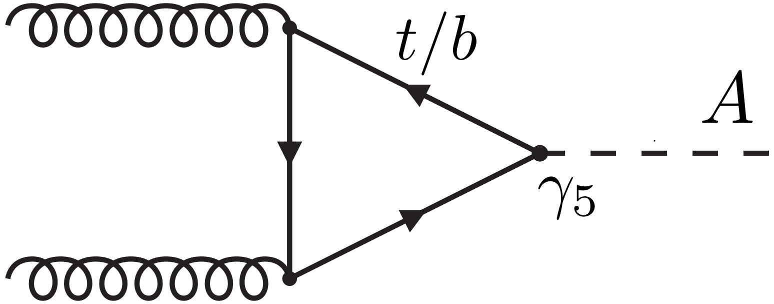
Pseudoscalar Higgs plus jet production at NNLO in QCD

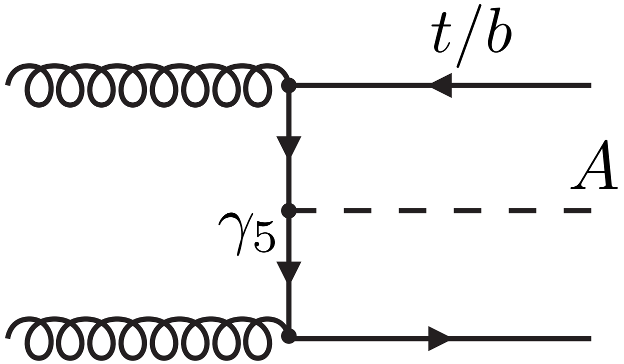
Based on (2405.02210) with Youngjin Kim

University at Buffalo The State University of New York

Overview

Pseudoscalar - Setup



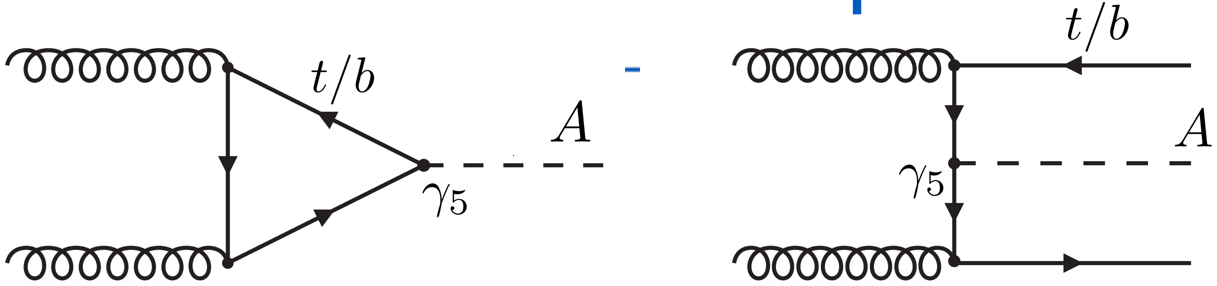


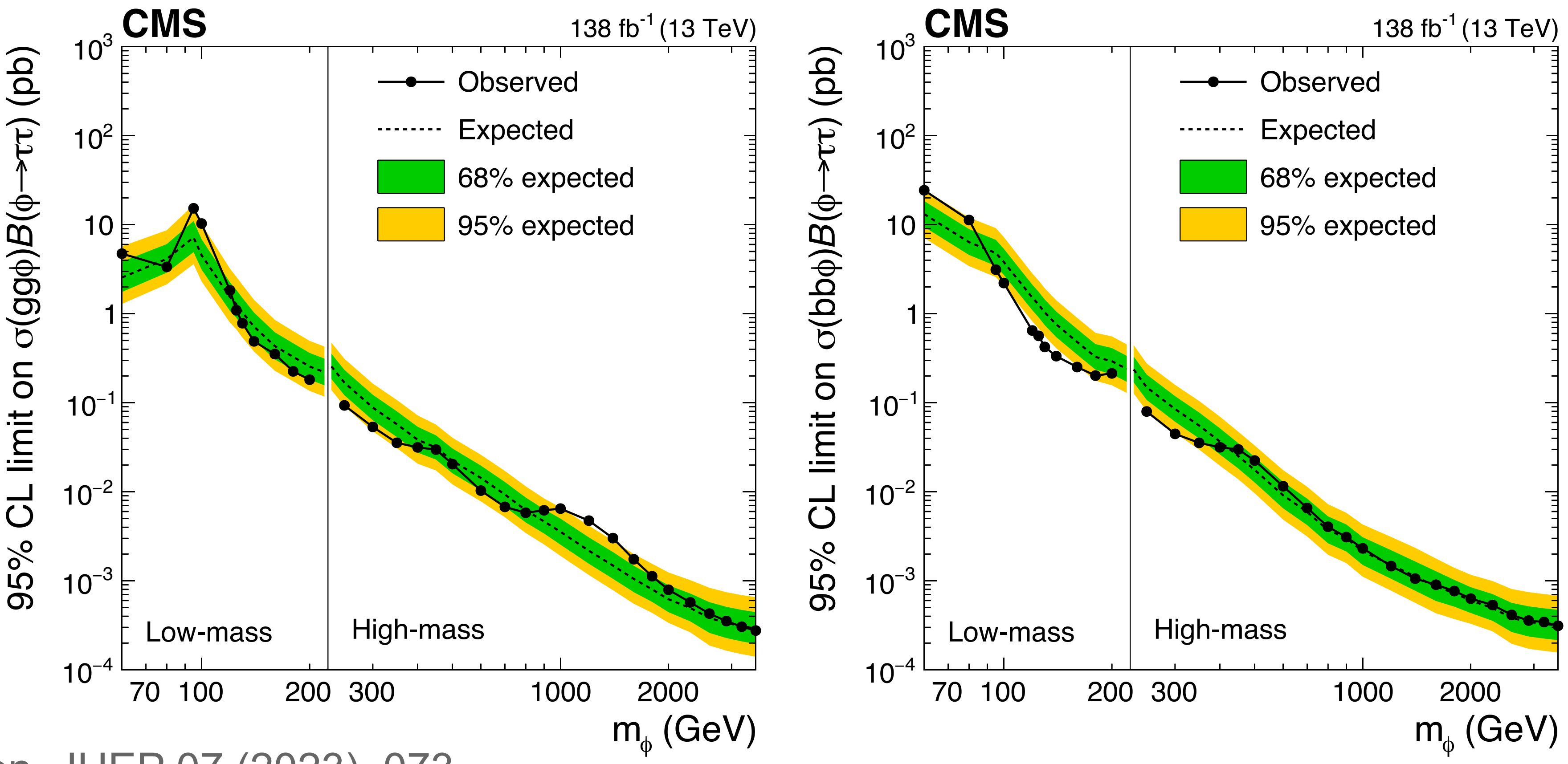
Many BSM models include an extended Higgs sector, with two (or more) scalar doublets included (e.g. 2HDM, MSSM etc etc.). These naturally produce both CP even (h, H) and CP odd (A) neutral scalar bosons.

An important point is that the A boson will not couple to up and down type quarks equally.

This introduces a new parameter typically defined as $\tan\beta$ which sets the ratio of the two vevs.

Some Recent results from LHC for psuedoscalars



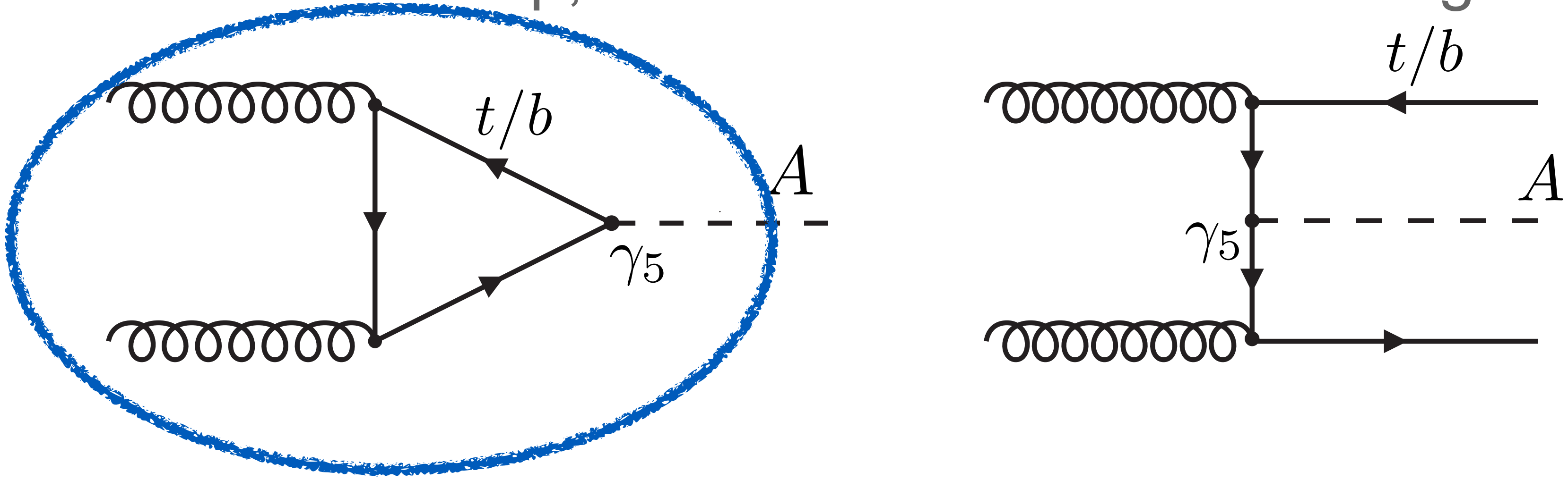


CMS Collaboration, JHEP 07 (2023), 073

Pseudoscalar - Setup

Clearly the pheno is rich and varied based on the specific values of $\tan\beta$ and other new interactions.

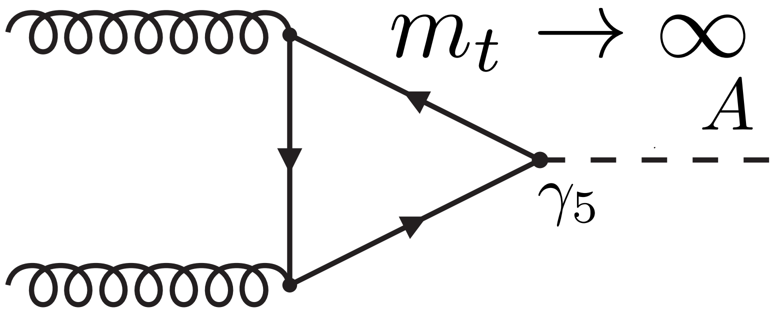
Today's talk will focus on the ggA contribution, and in particular the contributions from the top, where we will use an EFT to go to NNLO.



We therefore dont want a super large coupling to bottoms or a very heavy pseudoscalar

The Effective Lagrangian

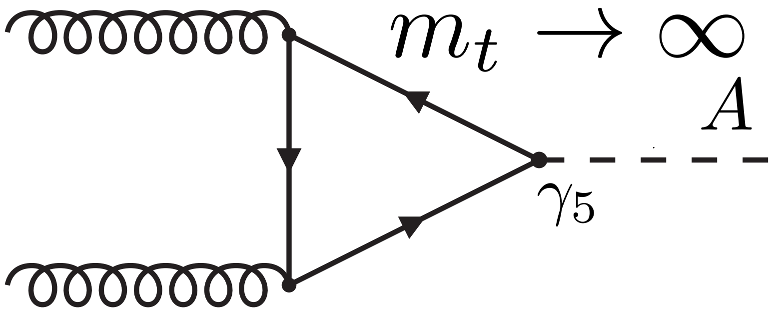
Taking the mass of the top to infinity introduces the effective Lagrangian



$$\mathcal{L}_{\text{eff}}^{A} = -A\Big[C_G O_G(x) + C_J O_J(x)\Big]$$

The Effective Lagrangian

Taking the mass of the top to infinity introduces the effective Lagrangian



$$\mathcal{L}^{A}_{\text{eff}} = -A\left[C_G O_G(x) + C_J O_J(x)\right]$$

Specifically,

$$O_G(x) = G^{\mu\nu}_a \tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu}_a G^{\rho\sigma}_a \,, \qquad O_J(x) = \partial_\mu \left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

Wilson Coefficients

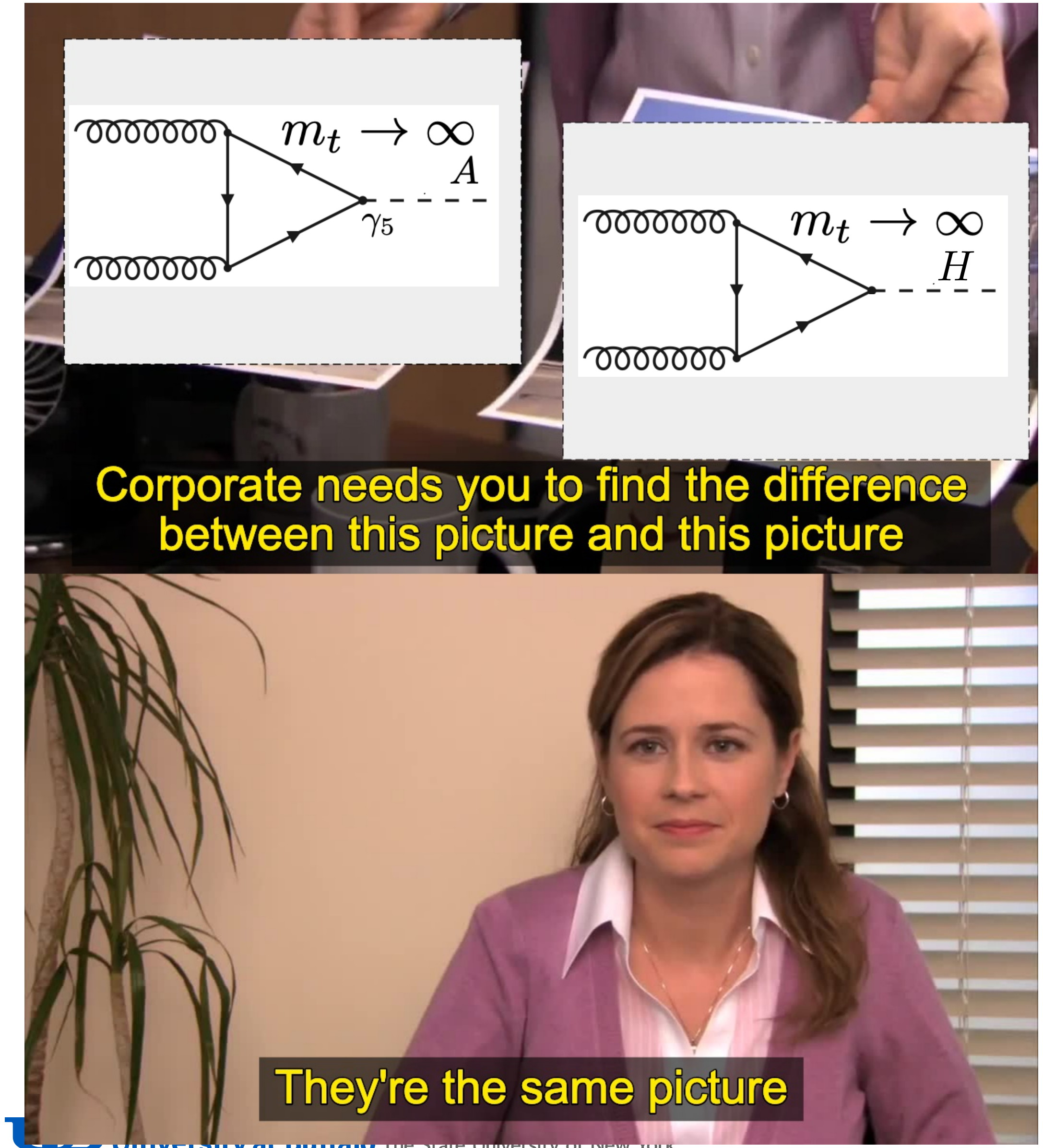
$$\mathcal{L}_{\text{eff}}^{A} = -A\Big[C_G O_G(x) + C_J O_J(x)\Big]$$

The two Wilson coefficients are known to sufficiently high orders in α_s for our purposes

$$C_G = -\frac{\alpha_s}{2\pi}\frac{1}{v}\left(\frac{1}{8}\right), \quad \text{when} \quad \tan\beta \sim 1$$

$$C_J = -\left[\left(\frac{\alpha_s}{2\pi}\right)\frac{C_F}{4}\left(\frac{3}{2} - 3\ln\frac{\mu_R^2}{m_t^2}\right) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_J^{(2)} + \cdots\right] C_G\,.$$

Higgs and Pseudoscalar are kind of the same



While the pseudoscalar is a hypothetical state with a broad range of phenomenological possibilities, it has a rather well known cousin.

At many points in this talk we'll leverage the vast knowledge of the H(125) to complete/check/simplify our results.

Higgs and Pseudoscalar are kind of NOT the same

A closer look at the pictures does reveal a rather non trivial difference between the two, our old friend γ_5

Famously γ_5 does not play nicely with dimensional regulation since its inherently a four-dimensional object.

No free lunch here, we follow the Larin prescription where

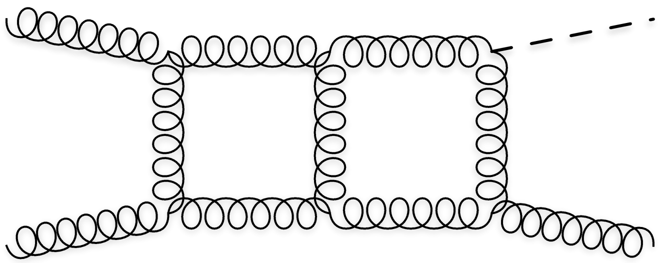
$$\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \quad \text{and} \quad \varepsilon_{\mu_1\nu_1\rho_1\sigma_1}\,\varepsilon^{\mu_2\nu_2\rho_2\sigma_2} = \begin{vmatrix} \delta^{\mu_2}_{\mu_1} & \delta^{\nu_2}_{\mu_1} & \delta^{\rho_2}_{\mu_1} & \delta^{\sigma_2}_{\mu_1} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\nu_2}_{\nu_1} & \delta^{\rho_2}_{\nu_1} & \delta^{\sigma_2}_{\nu_1} \\ \delta^{\mu_2}_{\rho_1} & \delta^{\nu_2}_{\rho_1} & \delta^{\rho_2}_{\rho_1} & \delta^{\sigma_2}_{\rho_1} \\ \delta^{\mu_2}_{\sigma_1} & \delta^{\nu_2}_{\sigma_1} & \delta^{\rho_2}_{\sigma_1} & \delta^{\sigma_2}_{\sigma_1} \end{vmatrix}$$

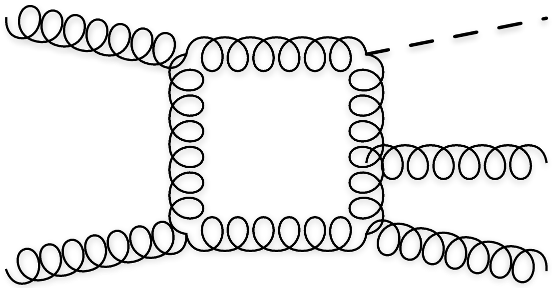
This provides a clean implementation in d-dimensions, but violates the Ward identity. This is restored through a (finite) renormalization of γ_5

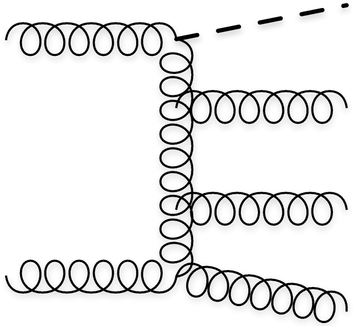
A (brief and incomplete) list of Higgs and A + jets predictions

SM Higgs	Pseudo-Higgs
h@NLO: [Dawson. (91)], [Djouadi, Spira, Zerwas. (91)] h+jet@NLO: [Ravindran, Smith, Neerven. (02)] h@NNLO: [Harlander, Kilgore. (02)], [Anastasiou, Melnikov. (02)], [Ravindran, Smith, Neerven. (03)] h+2jet@NLO: [Campbell, Ellis, CW (10)] h+jet@NNLO: [Boughezal,Caola,Melnikov, Petriello,Schulze. (13)], [Chen,Gehrmann,Glover,Jaquier. (15)], [Boughezal,Caola,Melnikov,Petriello,Schulze. (15)], [Boughezal,Focke,Giele,Liu,Petriello.(15)], [F. Caola, K. Melnikov and M. Schulze.(15)], [Chen, Martinez,Gehrmann, Glover, Jaquier. (16)] [Campbell, Ellis, Seth (19)] h@N3LO: [Anastasiou, Duhr, Dulat, Herzog, Mistlberger. (15)], [Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog, Lazopoulos, Mistlberger. (16)]	A@NLO: [Kauffman, Schaffer. (94)] A+jet@NLO: [Field, Smith, Yeomans. (03)] A@NNLO: [Harlander, Kilgore. (02)] [Anastasiou, Melnikov. (03)], [Ravindran, Smith, Neerven. (03)] A+2jet@NLO: [Demartin, Maltoni, Kentarou, Page, Zaro. (14)] A@N3LO(partially, based on h@N3LO): [Ahmed, Kumar, Mathews, Rana, Ravindran. (16)], [Ahmed, Bonvini, Kumar, Mathews, Rana, Ravindran, Rottoli. (16)] **A+jet@NNLO This talk!**

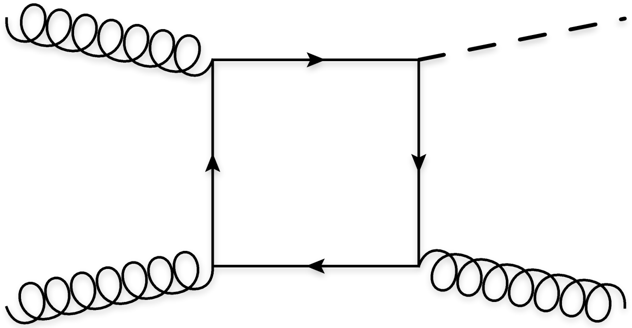
A+j at NNLO

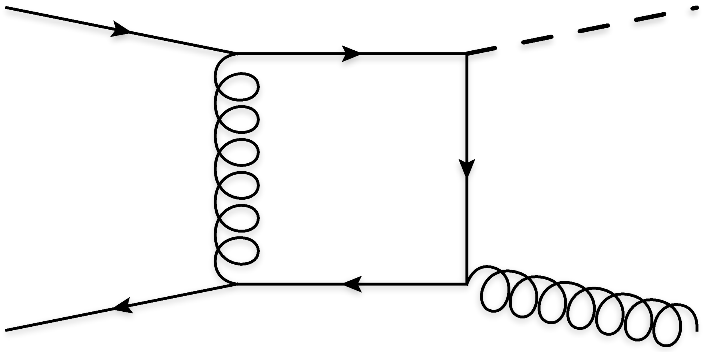






The aim of this talk is to present the results for A+j @ NNLO ($\mathcal{O}(\alpha_s^5)$), for which we have two types of topologies, those from O_G (above) and O_J (below)*



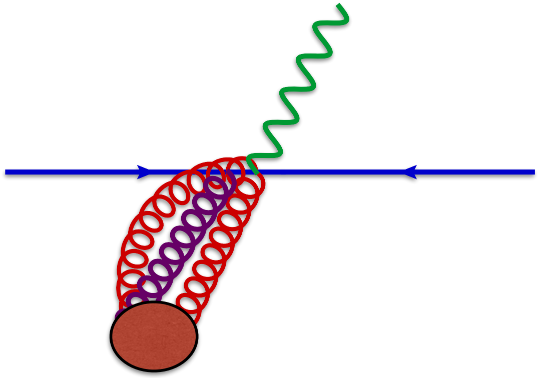


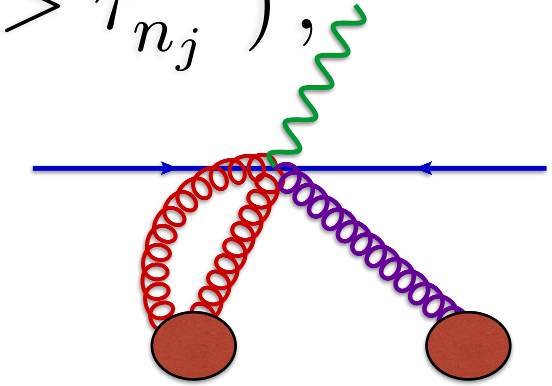
*Of course the operators annoyingly mix under renormalization at this order causing a huge headache, but I'll suppress that for narrative flow…

Slicing @ NNLO

Idea behind a slicing approach is to split the phase space into two based on some suitable variable

$$\sigma^{\mathrm{NNLO}} = \sigma(\tau_N \leq \tau_{n_j}^{\mathrm{cut}}) + \sigma(\tau_N > \tau_{n_j}^{\mathrm{cut}}) \,,$$





Should contain all double unresolved limits, and be accessible via simplified result (i.e. factorization theorem)

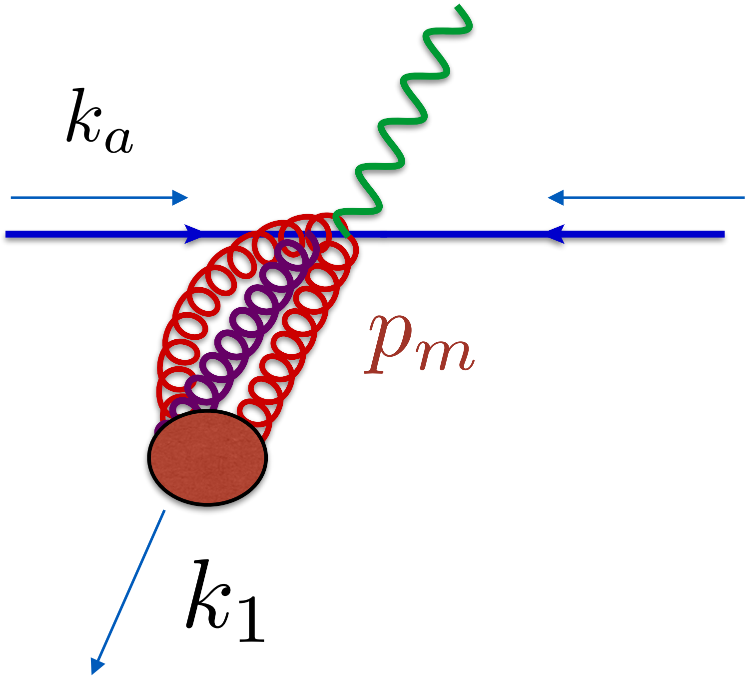
Should contain at most singly unresolved limits, (i.e. an NLO + extra parton) directly compute with suitable Monte Carlo codes

N-jettiness slicing

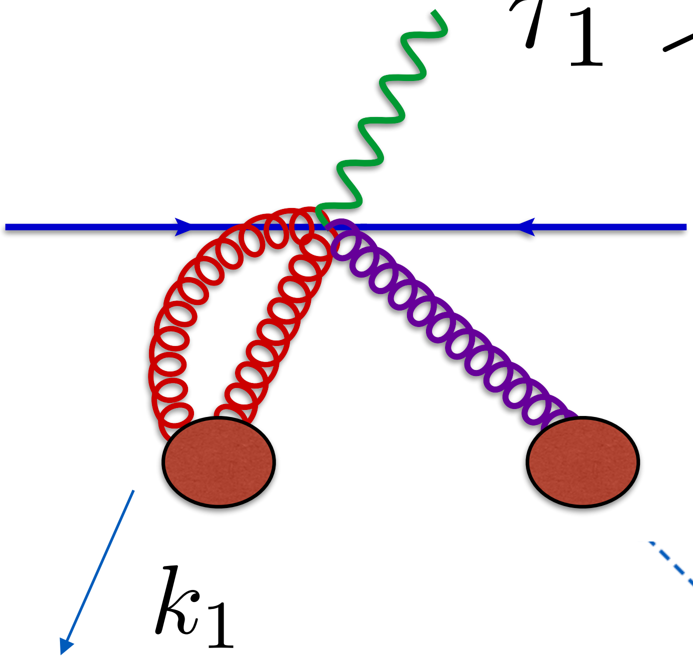
We use the N-jettiness event shape variable **(Stewart, Tackmann Waalewijn 09)** to split the regions

$$\tau_1 = \sum_m \min_i \frac{2p_m \cdot k_i}{P_i} \, ,$$

$\tau_1 \to 0$



$\tau_1 > 0$



SCET factorization

To compute the below-cut piece we can use the following factorization theorem, derived from SCET

$$\sigma(\tau \leq \tau_{n_j}^{\mathrm{cut}}) = \int_0^{\tau_{n_j}^{\mathrm{cut}}} d\tau \left(\mathcal{S} \otimes \prod_{i=1}^{n_\mathrm{j}} \mathcal{J}_i \otimes \prod_{a=1,2} \mathcal{B}_a \otimes \mathcal{H} \right) + \mathcal{F}(\tau_{n_j}^{\mathrm{cut}}),$$

At $\mathcal{O}(\alpha_s^2)$ the various pieces needed are :

- $\mathcal{S}$ - Soft function (for 3 partons) **(Boughezal Liu Petriello 15, Campbell Ellis Mondini CW 17)**
- $\mathcal{J}_i$, $\mathcal{B}_a$ - Jet and beam functions (collinear behavior) **(Becher Bell 10, Gaunt Stahlhofen Tackmann 14)**
- $\mathcal{H}$ - Hard function - process specific finite function.

H+j at NNLO (in MCFM)

Throughout our calculation we will make extensive use of the corresponding Higgs plus jet result implemented into MCFM, presented in **Campbell, Ellis, Seth 19.**

This calculation (based upon **Boughezal, Focke, Giele, Liu, Petriello 15)** uses N-jettiness slicing and **(Campbell, Ellis, Seth 19)** provides a detailed comparison with other existing methods based on Antenna subtraction and Sector decomposition.

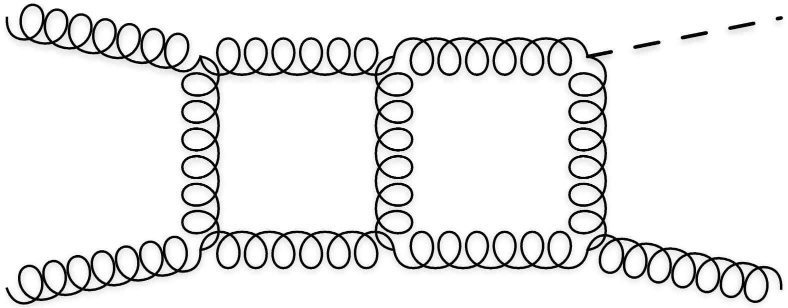
The one sentence summary is that it's hard to get the slicing parameter small enough to exactly match Antenna subtraction results, but using finite τ_1 and performing a fit results in excellent agreement between methods.

Using slicing for the (putative) pseudoscalar phenomenology is therefore reasonable.

Calculation

Two-loop Hard Function

A calculation for $A \to 3$ partons exists **(Banerjee, Dhani, Ravidran 17)**, but since we needed to cross it for LHC kinematics we did a fully independent calculation. (Re)calculated Higgs amplitudes too as a cross check.



Initial diagram generation and Feynman rules done with two independent implementations.

Diagrams then reduced to MI's via LiteRed **(Lee 14)** + Kira **(Maierhöfer Usovitsch, Uwer 18)**

Then UV renormalization ($\alpha_s, \gamma_5,$ and operator mixing), then extraction of IR poles through Catani's operator.

Then starting from initial decay kinematics we cross partons to the initial state using the coproduct method **(Duhr 12 & 14)**.

We checked each crossing numerically using AMFlow **(Liu, Ma 22)**

Collinear test of the two-loop hard function.

We checked against the known collinear limits of two-loop QCD amplitudes (Badger, Glover 04) in order to further validate our result.

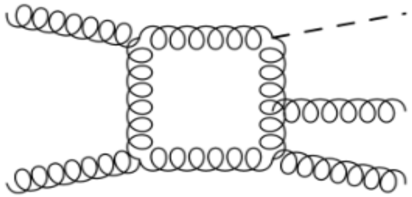
$$\hat{\mathcal{M}}_f^{G,(2)}\hat{\mathcal{M}}_f^{G,(0)*} \to C_f^{(2)} = P_f^{(0)} \cdot \hat{\mathcal{M}}_{A\to gg}^{G,(2)}\hat{\mathcal{M}}_{A\to gg}^{G,(0)*} + P_f^{(1)} \cdot \hat{\mathcal{M}}_{A\to gg}^{G,(1)}\hat{\mathcal{M}}_{A\to gg}^{G,(0)*} + P_f^{(2)} \cdot \hat{\mathcal{M}}_{A\to gg}^{G,(0)}\hat{\mathcal{M}}_{A\to gg}^{G,(0)*}.$$

Coefficient	$y\,C_{ggg}^{(2)}$	$y\,\hat{\mathcal{M}}_{ggg}^{G,(2)}\hat{\mathcal{M}}_{ggg}^{G,(0)*}$	$x\,C_{q\bar{q}g}^{(2)}$	$x\,\hat{\mathcal{M}}_{q\bar{q}g}^{G,(2)}\hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)*}$
ϵ^{-4}	$1.20981960 \cdot 10^6$	$1.20981960 \cdot 10^6$	$4.05026555 \cdot 10^2$	$4.05026555 \cdot 10^2$
ϵ^{-3}	$1.58228295 \cdot 10^7$	$1.58228295 \cdot 10^7$	$-2.59019027 \cdot 10^3$	$-2.59019027 \cdot 10^3$
ϵ^{-2}	$2.36283980 \cdot 10^8$	$2.36283980 \cdot 10^8$	$-1.20976857 \cdot 10^4$	$-1.20976857 \cdot 10^4$
ϵ^{-1}	$2.58965014 \cdot 10^9$	$2.58966527 \cdot 10^9$	$5.16726263 \cdot 10^4$	$5.16726262 \cdot 10^4$
ϵ^{0}	$2.19247701 \cdot 10^{10}$	$2.19253448 \cdot 10^{10}$	$2.38532152 \cdot 10^5$	$2.38475465 \cdot 10^5$

Also able to confirm our results with the literature (Banerjee, Dhani, Ravidran 17) for decay kinematics

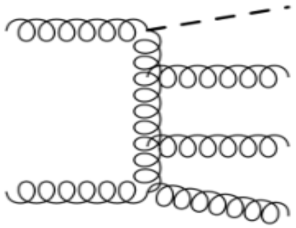
Also fully reproduced H+j 2-loop amplitudes too (Gehrmann, Jaquier, Glover, Koukoutsakis 11)

Above Cut: A+2j@NLO with C.S



(b) Real-Virtual

RR radiation;
obtained compact expressions with BCFW recursion

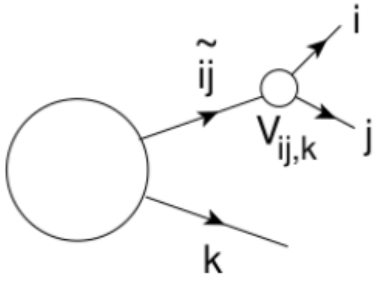


(c) Real-Real

RV radiation;
Process specific. Calculated using generalized unitarity

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^{R}\Big|_{\varepsilon=0} - d\sigma^{A}\Big|_{\varepsilon=0} \right] + \int_{m} \left[d\sigma^{V} + \int_{1} d\sigma^{A} \right]_{\varepsilon=0}$$

Dipole subtraction. [Catani, Seymour. (97)]

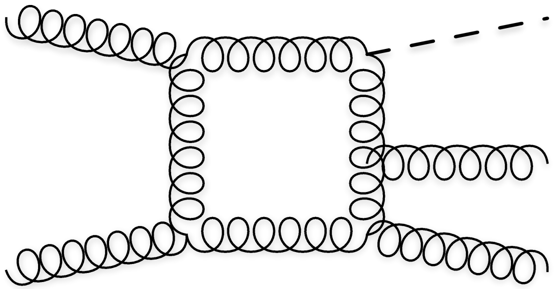


Dipole sub. terms;
Universal;

C.S insertion OP.;
Universal

Calculation of the real-virtual terms

$$\sigma^{NLO} = \int_{m+1}\left[d\sigma^R\Big|_{\varepsilon=0} - d\sigma^A\Big|_{\varepsilon=0}\right] + \int_m \left[\boxed{d\sigma^V} + \int_1 d\sigma^A\right]_{\varepsilon=0}$$



$$\mathcal{M}^{(1)}(H;\{p_k\}) = \mathcal{M}^{(1)}(\phi;\{p_k\}) + \mathcal{M}^{(1)}(\phi^\dagger;\{p_k\})$$

$$\phi = \frac{H+iA}{2}, \qquad \phi^\dagger = \frac{H-iA}{2}$$

[Dixon, Glover, Khoze. 04],
[Badger, Glover 06],
[Dixon, Sofianatos 09],
[Badger, Glover, Mastrolia, C.W 10],
[Badger, Campbell, Ellis, C.W. 09]

$$\mathcal{M}^{(1)}(A;\{p_k\}) = \frac{1}{i}\left(\mathcal{M}^{(1)}(\phi;\{p_k\}) - \mathcal{M}^{(1)}(\phi^\dagger;\{p_k\})\right)$$

Validation of the Above Cut

$$\sigma^{NLO} = \int_{m+1} \left[\boxed{d\sigma^R}_{\varepsilon=0} - d\sigma^A \Big|_{\varepsilon=0} \right] + \int_m \left[\boxed{d\sigma^V} + \int_1 d\sigma^A \right]_{\varepsilon=0}$$

```
# -----------------------
# Renormalisation scale:
# mu =    125.0000000000000
# -----------------------
#                LO:    9.506392576786364
# NLO, finite part:    51.88802421211302
# NLO, single pole:    6.845872960069436
# NLO, double pole:   -12.00000000000001
# IR,  single pole:    6.845872960052783
# IR,  double pole:   -12.00000000000000
#   Time/Event [ms]:                  1071.299
youngjin@youngjin-ThinkPad-X270:~/Research/Ato3P/GoSam/gg_Agg_OneLoop_MyGosam/vi
rtual/matrix$
```

GOSAM

```
|==============================================================================
|| Results for process gg > x0gg
|==============================================================================
|| Phase-Space point specification (E,px,py,pz)
|
|  2.500000000000000e+02  0.000000000000000e+00  0.000000000000000e+00  2.500000000000000e+02
|  2.500000000000000e+02  0.000000000000000e+00  0.000000000000000e+00 -2.500000000000000e+02
|  1.436778510616080e+02  5.166336491841381e+01 -2.254713401226180e+01  4.290510877298324e+01
|  1.902031886378761e+02 -1.533611083047499e+02 -1.082357859069662e+02 -3.070241157719544e+01
|  1.661189603005159e+02  1.016977433863362e+02  1.307829199192279e+02 -1.220269719578784e+01
|
| Unknown numerical stability because MadLoop is in the initialization stage.
|
|| Total(*) Born contribution (GeV^-2):
|    Born        =  1.2326332666199839e-07
|| Total(*) virtual contribution normalized with born*alpha_S/(2*pi):
|    Finite      =  5.1888024212099019e+01
|    Single pole =  6.8458729600371520e+00
|    Double pole = -1.2000000000000041e+01
|  (*) The results above sum all starred contributions below
|
|| All Born contributions are of split orders *(QCD=4)
|
|| All virtual contributions are of split orders *(QCD=6)
|==============================================================================
```

MadGraph

```
gg_Agg_lo w/o any factor:
  2433.6364996573238
qqbar_Agg_lo w/o any factor:
  3.5454810552182510
qqbar_AQQbar_lo w/o any factor:
 0.33504344652285434
qqbar_Aqqbar_lo w/o any factor:
  10.585098640980345
```

```
gg_Agg_oneloop w/o any factor:
  126276.58961768115
qqbar_Agg_oneloop w/o any factor:
  19.977526515793116
qqbar_AQQbar_oneloop w/o any factor:
 0.28125907643039583
qqbar_Aqqbar_oneloop w/o any factor:
  668.68877648164994
```

MCFM

```
In[1]:= mcfmLO = 2433.6364996573238;
        mcfmOneLoop = 126 276.58961768115;
        mcfmOneLoopNormalized = mcfmOneLoop / mcfmLO

Out[3]= 51.888
```

Comparisons: Good!

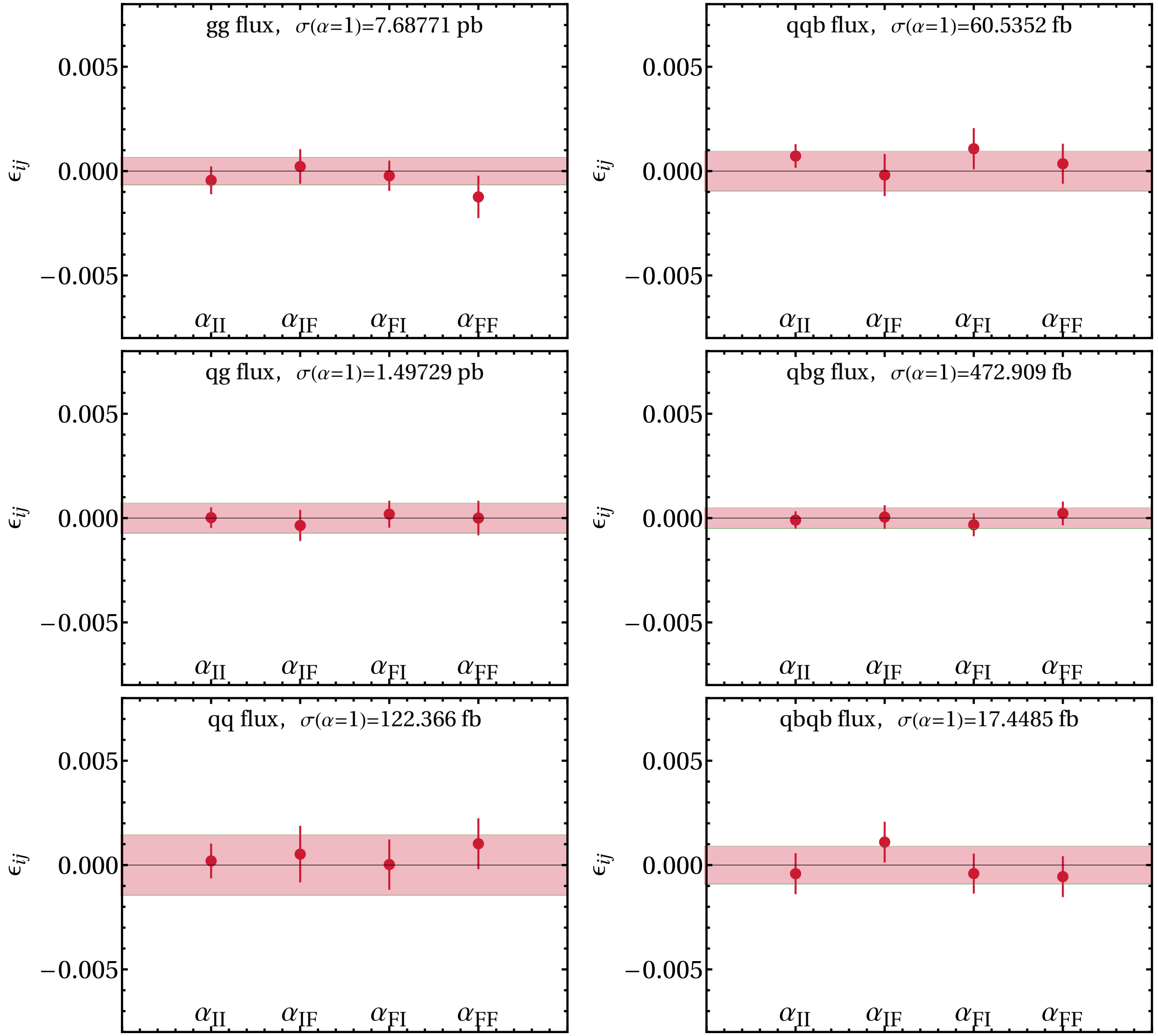
```
In[12]:= gosam = 51.88802421211302;
         madgraph = 5.1888024212099019 * 10^1;
         gosam - mcfmOneLoopNormalized
         madgraph - mcfmOneLoopNormalized

Out[14]= 8.23519 × 10^-12

Out[15]= -5.76961 × 10^-12
```

Agreed!

Checking A+2j@NLO

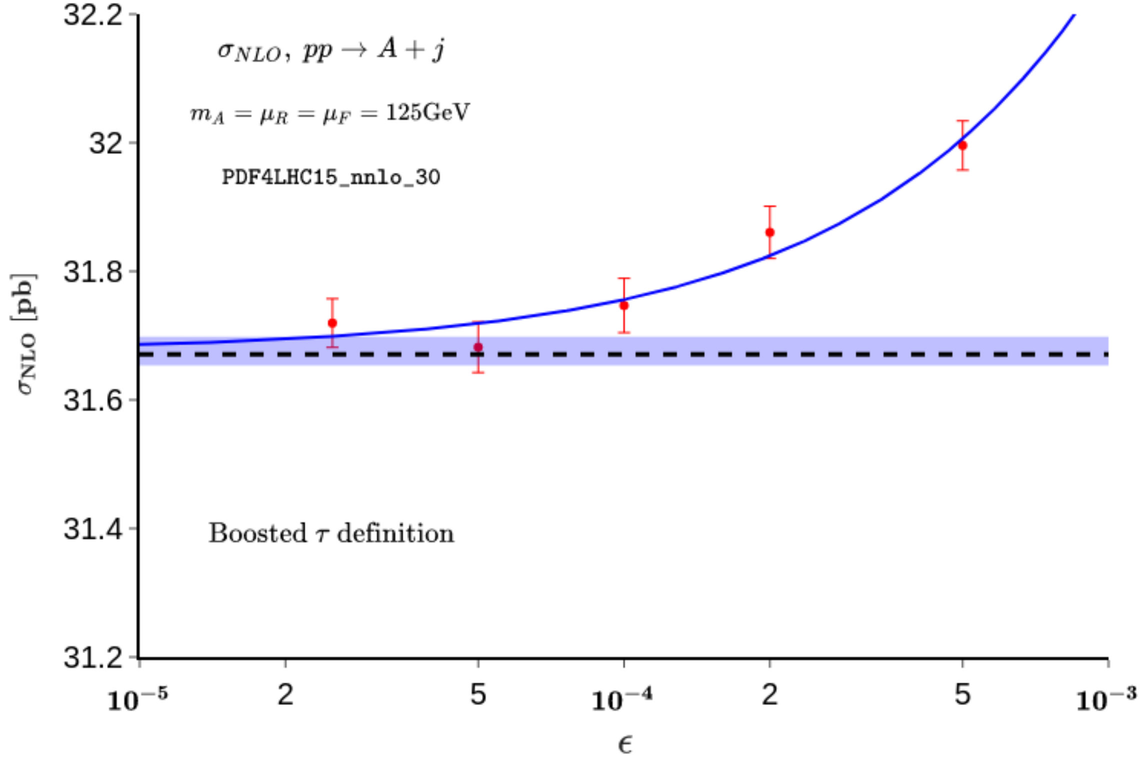


As a final check of our above cut contribution we can test the dipole cancelation across the two phase spaces by varying the unphysical “alpha” parameters associated with each dipole configuration.

$$\epsilon^{ab} = \frac{\sigma(\alpha_{ab} = 1) - \sigma(\alpha_{ab} = 0.01)}{\sigma(\alpha_{ab} = 1)},$$

Results for A+j

Validation at NLO



$$\text{LHC}, \ \sqrt{s} = 13 \text{ TeV}, \qquad \mu_R = \mu_F = m_A = 125 \text{ GeV},$$
$$p_T^{\rm jet} > 20 \text{ GeV}, \qquad \Delta R = 0.4$$
$$\text{anti}-k_T, \qquad \text{no explicit cut on rapidities}.$$

The first thing to do, is to check the dependence on the 1-jettiness cut at NLO, and compare to the dipole result.

$$\tau_1^{\rm cut} = \epsilon \times \sqrt{m_A^2 + \left(p_T^{j_1}\right)^2}.$$

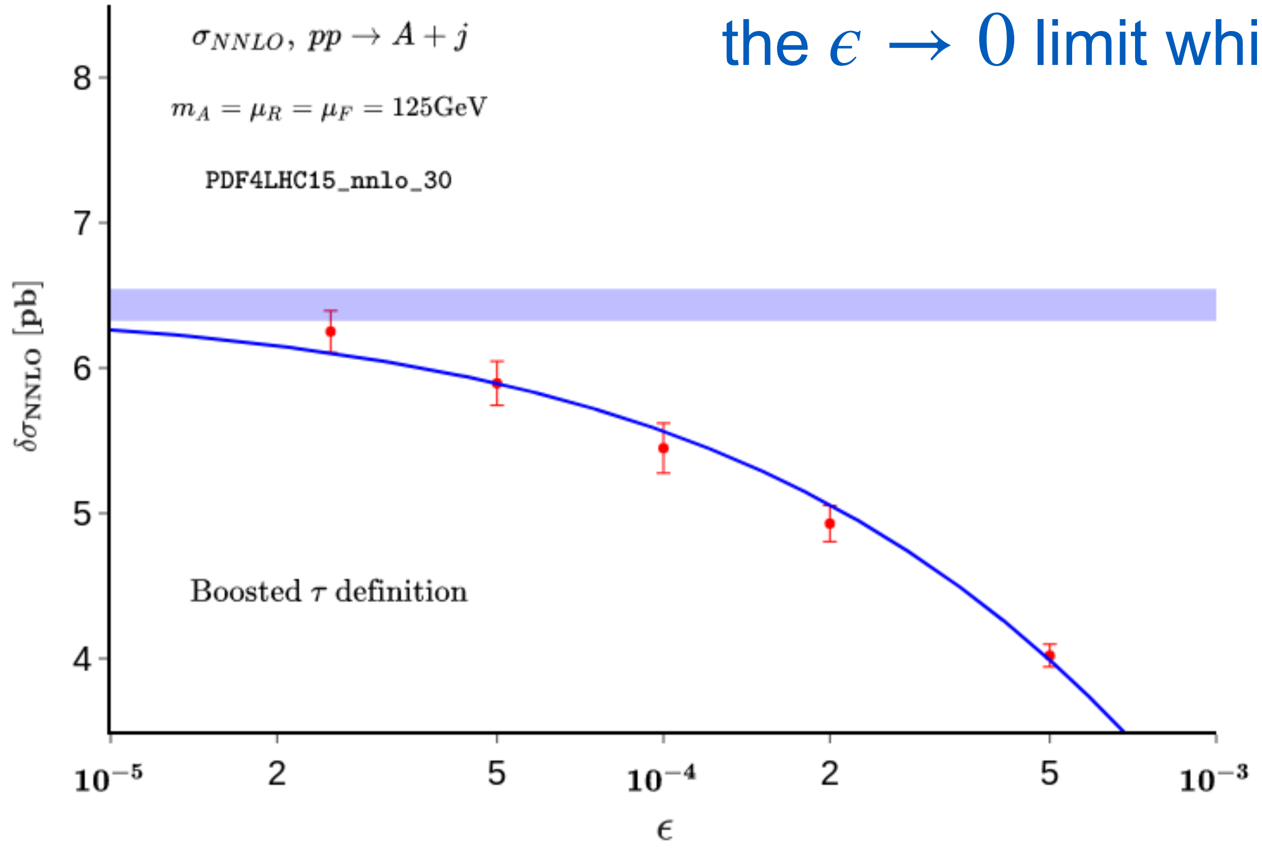
Asymptotic behavior is confirmed and the two results are in perfect agreement, writing,

$$\sigma_{NLO}(\epsilon) = \sigma_{NLO}^0 + c_0 \epsilon \log(\epsilon) + \cdots,$$

We find

$$\sigma_{NLO}^0 = 31.674 \pm 0.022 \text{ pb},$$
$$\sigma_{NLO}^{dipole} = 31.675 \pm 0.031 \text{ pb}.$$

Validation at NNLO



Validation at NNLO is trickier, since we have to extract the $\epsilon \to 0$ limit while fighting rising MC uncertainties.

Defining,

$$\delta\sigma_{NNLO}(\epsilon) = \delta\sigma^0_{NNLO} + c_0 \epsilon \log^3(\epsilon) + \cdots$$

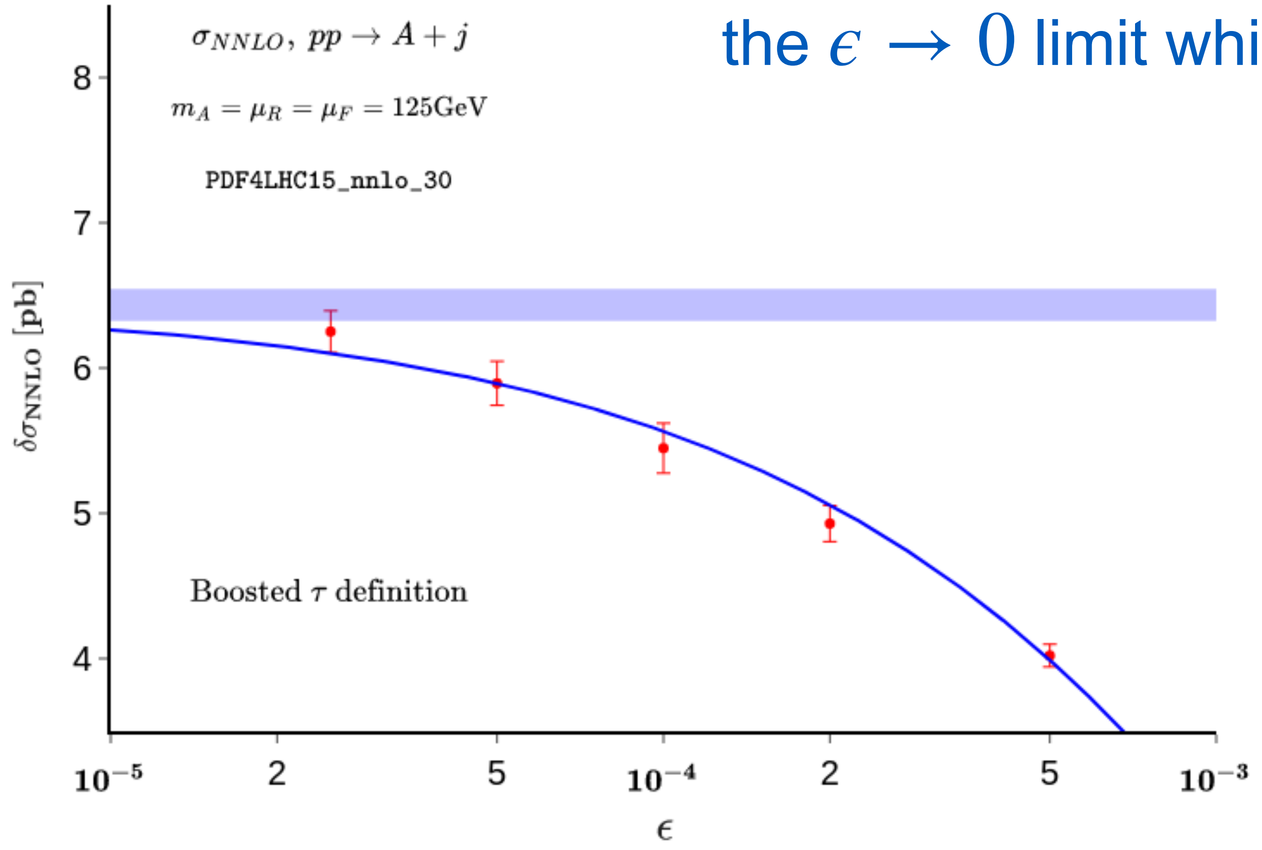
We find, $\delta\sigma^0_{NNLO} = 6.435 \pm 0.083 \text{ pb}\,.$

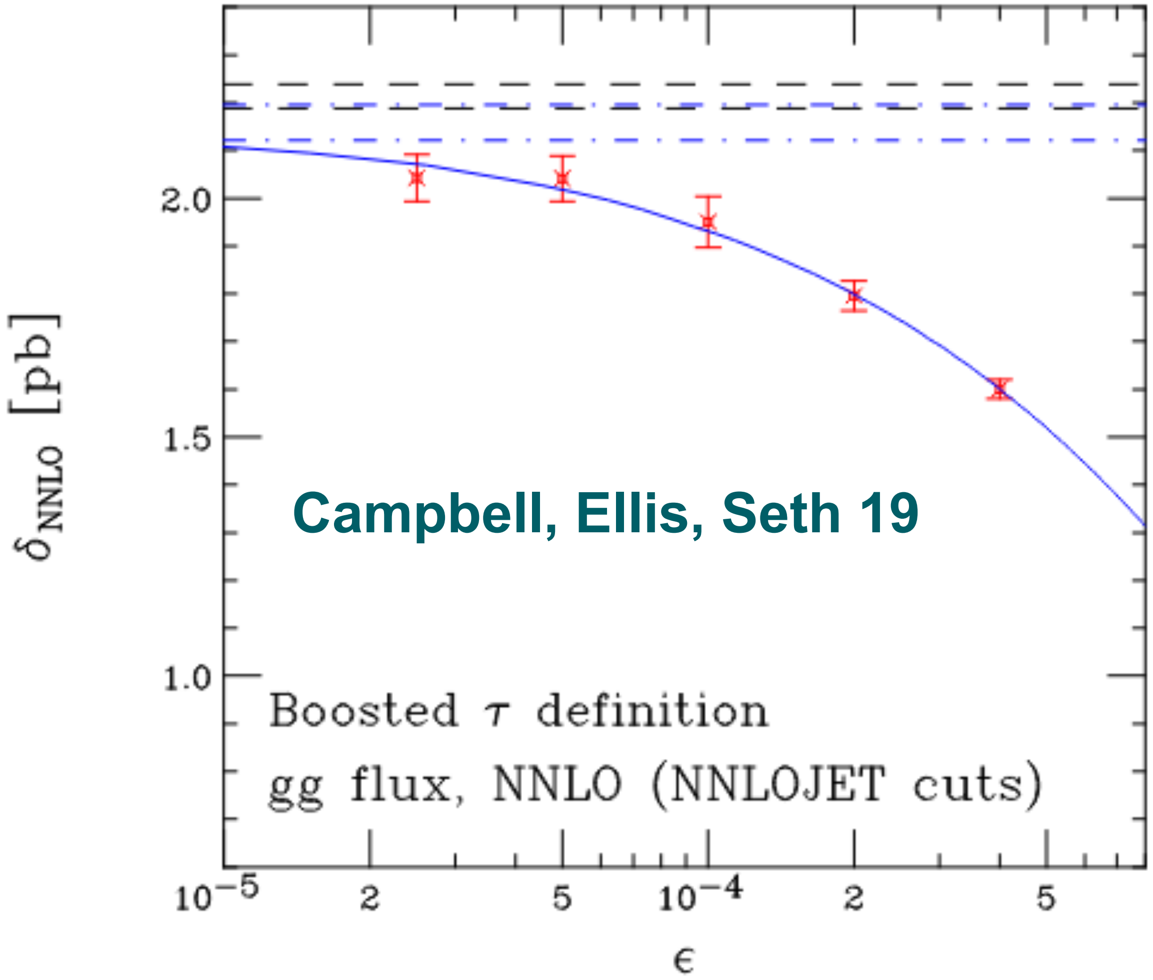
With agreement between fitting and MC uncertainties in the region $\epsilon \sim 2 - 3 \times 10^{-5}$

More physical;
But computationally more expensive and unstable

Validation at NNLO

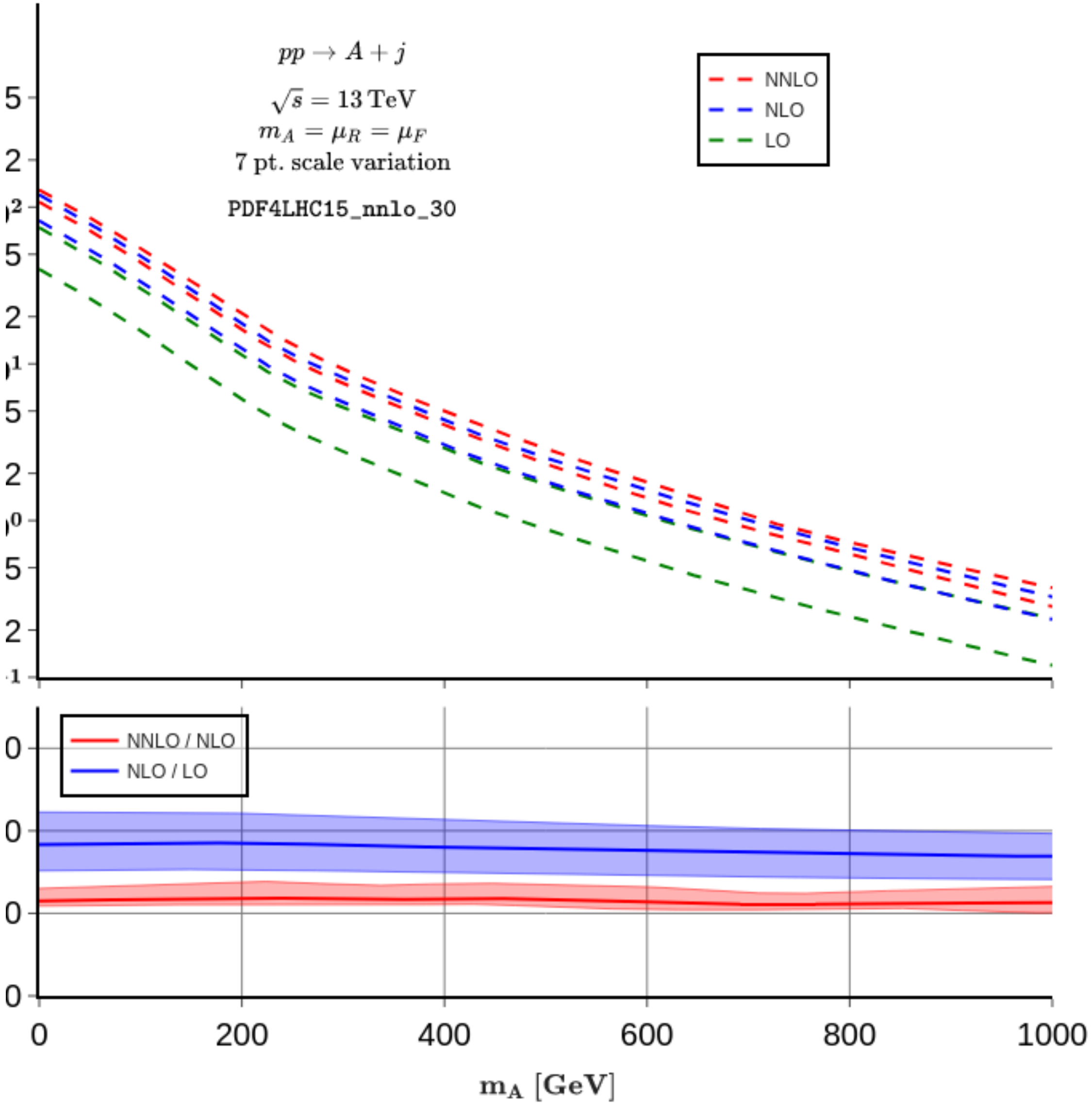
Validation at NNLO is trickier, since we have to extract the $\epsilon \to 0$ limit whil





More physical;
But computationally more expensive and unstable

Cross section as a function of mass



We begin by studying the cross section as a function of pseudoscalar mass through NNLO

As expected from the scalar Higgs case, the NLO to NNLO ratio is sizable (around 1.2)

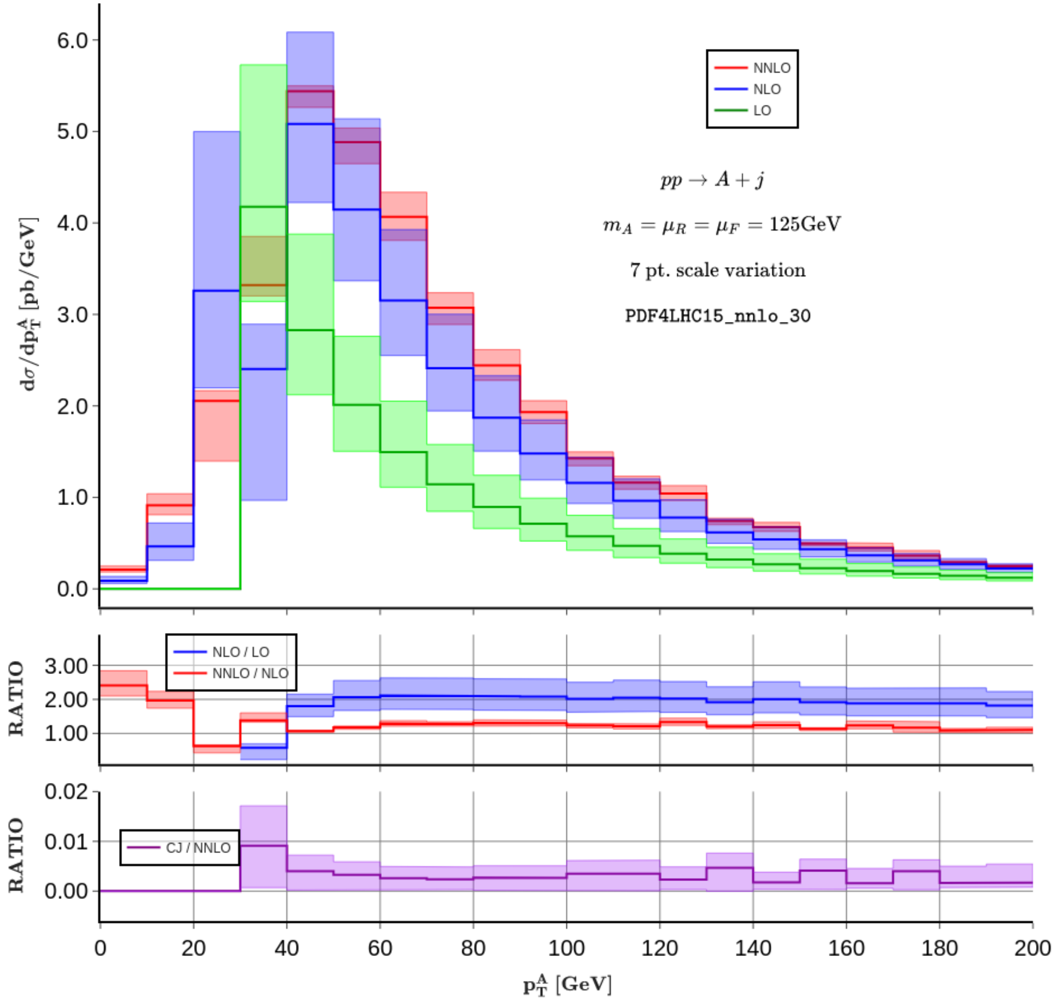
Scale variation is also notable reduced as expected.

Transverse momentum

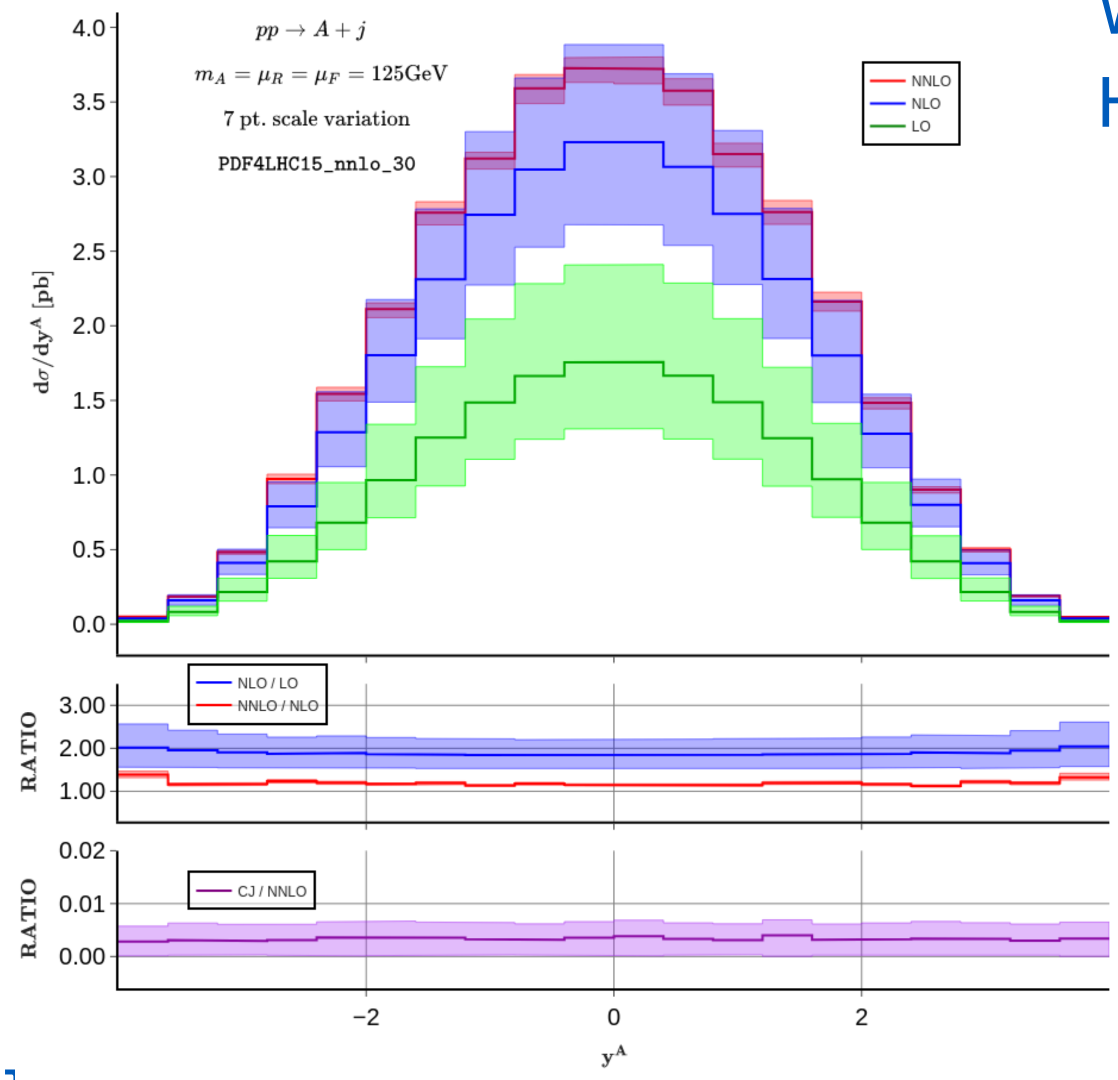
Next we turn to differential qualities, setting on m_A=125 GeV to provide a clean analog of the scalar Higgs case.

The correction from NLO -> NNLO is pretty similar to that observed in the Higgs case. The "Sudkov shoulder" at NLO is partly filled in by the NNLO corrections.

We get access to the O_J pieces at NNLO, they come in around 0.5% and are fairly flat across the phase space



Rapidity with $m_A = 125$ GeV



We also produced the distribution for the Higgs rapidity. Again for $m_A = 125$ GeV.

Here the NNLO corrections follow a similar pattern to NLO in that they are fairly flat but significantly decrease the scale variation.

Again the O_J pieces are pretty small, effecting the cross section around the 0.5% level.

Conclusions

- We presented a NNLO calculation of a pseudo scalar produced in association with an additional jet.
- We calculated all of the relevant amplitudes for the two- and one-loop as well as the tree-level result and were able to test our amplitudes in all cases. All amplitudes + Higgs checks are publicly available.
- We used N-jettiness slicing to regulate the IR divergences.
- We produced some initial phenomenological studies to quantify the impact of the NNLO corrections and produce total cross sections at this order.
- A natural future study would be to include the decays and to work in a more specific model related to tie into and update LHC constraints.

Thank you!

Thank you for listening!

But more importantly, on behalf of the Loopfest Advisory committee thanks to Fred, Pavel and SMU for hosting Loopfest and giving us such a warm welcome to Texas.

Thank you!

Thank you for listening!

But more importantly, on behalf of the Loopfest Advisory committee thanks to Fred, Pavel and SMU for hosting Loopfest and giving us such a warm welcome to Texas.

EXCEPT REGISTRATION FEES!

Backup

UV-Renormalization

Strong coupling renormalization:

$$\hat{\alpha}_s = S_\epsilon \left(\frac{\mu_R^2}{\mu^2}\right)^\epsilon Z_{\alpha_s}\alpha_s \qquad S_\epsilon = \frac{\exp(\epsilon\gamma_E)}{(4\pi)^\epsilon}$$

$$Z_\alpha = 1 + \left(\frac{\alpha_s}{2\pi}\right) r_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 r_2 + \mathcal{O}(\alpha_s^3)$$

$$r_1 = -\frac{\beta_0}{\epsilon}$$

$$r_2 = \frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}$$

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_R N_f$$

$$\beta_1 = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_R N_f - 4C_F T_R N_f$$

Operator renormalizations:

$$Z_{GG} = 1 + \frac{\alpha_s}{2\pi} z_{GG1} + \left(\frac{\alpha_s}{2\pi}\right)^2 z_{GG2}$$

$$Z_{GJ} = \frac{\alpha_s}{2\pi} z_{GJ1} + \left(\frac{\alpha_s}{2\pi}\right)^2 z_{GJ2},$$

$$Z_{JG} = 0,$$

$$Z_{JJ} = 1 + \frac{\alpha_s}{2\pi} z_{JJ1} + \left(\frac{\alpha_s}{2\pi}\right)^2 z_{JJ2},$$

$$z_{GG1} = -\frac{11}{6\epsilon}C_A + \frac{1}{3\epsilon}N_f,$$

$$z_{GG2} = \frac{1}{\epsilon^2}\left(\frac{121}{36}C_A^2 - \frac{22}{18}C_A N_f + \frac{1}{9}N_f^2\right) + \frac{1}{\epsilon}\left(-\frac{17}{12}C_A^2 + \frac{5}{12}C_A N_f + \frac{1}{4}C_f N_f\right)$$

$$z_{GJ1} = \frac{6}{\epsilon}C_F,$$

$$z_{GJ2} = \frac{1}{\epsilon^2}(-11C_A C_F + 2C_F N_f) + \frac{1}{\epsilon}\left(\frac{142}{12}C_A C_F - \frac{21}{2}C_F^2 - \frac{1}{3}C_F N_f\right),$$

$$z_{JJ1} = -2C_F,$$

$$z_{JJ2} = \frac{1}{\epsilon}\left(\frac{11}{6}C_A C_F + \frac{5}{12}C_F N_f\right) + \frac{11}{2}C_F^2 - \frac{107}{36}C_A C_F + \frac{31}{72}C_F N_f.$$

UV-renormalized amplitudes in terms of bare amplitudes.

$$A(q) \to g(p_1) + g(p_2) + g(p_3)$$

$$\langle \mathcal{M}_{ggg}^{G,(0)} | \mathcal{M}_{ggg}^{G,(1)} \rangle = \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(1)} \rangle + \left(\frac{1}{2} r_1 + z_{GG1} \right) \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(0)} \rangle$$

$$\langle \mathcal{M}_{ggg}^{G,(0)} | \mathcal{M}_{ggg}^{G,(2)} \rangle = \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(2)} \rangle + \left(\frac{3}{2} r_1 + z_{GG1} \right) \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(1)} \rangle$$
$$+ z_{GJ1} \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{J,(1)} \rangle + \left(-\frac{1}{8} r_1^2 + \frac{1}{2} r_2 + \frac{1}{2} r_1 z_{GG1} + z_{GG2} \right) \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(0)} \rangle$$

$$\langle \mathcal{M}_{ggg}^{G,(1)} | \mathcal{M}_{ggg}^{G,(1)} \rangle = \left(\frac{1}{2} r_1 + z_{GG1} \right) 2\mathrm{Re} \left[\langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(1)} \rangle \right] + \left(\frac{1}{4} r_1^2 + r_1 z_{GG1} + z_{GG1}^2 \right) \langle \hat{\mathcal{M}}_{ggg}^{G,(0)} | \hat{\mathcal{M}}_{ggg}^{G,(0)} \rangle$$

$$A(q) \to q(p_1) + \bar{q}(p_2) + g(p_3)$$

$$\langle \mathcal{M}_{q\bar{q}g}^{G,(0)} | \mathcal{M}_{q\bar{q}g}^{G,(1)} \rangle = \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(1)} \rangle + \left(\frac{1}{2} r_1 + z_{GG1} \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} \rangle + z_{GJ1} \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} \rangle$$

$$\langle \mathcal{M}_{q\bar{q}g}^{G,(0)} | \mathcal{M}_{q\bar{q}g}^{G,(2)} \rangle = \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(2)} \rangle + \left(\frac{3}{2} r_1 + z_{GG1} \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(1)} \rangle + z_{GJ1} \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{J,(1)} \rangle$$
$$+ \left(-\frac{1}{8} r_1^2 + \frac{1}{2} r_2 + \frac{1}{2} r_1 z_{GG1} + z_{GG2} \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} \rangle$$
$$+ \left(\frac{1}{2} r_1 z_{GJ2} + z_{GJ2} \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} \rangle$$

$$\langle \mathcal{M}_{q\bar{q}g}^{G,(1)} | \mathcal{M}_{q\bar{q}g}^{G,(1)} \rangle = \left(\frac{1}{2} r_1 + z_{GG1} \right) 2\mathrm{Re} \left[\langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(1)} \rangle \right] + \left(\frac{1}{4} r_1^2 + r_1 z_{GG1} + z_{GG1}^2 \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} \rangle$$
$$+ z_{GJ1} 2\mathrm{Re} \left[\langle \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(1)} \rangle \right] + \left(\frac{1}{2} r_1 z_{GJ1} + z_{GG1} z_{GJ1} \right) 2\mathrm{Re} \left[\langle \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{G,(0)} \rangle \right]$$
$$+ z_{GJ1}^2 \langle \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{J,(0)} \rangle$$

Compare to the SM Higgs case:

$$\langle \mathcal{M}_{q\bar{q}g}^{(0)} | \mathcal{M}_{q\bar{q}g}^{(1)} \rangle = \langle \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{(1)} \rangle - \frac{3\beta_0}{2\epsilon} \langle \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} \rangle$$

$$\langle \mathcal{M}_{q\bar{q}g}^{(0)} | \mathcal{M}_{q\bar{q}g}^{(2)} \rangle = \langle \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{(2)} \rangle - \frac{5\beta_0}{2\epsilon} \langle \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{(1)} \rangle - \left(\frac{5\beta_1}{4\epsilon} - \frac{15\beta_0^2}{8\epsilon^2} \right) \langle \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} | \hat{\mathcal{M}}_{q\bar{q}g}^{(0)} \rangle$$

CJ interfered amplitudes play important roles in UV-renormalization. Note all finite terms of CJ tree-level are zero while there are terms from $\mathrm{O}(\epsilon)$. These higher order terms participate in UV-renormalization.

Distributions @ 700 GeV