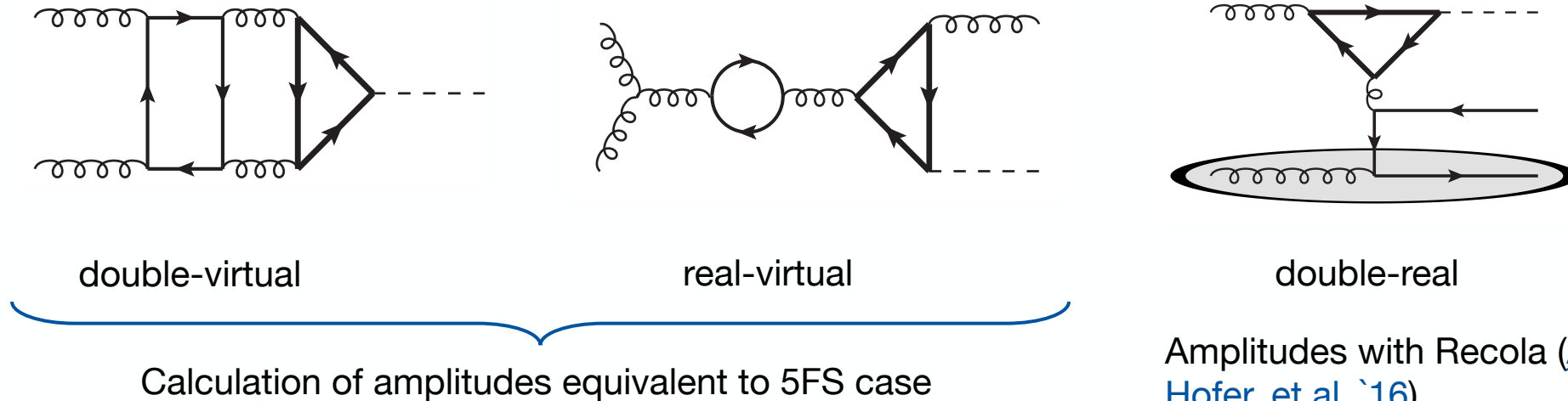
[Czakon, FE, Niggetiedt, Poncelet, Schellenberger `23](#)

- Interference effects much larger than pure top mass effect
- Interference effect at NNLO cancels against NLO
- Interference effect at NNLO larger than NLO scale variation (similar in HEFT but less severe)
- Interference NNLO scale variation increases compared to NLO
- Similar effects for different top quark mass ( $m_t \approx 170.979$  GeV)

# Flavor scheme

# 5-flavor scheme vs. 4-flavor scheme

- So far: b-quarks not coupled to the Higgs assumed as massless (5FS)
- While gauge-invariant, testing the effect of the neglected mass desirable  $\rightarrow$  4FS



Amplitudes with RecoLa ([Actis, Denner, Hofer, et al. '16](#))

Phase-space integration as separate calculation of LO Hbb production

- Exclude b-quark from initial state
- Infrared singularities regulated by finite  $m_b$ , only sum of contributions free of  $\log(m_b)$  divergences

# HEFT in the 4-flavor scheme

- Effects of 4FS concern b-quark *not* coupled to Higgs → also studiable within HEFT
- More stable calculation → can numerically test pole cancellation for small  $m_b$

Order	$\sigma_{\text{HEFT}}$ [pb]			
	$\sqrt{s} = 13 \text{ TeV}$			
	5FS	4FS ( $m_b = 0.01 \text{ GeV}$ )	4FS ( $m_b = 0.1 \text{ GeV}$ )	4FS ( $m_b = 4.78 \text{ GeV}$ )
$\mathcal{O}(\alpha_s^2)$	+16.30	+16.27	+16.27	+16.27
LO	$16.30^{+4.36}_{-3.10}$	$+16.27^{+4.63}_{-3.22}$	$+16.27^{+4.63}_{-3.22}$	$+16.27^{+4.63}_{-3.22}$
$\mathcal{O}(\alpha_s^3)$	+21.14	+20.08(3)	+20.08(3)	+20.08(3)
NLO	$37.44^{+8.42}_{-6.29}$	$+36.35(3)^{+8.57}_{-6.32}$	$+36.35(3)^{+8.57}_{-6.32}$	$+36.35(3)^{+8.57}_{-6.32}$
$\mathcal{O}(\alpha_s^4)$	+9.72	+10.8(4)	+11.2(4)	+9.5(2)
NNLO	$47.16^{+4.21}_{-4.77}$	$47.2(4)^{+5.4}_{-5.4}$	$+47.5(4)^{+5.4}_{-5.5}$	$+45.9(2)^{+4.3}_{-4.9}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger, **preliminary**

4FS PDF set: NNPDF31\_nnlo\_as\_0118\_nf\_4

- Convergence for small  $m_b$ , seems like numbers approach 5FS value
- Effect of finite  $m_b$  is  $\sim 3\%$ , order of magnitude as estimated in [Pietrullewicz, Stahlhofen `23](#)

# Results: Part II

- Top-bottom interference in the 5FS vs. 4FS

Order	$\sigma_{t \times b}$ [pb]	
	$\sqrt{s} = 13$ TeV	
	5FS	4FS
$\mathcal{O}(\alpha_s^2)$	-1.975	-1.971
LO	$-1.98^{+0.37}_{-0.53}$	$-1.97^{+0.39}_{-0.56}$
$\mathcal{O}(\alpha_s^3)$	-0.447(4)	-0.455(4)
NLO	$-2.42^{+0.19}_{-0.12}$	$-2.43^{+0.21}_{-0.13}$
$\mathcal{O}(\alpha_s^4)$	+0.434(8)	+0.389(11)
NNLO	$-1.99(1)^{+0.30}_{-0.15}$	$-2.04(1)^{+0.29}_{-0.14}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger, **preliminary**

4FS PDF set: NNPDF31\_nnlo\_as\_0118\_nf\_4

- Difference between schemes well below scale uncertainties
- Effect at the percent-level of the already small contribution → Use of (simpler) 5FS justified



# Mass renormalization

# On-shell to $\overline{\text{MS}}$ mass renormalization

$$Z_m^{\overline{\text{MS}}} \overline{m} = Z_m^{\text{OS}} m^{\text{OS}} \quad \longrightarrow \quad m^{\text{OS}} = \overline{m} \left( 1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \mathcal{O}(\alpha^3) \right)$$

[Gray, Broadhurst, Grafe, Schilcher '90](#)

$$\mathcal{M}^{\overline{\text{MS}}} = \mathcal{M}^{\text{OS}} + \delta\mathcal{M}$$

$$\delta\mathcal{M}^{(1)} = \overline{m} c_1 \frac{\alpha_s}{\pi} \left. \frac{d\mathcal{M}^{\text{OS},(0)}}{dm} \right|_{m=\overline{m}}$$

$$\delta\mathcal{M}^{(2)} = \overline{m} \left[ c_1 \frac{\alpha_s}{\pi} \left. \frac{d\mathcal{M}^{\text{OS},(1)}}{dm} \right|_{m=\overline{m}} + c_2 \left( \frac{\alpha_s}{\pi} \right)^2 \left. \frac{d\mathcal{M}^{\text{OS},(0)}}{dm} \right|_{m=\overline{m}} \right] + \frac{1}{2} \left( \overline{m} c_1 \frac{\alpha_s}{\pi} \right)^2 \left. \frac{d^2\mathcal{M}^{\text{OS},(0)}}{dm^2} \right|_{m=\overline{m}}$$

Two possibilities:

1. Only Yukawa coupling in  $\overline{\text{MS}}$   $\rightarrow$  Derivatives trivial since amplitude linear
2. Bottom mass always in  $\overline{\text{MS}}$   $\rightarrow$  Requires some work (derivatives, different mass value in integrals)

# Results: Part III

$$\bar{m}_b(\bar{m}_b) = 4.18 \text{ GeV}$$

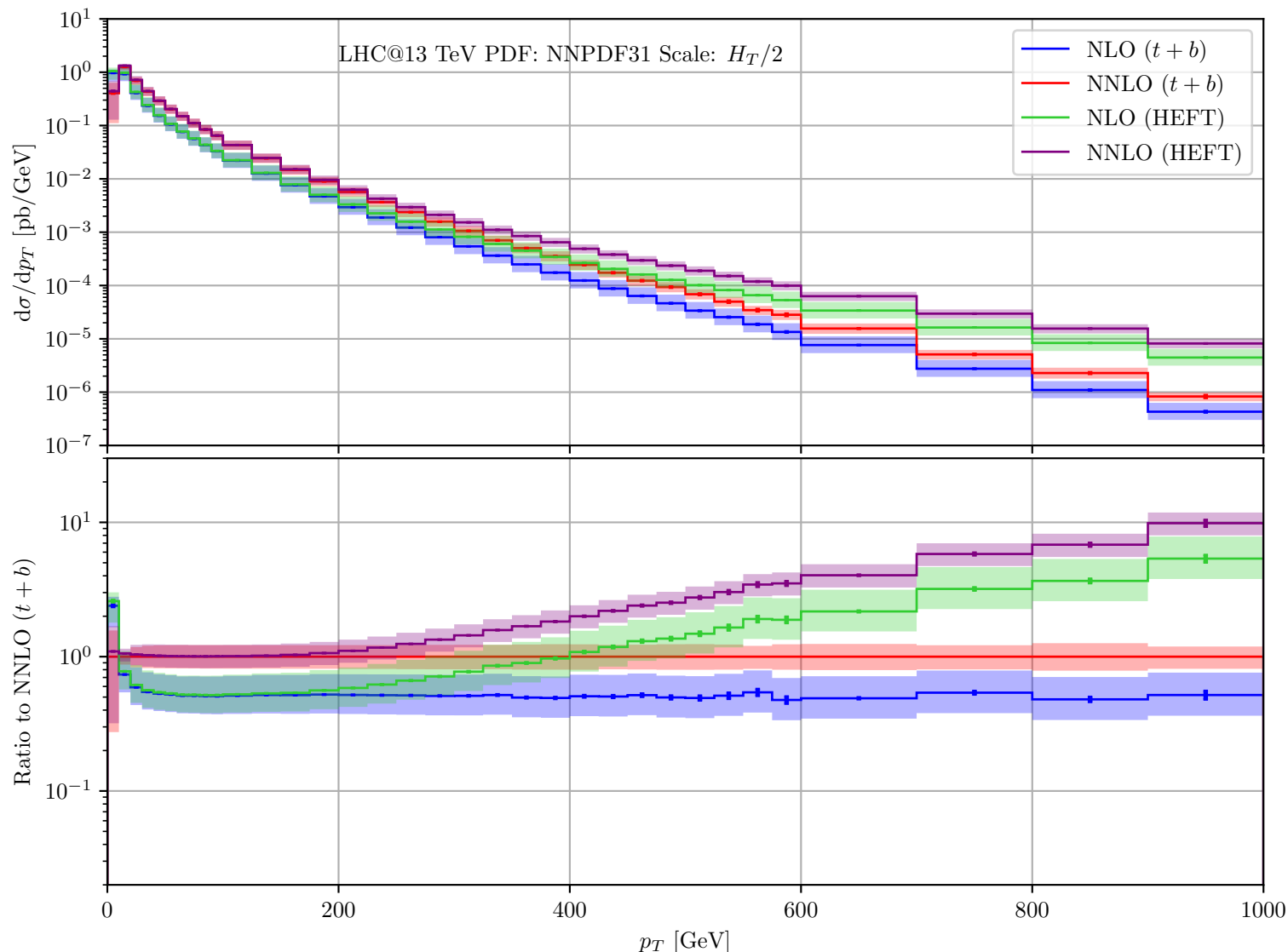
Order	$\sigma_{\text{HEFT}}$ [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]	$\sigma_{t \times b}$ [pb]	$\sigma_{t \times b}(Y_{b, \overline{\text{MS}}})$ [pb]	$\sigma_{t \times b}(\bar{m}_b)$ [pb]
$\sqrt{s} = 13 \text{ TeV}$					
$\mathcal{O}(\alpha_s^2)$	+16.30	–	–1.975	–1.223	–1.118
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	$-1.22^{+0.29}_{-0.44}$	$-1.118^{+0.28}_{-0.43}$
$\mathcal{O}(\alpha_s^3)$	+21.14	–0.303	–0.446(1)	–0.623(1)	–0.647
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-1.85^{+0.26}_{-0.26}$	$-1.76^{+0.27}_{-0.28}$
$\mathcal{O}(\alpha_s^4)$	+9.72	+0.147(1)	+0.434(8)	+0.019(5)	+0.02(1)
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-1.83(1)^{+0.08}_{-0.03}$	$-1.74(2)^{+0.13}_{-0.01}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger, **preliminary**

- Much better convergence behavior in  $\overline{\text{MS}}$
- Using  $\overline{\text{MS}}$  only for coupling vs. everywhere leads to similar results
- At NNLO, values for the first time compatible between  $\overline{\text{MS}}$  and OS

# Results Part IV: Distributions

# Transverse-momentum spectrum



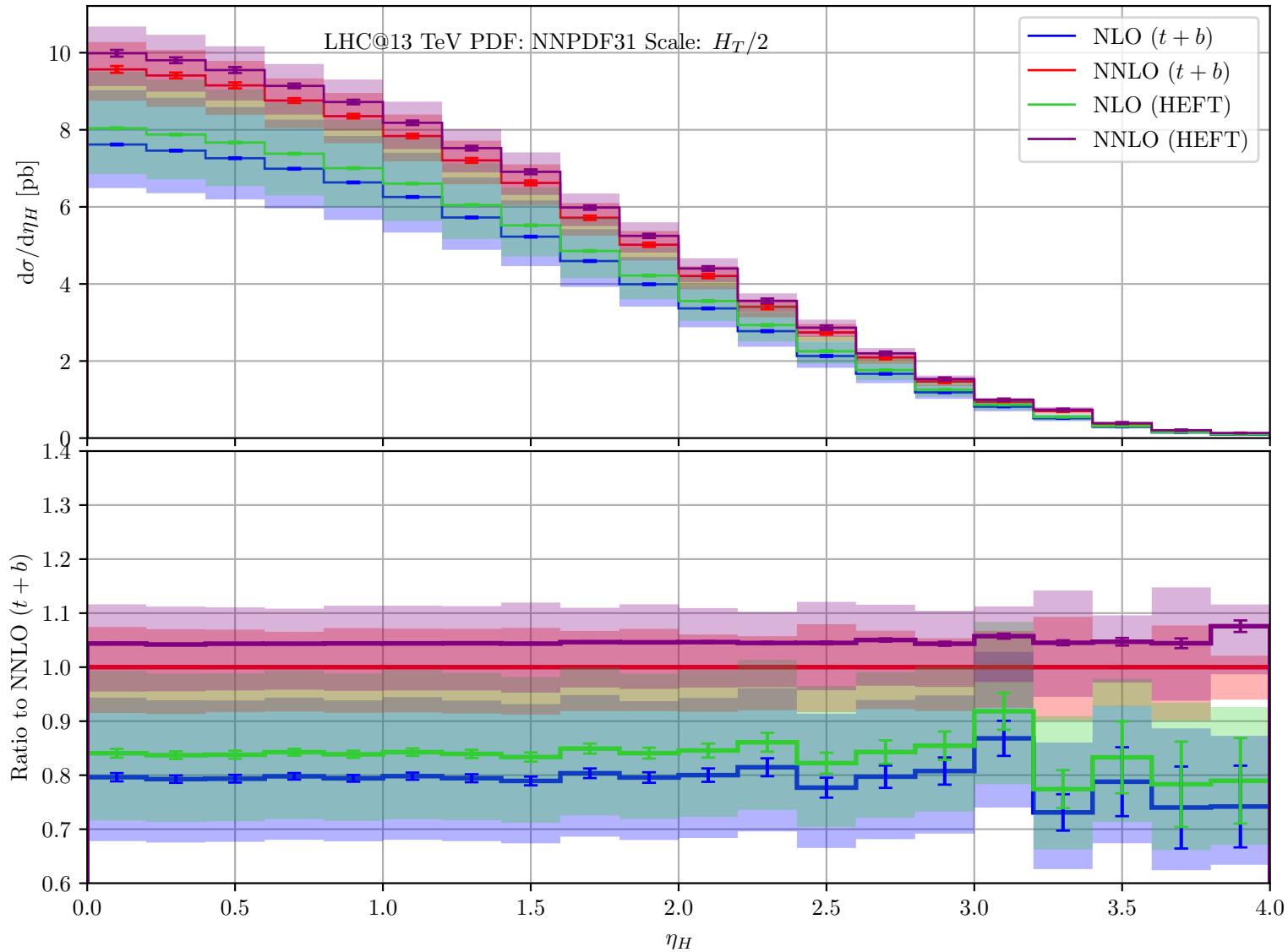
$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$  (central scale), seven-point variation

$m_H = 125$  GeV,  $m_t^{OS} \approx 173.055$  GeV, and  $m_b(m_b) = 4.18$  GeV

- Except for first bin (only there three-loop is needed), known from previous calculations of quark-mass effects on Higgs+jet at NLO [Lindert, Melnikov, Tancredi, Wever `17](#) [Caola, Lindert, Melnikov, et al. `18](#) [Bonciani, Del Duca, Frellesvig, et al. `22](#)
- Find agreement with known results
- Full theory: At very high  $p_T$ , only scale is  $p_T$   
 $\rightarrow d\sigma/dp_T^2 \sim 1/p_T^4$
- Effective theory: dimensionful coupling  
 $\rightarrow d\sigma/dp_T^2 \sim 1/(v^2 p_T^2)$

# Rapidity spectrum

Czakon, FE, Niggetiedt, Poncelet,  
Schellenberger, preliminary

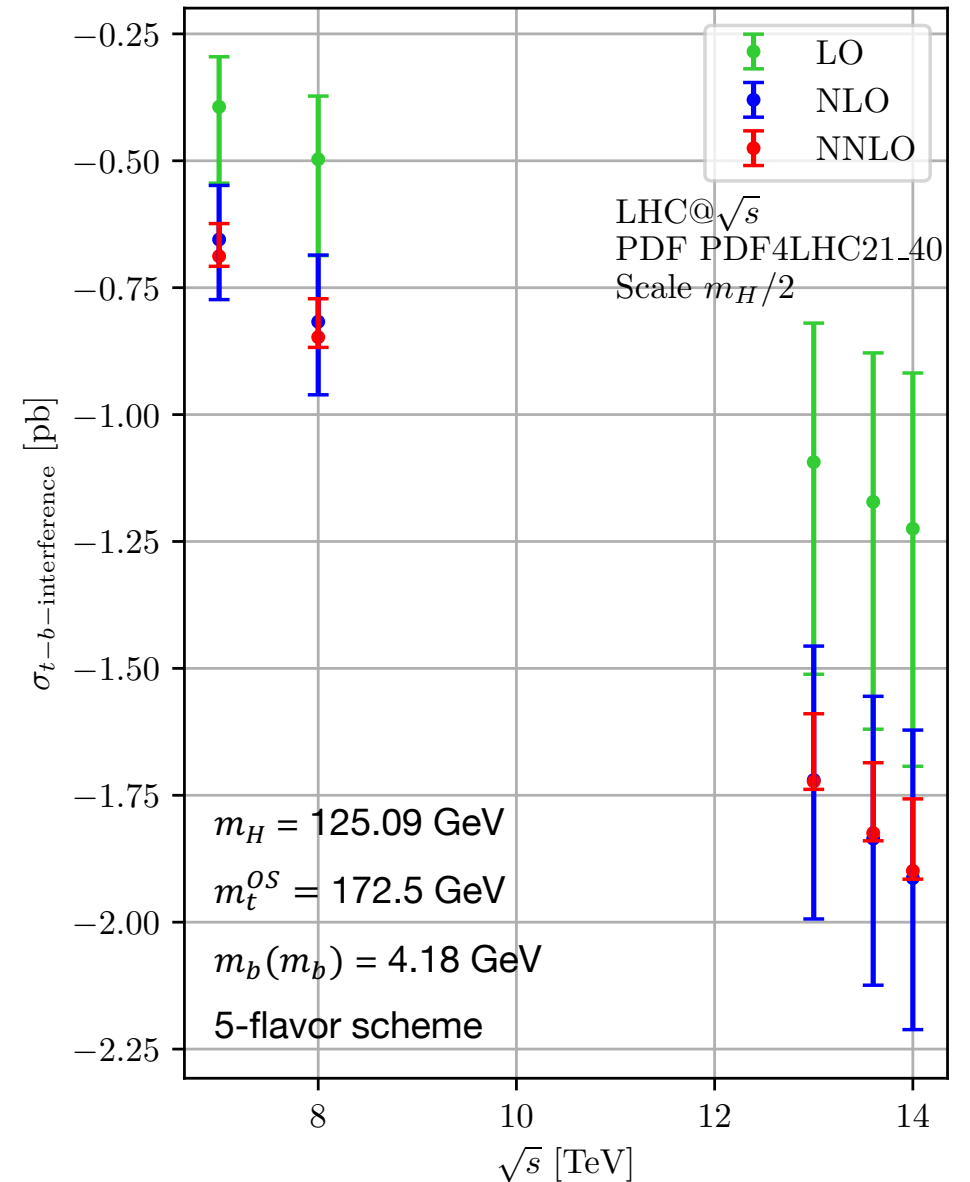


$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$  (central scale),  
seven-point variation  
 $m_H = 125$  GeV,  $m_t^{OS} \approx 173.055$  GeV, and  $m_b(m_b) = 4.18$  GeV

- New result, all bins contain genuine three-loop corrections
- Mass-effects barely affect shape of distribution, but shift it downwards

# Conclusions

- Complete analysis of top-bottom-interference effects on the Higgs production cross section at NNLO
- Addresses one of the leading theory uncertainties
- $\overline{MS}$  scheme shows better perturbative convergence than OS scheme
- Good agreement between 4- and 5-flavor scheme
- Differential distributions, including novel rapidity spectra

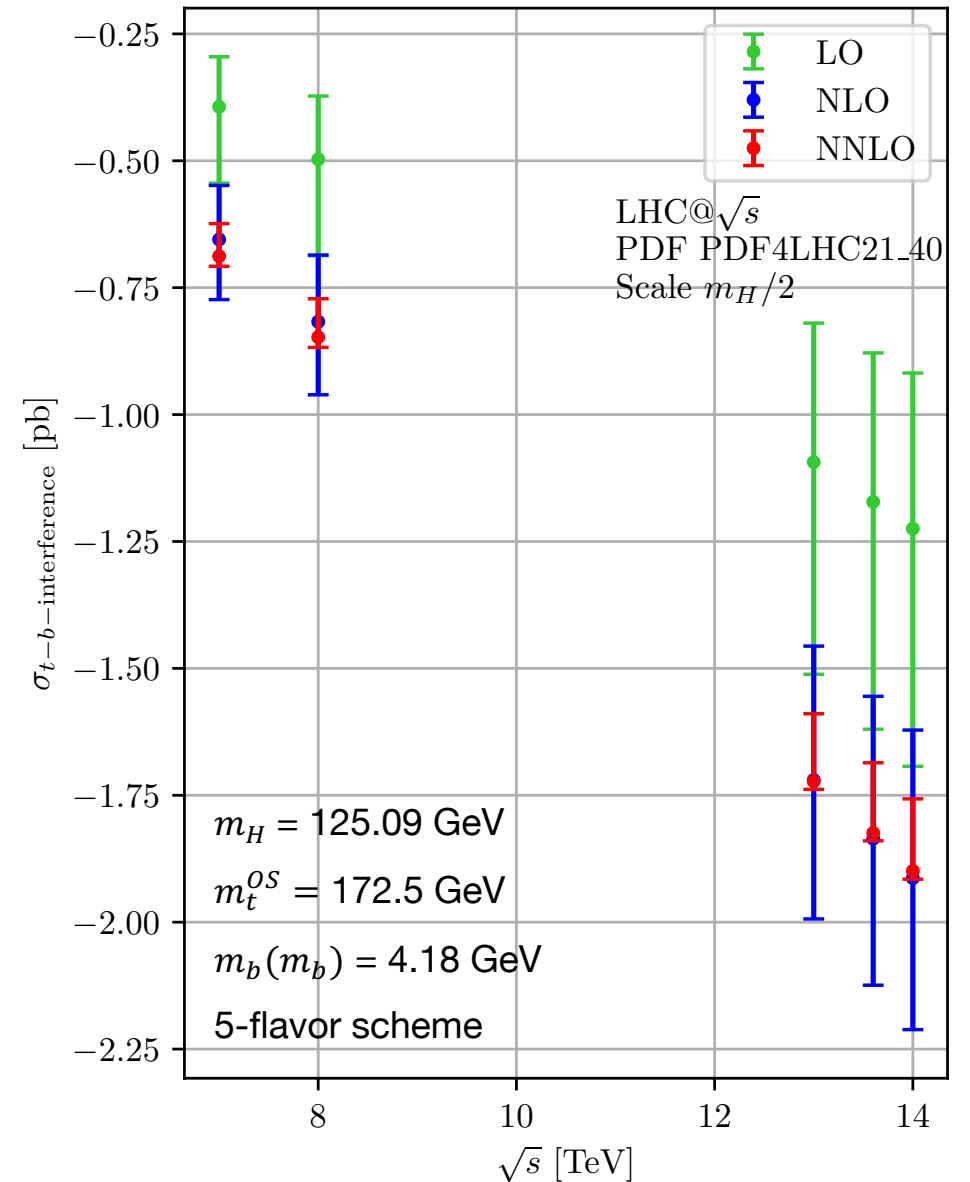


Czakon, FE, Niggetiedt, Poncelet, Schellenberger, preliminary

# Conclusions

- Complete analysis of top-bottom-interference effects on the Higgs production cross section at NNLO
- Addresses one of the leading theory uncertainties
- $\overline{MS}$  scheme shows better perturbative convergence than OS scheme
- Good agreement between 4- and 5-flavor scheme
- Differential distributions, including novel rapidity spectra

## Thank you for your attention!



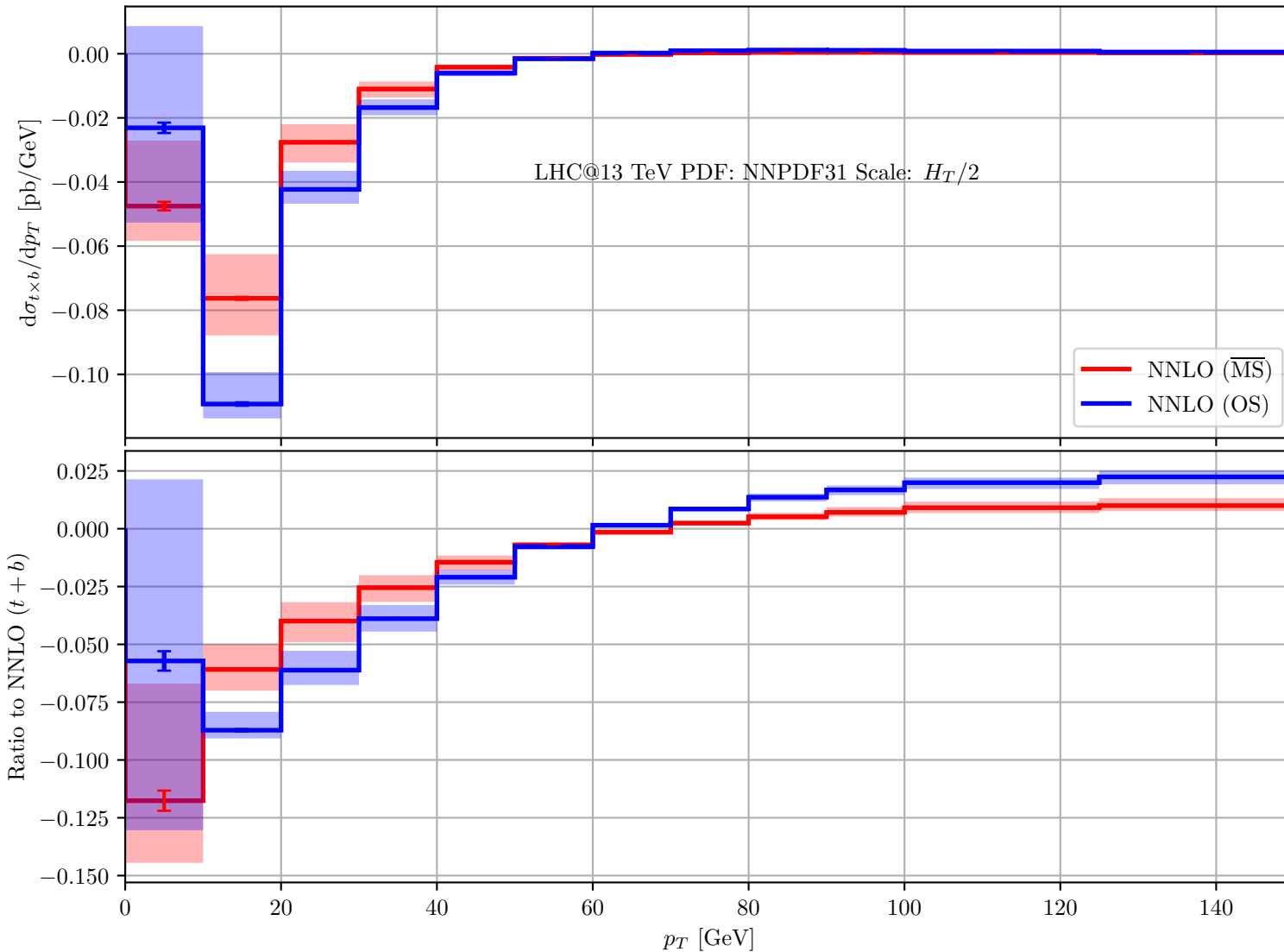
Czakon, FE, Niggetiedt, Poncelet, Schellenberger, preliminary



**Backup**

# Transverse-momentum spectrum

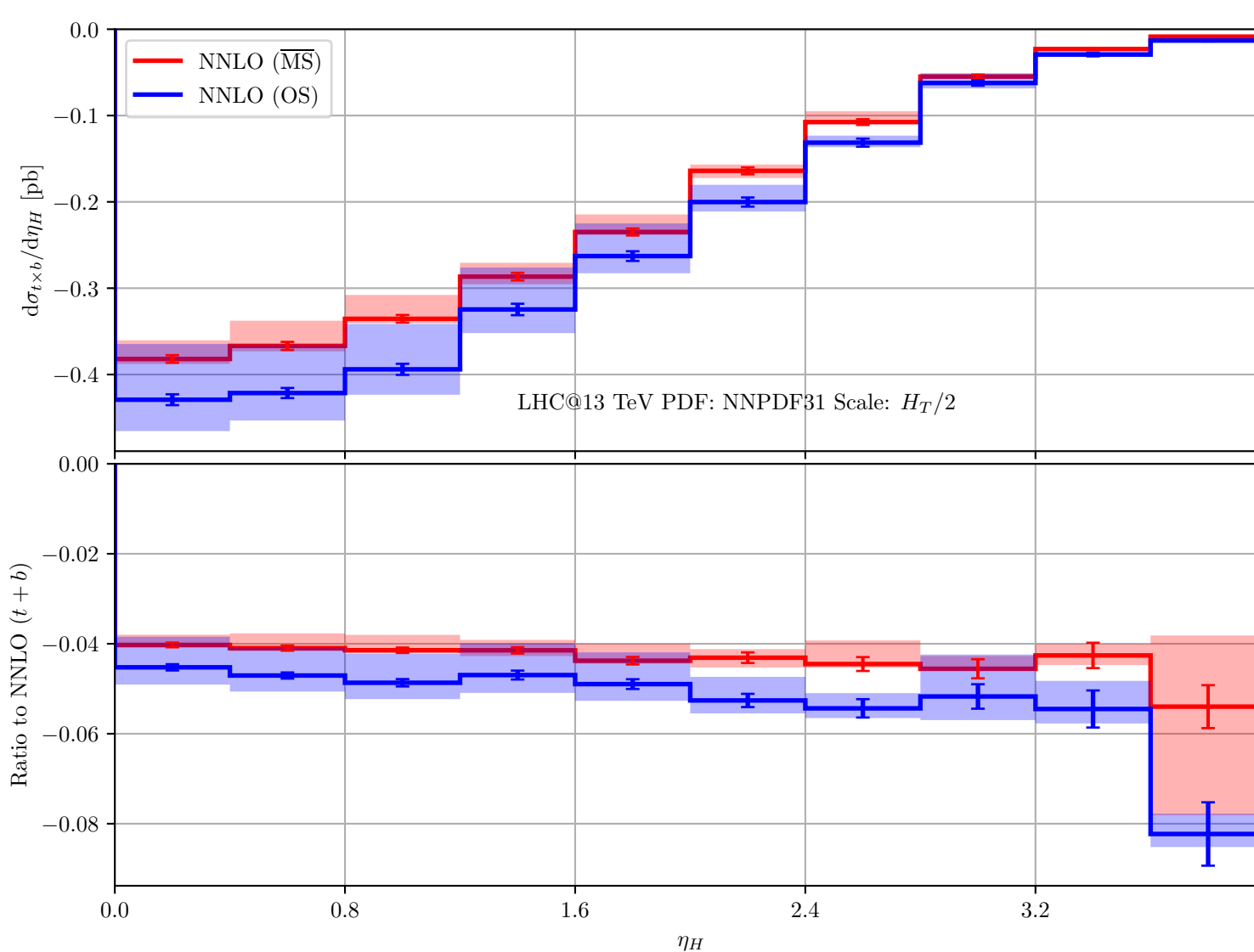
Czakon, FE, Niggetiedt, Poncelet,  
Schellenberger, preliminary



$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$  (central scale),  
seven-point variation  
 $m_H = 125$  GeV,  $m_t^{OS} \approx 173.055$  GeV and  $m_b(m_b) = 4.18$  GeV,  
 $m_b^{OS} = 4.78$  GeV

- Comparison of pure top-bottom interference effects between OS and  $\overline{MS}$

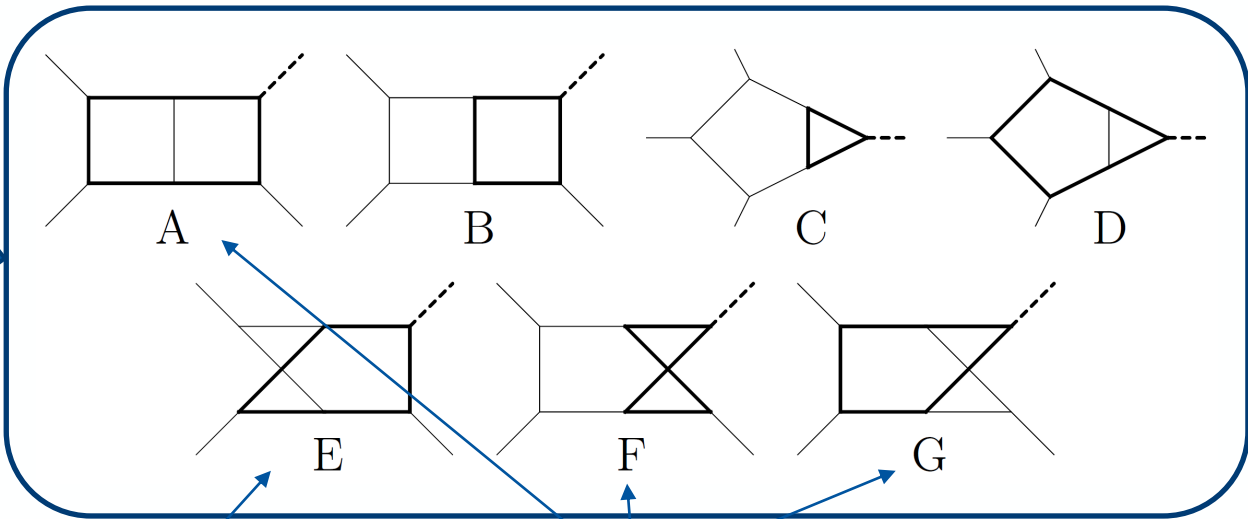
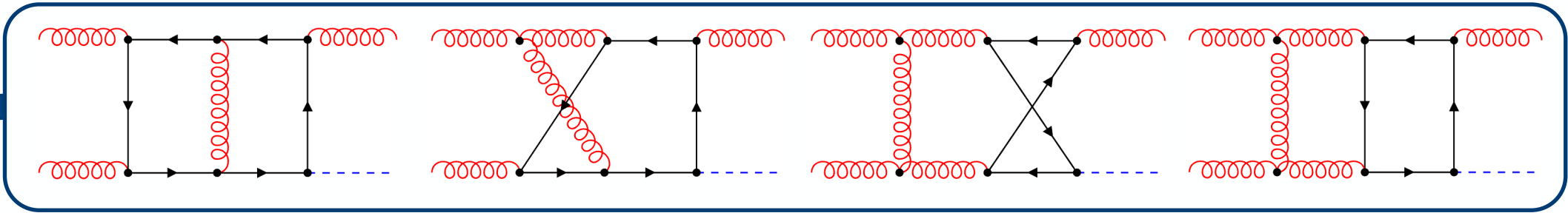
# Rapidity spectrum



$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$  (central scale), seven-point variation  
 $m_H = 125$  GeV,  $m_t^{OS} \approx 173.055$  GeV and  $m_b(m_b) = 4.18$  GeV,  
 $m_b^{OS} = 4.78$  GeV

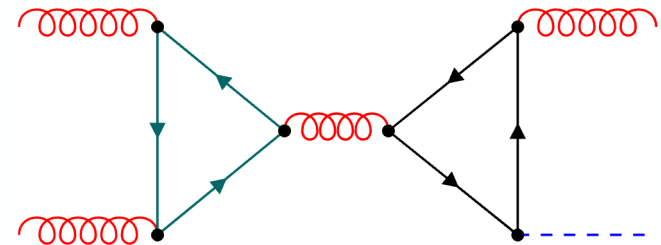
- Comparison of pure top-bottom interference effects between OS and  $\overline{MS}$

# Real-virtual corrections



A,B,C,D: [Bonciani, Del Duca, Frellesvig, et al. `16](#)  
 F: [Bonciani, Del Duca, Frellesvig, et al. `19](#)  
 G: [Frellesvig, Hidding, Maestri, et al. `19](#)

Contributions with two closed fermion chains are always factorizable:



vanishing color factor

Elliptic sector

# Parametrization

- Variables:  $\hat{s}$ ,  $\hat{t}$ ,  $\hat{u}$ ,  $m_H^2$ ,  $m_q^2$
- Introduce dimensionless variables and fix ratio  $m_q^2/m_H^2$ 
  - $z$  parametrizes **soft** limit
  - $\lambda$  parametrizes **collinear** limit

$$\hat{t}/\hat{s} = z \lambda$$

$$\hat{u}/\hat{s} = z (1-\lambda)$$

$$z = 1 - m_H^2/\hat{s}$$

$$\lambda = \hat{t}/(\hat{t} + \hat{u})$$

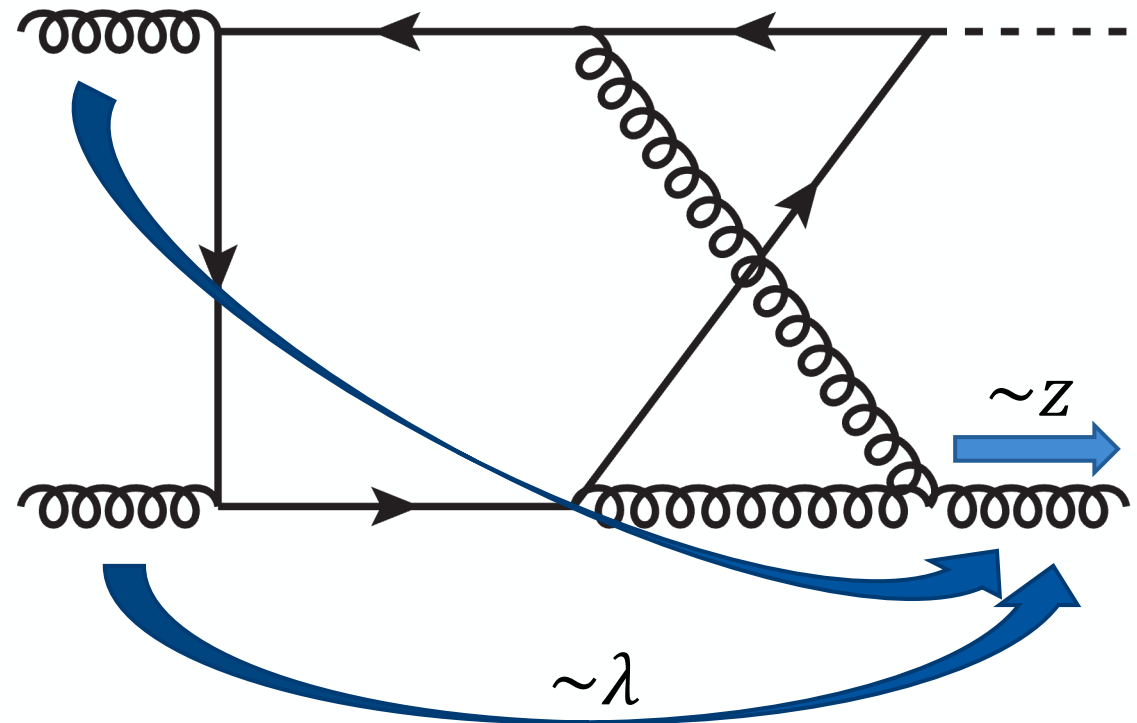
$$z = 1 - m_H^2/\hat{s}$$

$$\lambda = \hat{t}/(\hat{t} + \hat{u})$$

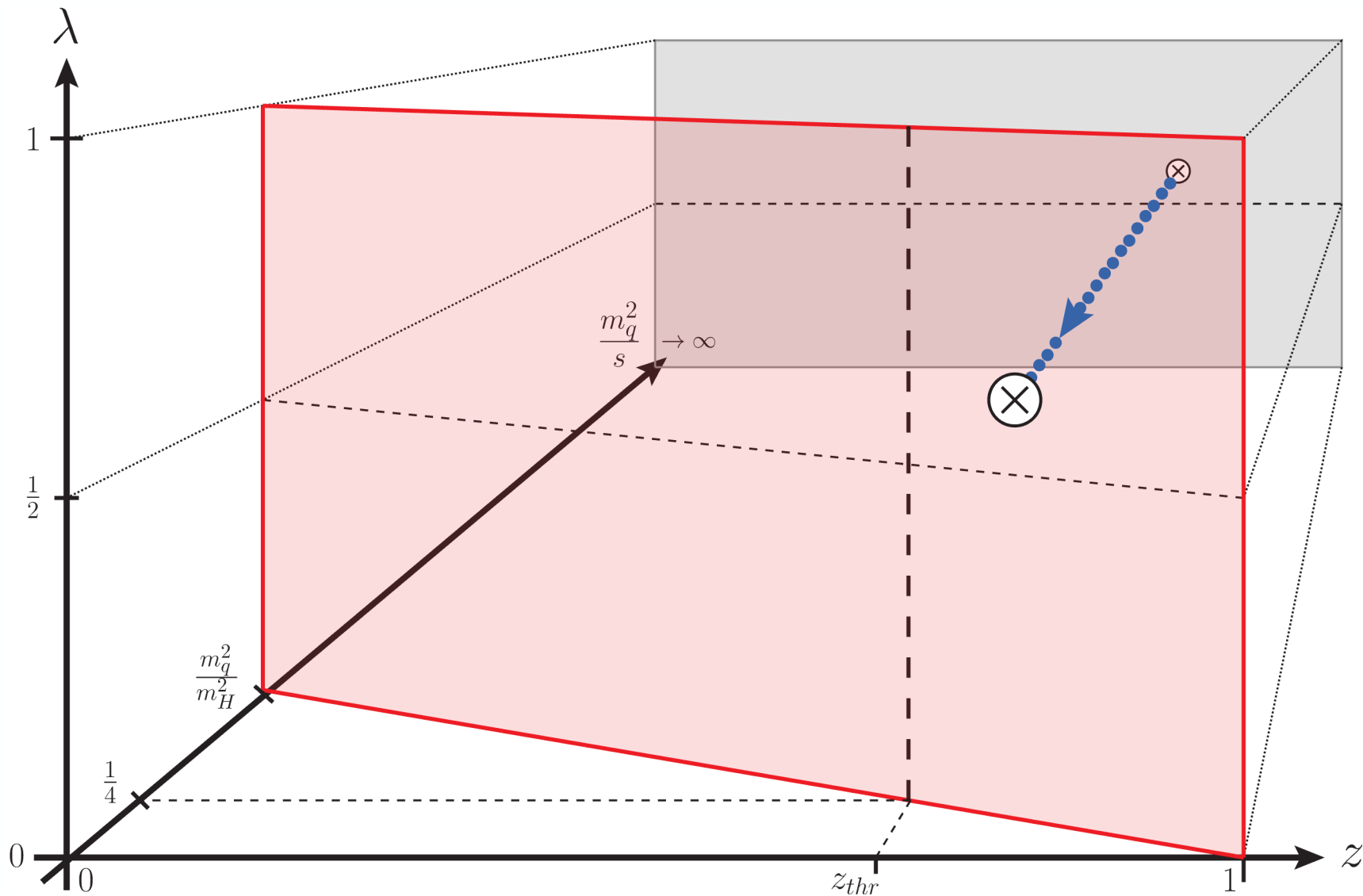
$$m_t^2/m_H^2 = 23/12$$

$$m_b^2/m_H^2 = 1/684$$

- Range of parameters:
- $\lambda \in (0,1)$
  - $z \in (0,1)$



# Evolution of differential equations



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2 / m_H^2 = 23/12$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

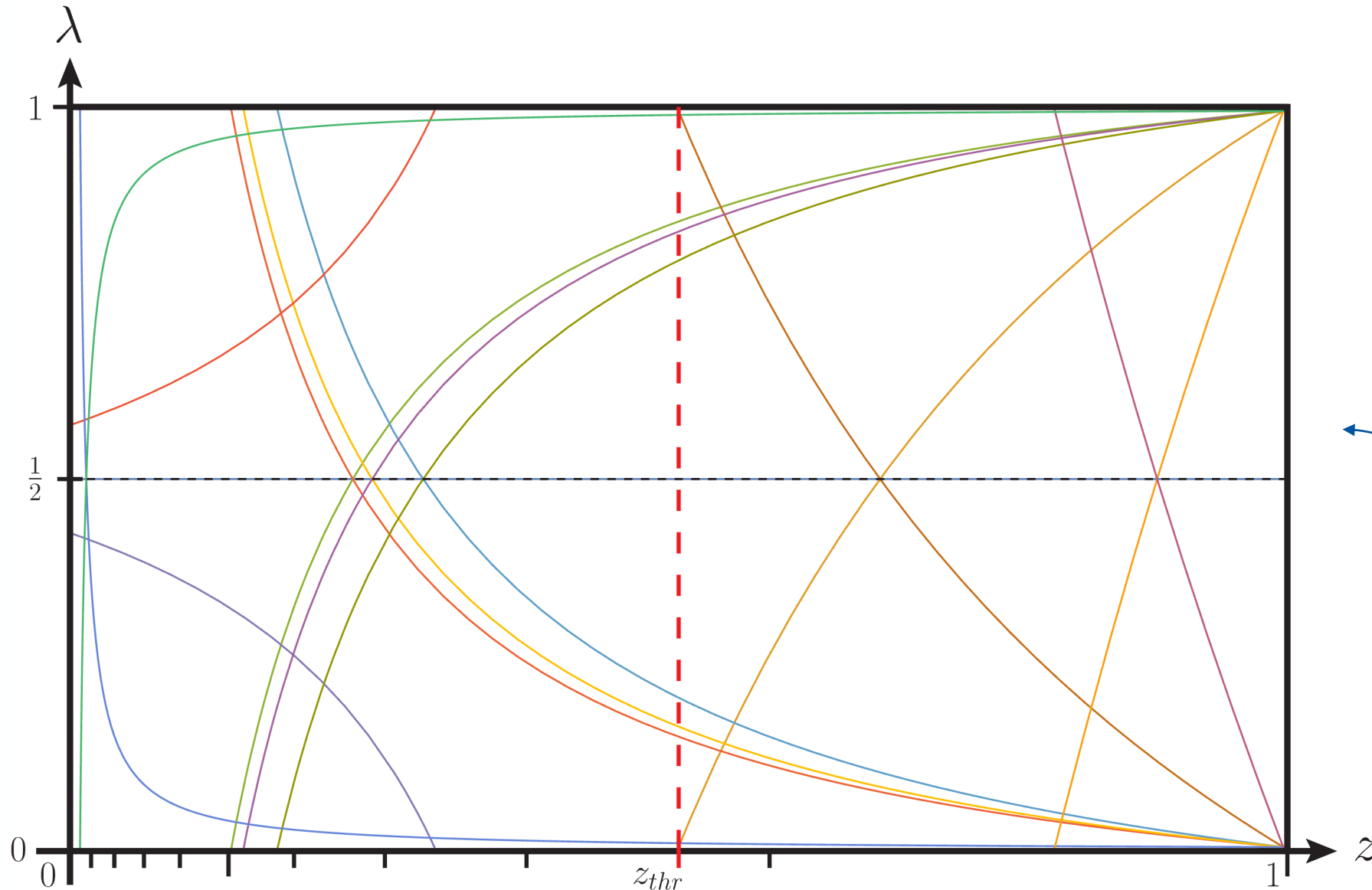
$$z_{thr} = 1 - \frac{m_H^2}{4m_q^2}$$

For  $m_t^2 / m_H^2 =$

23/12:

$$z_{thr} = 20/23 \approx 0.87$$

# Evolution in the $(z, \lambda)$ -plane



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

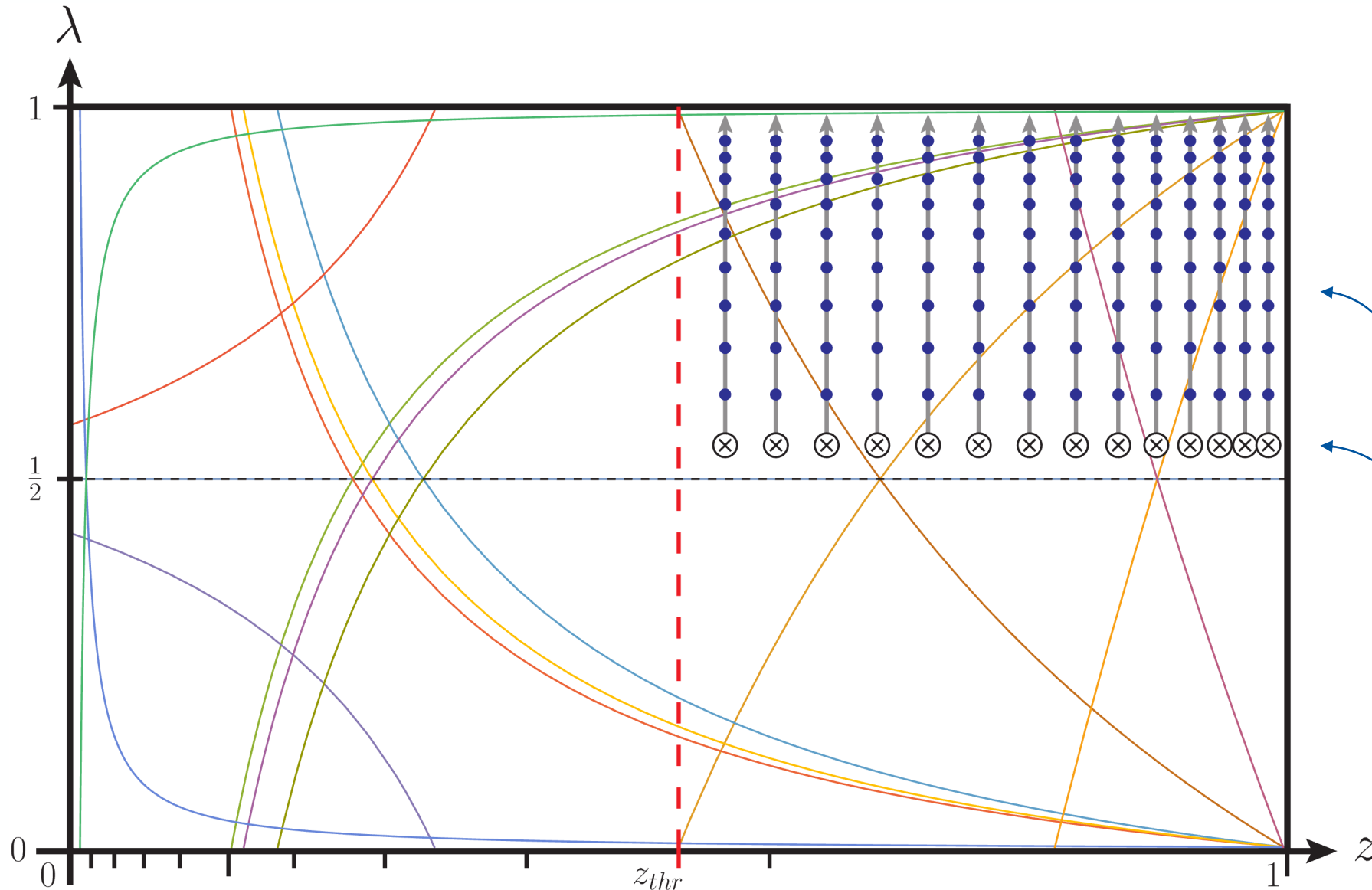
$$m_t^2 / m_H^2 = 23/12$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Poles of differential equations in  $\lambda$

# Evolution in the $(z, \lambda)$ -plane



$$z = 1 - m_H^2 / \hat{s}$$
$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$
$$m_t^2 / m_H^2 = 23/12$$

Range of parameters:

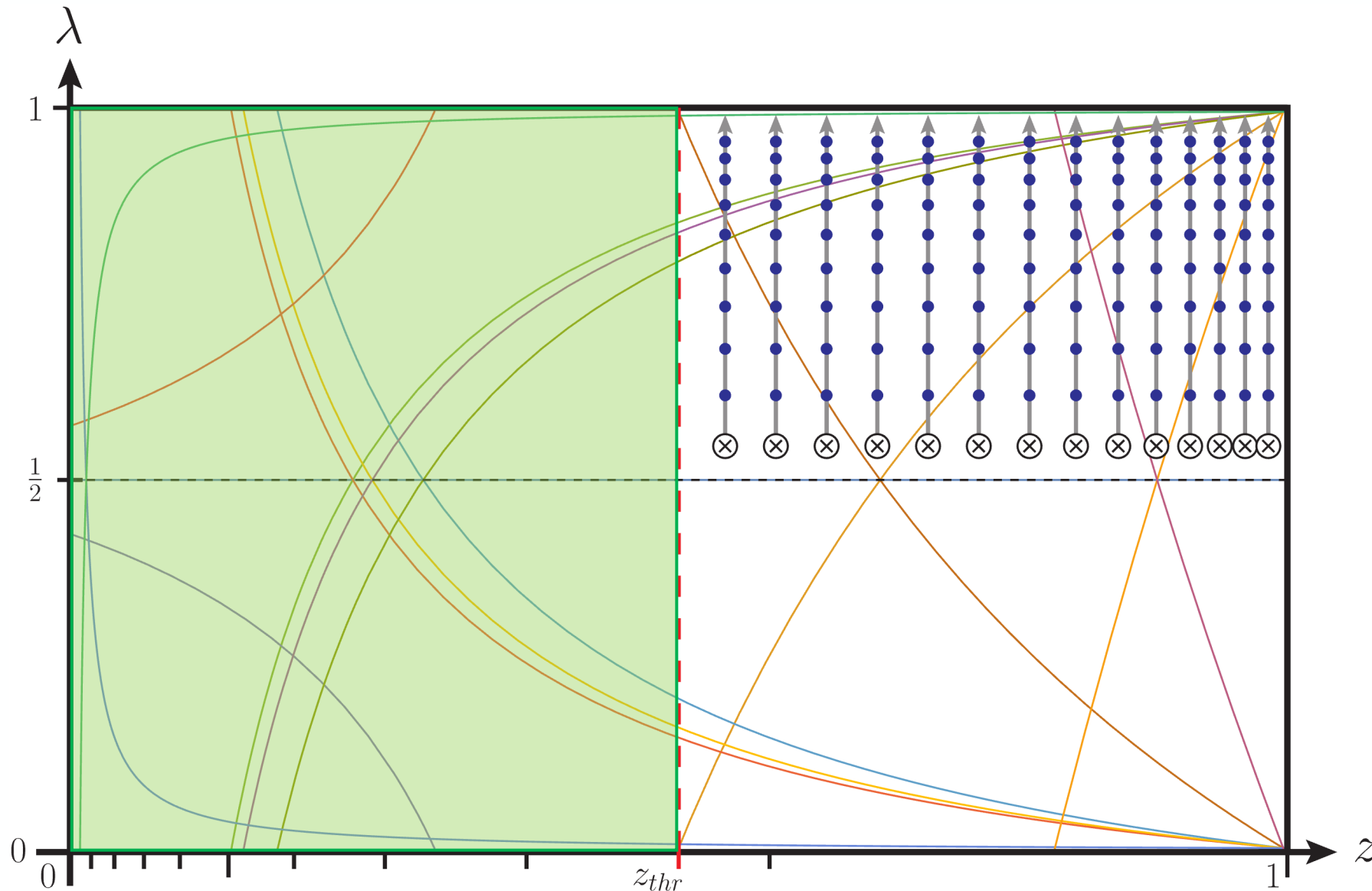
- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Collect numerical samples for MI along straight integration contours

Boundaries from numerical integration in the mass



# Evolution in the $(z, \lambda)$ -plane



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2 / m_H^2 = 23/12$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

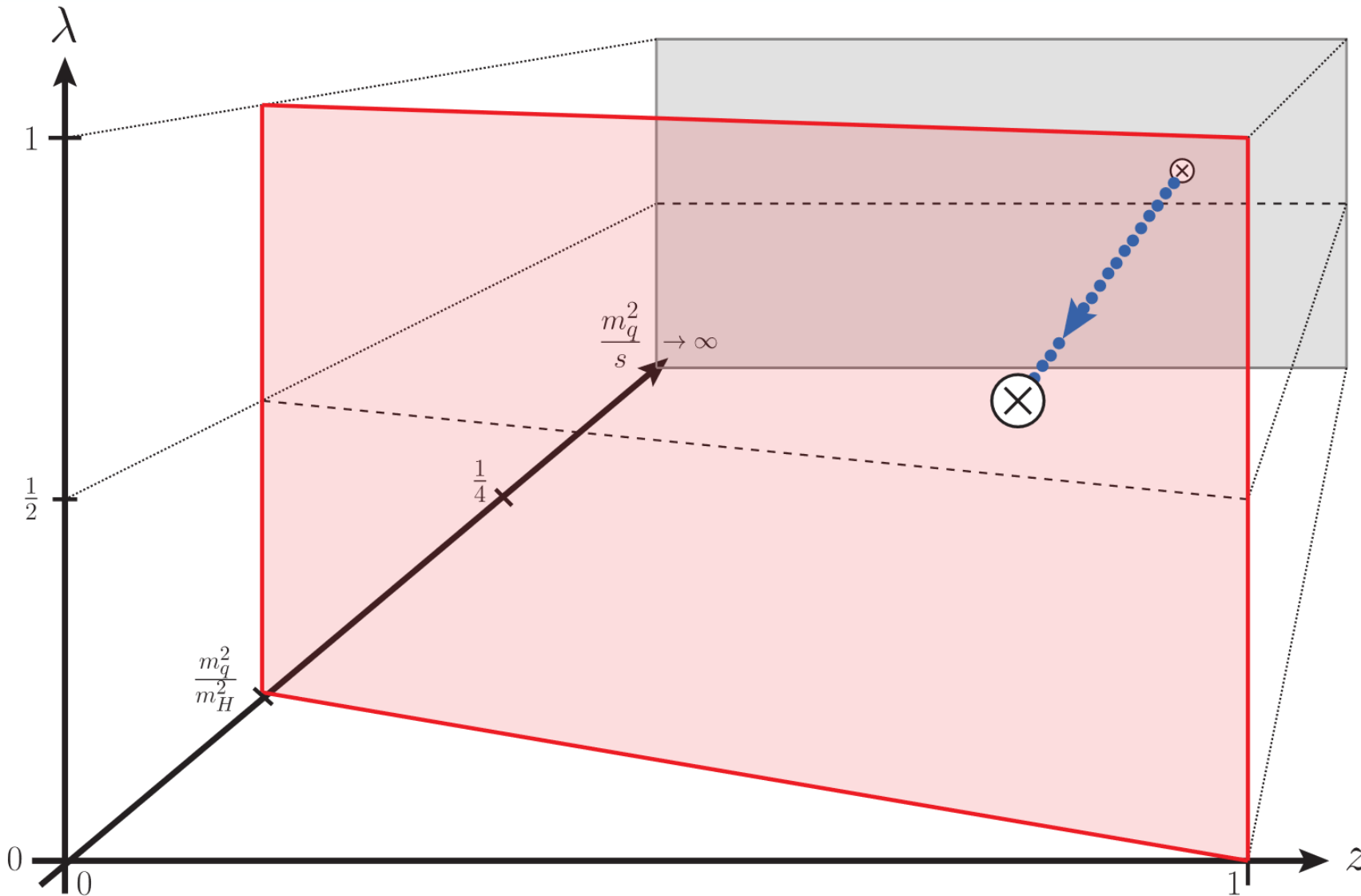
Region below  
threshold  
covered by

LME

LME

$$\mathcal{O}((1/m_q^2)^{40})$$

# Evolution of differential equations



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

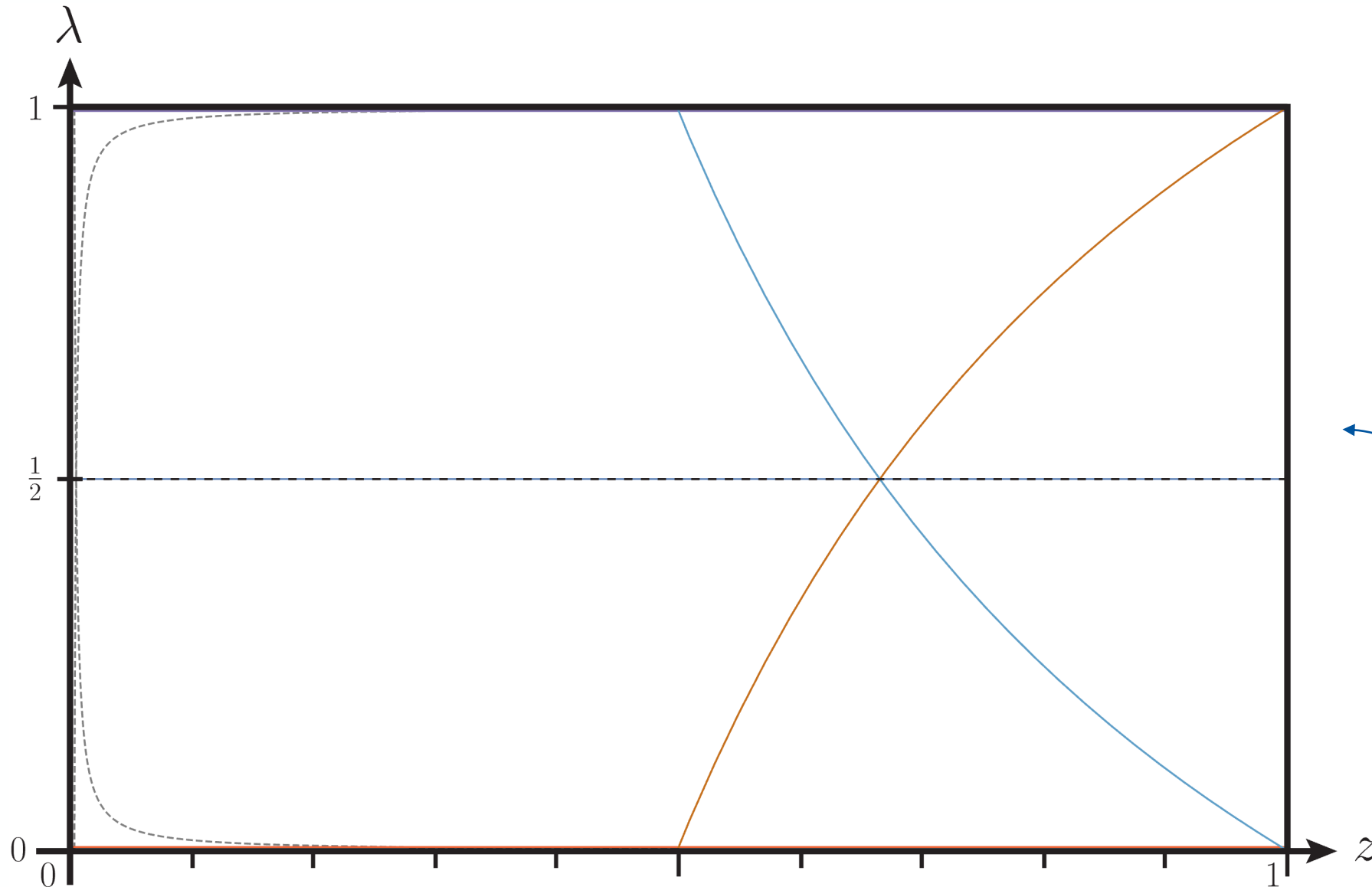
$$m_b^2 / m_H^2 = 1/684$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

$$z_{thr} = 1 - \frac{m_H^2}{4m_q^2} < 0$$

# Evolution in the $(z, \lambda)$ -plane



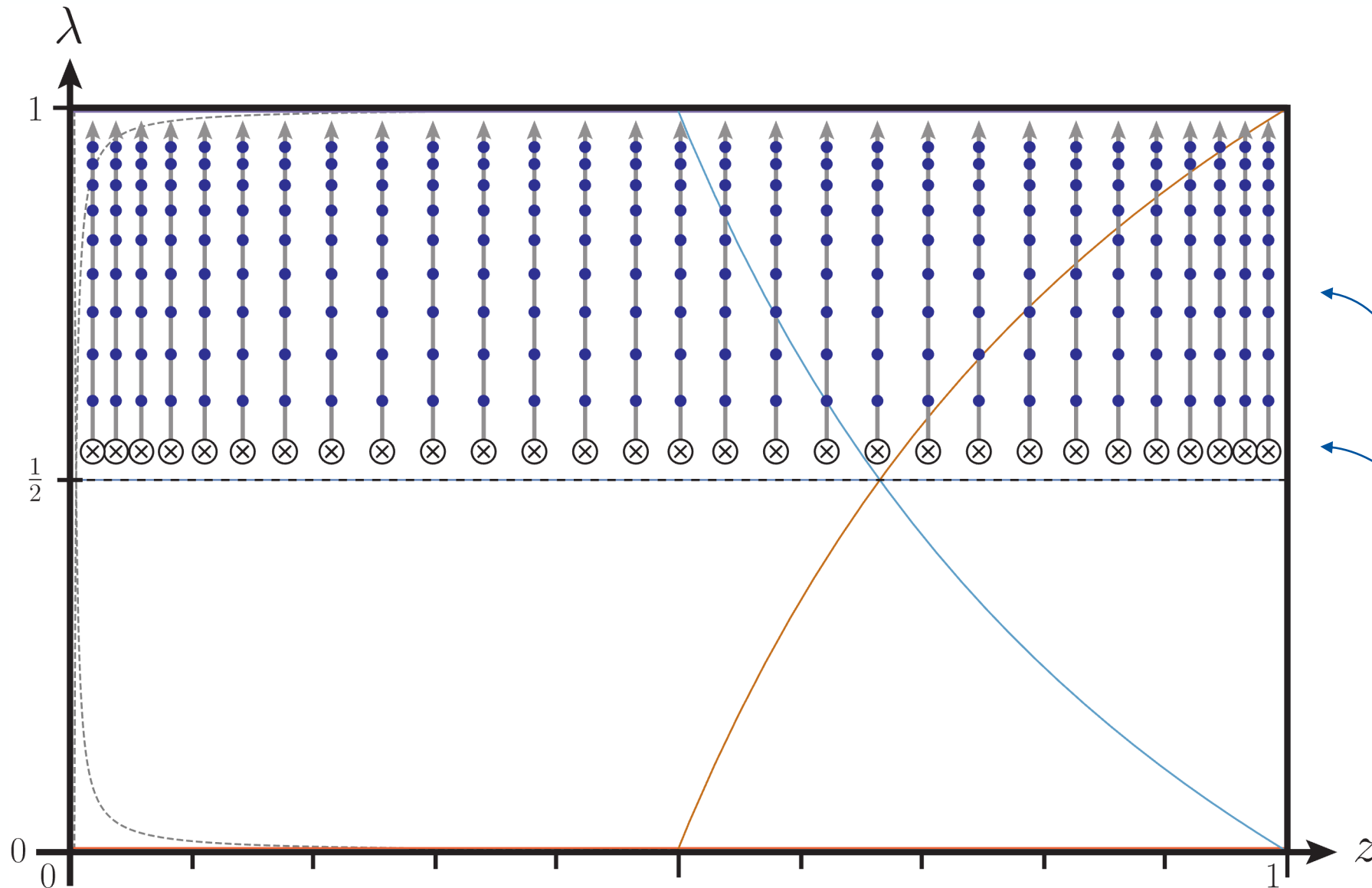
$$z = 1 - m_H^2 / \hat{s}$$
$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$
$$m_b^2 / m_H^2 = 1/684$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Poles of  
differential  
equations  
in  $\lambda$

# Evolution in the $(z, \lambda)$ -plane



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_b^2 / m_H^2 = 1/684$$

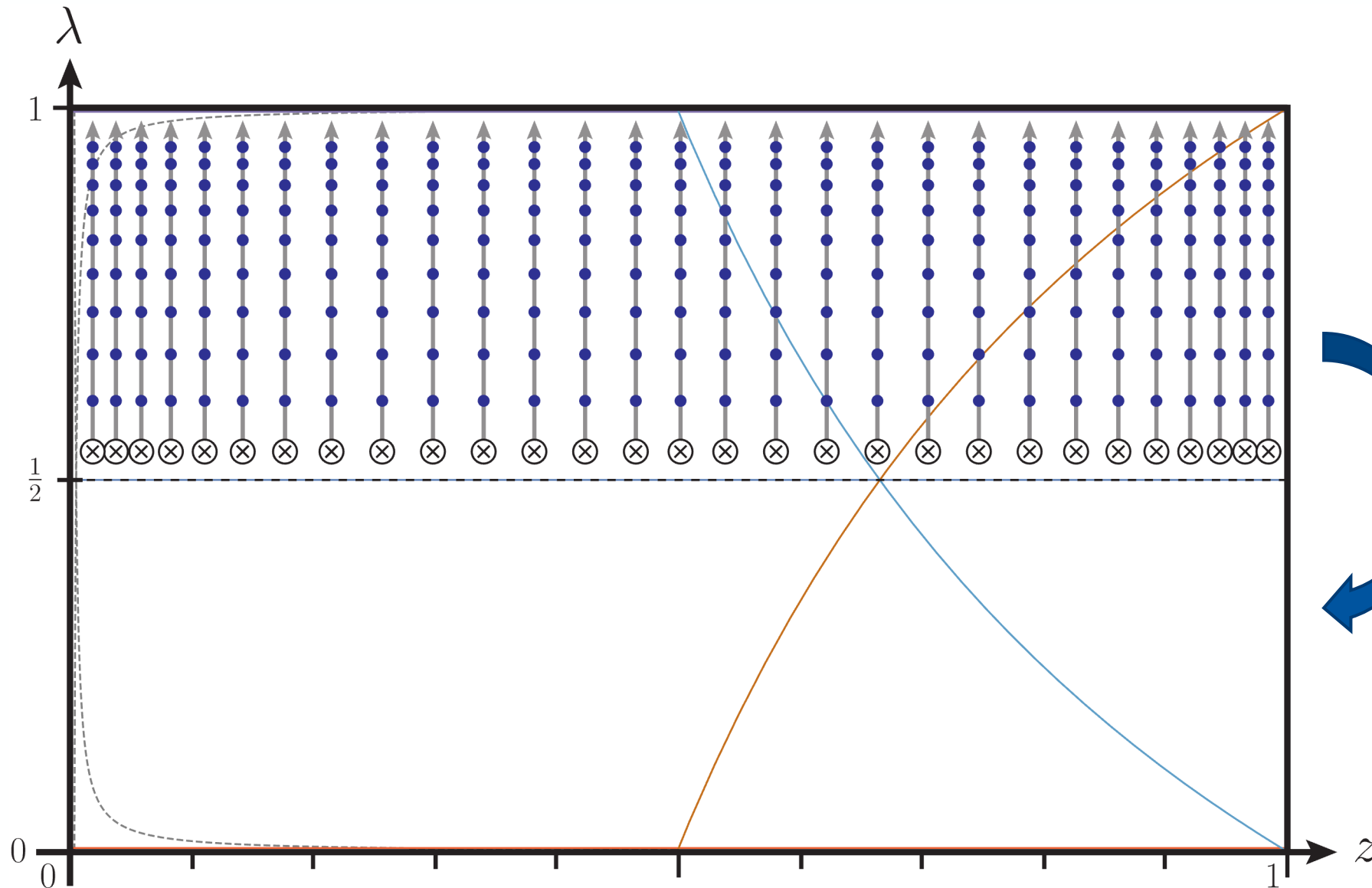
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Collect numerical samples for MI along straight integration contours

Boundaries from numerical integration in the mass

# Evolution in the $(z, \lambda)$ -plane



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_b^2 / m_H^2 = 1/684$$

Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Exploit symmetry of the problem at the amplitude level!

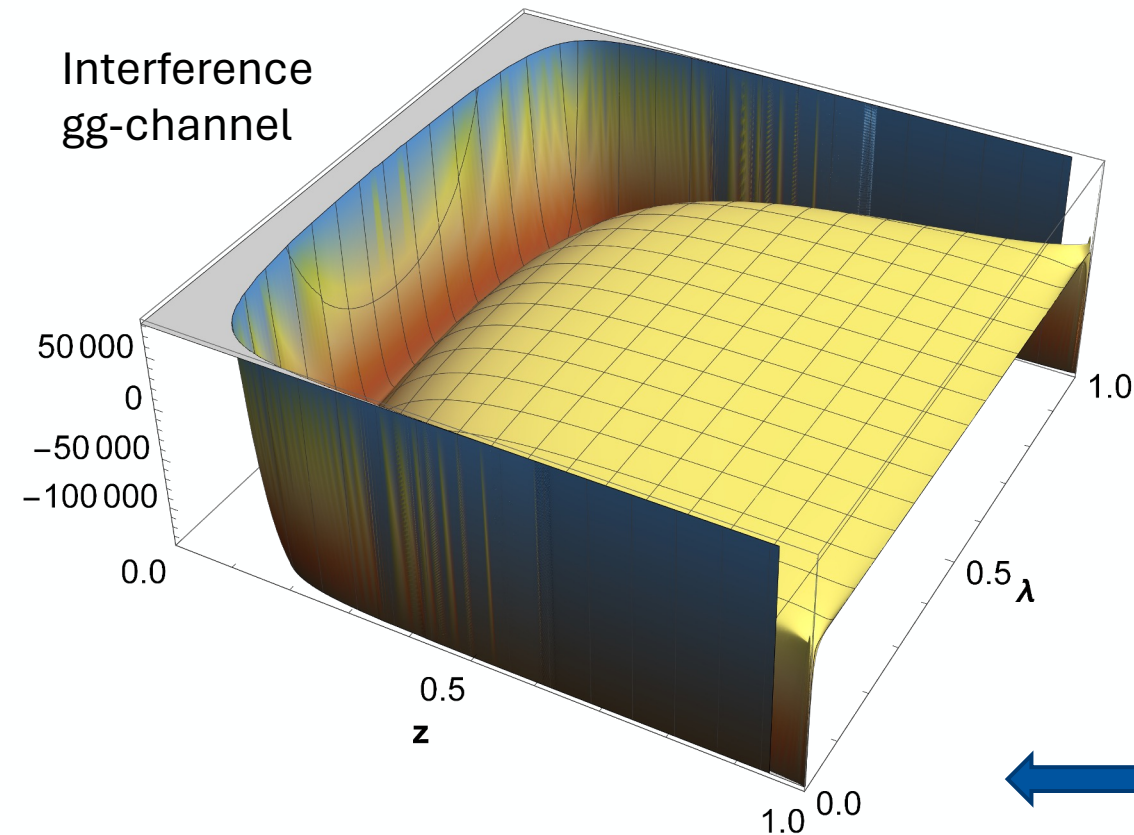
# Construction of amplitudes

- Collected  $2 \times 10^6$  numerical samples for MIs at  $m_t^2/m_H^2$  by evaluation of the LME and numerical evolution above threshold
- Collected  $1 \times 10^6$  numerical samples for MIs at  $m_b^2/m_H^2$  via numerical evolution in the entire phase space

Insert into form factors and construct helicity amplitudes



Interference  
gg-channel



$$\mathcal{M}_{+++} = \frac{1}{\sqrt{2}} \frac{s_{12}s_{23}}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \left( F_1 + \frac{s_{13}}{s_{23}} F_2 + \frac{s_{13}}{s_{12}} F_3 + \frac{s_{13}}{2} F_4 \right)$$

$$\mathcal{M}_{---} = \mathcal{M}_{+++} \Big|_{\langle ij \rangle \leftrightarrow [ji]} = -\frac{1}{\sqrt{2}} \frac{s_{12}s_{23}}{[12][23][31]} \left( F_1 + \frac{s_{13}}{s_{23}} F_2 + \frac{s_{13}}{s_{12}} F_3 + \frac{s_{13}}{2} F_4 \right)$$

$$\mathcal{M}_{++-} = \frac{1}{\sqrt{2}} \frac{[12]^3}{[13][23]} \frac{s_{23}}{s_{12}} \left( F_1 + \frac{s_{13}}{2} F_4 \right)$$

$$\mathcal{M}_{--+} = \mathcal{M}_{++-} \Big|_{\langle ij \rangle \leftrightarrow [ji]}$$

$$\mathcal{M}_{+-+} = \frac{1}{\sqrt{2}} \frac{[13]^3}{[12][23]} \frac{s_{12}}{s_{13}} \left( F_2 + \frac{s_{23}}{2} F_4 \right)$$

$$\mathcal{M}_{-+-} = \mathcal{M}_{+-+} \Big|_{\langle ij \rangle \leftrightarrow [ji]}$$

$$\mathcal{M}_{-++} = \frac{1}{\sqrt{2}} \frac{[23]^3}{[12][13]} \frac{s_{13}}{s_{23}} \left( F_3 + \frac{s_{12}}{2} F_4 \right)$$

$$\mathcal{M}_{+--} = \mathcal{M}_{-++} \Big|_{\langle ij \rangle \leftrightarrow [ji]}$$

$$|\mathcal{M}|^2 = \sum_{h \in Hel.} |\mathcal{M}_h|^2$$