Improving α_s extractions from collider data

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LoopFest XXII

Based on:

- 2306.08033 [e^+e^-, Heavy Jet Mass]:
  with Arindam Bhattacharya [Harvard], Johannes K.L. Michel [Amsterdam], Matthew D. Schwartz [Harvard] and Iain W. Stewart [MIT]
- 2307.07510 [$pp \to jj$, Energy Correlators]:
  with Wen Chen [South China Normal], Jun Gao [Shanghai Jiao Tong], Yibei Li [Zhejiang], Zhen Xu [Zhejiang] and HuaXing Zhu [Peking]

Outline

- Motivation
- Part I: Improving the perturbative prediction of heavy jet mass
    - Sudakov shoulder
    - Factorization theorem
    - Position-space resummation
    - NNLL shoulder resummation
- Part II: α_s measurements from $pp \to jj$ using three-point energy correlator
    - Collinear factorization
    - Two-loop jet functions
    - NNLL resummation for e^+e^-
    - NNLL resummation for $pp \to jj$

Motivation

- The precise measurement of α_s is one of the most important problems in QCD

Asymptotic freedom: (50+1) years of QCD

[Gross, Wilczek, Politzer, 1973]

$$\frac{d\alpha_s(\mu)}{d\ln\mu} = \beta\left(\alpha_s(\mu)\right), \quad \beta(\alpha) = -2\alpha\left[\left(\frac{\alpha_s}{4\pi}\right)\beta_0 + \cdots\right]$$

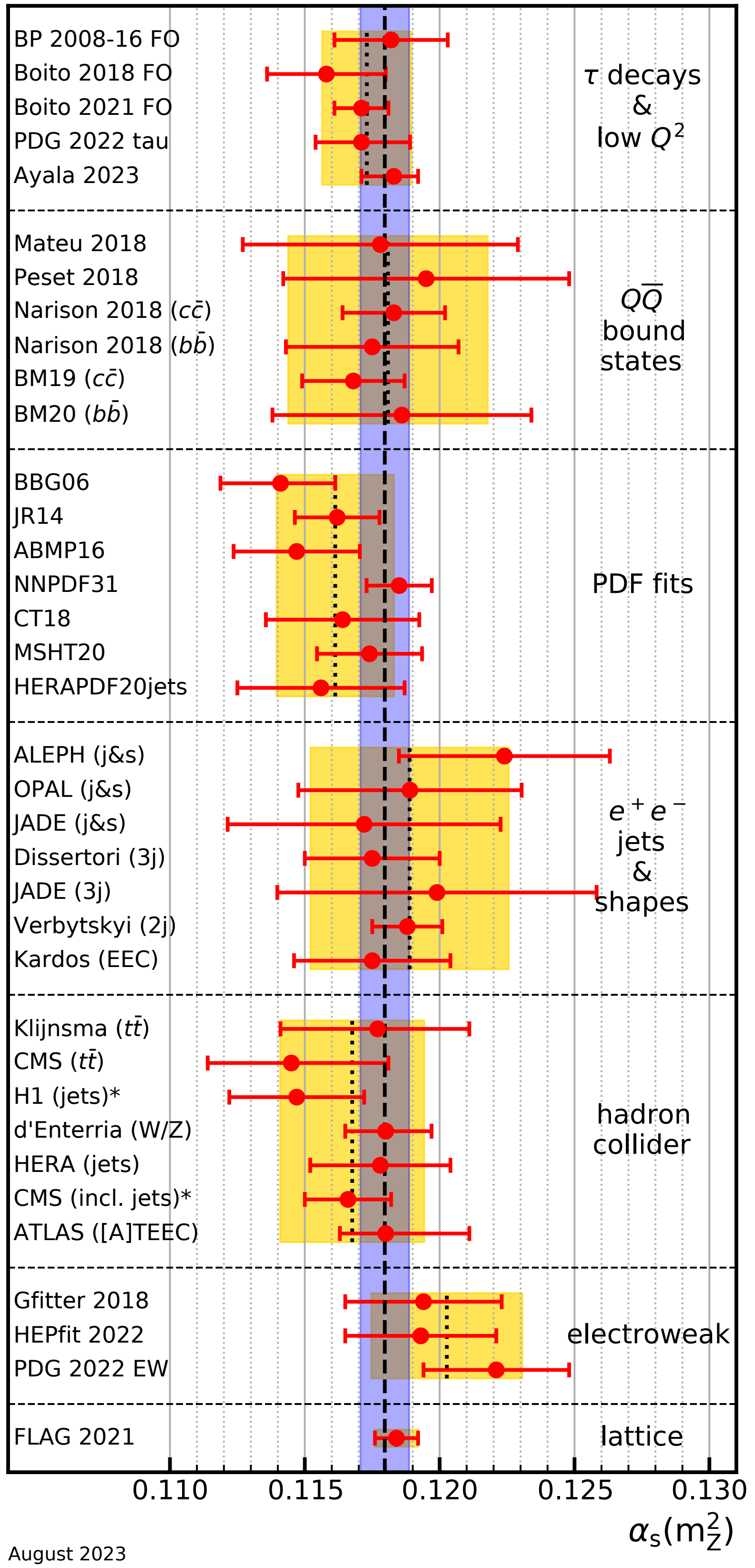
$$\Rightarrow \alpha_s(\mu) = \alpha_s(\mu_0)\left[1 + \frac{\alpha_s(\mu_0)}{2\pi}\beta_0 \ln\frac{\mu}{\mu_0}\right]^{-1}$$

- PDG 2023 average: $\alpha_s(m_Z) = 0.1180 \pm 0.0009$

- Discrepancy between different measurements:

PDF fit:	0.1161 ± 0.0022
e^+e^- collision:	0.1189 ± 0.0037
Hadron collision:	0.1168 ± 0.0027
Lattice FLAG:	0.1184 ± 0.0008

[Huston, Rabbertz, Zanderighi, PDG QCD review 2312.14015]



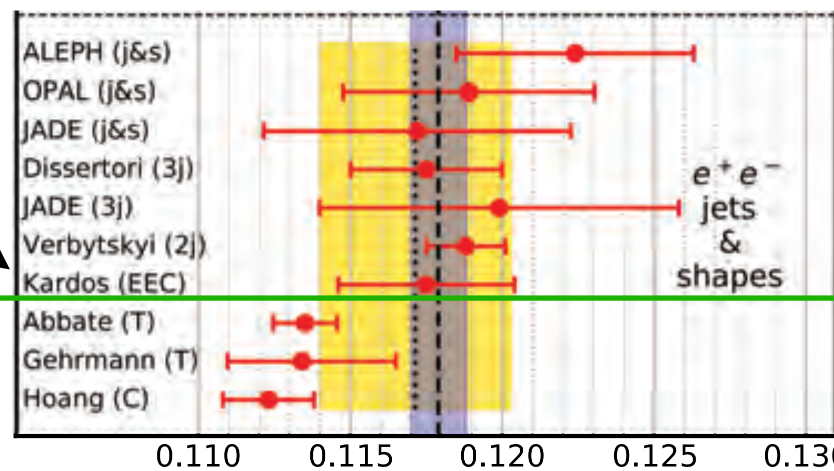
Motivation: e^+e^-

- For e^+e^- colliders, different treatments of power corrections lead to different result.

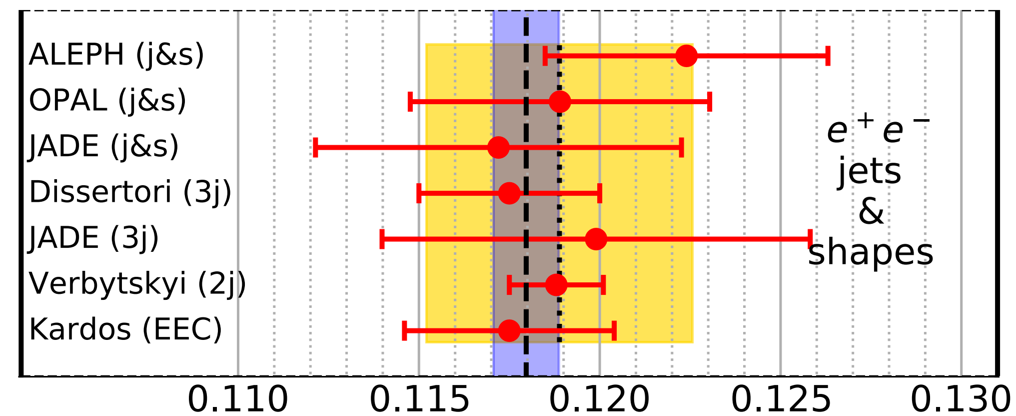
e^+e^- collision (2021): 0.1171 ± 0.0031

e^+e^- collision (2023): 0.1189 ± 0.0037

EEC: MC power corrections



Thrust/C parameter: theory power corrections



[Huston, Rabbertz, Zanderighi, PDG QCD review, 2312.14015]

the Durham algorithm. The key finding in Ref. [688] is that non-perturbative corrections computed in the three-jet region significantly deviate from those computed in the two-jet limit and hence the aforementioned fits based on power corrections in the two-jet limit result in smaller values of $\alpha_s(m_Z^2)$. Another important observation is that the inclusion of resummation effects introduces a relatively substantial ambiguity outside the two-jet limit. Additionally, other factors such as the choice of mass-scheme used to extend the definition of event shapes to massive hadrons can have significant effects.

These findings are inconsistent with the very small experimental, hadronization, and theoretical uncertainties of only 2, 5, and 9 per-mille, respectively, as reported in Refs. [683, 685]. For these reasons, we exclude the results of Refs. [683–685] from the average. Determinations based on

- However, thrust with only dijet fit range still give the same result with good stability

[Benitez-Rathgeb, Hoang, Mateu, Stewart, Vita, 2024]

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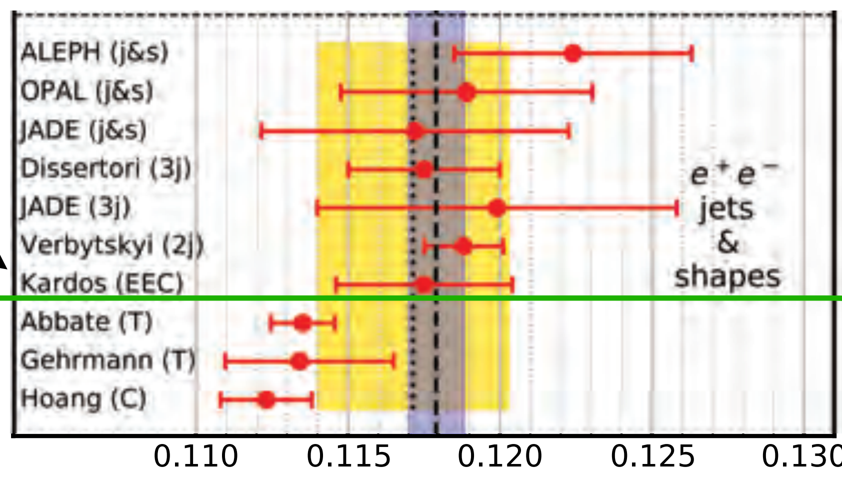
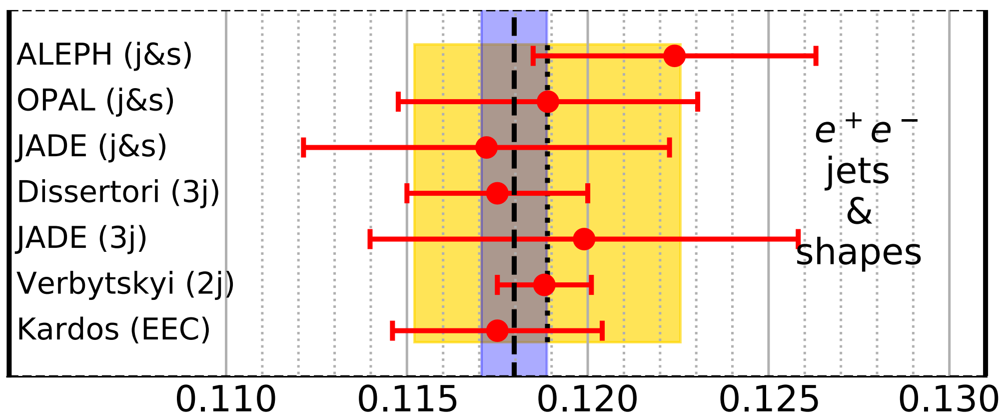


e^+e^- collision (2023): 0.1189 ± 0.0037



- Need a better understanding of power corrections (MC vs analytic models) and why the latter leads to a lower value of α_s
- There is also another observable, heavy jet mass, that
  - gives very low α_s when including the power corrections [Chien, Schwartz, 1005.1644]
  - has very different tail behavior compared to thrust

Motivation: pp

- For pp colliders, there are not many α_s data points with high precisions.
- Many jet (substructure) observables used for α_s determination at the LHC haven't reached NNLO accuracy.

$t\bar{t}$ production: $0.1145^{+0.0036}_{-0.0031}$

Transverse EEC: $0.1180^{+0.0031}_{-0.0017}$

[CMS collaboration, 1812.10505]

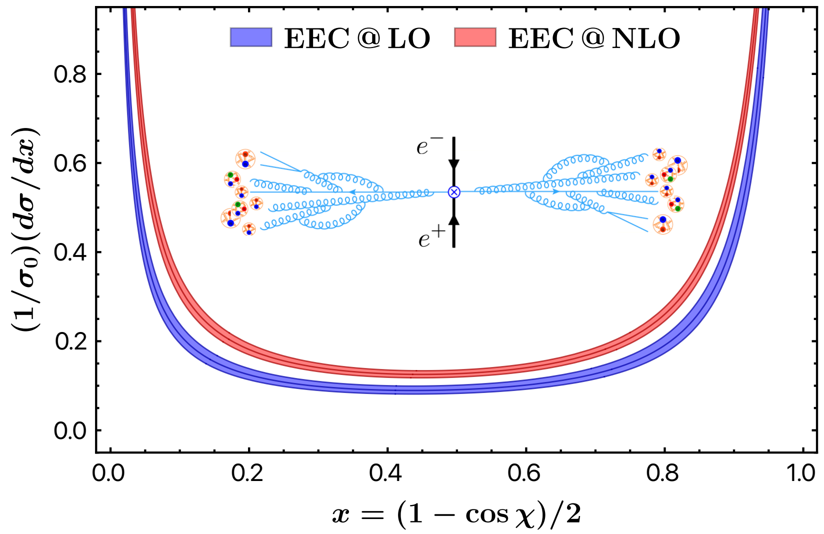
[ATLAS collaboration, 1508.01579, 1707.02562, 2301.09351]

- A competitive candidate in recent years: energy correlators
  - Energy-weighted cross section as a function of angles among any two detectors

Energy-energy correlation (2-point)

[Basham, Brown, Ellis, Love, 1978]

$$\mathrm{EEC}(\chi) = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi)$$



N-point energy correlator

[Chen, Luo, Moult, Yang, **XYZ**, Zhu, 1912.11050]

$$\frac{d\sigma}{dx_{12}\cdots dx_{(n-1)n}} = \sum_m \sum_{1\le i_1,\cdots i_n \le m} \int d\sigma_m \times \prod_{1\le k\le n} \frac{E_{i_k}}{Q} \prod_{1\le j<l\le n} \delta\left(x_{jl} - \frac{1-\cos\theta_{i_j i_l}}{2}\right)$$

Energy correlator has the potential to be one of the most precise observables at the LHC

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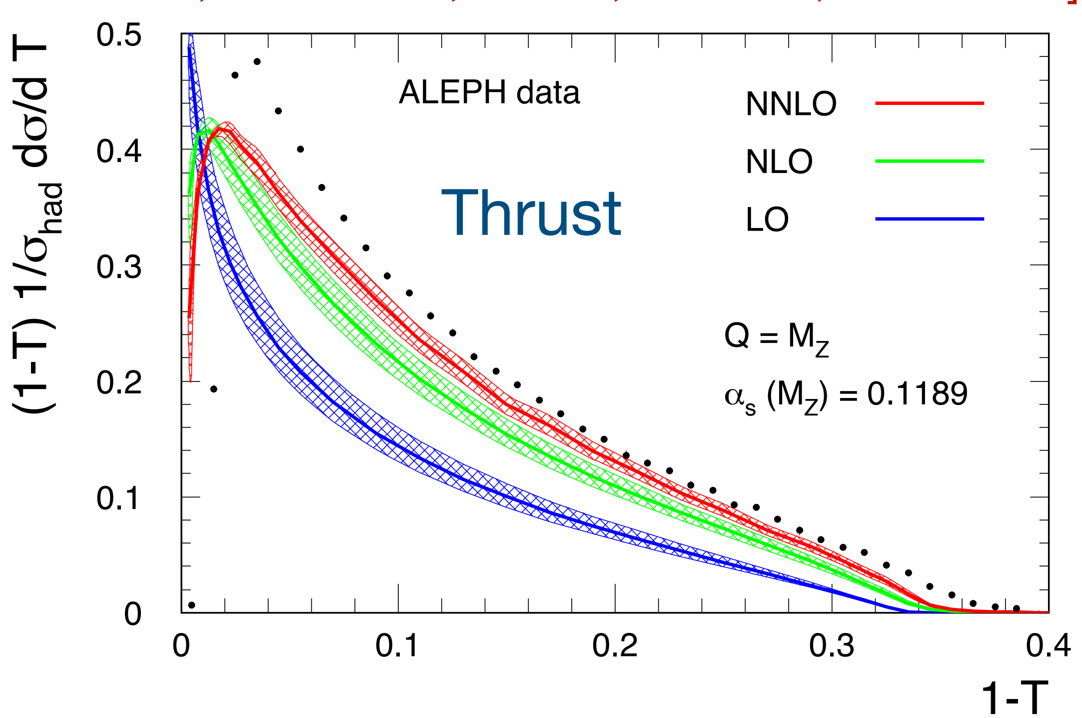
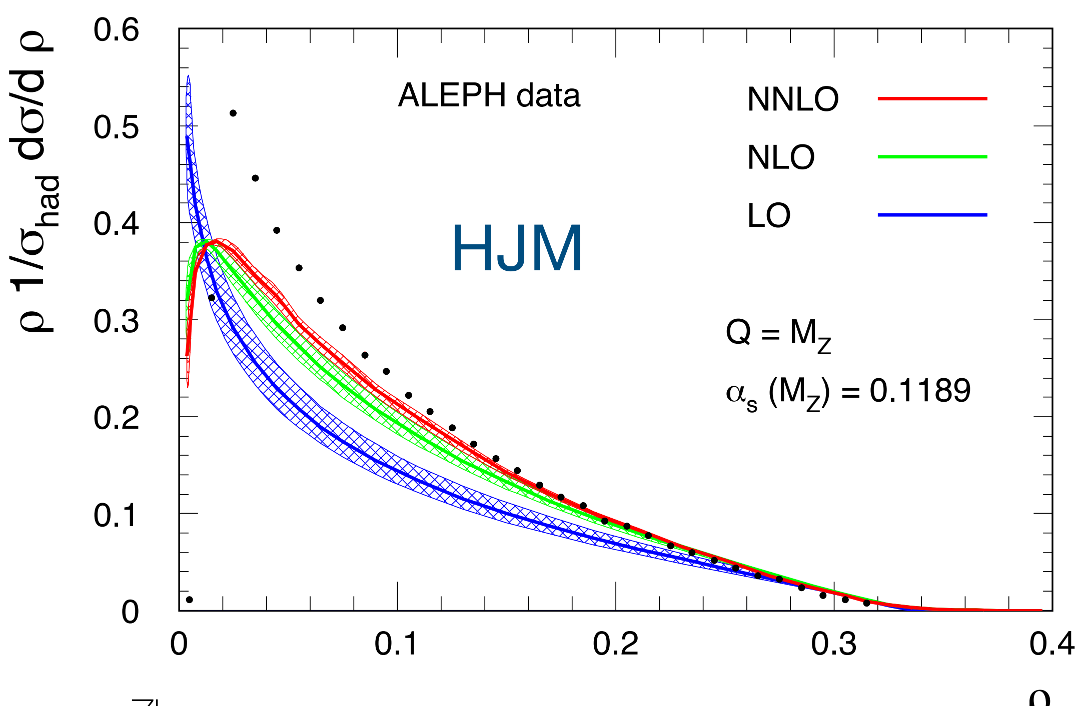
Heavy jet mass (HJM)

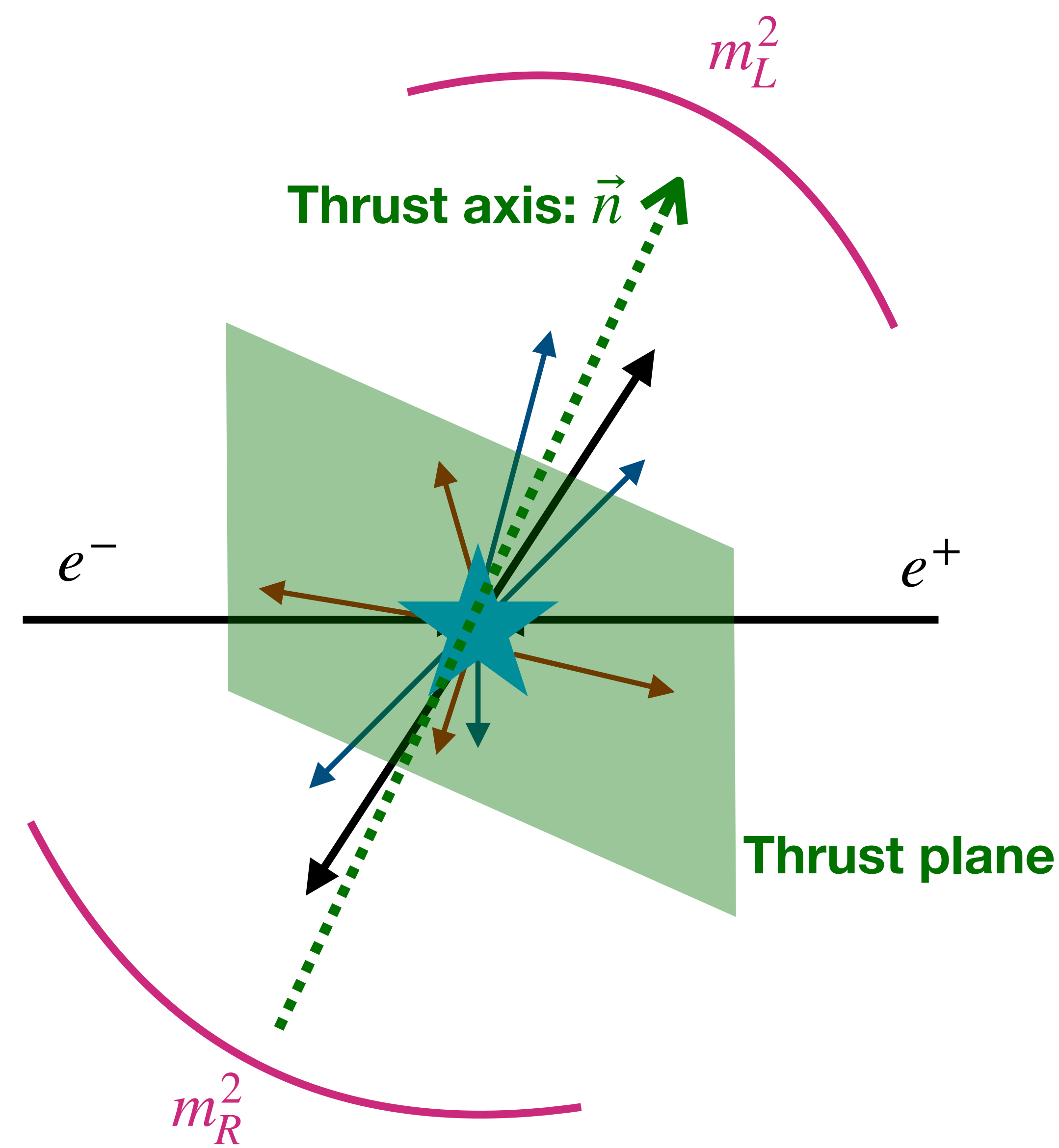
- Definition: Thrust: $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$ HJM: $\rho = \frac{1}{Q^2} \max\{m_L^2, m_H^2\}$

[De Ridder, Gehrmann, Glover, Heinrich, 0711.4711]

$\tau = 1 - T$





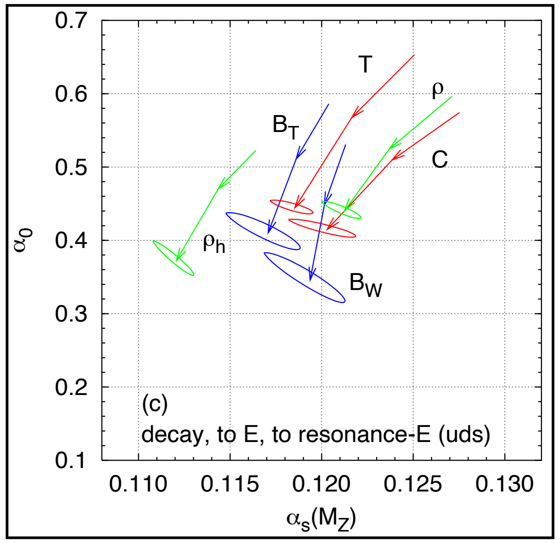


Heavy jet mass (HJM)

- Remaining problem in α_s measurement with HJM:

20 years ago: [Salam, Wicke, hep-ph/0102343]

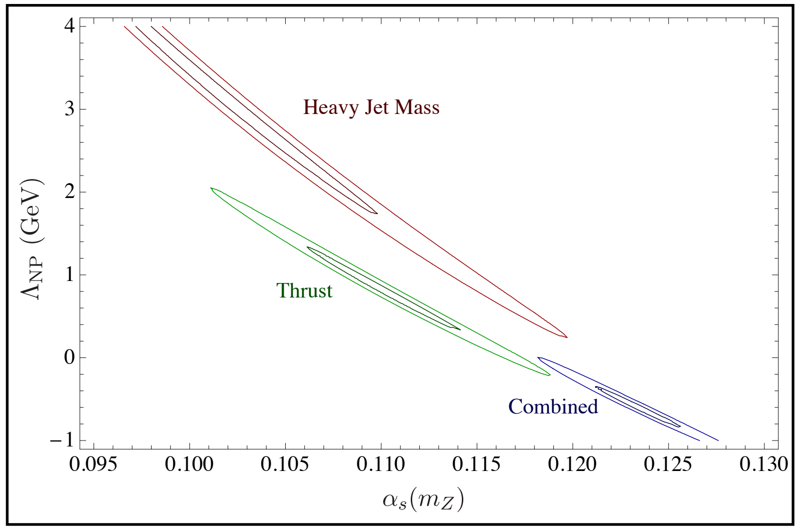
> Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo simulations that hadronisation corrections for ρ_h have unusual characteristics: in contrast to what is seen in more inclusive variables, the hadronisation depends strongly on the underlying hard configuration. There is therefore a need to develop techniques allowing a more formal approach to the study of such problems.



10 years ago: [Becher, Schwartz, 0803.0342] [Chien, Schwartz, 1005.1644]

$\mathrm{N^3LL}$ dijet resummation + NNLO + power correction:

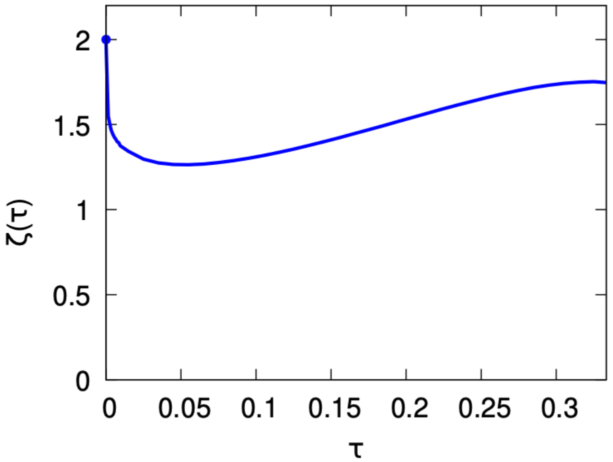
Inconsistence between thrust and heavy jet mass

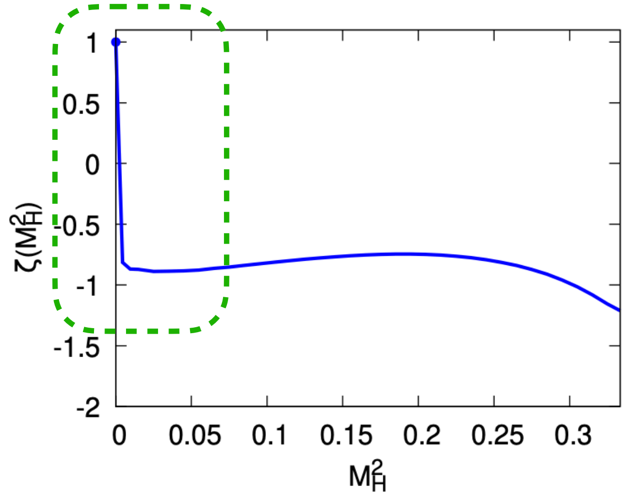


Today: [Caola et al, 2204.02247] [Nason, Zanderighi, 2301.03607]

Recent progress on trijet power corrections

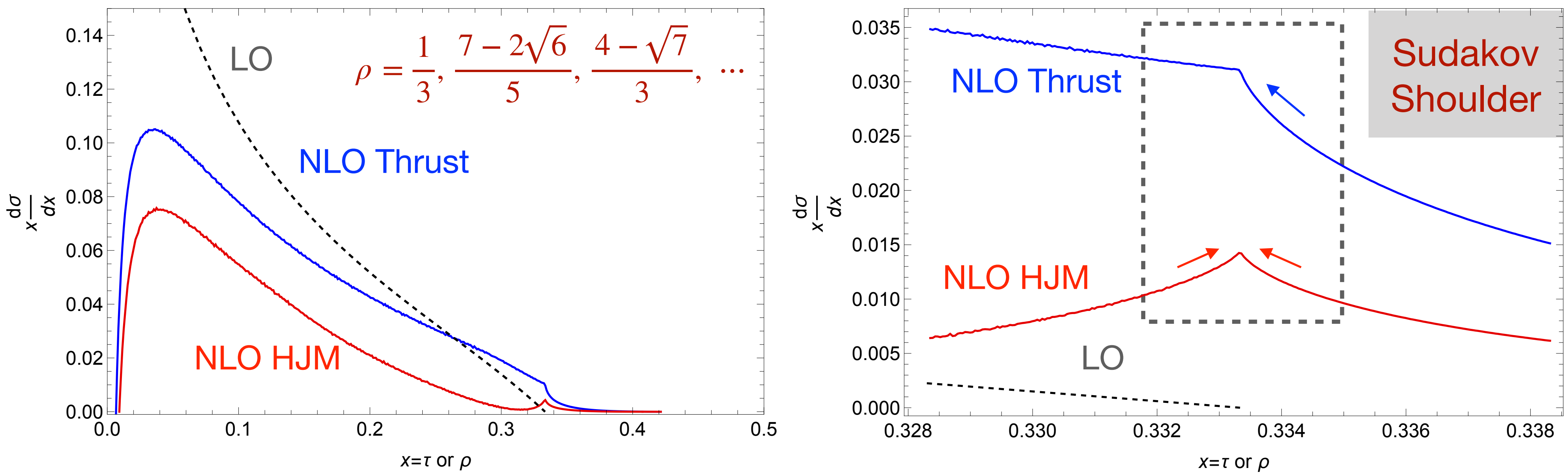
- Thrust: positive everywhere
- HJM: negative almost everywhere but positive near $\rho \to 0$





Thrust vs HJM

- Thrust and HJM have different kinks order by order in perturbation theory

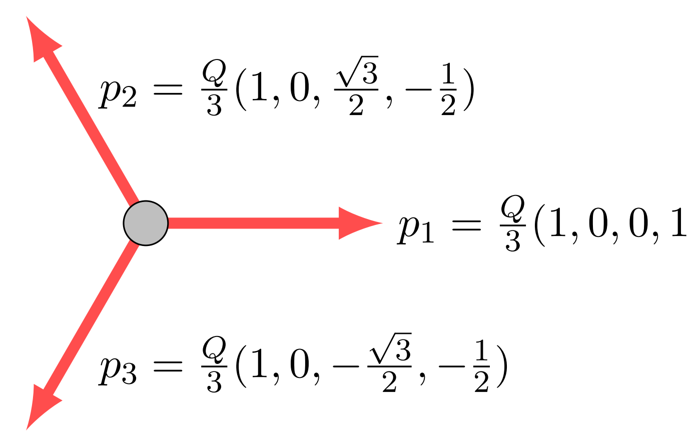


[Catani, Webber, hep-ph/9710333]

- Sudakov shoulders arise from incomplete cancellations between the virtual corrections and real emissions, where the range of event shape grows order-by-order in perturbation theory.
- Start with 3-parton configuration, the event shapes are restricted at each order:

Tree, one-loop virtual: $\tau, \rho \leq \dfrac{1}{3}$

Real emission: $\tau, \rho \leq \dfrac{7-2\sqrt{6}}{5} \approx 0.42$



Incomplete cancellation $\Rightarrow$ divergence, kinks, etc. $\Rightarrow$ large logarithms

Thrust vs HJM

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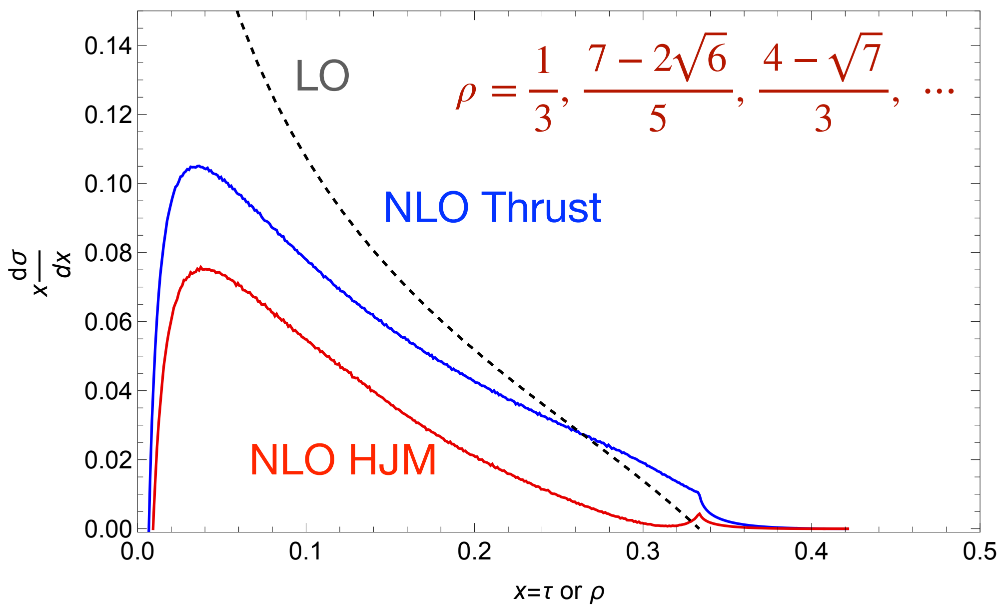
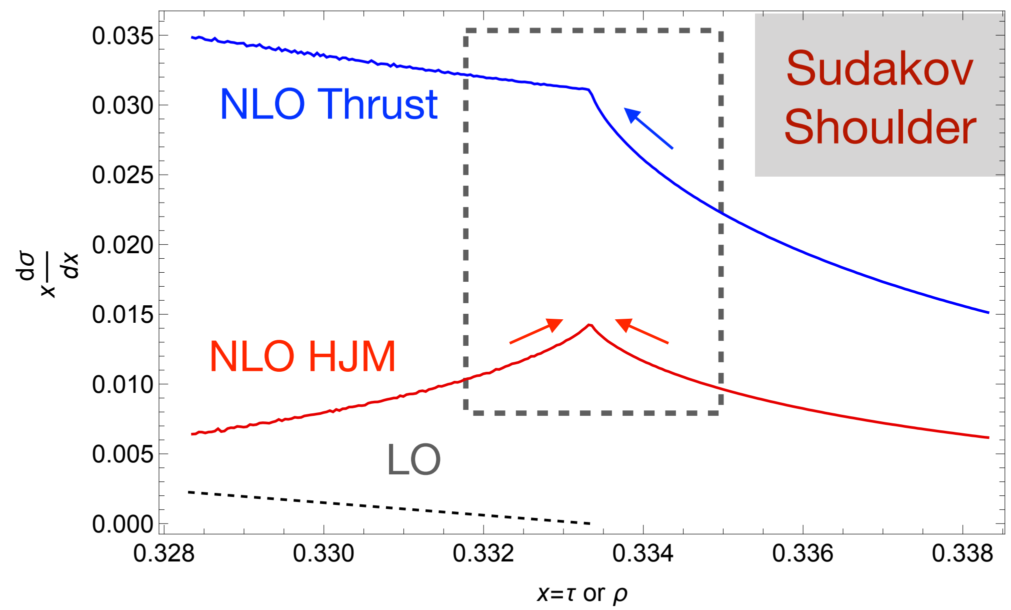




Fixed-order calculations: [Bhattacharya, Schwartz, **XYZ**, 2205.05702]

- Thrust: only right shoulder $\quad t = \tau - \frac{1}{3}$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma}{d\tau} = \frac{\alpha_s}{4\pi}\theta(t)\left\{-6\left(2C_F + C_A\right) t\ln^2 t + \left[6C_F\left(1-4\ln 3\right) + C_A\left(1-12\ln 3\right) + 4n_f T_F\right] t\ln t\right\}$$

- HJM: left shoulder (affects the α_s fit!) and right shoulder $\quad r = \frac{1}{3} - \rho$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma}{d\rho} = \frac{\alpha_s}{4\pi}\theta(r)\left\{-2\left(2C_F + C_A\right) r\ln^2 r + \left[2C_F\left(1+4\ln\frac{4}{3}\right) + C_A\left(\frac{1}{3}+4\ln\frac{4}{3}\right) + \frac{4}{3}n_f T_F\right] r\ln r\right\}$$

$$+\frac{\alpha_s}{4\pi}\theta(-r)\left\{-4\left(2C_F + C_A\right)(-r)\ln^2(-r) + \left[4C_F\left(1-4\ln 6\right) + 2C_A\left(\frac{1}{3}-4\ln 6\right) + \frac{8}{3}n_f T_F\right](-r)\ln(-r)\right\}$$

Shoulder factorization theorem

- Motivated from the fixed-order calculation, we derive the factorization in Soft-Collinear Effective theory (SCET):

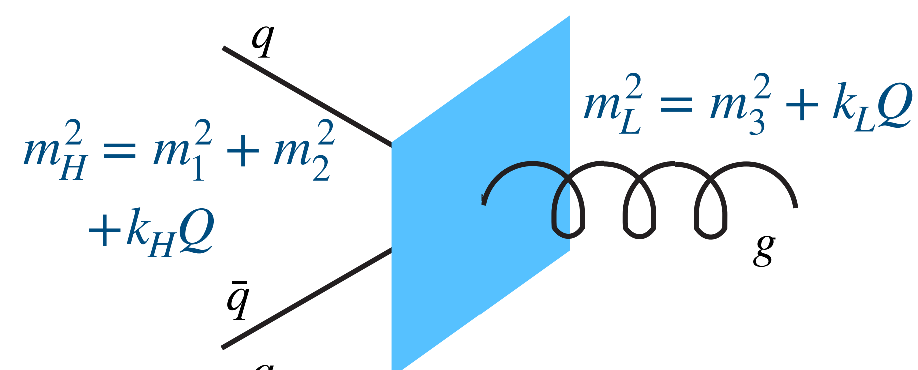
$$\frac{d\sigma_i}{dx} = \sigma_{LO} H(Q) \int dm_1^2 dm_2^2 dm_3^2 dk_L dk_H J_q(m_1^2) J_q(m_2^2) J_g(m_3^2) S_i^{(x)}(k_L, k_H) \times M_t^{(x)} \Theta\left(M_t^{(x)}\right)$$

LO phase space | Trijet hard function | Inclusive jet function | Trijet hemisphere soft function | Measurement

- Trijet kinematics:

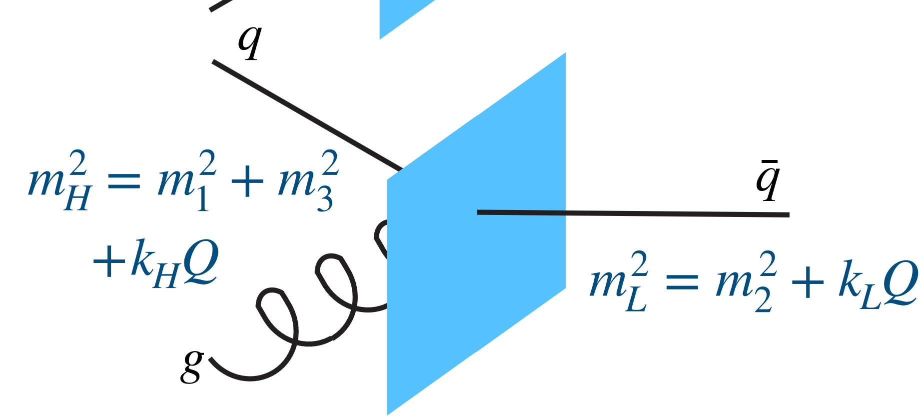
HJM: $M_t^{(\rho)} = r - (m_L^2 - m_H^2)$

$i = g$:



Thrust: $M_t^{(\tau)} = t - (m_L^2 + m_H^2)$

$i = q, \bar{q}$:



- New ingredient needed: six-directional differential soft function, integrated to the trijet hemisphere soft function

Trijet hemisphere soft function

- Definition of differential soft function

$$S_{6i}(q_i) = 2g_s^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^{d-1}}\delta^+(k^2)\mathscr{H}(k,q_i)\times\left[C_{23}\frac{n_2\cdot n_3}{(n_2\cdot k)(n_3\cdot k)}+C_{12}\frac{n_1\cdot n_2}{(n_1\cdot k)(n_2\cdot k)}+C_{13}\frac{n_1\cdot n_3}{(n_1\cdot k)(n_3\cdot k)}\right]$$

- From thrust axis constraint (trijet kinematics):

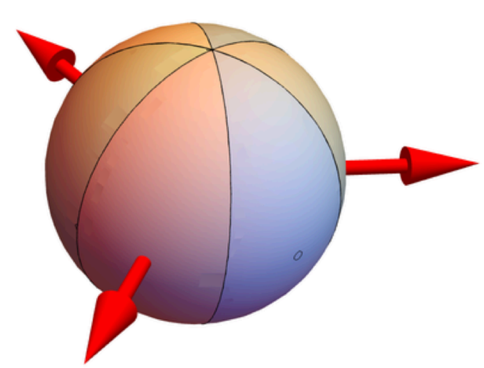
$$m_1^2+\frac{2Q}{3}\left(n_1\cdot k_1+N_2\cdot k_{\bar{2}}+N_3\cdot k_{\bar{3}}\right)<\frac{1}{3}-\rho+m_2^2+m_3^2+\frac{2Q}{3}\left(n_2\cdot k_2+n_3\cdot k_3+\bar{n}_1\cdot k_{\bar{1}}\right)$$

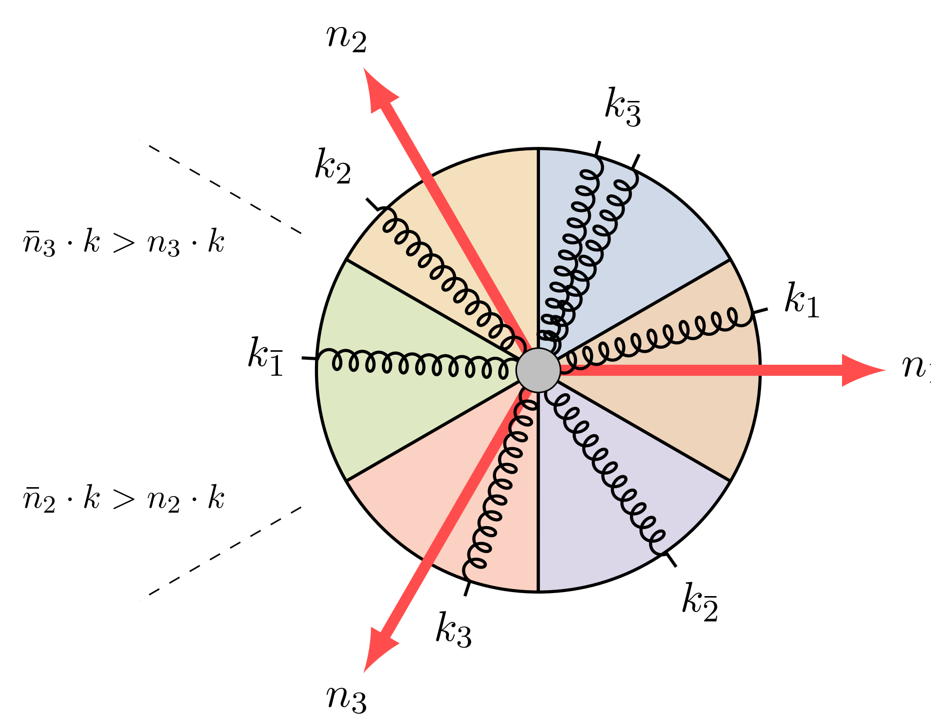
$\Rightarrow$ soft projections

$$\mathscr{H}(k,q_i)=\theta\left(n_2\cdot k-\bar{n}_2\cdot k\right)\theta\left(n_3\cdot k-\bar{n}_3\cdot k\right)\delta\left(q_1-\frac{2}{3}n_1\cdot k\right)+\text{other five terms}$$

- For HJM, $N_2=\left(1,0,+\frac{\sqrt{3}}{2},\frac{3}{2}\right),\quad N_3=\left(1,0,-\frac{\sqrt{3}}{2},\frac{3}{2}\right)$
- For thrust, $N_2=\bar{n}_2,\quad N_3=\bar{n}_3$

Similar to 3-jettiness soft function





Trijet hemisphere soft function

Non-global logs beyond NNLL

- Integrating the differential soft function over hemispheres

$$S_i(q_L, q_H, \mu) = \int d^6 q_i S_{6i}(q_i, \mu) \delta\left(q_L - q_1 - q_{\bar{2}} - q_{\bar{3}}\right) \delta\left(q_H - q_{\bar{1}} - q_2 - q_3\right) = S_{iL}(q_L, \mu) S_{iH}(q_H, \mu) S_f(q_L - q_H)$$

- One-loop result:

$$S_{iL}(q_L, \mu) = \delta(q_L)\left(1 + \frac{\alpha_s}{4\pi} c_{iL}\right) + \frac{\alpha_s}{4\pi}\left[\frac{-2C_A \Gamma_0 \ln\frac{q_L}{\mu} + 2\gamma_{si}}{q_L}\right]_\star$$

$$S_{iH}(q_H, \mu) = \delta(q_H)\left(1 + \frac{\alpha_s}{4\pi} c_{iH}\right) + \frac{\alpha_s}{4\pi}\left[\frac{-4C_F \Gamma_0 \ln\frac{q_H}{\mu} + 2\gamma_{si}}{q_H}\right]_\star$$

$$\gamma_{sg} = 8C_F \ln 2 - 4C_A \ln 3, \quad \gamma_{sqq} = -8C_F \ln 6\,,$$

$$\gamma_{sq} = 4C_A \ln 2 + 4C_F \ln\frac{2}{3}, \quad \gamma_{sqg} = -4(C_A + C_F)\ln 6\,,$$

$$c_{gL} = -4.44002C_A + 1.68285C_F,$$
$$c_{gH} = -0.210218C_A + 5.10882C_F,$$
$$c_{qL} = 0.841426C_A - 3.59860C_F,$$
$$c_{qH} = 2.55411C_A + 2.34389C_F\,.$$

$$\frac{d\sigma_g}{d\rho} = \sigma_{LO} H(Q) \int dm_L^2 dm_H^2 \underbrace{\int dm_{1,2}^2 dk_H J_q(m_1^2) J_q(m_2^2) S_{iH}^{(\rho)}(k_H) \delta(m_H^2 - m_1^2 - m_2^2 - k_H Q)}_{K_H(m_H^2)}$$
$$\times \underbrace{\int dm_3^2 dk_L J_g(m_3^2) S_{iL}^{(\rho)}(k_L) \delta(m_L^2 - m_3^2 - k_L Q)}_{K_L(m_L^2)} \times \left(\frac{1}{3} - \rho - m_L^2 + m_H^2\right) \Theta\left(\frac{1}{3} - \rho - m_L^2 + m_H^2\right)$$

where $K_{L,H}(m^2)$ RGE can be solved in the Laplace space respectively

Sudakov Landau poles

- Resummation in the momentum space

Left shoulder: $r = \frac{1}{3} - \rho > 0$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma_i}{d\rho} = \Pi_i(\partial_{\eta_l}, \partial_{\eta_h}) r \left(\frac{rQ}{\mu_s e^{-\gamma_E}}\right)^{\eta_l+\eta_h} \frac{\sin(\pi\eta_l)}{\pi}\Gamma\left(-1-\eta_l-\eta_h\right)$$

Right shoulder: $s = \rho - \frac{1}{3} > 0$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma_i}{d\rho} = \Pi_i(\partial_{\eta_l}, \partial_{\eta_h}) s \left(\frac{sQ}{\mu_s e^{-\gamma_E}}\right)^{\eta_l+\eta_h} \frac{\sin(\pi\eta_h)}{\pi}\Gamma\left(-1-\eta_l-\eta_h\right)$$

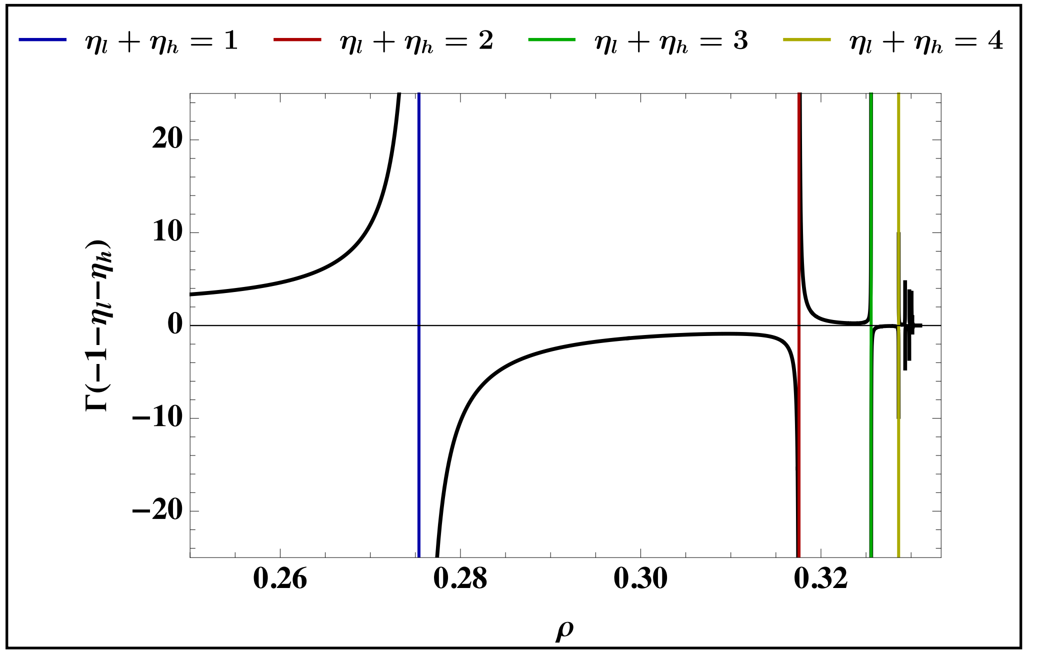
$$\eta_l^{(g)} = 2C_A A_\Gamma(\mu_j, \mu_s)$$
$$\eta_h^{(g)} = 4C_F A_\Gamma(\mu_j, \mu_s)$$

With RG kernel following

$$\Pi_g(\partial_{\eta_l}, \partial_{\eta_h}) = \exp\left[4C_F S(\mu_h, \mu_j) + 4C_F S(\mu_s, \mu_j) + 2C_A S(\mu_h, \mu_j) + 2C_A S(\mu_s, \mu_j)\right] \exp\left[2A_{\gamma_{sg}}(\mu_s, \mu_h) + 2A_{\gamma_{sqq}}(\mu_s, \mu_h) + 2A_{\gamma_{jg}}(\mu_j, \mu_h) + 4A_{\gamma_{jq}}(\mu_j, \mu_h)\right]$$
$$\times \left(\frac{Q^2}{\mu_h^2}\right)^{-2A_\Gamma(\mu_h,\mu_j)} H(Q, \mu_h)\tilde{j}_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{j}_{\bar{q}}\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{j}_g\left(\partial_{\eta_l} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\tilde{s}_{gL}(\partial_{\eta_l})\tilde{s}_{gH}(\partial_{\eta_h})$$

- The Γ function has an infinite number of poles in the r space (referred as Sudakov Landau pole):

$$-1-\eta_l-\eta_h = \underbrace{0, -1,}_{\rho<0} \underbrace{-2, -3, \cdots}_{0<\rho<\frac{1}{3}}$$



Sudakov Landau poles

- Our shoulder HJM measurement $\frac{d\sigma}{d\rho} \sim \left(r - m_L^2 + m_H^2\right)\theta\left(r - m_L^2 + m_H^2\right), \quad r \sim \lambda^2$
- Power counting: possible hierarchies between hemisphere masses

where EFT is valid	irrelevant regions	
$m_L^2 \sim \lambda^2,\ m_H^2 \sim \lambda^2;\ r \sim \lambda^2$	$m_L^2 \sim 1,\ m_H^2 \sim \lambda^2;\ r \sim 1$	
$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim \lambda^2$	$m_L^2 \sim \lambda^2,\ m_H^2 \sim 1;\ r \sim 1$	$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim 1$

our factorization includes

Our resummation contains non-EFT contributions!

Sudakov Landau poles

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where EFT is valid | irrelevant regions

$m_L^2 \sim \lambda^2,\ m_H^2 \sim \lambda^2;\ r \sim \lambda^2$	$m_L^2 \sim 1,\ m_H^2 \sim \lambda^2;\ r \sim 1$	
$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim \lambda^2$	$m_L^2 \sim \lambda^2,\ m_H^2 \sim 1;\ r \sim 1$	$m_L^2 \sim 1,\ m_H^2 \sim 1;\ r \sim 1$

our factorization includes

Our resummation contains non-EFT contributions!

Laplace $\neq$ Fourier:

Fourier space is the only space that diagonalizes the δ function and allows us to suppresses the non-EFT region

$$\sigma_i(r) \propto f_i(r) = \frac{1}{\Gamma(\eta_l)\Gamma(\eta_h)} \int_0^\infty dm_L^2 \int_0^\infty dm_H^2\, (m_L^2)^{\eta_l - 1} (m_H^2)^{\eta_h - 1} \delta\left(r - m_L^2 + m_H^2\right)$$

Fourier transformation

$$\tilde{f}_i(z) = \int_{-\infty}^{+\infty} dr\, e^{izr} f_i(r) = \underbrace{\int_0^\infty dm_L^2\, (m_L^2)^{\eta_l - 1} \boxed{e^{im_L^2 z}}}_{\mathrm{Im}(z)>0} \underbrace{\int_0^\infty dm_H^2\, (m_H^2)^{\eta_h - 1} \boxed{e^{-im_H^2 z}}}_{\mathrm{Im}(z)<0}$$

$e^{-\epsilon(m_L^2 + m_H^2)}$ suppresses the non-EFT region

$$= (-iz_+)^{-\eta_l} (+iz_-)^{-\eta_h} \qquad z_\pm = z \pm i\epsilon$$

Fourier/position space scale-setting

- Resummed second derivative

$$\tilde{\sigma}_i(z) = \int_{-\infty}^{+\infty} dr\ e^{izr}\sigma_i(r) = \Pi_i(\partial_{\eta_l}, \partial_{\eta_h}) \times \left(-\frac{ize^{\gamma_E}\mu_s}{Q}\right)^{-\eta_l} \left(+\frac{ize^{\gamma_E}\mu_s}{Q}\right)^{-\eta_h}$$

- The canonical scale in the Fourier space is (freeze the soft scale at $\mu_s^{\min} = 2\mathrm{GeV}$)

$$\mu_h^{\mathrm{can}} = Q, \quad \mu_s^{\mathrm{can}}(z) = \sqrt{\left(\frac{Qe^{-\gamma_E}}{|z|}\right)^2 + (\mu_s^{\min})^2}, \quad \mu_j^{\mathrm{can}}(z) = \sqrt{\mu_h^{\mathrm{can}}\mu_s^{\mathrm{can}}}$$

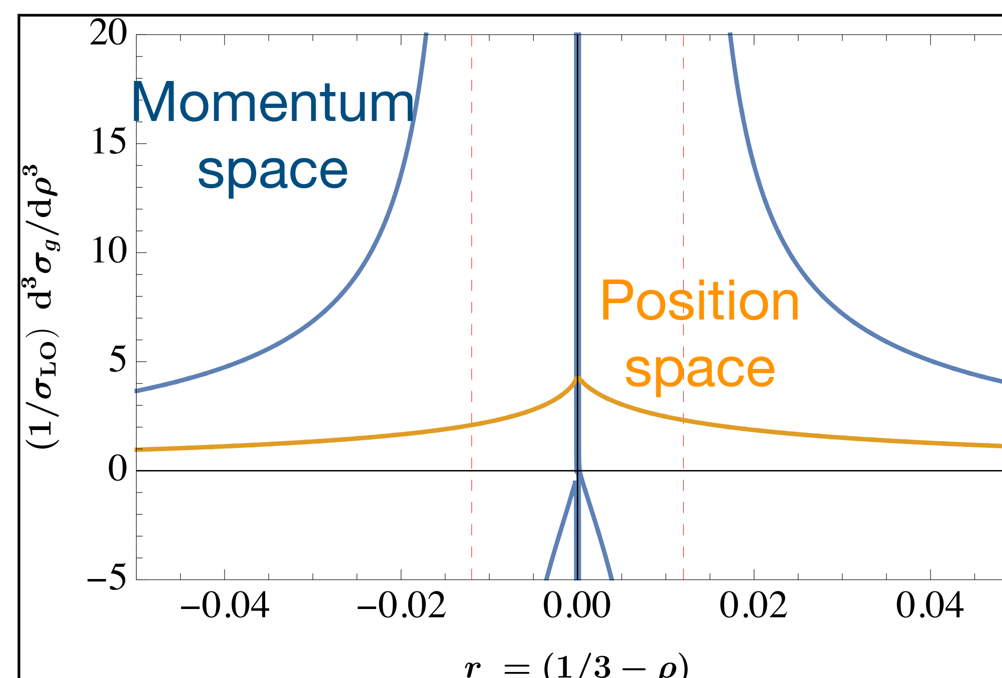
- Back to momentum space via Inverse Fourier transformation:

$$\left(\frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3}\right)_{\mathrm{pos}} = \int_{-\infty}^{+\infty} \frac{dz}{2\pi} e^{-izr}\tilde{\sigma}_i(z) = 2\Re\left[\int_0^{\infty} \frac{dz}{2\pi} e^{-izr}\tilde{\sigma}_i(z)\right], \quad \tilde{\sigma}_i^{\star}(-z) = \tilde{\sigma}_i(z)$$

Γ_0 approximation: LL with $\beta_0 = 0$

$$\left(\frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3}\right)_{\mathrm{pos}} = e^{-\frac{1}{2}\hat{\alpha}_s\Gamma_0(C_A\partial^2_{\eta_l}+2C_F\partial^2_{\eta_h})}\left(\frac{1}{\sigma_{LO}}\frac{d^3\sigma}{d\rho^3}\right)_{\mathrm{mom}}$$

- Derivatives $\partial_\eta \sim \partial_r$ blow up at Sudakov Landau poles
- $e^{-\partial^2}$ suppresses the divergences



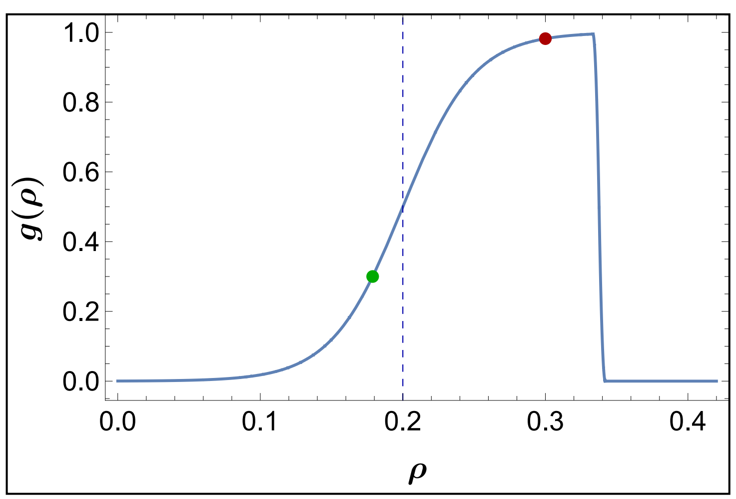
NNLL shoulder resummation

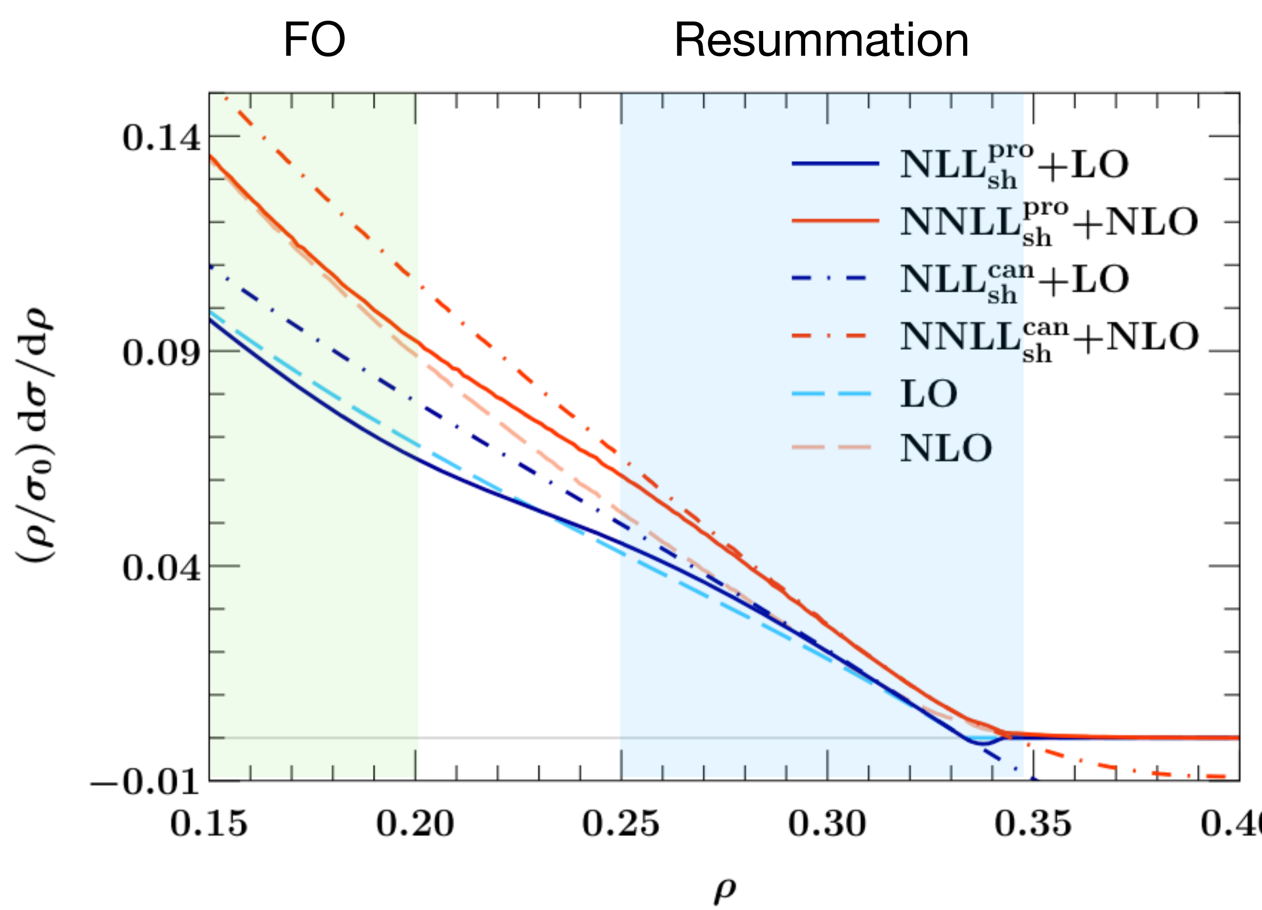
- Resummation matched to fixed-order:

$$\frac{d\sigma^{\rm sh,match}}{d\rho} = \frac{d\sigma^{\rm FO}}{d\rho} + \sigma_{\rm LO}\left[\frac{d\sigma^{\rm resum}(\mu^{\rm pro}_{h,j,s})}{d\rho} - \frac{d\sigma^{\rm resum}(\mu^{\rm FO}_{h,j,s}=Q)}{d\rho}\right]$$

- Profile scale: turn on the resummation when $\rho \to 1/3$ and turn it off when away from shoulder point

$$\mu^{\rm pro}_{j,s} = \mu_h^{1-g(\rho)}\left[\mu^{\rm can}_{j,s}(z)\right]^{g(\rho)}$$



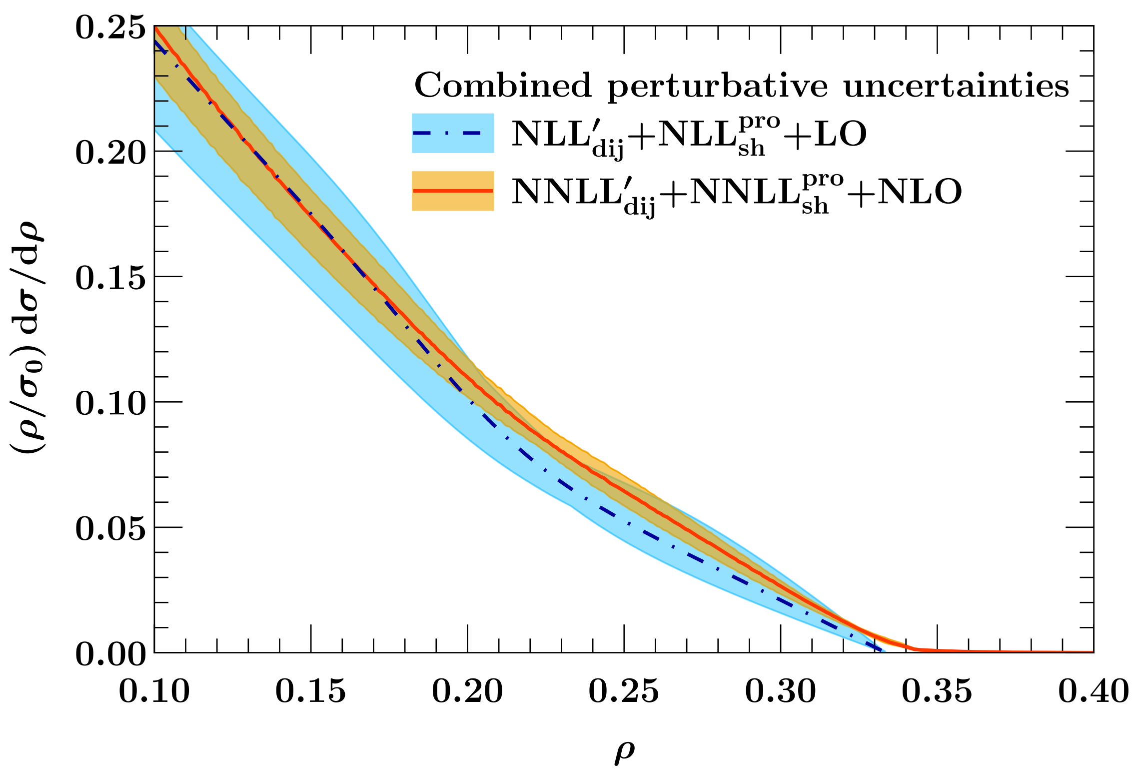


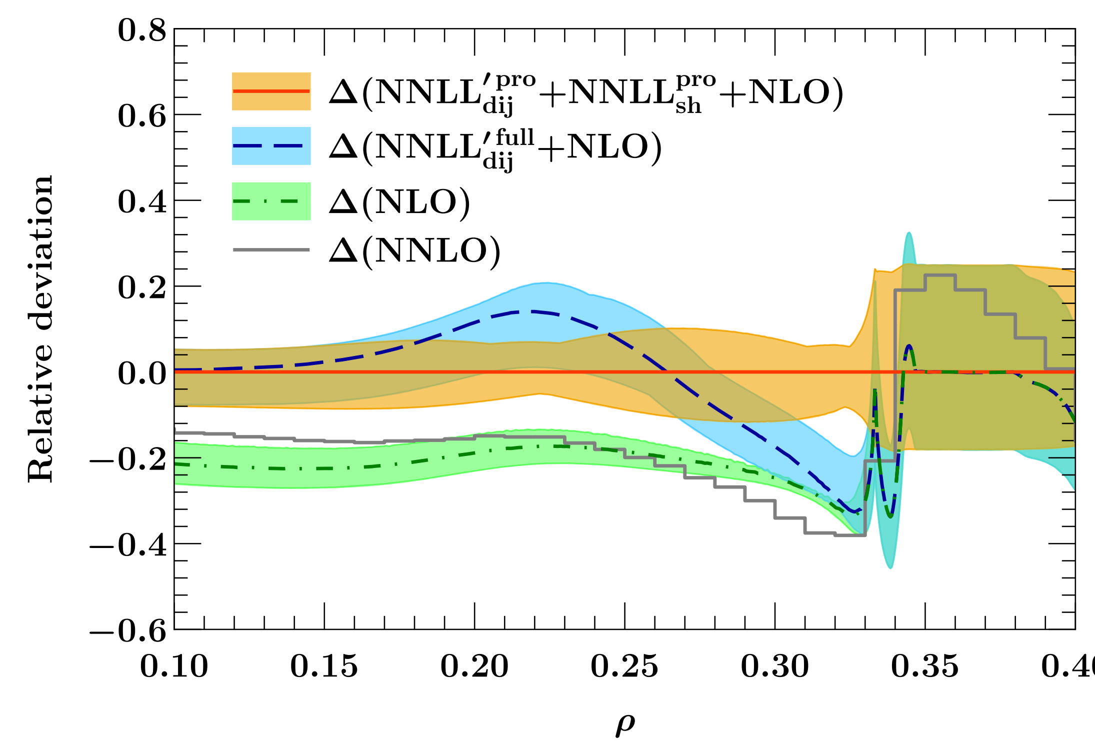
First-time position-space scale is critical for non-TMD (SCET-I) observable

Matching to dijet resummation

- Combine dijet ($\rho \to 0$) resummation and shoulder ($\rho \to 1/3$) resummation:

$$\frac{d\sigma^{\mathrm{match}}}{d\rho} = \frac{d\sigma^{\mathrm{dij}}(\mu_{\mathrm{dij}}^{\mathrm{pro}})}{d\rho} + \frac{d\sigma^{\mathrm{sh}}(\mu_{\mathrm{sh}}^{\mathrm{pro}})}{d\rho} + \left[\frac{d\sigma^{\mathrm{FO}}(\mu_{\mathrm{FO}})}{d\rho} - \frac{d\sigma^{\mathrm{dij}}(\mu_{\mathrm{FO}})}{d\rho} - \frac{d\sigma^{\mathrm{sh}}(\mu_{\mathrm{FO}})}{d\rho}\right]$$





- Conclusion: shoulder resummation provides significant corrections, which could affect the α_s extraction
- The shoulder factorization provides us a way to study trijet non-perturbative power corrections from EFT/SCET

[Benitez-Rathgeb, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, **XYZ**, in progress]

Outline

- Motivation
- Part I: Improving the perturbative prediction of heavy jet mass
  - Sudakov shoulder
  - Factorization theorem
  - Position-space resummation
  - NNLL shoulder resummation
- Part II: α_s measurements from $pp \to jj$ using three-point energy correlator
  - Collinear factorization
  - Two-loop jet functions
  - NNLL resummation for e^+e^-
  - NNLL resummation for $pp \to jj$

Projected energy correlators

- Projecting the N-point energy correlator into a one-dimensional distribution: projected N-point correlator (ENC) [Chen, Moult, **XYZ**, Zhu, 2004.11381]

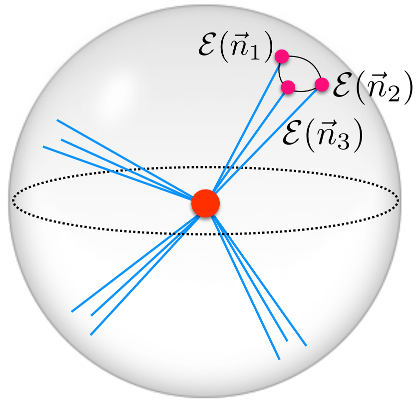
$$\frac{d\sigma^{[N]}}{dx_L} = \sum_m \sum_{1\le i_1,\cdots i_N \le m} \int d\sigma_m \left(\prod_{1\le k\le N} \frac{E_i}{Q} \right) \delta\left(x_L - \max\{x_{i_1,i_2}, x_{i_1,i_3}, \cdots x_{i_{N-1},i_N}\}\right)$$

m is the number of final-state particles

$x_L = (1-\cos\theta_L)/2$ being the largest angular distance

- Can be used as an event-shape observable at e^+e^- colliders and a jet substructure observable at pp colliders

Our work: Collinear limit of projected N=3 correlator provides another α_s data point for $pp \to jj$

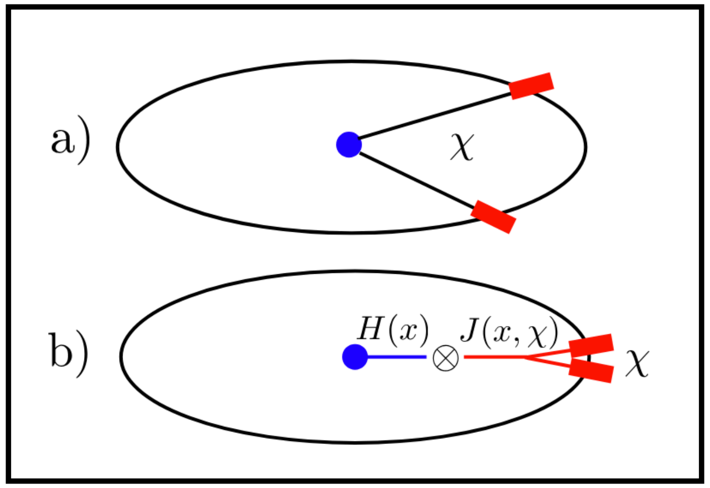


- In perturbation theory, the collinear limit $x_L \to 0$ behaves

$$\frac{d\sigma^{[N]}}{dx_L} = \sum_{L=1}^{\infty} \sum_{j=-1}^{L-1} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^L c_{L,j}\mathcal{L}^j(x_L) + \cdots, \quad \mathcal{L}^{-1}(x_L) = \delta(x_L), \quad \mathcal{L}^j(x_L) = \left[\ln^j(x_L)/x_L\right]_+$$

Single log structure because of no soft divergences

Collinear factorization



- The factorization proposed for EEC at e^+e^- colliders

  [Dixon, Moult, Zhu, 1905.01310]

$$\Sigma^{[N]}\left(x_L, \ln\frac{Q^2}{\mu^2}\right) = \frac{1}{\sigma_{\rm tot}}\int_0^{x_L} dx'_L \frac{d\sigma^{[N]}}{dx'_L}\left(x'_L, \ln\frac{Q^2}{\mu^2}\right)$$

$$\Sigma^{[2]}_{ee}\left(x_L, \ln\frac{Q^2}{\mu^2}\right) = \int_0^1 dx\, x^2 \vec{J}^{[2]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}\right)\cdot \vec{H}_{ee}\left(x, \ln\frac{Q^2}{\mu^2}\right)$$

jet function hard function

$$\vec{J}^{[N]} = \{J_q^{[N]}, J_g^{[N]}\}$$
$$\vec{H}_{ee} = \{H_{ee,q}, H_{ee,g}\}$$

- Generalized to N-point: [Chen, Moult, **XYZ**, Zhu, 2004.11381], [Lee, Meçaj, Moult, 2205.03414]

$$\Sigma^{[N]}_{ee}\left(x_L, \ln\frac{Q^2}{\mu^2}\right) = \int_0^1 dx\, x^N \vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}\right)\cdot \vec{H}_{ee}\left(x, \ln\frac{Q^2}{\mu^2}\right)$$

$$\Sigma^{[N]}_{\rm had}\left(R_0, R_L, \ln\frac{p_T^2}{\mu^2}\right) = \int_0^1 dx\, x^N \vec{J}^{[N]}\left(\ln\frac{R_L^2 x^2 p_T^2}{\mu^2}\right)\cdot \vec{H}_{\rm had}\left(R_0, x, \ln\frac{p_T^2}{\mu^2}\right)$$

For $pp \to jj$, the largest distance x_L is replaced with the rapidity-azimuth distance $R_L = \max_{i,j\in X_E}\sqrt{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2}$.

The hard function also depends on the jet radius R_0 parameter. Collinear limit corresponds to $R_L \ll R_0$

Hard function

- For e^+e^- collisions, extract from semi-inclusive hadron fragmentation function:

$$\vec{H}^{(0)}_{ee}(x) = \{2\delta(1-x), 0\}$$

$$\frac{1}{2}H^{(1)}_{ee,q}(x) = \frac{\alpha_s}{4\pi}C_F\left[\left(\frac{4\pi^2}{3}-9\right)\delta(1-x) + 4\left[\frac{\ln(1-x)}{1-x}\right]_+ + \left(4\ln(x)-\frac{3}{2}\right)\left(2\frac{1}{[1-x]_+}-x-1\right) - \frac{9x}{2} - 2(x+1)\ln(1-x) + \frac{7}{2}\right]$$

Known to NNLO

[Mitov, Moch, Vogt, hep-ph/0604053]

[Gehrmann, Schürmann, 2201.06982]

- RGE (DGLAP): $$\frac{d\vec{H}(x, \ln\frac{Q^2}{\mu^2})}{d\ln\mu^2} = -\int_x^1 \frac{dy}{y}\widehat{P}(y)\cdot\vec{H}\left(\frac{x}{y}, \ln\frac{Q^2}{\mu^2}\right)$$

$$\widehat{P}(y) = \begin{bmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{bmatrix}$$

Singlet timelike splitting matrix

- For $pp \to jj$ collision, we absorb PDFs and anti-k_t algorithms into hard function.
- Selection cuts: $R_0 = 0.4, \quad p_T > 15\text{GeV}, \quad |\eta| < 1.5$
- The two leading jets are further subject to
  - $|\Delta\phi(j_1, j_2)| > 2, \quad |p_T^1 - p_T^2|/(p_T^1 + p_T^2) < 0.5$
- Calculation: MadGraph + LHAPDF

[Liu, Shen, Zhou, Gao, 2305.14620]

(NNPDF31_nnlo_as_0112)

(NNPDF31_nnlo_as_0118)

(NNPDF31_nnlo_as_0124)

Jet functions

- From RG invariance, the jet RGE is

$$\frac{d\vec{J}^{[N]}(\ln\frac{x_L Q^2}{\mu^2})}{d\ln\mu^2} = \int_0^1 dy\, y^N \vec{J}^{[N]}\left(\ln\frac{x_L y^2 Q^2}{\mu^2}\right)\cdot\widehat{P}(y)$$

- At LL, we can solve the RGE exactly:

$$\frac{d\vec{J}^{[N]}_{\text{LL}}(\ln\frac{x_L Q^2}{\mu^2})}{d\ln\mu^2} = \vec{J}^{[N]}_{\text{LL}}(\ln\frac{x_L Q^2}{\mu^2})\cdot\frac{\alpha_s}{4\pi}\int_0^1 dy\, y^N\hat{P}^{(0)}(y) = -\vec{J}^{[N]}_{\text{LL}}(\ln\frac{x_L Q^2}{\mu^2})\cdot\frac{\alpha_s}{4\pi}\gamma_T^{(0)}(N+1)$$

Anomalous dimension:

$$\gamma_T(N) \equiv -\int_0^1 dy\, y^N\hat{P}(y) = \left(\frac{\alpha_s}{4\pi}\right)\gamma_T^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_T^{(1)} + \cdots$$

$$\vec{J}^{[N]}_{\text{LL}} = 2^{-N}(1,1)\cdot\exp\left[-\frac{\gamma_T^{(0)}}{\beta_0}\ln\frac{\alpha_s\left(\sqrt{x_L}Q\right)}{\alpha_s(\mu)}\right]$$

- Beyond LL, we use a truncated solution in α_s for DGLAP (called expanded solution)

Ansatz:

$$\vec{J}^{[N]} = \underbrace{\sum_{i=0}^{\infty}\alpha_s^i L^i\vec{c}_{i,i}}_{\text{LL}} + \underbrace{\sum_{i=1}^{\infty}\alpha_s^i L^{i-1}\vec{c}_{i,i-1}}_{\text{NLL}} + \underbrace{\sum_{i=2}^{\infty}\alpha_s^i L^{i-2}\vec{c}_{i,i-2}}_{\text{NNLL}} + \cdots \qquad L \equiv \ln\frac{x_L Q^2}{\mu^2}$$

- The only missing ingredients: two-loop jet function constants

Two-loop jet functions

- We extract the two-loop jet functions from E3C fixed-order calculation
  - $e^+e^- \to q\bar{q}$ process ⇒ quark jet function
  - $\mathrm{Higgs} \to gg$ process ⇒ gluon jet function

Finite term

$$\frac{1}{\sigma_0}\frac{d\sigma^{[3]}}{dx_L} = \underbrace{\sum_{1\le i_1\neq i_2\neq i_3\le 4}\int d\mathrm{LIPS}_4\,|\mathcal{M}_4|^2\frac{E_{i_1}E_{i_2}E_{i_3}}{Q^3}\delta(x_L-\max\{x_{i_1,i_2},x_{i_1,i_3},x_{i_2,i_3}\})}_{\text{Finite term}}$$
$$+\underbrace{\sum_{n\in\{3,4\}}\sum_{1\le i_1\neq i_2\le n}\int d\mathrm{LIPS}_n\,|\mathcal{M}_n|^2\frac{E_{i_1}^2E_{i_2}}{Q^3}\delta(x_L-x_{i_1,i_2}) + \sum_{n\in\{2,3,4\}}\sum_{1\le i_1\le n}\int d\mathrm{LIPS}_n\,|\mathcal{M}_n|^2\frac{E_{i_1}^3}{Q^3}\delta(x_L)}_{\text{Boundary term}}$$

Boundary term

- For Boundary term, use standard IBP+Differential equation (same as EEC)
- For finite term, we take two steps:

[Dixon, Luo, Shtabovenko, Yang, Zhu, 1801.03219]

$$\frac{d\sigma^{[3]}}{dx_L} \sim \int dx_{i_1,i_2}dx_{i_1,i_3}dx_{i_2,i_3}\delta(x_L-\max\{x_{i_1,i_2},x_{i_1,i_3},x_{i_2,i_3}\})\frac{d\sigma^{[3]}}{dx_{i_1,i_2}dx_{i_1,i_3}dx_{i_2,i_3}}$$

Monte Carlo with proper infrared subtraction

(1) Brute-force integration;
(2) parametric IBP+Differential equation

[Chen, 1902.10387, 1912.08606, 2007.00507]

Two-loop jet functions

- The logarithmic terms are predicted by the jet RGE.
- New result: two-loop jet constants

$$j_2^{q,[3]} = 12.3020\, C_F T_F n_f - 26.2764\, C_A C_F + 21.3943\, C_F^2$$

$$j_2^{g,[3]} = 17.5487\, C_A T_F n_f - 2.05342\, C_F T_F n_f - 5.97991\, C_A^2 + 0.904693\, n_f^2 T_F^2$$

- We are now ready for NNLL resummation

$$\frac{d\sigma^{\rm match}}{dx_L} = \frac{d\sigma^{\rm resum}}{dx_L} - \frac{d\sigma^{\rm sing}}{dx_L} + \frac{d\sigma^{\rm FO}}{dx_L}$$

resummation order	$\hat{P}(y)$	$\vec{H}$, $\vec{J}$ constants	$\beta[\alpha_s]$	fixed-order matching
LL	tree	tree	1-loop	LO
NLL	1-loop	1-loop	2-loop	NLO
NNLL	2-loop	2-loop	3-loop	NNLO

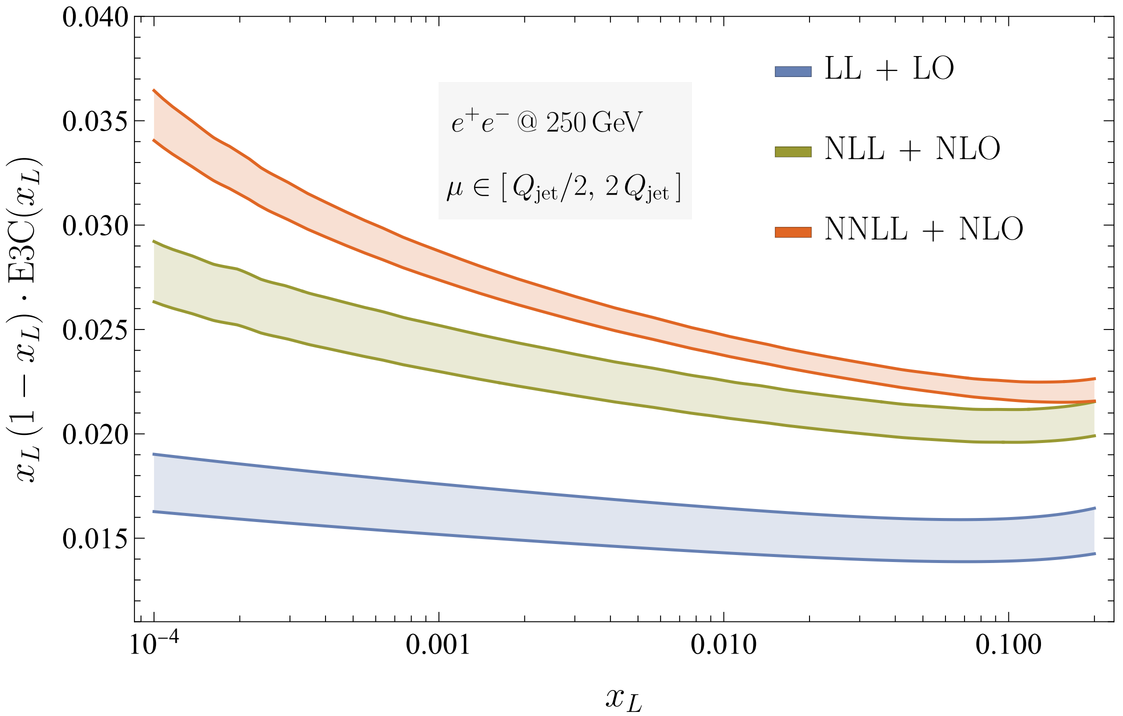
Since we are interested in the collinear limit (as a jet substructure observable), the non-singular distribution has tiny effect

For $pp \to jj$, we don't have the two-loop hard constant. So we use the Padé approximation:

$$a_s^2 h_0^{(2)} \approx \kappa \frac{(a_s h_0^{(1)})^2}{h_0^{(0)}}, \quad \kappa \in [0,1/2]$$

NNLL resummation: e^+e^- collisions

- Perturbative convergence is not satisfactory due to presence of renormalon
- One could do the renormalon subtraction: [Schindler, Stewart, Sun, 2305.19311]
-



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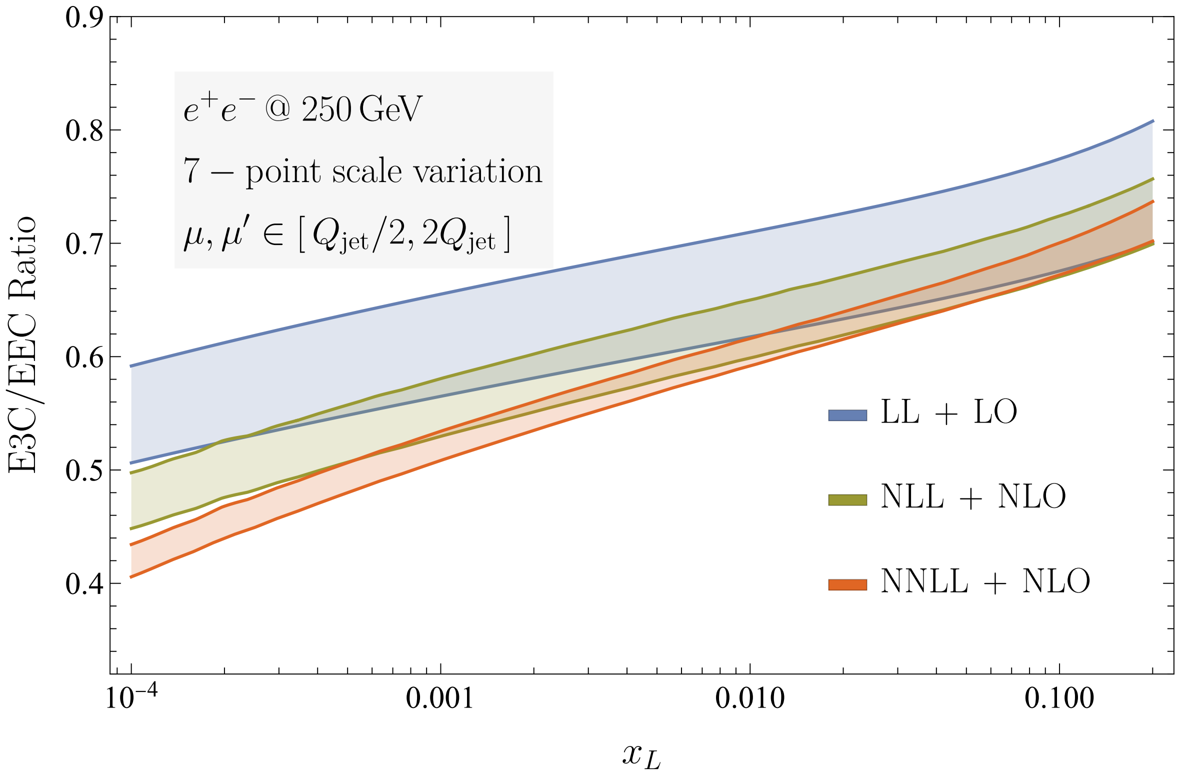
- One could do the renormalon subtraction: [Schindler, Stewart, Sun, 2305.19311]
- A easier way is to consider the ratio

$$\Delta_{m,n}(x_L, \mu, \mu') \equiv \frac{d\sigma^{[m]}/dx_L}{d\sigma^{[n]}/dx_L}, \quad m, n \geq 2$$

m-point correlator/n-point correlator



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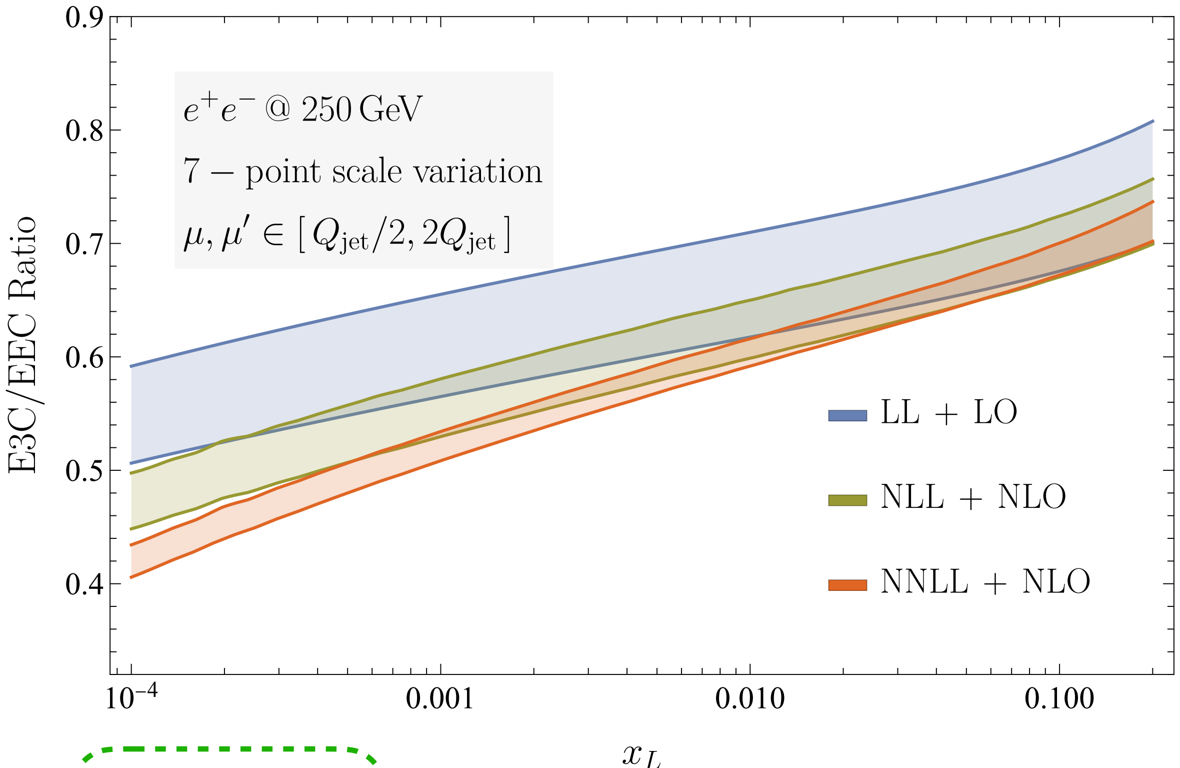
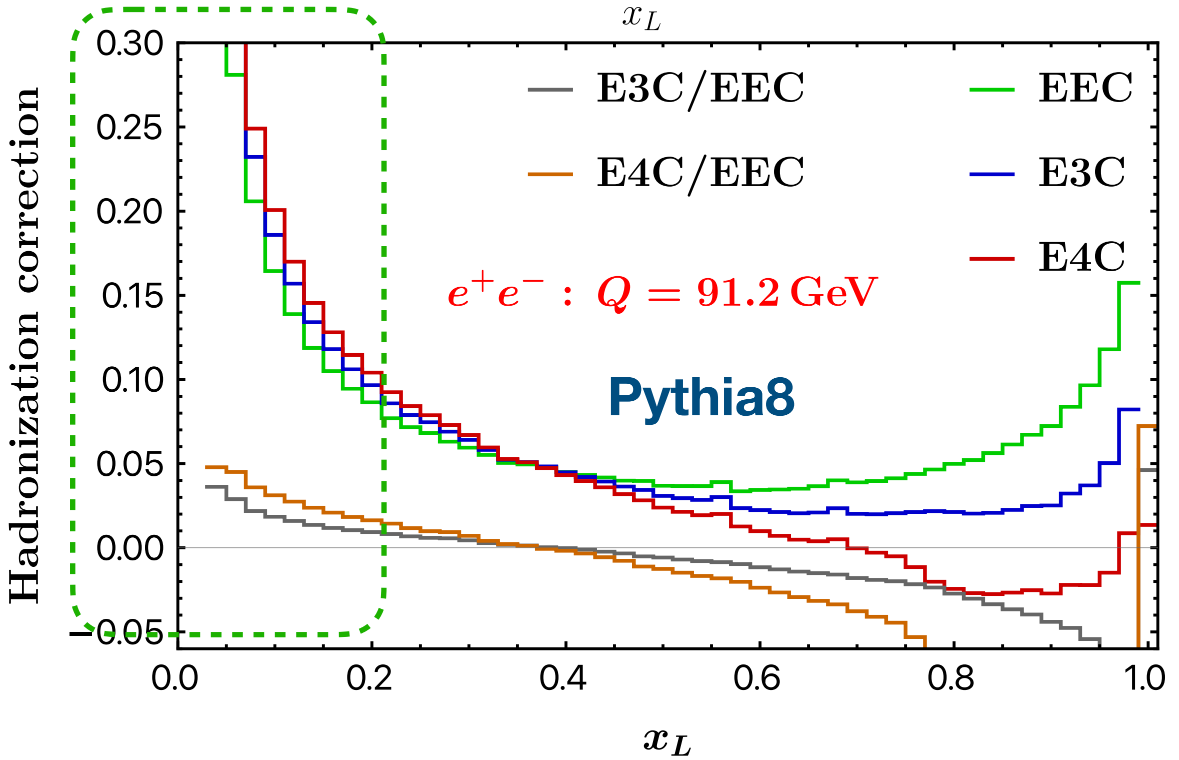
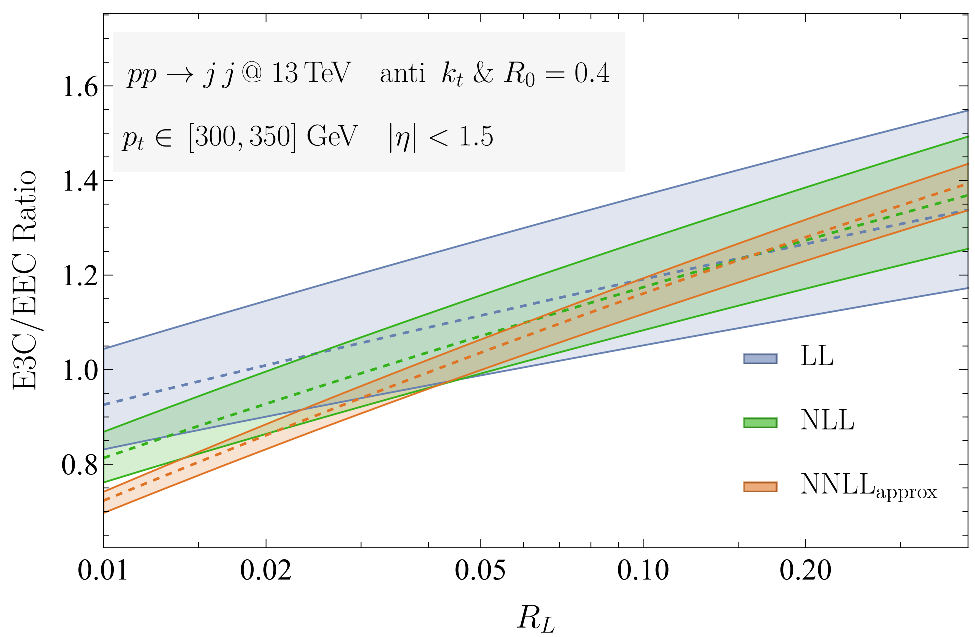


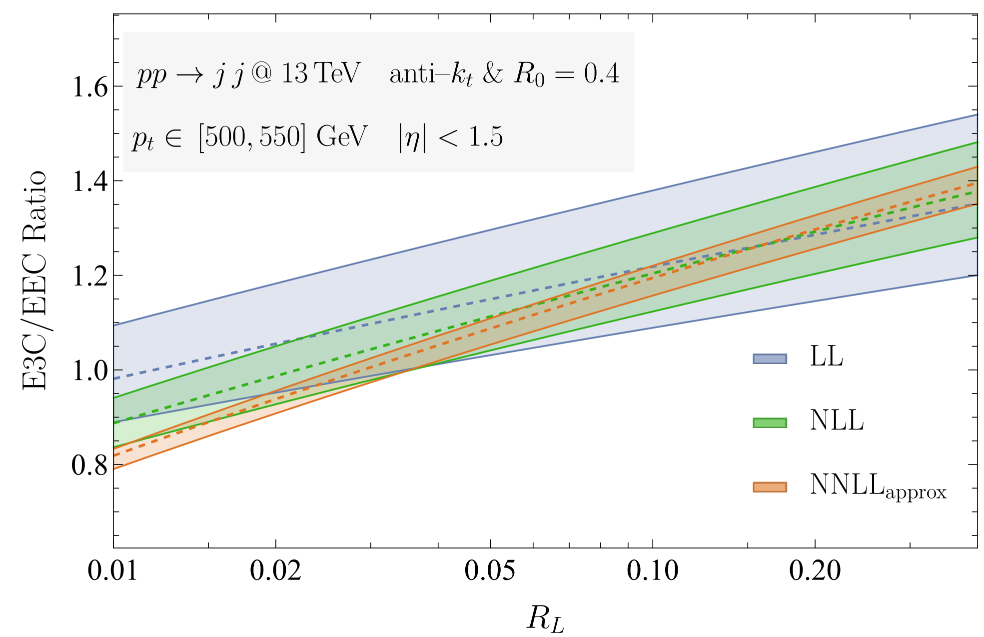
- We also find smaller hadronization correction in the ratio of energy correlators



Approximate NNLL resummation: $pp \to jj$

- We consider $\Delta_{3,2}(R_L)$ with two jet p_t range: [300, 350] GeV and [500,550] GeV
    - Normalize the E3C/EEC distribution in the range of $R_L \in [0.01, 0.4]$
    - Include the $\kappa \in [0, 1/2]$ from two-loop hard constant as additional uncertainty



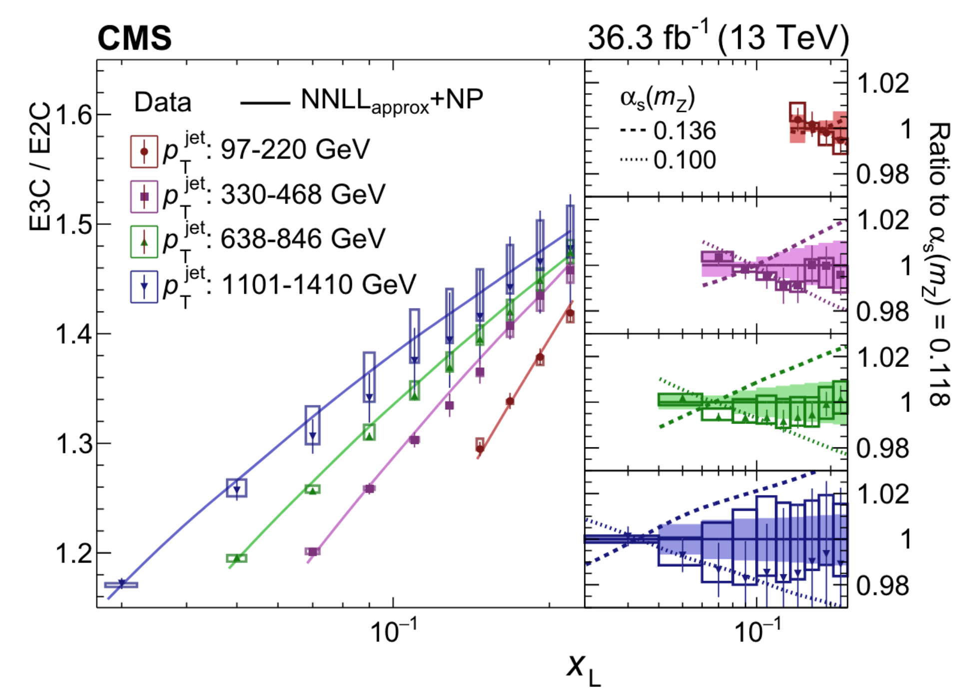


- The value of α_s from CMS collaboration:

[CMS collaboration, 2402.13864]

$$\alpha_s(m_Z) = 0.1229^{+0.0014}_{-0.0012}(\text{stat})^{+0.0030}_{-0.0033}(\text{theo})^{+0.0023}_{-0.0036}(\text{exp})$$

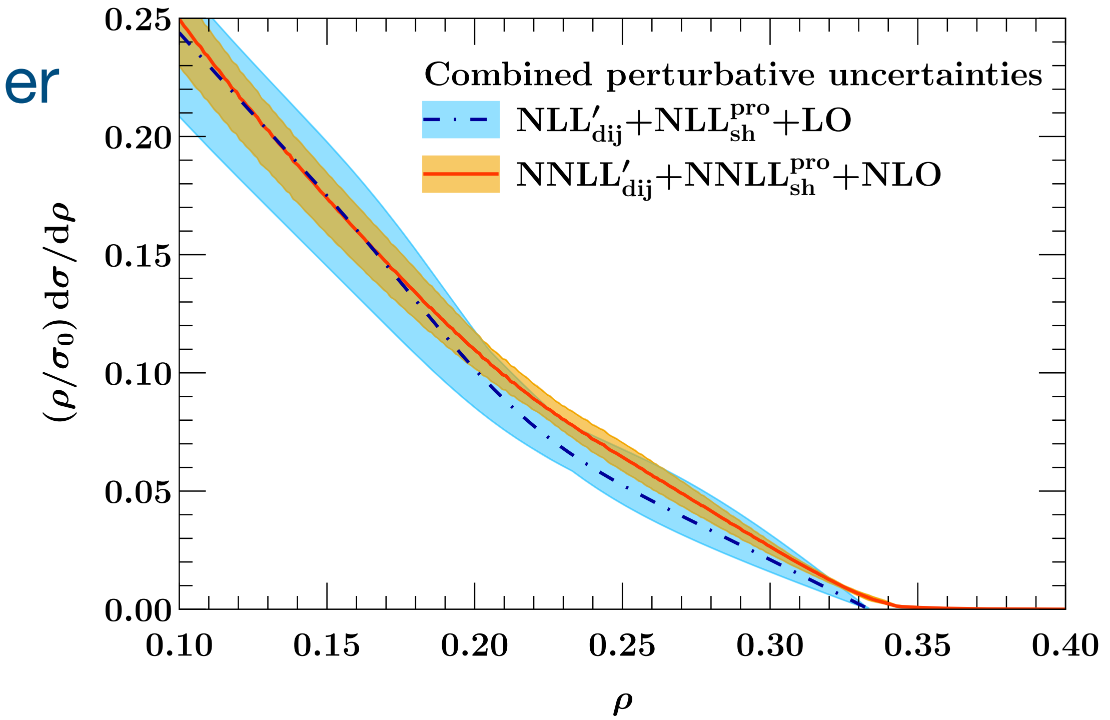
The most precise α_s using jet substructure observable



Conclusion

- There is a long-standing discrepancy between low-energy α_s extractions in lattice QCD and high-energy collider measurements.

- For e^+e^- colliders, we study the Sudakov shoulder resummation for heavy jet mass
    - Position-space scale setting is critical
    - Provide significant corrections to the perturbative predictions



- For $pp \to jj$, we study the collinear resummation of projected three-point energy correlators
    - The two-loop jet constants are calculated using modern multi-loop techniques.
    - The most precise α_s measurement from jet substructure: $\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$.