# NNLO+PS predictions for Higgs production in bottom quark fusion with MiNNLO<sub>PS</sub>

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   (y<sub>b</sub>) in production
- Bottom Yukawa coupling: Important due to its enhancement in New Physics models like minimal supersymmetric extensions of the SM
- bbH enters as a background in other Higgs searches (notably HH)

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[Image courtesy : C. Biello]



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- > Active parton inside the proton.
- Included in the parton distribution functions (PDFs) of the proton.
- It is taken to be massless except in the Yukawa coupling

[Image courtesy : C. Biello]

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- It is taken to be massless except in the Yukawa coupling



- Considered as a heavy quark
- The bottom quark's contribution is neglected in the PDFs.
- A massive bottom quark is produced from gluon splitting

[Image courtesy : C. Biello]



- Computing higher orders is easier
- The DGLAP evolution resums initial state collinear logs into the bottom PDFs
- Neglects power-suppressed terms of the O(m<sub>b</sub>/m<sub>H</sub>)

[Image courtesy : C. Biello]



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- The DGLAP evolution resums initial state collinear logs into the bottom PDFs
- Neglects power-suppressed terms of the O(m<sub>b</sub>/m<sub>H</sub>)



- Computing **higher orders** is more **difficult** due to higher multiplicity & also due to the massive bottom
- It does not resum possibly large collinear logs
- Full kinematics of the massive bottom quark is taken into account already at LO

[Image courtesy : C. Biello]

### **STATE OF THE ART:**

N3LO for the total cross section in the 5FS

[Duhr, Dulat, Mistlberger (1904.09990)]

- N3LO matched to NLO in the 4FS by a prescription, namely, FONLL [Duhr, Dulat, Hirschi, Mistlberger (2004.04752)] [Forte, Napoletano, Ubiali [1508.01529, (1607.00389)]
- > NLO+PS in the 4FS (MADGRAPH5\_AMC@NLO framework) [Wiesemann, Frederix, Frixione, Hirschi, Maltoni, Torrielli (1409.5301)]
- > NLO+PS in the 4FS using POWHEG+PYTHIA6
- > NLO-QCD+PS combined with NLO-EW in the 4FS
- > Full NLO-QCD  $(y_b^2, y_t^2 \& y_b y_t)$  +PS in the 4FS

[Jäger, Reina, Wackeroth (1509.05843)]

[Pagani, Shao, Zaro (2005.10277)]

[Manzoni, Mazzeo, Mazzitelli , Wiesemann, Zaro (2307.09992)]

# **bbH** simulation

Precise and realistic LHC phenomenology requires full-fledged event simulations.



# **bbH** simulation

**Precise and realistic LHC phenomenology requires full-fledged event simulations.** 



- MiNLO' + reweighting [Hamilton, Nason, Zanderighi (1212.4504)]
- Geneva [Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi (1211.7049)]
- UNNLOPS [Höche, Prestel (1507.05325)]



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### $\textbf{MINNLO}_{\text{PS}}$

- **2->1:** [Monni, Nason, Re, Wisemann, Zanderighi (1908.06987)] [Monni, Re, Wiesemann (2006.04133)]
- 2->2 : [Lombardi, Wiesemann, Zanderighi (2010.10478)]
- tt: [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi (2012.14267)]
- **bbZ** : [Mazzitelli, Sotnikov, Wiesemann (2404.08598)]

Talk by M. Wiesemann



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	F	F+J	F+JJ
F@MiNNLO <sub>PS</sub>	NNLO	NLO	LO



	F	F+J	F+JJ
F@MiNNLO <sub>PS</sub>	NNLO	NLO	LO

- No computationally intense reweighting
- No unphysical merging scale
- Leading-log (LL) accuracy of the shower preserved

Talk by M. Wiesemann

Numerically efficient



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- > The matching to the parton shower is performed according to the **POWHEG** method [P. Nason (0409146)]
- The **POWHEG** approach: we generate the **hardest radiation** (i.e. the largest p<sub>T</sub>) **first** with **NLO** accuracy, then attaching a **parton shower** with **softer** emissions.



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> MiNNLO<sub>PS</sub> in POWHEG framework: we start from a differential description of the production of the colour singlet and a jet ( $pp \rightarrow F + J$ ). POWHEG Sudakov form factor

 $d\sigma_{F}^{MiNNLO_{PS}} = d\Phi_{FJ}\bar{B}^{MiNNLO_{PS}} \times \left\{ \Delta_{pwg}(\Lambda_{pwg}) + \int d\Phi_{rad}\Delta_{pwg}(p_{T,rad})\frac{R_{FJ}}{B_{FJ}} \right\}$ Describes the generation of the 1<sup>st</sup> radiation Describes the generation of the 2<sup>nd</sup> radiation according to the **POWHEG** method

# $\label{eq:central ingredient of Minnlops} Very simplified notation! $$ \mu_{R} = \mu_{F} = p_{T}$$ $$ \overline{B}^{MiNNLO_{PS}} \sim e^{-\widetilde{S}} \left\{ d\sigma_{FJ}^{(1)} \big(1 + \widetilde{S}^{(1)}\big) + d\sigma_{FJ}^{(2)} + \big(D - D^{(1)} - D^{(2)}\big) \right\} $$ $$$

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Sudakov form factor suppresses  $\overline{B}$  at low  $p_T$ 









For bbH: We revised the original **MiNNLO**<sub>PS</sub> method to account for the **Yukawa** coupling in MS **scheme** 

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# **The computation**

Sample Feynman diagrams for Higgs production in association with bottom quarks



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# We focus on the 5FS & 4FS calculation of the $b\bar{b}\text{H}$ process proportional to $Y^2_b~$ at NNLO+PS

NNLO+PS predictions for Higgs production in bottom-quark fusion with MiNNLO<sub>PS</sub>

# The computation (5FS)

- MiNNLO<sub>PS</sub>  $b\bar{b} \rightarrow H$  generator implemented in the Powheg-Box-Res
- First, we implemented a **NLO+PS** generator for **HJ** production in bottom ٠ fusion using the **Powheg** method

• Tree-level amplitudes of the **HJ & HJJ** : **OPENLOOPS** 

[F. Buccioni, S. Pozzorini and M. Zoller (1710.11452), F. Buccioni et al (1907.13071)]

- substantially improve the numerical performance of the code
- In a second step, we extended the **HJ NLO+PS** implementation to **NNLO accuracy** through the MiNNLO<sub>PS</sub> method

[Monni, Nason, Re, Wisemann, Zanderighi (1908.06987)] [Monni, Re, Wiesemann (2006.04133)]

[R.V. Harlander et al (1007.5411)]

Virtual corrections : Analytic results

The POWHEG BOX



[T. Ježo and P. Nason (1509.09071)]

[P. Nason (0409146), S. Alioli et al (1002.2581), S. Frixione et al (0709.2092)]

# Phenomenological Results for bbH (5FS)

# **The Setup**

### Inputs:

- Center-of-mass energy: **13 TeV** at LHC.
- Higgs boson mass ( $m_H$ ): **125 GeV**,  $\Gamma_H$  (decay width): 0 GeV.
- Default PDF: NNPDF40\_nnlo\_as\_01180 with 5 active flavours.
- Central  $\mu_R$  and  $\mu_F$  scales set via **Minnlo**<sub>Ps</sub> method [ $\mu_R \sim \mu_F \sim p_T$ ].
- Yukawa coupling renormalized in  $\overline{MS}$  scheme [Y<sub>b</sub>(m<sub>b</sub>=4.18 GeV) -> Y<sub>b</sub>(m<sub>H</sub>) = 2.79].

### Scale Settings and Uncertainties:

- Scale uncertainities assessed through customary **7-point**  $\mu_R$  and  $\mu_F$  variation.
- Matching to Parton Shower:
  - Predictions matched to parton shower using Pythia8 with leading-log (LL) accuracy.
- Exclusion of Effects:
  - Hadronization, multi-parton interactions (MPI), and QED radiation effects are switched off.

Comparison of the total inclusive cross section of **MiNLO'** and **MiNNLO**<sub>PS</sub> predictions with fixedorder results at NLO and NNLO obtained with the public code **SusHi** [with  $\mu_R$  and  $\mu_F$  set to  $m_H$ ]

[Harlander, Liebler, Mantler (1212.3249)]

Process NLO (SUSHI) NNI		NNLO (SUSHI)	MINLO'	MINNLO <sub>PS</sub>
$b\bar{b} \rightarrow H$	$0.646(0)^{+10.4\%}_{-10.9\%} \mathrm{pb}$	$0.518(2)^{+7.2\%}_{-7.5\%}{ m pb}$	$0.571(1)^{+17.4\%}_{-22.7\%}  \mathrm{pb}$	$0.509(8)^{+2.9\%}_{-5.3\%}$ pb

[Biello, AS, Wiesemann, Zanderighi (2402.04025)]

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[Biello, AS, Wiesemann, Zanderighi (2402.04025)]

- NNLO QCD corrections reduce cross section by > 10%
- Scale uncertainities significantly reduced with NNLO QCD corrections
- > Our MiNNLOps predictions are in agreement with NNLO QCD cross section within quoted uncertainties

### Transverse-momentum spectrum of the Higgs boson ( $p_{T,H}$ )

Les Houches level (LHE)



NNLO [Harlander, Tripathi, Wiesemann (1403.7196)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]

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### Transverse-momentum spectrum of the Higgs boson ( $p_{T,H}$ )

Les Houches level (LHE)



- Full agreement in large p<sub>T,H</sub> regime with fixed-order predictions within quoted uncertainities
- > Fixed-order calculations diverge for  $p_{T,H} \rightarrow 0$ MiNNLO<sub>PS</sub> prediction remains finite

NNLO [Harlander, Tripathi, Wiesemann (1403.7196)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]

### Rapidity distribution of the Higgs boson $(y_H)$

PY8 level



NNLO [Mondini, Williams (2102.05487)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]



### Rapidity distribution of the Higgs boson $(y_H)$

PY8 level



- A good agreement, both in terms of normalization and in terms of shape, between the two central predictions.
- The bands of MiNNLO<sub>PS</sub> result are more symmetric & slightly smaller than the NNLO ones.

NNLO [Mondini, Williams (2102.05487)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]

# Transverse-momentum spectrum of the Higgs boson (p<sub>T,H</sub>)



[Biello, AS, Wiesemann, Zanderighi (2402.04025)]

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NNLO+PS predictions for Higgs production in bottom-quark fusion with MiNNLO<sub>PS</sub> PY8 level

# Transverse-momentum spectrum of the Higgs boson (p<sub>T,H</sub>)



PY8 level

- At small p<sub>T</sub>, MiNNLO<sub>PS</sub>
   significantly dampens
   distributions, reduces scale
   uncertainties.
- At large p<sub>T</sub>, MiNLO' &
   MiNNLO<sub>PS</sub> predictions coincide, both NLO accurate.

[Biello, AS, Wiesemann, Zanderighi (2402.04025)]

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### Rapidity distribution of the Higgs (y<sub>H</sub>)

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PY8 level

- At large p<sub>T</sub>, MiNLO' & MiNNLO<sub>PS</sub> predictions coincide, both NLO accurate.
- y<sub>H</sub> distribution: MiNNLO<sub>PS</sub> introduces a flat 12% negative correction, reduces scale uncertainties.

[Biello, AS, Wiesemann, Zanderighi (2402.04025)]

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NNLO+NNLL [Harlander, Tripathi, Wiesemann (1403.7196)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]

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At large p<sub>T,H</sub>: MiNNLO<sub>PS</sub> shifted 10% up, well within the given scaleuncertainty bands.

 At small p<sub>T,H</sub>: slightly worsen the agreement.
 MiNNLO<sub>PS</sub> uncertainities are underestimated.

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- At small p<sub>T,H</sub>: slightly worsen the agreement.
   MiNNLO<sub>PS</sub> uncertainities are underestimated.
- Massless approximation misses potentially relevant mass effects at small p<sub>T</sub>, need to combine with massive 4FS calculation.

NNLO+NNLL [Harlander, Tripathi, Wiesemann (1403.7196)] MiNNLO<sub>PS</sub> [Biello, **AS**, Wiesemann, Zanderighi (2402.04025)]

We implemented **NLO+PS** for **Hbb** in **POWHEG** and compared it against **MiNLO**' obtained from a **Hbbj** generator

$(\mu_{ m \scriptscriptstyle R}^{(0),lpha},\mu_{ m \scriptscriptstyle R}^{(0),y})$	$\rm NLO_{PS}$	MiNLO'
$(rac{H_{ m T}}{4},m_{H})$	$0.381(2)^{+20.2\%}_{-15.9\%}{ m pb}$	$0.277(5)^{+34.5\%}_{-27.0\%}\mathrm{pb}$
$(rac{H_{\mathrm{T}}}{4},rac{H_{\mathrm{T}}}{4})$	$0.406(4)^{+16.6\%}_{-14.3\%}\mathrm{pb}$	$0.315(3)^{+30.6\%}_{-27.5\%}{ m pb}$
$\boxed{\frac{H_T}{4} = \frac{1}{4} \sum_{i \in \text{final}} \sqrt{m^2(a)}}$	$\frac{1}{p_T(i)} = \frac{\text{MiNLO' m}}{\text{than NLO}}$	ore than 20% less



[Biello, Mazzitelli, AS, Wiesemann, Zanderighi (in progress)]

20/05/24

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$\frac{H_T}{4} = \frac{1}{4} \sum \sqrt{m^2(i) + p_T^2(i)}$ MiNLO' more than 20% less than NLO				

- In MiNLO', the large log(m<sub>b</sub>) terms in RV & RR contributions are not balanced.
- We need the **double virtual** (VV) to **cancel** this quasi-collinear **divergence**.

[Biello, Mazzitelli, AS, Wiesemann, Zanderighi (in progress)]



## **Double virtual Amplitude**

The **VV correction** for a **massive bottom** pair and Higgs production is not known: Approximation using the **massification procedure: leading mass corrections** are restored

Collinear poles in 5FS Logs of  $m_b$  in 4FS  $\mathscr{A}^{(2)} = \log(m_b) \text{-terms} + \text{const.} + \mathscr{O}\left(\frac{m_b}{Q}\right)$   $\mathscr{F}^{(2)}\mathscr{A}^{(0)}_{m_b=0} + \mathscr{F}^{(1)}\mathscr{A}^{(1)}_{m_b=0} + \mathscr{F}^{(0)}\mathscr{A}^{(2)}_{m_b=0}$ Massification coefficients Massless double virtual amplitude

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**Collinear poles in 5FS** 

### Logs of m<sub>b</sub> in 4FS

Predictions using

**MiNNLO**<sub>PS</sub>

$\mathscr{A}^{(2)} =$	$\log(m_b)$ -terms + const. + $O$	$\left(\frac{m_b}{Q}\right)$

$$\mathscr{F}^{(2)}\mathscr{A}^{(0)}_{m_b=0} + \mathscr{F}^{(1)}\mathscr{A}^{(1)}_{m_b=0} + \mathscr{F}^{(0)}\mathscr{A}^{(2)}_{m_b=0}$$

Massification coefficients

Massless double virtual amplitude

$(\mu_{ m \scriptscriptstyle R}^{(0),lpha},\mu_{ m \scriptscriptstyle R}^{(0),y})$	NLO <sub>PS</sub>	MiNLO'	$\mathrm{MiNNLO}_{\mathrm{PS}}\left(\mathcal{F}^{(0)}=0 ight)$
$(rac{H_{\mathrm{T}}}{4},m_{H})$	$0.381(2)^{+20.2\%}_{-15.9\%}\mathrm{pb}$	$0.277(5)^{+34.5\%}_{-27.0\%}\mathrm{pb}$	$0.434(1)^{+6.4\%}_{-9.9\%}{ m pb}$
$\left(\frac{H_{\mathrm{T}}}{4}, \frac{H_{\mathrm{T}}}{4}\right)$	$0.406(4)^{+16.6\%}_{-14.3\%}{ m pb}$	$0.315(3)^{+30.6\%}_{-27.5\%}{ m pb}$	$0.443(9)^{+4.0\%}_{-8.7\%}{ m pb}$

[Biello, Mazzitelli, **AS**, Wiesemann, Zanderighi (in progress)]

**MINNLO**<sub>PS</sub> with **only logarithmic** contributions in the 2-loop predicts a total cross-section **bigger** than the **NLO+PS** one.

# **Double virtual Amplitude**

• We used analytic VV amplitudes for massless bottoms computed in the leading color approximation

$$\mathcal{F}^{(2)} \mathscr{A}^{(0)}_{m_b=0} + \mathcal{F}^{(1)} \mathscr{A}^{(1)}_{m_b=0} + \mathcal{F}^{(0)} \mathscr{A}^{(2)}_{m_b=0} \quad \text{[Badger, Hartanto, Kryś, Zoia (2107.14733)]}$$

- Evaluation of special functions through **PentagonFunctions++** [Chicherin, Sotnikov, Zoia (2110.10111)]
- C++ code interfaced with POWHEG
- We cross-checked against the Zurich implementation (Chiara Savoini)

# **Massification procedure**



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# **Total cross-section**



**Large differences** in the predictions were first observed at the **LO**: the effect of collinear resummation is extremely large.

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### NLO: 5FS = 1.78 \* 4 FS

NLO+PS (5FS)	NLO+PS (4FS)
$0.677(2)^{+11\%}_{-11\%}  \mathrm{pb}$	$0.381(0)^{+20\%}_{-16\%}\mathrm{pb}$

# **Total cross-section**



[Biello, Mazzitelli, AS, Wiesemann, Zanderighi (in progress)]

# **Higgs rapidity**





[Biello, Mazzitelli, AS, Wiesemann, Zanderighi (in progress)]

# Higgs p<sub>T</sub> spectrum



[Biello, Mazzitelli, AS, Wiesemann, Zanderighi (in progress)]



# **Summary & Outlook**

- Presented the first NNLO+PS computation for bbH in both 5FS & 4FS at the LHC by using MiNNLO<sub>PS</sub> method.
- Extensive validation of 5FS predictions against fixed-order results from literature, showcasing consistency in relevant kinematical regions.
- For the 4FS, approximation of the double virtual using the massification procedure
- > Theoretical **tension** between the **4FS** & **5FS** predictions seem to stabilise **at NNLO**.
- Future directions include combination of full 4FS-5FS at NNLO+PS and also b-tagging of the MiNNLO<sub>PS</sub> events.





# Backup slides.....



**At high р**т,н: they coincide again

At small  $p_{T,H}$ : Acceptable agreement





- Very similar shapes for MiNLO' & MiNNLO<sub>PS</sub> results
- MiNLO' & MiNNLO<sub>PS</sub>: fully consistent within the quoted scale uncertainties



- → In 4FS, the phase-space integration is performed with  $m_b \neq 0$ .
- → The massless amplitudes must be evaluated on on-shell phase-space points  $P_0$  with  $m_b = 0$ .

$$\mathcal{F}^{(2)} \mathscr{A}^{(0)}_{m_b=0} + \mathcal{F}^{(1)} \mathscr{A}^{(1)}_{m_b=0} + \mathcal{F}^{(0)} \mathscr{A}^{(2)}_{m_b=0}$$

- → We need an explicit mapping of massive phase-space points P ,  $\eta$  : P → P<sub>0</sub>, such that  $\eta$ (P ) = P<sub>0</sub> + O(m<sub>b</sub> /m<sub>H</sub>).
- We have to ensure that  $\eta$  does not cause amplitudes to be evaluated near their singularities.
- → Since the quark- and gluon-initiated channels have distinct leading order momentum flows, we use dedicated mappings  $\eta_{q\bar{q}}$ ,  $\eta_{gg}$  for each of the channels.

For  $\eta_{q\bar{q}}$ , we perform the simultaneous light-cone decomposition of the massive bottom and anti-bottom momenta  $p_b$  and  $p_{\bar{b}}$ , respectively, and determine the massless momenta  $\hat{p}_b$  and  $\hat{p}_{\bar{b}}$  as

$$\hat{p}_{b} = \alpha^{+} p_{b} - \alpha^{-} p_{\bar{b}}, \qquad \alpha^{\pm} = \frac{1}{2} \left( 1 \pm \left( 1 - 4 \frac{m_{b}^{2}}{m_{b\bar{b}}} \right)^{-\frac{1}{2}} \right)$$
$$\hat{p}_{\bar{b}} = \alpha^{+} p_{\bar{b}} - \alpha^{-} p_{b},$$

which preserves the total momentum  $\hat{p}_{b\overline{b}} \equiv p_{b\overline{b}}$  of the  $b\overline{b}$  system and prevents a collinear  $g \rightarrow b\overline{b}$  splitting in the quark channel.

The mapping  $\eta_{q\bar{q}}$  is minimal in the sense that only the bottom-quark momenta are modified.

An side effect of the mapping  $\eta_{q\bar{q}}$  (when applied in the gluon channel) is that  $p_b$  or  $p_{\bar{b}}$  can become collinear to the initial state momenta  $p_1$  or  $p_2$  when the  $b\bar{b}$  pair is produced at the threshold.

In the gluon channel this introduces a collinear singularity, and we therefore construct  $\eta_{gg}$  such that it avoids these configurations.

First, we set the massless momenta to

$$\begin{split} \hat{p}_x &= p_x + \left(\sqrt{1 - \frac{m_b^2 n_x^2}{(p_x \cdot n_x)^2}} - 1\right) \frac{(p_x \cdot n_x)}{n_x^2} \ n_x \quad \text{with } x \in \{b, \bar{b}\}\\ n_x &= p_x - p_1 \frac{(p_2 \cdot p_x)}{(p_1 \cdot p_2)} - p_2 \frac{(p_1 \cdot p_x)}{(p_1 \cdot p_2)}, \end{split}$$

where  $n_x$  are transverse to both p1 and p2 .

[Mazzitelli, Sotnikov, Wiesemann (2404.08598)]

Then to restore momentum conservation we consider two options:

1. We redistribute  $\Delta p_{b\bar{b}} = p_b + p_{\bar{b}} - \hat{p}_b - \hat{p}_{\bar{b}}$  into  $\hat{p}_1$  and  $\hat{p}_2$ , such that  $\hat{p}_{12} = \hat{p}_1 + \hat{p}_2 = p_1 + p_2 - \Delta p_{b\bar{b}}$ , by performing a Lorentz boost on  $p_1$  and  $p_2$  in the direction  $-\hat{p}_{12}$  followed by rescaling with  $\sqrt{\hat{p}_{12}^2/p_{12}^2}$ 

### OR

2. we redistribute  $\Delta p_{b\overline{b}}$  into the Higgs momentum instead.

# **Cross-section details (4FS)**

	K <sub>R</sub>	K <sub>F</sub>	MINLO'	MINNLO <sub>PS</sub> (Orig. Mass.)	MINNLO <sub>PS</sub> (Gen. Mass.)
	1	1	0.277(0)	0.460(7)	0.464(9)
	1	2	0.268(8)	0.465(2)	0.470(7)
	2	1	0.192(5)	0.403(0)	0.408(1)
	2	2	0.195(5)	0.407(0)	0.412(1)
	1	$\frac{1}{2}$	0.258(9)	0.457(8)	0.466(0)
	$\frac{1}{2}$	1	0.382(7)	0.520(7)	0.527(4)
	$\frac{1}{2}$	$\frac{1}{2}$	0.375(3)	0.519(3)	0.525(1)
Π			$0.277(0)^{+34\%}_{-27\%}{ m pb}$	$0.460(7)^{+13\%}_{-13\%}{ m pb}$	$0.464(9)^{+14\%}_{-13\%}\mathrm{pb}$

# **Before the two-loop | 4FS**



# **FONLL** matching

- FONLL matches the flavour schemes  $\sigma^{FONNL} = \sigma^{4FS} + \sigma^{5FS}$  double couting. For a consistent subtraction, we have to express the two cross-sections in terms of the same  $\alpha_s$  and PDFs.
- Currently, the flavour matching for bbH is performed at

 $FONNL_C := N^3 LO_{5FS} \oplus NLO_{4FS}$ .