Resummation in Lattice QCD Calculation of Generalized Parton Distributions in Large Momentum Effective Theory

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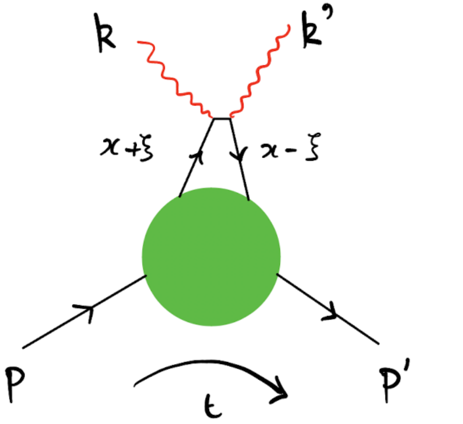
Loopfest XXII, May 20-22, Dallas, TX



QUARK-GLUON TOMOGRAPHY COLLABORATION

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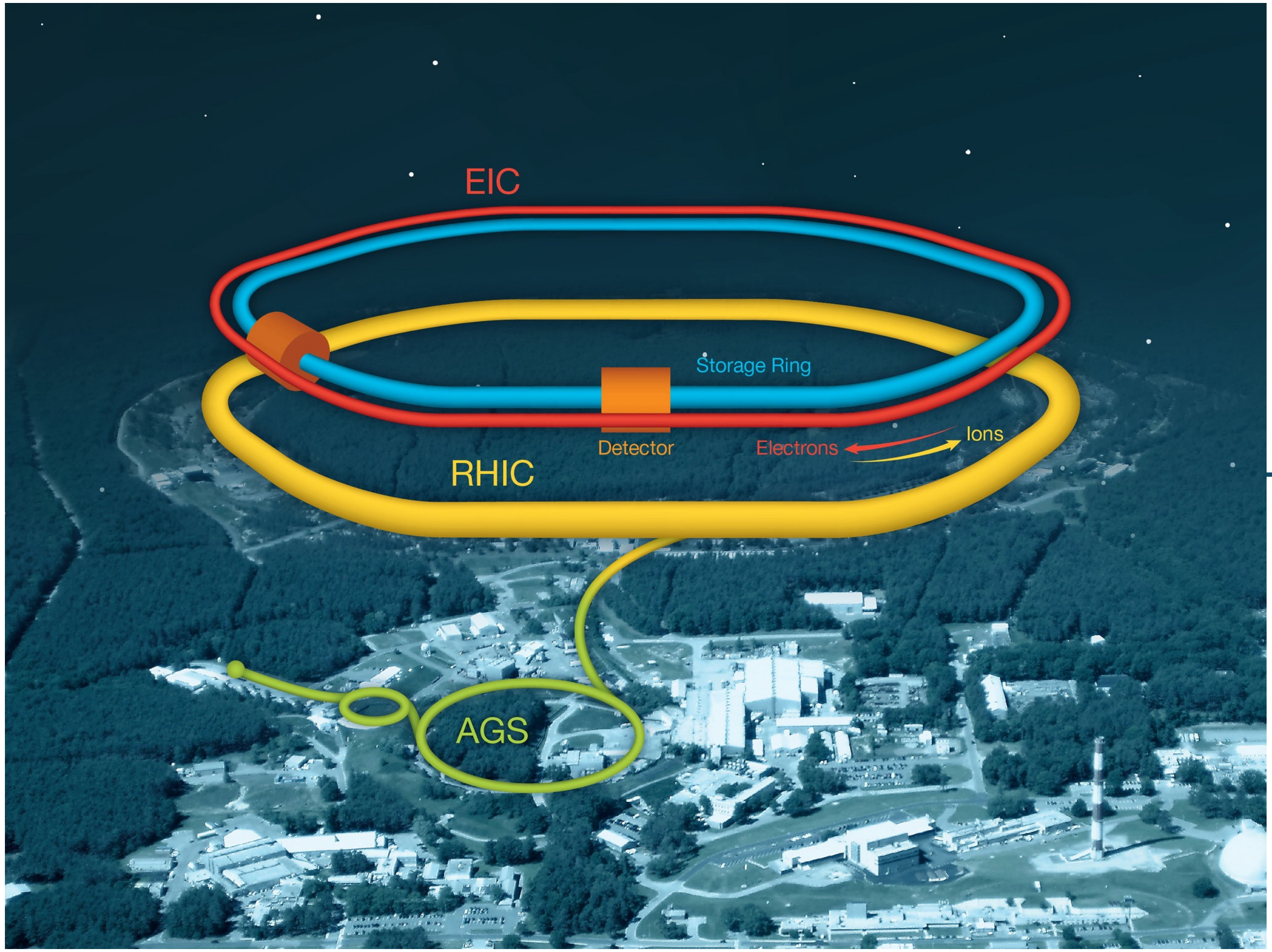
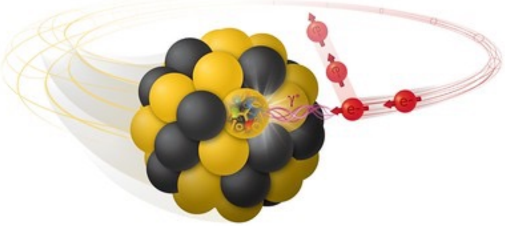
In collaboration with Jack Holligan, Xiang Gao , and Yong Zhao

EIC physics



By BNL



- 3D structure of protons and nuclei
- Solving the mystery of proton spin
- Gluon saturation and the color glass condensate
- Quark and gluon confinement

GPD (3D structure of protons and nuclei; Solving the mystery of proton spin)

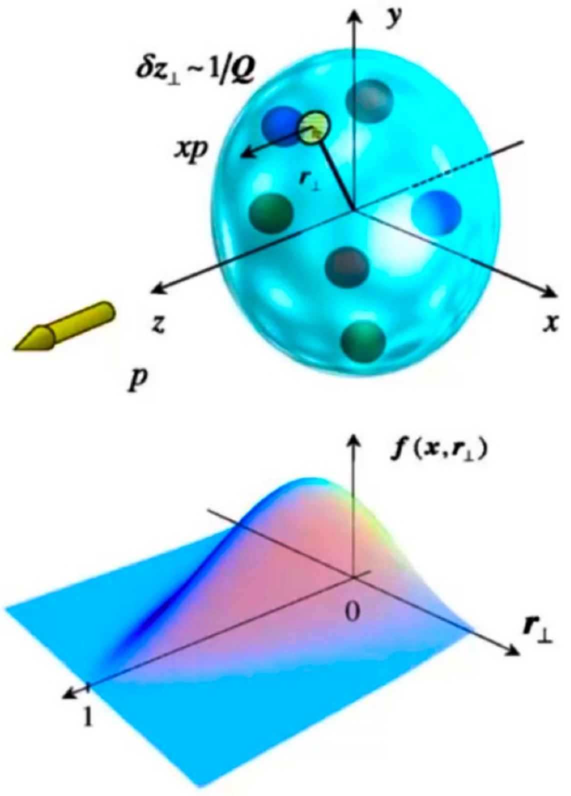
Generalized Parton Distributions

GPD offers insights into the 3D image of hadrons

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

$\xi \to 0, \int d\Delta_\perp e^{ir\cdot\Delta}$

$$P^+ = \frac{p'^+ + p^+}{2}, \Delta = p' - p, \xi = \frac{\Delta^+}{2P^+}$$

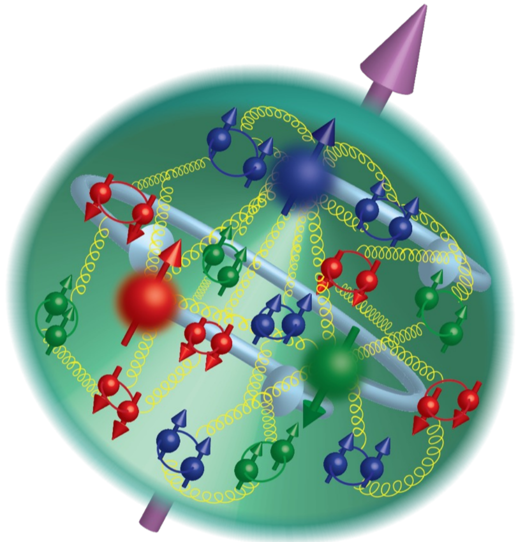


Belitsky and Radyushkin: Phys.Rept.(2005)

GPD and hadronic properties

- Connection to Gravitational Form Factors:
  - $\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[A(t)\frac{P^\mu P^\nu}{m} + B(t)\frac{P^{(\mu}\sigma^{\nu)\Delta}}{2m} + D(t)\frac{\Delta^\mu\Delta^\nu}{4m} + m\bar{c}g^{\mu\nu}\right]u(p)$
  - $\int_{-1}^{1} dx\ \ xH(x,\xi,t) = A + \xi^2 D$
  - $\int_{-1}^{1} dx\ \ xE(x,\xi,t) = B - \xi^2 D$

- Connection to quark & gluon angular momentum:
  - Ji's sum rule: $J_P = J_{q,\bar{q}} + J_g$ Ji, PRL (1997)
  - $J_i = \lim_{t\to 0} \int_{-1}^{1} dx\ \ x[H_i(x,\xi,t) + E_i(x,\xi,t)]$



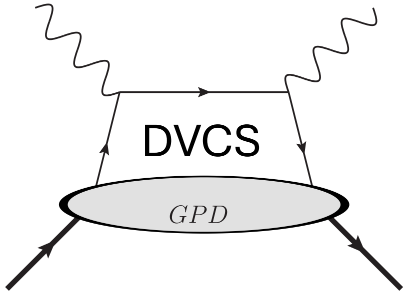
by ANL

Challenge in extracting GPD from experiment

- Multi-dimensionality $F(x, \xi, t)$
- Observables appear at the amplitude level
- x is always integrated over

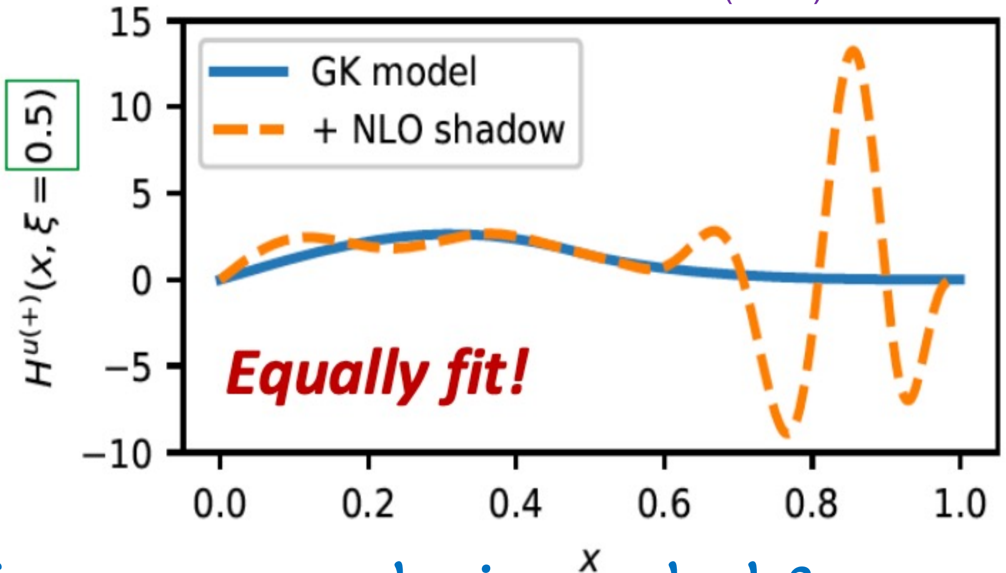
  $iM \propto \int dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$
- Shadow GPDs (degenerate solutions)
  - $\int dx \frac{\Delta F(x,\xi,t)}{x-\xi+i\epsilon} = 0$

Ji, PRD(1997)



Hall A @JLAB
CLAS@JLAB
H1@HERA

Bertone et al. PRD (2021)



A direct x-dependence calculation from non-perturbative methods?

Lattice QCD

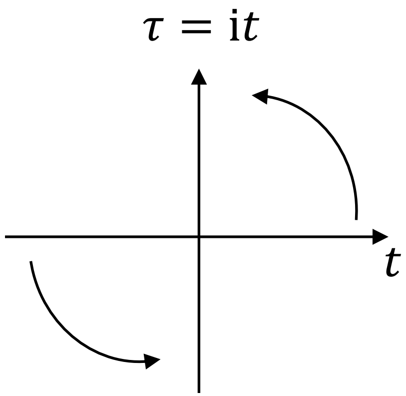
- Path integral formalism

$$\langle O \rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\; O\; e^{iS_{\text{QCD}}(A,\psi,\bar{\psi})}$$

Wick Rotation

$$\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\; O\; e^{-S^E_{\text{QCD}}}$$

Positive
Bounded



- Ideal for a Monte Caro sampling:

$$P(A,\psi,\bar{\psi}) = \frac{e^{-S^E_{\text{QCD}}}}{\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S^E_{\text{QCD}}}}$$

$$\langle O \rangle = \sum_n O_n / n$$

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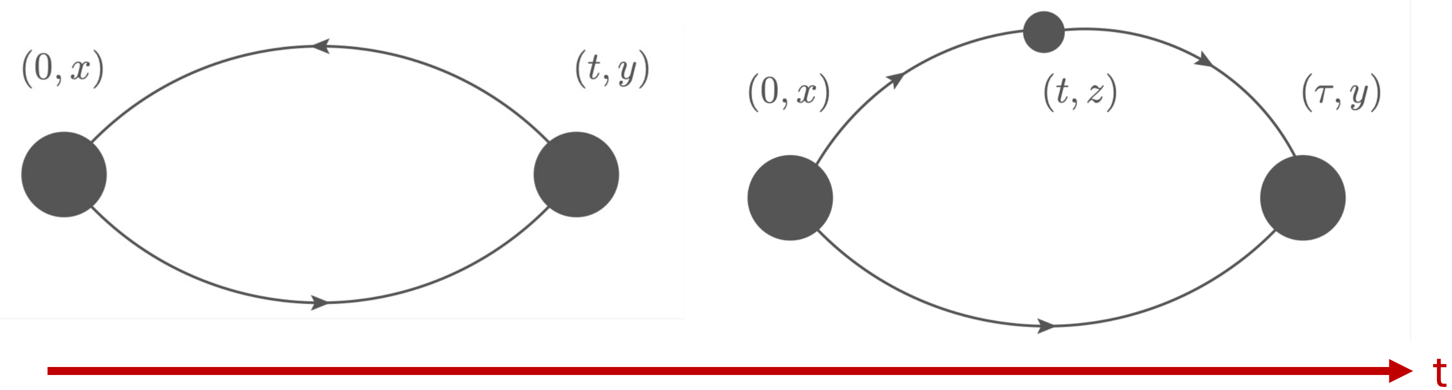
Lattice QCD

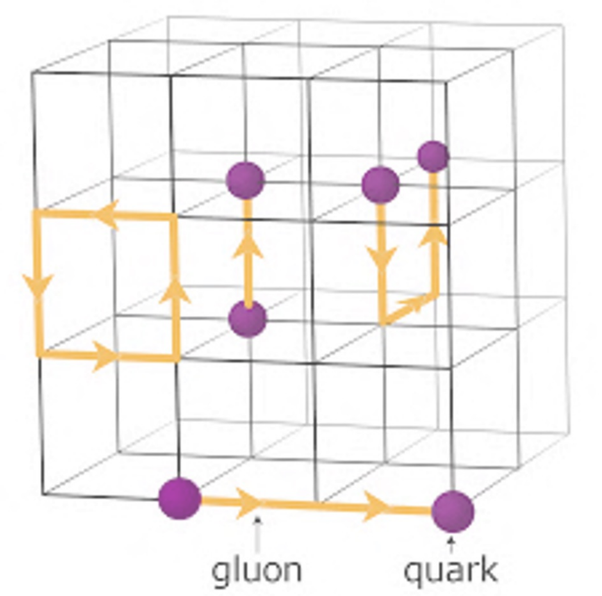
K.G. Wilson, Nobel Prize Winner (1982)

- Discretization of QCD action:
- Construction of correlators:

$C_2(t) = \langle \chi_{src}(0) | \chi_{snk}(t) \rangle$ $\quad$ $C_3(t) = \langle \chi_{src}(0) | O(t) | \chi_{snk}(\tau) \rangle$



Euclidean 4D spacetime



Savage, NNPSS (2015)

- Extraction of matrix elements:

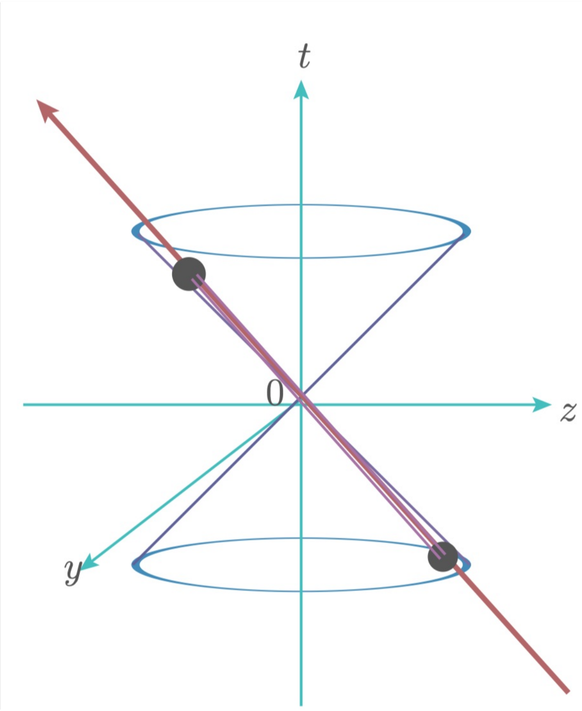
$$C_2(t) = \sum |c_n|^2 e^{-E_n t}$$
$$C_3(t,\tau) = \sum c_m^* c_n \langle m|O|n\rangle e^{-E_m(\tau - t)} e^{-E_n t}$$

Large Momentum Effective Theory (LaMET)

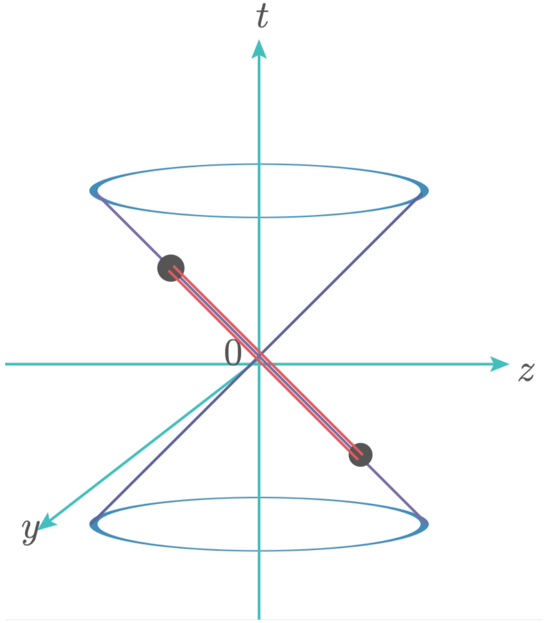
Ji, PRL (2013)
Ji, SCPMA(2014)

Applicable to PDF, DA, GPD, TMDPDF...

Momentum distribution



Large P_z Expansion



$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$

Quasi-GPD: $\widetilde{H}(x,\xi,t,P_z) =$

$$\int \frac{dzP_z}{2\pi} e^{-ixzP_z} \langle P' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma_t U(0,z) q\left(\frac{z}{2}\right)\right| P\rangle$$

$C\,(x,y,\xi,\mu,P_z) \otimes H(y,\xi,t,\mu) \quad + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(\xi \pm x)^2 P_z^2}\right)$

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$\xi \neq 0$ Matching kernel

Ji, Yao & Zhang JHEP (2023)

$$
\begin{aligned}
C(x, y, \xi, \mu, P_z) &= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\Bigg[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right) - 1\right) \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right) - 1\right) \\
&+ \left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4(x-y)^2 P_z^2}{\mu^2}\right) - 1\right)\Bigg]
\end{aligned}
$$

Only one-loop results have been calculated, but we can examine higher-order effects through resummation of logarithms.

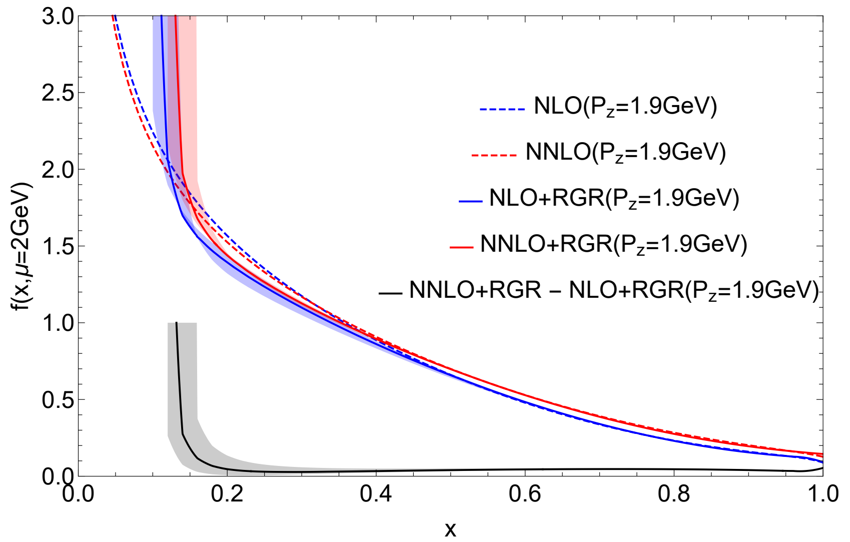
Resum quark momentum log in $\xi = 0$

- Matching of quasi-GPD $\xrightarrow{\xi=0}$ Matching of quasi-PDF
- Analogy to DIS: $Q = 2xP_z$
- $C^{RGR}\left(\frac{x}{y}, \mu, xP_z\right) = C^{FO}\left(\frac{x}{z}, 2xP_z, xP_z\right) \exp[\int_{2xP_z}^{\mu} d\mu' \hat{P}\left(\frac{z}{y}, \mu'\right)]$

DGLAP evolution

- Non-negligible effect for $x < 0.3$
- Scale variation blows up when $x \to 0$

Su, et al., NPB (2023)



- Threshold logarithm $\ln\left(1 - \frac{x}{y}\right)$ resummation

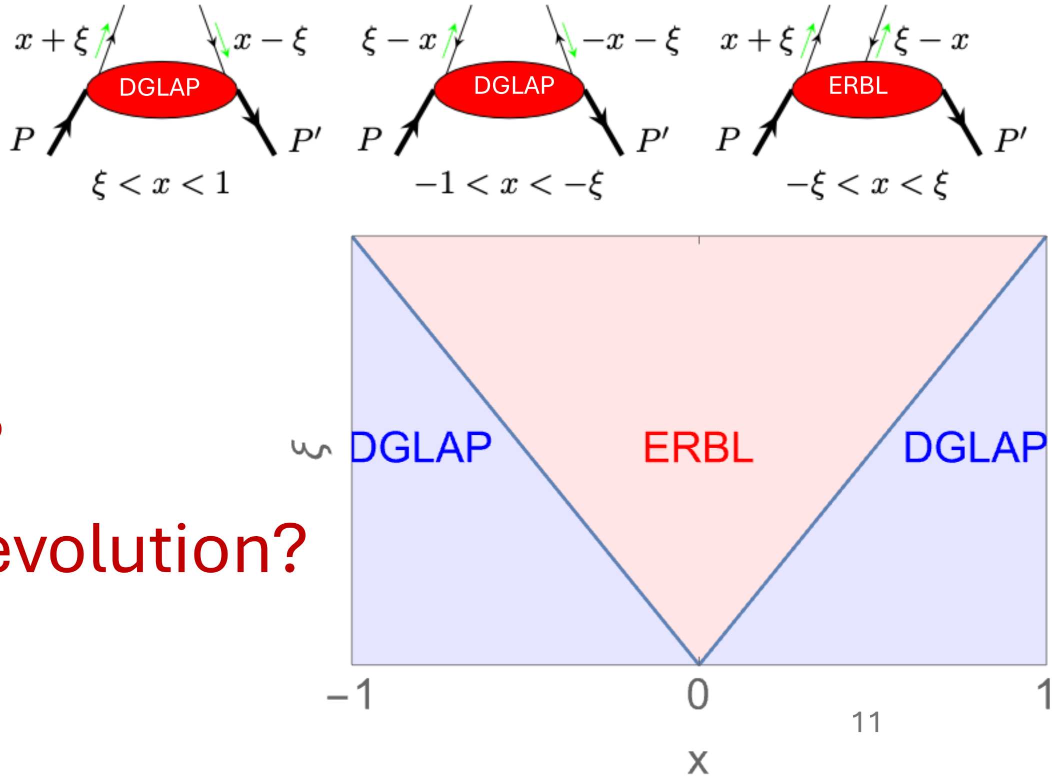
Ji, Liu & Su JHEP (2023)

Numerical implementation in progress

μ dependence in the GPD matching kernel

- ERBL region $|x| < \xi$:
  - $\theta(x-y)\frac{(x-\xi)(x-y+2\xi)}{(x-y)(y-\xi)\xi}\ln\frac{Q_1^2}{\mu^2} + \theta(y-x)\frac{(x+\xi)(x-y-2\xi)}{(x-y)(y+\xi)\xi}\ln\frac{Q_2^2}{\mu^2}$

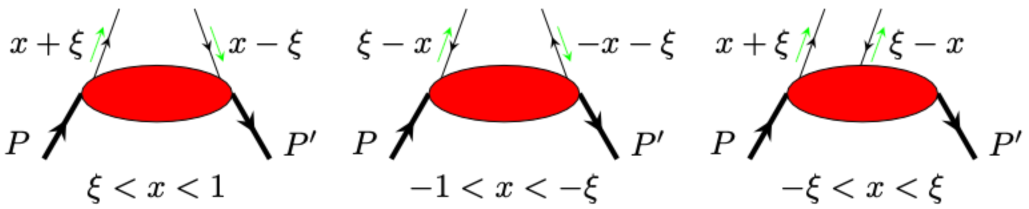
Ji, et al. RMP (2022)



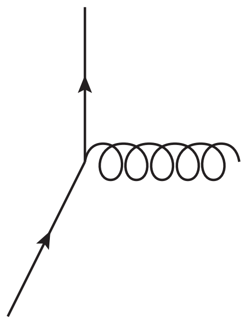
- DGLAP region $|x| > \xi$:
  - $\theta(|y|-|x|)\frac{2(x^2+y^2-2\xi^2)}{(x-y)(y^2-\xi^2)}\ln\frac{Q^2}{\mu^2}$

- What are the physical scales Q_1, Q_2, Q?
- Can we resum the logarithms w/ GPD evolution?

Full logarithms



- Outgoing quark/Incoming antiquark momentum
  - $\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right)$
  - Suppressed when $(\xi + x) \to 0$ except for $x \to y$ or $\xi \to 0$
- Incoming quark/Outgoing antiquark momentum
  - $\left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right)$
  - Suppressed when $(\xi - x) \to 0$ except for $x \to y$ or $\xi \to 0$
- Gluon emission momentum
  - $\left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\ln\left(\frac{4(y-x)^2 P_z^2}{\mu^2}\right)$
  - Enhanced when $x \to y$



In the ERBL region, logarithms are important only in the threshold limit $x \longrightarrow y$!

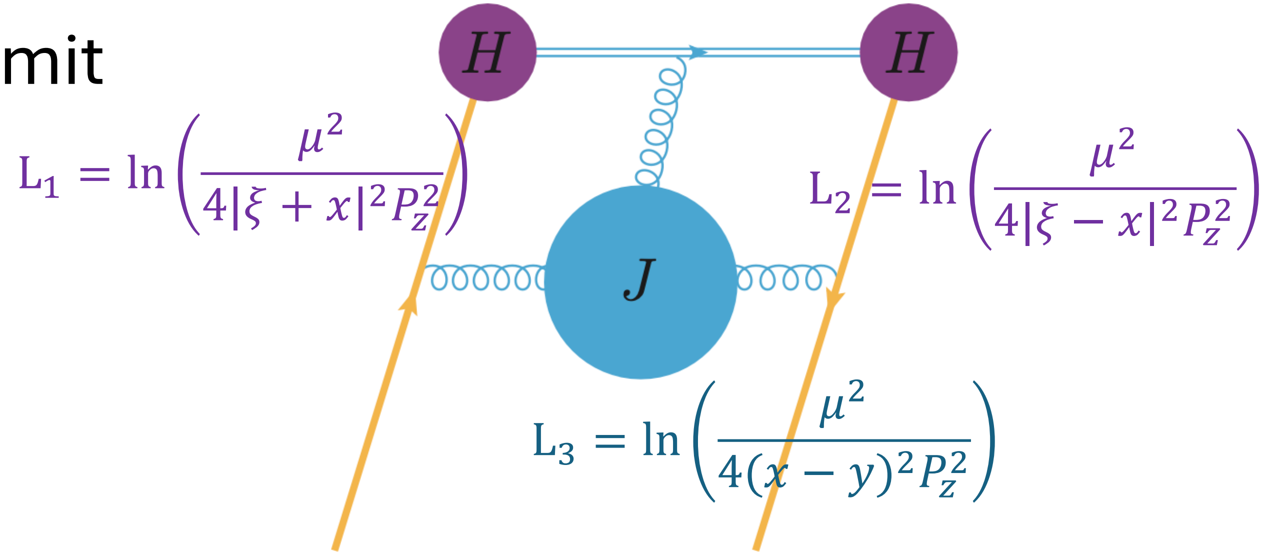
What's magical in the threshold limit $x \longrightarrow y$?

Factorizing Hard and "Soft" scales

- In ERBL region, logarithms are important only in the threshold limit
  - $x - y \to 0$, soft gluon emission
- Integrate out hard modes
  - Sudakov factor H
    - Quark component
    - Anti-quark component
- Integrate out hard collinear modes
  - Jet function J

Becher, Neubert & Pecjak JHEP(2007)

$$\mathcal{C}(x, y, \mu, P_z) \xrightarrow{x \to y} H(xP_z, \bar{x}P_z, \mu) \otimes J(|x-y|P_z, \mu)$$



Threshold log: soft gluon

Ji, Liu & Su JHEP (2023)

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Separating all three scales

- $C(x \to y, \xi, \mu, P) \approx H(|\xi + x|P, \mu) H(|\xi - x|P, \mu) J(|x - y|P, \mu)$
- $H\left(L_z^{\pm} = \ln\left(\frac{2xP}{\mu}\right)^2 + i\pi\,\mathrm{sgn}(zx), \mu\right) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi}\left[-\frac{1}{2}(L_z^{\pm})^2 + L_z^{\pm} - 2 - \frac{5\pi^2}{12}\right]$

Vladimirov & Schafer PRD (2020)
Ji & Liu PRD (2022)

- $J\left(l_z = \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}, \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{2}l_z^2 + l_z + \frac{\pi^2}{12} + 2\right)$

Ji, Liu & Su JHEP (2023)

- Double logarithm come from soft and colinear divergences
- Cancellation of $\ln^2 \mu^2$ between H and J happens at all orders

Correcting the matching kernel

- Resummed Sudakov factor: $H = |H|e^{i\hat{A}}$

$$H(p,\mu,\pm) = |H|(p,\mu_h)e^{S(\mu_h,\mu)-a_H(\mu_h,\mu)}e^{iA_\pm(\mu_h)\mp i\frac{\pi}{2}a_\Gamma(\mu_h,\mu)}\left(\frac{\mu_h}{2p}\right)^{a_\Gamma(\mu_h,\mu)}$$

- Resummed Jet function:

Becher, Neubert & Pecjak JHEP(2007)

$$J(\Delta,\mu) = e^{[-2S(\mu_i,\mu)+a_J(\mu_i,\mu)]}\tilde{J}_z(l_z=-2\partial_\eta,\alpha_s(\mu_i))\left[\frac{\sin(\eta\pi/2)}{|\Delta|}\left(\frac{2|\Delta|}{\mu_i}\right)^\eta\right]_* \frac{\Gamma(1-\eta)e^{-\eta\gamma_E}}{\pi}\Bigg|_{\eta=2a_\Gamma(\mu_i,\mu)}$$

- $C_{TR} = (H\otimes J)_{TR}\otimes(H\otimes J)^{-1}_{NLO}\otimes C_{NLO}$
- Inverse matching:

$$C^{-1}_{TR} = C^{-1}_{NLO}\otimes(H\otimes J)_{NLO}\otimes(H\otimes J)^{-1}_{TR}$$

What are the scale choices of $\mu_{1,2}$ and μ_i?

Scale choices of resummation

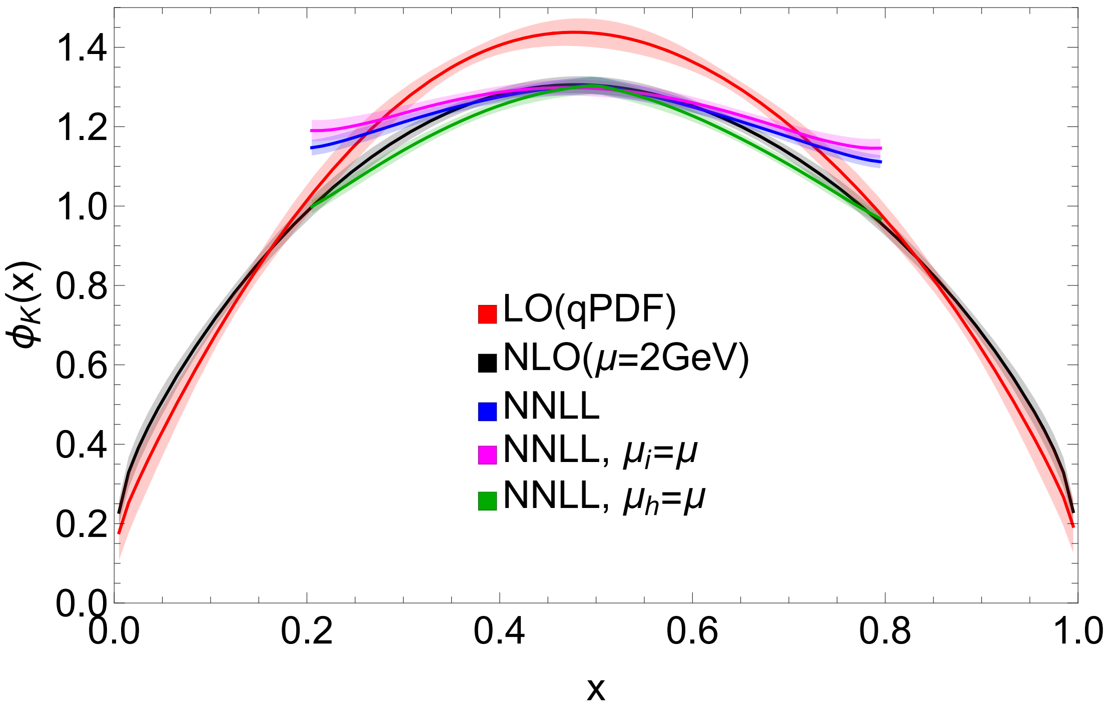
- Hard scale:
  - $H(|\xi + x|P, \mu)$: quark momentum $\mu_{h_1} = 2|\xi + x|P$
  - $H(|\xi - x|P, \mu)$: anti-quark momentum $\mu_{h_2} = 2|\xi - x|P$
- Semi-hard scale:
  - $J(|y - x|P, \mu)$: gluon momentum $\mu_i = 2|y - x|P$?
  - This scale choice is not applicable because $\mu_i \to 0$ hits the Landau Pole for any given x!

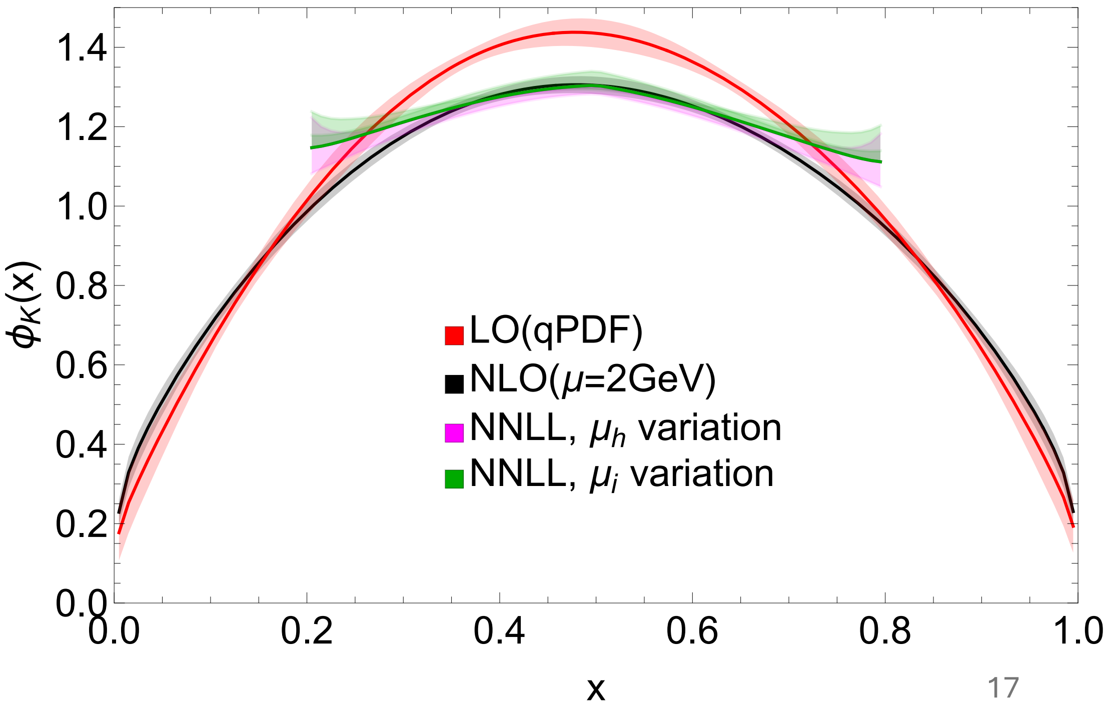
Becher, Neubert & Pecjak JHEP(2007)

- Actual semi-hard scale choice turns out to be
  - $2|\xi + x|P$ when $x \to -\xi$ in ERBL region
  - $2|\xi - x|P$ when $x \to \xi$ in ERBL region
  - $2(1 - |x|)P$ when $x \to \pm 1$ in DGLAP region

An application to distribution amplitude

- DA is very similar to $\xi \to 1$ case in GPD $(x \to (1+x)/2)$
- Resummation effect of H and J are opposite
- Scale variation is controllable in $x \in [0.2, 0.8]$





Logarithms in DGLAP region

- Outgoing quark momentum
  - $A = \left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right) \ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right)$
- Incoming quark/Outgoing antiquark momentum
  - $B = \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right) \ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right)$
- When $\xi \to 0$, the $\frac{1}{\xi}$ prefactor enhance the logarithm
  - $\lim_{\xi\to 0} A + B = \frac{x^2+y^2}{y^2(y-x)} \ln \frac{4x^2 P^2}{\mu^2}$
- The logarithm is not suppressed as $x \ln x$
- We cannot just resum in the threshold limit $x \to y$

Logarithms in DGLAP region

- Outgoing quark momentum
  - $\left(\frac{|\xi+x|}{2\xi(\xi+y)}+\frac{|\xi+x|}{(\xi+y)(y-x)}\right)\ln\left(\frac{4(\xi+x)^2P_z^2}{\mu^2}\right)$
- Incoming quark/Outgoing antiquark momentum
  - $\left(\frac{|\xi-x|}{2\xi(\xi-y)}+\frac{|\xi-x|}{(\xi-y)(x-y)}\right)\ln\left(\frac{4(\xi-x)^2P_z^2}{\mu^2}\right)$
- If we choose $\mu = 2|x|P_z$, then the logarithms are still suppressed in the $\xi \to 0$ limit!

When $\mu = 2xP_z$, in both DGLAP region and ERBL region, logarithms are important only in the threshold limit $x \to y$!

We can still resum the remaining logarithm in the threshold limit.

2-step resummation in DGLAP region

If we set $\mu = 2|x|P_z$, the logarithm are important only in threshold limit, threshold factorization works!

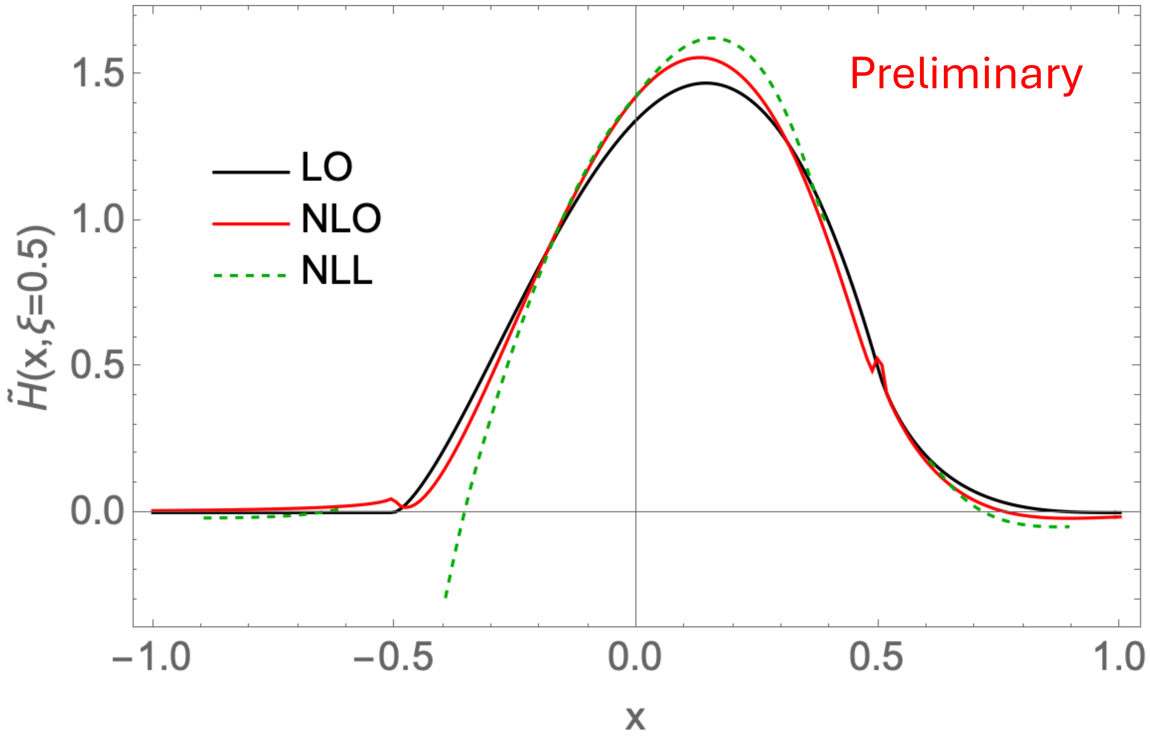
1. First evaluate the full kernel $C^{NLO}(x, y, \xi, P, \mu_h)$ at $\mu_h = 2|x|P_z$
2. Then resum logarithm in threshold limit at $\mu_h = 2|x|P_z$
   1. $H(\mu_h, x, \xi, P) = H\left(\mu_{h_1}, |\xi + x|P\right) H\left(\mu_{h_2}, |\xi - x|P\right) e^{S\left(\mu_{h_1},\mu_h\right)+S\left(\mu_{h_2},\mu_h\right)\dots}$
   2. $J(\mu_h, x, y, P) = J(\mu_i, |x - y|P)\, e^{-2S(\mu_i,\mu_h)\dots}$
3. Using the full evolution to evolve C^{TR} to scale μ
   1. $C(x, y, \xi, P, \mu) = C^{TR}(x, y, \xi, P, \mu_h)\hat{V}(\mu_h, \mu)$

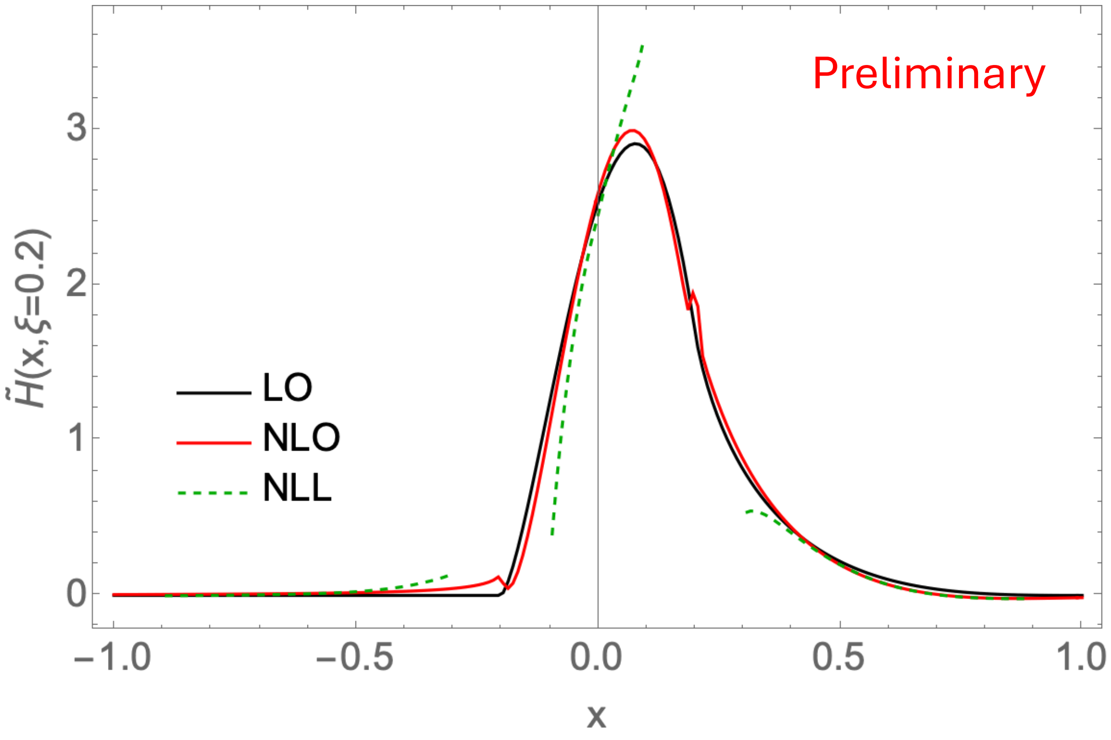
Preliminary test on a quark GPD model

- Double-distribution:

Belitsky and Radyushkin: Phys.Rept.(2005)

$$H(x,\xi) = \theta(x > -\xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x+\xi}{1+\xi}\right)^{\lambda}[\xi^2 - x + \lambda\xi(1-x)]$$
$$-\theta(x > \xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x-\xi}{1-\xi}\right)^{\lambda}[\xi^2 - x - \lambda\xi(1-x)],$$





Noticeable resummation effects in ERBL region when getting close to $x \to \xi$!

Summary and Outlook

Summary

- ❖ We found the logarithms in the pert matching of quasi-GPD to GPD are only important in the threshold limit or $\xi \to 0$ limit
- ❖ At zero skewness $\xi = 0$, the resummation is the same as PDF
- ❖ We develop a first formalism to resum the logarithms at non-zero skewness
- ❖ Resummation effect is small in DGLAP region, but noticeable in ERBL region

Outlook

- ➢ We will examine scale variation to determine the applicable range of perturbation theory
- ➢ We will apply the method to inverse matching, and apply to real lattice quasi-GPD data
- ➢ We will include the leading renormalon resummation to subtract linear power corrections from the renormalon in both jet function and the imaginary part of Sudakov factors

Zhang, et al., PLB(2023)
Liu, et al., JHEP(2023)

Thank you for listening!