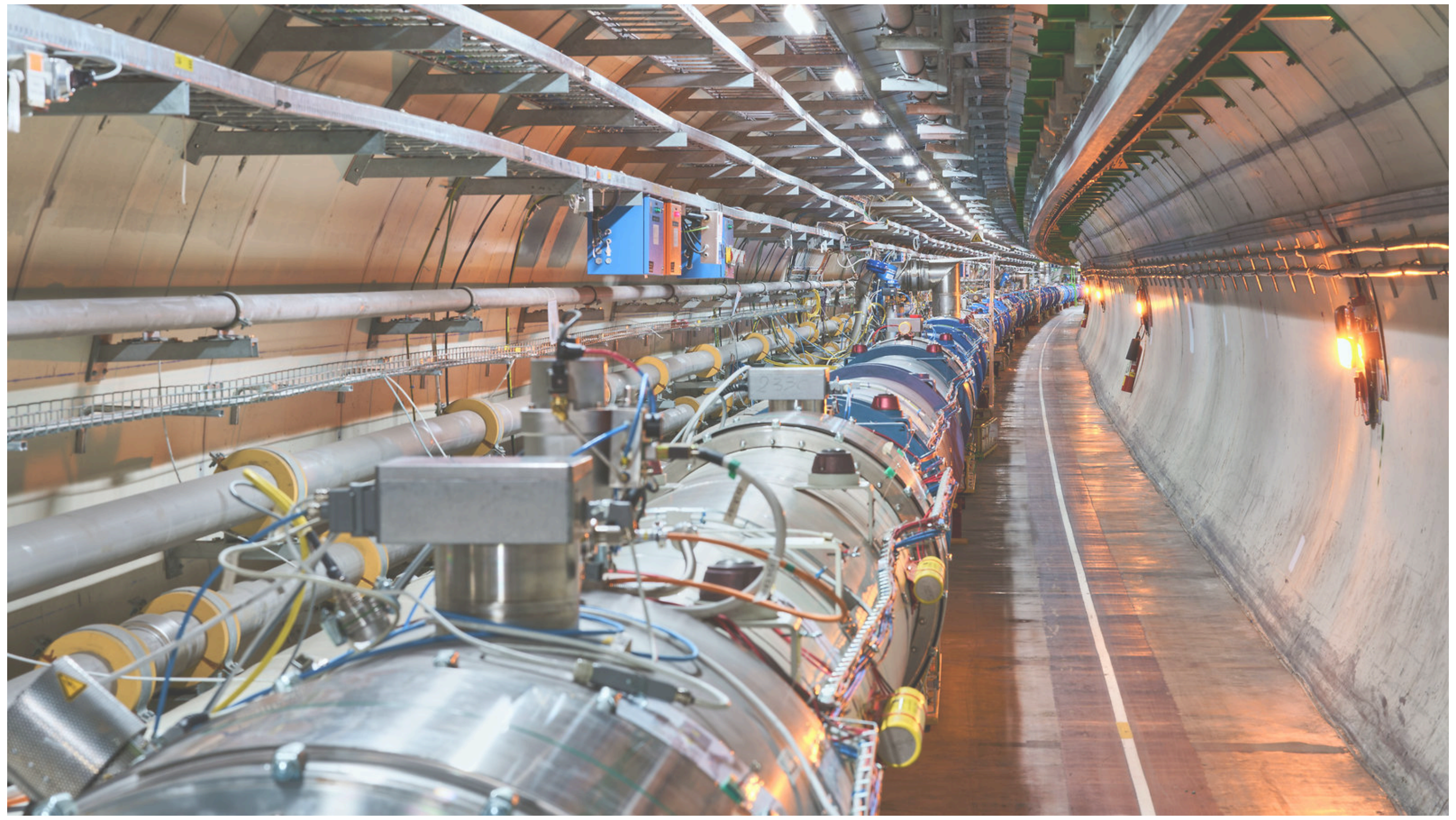
Perturbative calculations for first-principle PDFs

based on arXiv:2408.XXXXX
with Christopher Monahan

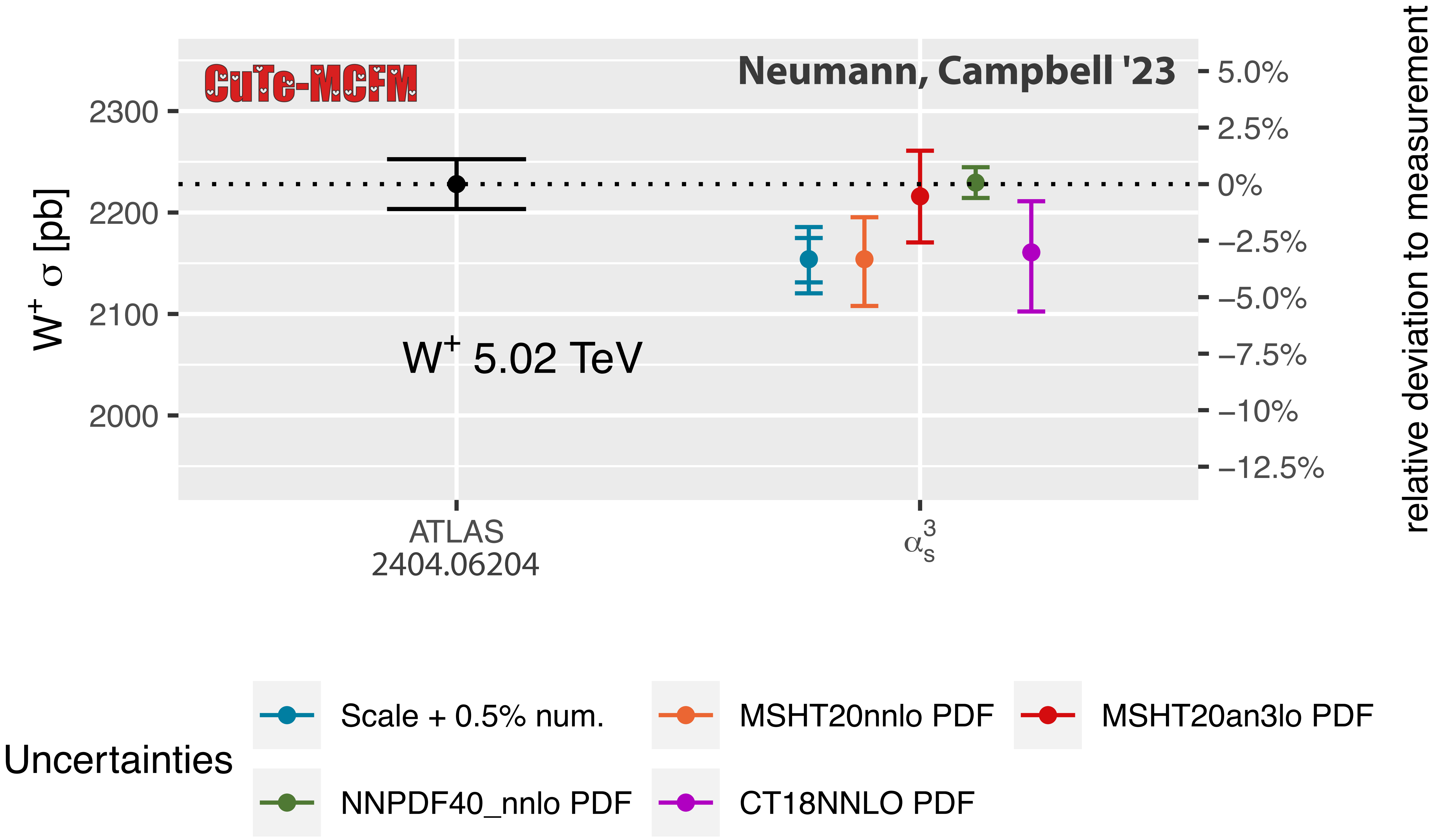


Tobias Neumann
William & Mary

$$\sigma = f \otimes f \otimes \hat{\sigma} + p.c.$$

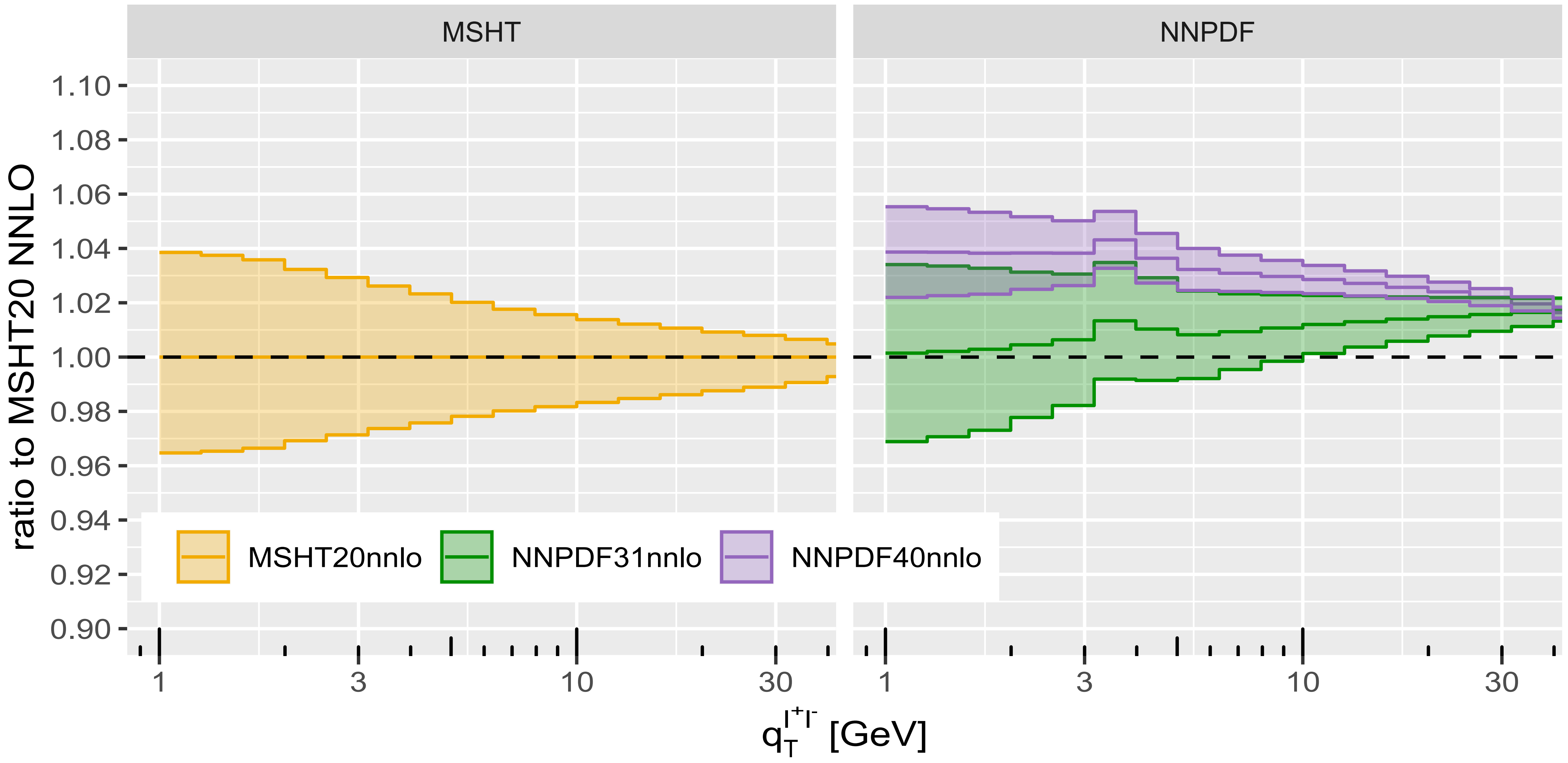
PDFs at the interface of experimental physics, data science and theory

$$x\,q(x) = A \cdot x^b (1-x)^c P(x)$$

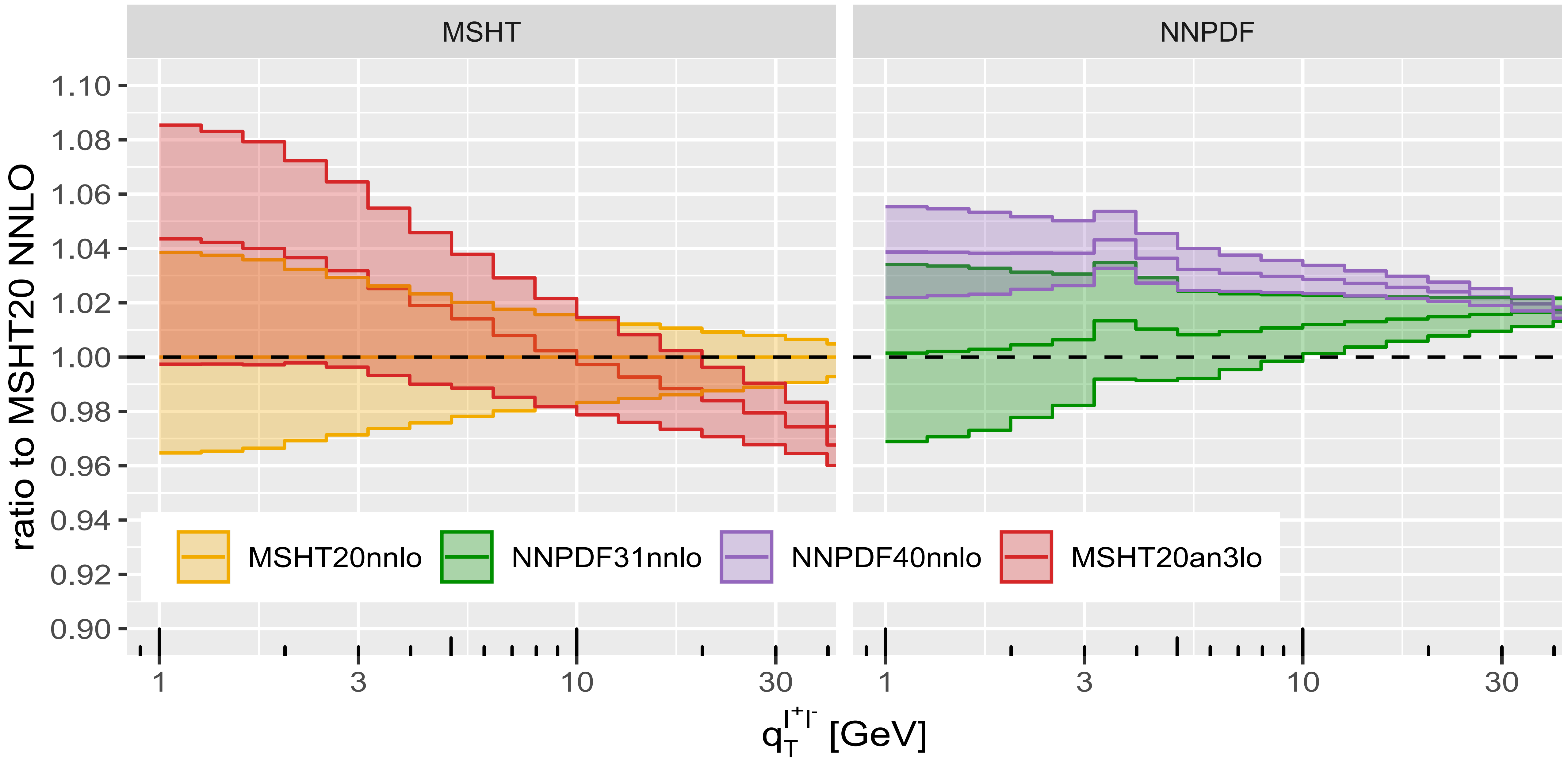


CuTe-MCFM
Neumann, Campbell '23
W+ σ [pb]
2300
2200
2100
2000
5.0%
2.5%
0%
−2.5%
−5.0%
−7.5%
−10%
−12.5%
relative deviation to measurement
W+ 5.02 TeV
ATLAS
2404.06204
αs3
Uncertainties
Scale + 0.5% num.
MSHT20nnlo PDF
MSHT20an3lo PDF
NNPDF40_nnlo PDF
CT18NNLO PDF

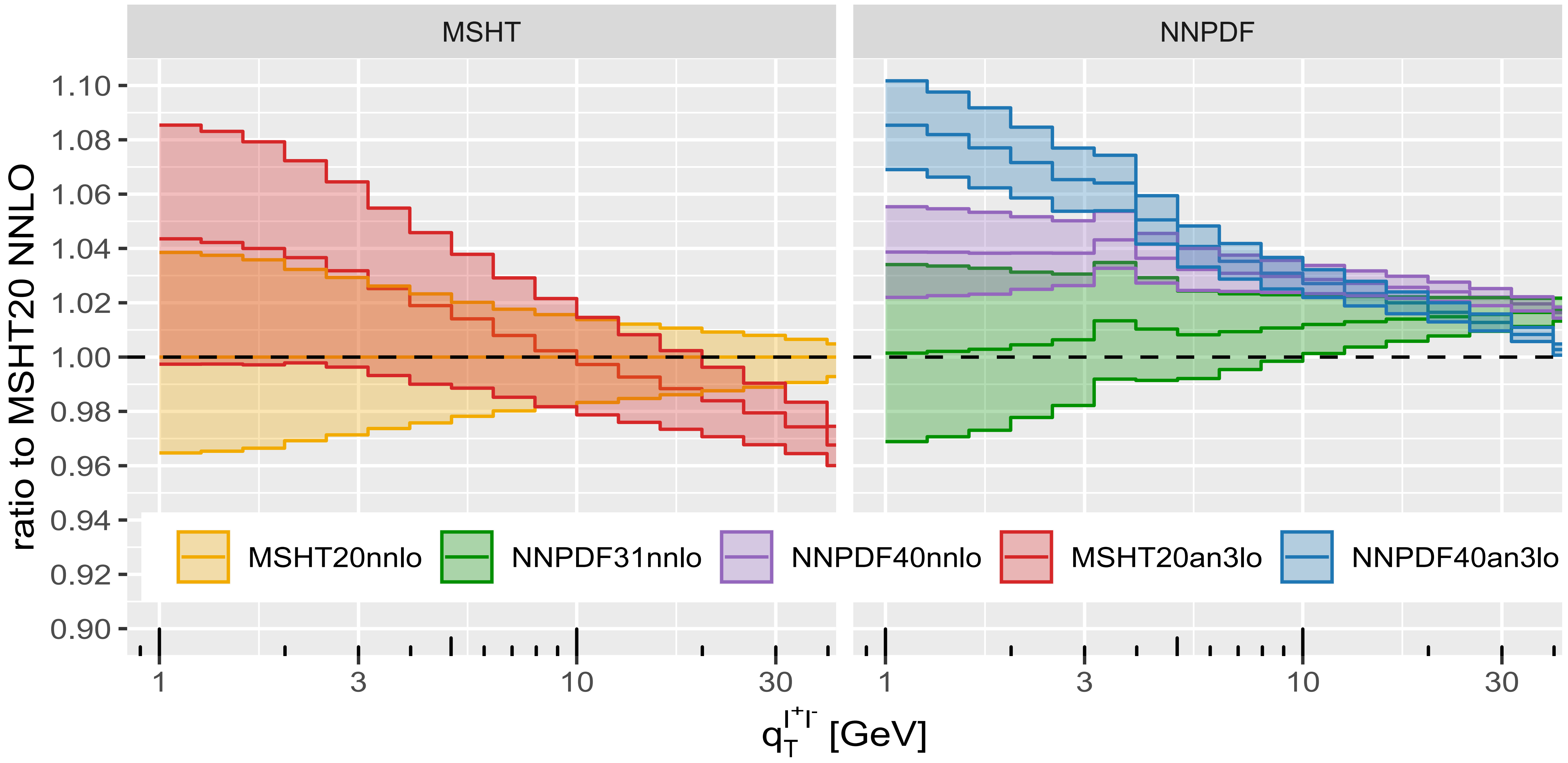
Impact of PDFs (e.g. in the Z transverse momentum)



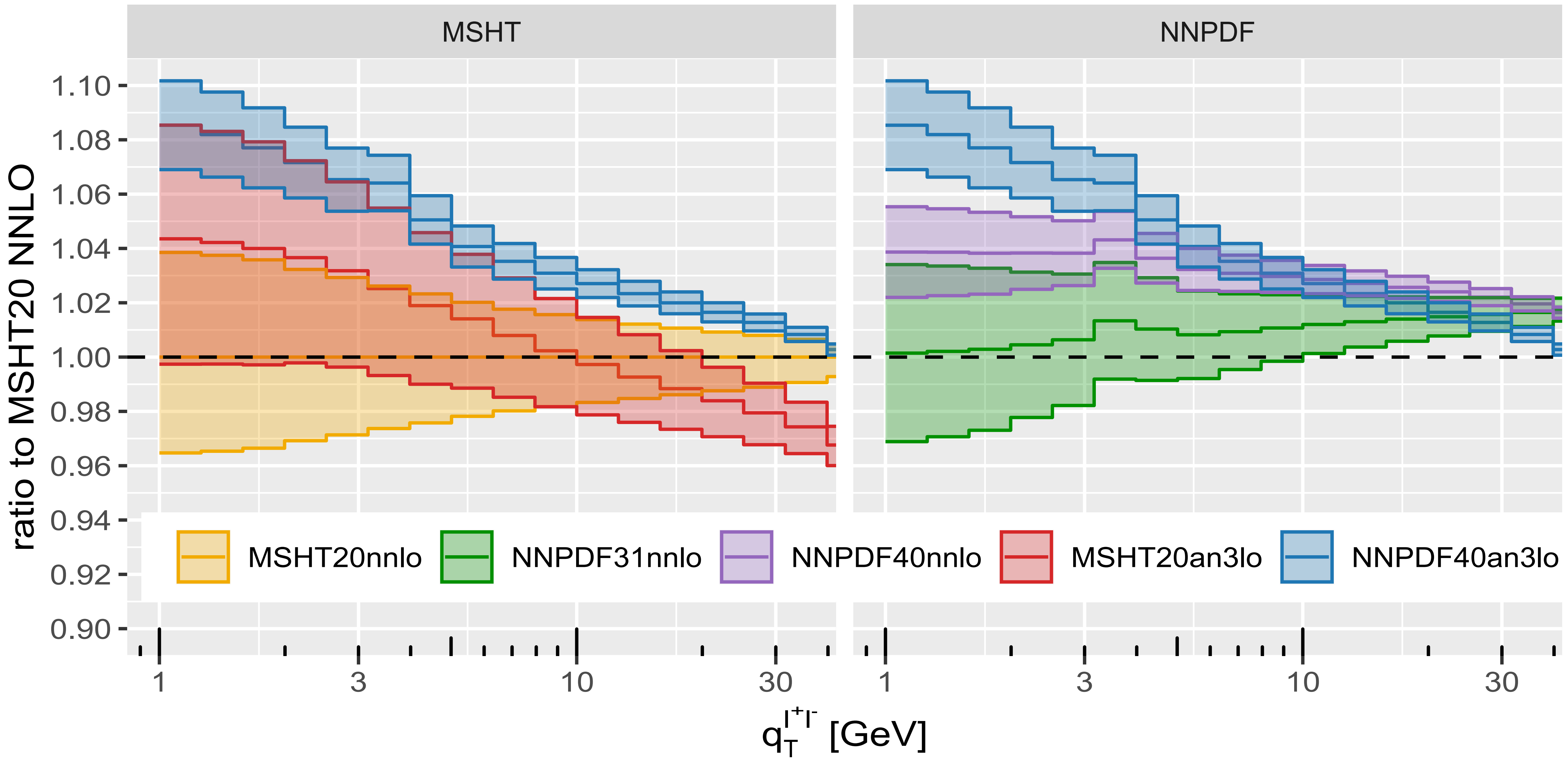
Impact of PDFs (e.g. in the Z transverse momentum)



Impact of PDFs (e.g. in the Z transverse momentum)



Impact of PDFs (e.g. in the Z transverse momentum)



FUTURE CIRCULAR COLLIDER

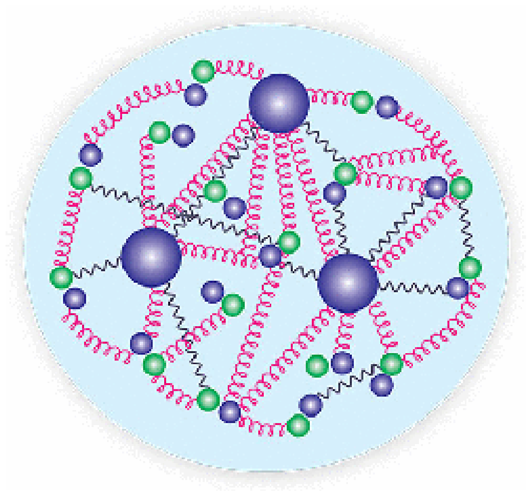
> *“At a lepton collider every event is a signal event,*
> *while at a hadron collider every event is a background event.”*
>
> *on a slide from Christoph Paus, FCC Week '23*

A NEW ERA OF DISCOVERY

THE 2023 LONG RANGE PLAN FOR NUCLEAR SCIENCE

- How are quarks distributed in the nucleon?
- Where does the proton spin come from?
- Three-dimensional imaging of the proton

Parton distirbution functions

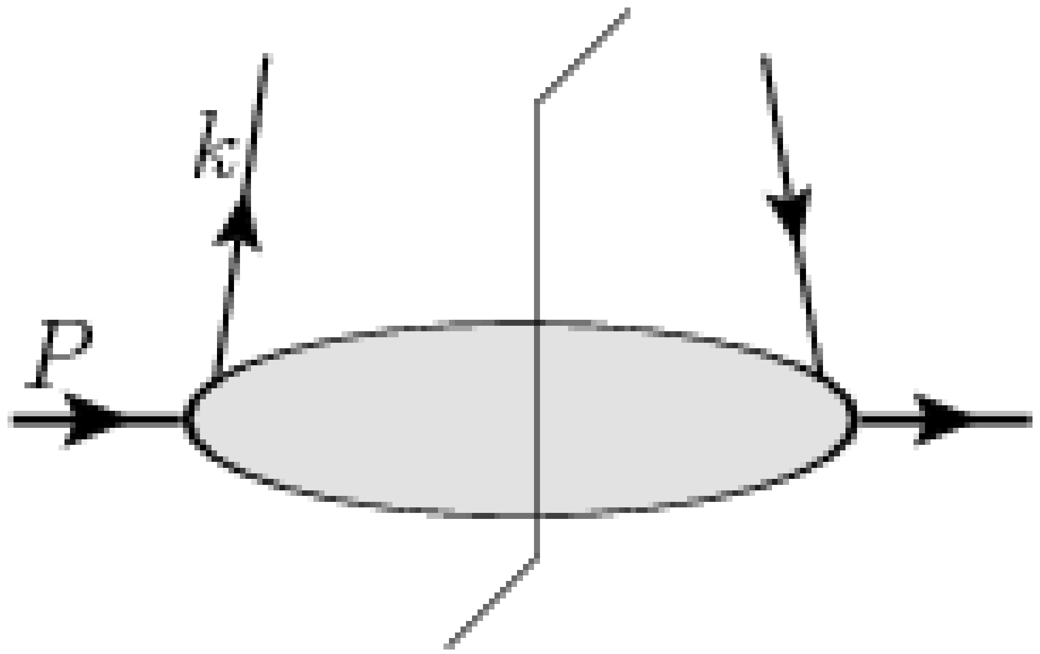


First ideas of a parton picture in the infinite momentum frame:
"Very high-energy collisions of hadrons", Feynman, 1969

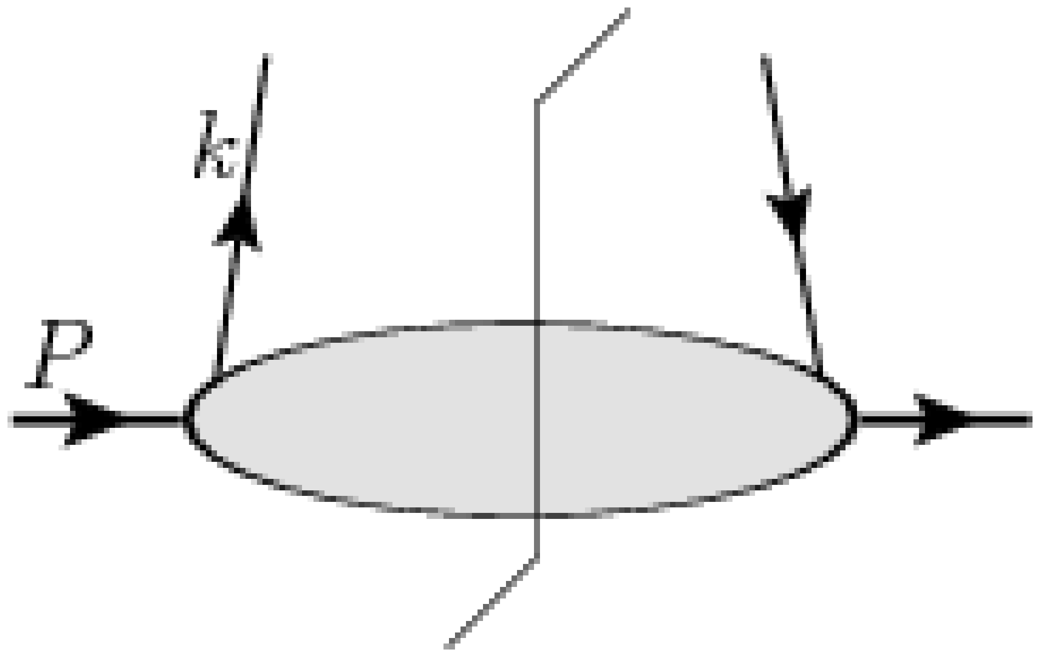
to

Formalization and operator definition:
"Parton distribution and decay functions", Collins, Soper, 1981

$$f_{j/H}(\xi) = \int \frac{dw^-}{2\pi}\, e^{-i\xi P^+ w^-} \langle P|\overline{\psi}_j(0, w^-, \mathbf{0}_\mathrm{T})\frac{\gamma^+}{2}\psi_j(0)|P\rangle$$



$$f_{j/H}(\xi) = \int \frac{dw^-}{2\pi}\, e^{-i\xi P^+ w^-} \langle P|\overline{\psi}_j(0, w^-, \mathbf{0}_\mathrm{T})\, \frac{\gamma^+}{2} \psi_j(0)|P\rangle$$



$$f_{j/H}(\xi) = \int \frac{dw^-}{2\pi}\, e^{-i\xi P^+ w^-} \langle P|\overline{\psi}_j(0, w^-, \mathbf{0}_\mathrm{T})\, \frac{\gamma^+}{2}\, \psi_j(0)|P\rangle$$

Non-perturbative, but also inherently non-Euclidean

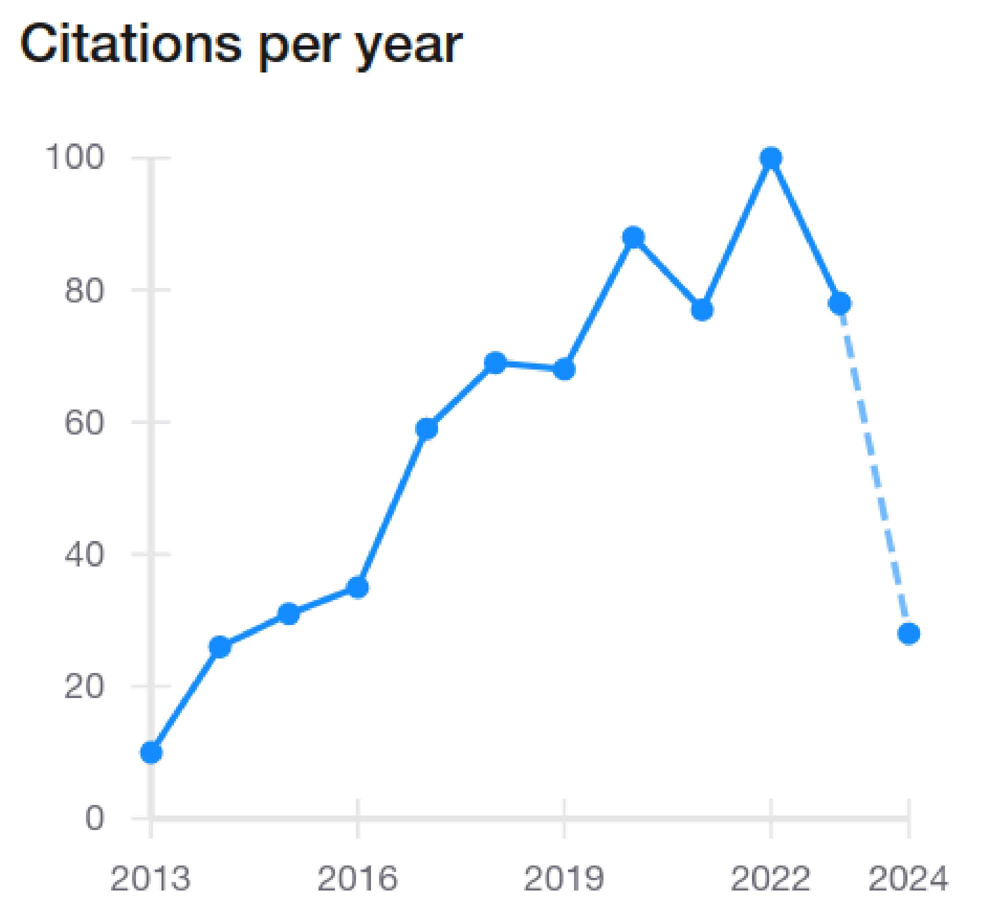
Large momentum effective theory (LaMET)

Parton Physics on a Euclidean Lattice

Xiangdong Ji (Maryland U. and Shanghai Jiaotong U.)
May 7, 2013

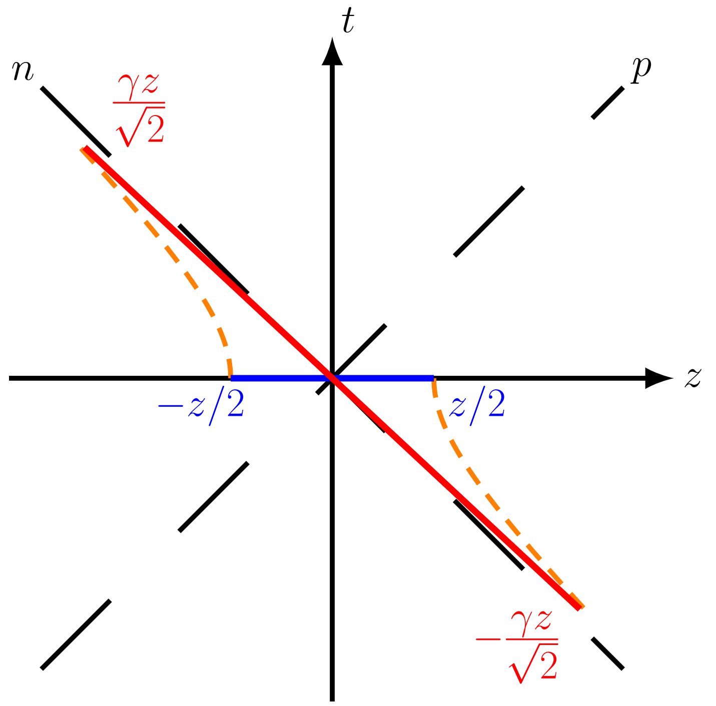
4 pages
Published in: *Phys.Rev.Lett.* 110 (2013) 262002
Published: Jun 26, 2013
e-Print: 1305.1539 [hep-ph]
DOI: 10.1103/PhysRevLett.110.262002
View in: OSTI Information Bridge Server, ADS Abstract Service

pdf cite claim reference search 669 citations



see X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, Y. Zhao '20, arXiv:2004.03543 for a review

Going back to Feynman's infinite momentum frame
Can't we just take $P \to \infty$?



$$P^z \gg \Lambda_{\mathrm{QCD}}$$

$$f(k^z, P^z) = f(x) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/P^z)^2$$

In QCD collinear factorization:

$$(\Lambda_{\mathrm{UV}} \ll P^z) \to \infty$$

In LaMET:

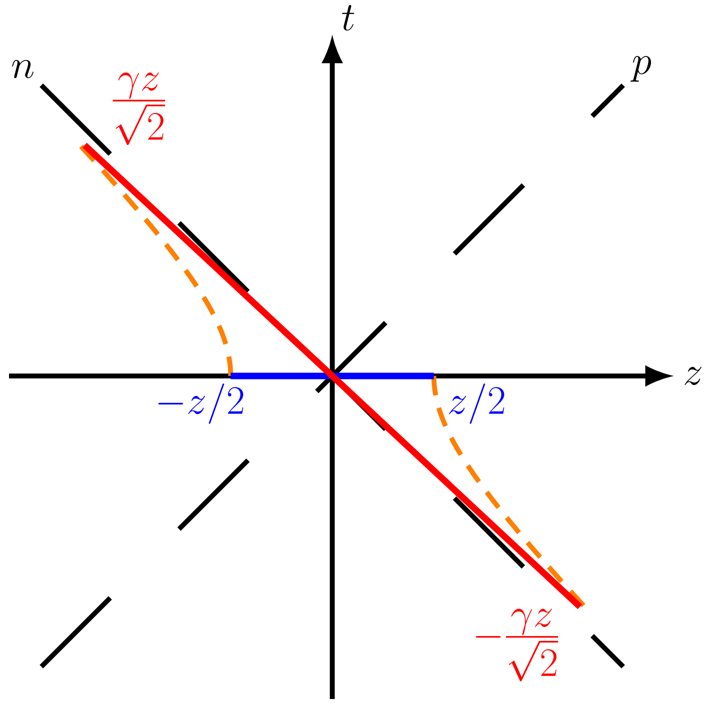
$$(P^z \ll \Lambda_{\mathrm{UV}}) \to \infty$$

Quasi- and pseudo PDFs based on Fourier transforms of equal-time correlator, e.g.

$$\tilde{q}(x, p_z) \sim \int \mathrm{d}z\, e^{ixzp^z} \langle p \mid \overline{\Psi}(z)\gamma^0 W(z,0)\Psi(0) \mid p\rangle$$

Factorization onto light-cone PDFs used in cross-sections:

$$\tilde{q}(x, p_z) = C(x, p_z) \otimes q + \mathcal{O}\left(\frac{M^2}{p_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 p_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 p_z^2}\right)$$



Perturbative matching kernel C

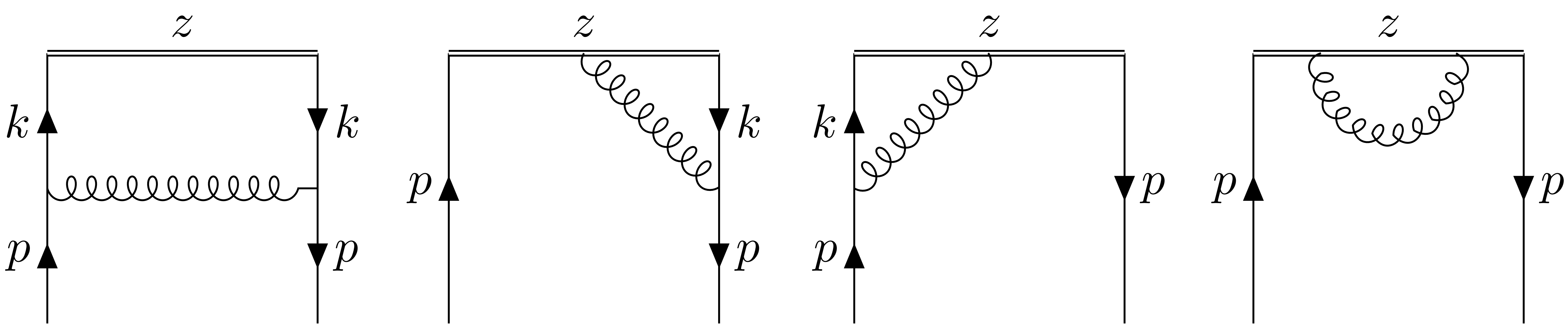
$\overline{MS}$ one-loop for quarks: Izubuchi, X. Ji, L. Jin, I. Stewart, Y. Zhao '18;
two-loop quark quasi: L.-B. Chen, W. Wang, R. Zhu '20; Z.-Y. Li, Y.-Q. Ma. J.-W. Qiu'20

Quark quasi PDF at one loop

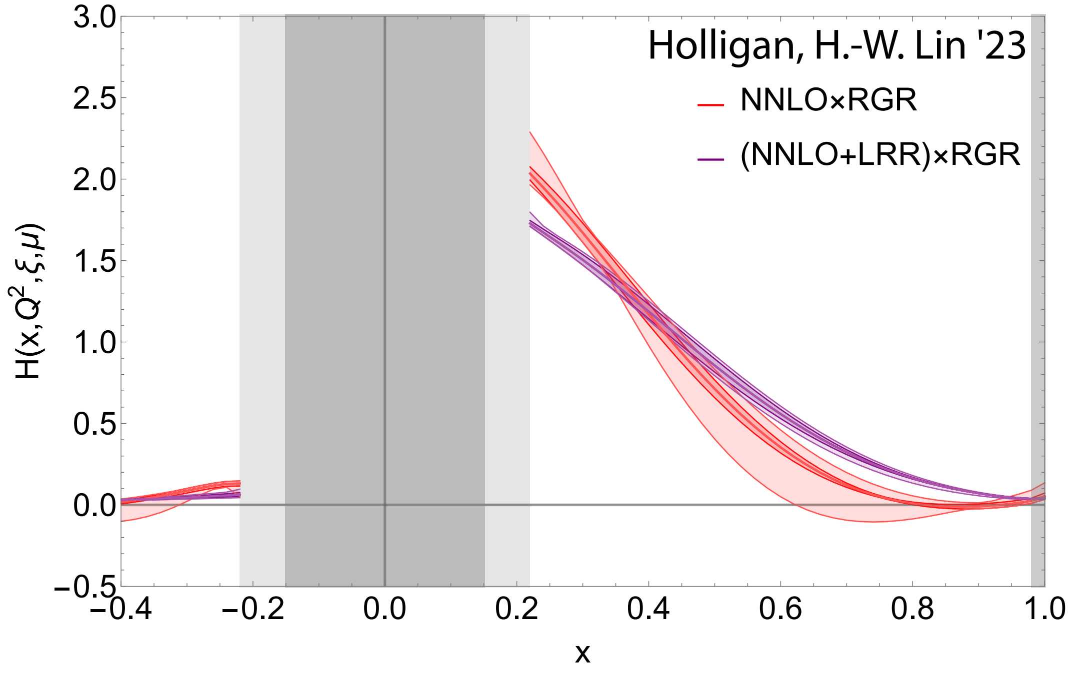
$$\tilde{q}^{(1)}(x,\mu/|p^z|,\epsilon_{\rm IR}) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+x^2}{1-x}\ln\frac{x}{x-1}+1+\frac{3}{2x}\right)_{+(1)}^{[1,\infty]} - \left(\frac{3}{2x}\right)_{+(\infty)}^{[1,\infty]} & x>1 \\ \left(\frac{1+x^2}{1-x}\left[-\frac{1}{\epsilon_{\rm IR}} - \ln\frac{\mu^2}{4p_z^2} + \ln\left(x(1-x)\right)\right] - \frac{x(1+x)}{1-x}\right)_{+(1)}^{[0,1]} & 0<x<1 \\ \left(-\frac{1+x^2}{1-x}\ln\frac{-x}{1-x} - 1 + \frac{3}{2(1-x)}\right)_{+(1)}^{[-\infty,0]} - \left(\frac{3}{2(1-x)}\right)_{+(-\infty)}^{[-\infty,0]} & x<0 \end{cases}$$

$$+\frac{\alpha_s C_F}{2\pi}\left[\delta(1-x)\left(\frac{3}{2}\ln\frac{\mu^2}{4p_z^2}+\frac{5}{2}\right)+\frac{3}{2}\gamma_E\left(\frac{1}{(x-1)^2}\delta^+\left(\frac{1}{x-1}\right)+\frac{1}{(1-x)^2}\delta^+\left(\frac{1}{1-x}\right)\right)\right].$$

Izubuchi, X. Ji, L. Jin, Stewart, Y. Zhao '18; C.-Y. Chou, J.-W. Chen '22



~ quasi quark


Holligan, H.-W. Lin '23
NNLO×RGR
(NNLO+LRR)×RGR
$H(x,Q^2,\xi,\mu)$
x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

Almost there!

Gluons

Fourier transform matrix element into Quasi- and Pseudo PDFs:

$$\mathcal{G}_{\mu\alpha;\lambda\beta}(z,p) = \langle P \mid G^a_{\mu\nu}(z) W(z,0) G^b_{\rho\sigma} \mid P \rangle$$

Gluons

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$$\mathcal{G}_{\mu\alpha;\lambda\beta}(z,p) = \langle P \mid G^a_{\mu\nu}(z) W(z,0) G^b_{\rho\sigma} \mid P \rangle$$

- Presence of a power divergence for $z \to 0$
  e.g. W. Wang, S. Zhao '17
- Proof of multiplicative renormalization
  J.-H. Zhang, X. Ji, A. Schäfer, W. Wang, S. Zhao '18; Z.-Y. Li, Y.-Q. Ma, J.-W. Qiu '18
- Discussion of higher-twist contamination
  e.g. I. Balitsky, W. Morris, A. Radyushkin '21

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- First one-loop calculation, in a cutoff scheme
  W. Wang, S. Zhao, R. Zhu '17
- One-loop in momentum subtraction scheme
  W. Wang, J.-H. Zhang, S. Zhao, R. Zhu '19
- One-loop in $\overline{\mathrm{MS}}$
  Balitsky, Morris, Radyushkin '19, '21
- Presence of a power divergence for $z \to 0$
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- Discussion of higher-twist contamination
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Our calculation

- We reproduce quark quasi- and pseudo- calculations in the $\overline{\mathrm{MS}}$ scheme
- Tensor decomposition for the gluon

$$\mathcal{G}_{\mu\alpha;\lambda\beta}(z,p) = \langle P \mid G^a_{\mu\nu}(z) W(z,0) G^b_{\rho\sigma} \mid P \rangle$$

$$\begin{aligned}
&\mathcal{G}_{\mu\alpha;\lambda\beta}(z,p) = \\
&\left(g_{\mu\lambda} p_\alpha p_\beta - g_{\mu\beta} p_\alpha p_\lambda - g_{\alpha\lambda} p_\mu p_\beta + g_{\alpha\beta} p_\mu p_\lambda\right) \frac{z^2}{(pz)^2} \mathcal{M}_{pp} \\
&\quad + \left(g_{\mu\lambda} z_\alpha z_\beta - g_{\mu\beta} z_\alpha z_\lambda - g_{\alpha\lambda} z_\mu z_\beta + g_{\alpha\beta} z_\mu z_\lambda\right) \mathcal{M}_{zz}/z^2 \\
&\quad + \left(g_{\mu\lambda} z_\alpha p_\beta - g_{\mu\beta} z_\alpha p_\lambda - g_{\alpha\lambda} z_\mu p_\beta + g_{\alpha\beta} z_\mu p_\lambda\right) \mathcal{M}_{zp}/(pz) \\
&\quad + \left(g_{\mu\lambda} p_\alpha z_\beta - g_{\mu\beta} p_\alpha z_\lambda - g_{\alpha\lambda} p_\mu z_\beta + g_{\alpha\beta} p_\mu z_\lambda\right) \mathcal{M}_{pz}/(pz) \\
&\quad + \left(p_\mu z_\alpha - p_\alpha z_\mu\right)\left(p_\lambda z_\beta - p_\beta z_\lambda\right) \mathcal{M}_{ppzz}/(pz)^2 \\
&\quad + \left(g_{\mu\lambda} g_{\alpha\beta} - g_{\mu\beta} g_{\alpha\lambda}\right) \mathcal{M}_{gg} \,.
\end{aligned}$$

- One loop, in the $\overline{\mathrm{MS}}$ scheme

Our calculation (II)

- Setup using reduction to master integrals
- Automatized; checks through R_ξ and general axial gauge

- Just three master integrals

$$\int_k \frac{e^{ikz}}{(k^2+i\epsilon)((k-p)z+i\eta)} \sim e^{ipz}(i\,pz)^{(d-3)}(-z^2)^{(1-d/2)}\Gamma(3-d,i\,pz)\ldots$$

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$$\int_k \frac{e^{ikz}}{(k^2+i\epsilon)((k-p)z+i\eta)} \sim e^{ipz}(i\,pz)^{(d-3)}(-z^2)^{(1-d/2)}\Gamma(3-d,i\,pz)\dots$$

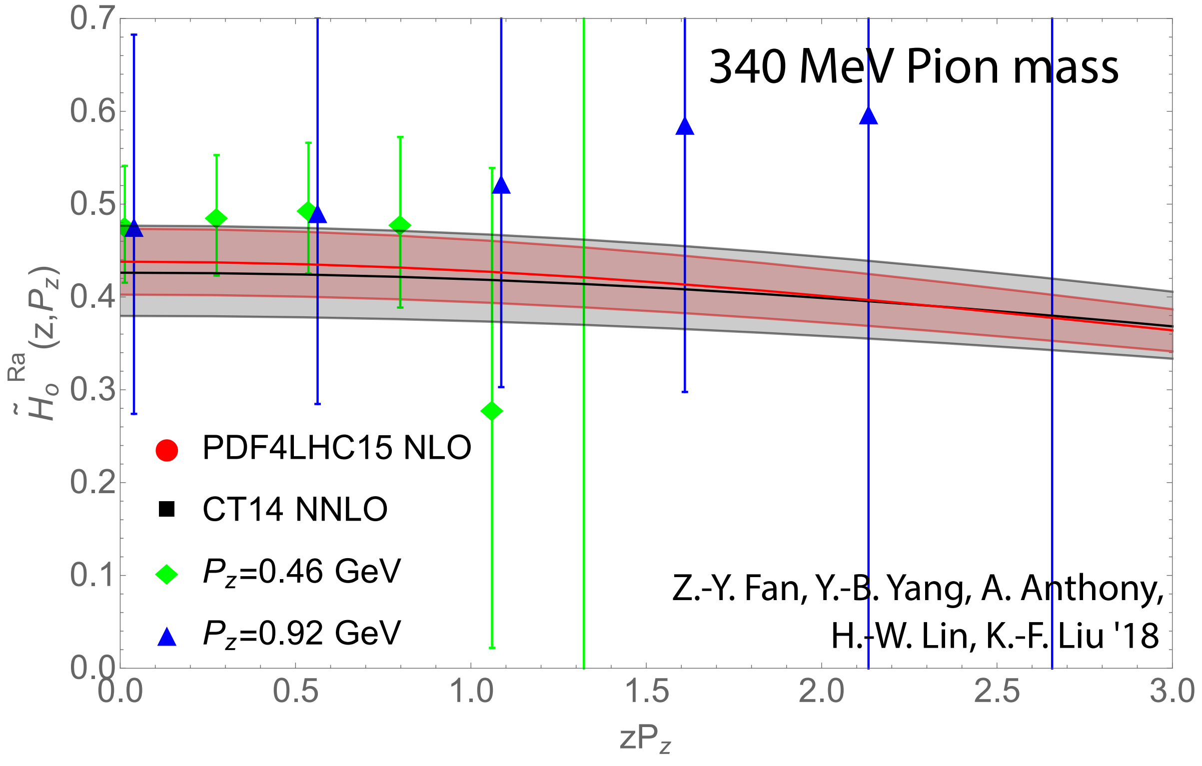
- A technical complication: Fourier transformation into quasi- and pseudo distributions

$$\int \mathrm{d}\zeta\, e^{i\zeta(x-1)}(-i\zeta)^{-2\epsilon}\Gamma(2\epsilon,-i\zeta)$$

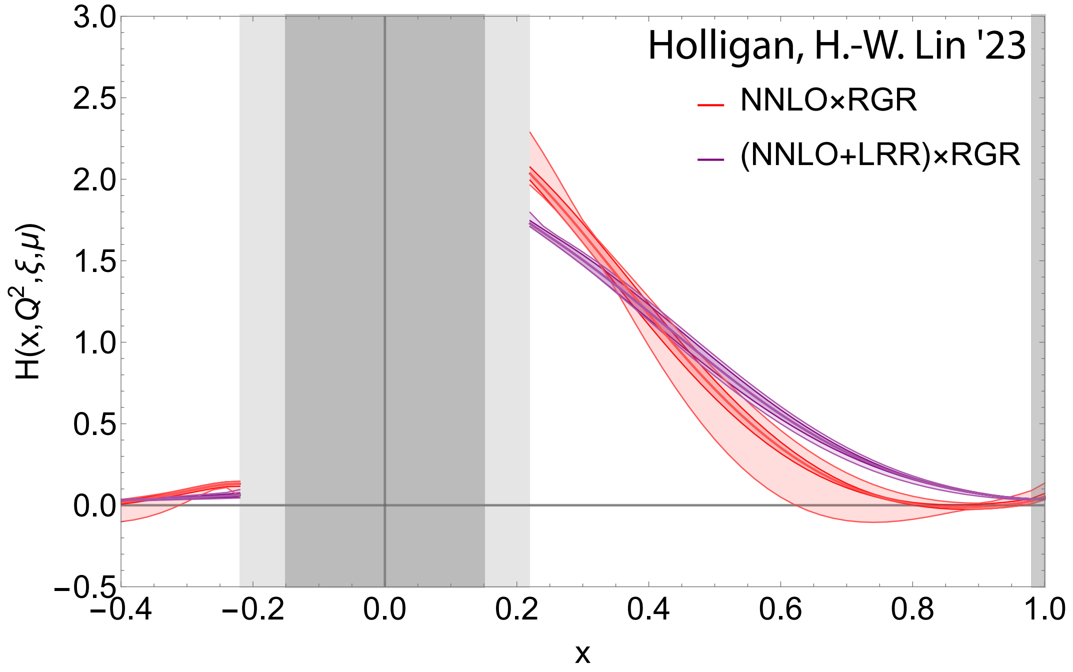
- ϵ-expansion and Fourier transform do not commute!

arXiv:2408.XXXXX with Chris Monahan

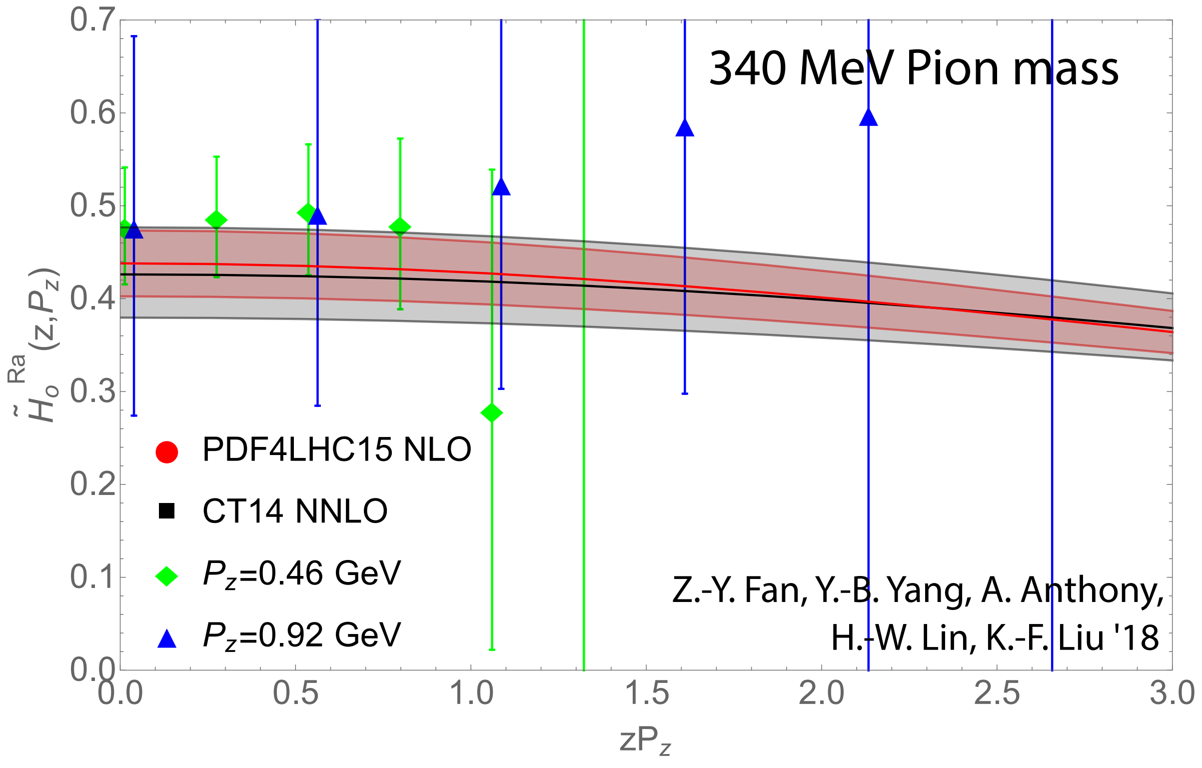
~ quasi gluon



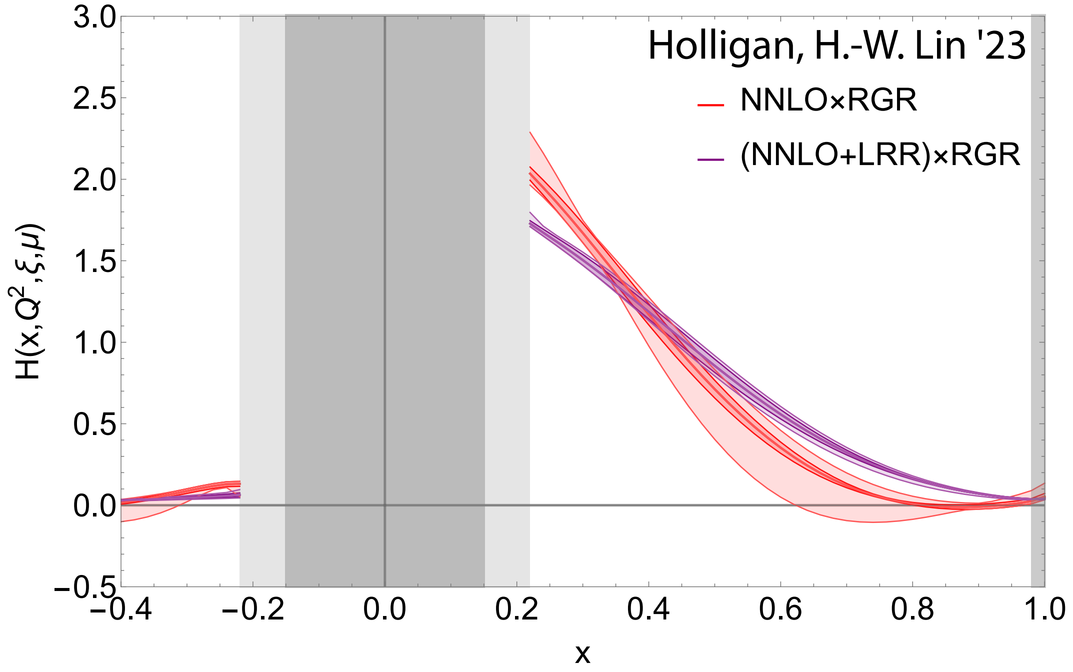
~ quasi quark



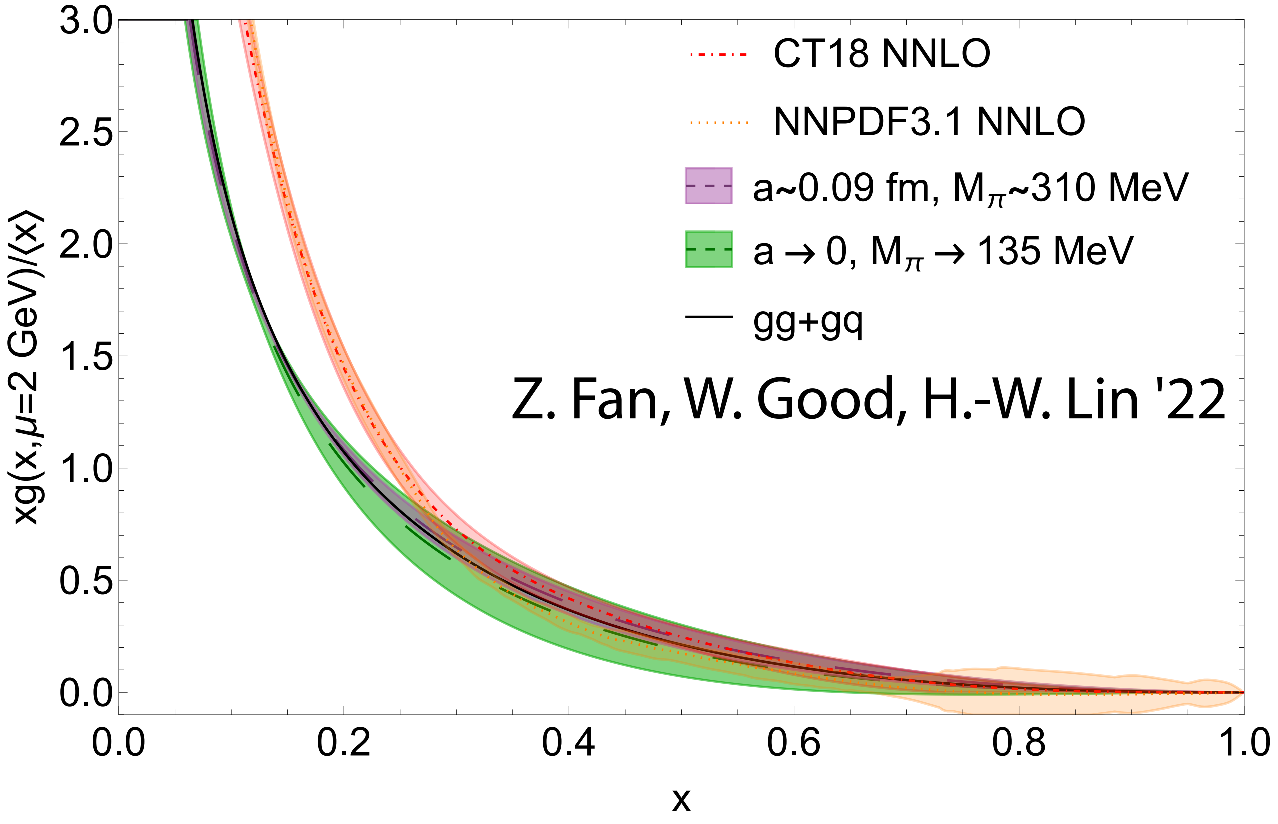
~ quasi gluon



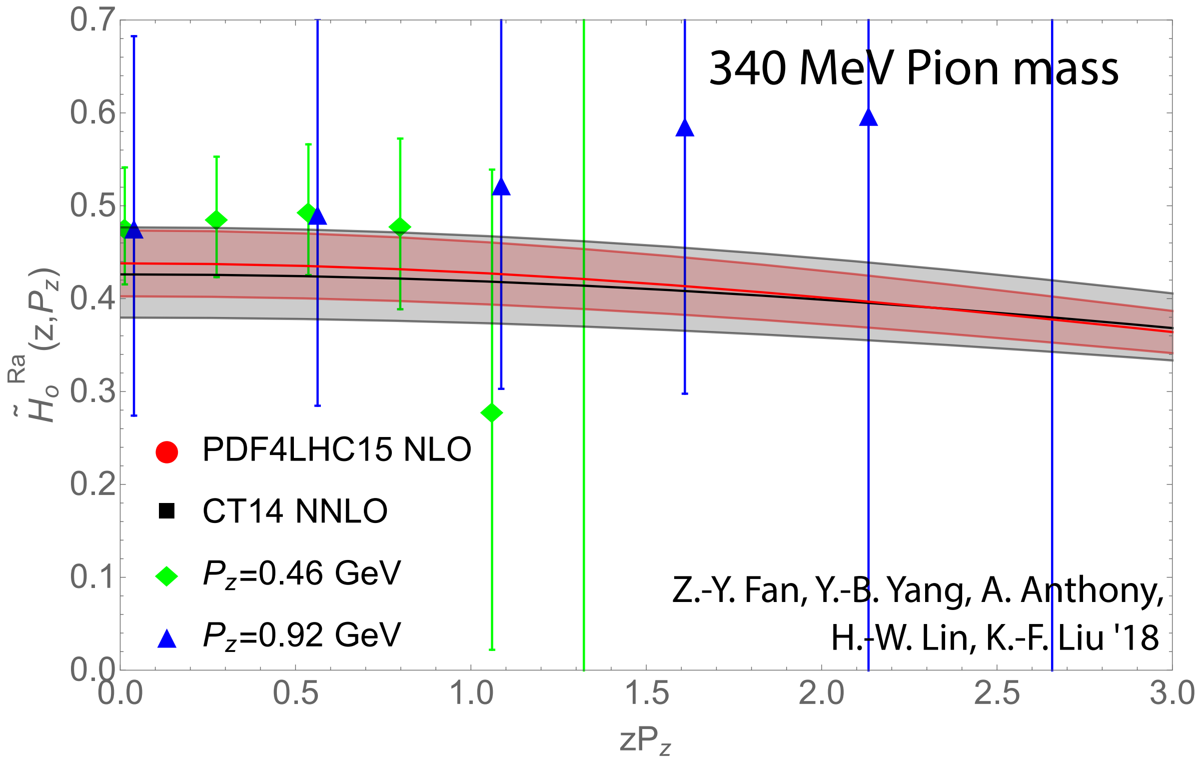
~ quasi quark



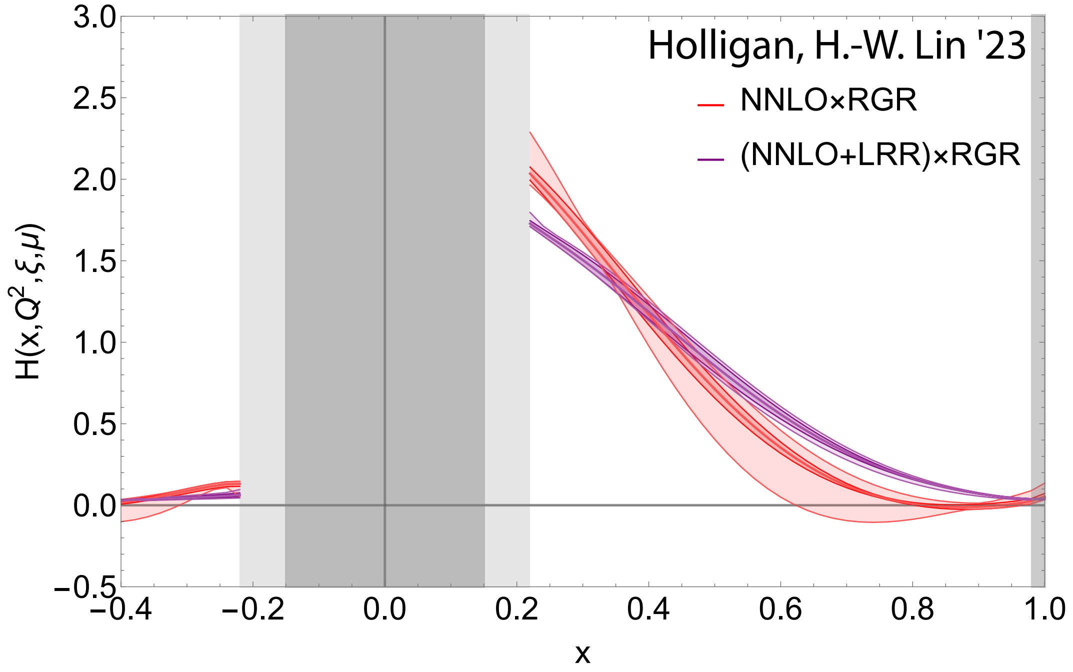
~ quasi gluon



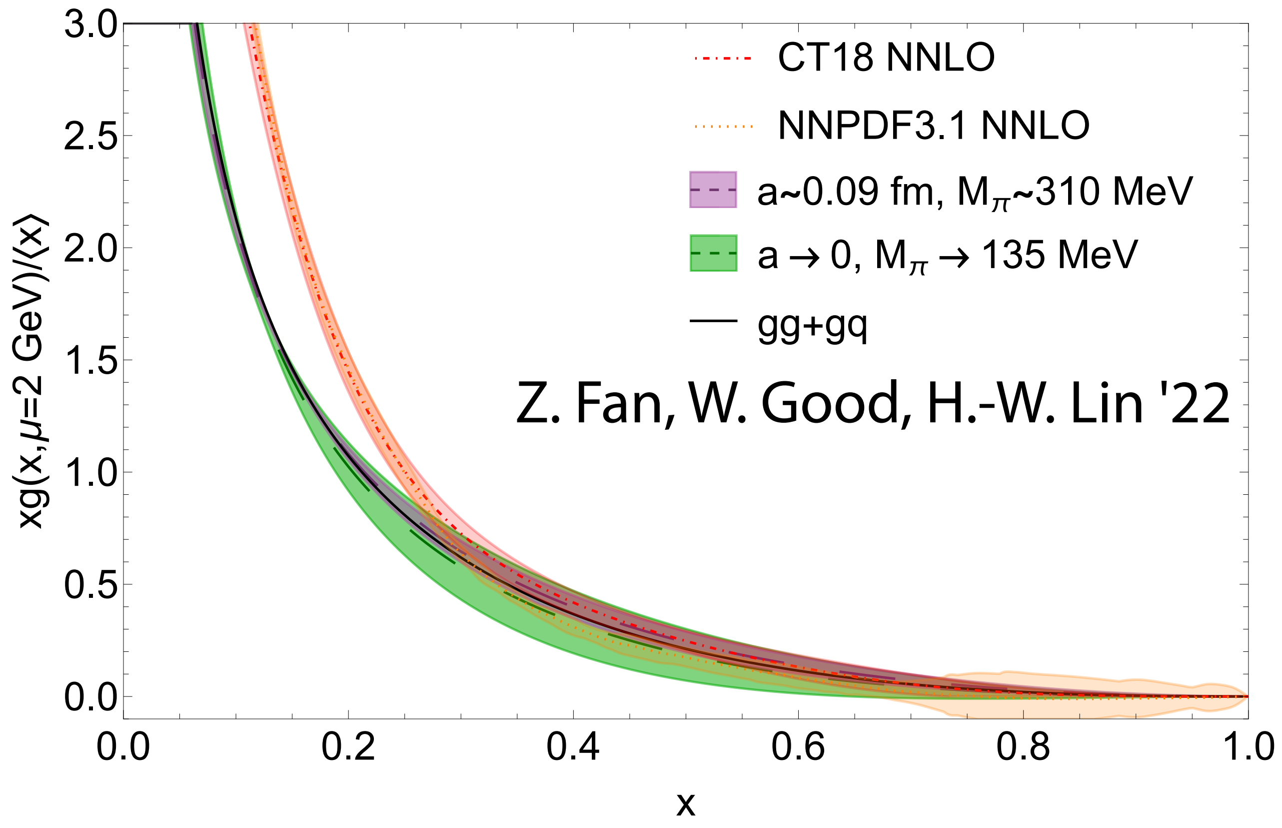
~ quasi gluon



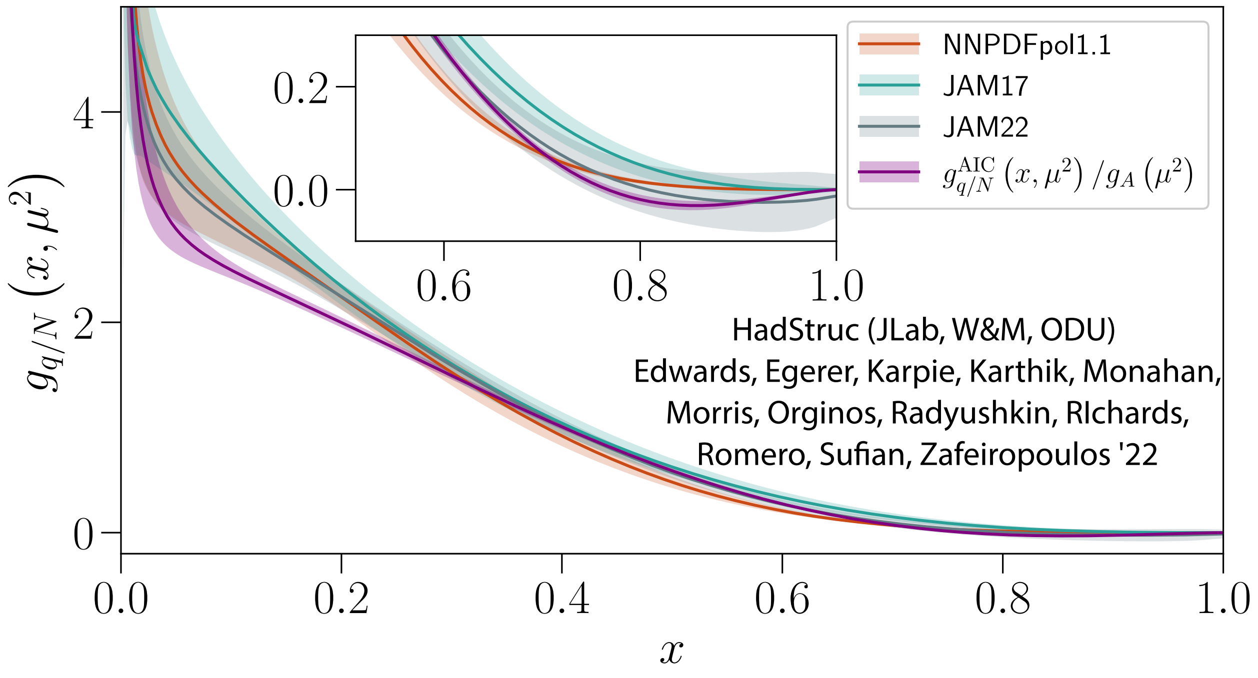
~ quasi quark



~ quasi gluon



~ quark helicity, pseudo approach



First-principle parton distribution functions

- PDFs are a bottleneck in current hadron collider predictions
  — but they are also of fundemental interest from a nuclear physics perspective
- Large Momentum Effective Theory (LaMET) opens the way for first-principle calculations
  - non-perturbative ingredient via lattice QCD
  - matching to light-cone PDFs via perturbation theory
- Quasi-quark: one- and two-loop calculations available in $\overline{\mathrm{MS}}$ scheme
- **My calculation with Chris Monahan**:
  - **One-loop gluon quasi- and pseudo distributions for generic tensor structure in $\overline{\mathrm{MS}}$**
  - Setup paves way for future matching in $\overline{\mathrm{MS}}$ gradient-flow scheme (next project)
  - Pseudo approach (+gradient flow) current method by W&M, JLab lattice groups
- Outlook: Extension to two-loop level for Pseudo distributions will require substantial developments (Fourier transform can be simplified for quasi distributions)