Heavy Quark Production at High Precision

The challenge of multi-scale problems

Fred Olness
SMU

*Thanks for substantial input
from my friends & colleagues*

nCTEQ
nuclear parton distribution functions

C T E Q

xFitter

SURGE
COLLABORATION

LoopFest 2024
20 - 22 May 2024

… an old problem

neutrino DIS

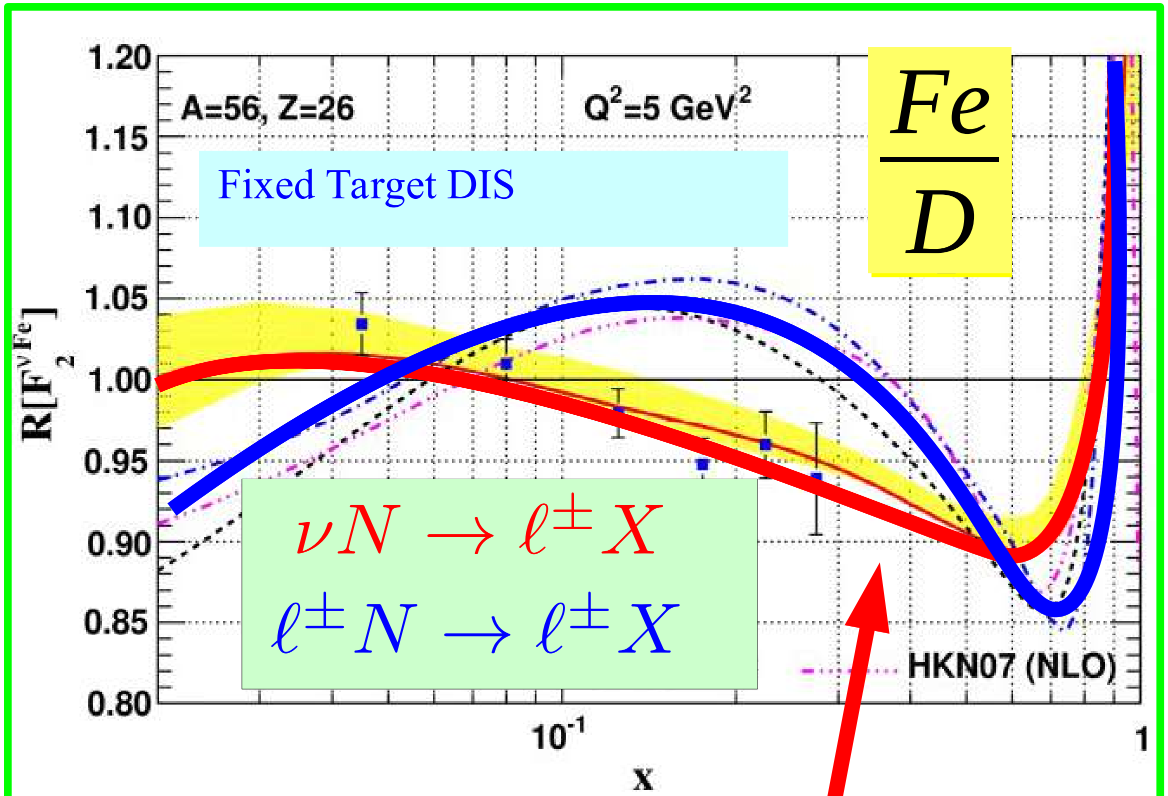
some new results

S-ACOT χ CC @ N2LO

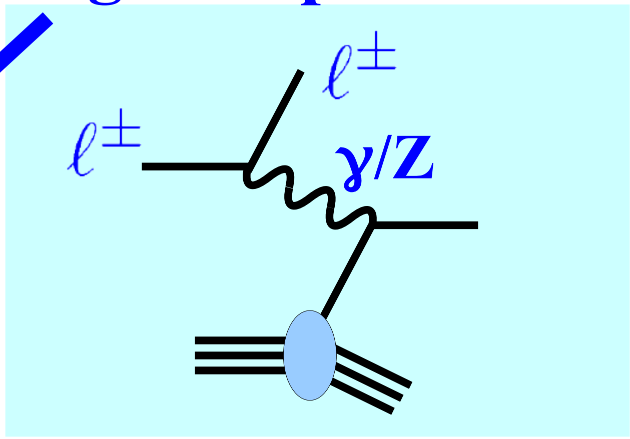
Thanks to Peter Risse & Valerio Bertone

Neutrinos: many outstanding puzzles & discrepancies

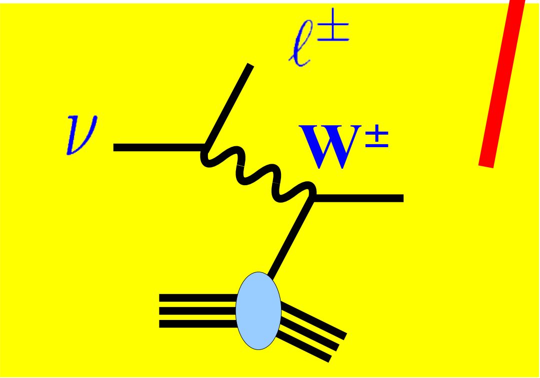
Puzzle: Split Personality … *What is the correct Nuclear ratio*



Charged Lepton DIS

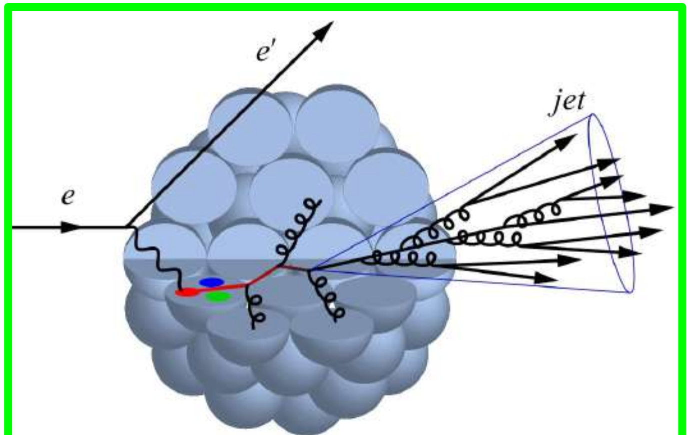


some caveats
… correlated errors



Neutrino DIS

Depends on nuclear corrections



Propagation of γ/W thru nuclei

QCD for Precision Neutrino Physics

Un-ki Yang
Seoul National University

Precision, Precision… but systematic effect: theory and experiment

- Discrepancy between CCFR (ν) and NMC(μ) data at low x region (0.01<x<0.1)
  - Resolved by the proper handling of massive charm treatment (VFS, FFS): Model Ind. CCFR F2, xF3, δxF, *Phys.Rev.Lett.* 86 (2001) 2742
- Discrepancy in QCD analysis between CCFR(ν) and CDHSW (ν)
  - Problem appeared in the CDHSW diff. cross section, overall level fine, but wrong y-dependence, *Phys.Rev.Lett.* 87 (2001) 251802
- Discrepancy in diff. cross section between CCFR(ν) and NuTeV (ν) at high x region (x>0.5), making a new discrepancy at high region
  - Problem appeared in the toroidal magnet calibration of the CCFR detector: *Phys.Rev.D* 74 (2006) 012008
- Different neutrino effect in neutrino: MINERvA saw a different nuclear effect?
- d/u at high x and asymmetry in strange sea
  - Updated d/u→0.2 or 0 at x=1
  - Asymmetry measurement in strange sea: correlated with d/u issue

Unki Yang DIS2024

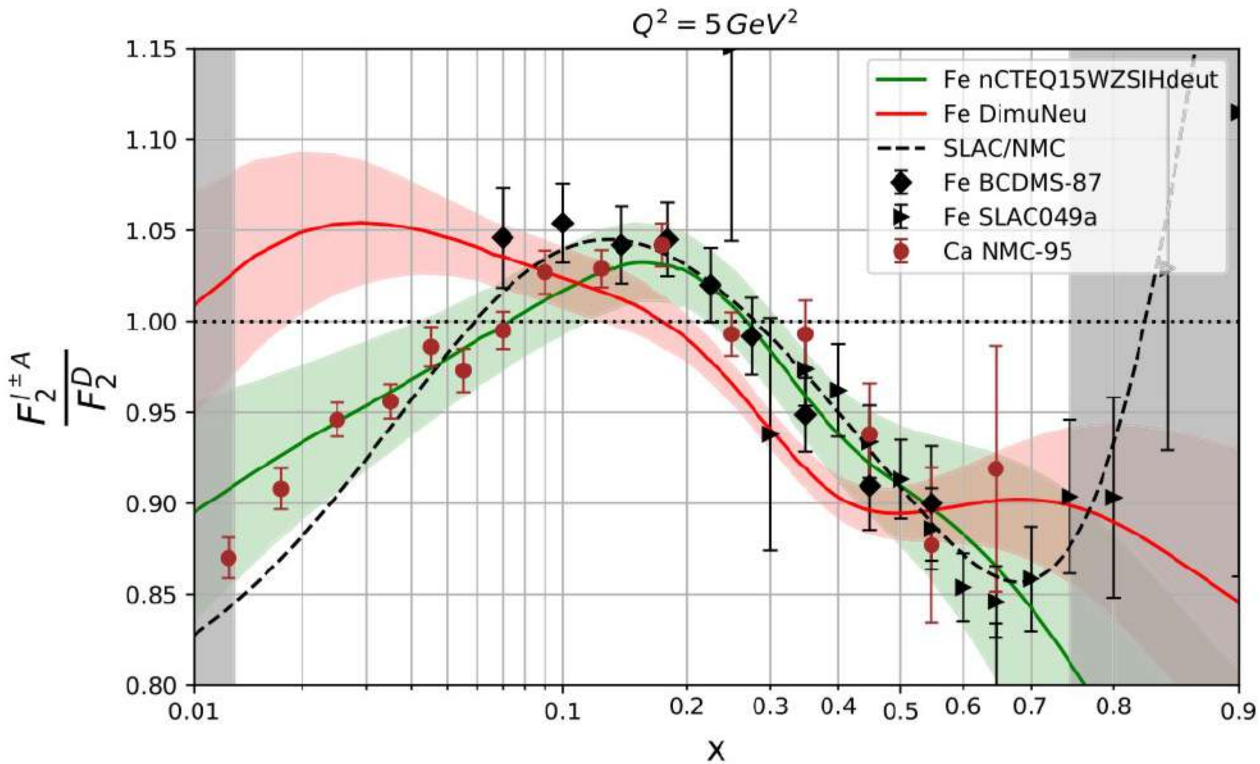
nCTEQ

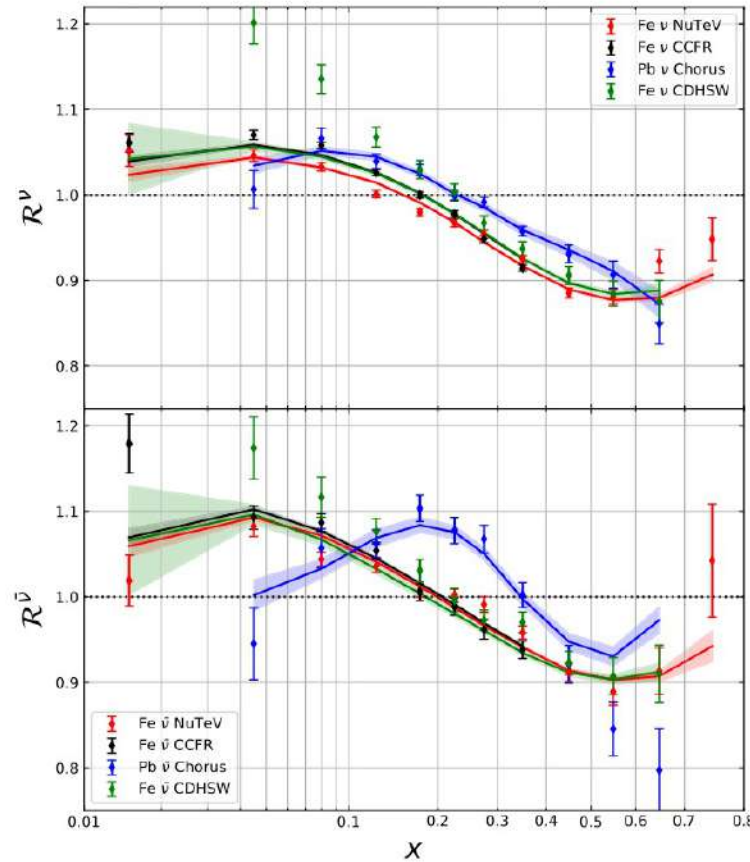
nuclear parton distribution functions

PHYSICAL REVIEW D **106,** 074004 (2022)

Compatibility of neutrino DIS data and its impact on nuclear parton distribution functions

K. F. Muzakka,[1,*] P. Duwentäster,[1] T. J. Hobbs,[2,3] T. Ježo,[1] M. Klasen,[1] K. Kovařík,[1,†] A. Kusina,[4] J. G. Morfín,[2] F. I. Olness,[5] R. Ruiz,[4] I. Schienbein,[6] and J. Y. Yu[5]



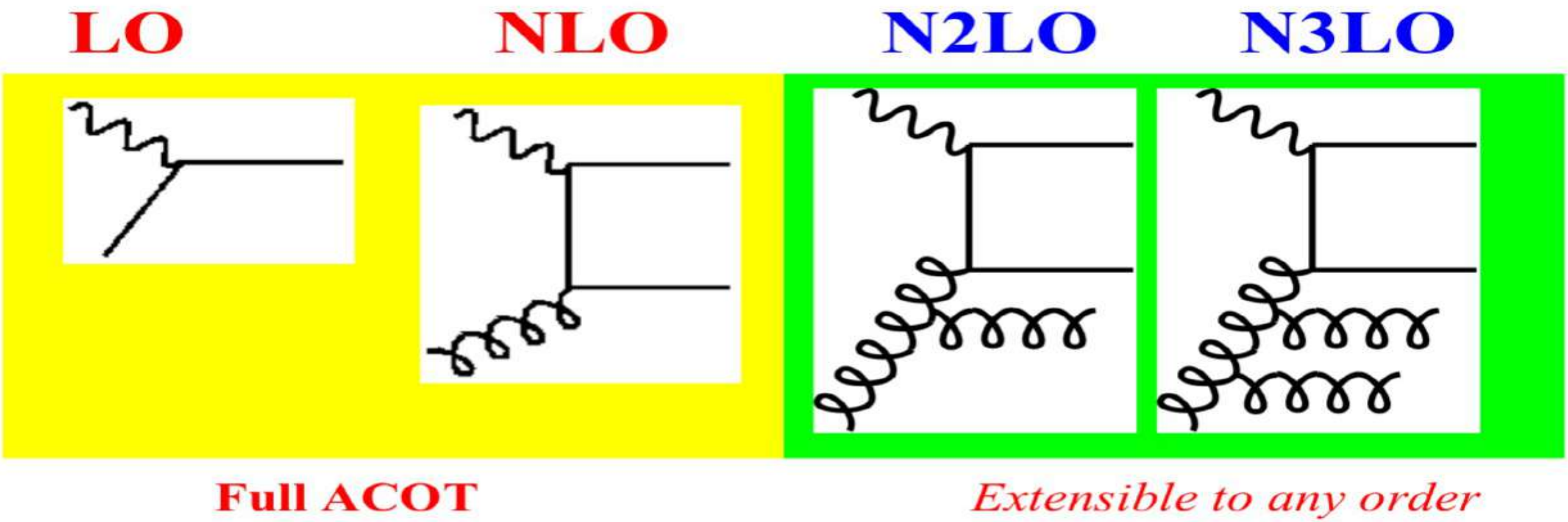


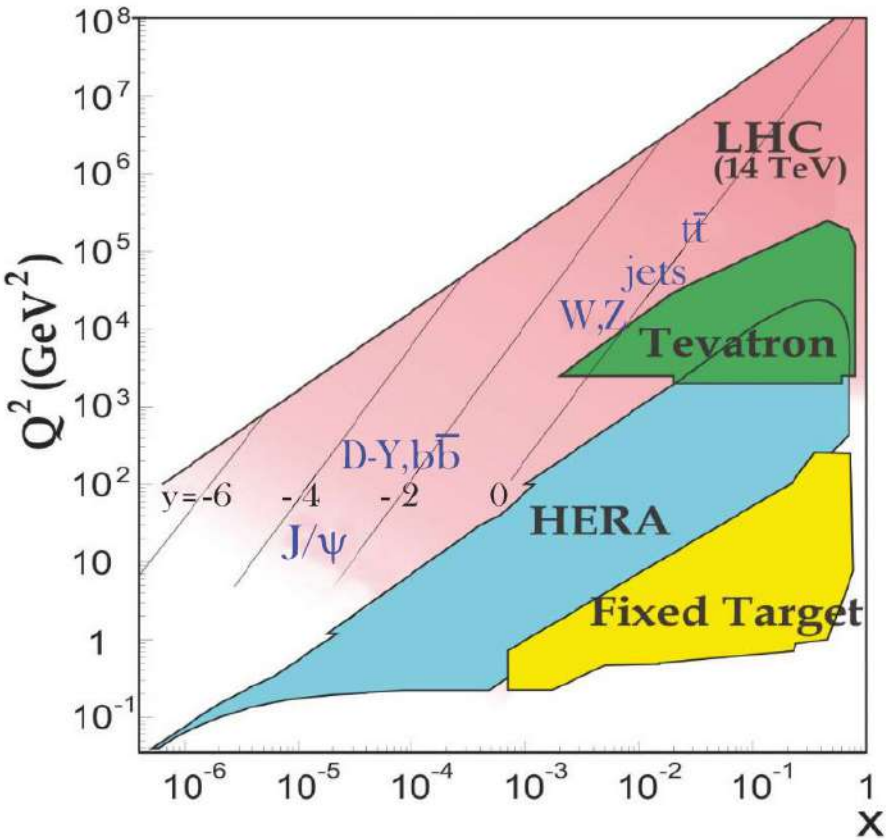
To Do List:

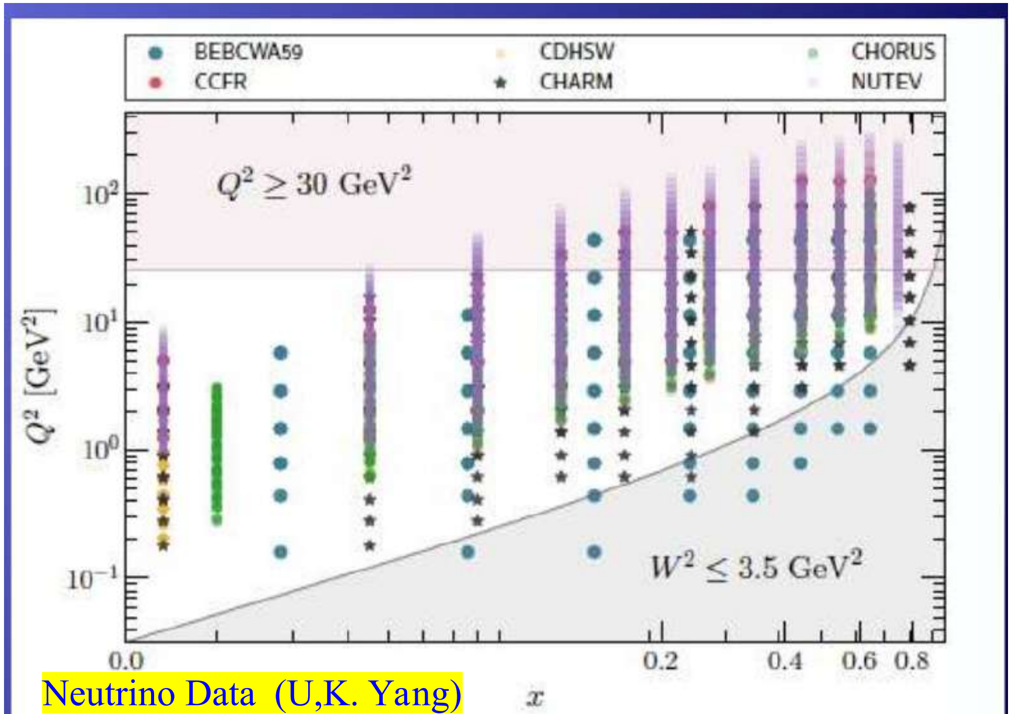
… more data

… improved predictions

Higher Order Calculations: *... crossing mass thresholds*

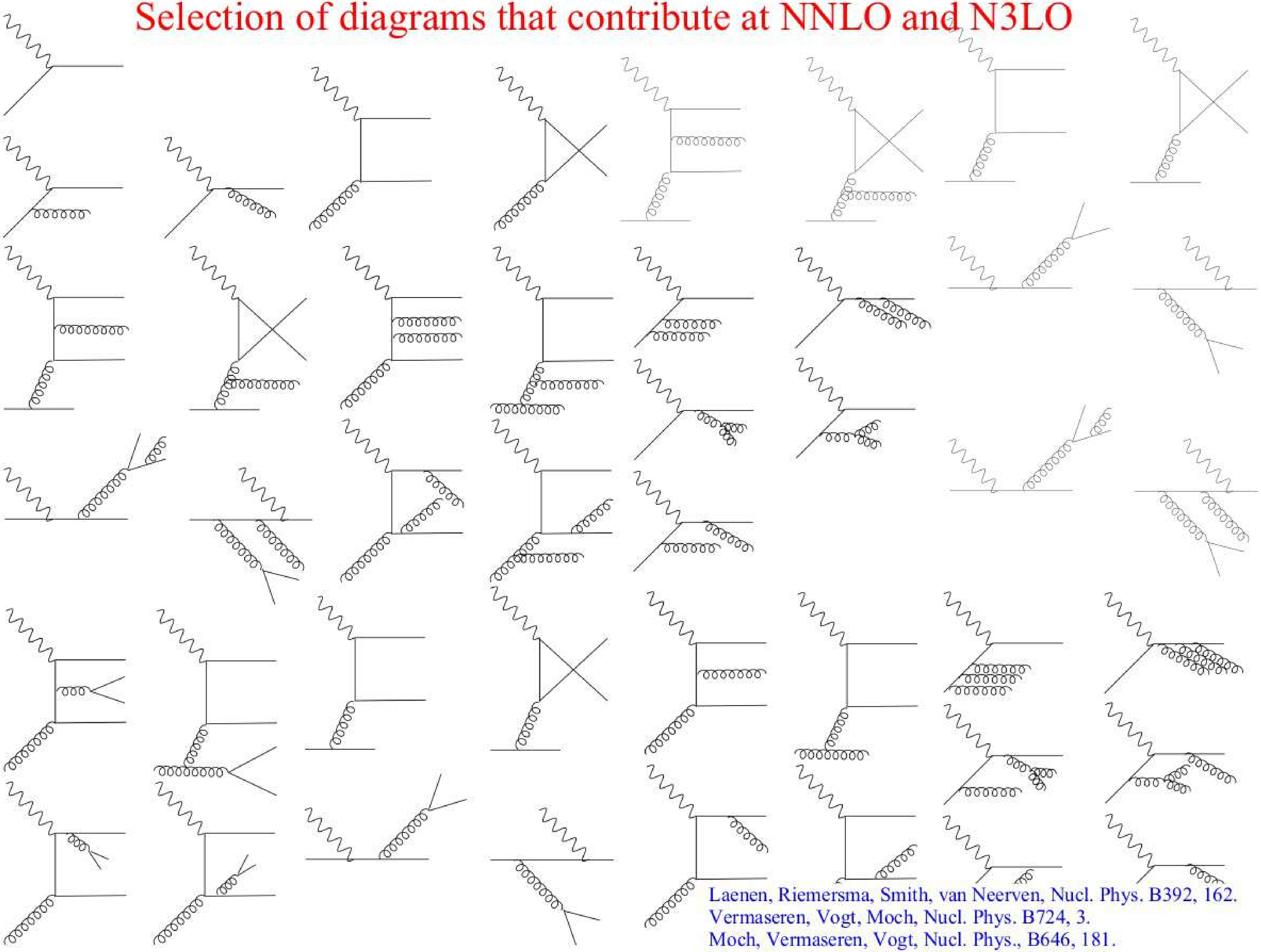






Neutrino Data (U,K. Yang)

Selection of diagrams that contribute at NNLO and N3LO



Laenen, Riemersma, Smith, van Neerven, Nucl. Phys. B392, 162.
Vermaseren, Vogt, Moch, Nucl. Phys. B724, 3.
Moch, Vermaseren, Vogt, Nucl. Phys., B646, 181.
Moch, Vermaseren, Vogt, Phys. Lett., B606, 123.
Blumlein, Hasselhuhn, Kovacikova, Moch, Phys.Lett. B700, (2011) 294.

Example #2: Multi-Scale Problems are Challenging

Two-Loop Total Cross Section: One Scale

$$\sigma(Q^2)=\sigma_0\left\{1+\frac{\alpha_s(Q^2)}{4\pi}(3C_F)+\left[\frac{\alpha_s(Q^2)}{4\pi}\right]^2\left[-C_F^2\left[\frac{3}{2}\right]+C_FC_A\left[\frac{123}{2}-44\zeta(3)\right]+C_FTn_f(-22+16\zeta(3))\right]\right.$$

Two-Loop Drell-Yan Cross Section: Two Scales

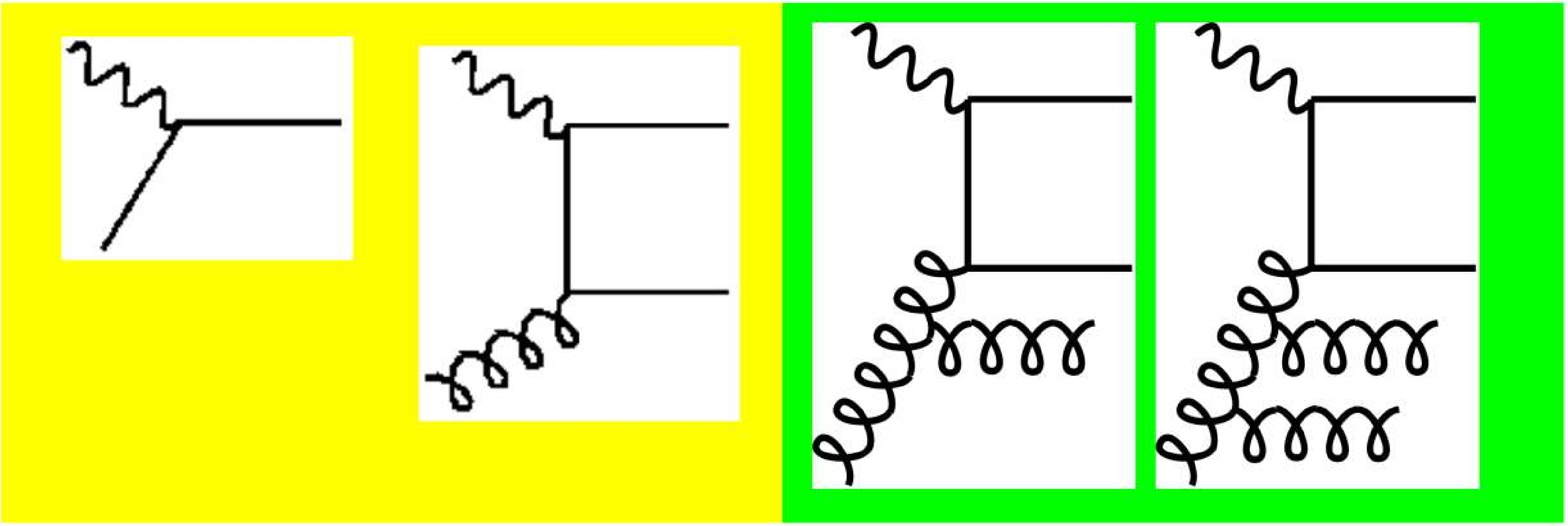
$$\begin{aligned}
H_{q\bar{q}}^{(2),S+V}(z)=&\left[\frac{\alpha_s}{4\pi}\right]^2\delta(1-z)\left\{C_AC_F\left[[\tfrac{193}{3}-24\zeta(3)]\ln\left[\frac{Q^2}{M^2}\right]-11\ln^2\left[\frac{Q^2}{M^2}\right]-\tfrac{12}{5}\zeta(2)^2+\tfrac{592}{9}\zeta(2)+28\zeta(3)-\tfrac{1535}{12}\right]\right.\\
&+C_F^2\left[[18-32\zeta(2)]\ln^2\left[\frac{Q^2}{M^2}\right]+[24\zeta(2)+176\zeta(3)-93]\ln\left[\frac{Q^2}{M^2}\right]\right.\\
&\left.+\tfrac{8}{5}\zeta(2)^2-70\zeta(2)-60\zeta(3)+\tfrac{511}{4}\right]\\
&\left.+n_fC_F\left[2\ln^2\left[\frac{Q^2}{M^2}\right]-\tfrac{34}{3}\ln\left[\frac{Q^2}{M^2}\right]+8\zeta(3)-\tfrac{112}{9}\zeta(2)+\tfrac{127}{6}\right]\right\}\\
&+C_AC_F\left[-\tfrac{44}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right]+\{[\tfrac{536}{9}-16\zeta(2)]\mathcal{D}_0(z)-\tfrac{176}{3}\mathcal{D}_1(z)\}\ln\left[\frac{Q^2}{M^2}\right]\right.\\
&\left.-\tfrac{176}{3}\mathcal{D}_2(z)+[\tfrac{1072}{9}-32\zeta(2)]\mathcal{D}_1(z)+[56\zeta(3)+\tfrac{176}{3}\zeta(2)-\tfrac{1616}{27}]\mathcal{D}_0(z)\right]\\
&+C_F^2\left[[64\mathcal{D}_1(z)+48\mathcal{D}_0(z)]\ln^2\left[\frac{Q^2}{M^2}\right]+\{192\mathcal{D}_2(z)+96\mathcal{D}_1(z)-[128+64\zeta(2)]\mathcal{D}_0(z)\}\ln\left[\frac{Q^2}{M^2}\right]\right.\\
&\left.+128\mathcal{D}_3(z)-(128\zeta(2)+256)\mathcal{D}_1(z)+256\zeta(3)\mathcal{D}_0(z)\right]\\
&+n_fC_F\left[\tfrac{8}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right]+[\tfrac{32}{3}\mathcal{D}_1(z)-\tfrac{80}{9}\mathcal{D}_0(z)]\ln\left[\frac{Q^2}{M^2}\right]+\tfrac{32}{3}\mathcal{D}_2(z)-\tfrac{160}{9}\mathcal{D}_1(z)+[\tfrac{224}{27}-\tfrac{32}{3}\zeta(2)]\mathcal{D}_0(z)\right].
\end{aligned}$$

Ref: CTEQ Handbook


Approximation
Tricks

ACOT Extension to Higher Orders

LO **NLO** **N2LO** **N3LO**



Full ACOT *Extensible to any order*

$$\sigma = f(\xi(x, m_{ps}), Q) \otimes \hat{\sigma}(m_{dyn})$$

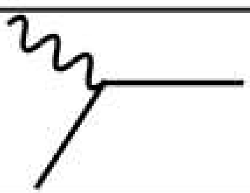
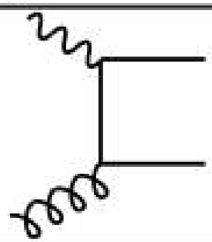
$$\xi(x, m_{ps}) = x \left(1 + \left[\frac{n\, m_{ps}}{Q} \right]^2 \right)$$

$$n = \{0, 1, 2\}$$

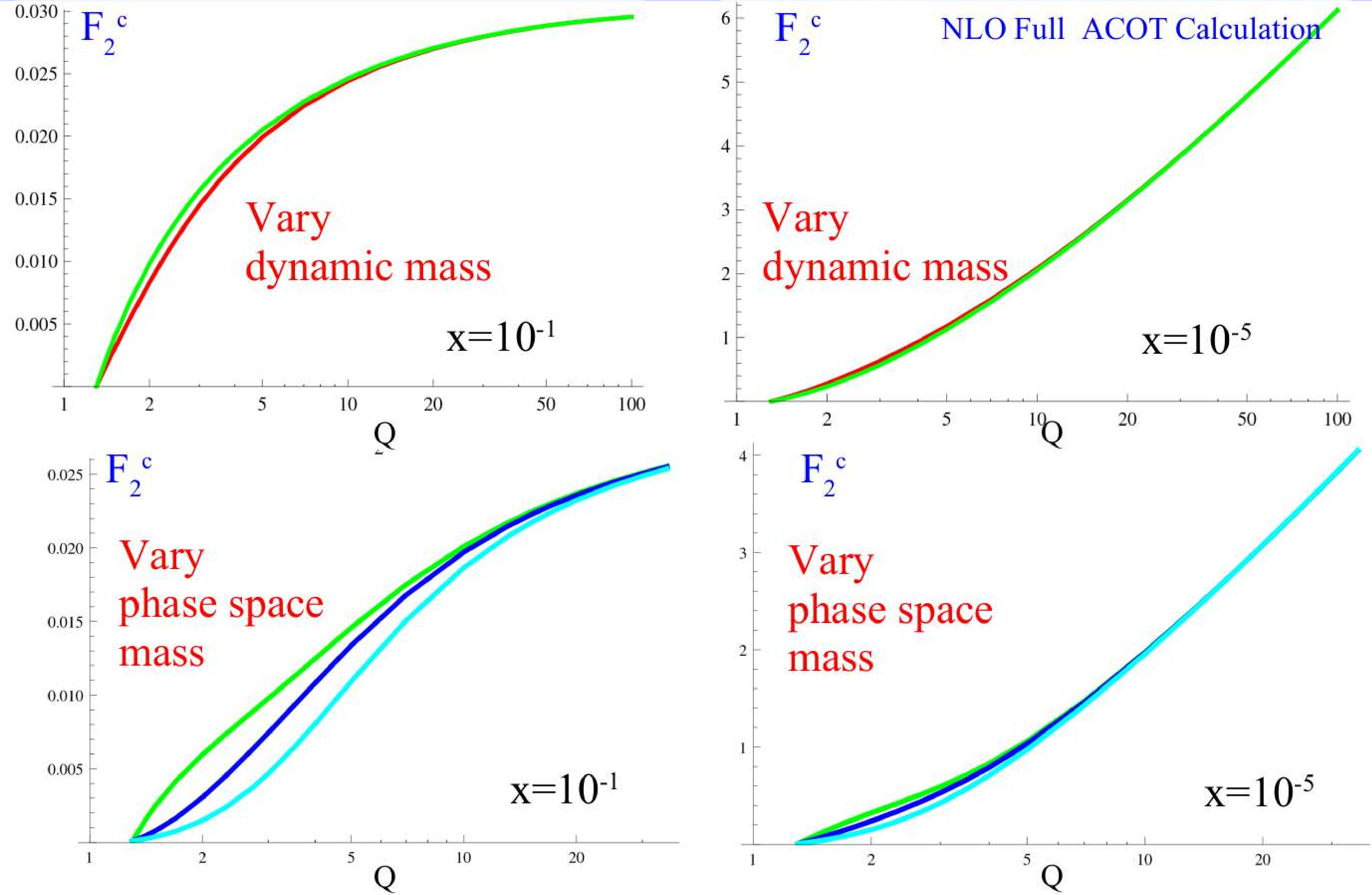
Distinguish:
"phase space" mass
"dynamic" mass

Demonstrate:
1) PS mass dominates
2) Estimated Error small

Approximation Tricks

ξ	General	$m_1 = m_2 = m$
	$\eta\left[\frac{Q^2-m_1^2+m_2^2+\Delta[-Q^2,m_1^2,m_2^2]}{2Q^2}\right]$	$\eta\left[1+\frac{m^2}{Q^2}\right]$
	$\eta\left[1+\left(\frac{m_1+m_2}{Q}\right)^2\right]$	$\eta\left[1+\frac{(2m)^2}{Q^2}\right]$

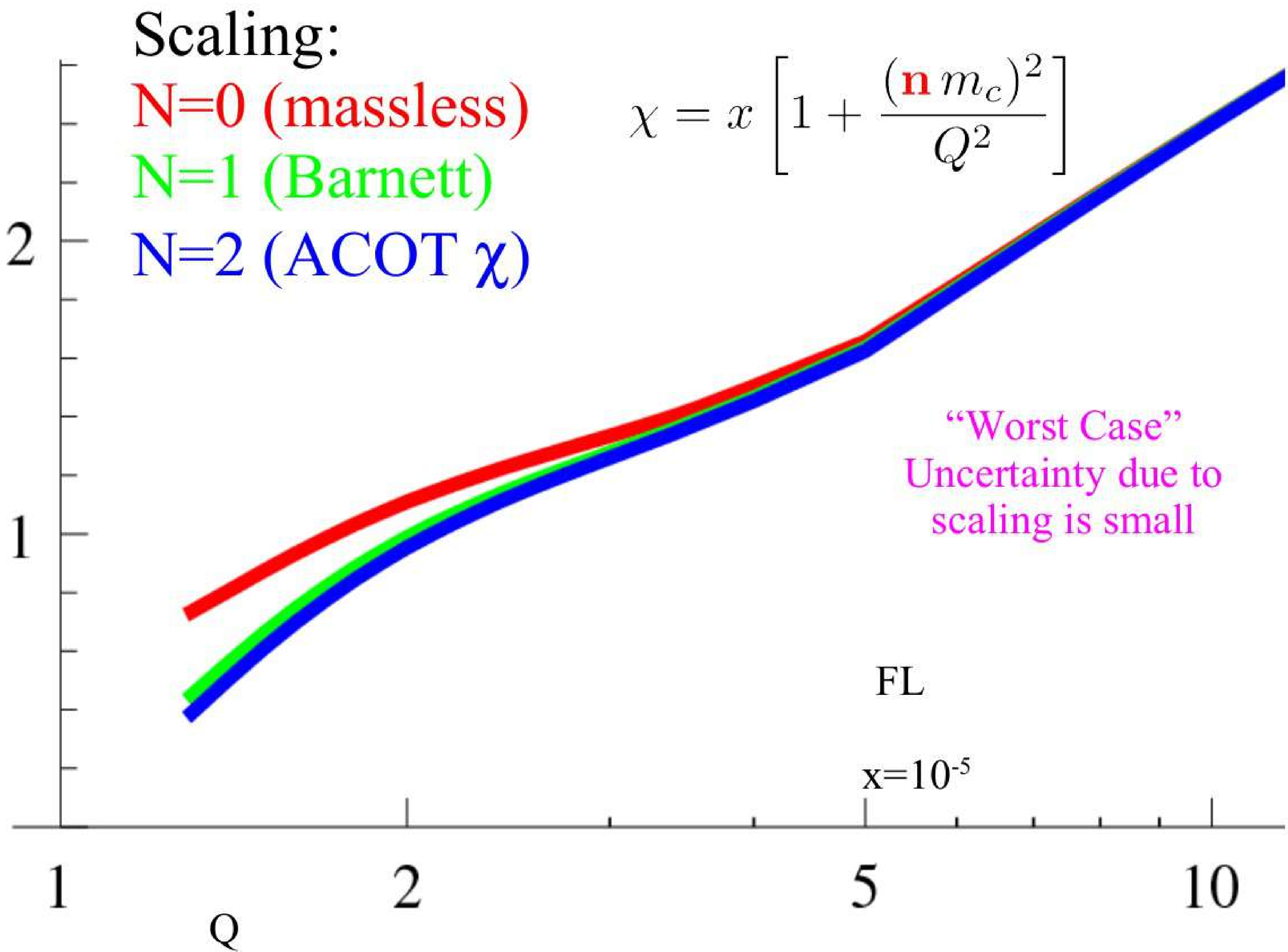
Identify Two Types of Mass Depencence: "dynamic" & "phase space"



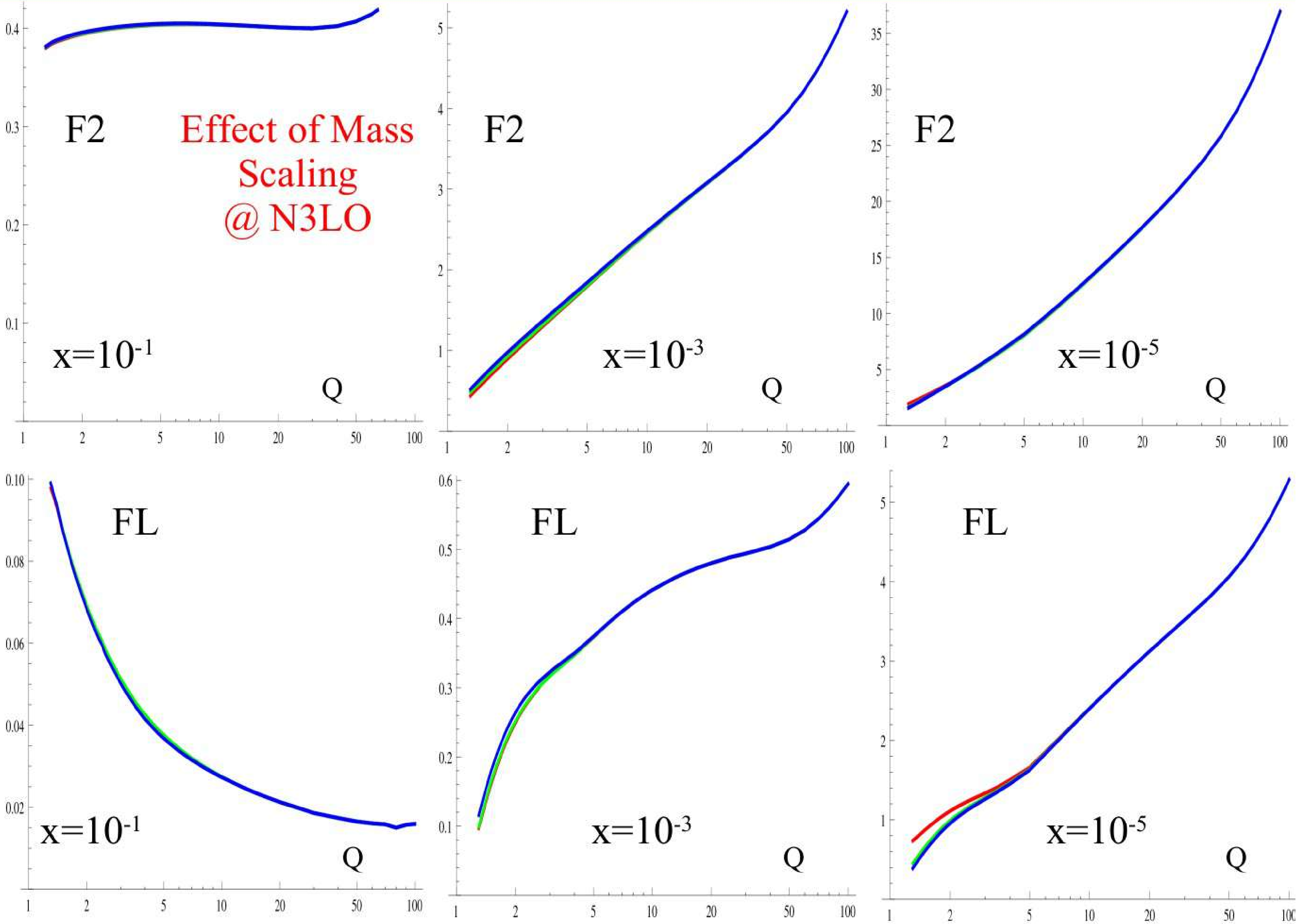
"phase space" mass yields larger variation. Not a proof, but ….

How can we estimate the uncertainty

EFFECT OF MASS SCALING @ N3LO (Phase Space Mass)

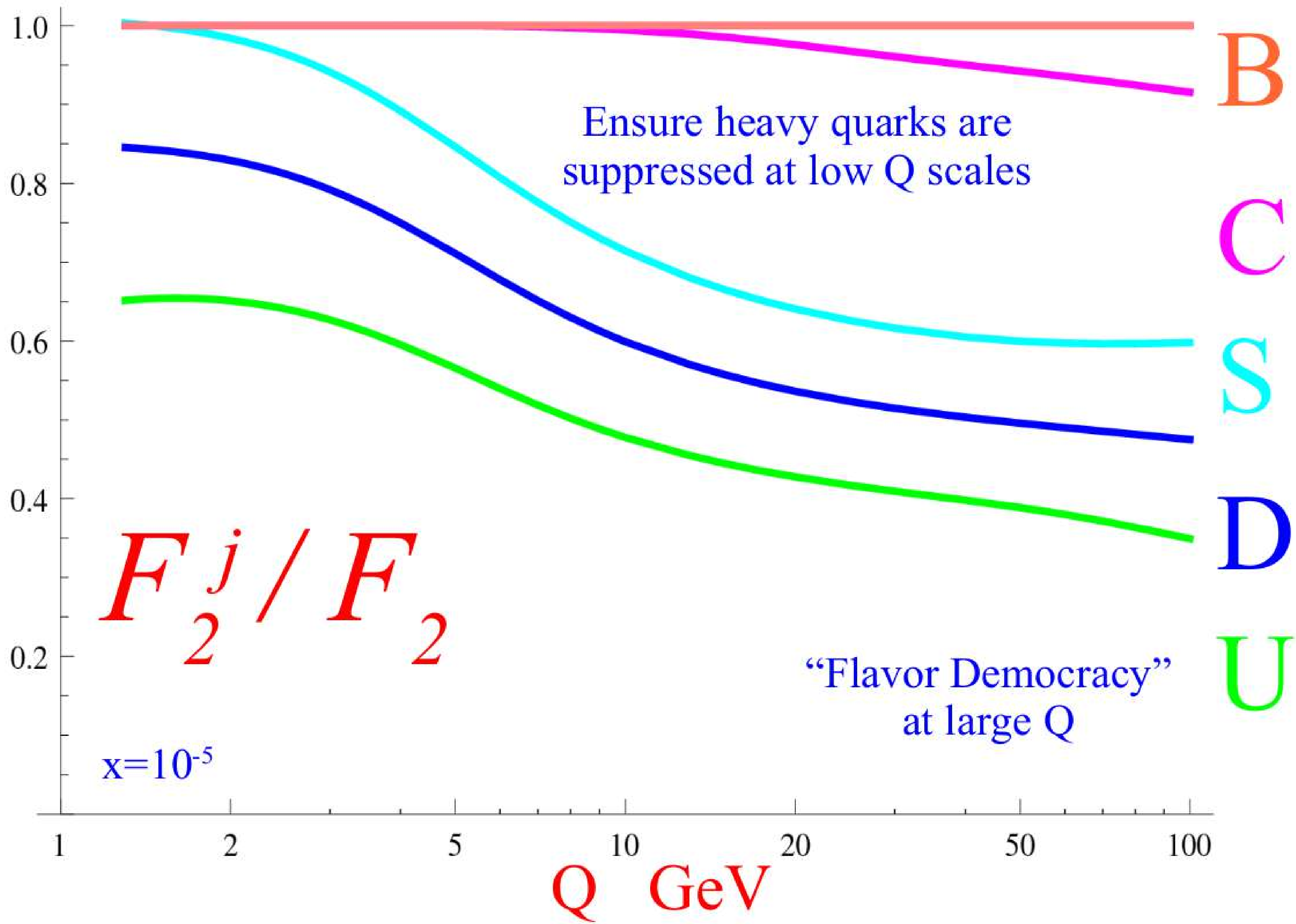


EFFECT OF MASS SCALING @ N3LO (Phase Space Mass)

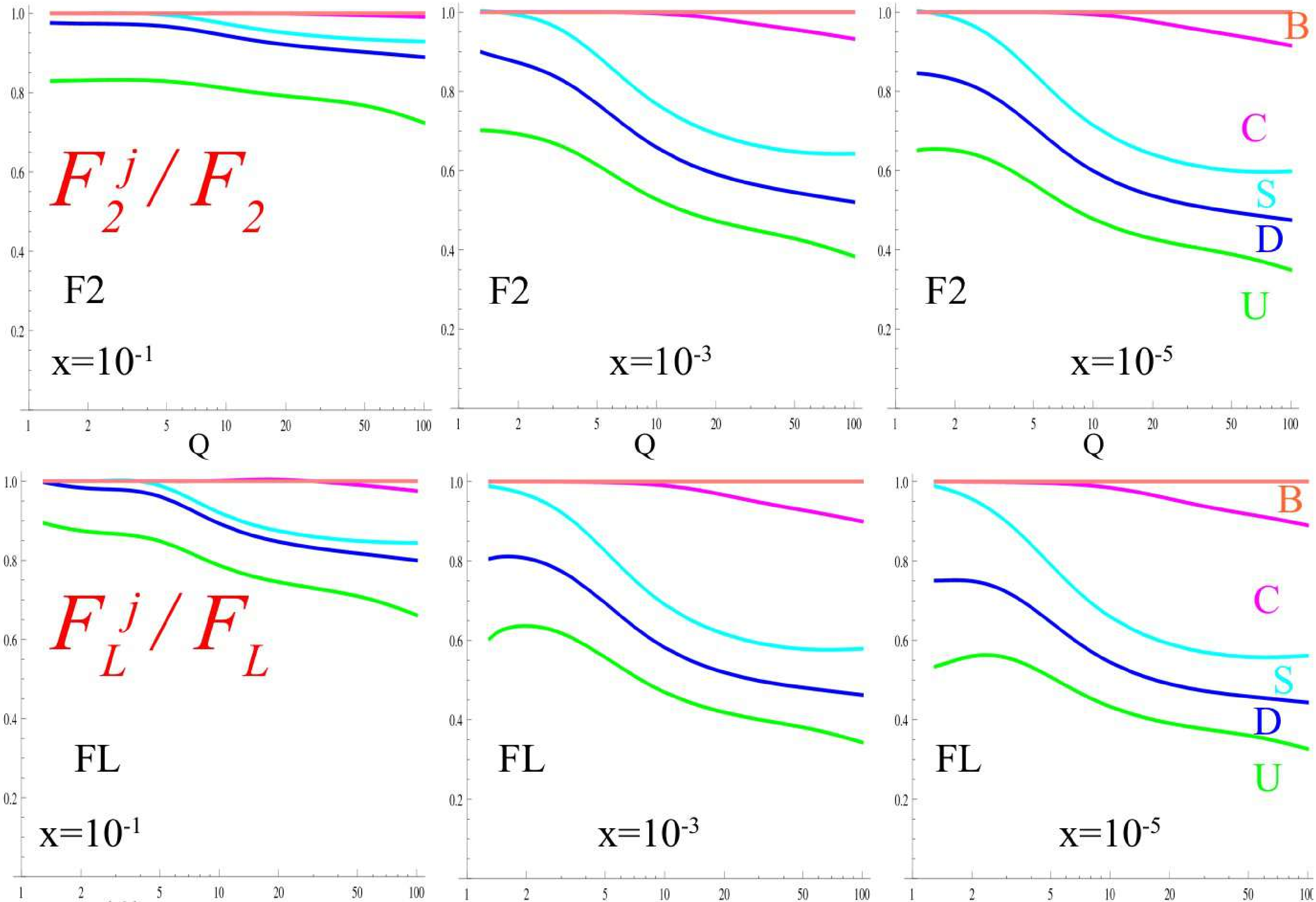


What about flavor thresholds

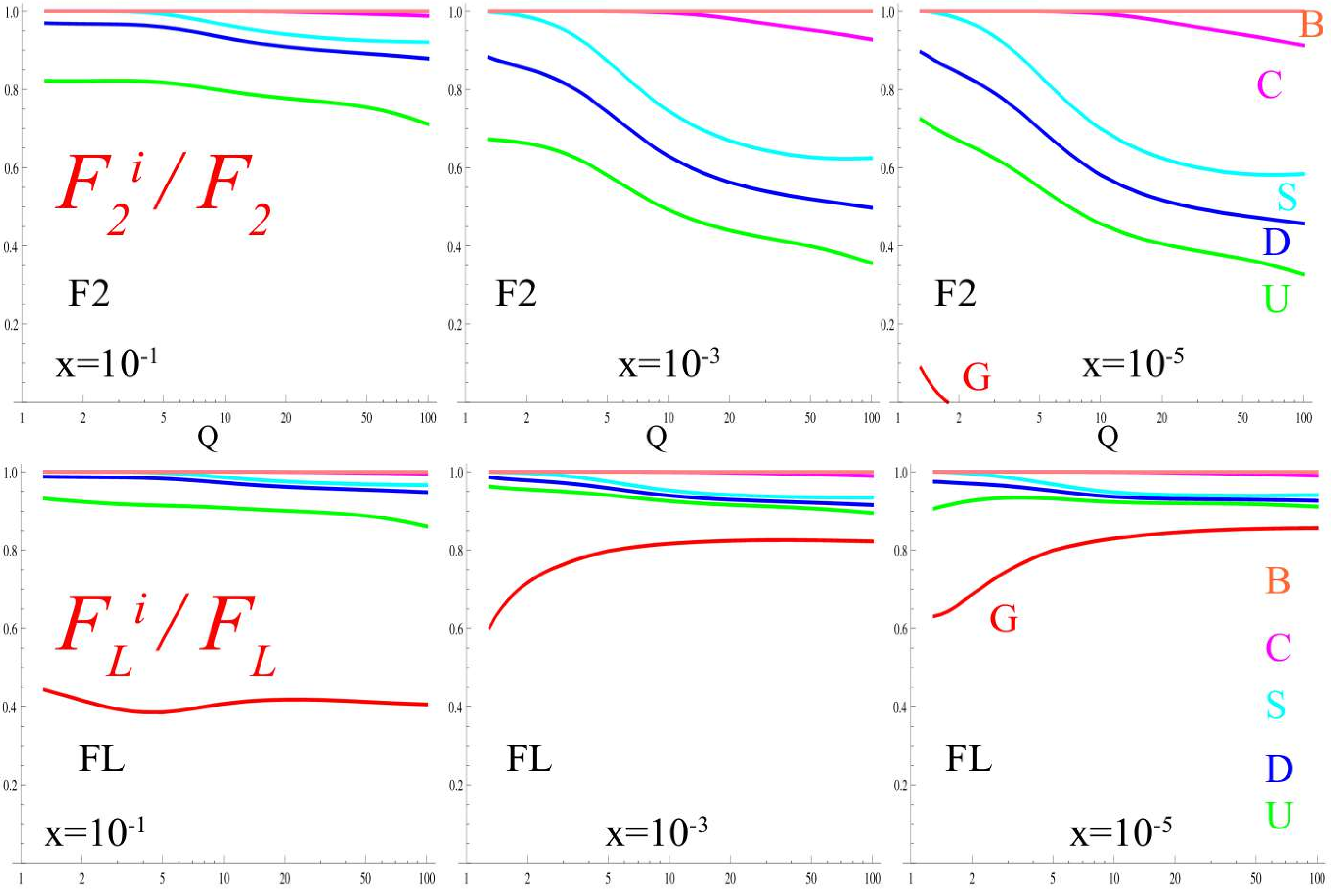
FLAVOR DECOMPOSITION



FLAVOR DECOMPOSITION: Final State Quark:

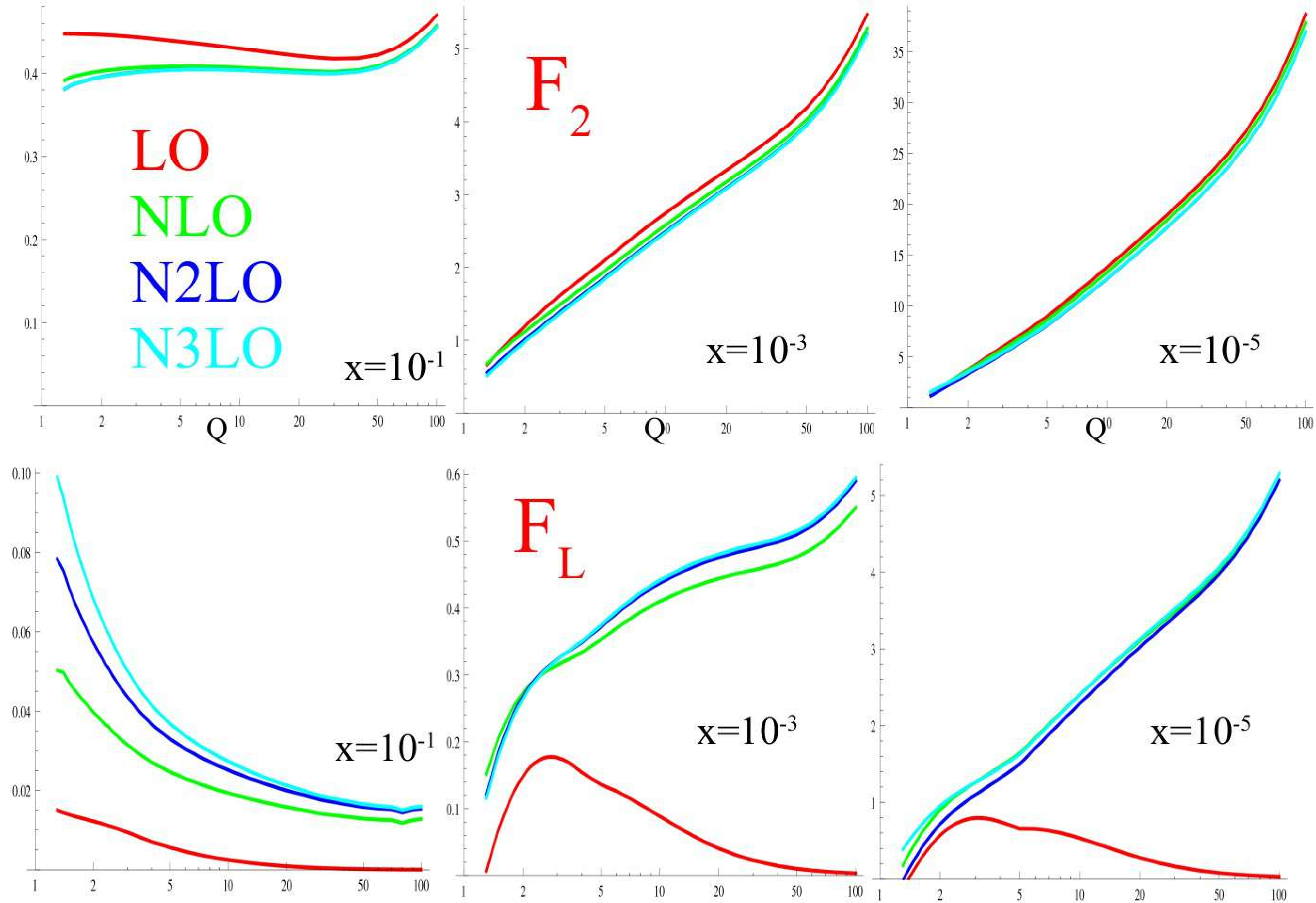


FLAVOR DECOMPOSITION: Initial State Quark:



Magnitude of
Higer Order Corrections

$F_{2,L}$ @ N3LO



Implementations

Neutral Current S-ACOT χ implemented at N2LO and N3LO

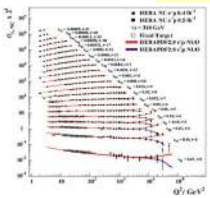
xFitter

PROTON NUCLEON MESON

www.xFitter.org

Sample data files:
- **LHC:** ATLAS, CMS, LHCb
- **Tevatron:** CDF, D0
- **HERA:** H1, ZEUS, Combined
- **Fixed Target:** …
- **User Supplied:** ...

Experimental Data



Data: HERA, Tevatron, LHC, fixed target experiments

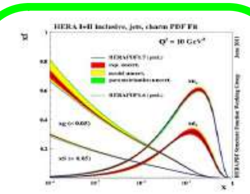
Processes: Inclusive DIS, Jets, Drell-Yan, Diffraction, Top production W and Z production

χ

Theory Calculations

HQ Schemes:	MSTW, NNPDF, ABM, ACOT
Jets, W, Z:	FastNLO, ApplGrid
Top:	Hathor
Evolution:	QCDNUM, APFEL, k_T
Other:	NNPDF reweighting TMDs, Dipole Model, ...

xFitter



Parton Distribution Functions: PDF, Updf, TMD

$\alpha_S(M_Z)$, m_c, m_b, m_t ...

Theoretical Cross Sections

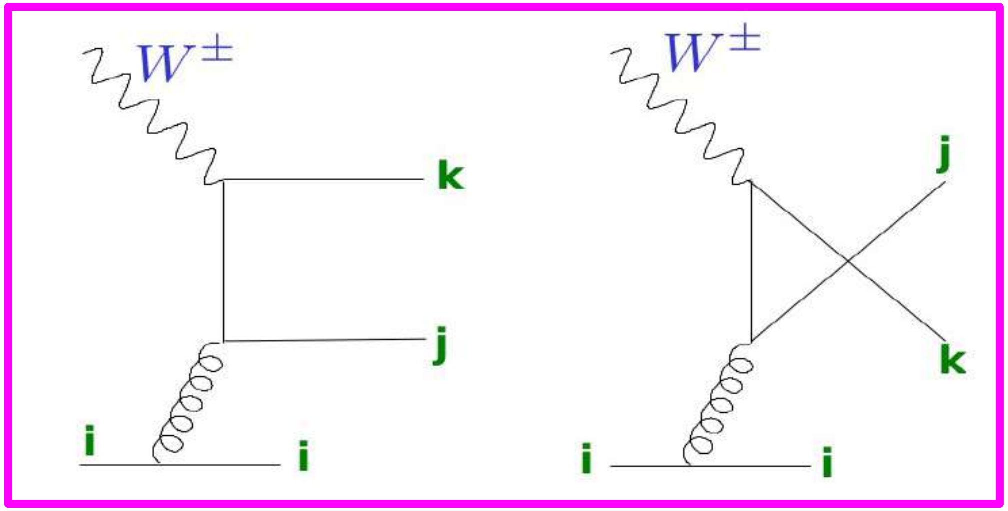
Comparisons to other PDFs (LHAPDF)

extensions include nuclear PDFs

Features & Recent Updates:
- **NNLO DGLAP**
- Photon PDF & **QED**
- Pole & MS-bar masses
- Profiling and Re-Weighting
- **BFKL interface**
- Heavy Quark Variable Treshold
- Improvements in χ^2 and correlations
- **TMD** PDFs (uPDFs)
- *... and many other*

xFitter 2.2.0 Future Freeze

ACOT Charged Current to N2LO: … Impacts Neutrino Data

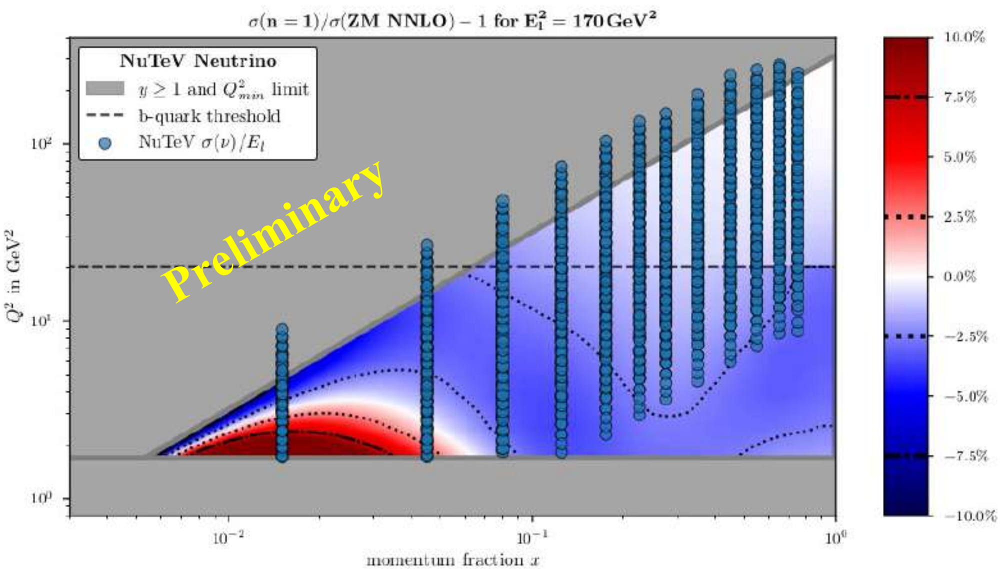


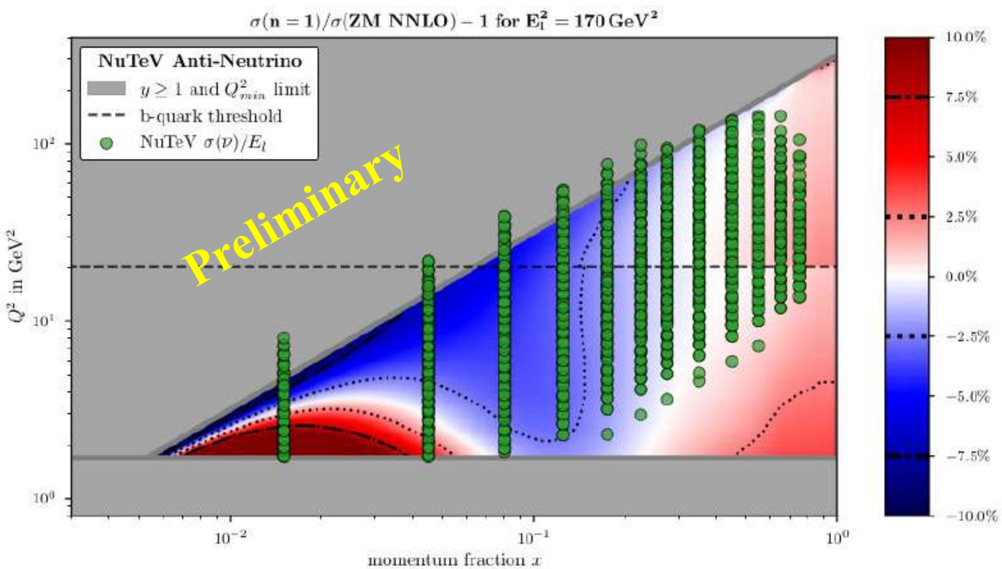
Fast evaluation of heavy-quark contributions to DIS in APFEL++

P. Risse[a,†], V. Bertone[b], T. Ježo[a], M. Klasen[a], K. Kovařík[a], F.I. Olness[c], I. Schienbein[d]

Peter Risse (Muenster)

arXiv:2307.08269v1





APFEL++ – A PDF evolution library in c++

Valerio Bertone

Peter Risse

APFEL++

Bertone, arXiv:1708.00911

Available schemes in APFEL++

scheme	$\mathcal{O}(\alpha_s)$		NC: F_2	NC: F_3	NC: F_L		CC: F_2	CC: F_3	CC: F_L
ZM	N2LO		✓	✓	✓		✓	✓	✓
FONLL-C	N2LO		✓	✗	✓		✗	✗	✗
ACOT	NLO		✓	✓	✓		✗	✗	✗
sACOT-χ	NLO		✓	✓	✓		✓	✓	✓
approx. sACOT-χ	N2LO		✓	✓	✓		✓	✓	✓

nCTEQ++ … a complete rewrite in C++

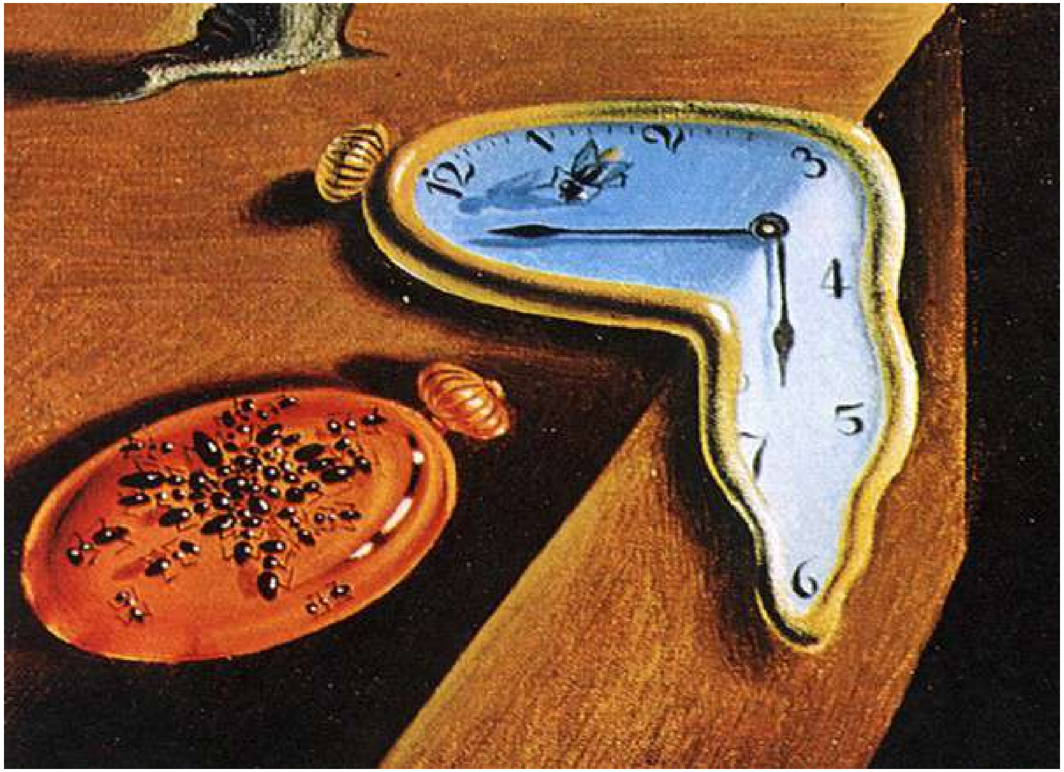
Code benchmark timings:

Original Fortran Code

contains multiple levels of integrals

New C++ Code

using modern grid techniques



Typical fits current run a few days to a week.
This will be reduced to a few hours.

High order DIS processes
(Peter Risse)

Precision, Precision... but systematic effect: theory and experiment

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Conclusions

Multi-scale problems are hard: **Thank you to those computing these results**

Proper mass treatment: essential to fit PDFs over large Q scales

Many outstanding issues related to neutrino DIS analysis

Improved calculations can help

Approximate S-ACOT-χ:

leverages N2LO and N3LO results

Neutral Current:

N2LO and N3LO available in xFitter (no grids)

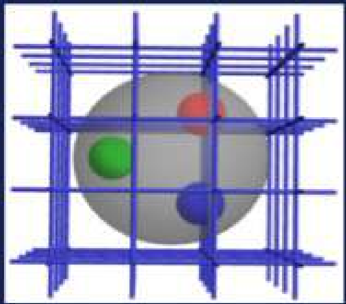
Preliminary

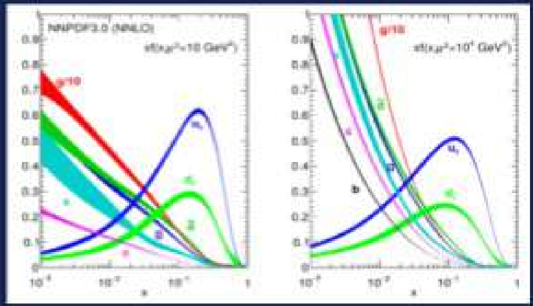
Results with APFEL++ Grids:

<u>BOTH</u> Charged Current & Neutral Current results

Speed increase of ~100×

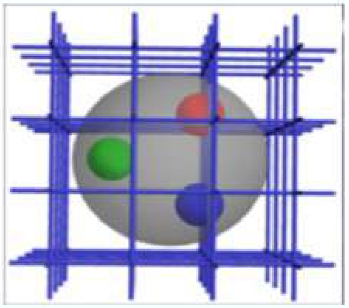
Peter Risse
Valerio Bertone



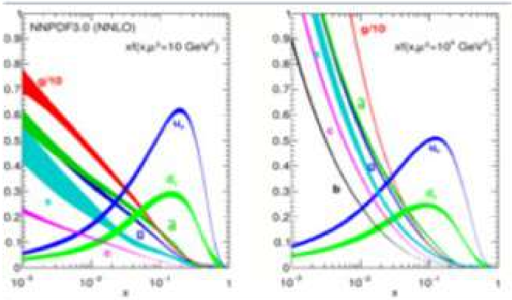


Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

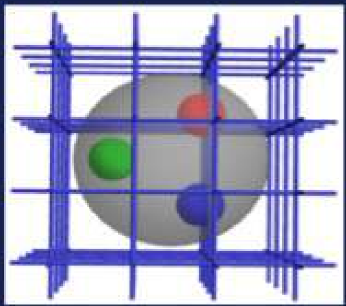
22-24 March 2017, Oxford, UK



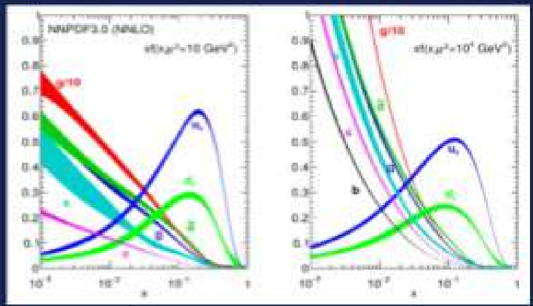
W. K . Kellogg Biological Station
MICHIGAN STATE UNIVERSITY



Parton Distributions and Lattice Calculations (PDFLattice 2019)



Jefferson Lab



Parton Distributions and Lattice Calculations (PDF Lattice 2024)

18-20 November 2024

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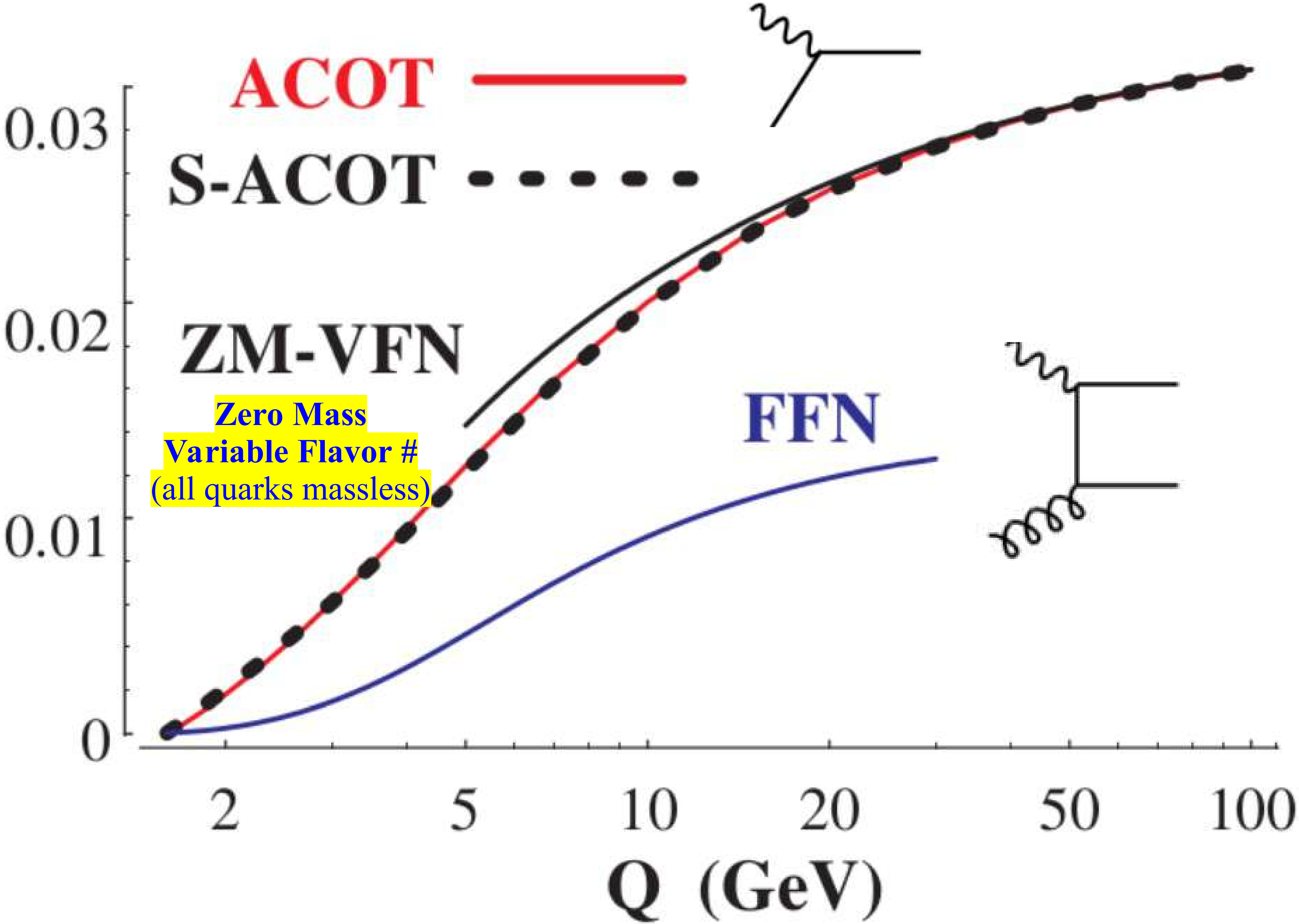
Speed increase of ~100×

Peter Risse
Valerio Bertone

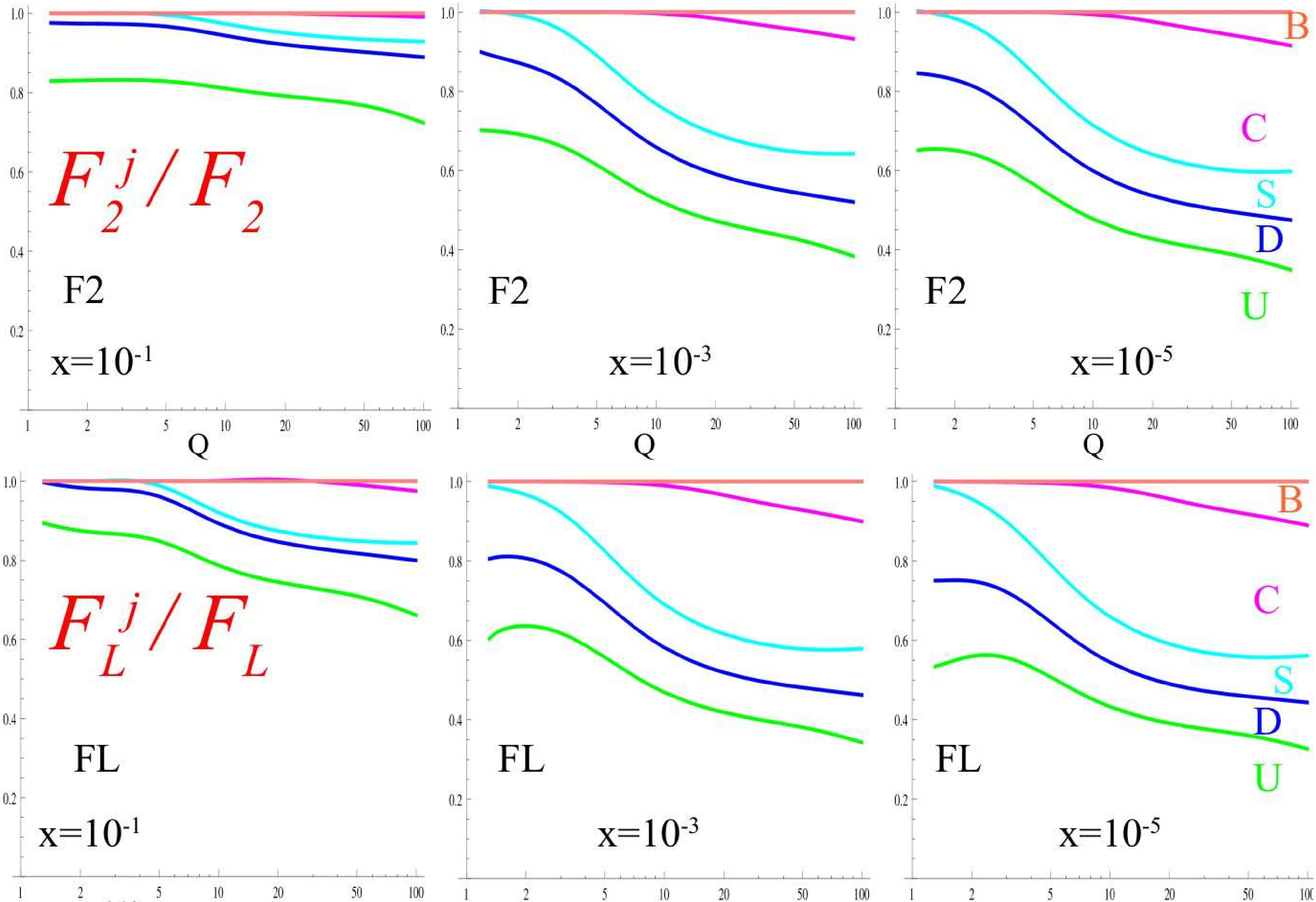
Neutrinos:

EXTRAS

Transition: Low to High Scales



FLAVOR DECOMPOSITION: Final State Quark:

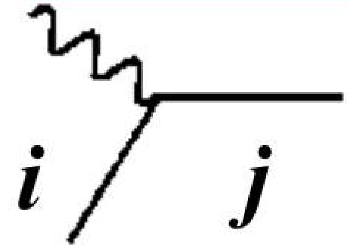


Master formula for decomposing the flavor components

T.P. Stavreva, I Schienbein

$$F = \sum_{i,j}^{6} F^{ij}$$

The Goal: Convert from {s, ns, ps} to {q,g, ...}

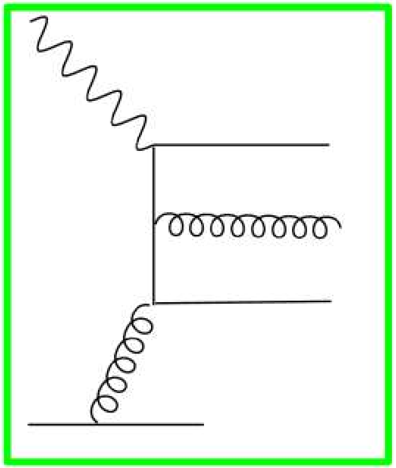


$$x^{-1} F_a^{ij} = q_i^+ \otimes \Big\{ e_i^2 \Big[C_{a,q}^{\rm ns}(n_f = 0)\ \delta_{ij} + C_{a,q}^{\rm ns}(j) - C_{a,q}^{\rm ns}(j-1) \Big] - \langle e^2 \rangle^{(j)}\ C_{a,q}^{\rm ps}(j) - \langle e^2 \rangle^{(j-1)}\ C_{a,q}^{\rm ps}(j-1) \Big\}$$

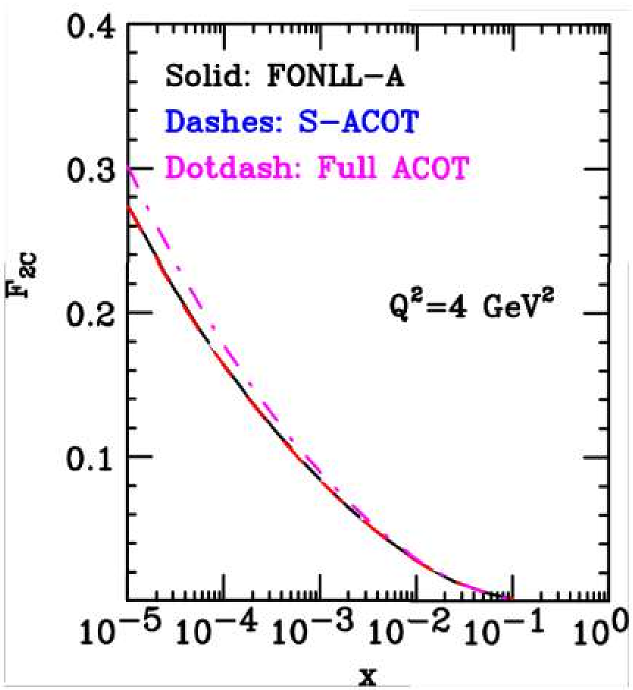
Issues: Flavor separation:

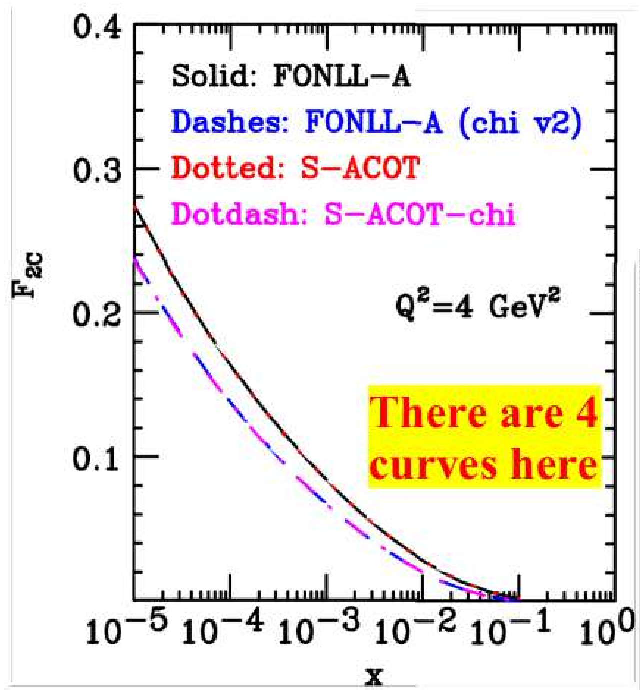
New diagrams at this order

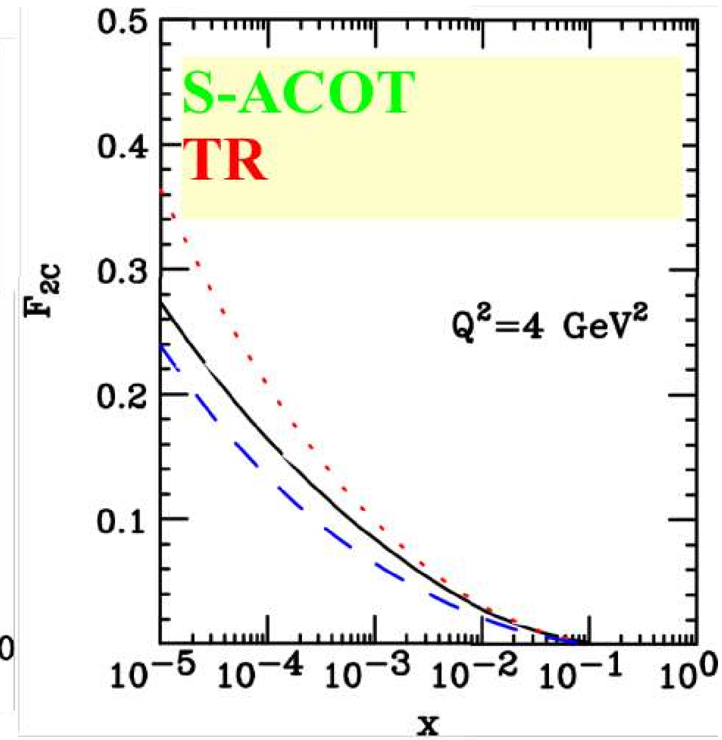
- c,b, goes down beam pipe
- both c & b in final state



Les Houches Comparative Study







ACOT & S-ACOT essentially identical

... scheme differences are higher order

FONNL & S-ACOT

Numerically similar

MSTW09

We can quantify <u>theoretical</u> scheme diffferences

The SM and NLO Multileg Working Group: Summary report.
J. Rojo, et al., e-Print: arXiv:1003.1241 [hep-ph]

Differences understood
Should reduce at higher order

ACOT m→ 0 limit cross check with QCDNUM at NLO

```
PATCH FOR TESTING: HMASS=0=   0.938000023
  IHADRON: SET TO HADRON= 1
  GZ and ZZ are for testing
```

	x	q	F_2	F_L
print x,q, ratios:	0.00319999992	12.2474487	**1.00092636**	**1.0012981**
print x,q, ratios:	0.00499999989	12.2474487	**1.00098575**	**1.00126809**
print x,q, ratios:	0.00800000038	12.2474487	**1.00106943**	**1.00153596**
print x,q, ratios:	0.00319999992	14.1421356	1.00092542	1.00125357
print x,q, ratios:	0.00499999989	14.1421356	1.00097202	1.00121532
print x,q, ratios:	0.00800000038	14.1421356	1.00104411	1.00146055
print x,q, ratios:	0.0130000003	14.1421356	1.00107382	1.0013549
print x,q, ratios:	0.0199999996	14.1421356	1.00114663	1.0014694
print x,q, ratios:	0.0320000015	14.1421356	1.00119237	1.00152525
print x,q, ratios:	0.0500000007	14.1421356	1.00117963	1.00131561
print x,q, ratios:	0.0799999982	14.1421356	1.00098036	1.00123239
print x,q, ratios:	0.00499999989	15.8113883	1.00095999	1.00117694
print x,q, ratios:	0.00800000038	15.8113883	1.0010229	1.00140587
print x,q, ratios:	0.0130000003	15.8113883	1.00104124	1.00130353
	0.0199999996	15.8113883	1.00110599	1.00140934
	0.0320000015	15.8113883	1.00114419	1.00146259
	0.0500000007	15.8113883	1.00113621	1.00125726
	0.0799999982	15.8113883	1.00095108	1.0011996
	0.129999995	15.8113883	1.00055001	1.00103563
print x,q, ratios:	0.25	15.8113883	0.99929117	1.0000816
print x,q, ratios:	0.400000006	15.8113883	0.997267345	0.998376607
print x,q, ratios:	0.00499999989	17.3205081	1.00094852	1.00114569
print x,q, ratios:	0.00800000038	17.3205081	1.00100525	1.00136309
print x,q, ratios:	0.0130000003	17.3205081	1.00101481	1.00118502
print x,q, ratios:	0.0199999996	17.3205081	1.00107357	1.00136459
print x,q, ratios:	0.0320000015	17.3205081	1.00110601	1.00140262

NLO Check with QCDNUM

~1E-3