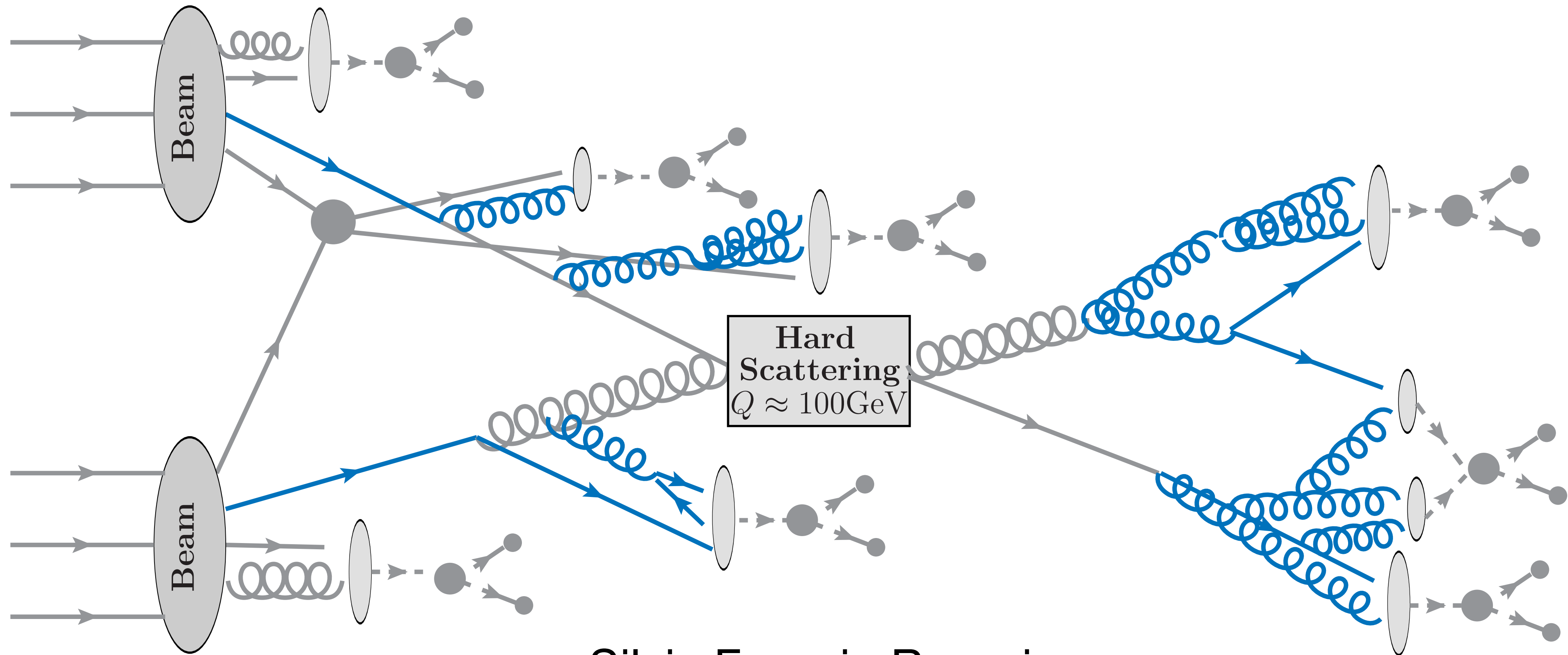


# Recent Progress in PanScales Parton Showers with higher logarithmic accuracy



Silvia Ferrario Ravasio

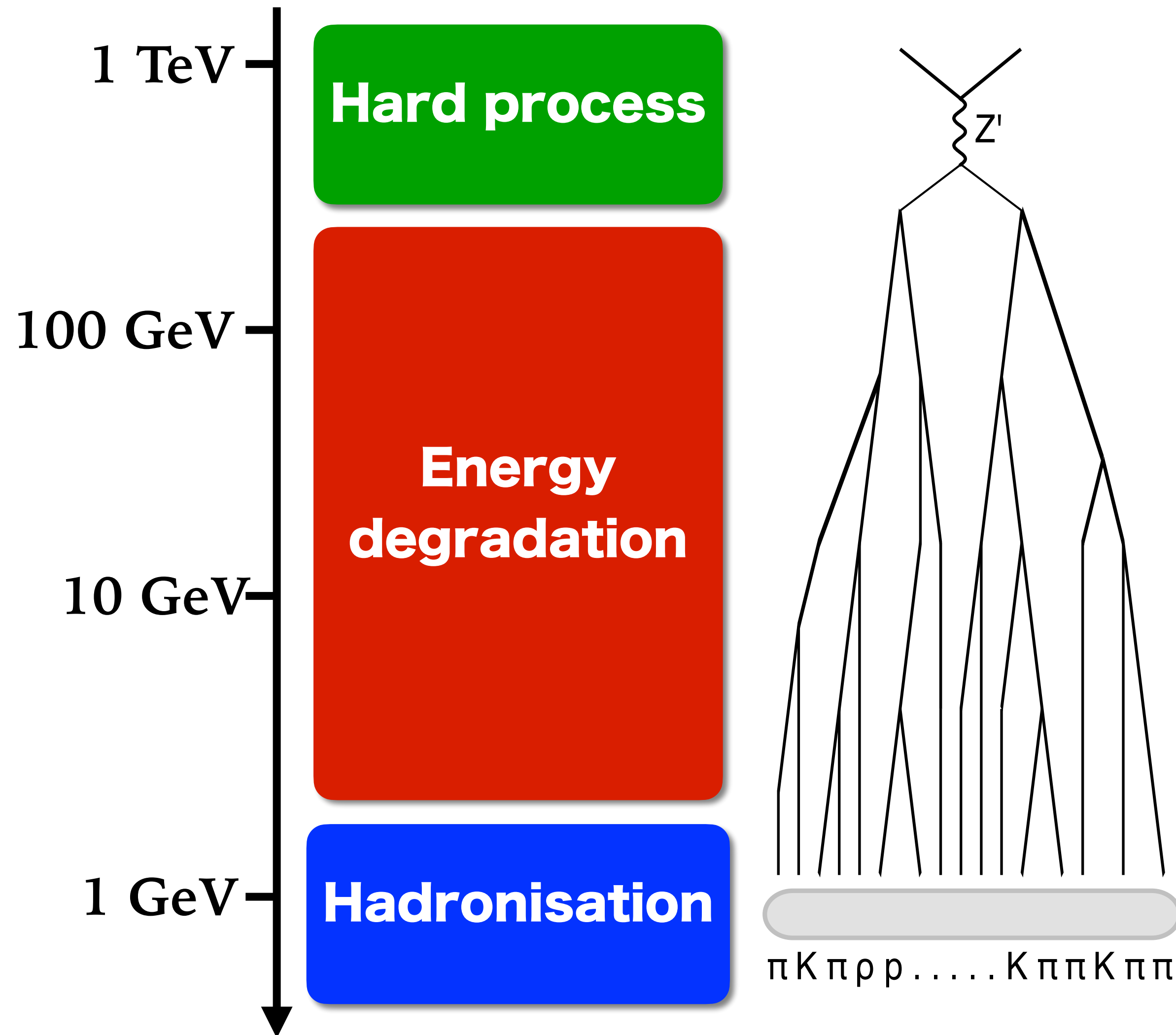
Loopfest XXII

22<sup>nd</sup> May 2024, Dallas Texas USA



# Shower Monte Carlo event generators

**SHOWER MONTE CARLO EVENT GENERATORS** = default tool for interpreting collider data

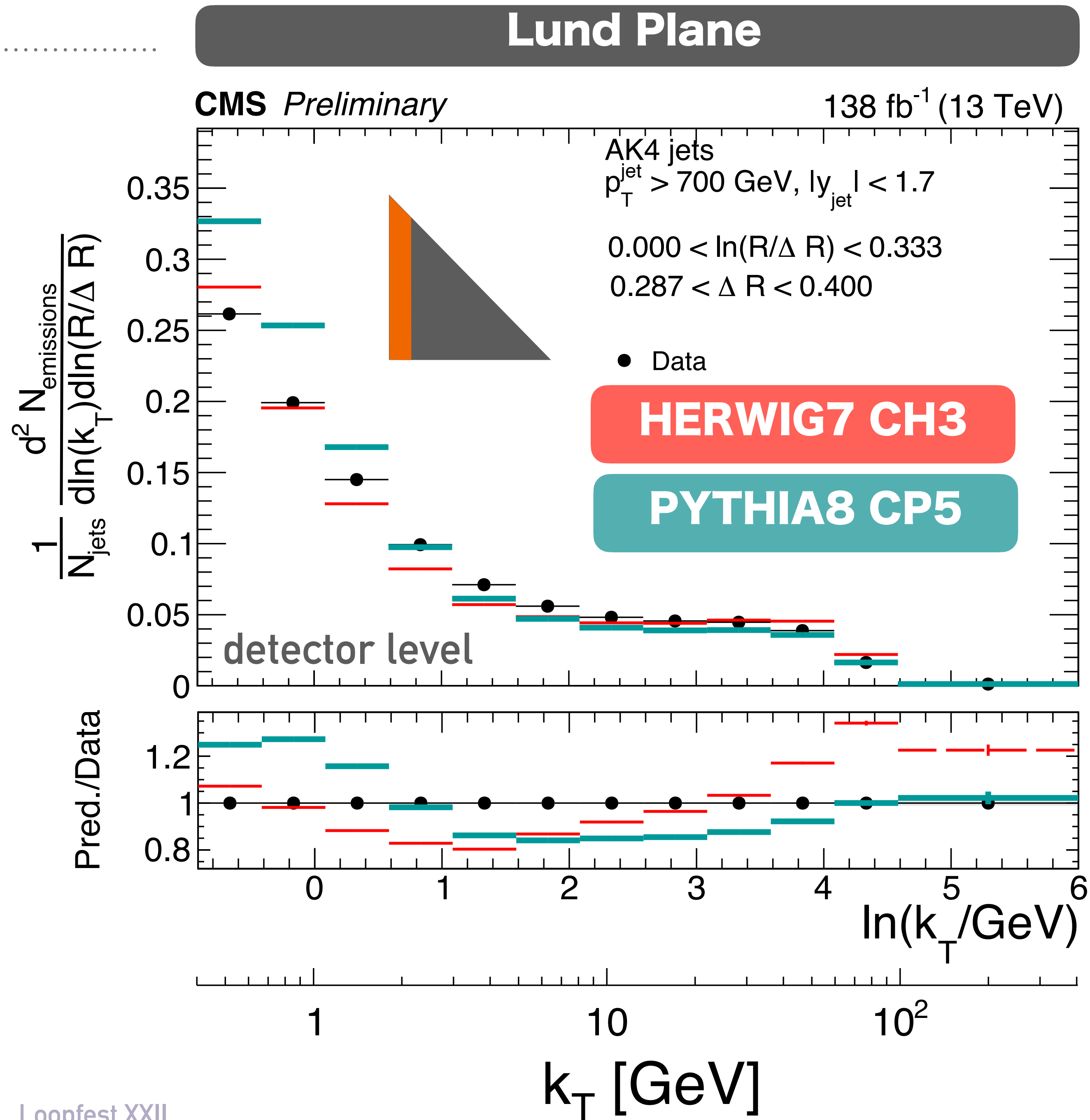


## Parton Showers

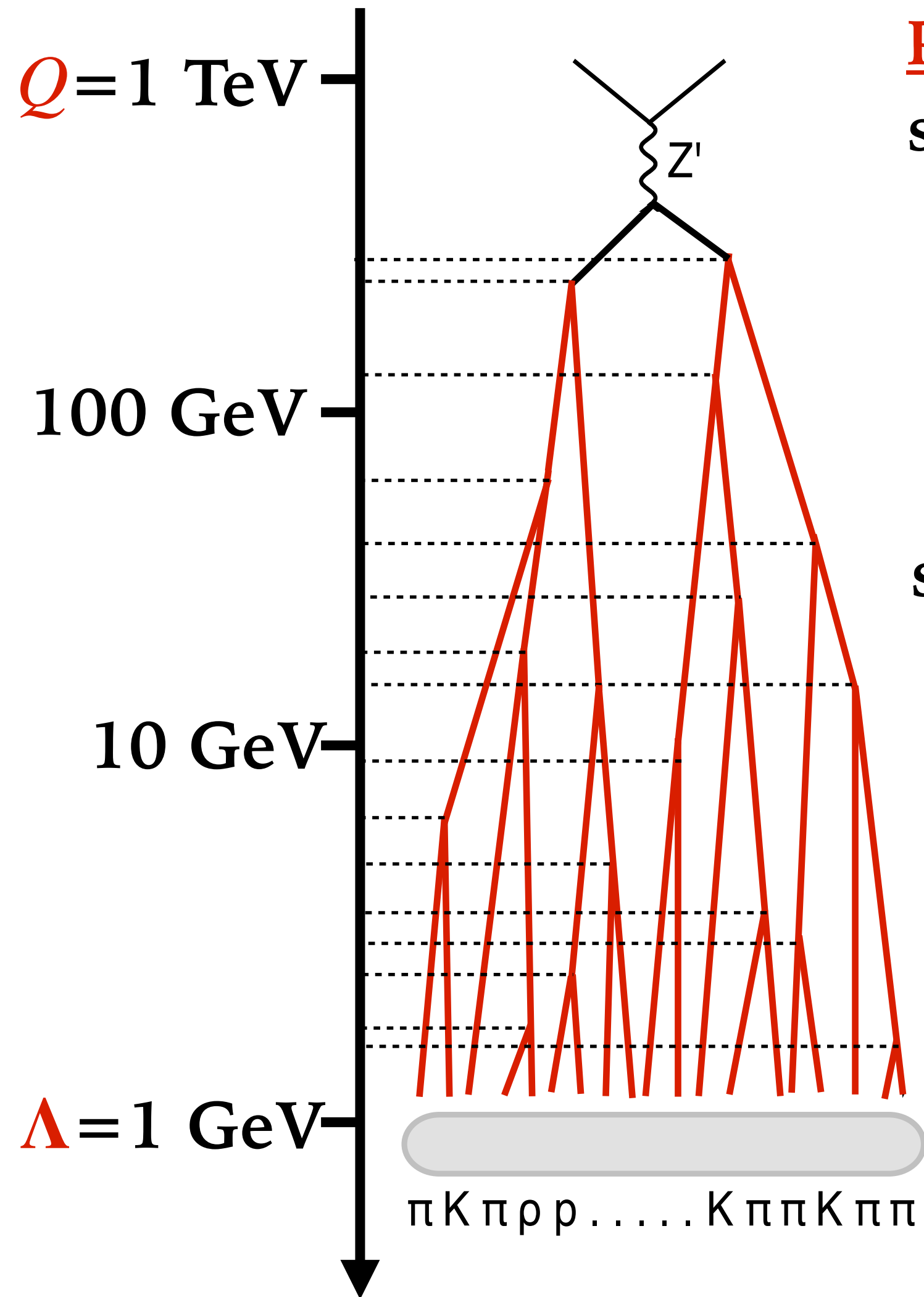
- Energy degradation of hard particles produced during the collision

# Are current showers good enough?

- showers do an amazing job on many observables for **LHC**
- various places see **10–30% discrepancies** between showers and data
- A lot of work is required to meet the **percent precision target!**



# Logarithmically-accurate Parton Showers



**PARTON SHOWERS** = energy degradation via an iterated sequence of softer and softer emissions

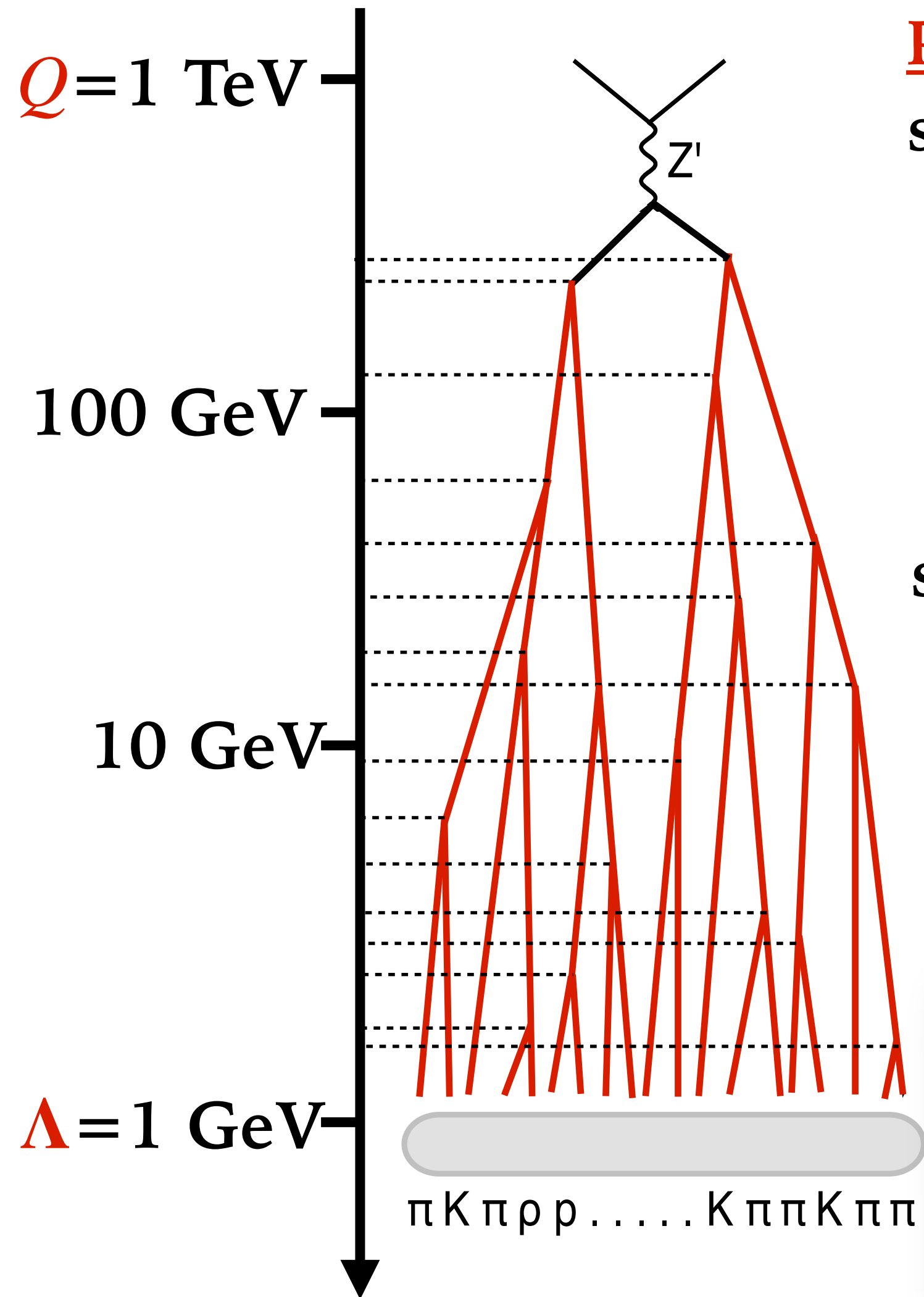
$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable!**

$$\Sigma(O < e^{-L}) = \exp \left( -L g_{LL}(\beta_0 \alpha_s L) + \dots \right)$$

**LL** = leading logs

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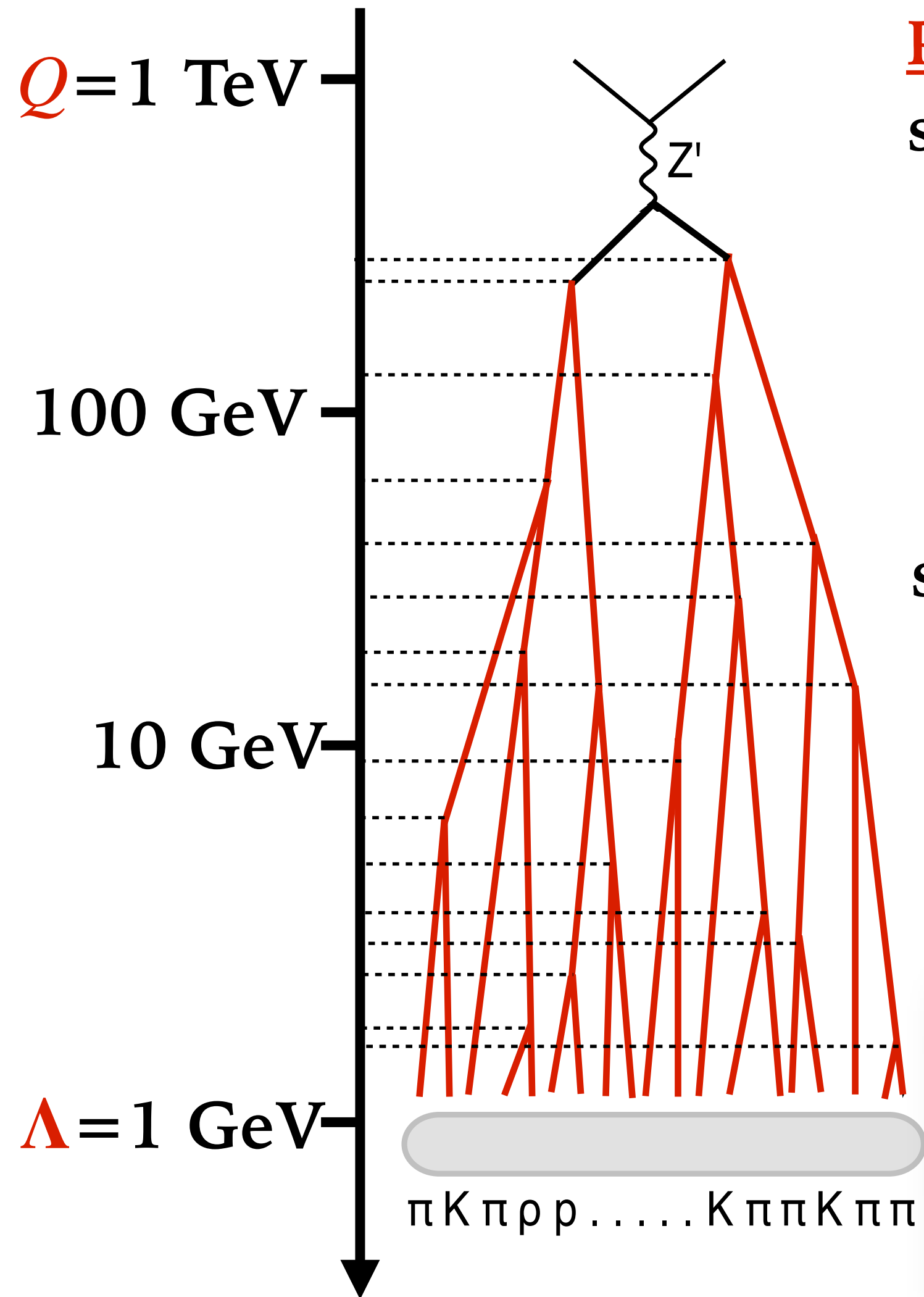
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For  $Q \sim 50 - 10000 \text{ GeV}$ ,  $\beta_0 \alpha_s L \sim 0.3 - 0.5$ :  
Next-to-Leading Logarithms needed for quantitative predictions!

# Logarithmically-accurate Parton Showers



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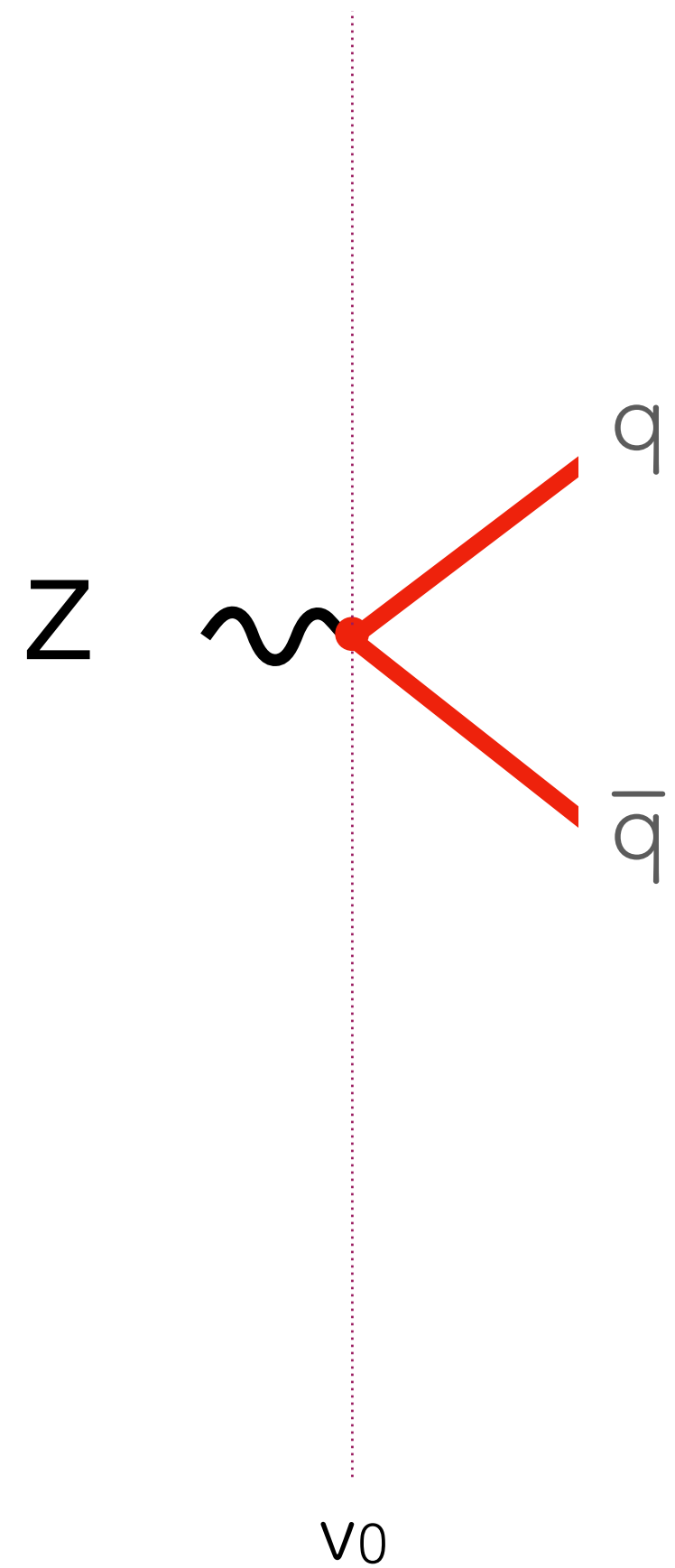
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# Parton Showers in a nutshell

---

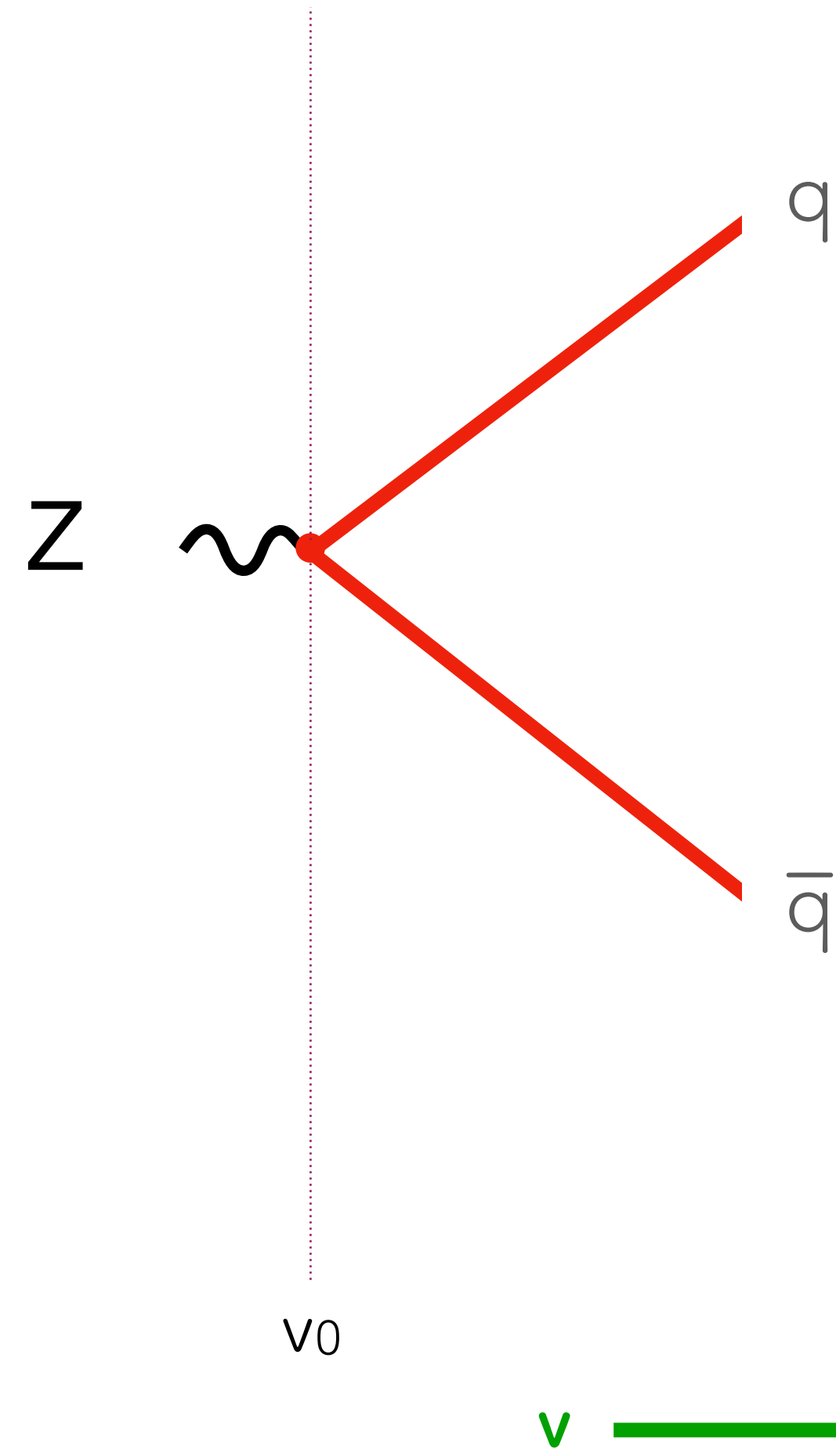
Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm

Start with  $q\bar{q}$  state produced at a hard scale  $v_0$ .



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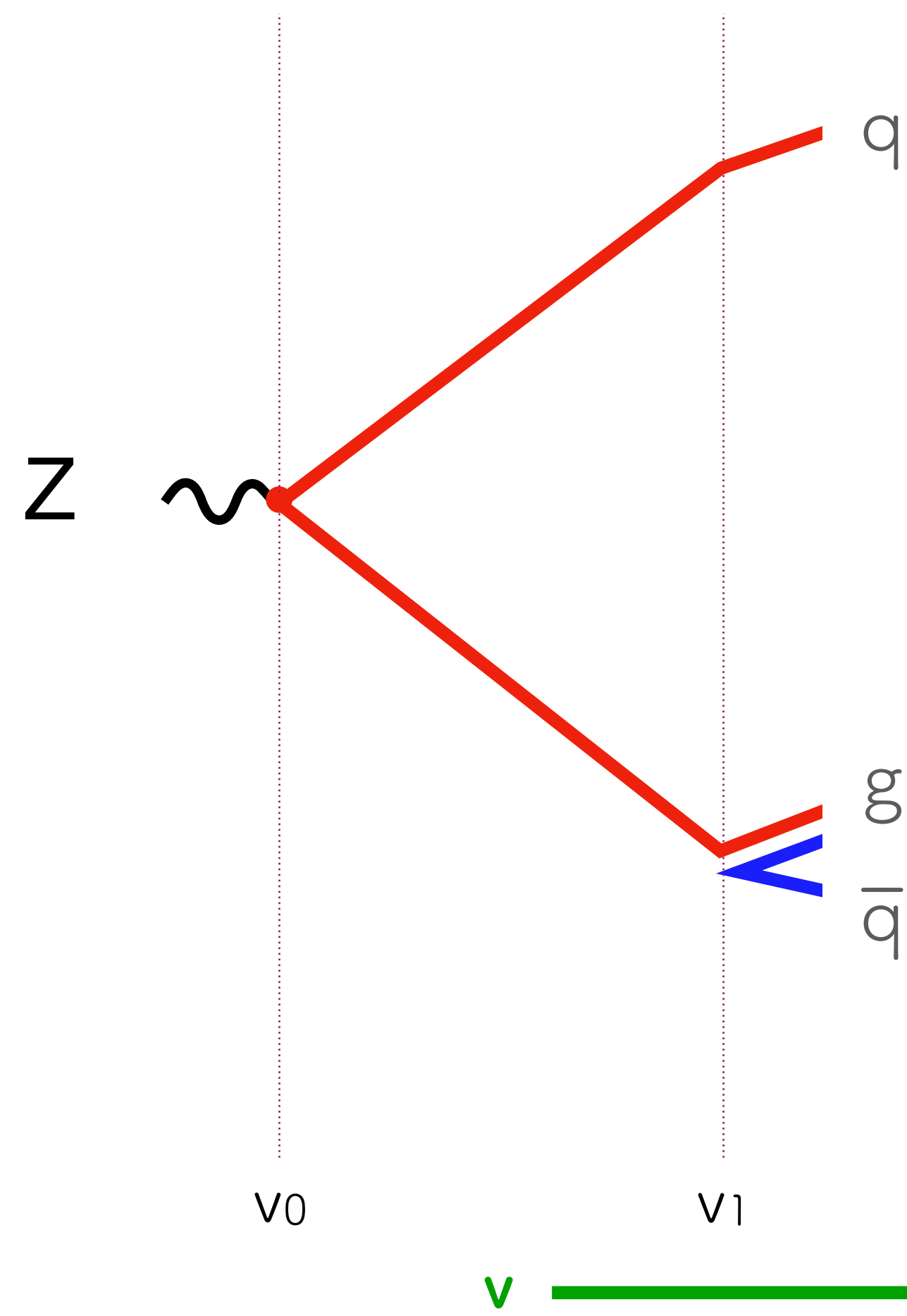
Throw a random number to determine down to what **scale** state persists unchanged

$$\Delta(v_0, v) = \exp \left( - \int_v^{v_0} dP_{q\bar{q}}(\Phi) \right)$$



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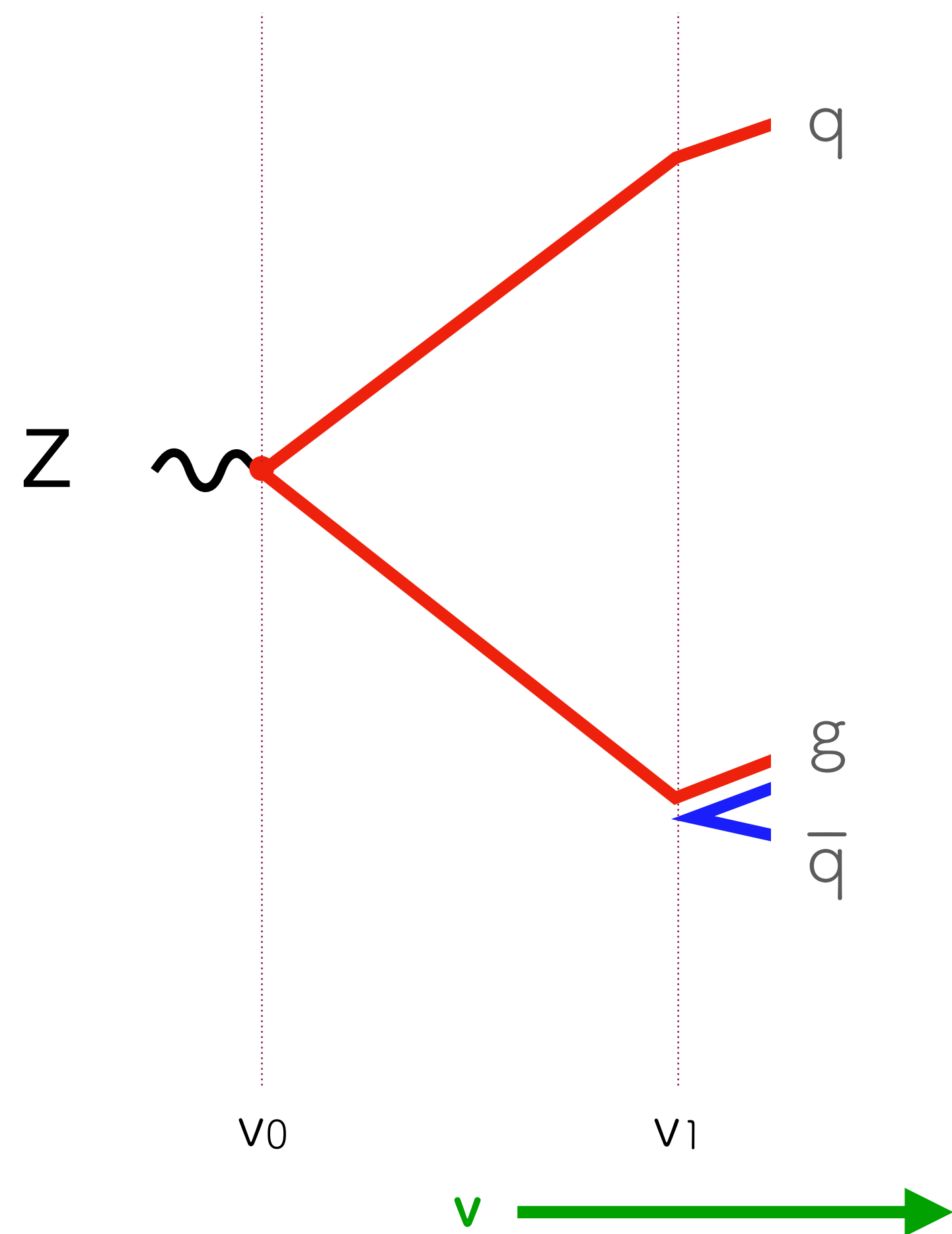
Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** ( $2 \rightarrow 3$ , i.e. emits gluon) at a scale  $v_1 < v_0$ . The kinematic (rapidity and azimuth) of the gluon is chosen according to

$$dP_{q\bar{q}}(\Phi(v_1)) \quad \Phi = \{v, \eta, \varphi\}$$

# Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



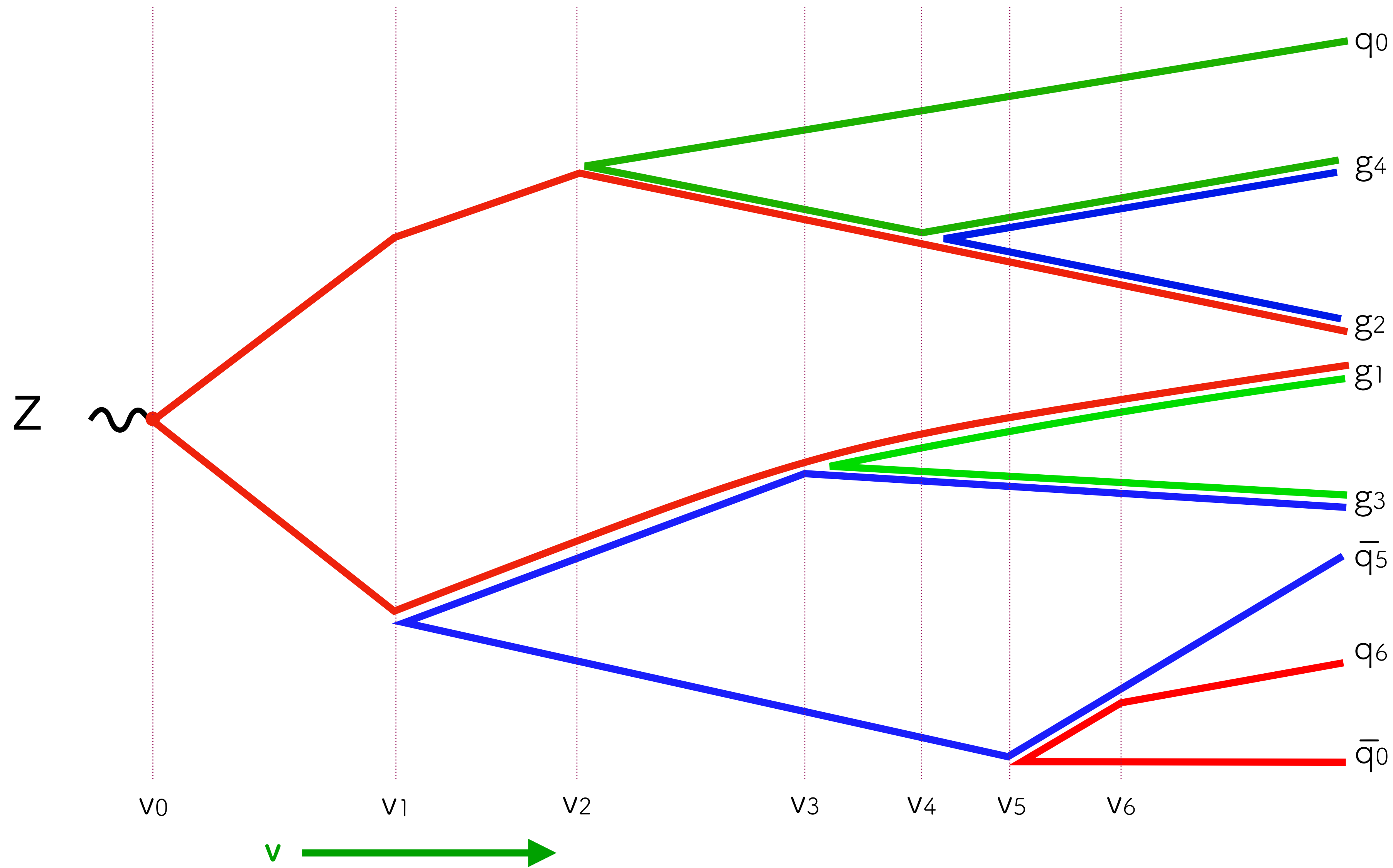
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At some point, **state splits** ( $2 \rightarrow 3$ , i.e. emits gluon) at a scale  $v_1 < v_0$ .

The gluon is part of two dipoles ( $qg$ ), ( $g\bar{q}$ ).

Iterate the above procedure for both dipoles independently, using  $v_1$  as starting scale.

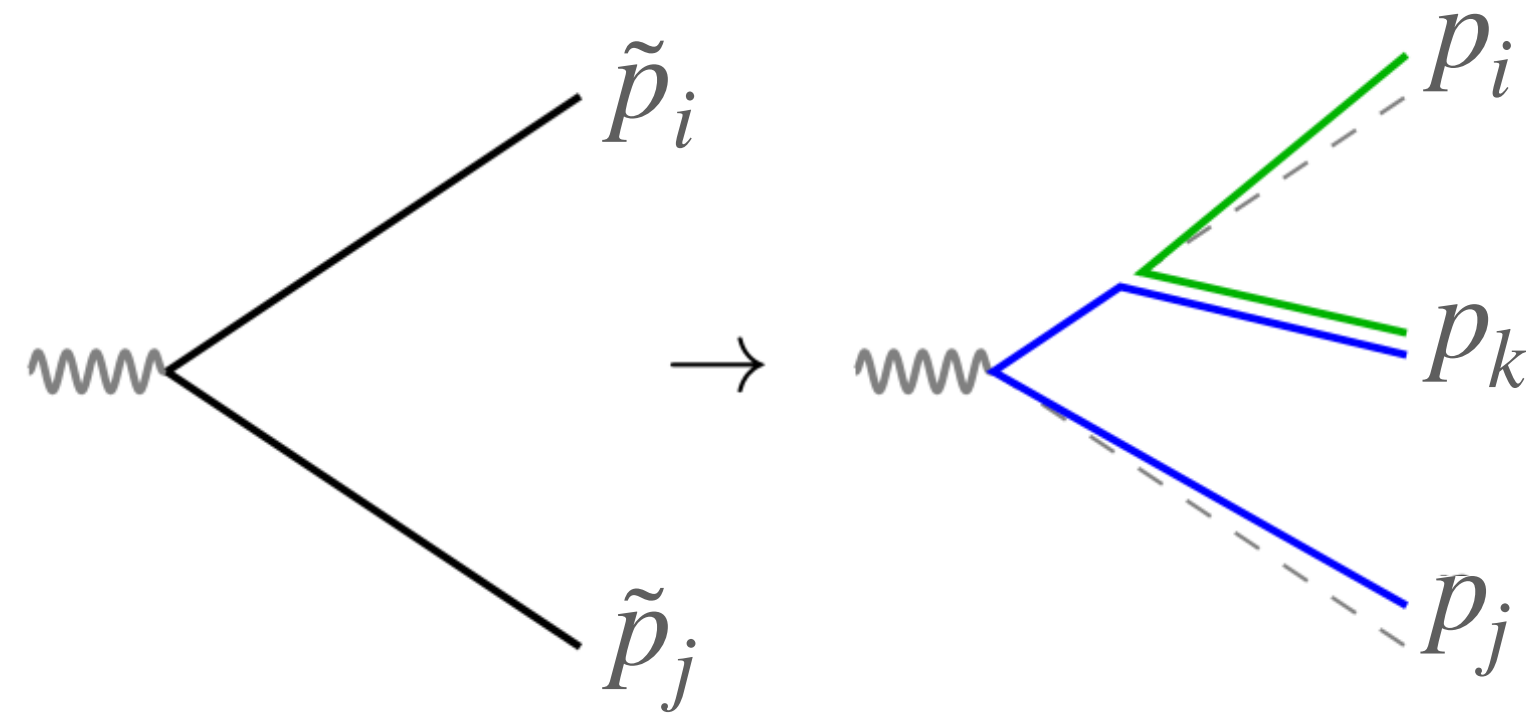


self-similar  
evolution  
continues until it  
reaches a non-  
perturbative  
scale

# Dissecting the parton shower emission probability

---

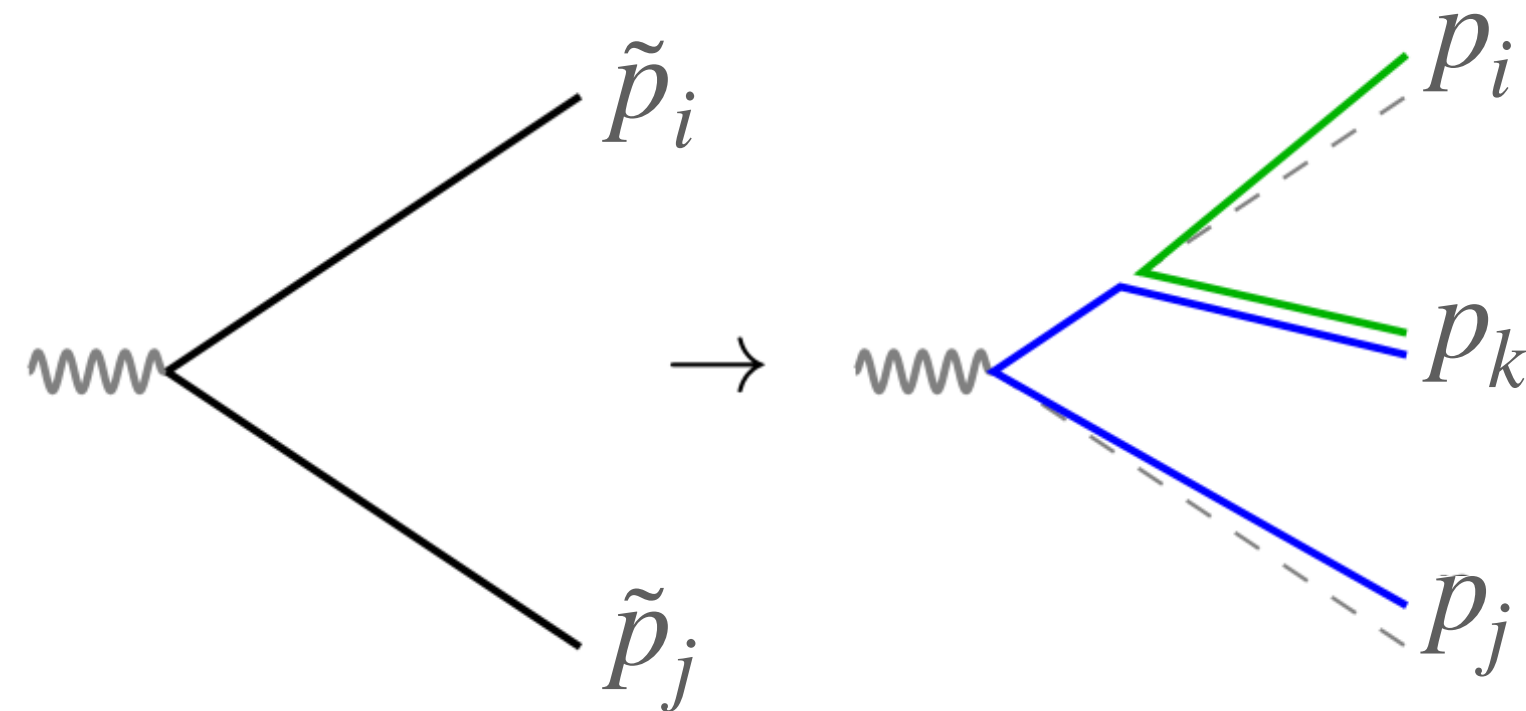
Starting from a  $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$  system, what is the splitting probability?



$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

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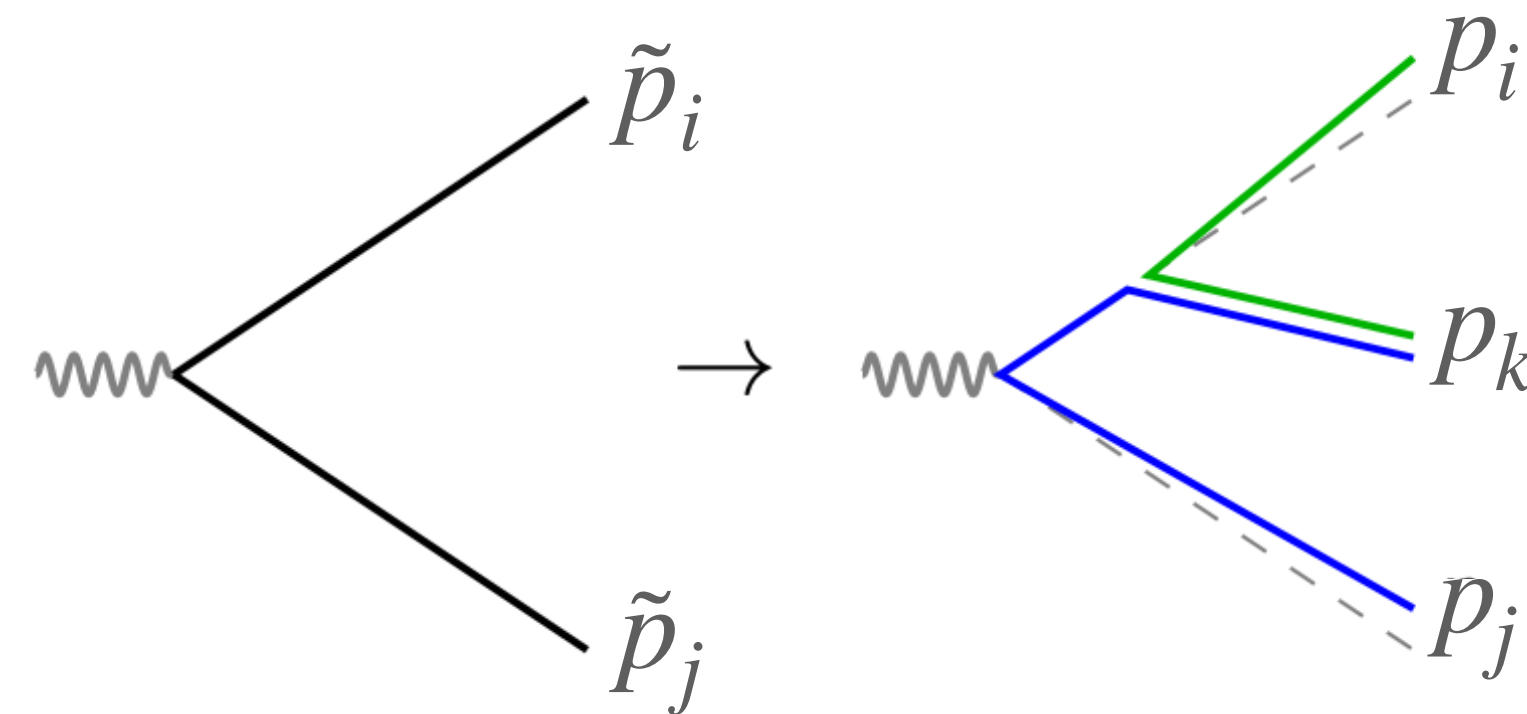


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**Matrix element** for emitting a parton  $k$  from a parton  $i$  (or  $j$ )

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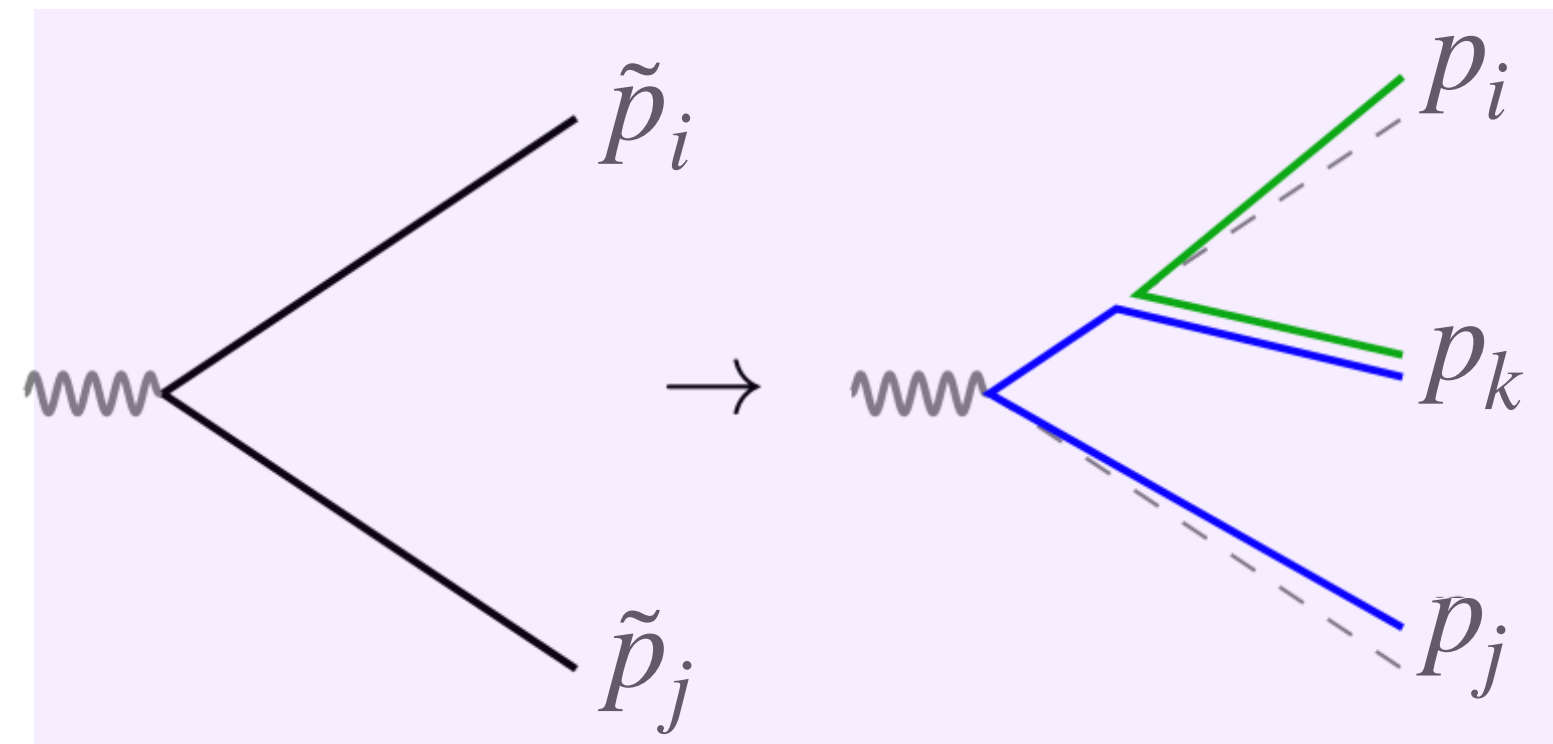
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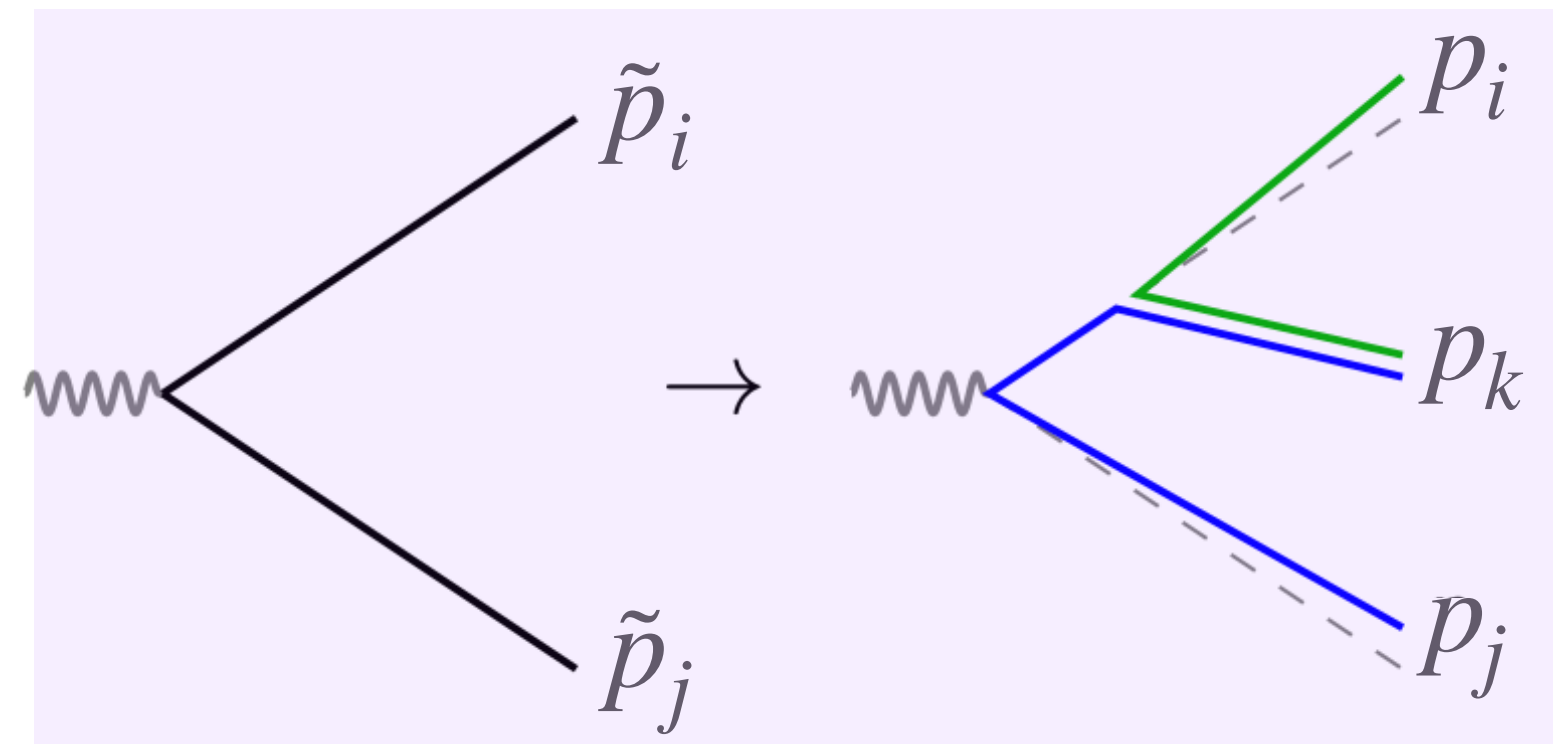
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Their interplay determines the shower **logarithmic accuracy**



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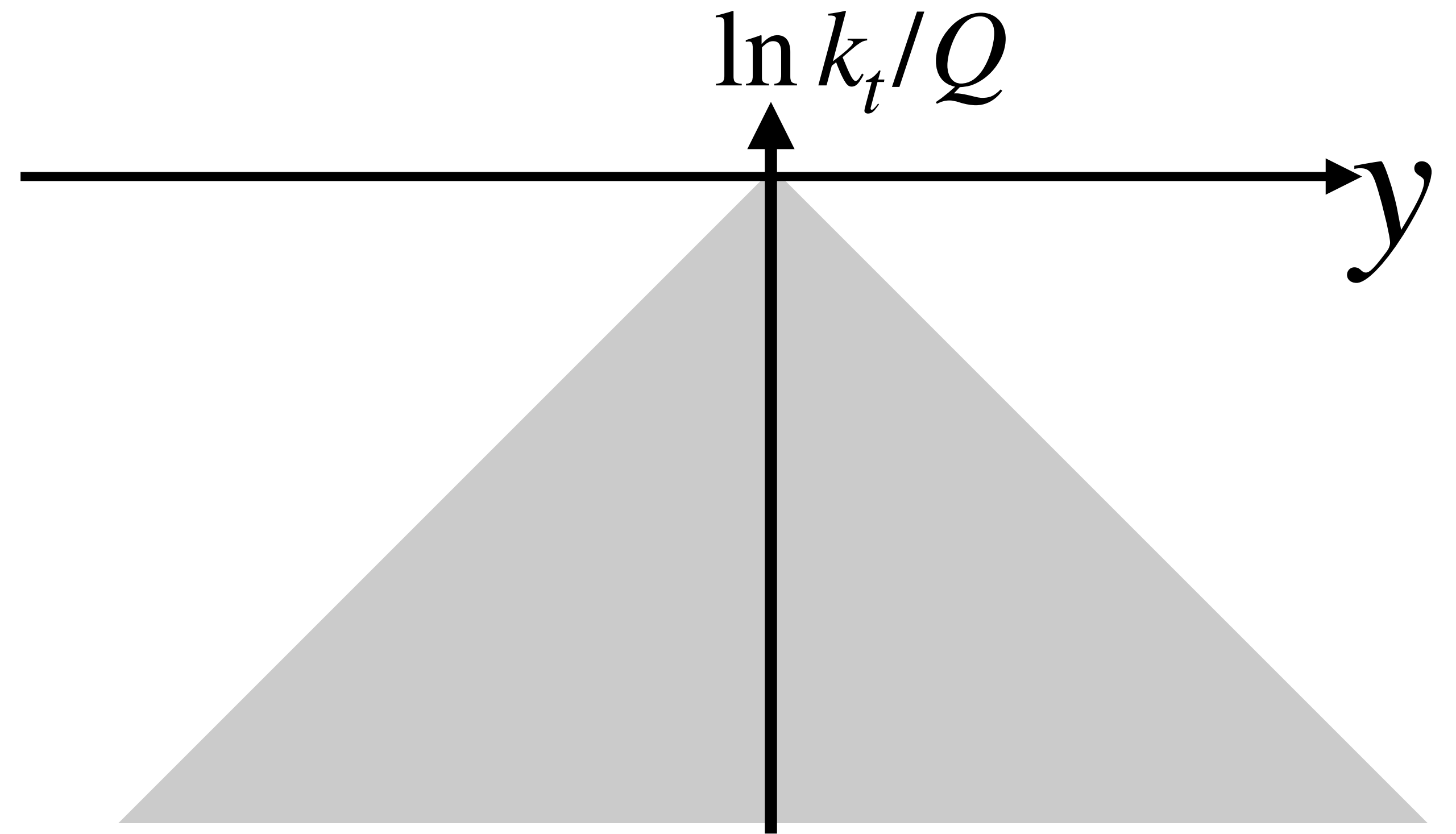
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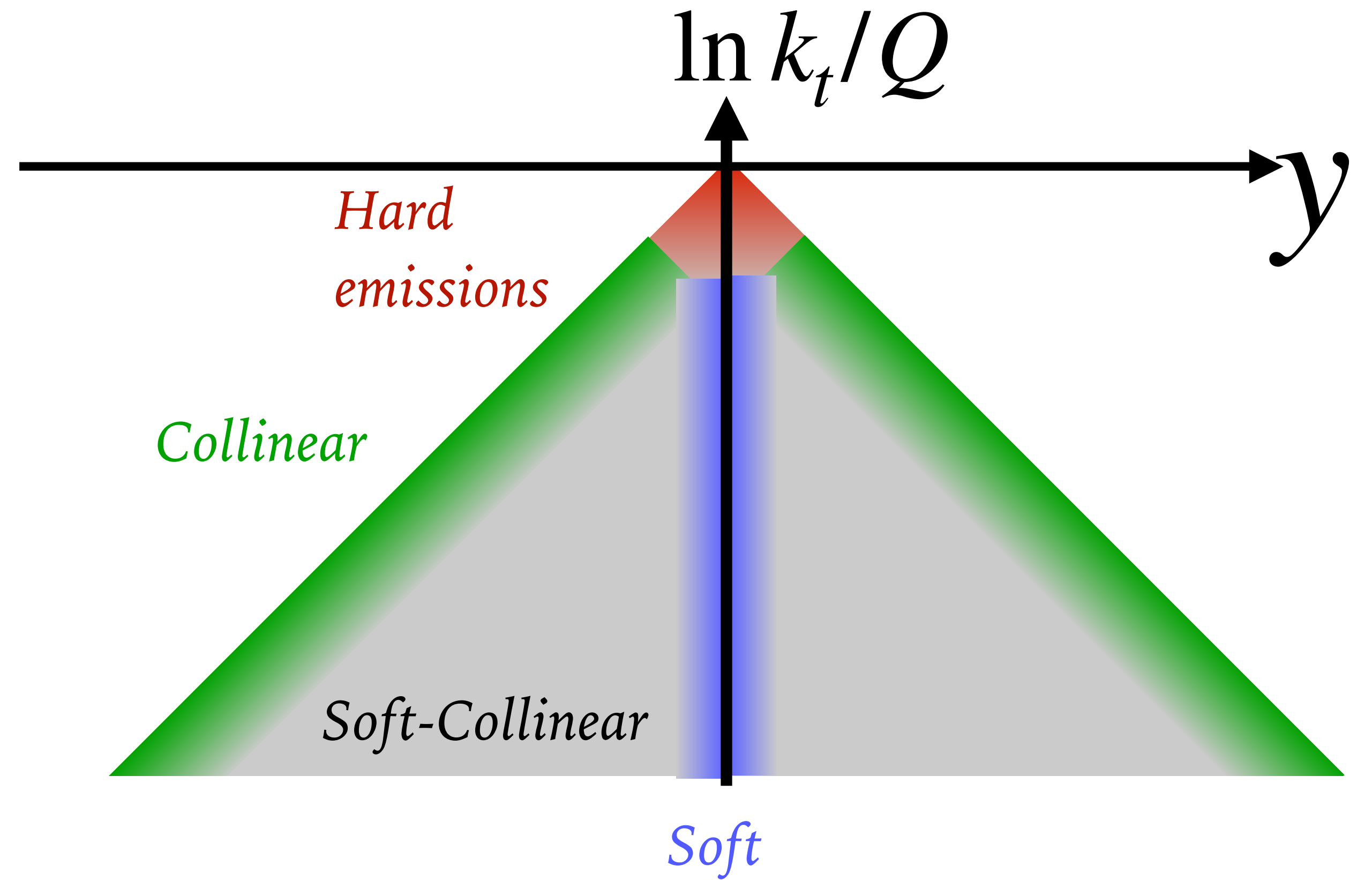
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- **The Lund plane:** diagnostic tools for resummation and parton showers



# How to build a logarithmically-accurate parton shower?

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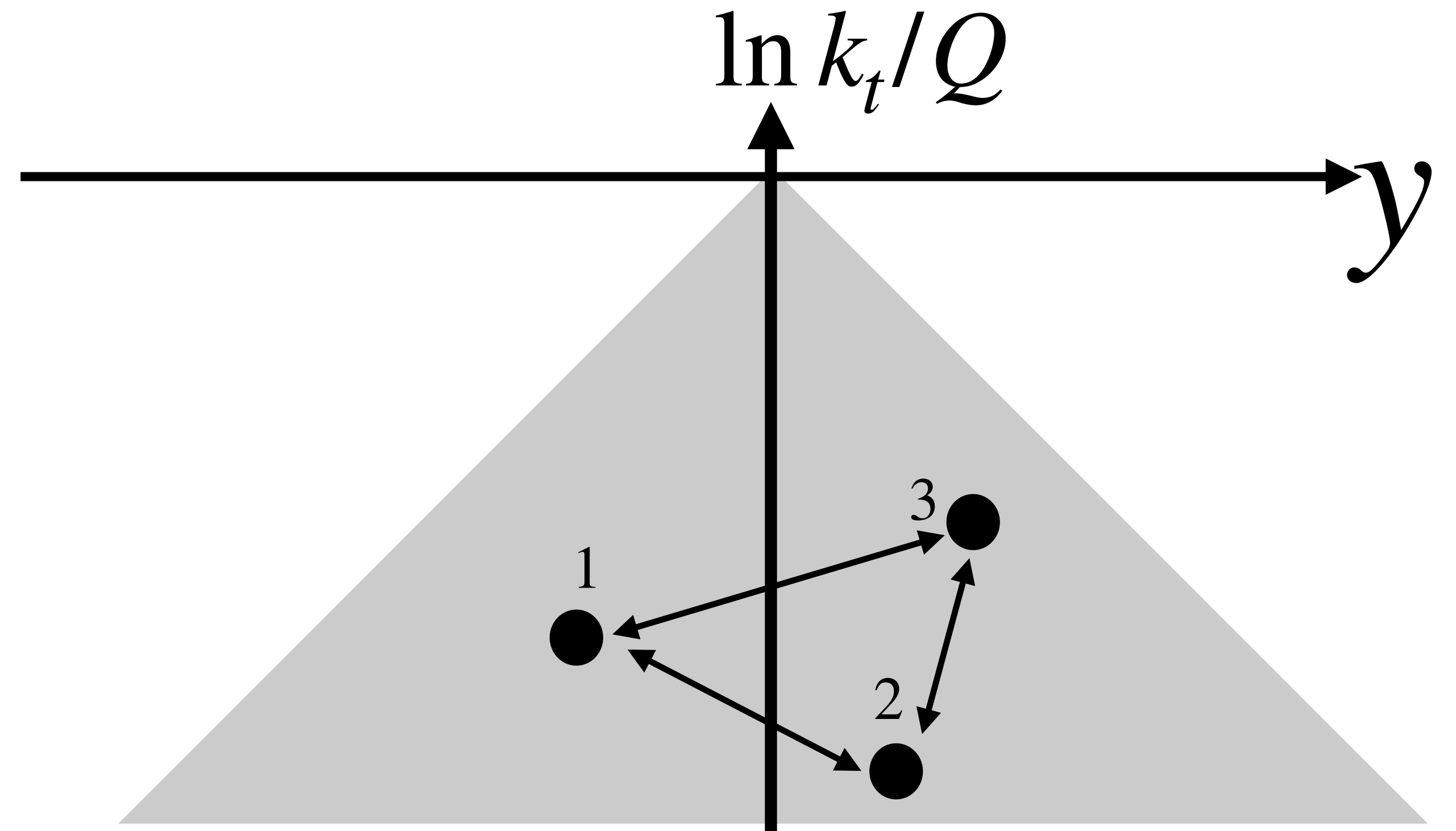
# How to build a LL parton shower?

- The Lund plane: diagnostic tools for resummation and parton showers
- At Leading Logarithmic accuracy we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz d \ln k_t$$

*One-loop QCD coupling constant at  $\mu_R = k_t$*

*LO soft splitting function*



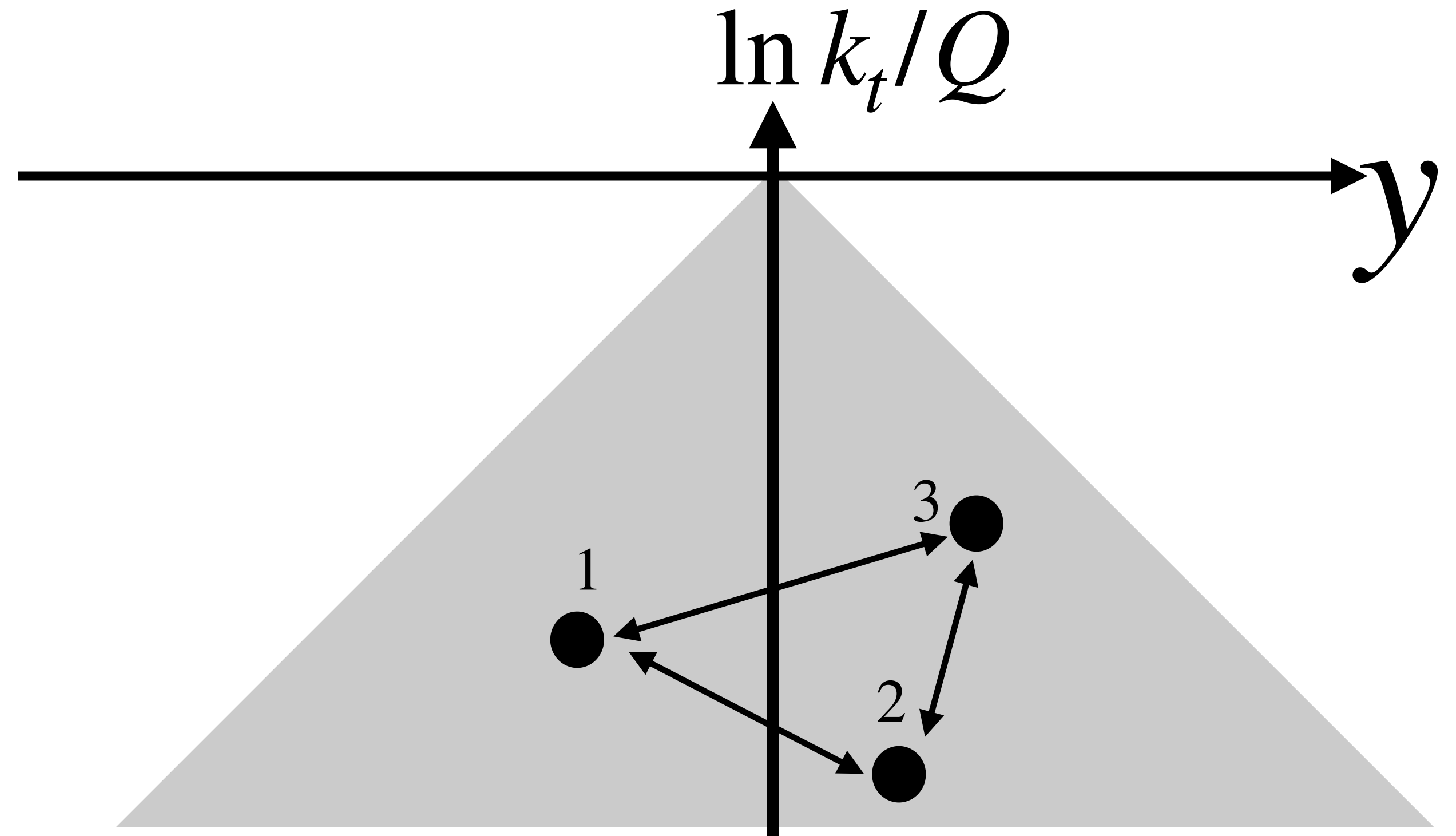
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This tells us what **matrix element** should we use to generate a new emission

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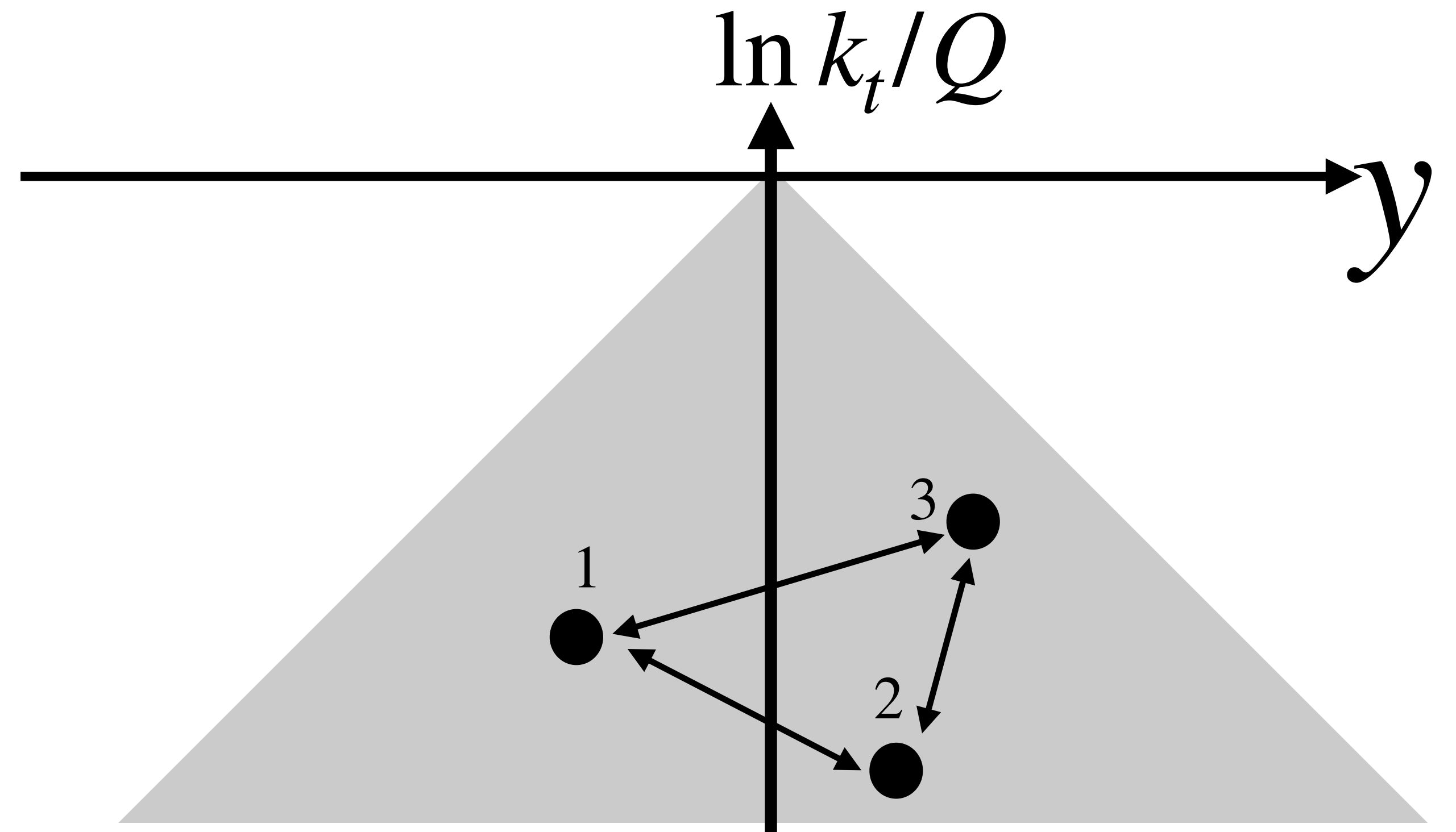
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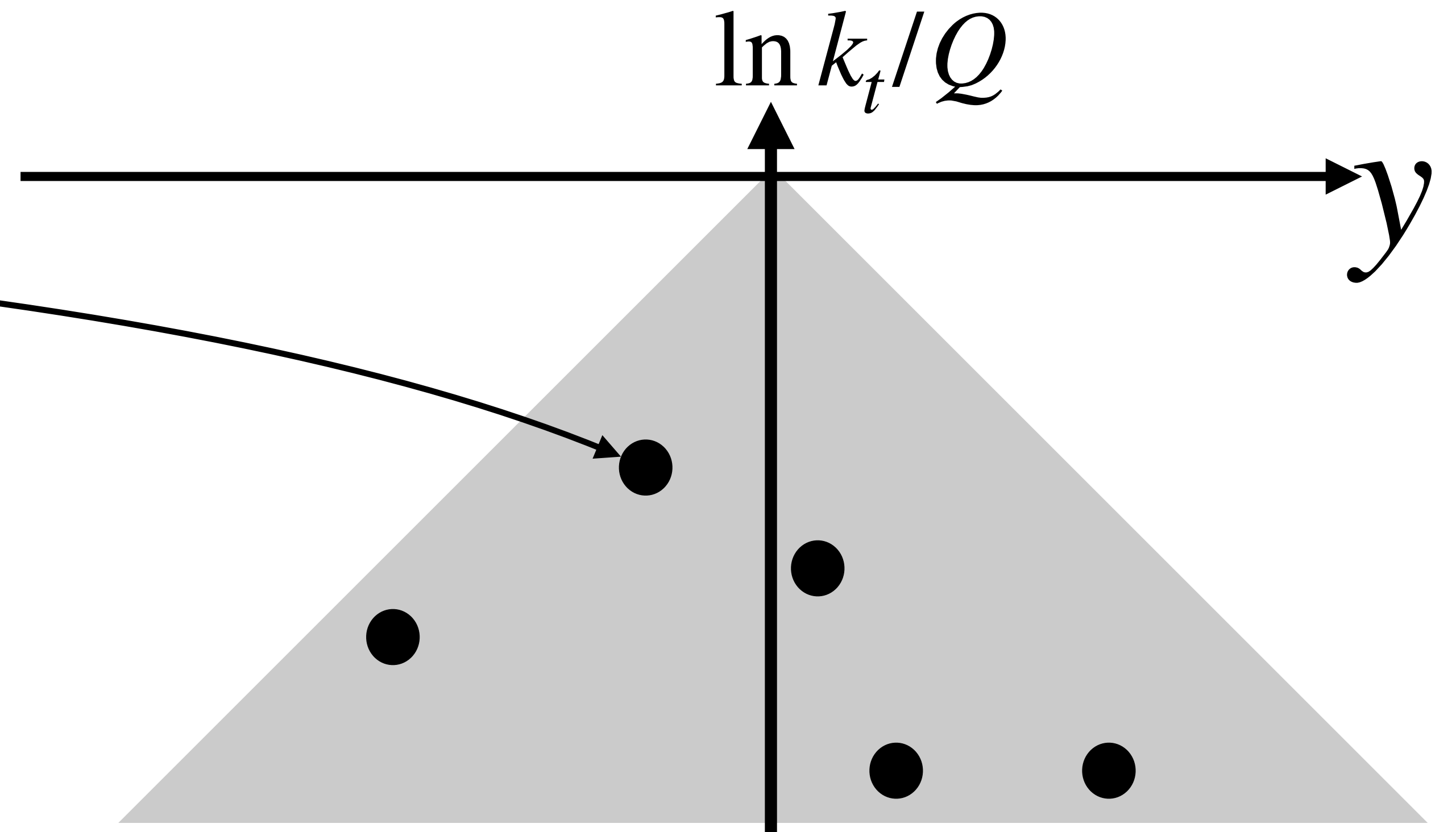
This constraints the **kinematic mapping**  $\Phi_n \rightarrow \Phi_{n+1}$  and the **ordering variable** choice: emissions well separated in rapidity and transverse momentum are independent from each others

# How to build a NLL parton shower?

At NLL accuracy:

- The rate for soft-collinear emissions must be correct at NLO

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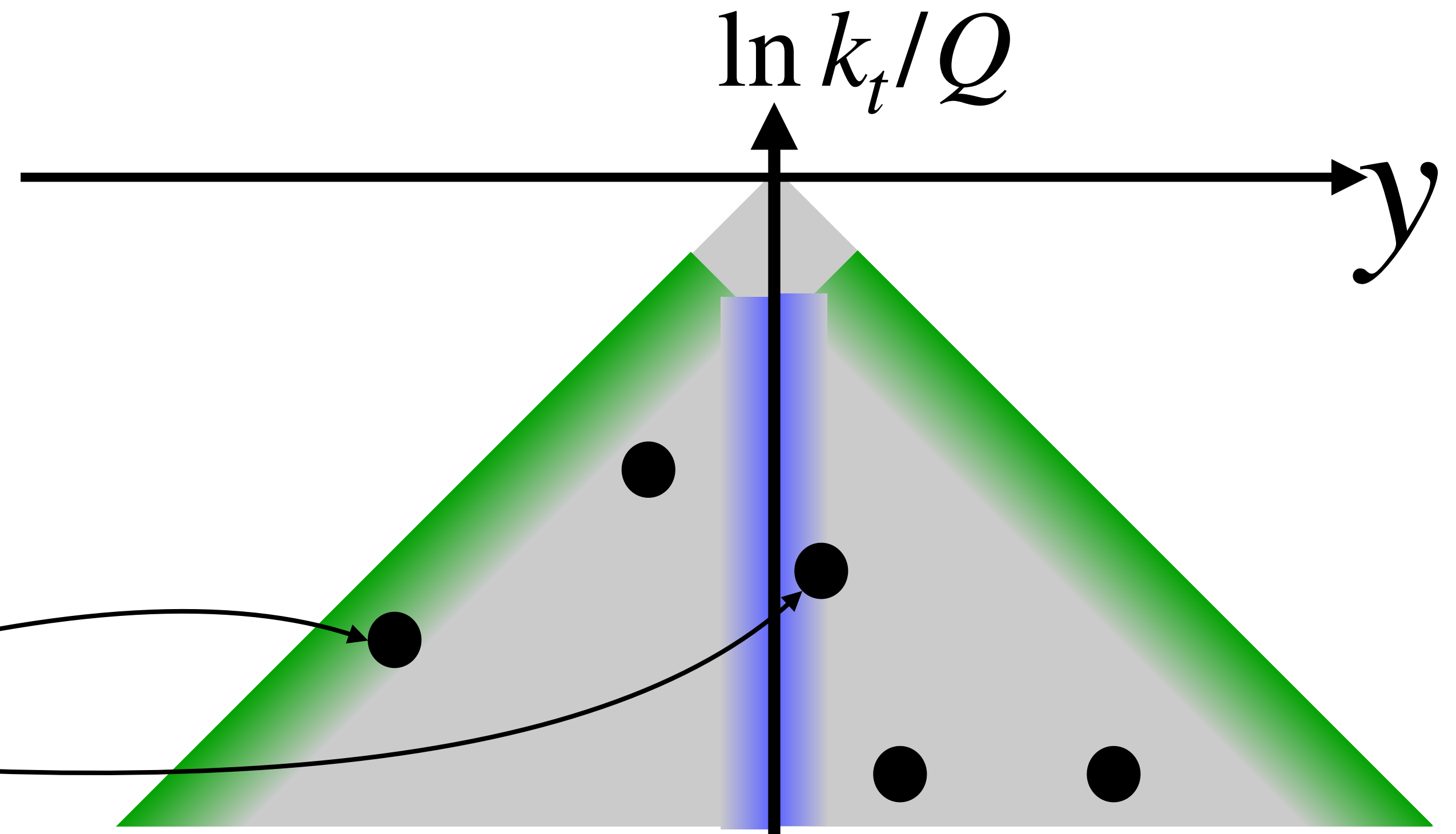
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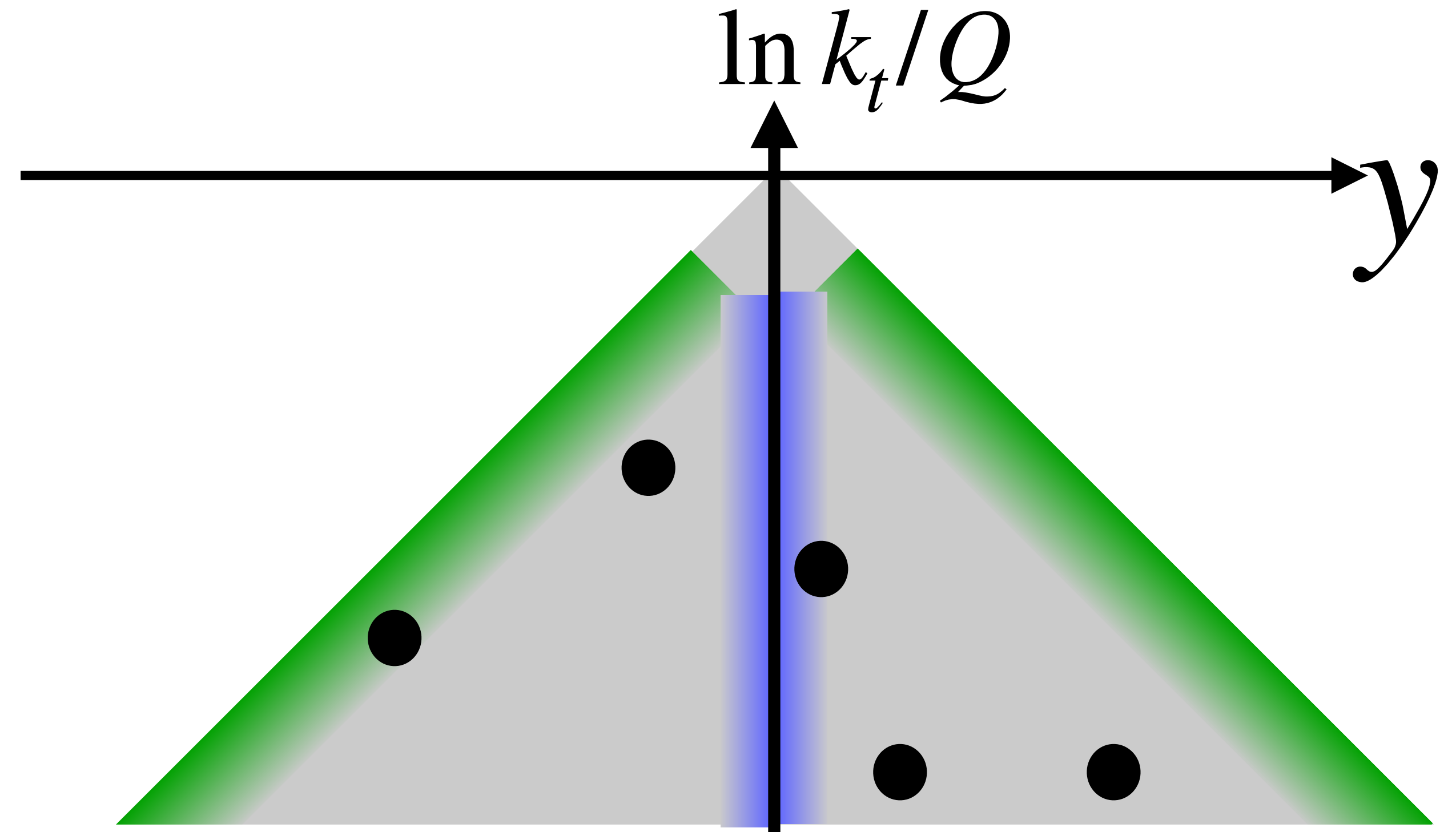
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*Catani, Marchesini, Webber '91*



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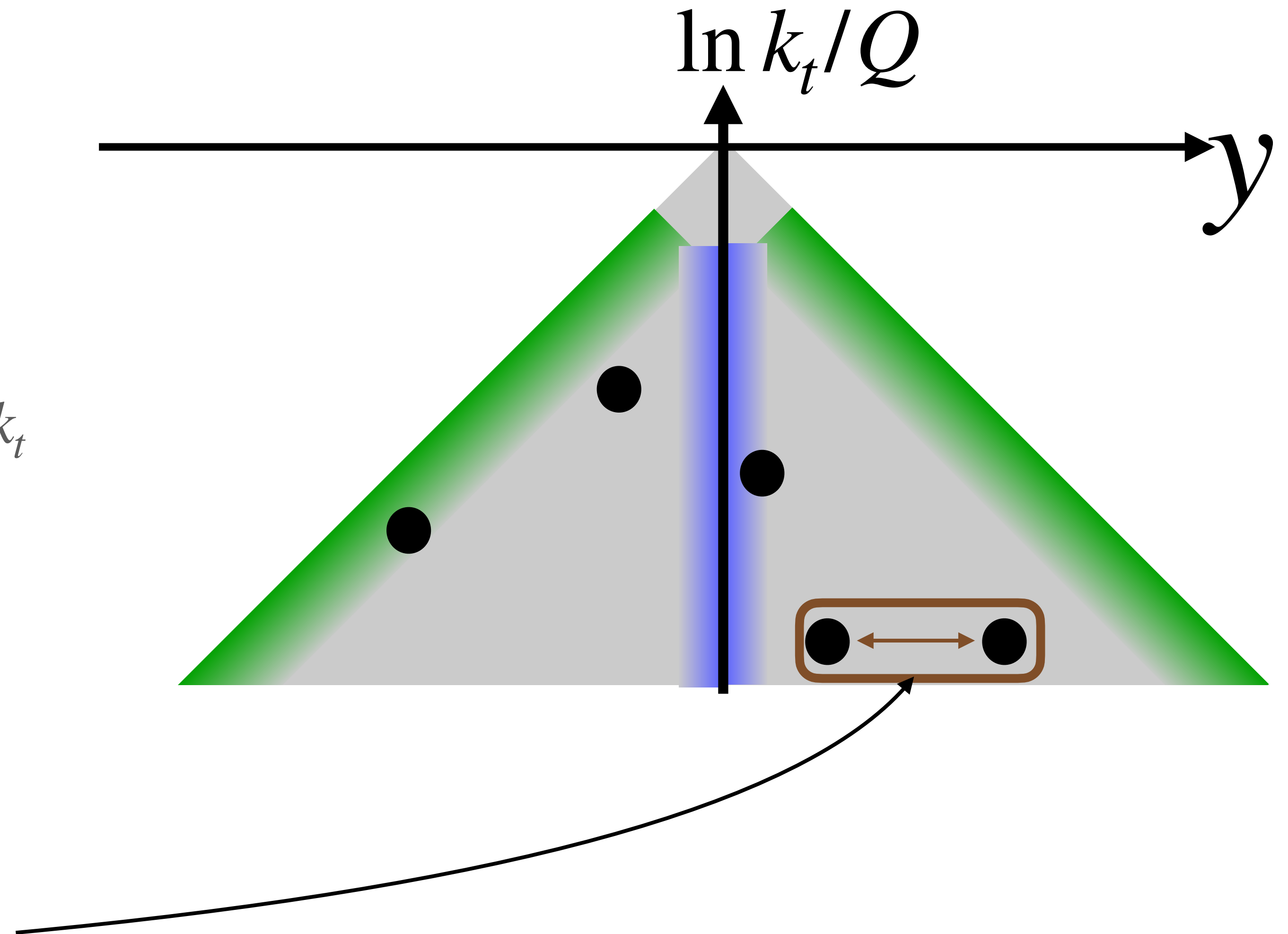
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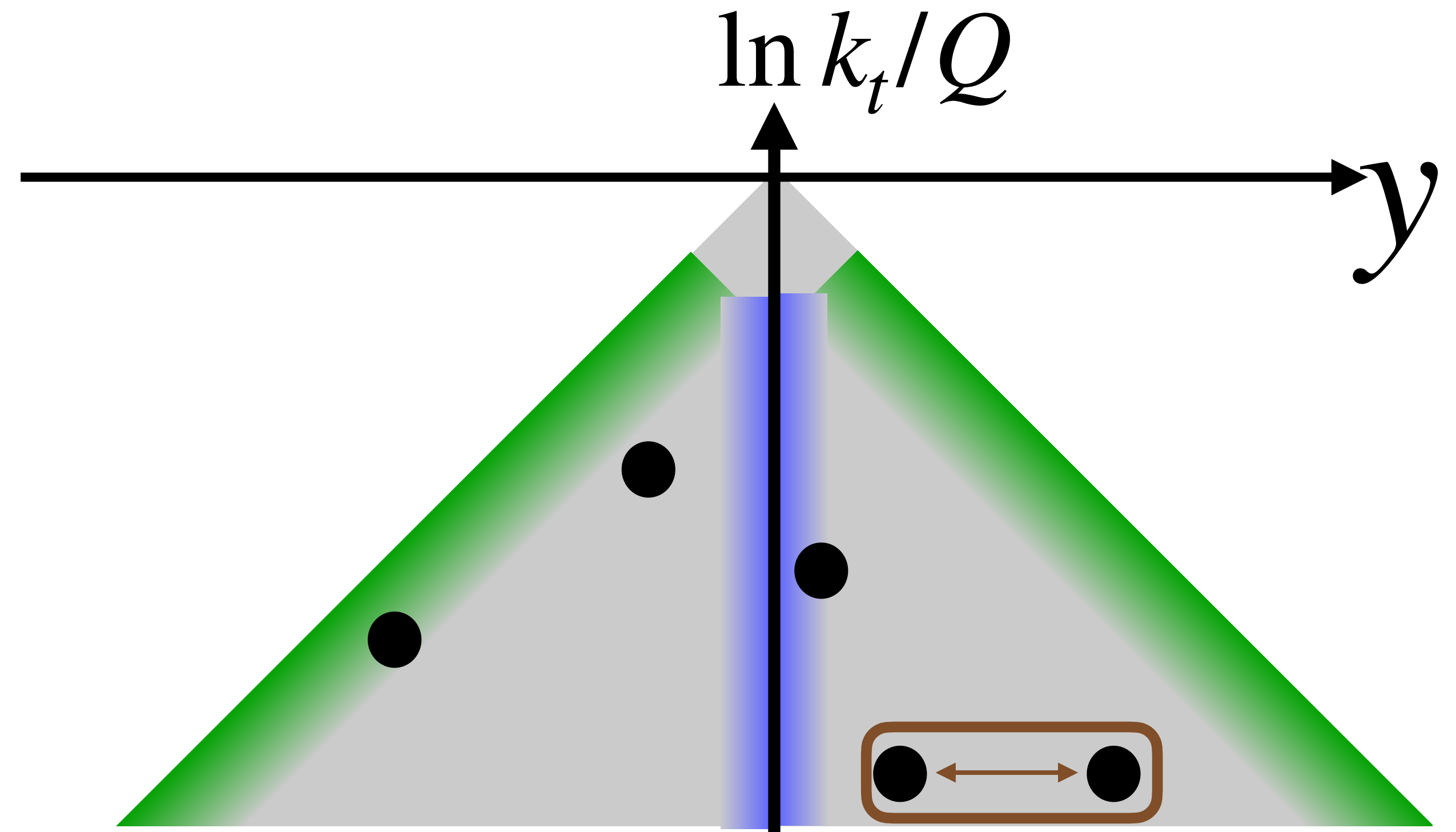
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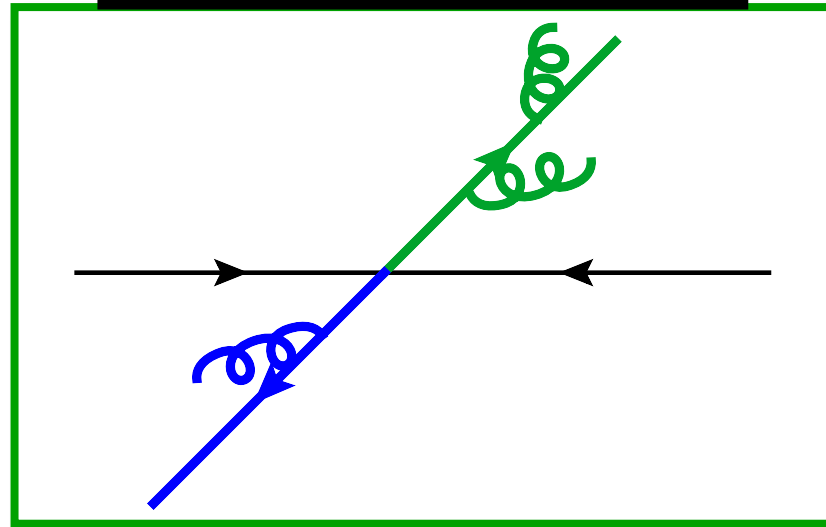
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*Dasgupta, Dreyer, Hamilton, Monni, Salam, 1805.09327 ;+ Soyez, 2002.11114*

# Status of NLL PanScales showers

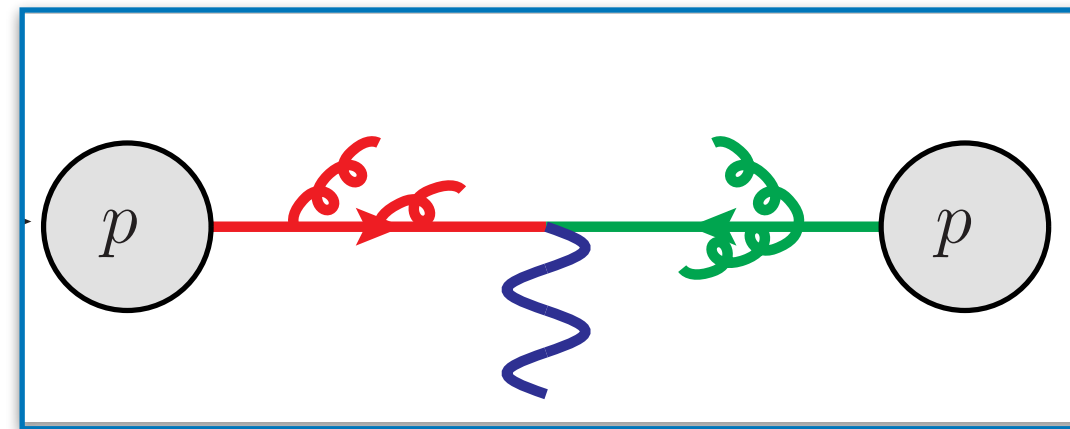
- This enabled the PanScales to devise the first showers with **general** NLL accuracy for

$$e^+e^- \rightarrow j_1j_2$$



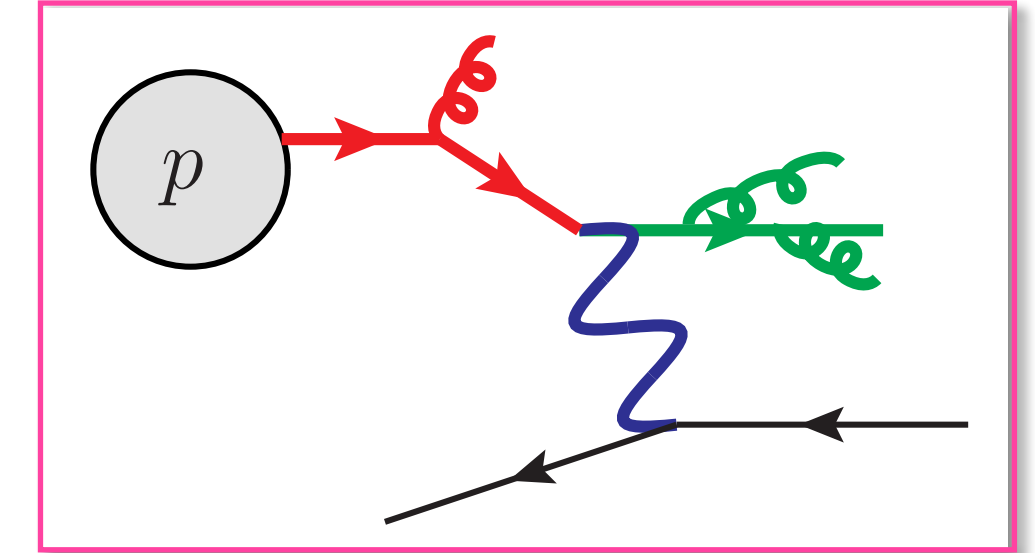
Dasgupta, Dreyer, Hamilton,  
Monni, Salam, Soyez,  
2002.11114

$$pp \rightarrow \text{colour singlet}$$



van Beekveld, SFR, Soto-Ontoso,  
Salam, Soyez, Verheyen, 2205.02237,  
+ Hamilton 2207.09467

DIS & VBF

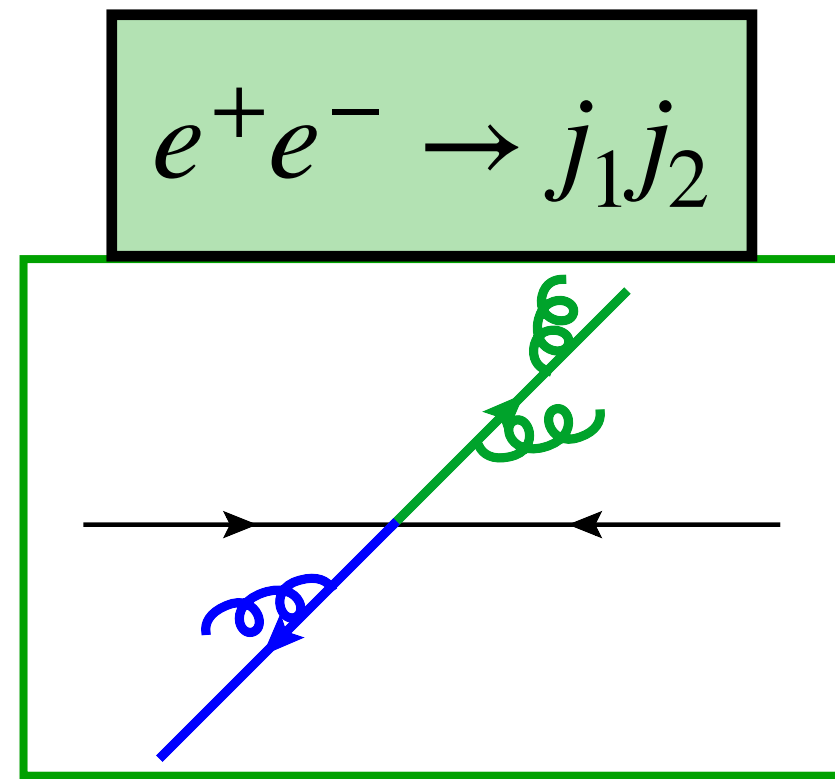


van Beekveld, SFR,  
2305.08645

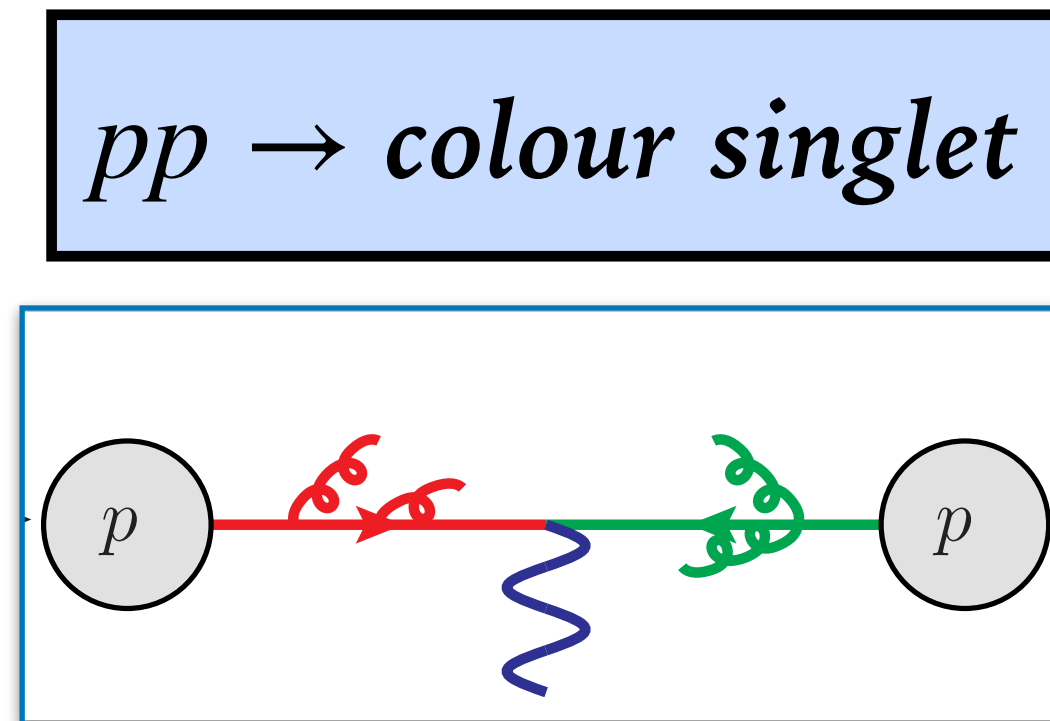
...with **subleading colour** (2011.10054) and **spin correlations** (2103.16526, 2111.01161)

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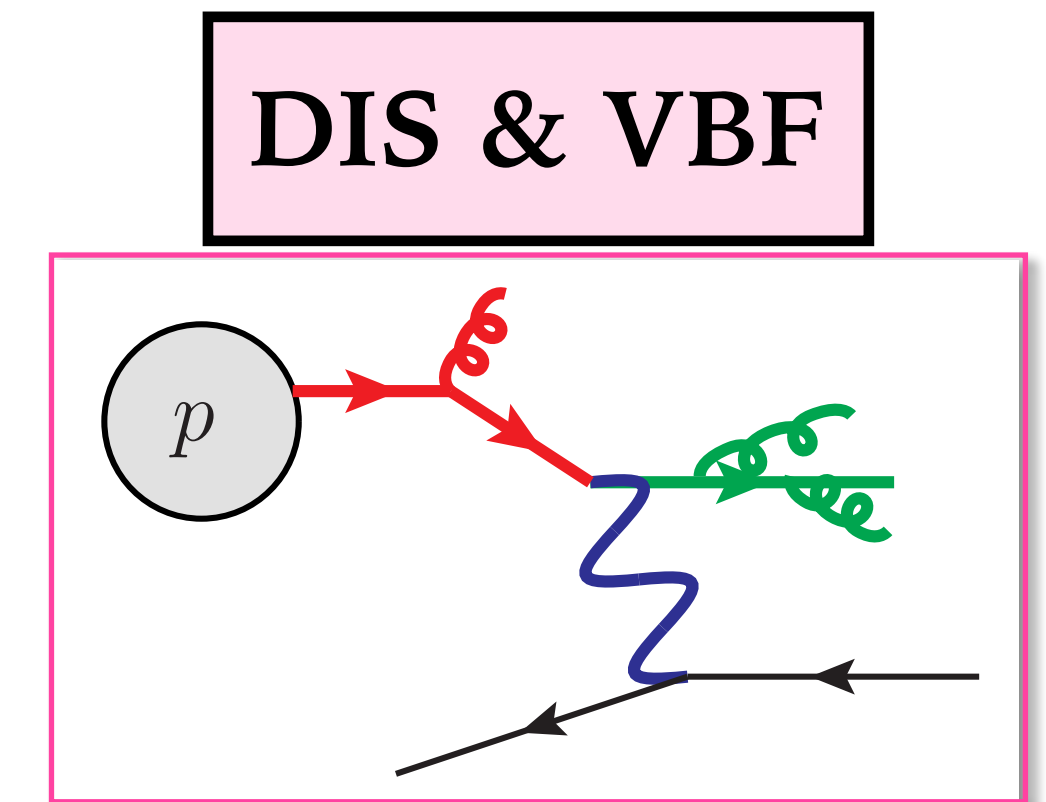
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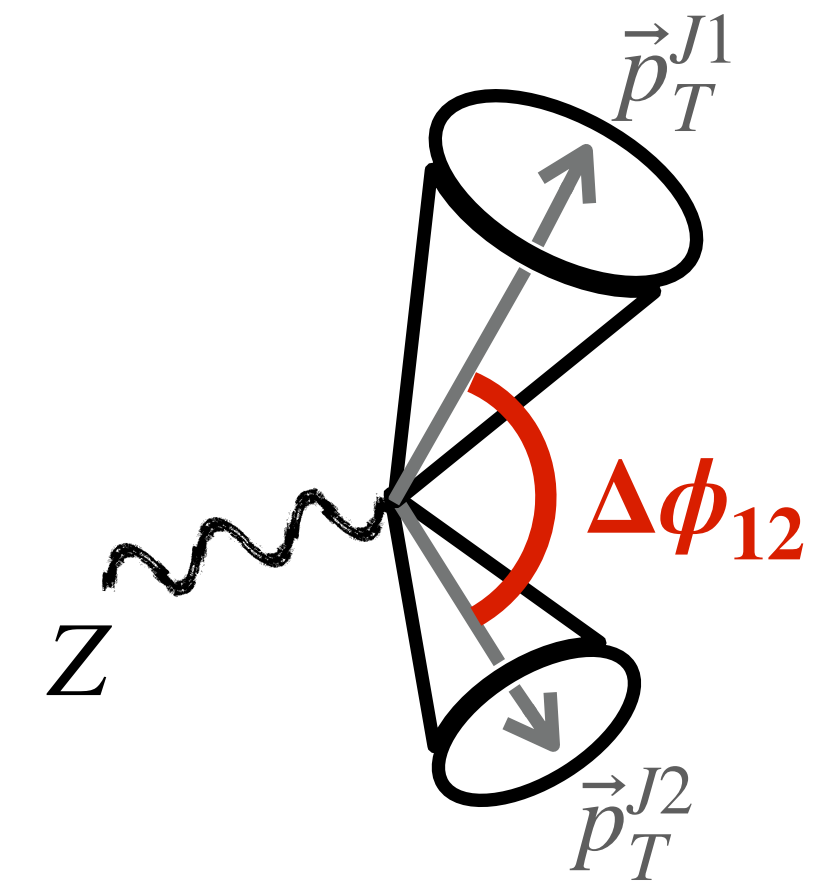
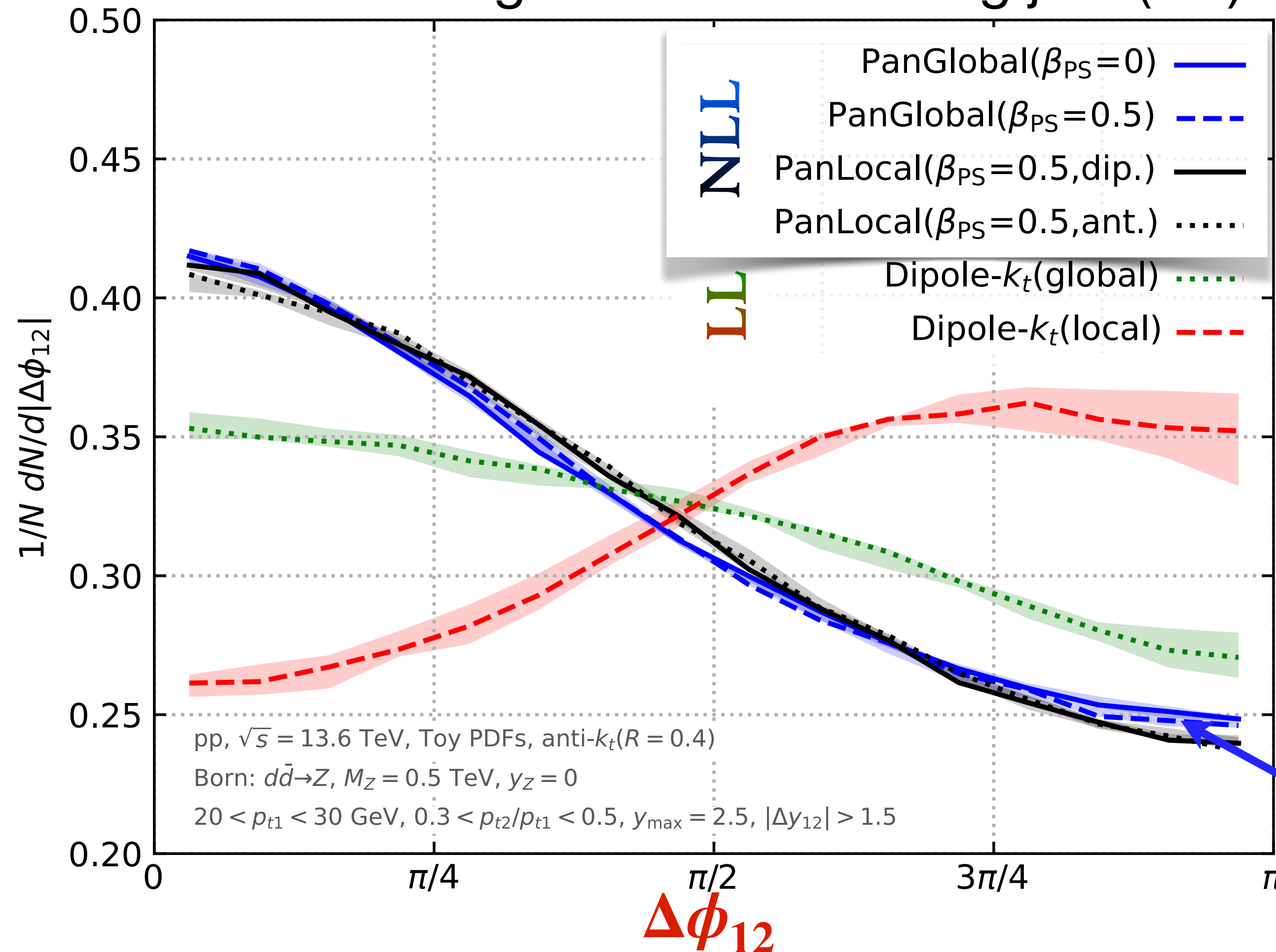
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- Herwig7 angular-ordered shower for the same processes is NLL but only for global event shapes (Bewick, SFR, Richardson, Seymour, 1904.11866, 2107.04051 )
- **Deductor** has been proven to be NLL at least for  $e^+e^- \rightarrow j_1j_2$  (Nagy, Soper 2011.04777)
- **Alaric** is NLL at leading colour for  $e^+e^- \rightarrow j_1j_2$  (2208.06057), recently extended to generic  $pp$  collisions (2404.14360) — expected to retain NLL accuracy for  $pp \rightarrow \text{colour singlet}$

# Exploratory phenomenology for high-mass Drell-Yan at the LHC

$$m_{\ell\ell} = 500 \text{ GeV}$$

Azimuthal angle between leading jets (DY)



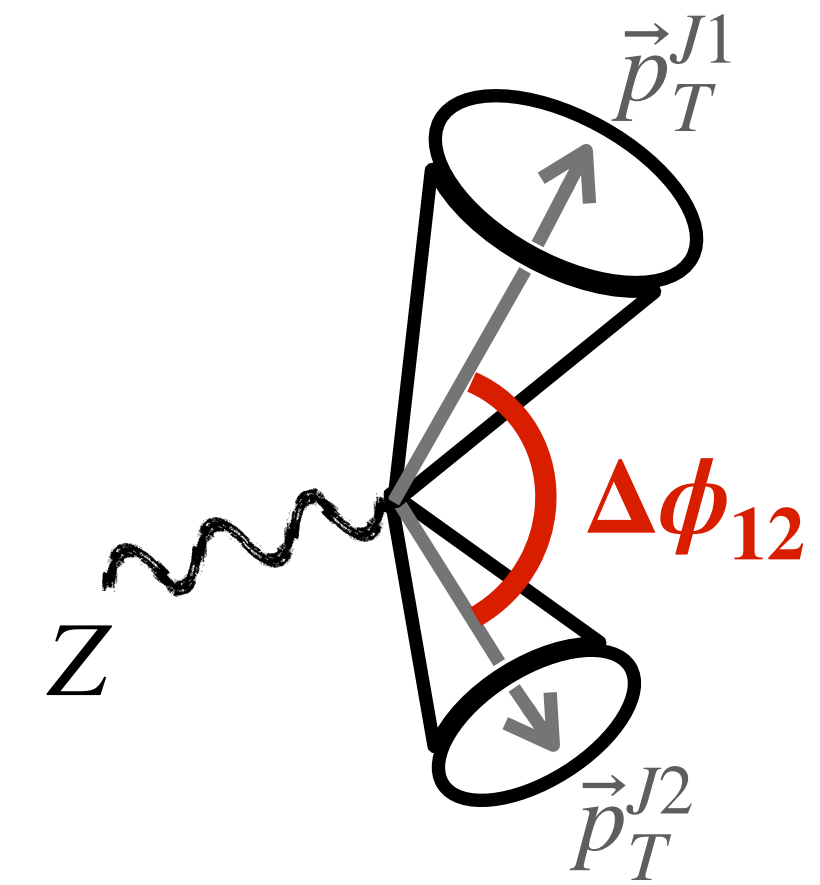
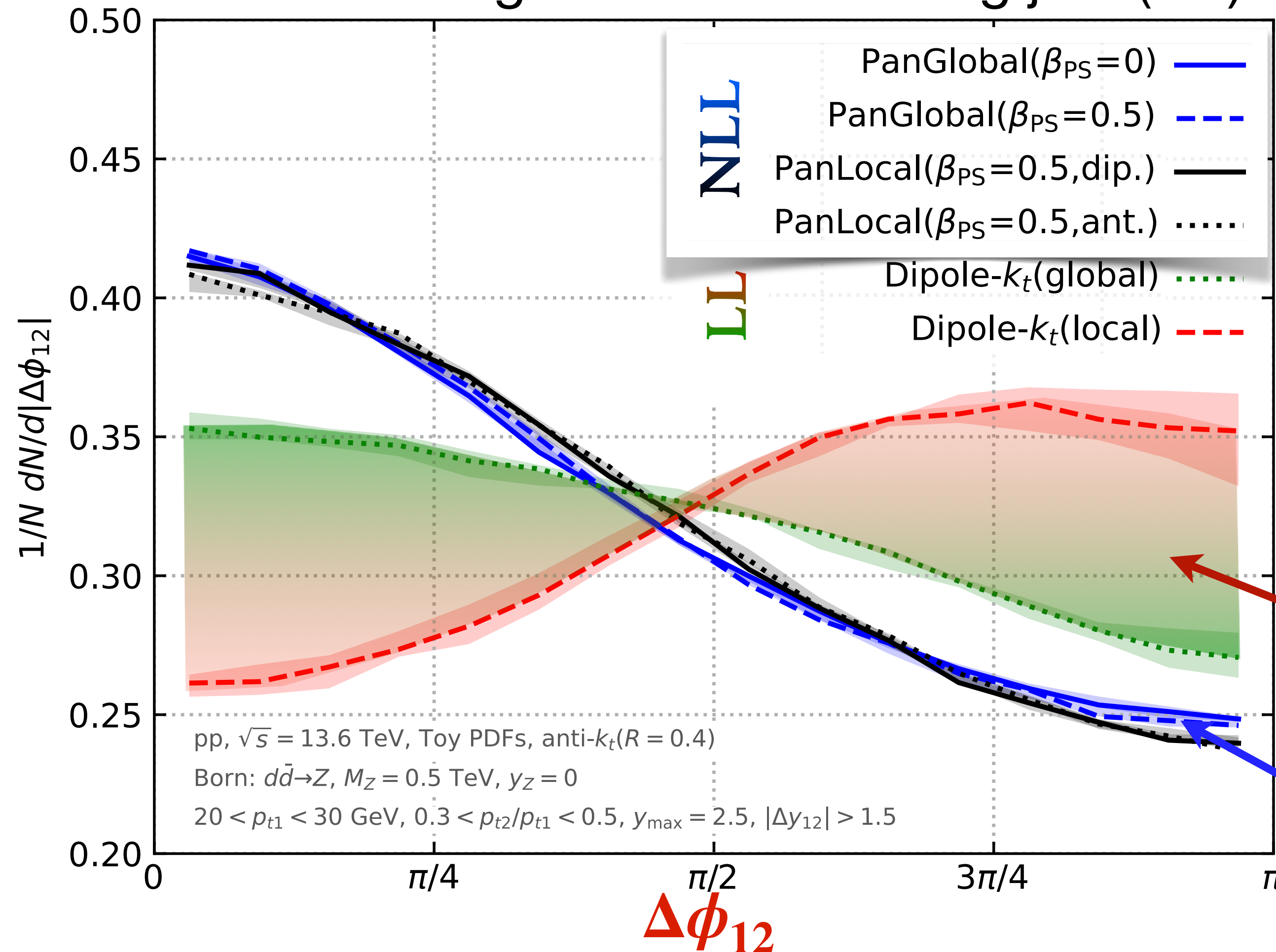
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**NLL showers**

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**NLL/LL discrepancies at larger scales**

**LL showers**

**NLL showers**

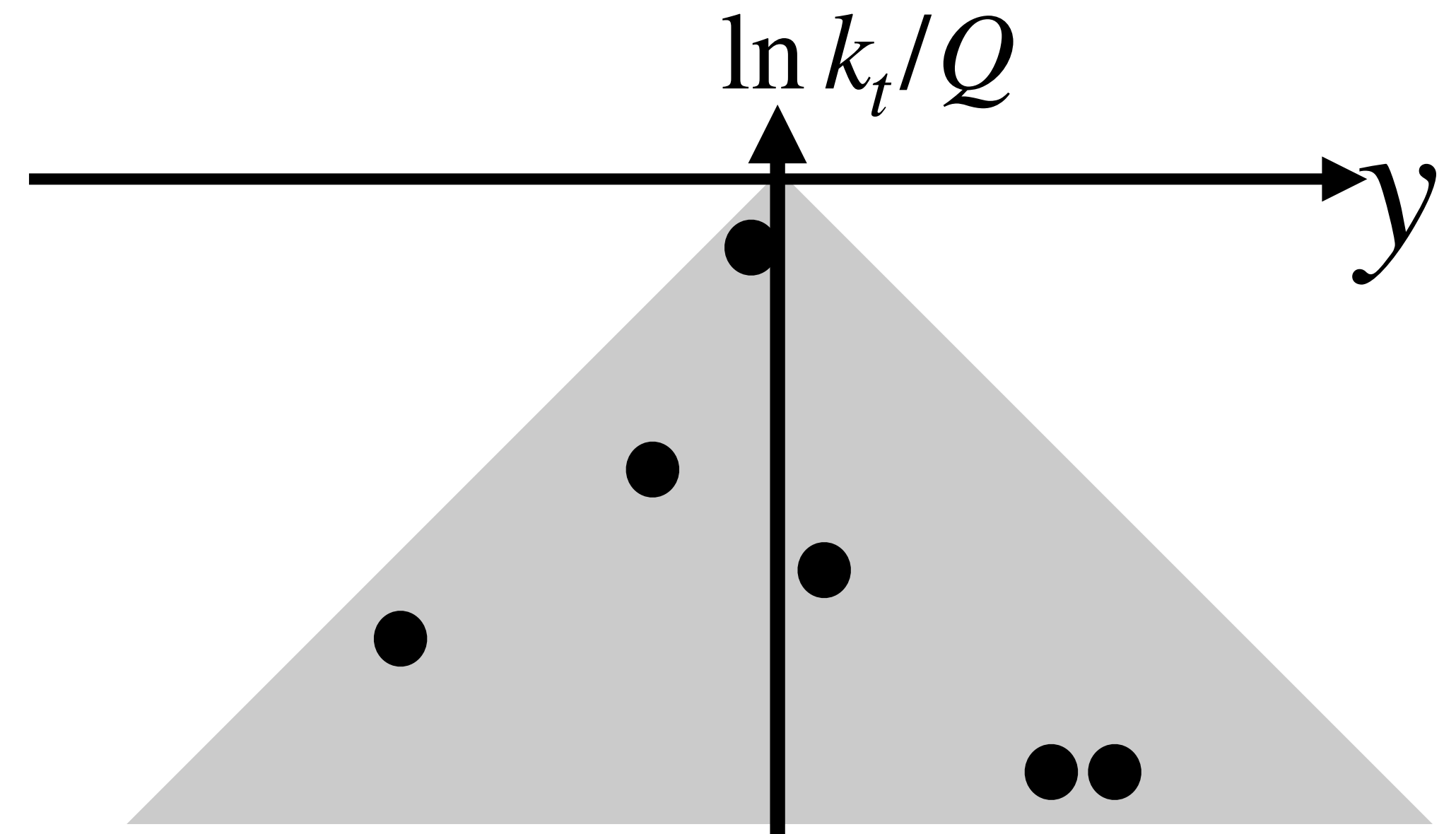
# How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

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NLL



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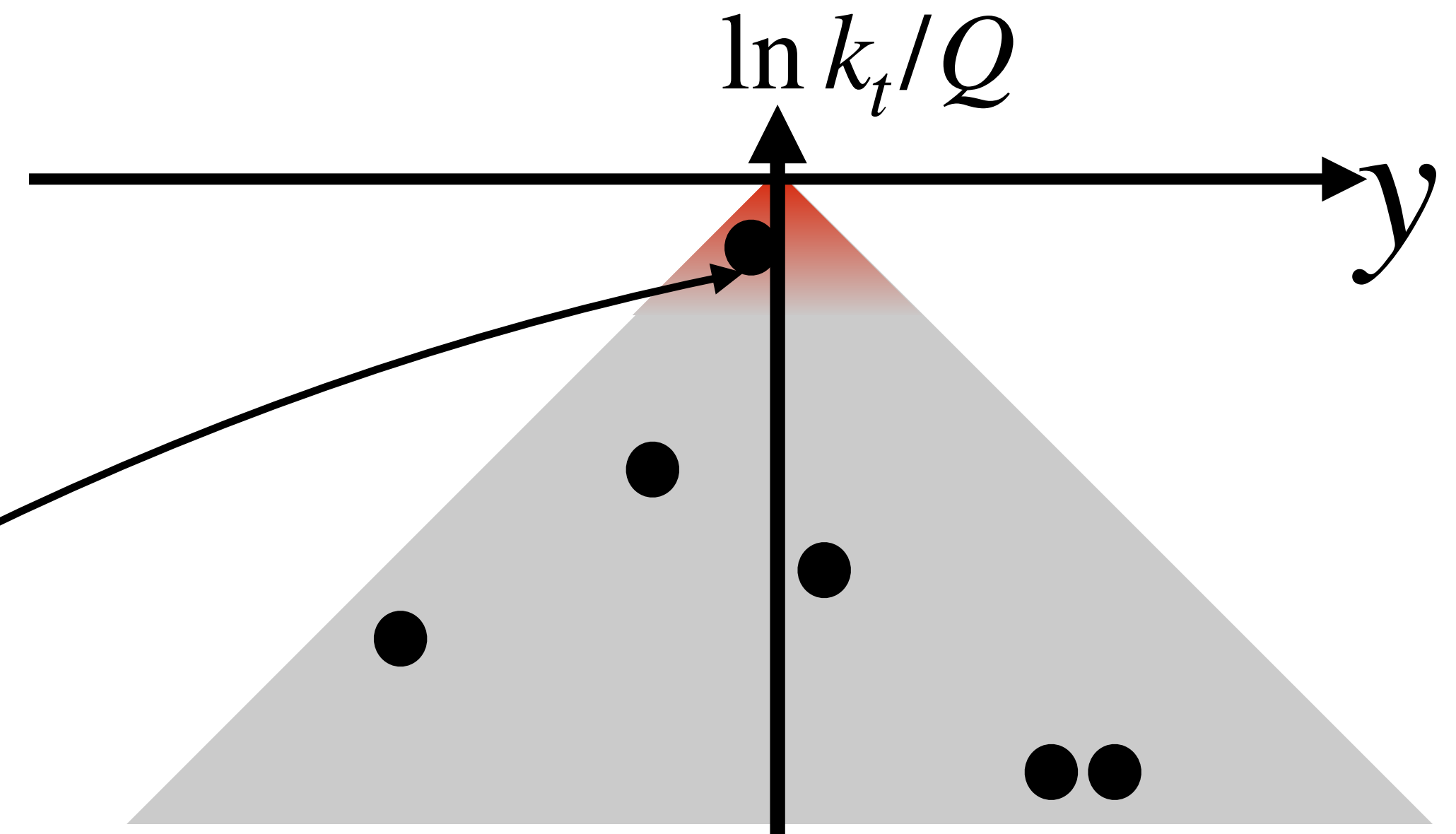
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[Hamilton, Karlberg, Scyboz, Salam, [2301.09645](#)]



NLL

NNLL

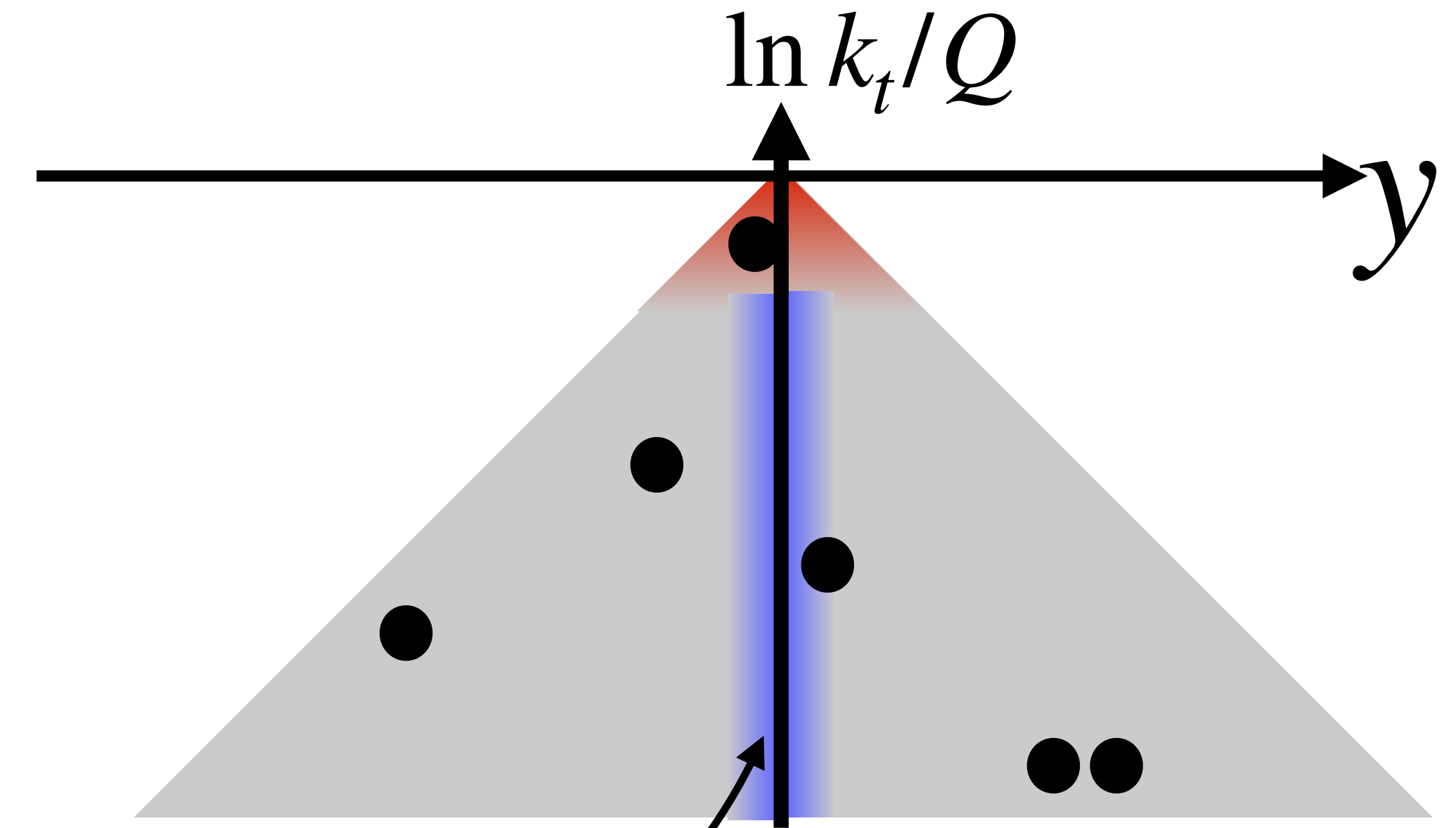


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NLL

NNLL

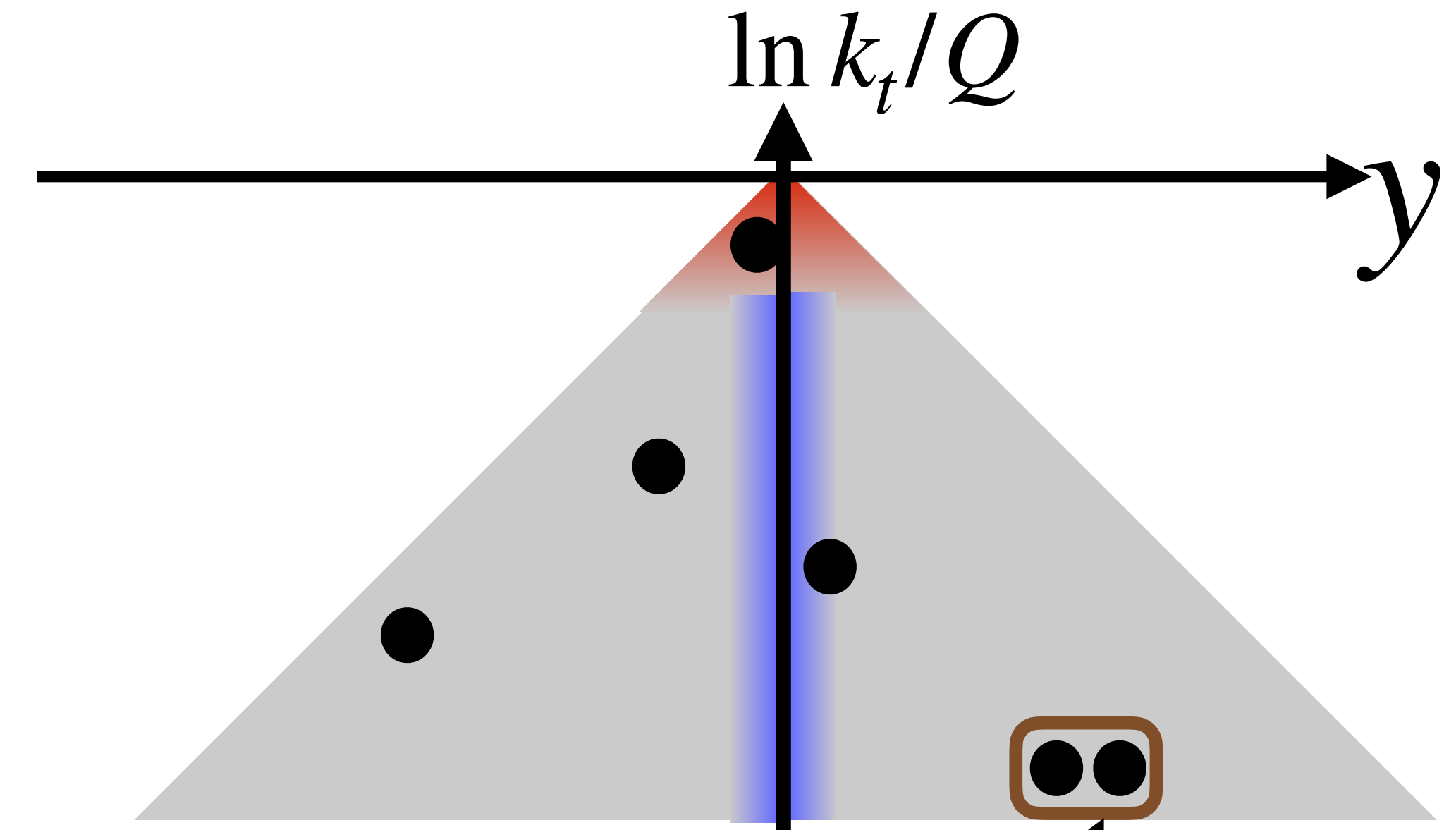
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- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for pair of emsns separated only in **one Lund coordinate**

- ✓ **Hard** emissions at LO
- ✓ Soft (large angle) emsns at NLO
- ✓ Correct rate for pair of emsns **close in the Lund plane**



NLL

NNLL

# How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

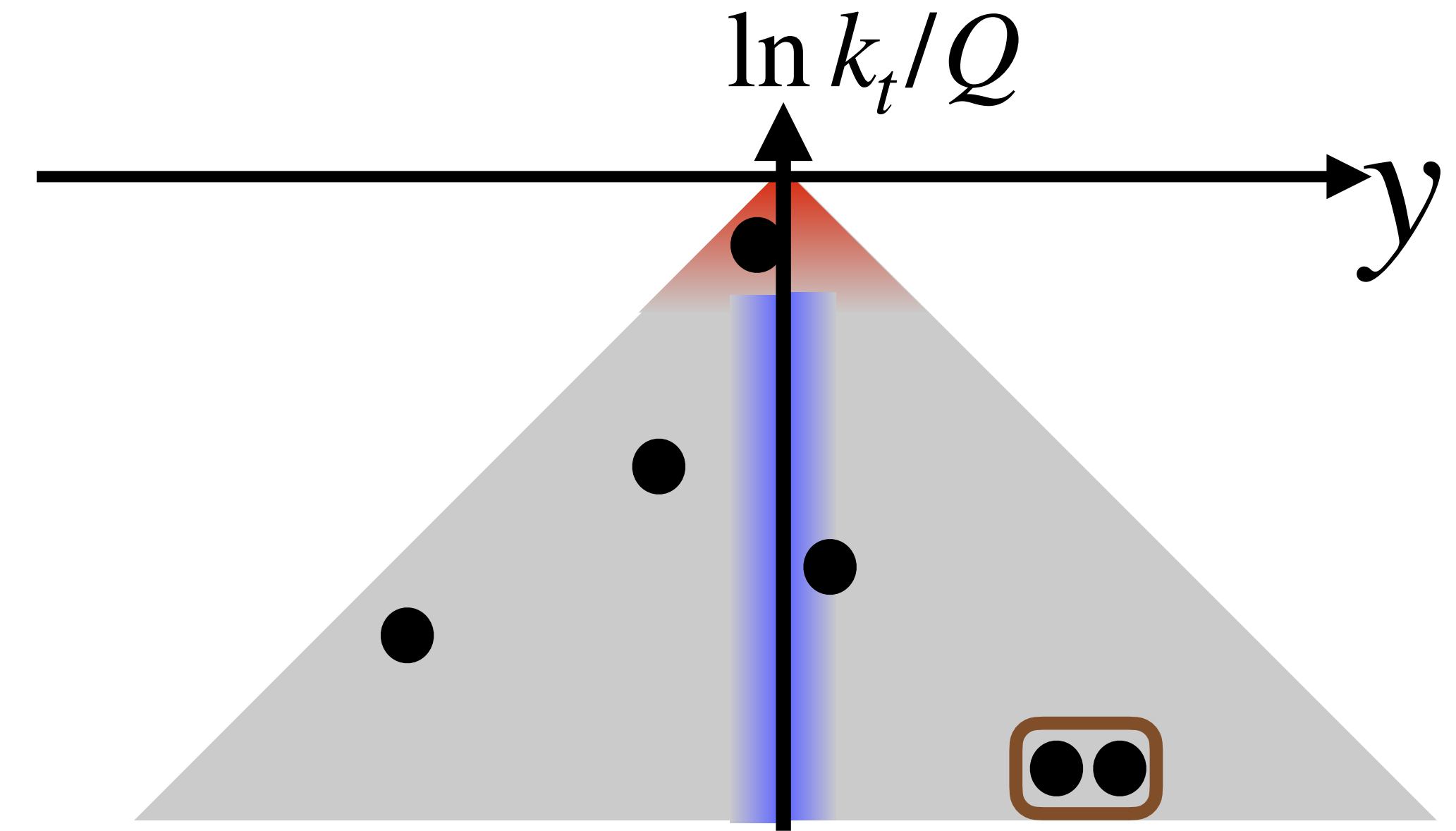
Focus on soft emissions

NLL

- ✓ Soft-collinear emsns at **NLO**
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NNLL

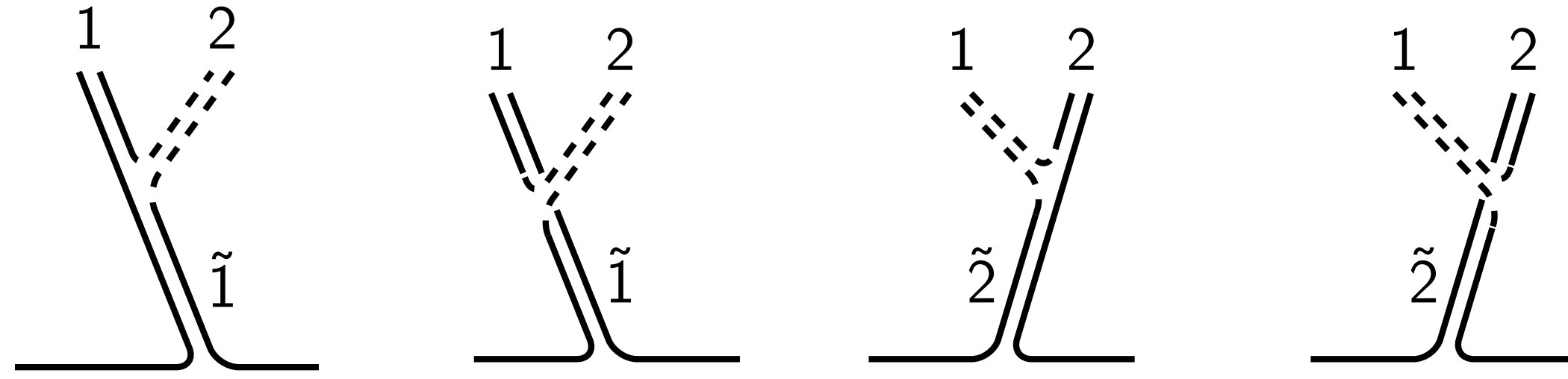
- ✓ **Hard** emissions at **LO**
- ✓ Soft (large angle) emsns at **NLO**
- ✓ Correct rate for pair of emsns **close in the Lund plane**
- ✓ ...



- **NNDL** for [subset] multiplicities, i.e.  $\alpha_s^n L^{2n}$ ,  $\alpha_s^n L^{2n-1}$ ,  $\alpha_s^n L^{2n-2}$
- **Next-to-Single-Log (NSL)** for non-global logarithms, e.g. energy in a slice, all terms  $\alpha_s^n L^n$  and  $\alpha_s^n L^{n-1}$

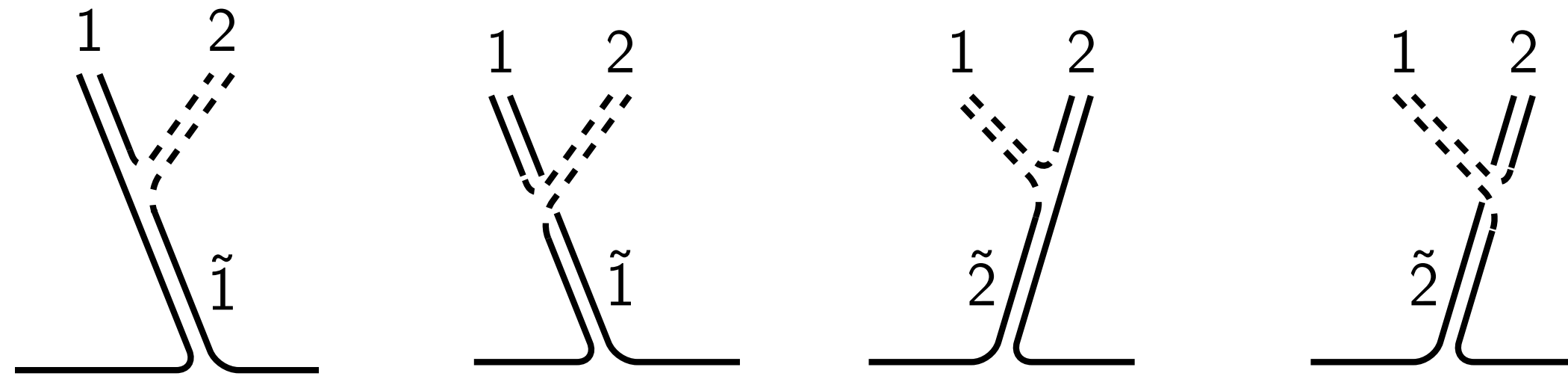
# Correct rate for **pairs** or soft emissions = **Real** corrections

---



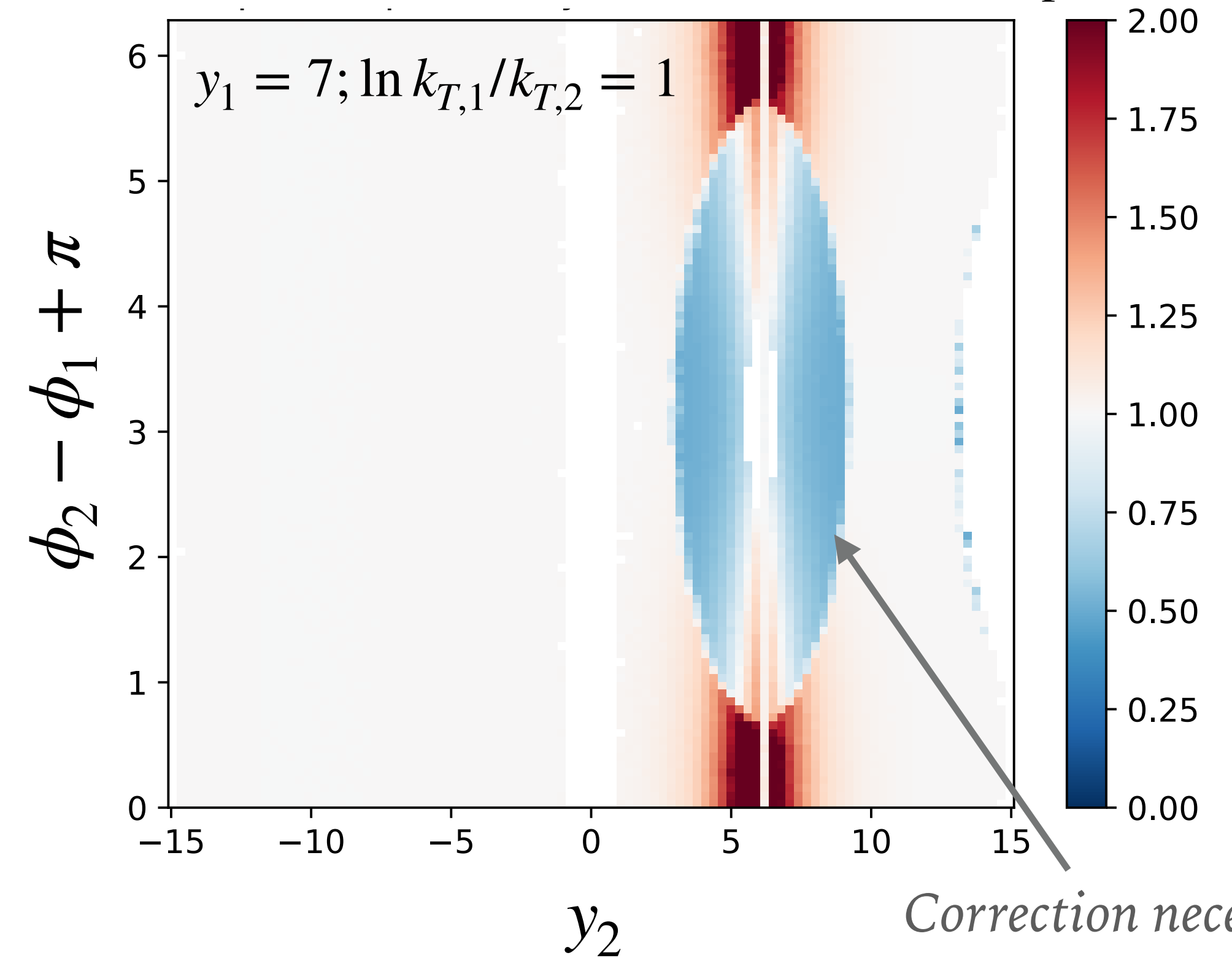
- a given two-emission configuration can come from several shower histories

# Correct rate for pairs or soft emissions = **Real** corrections



- a given two-emission configuration can come from several shower histories
- **accept a given emission with exact double-soft  $M_{\text{exact}}^{(\text{DS})}$  divided by shower's effective double-soft matrix element summed over the histories  $h$  that could have produced that configuration**

Double-soft acceptance  $P_{\text{accept}}$



*Correction necessary only for neighbouring emsn as the shower is already NLL*

$$P_{\text{accept}} = \frac{M_{\text{exact}}^{(\text{DS})}}{\sum_h M_{h,\text{PS}}^{(\text{DS})}}$$

# NLO corrections to a single soft emission: standard behaviour

► For a soft emission

The diagrammatic equation shows the sum of a tree-level soft emission and its NLO correction. On the left, a blue circle labeled 'V' is connected to a wavy line representing a soft emission, which is enclosed in an orange cone. This is added to an integral over a region 'R' (a green circle) of a similar soft emission diagram. The integral is labeled with 'y, p\_perp fixed'. The result is a yellow box containing the expression  $\frac{\alpha_s}{2\pi} K_1$ .

$$\text{Tree-level soft emission} + \int_{\text{Region R}} \text{NLO correction} \quad (y, p_{\perp} \text{ fixed}) = \frac{\alpha_s}{2\pi} K_1$$

► If this happens also in a **parton shower** simulation, we have the emission rate correct at  $\mathcal{O}(\alpha_s^2)$

# NLO corrections to a single soft emission: standard behaviour

► For a soft emission

$$\begin{array}{c} \text{V} \end{array} + \int \begin{array}{c} \text{R} \\ y, p_{\perp} \\ \text{fixed} \end{array} = \frac{\alpha_s}{2\pi} K_1$$

► If this happens also in a **parton shower** simulation, we have the emission rate correct at  $\mathcal{O}(\alpha_s^2)$

► In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)

$$\begin{array}{c} \text{V}_{\text{PS}} \end{array} \equiv - \int \begin{array}{c} \text{R}_{\text{PS}} \end{array} \quad \text{At fixed "shower variables",} \\ \text{but the rapidity and } p_{\perp} \text{ of} \\ \text{the jet can vary}$$

# NLO corrections to a single soft emission: standard behaviour

► For a soft emission

$$V + \int_{y, p_\perp \text{ fixed}} R = \frac{\alpha_s}{2\pi} K_1$$

► If this happens also in a **parton shower** simulation, we have the emission rate correct at  $\mathcal{O}(\alpha_s^2)$

► In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)

$$V_{PS} \equiv - \int R_{PS}$$

*At fixed “shower variables”,  
but the rapidity and  $p_\perp$  of  
the jet can vary*

► **Catani**, **Marchesini** and **Webber** defined the “CMW” scheme for the coupling in the shower

[*Nucl.Phys.B* 349 (1991) 635-654]

$$\alpha_s^{\text{CMW}} = \alpha_s \left( 1 + \frac{\alpha_s}{2\pi} K_1 \right)$$

*Additional virtual correction added directly to the splitting function*

Ensures “on average”

$$V_{PS} + \int R_{PS} = \frac{\alpha_s}{2\pi} K_1$$



# Revisiting **virtual** corrections to a single soft emission

---

- With our double soft acceptance we have  $\mathbf{R}_{\text{PS}} = \mathbf{R}$ . This yields

$$\text{Diagram 1} = \frac{\alpha_s}{2\pi} K_1 - \int \text{Diagram 2} \quad \text{Fixed shower variables}$$

The diagram on the left shows a blue circle labeled  $V_{\text{PS}}$  on a horizontal line, with a brown cone extending upwards and a wavy line inside it. The diagram on the right shows a pink cone on a horizontal line, with a green circle labeled  $\mathbf{R}$  inside it and wavy lines extending from it.

# Revisiting virtual corrections to a single soft emission

- ▶ With our double soft acceptance we have  $\mathbf{R}_{\text{PS}} = \mathbf{R}$ . This yields

$$\underbrace{V_{\text{PS}}}_{\text{blue circle}} = \frac{\alpha_s}{2\pi} K_1 - \int \underbrace{\text{[diagram]}}_{\text{Fixed shower variables}}$$

- ▶ We modify the CMW scheme

$$K_1 \rightarrow K_1 + \Delta K_1(\Phi_{\text{PS}}^{(1)})$$

$$\frac{\alpha_s}{2\pi} \Delta K_1(\Phi_{\text{PS}}^{(1)}) = \int \underbrace{\text{[diagram]}}_{\text{Fixed shower variables}} - \int \underbrace{\text{[diagram]}}_{y, p_{\perp} \text{ fixed}}$$

# Revisiting virtual corrections to a single soft emission

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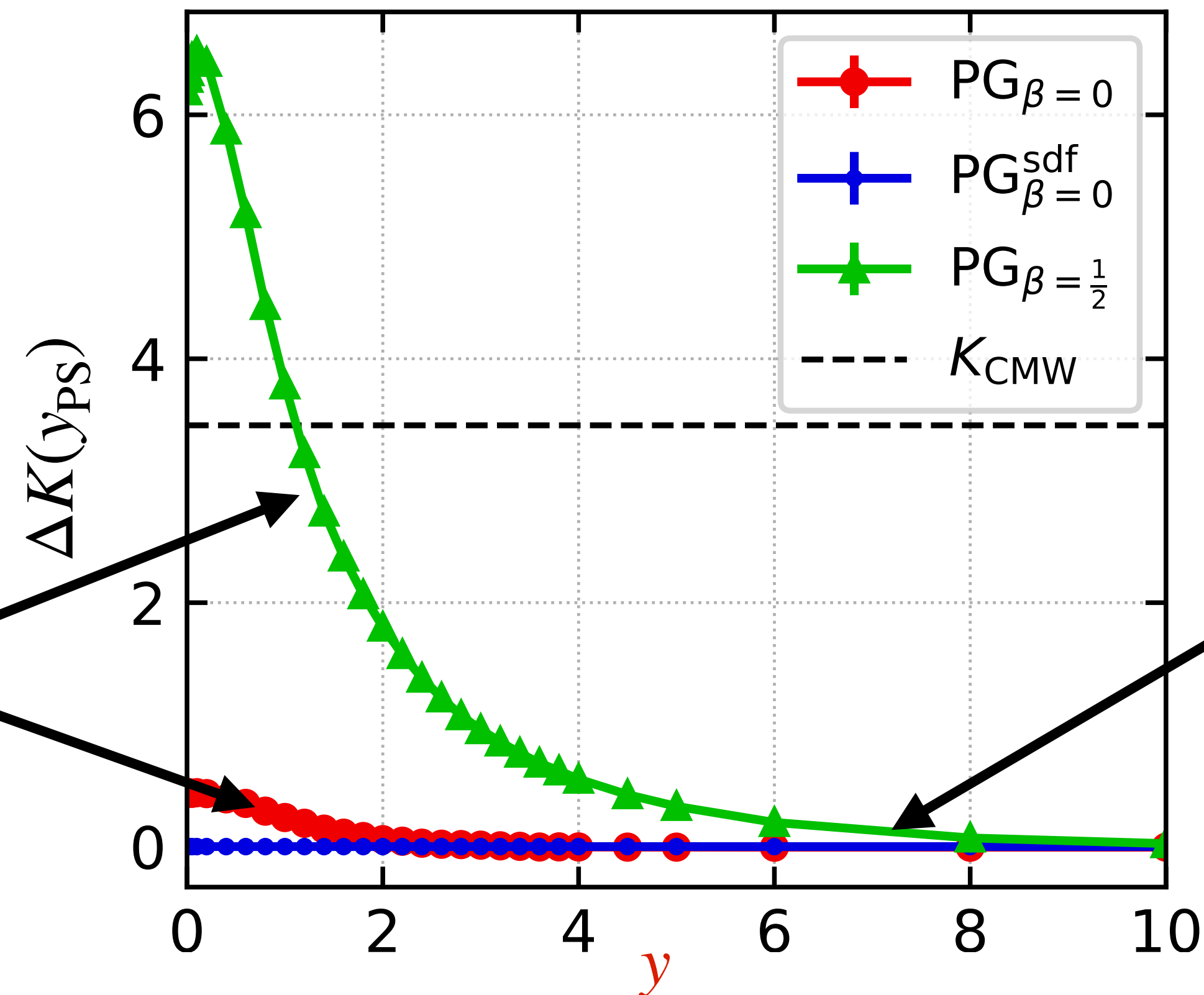
- ▶ ...so to have

$$\underbrace{\text{Diagram}}_{V_{\text{PS}}} = \frac{\alpha_s}{2\pi} K_1 - \int \underbrace{\text{Diagram}}_{\mathbf{R}} \text{ } y, p_{\perp} \text{ fixed}$$

# Virtual corrections to a single soft emission

$$= \frac{\alpha_s}{2\pi} \left( K_1 + \Delta K_1(\Phi_{PS}^{(1)}) \right) - \int \text{Fixed shower variables}$$

example  $\Delta K_1$  correction



Soft large-angle emissions can require a “large”  $\Delta K_1$

Soft-collinear emissions are already OK (because the shower is NLL)

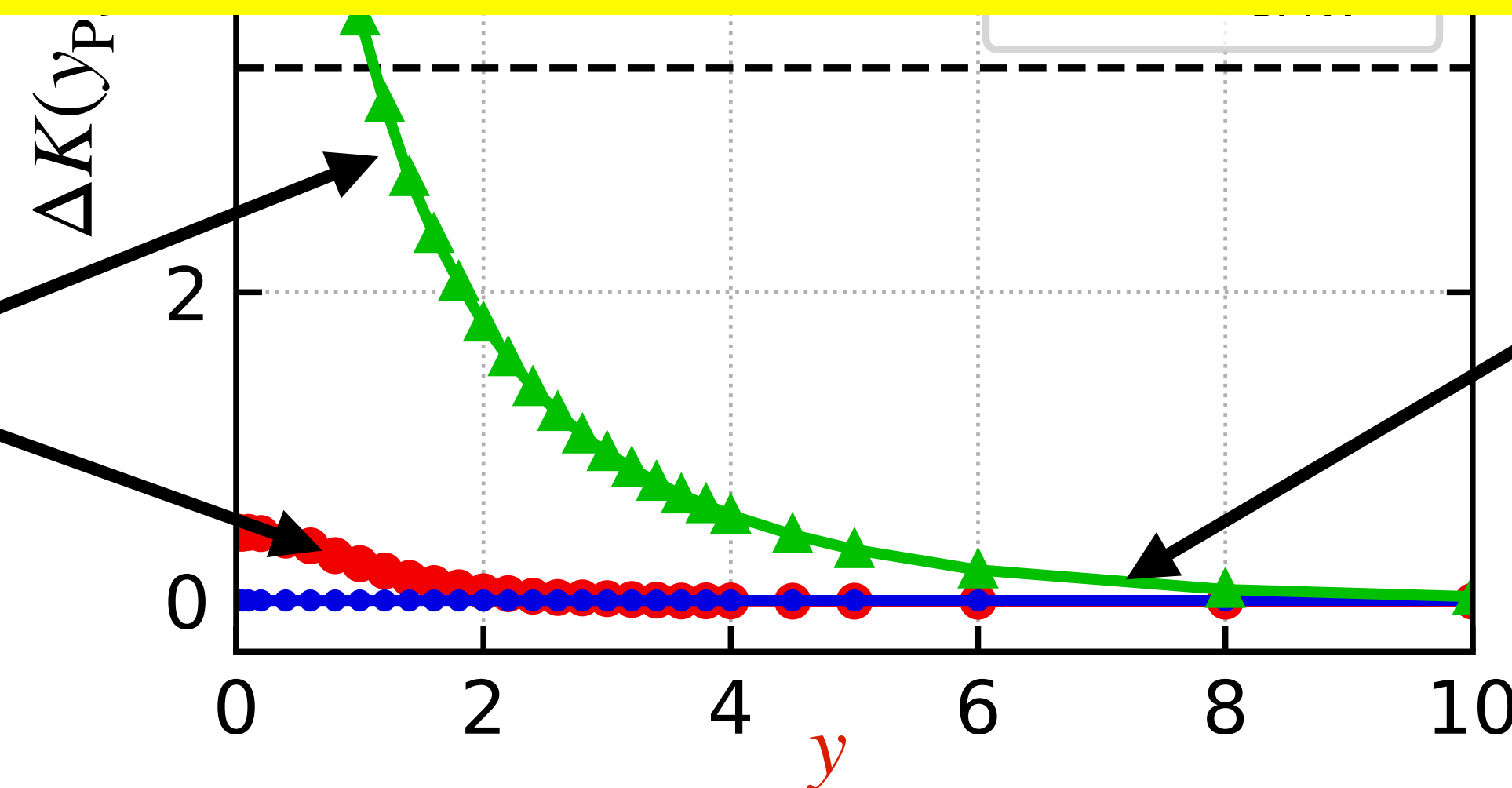
# Virtual corrections to a single soft emission



$$= \frac{\alpha_s}{2\pi} \left( K_1 + \Delta K_1(\Phi_{PS}^{(1)}) \right) - \int \text{Fixed shower variables}$$

Augmenting the order of the splitting function used is not sufficient to achieve superior logarithmic accuracy: one first needs to remove the mistakes a shower is making at a given order!

Soft large-angle emissions can require a “large”  $\Delta K_1$



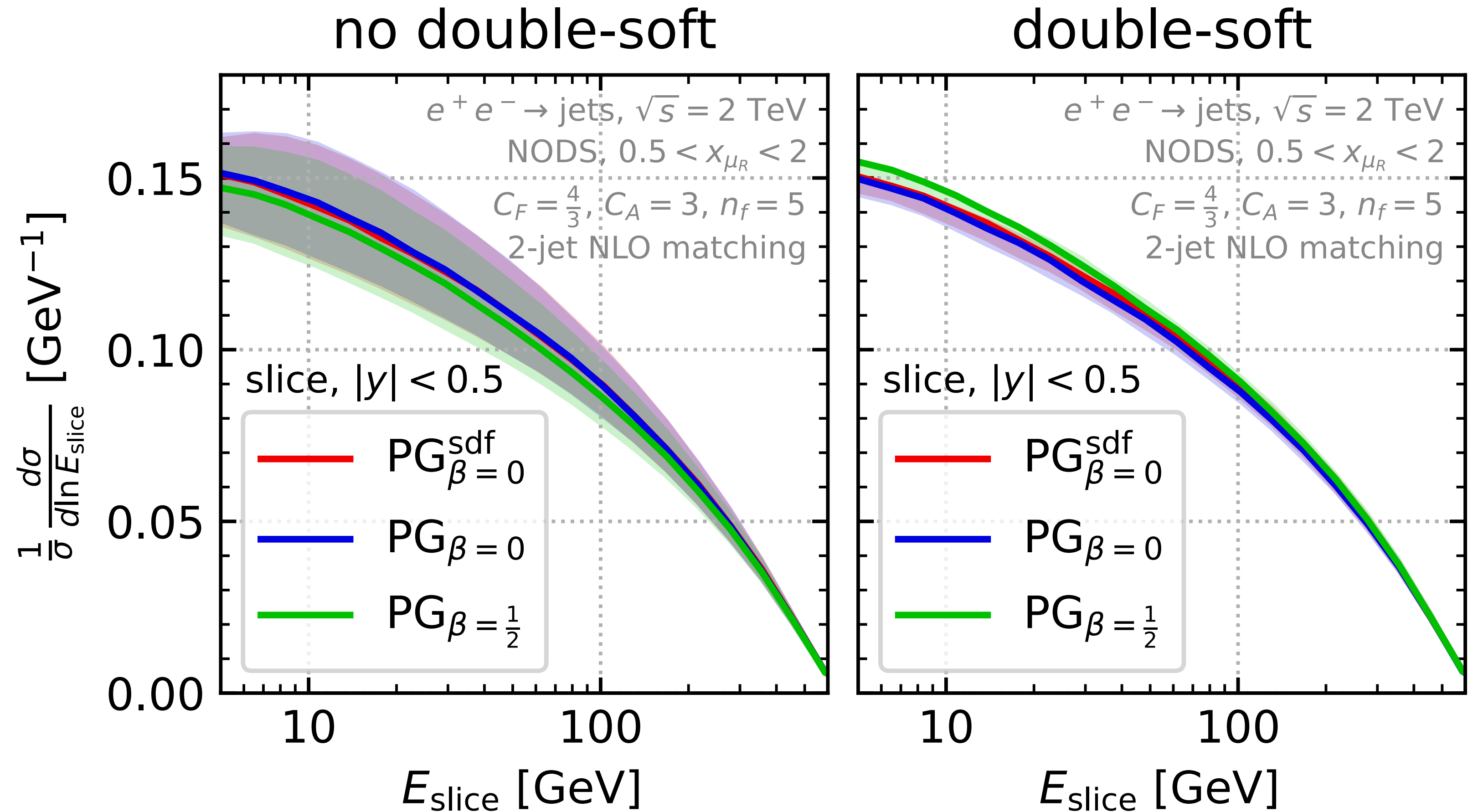
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# NSL Pheno outlook

S.F.R., Hamilton,  
Karlberg, Salam,  
Scyboz, Soyez  
[2307.11142](https://arxiv.org/abs/2307.11142)

- Energy flow in slice between two 1 TeV jets
- **Double-soft reduces uncertainty band**

Uncertainty here is estimated varying the renormalisation scale



$$\alpha_s^{\text{CMW}}(k_t; x_R) = \alpha_s(x_R k_t) \left( 1 + \frac{\alpha_s(x_R k_t)}{2\pi} (K_1 + \Delta K_1(\Phi)) + 2\alpha_s(x_R k_t) b_0 (1-z) \ln x_R \right)$$

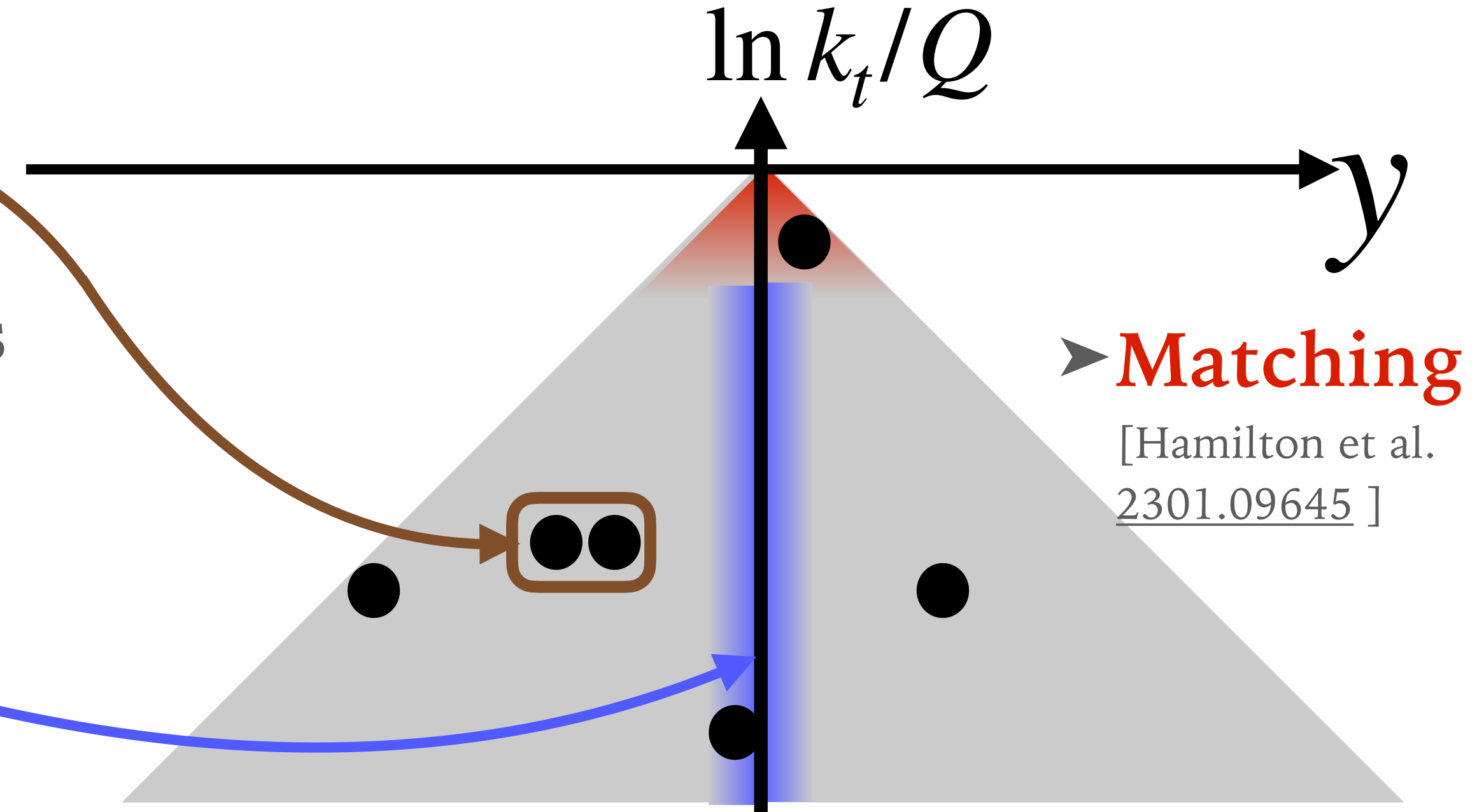
# Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft “reweighting”** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left( 1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

Catani, Marchesini,  
Webber, '91



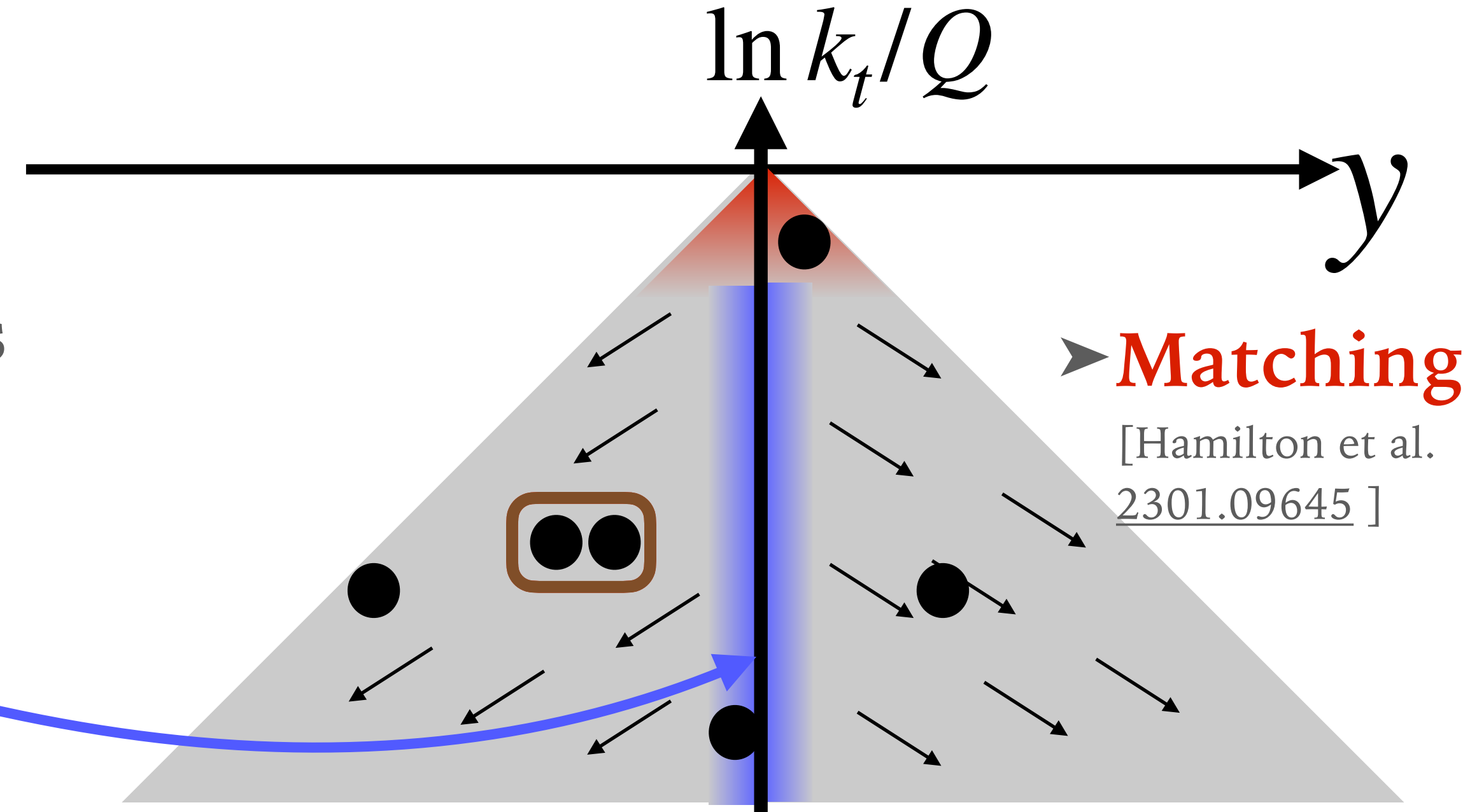
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➤ **Matching**  
[Hamilton et al. 2301.09645]

Drift in rapidity of an emission when it further branches

$$\int 2C_F d\eta \Delta K_1(\eta) \propto \langle \Delta y \rangle$$

⇒ *correct the shower mistake*



# Building a NNLL shower

SFR et al, 2307.11142

► **Double-soft “reweighting”** for neighbouring soft-collinear emsns

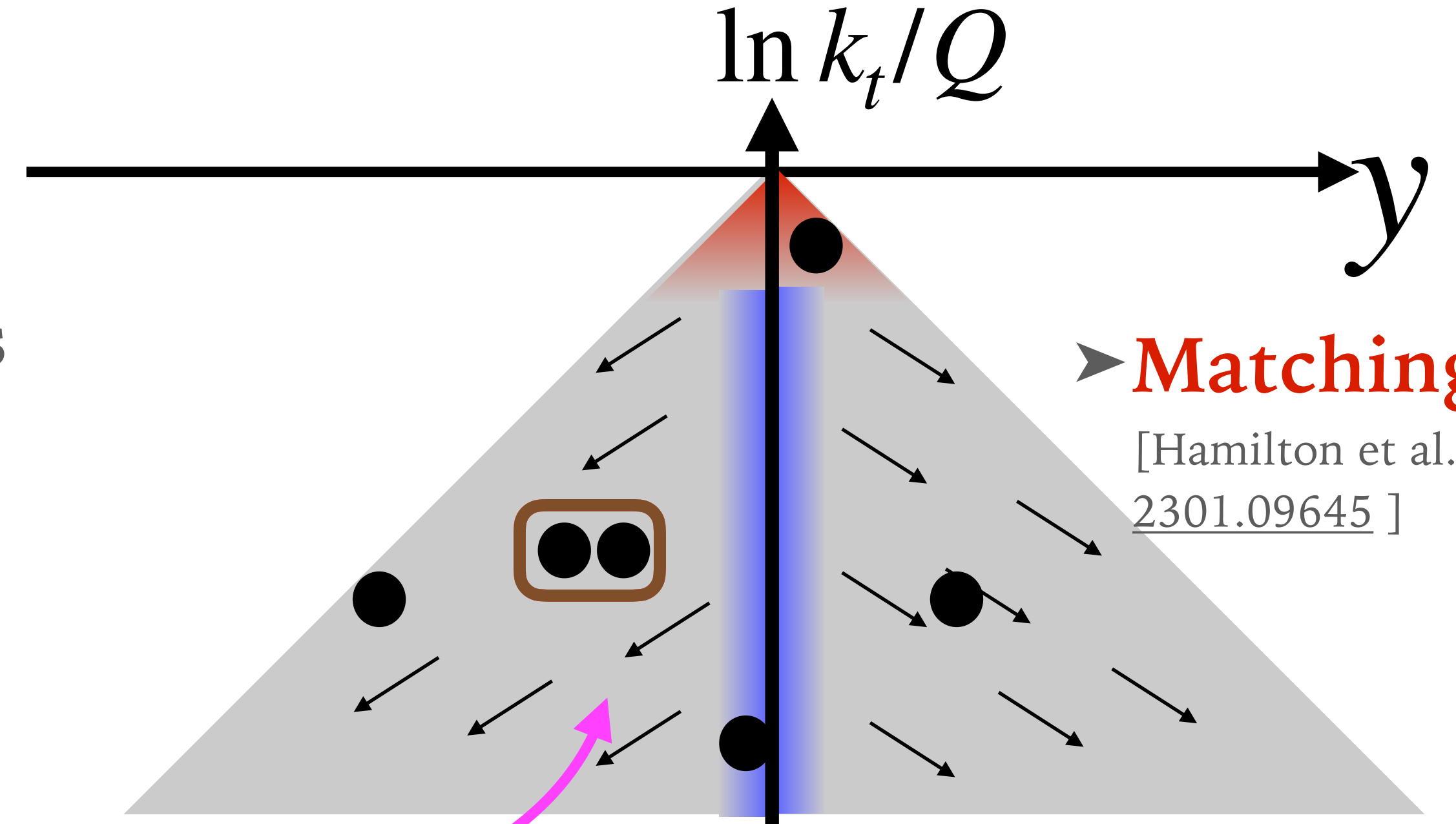
► NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left( 1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

► **NNLO corrections** for soft-collinear emsns

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left( \dots + \frac{\alpha_s^2(k_t)}{4\pi^2} (K_2 + \Delta K_2) \right)$$

Banfi, El-Menoufi,  
Monni, 1807.11487



► **Matching**

[Hamilton et al.  
2301.09645 ]

Drift in  $\ln k_t$  of an emission when it further branches

$$\Delta K_2 \propto \beta_0 \langle \Delta \ln k_t \rangle$$

⇒ **correct the shower mistake**

At this accuracy, it is sufficient to get the average

# Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft “reweighting”** for neighbouring soft-collinear emsns

- NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left( 1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

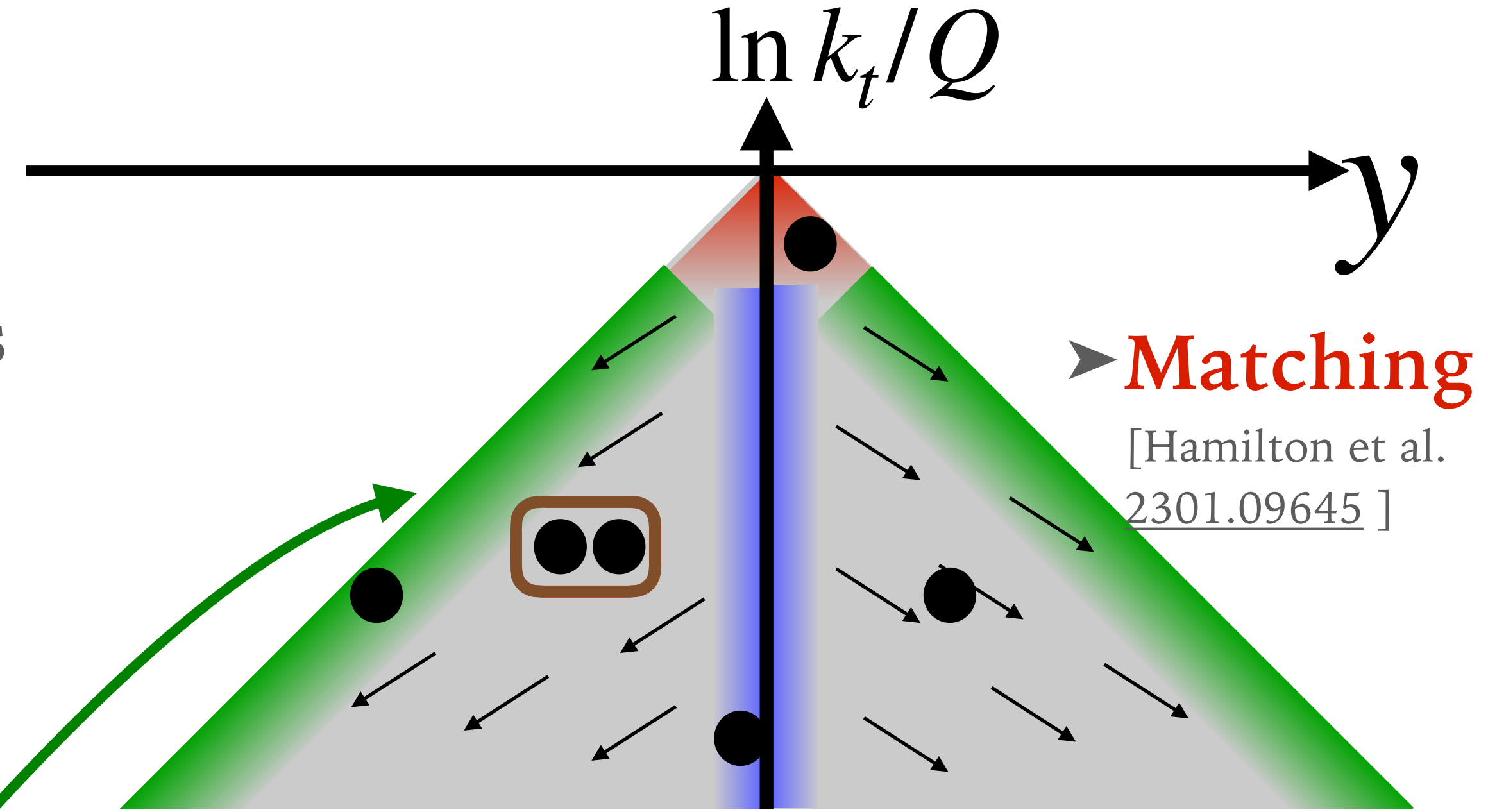
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- **NLO corrections** for collinear emsns

$$d\mathcal{P}_{\text{coll}} \propto P(z) \left( 1 + \frac{\alpha_s}{2\pi} (B_2(z) + \Delta B_2(z)) \right)$$

Dasgupta, El-Menoufi 2109.07496,  
+ van Beekveld, Helliwell, Monni 2307.15734,  
++ Karlberg 2402.05170



➤ **Matching**

[Hamilton et al. 2301.09645]

Drift in  $\ln z = \ln k_t + y$  of an emission when it further branches

$$\int P(z) dz \Delta B_2(z) \propto - \langle \Delta z \rangle$$

⇒ **correct the shower mistake**

At this accuracy, it is sufficient to get the integral right, not the functional form of  $\Delta B_2(z)$

## A new standard for the logarithmic accuracy of parton showers

Melissa van Beekveld,<sup>1</sup> Mrinal Dasgupta,<sup>2</sup> Basem Kamal El-Menoufi,<sup>3</sup> Silvia Ferrario Ravasio,<sup>4</sup> Keith Hamilton,<sup>5</sup> Jack Helliwell,<sup>6</sup> Alexander Karlberg,<sup>4</sup> Pier Francesco Monni,<sup>4</sup> Gavin P. Salam,<sup>6,7</sup> Ludovic Scyboz,<sup>3</sup> Alba Soto-Ontoso,<sup>4</sup> and Gregory Soyez<sup>8</sup>

We report on a major milestone in the construction of logarithmically accurate final-state parton showers, achieving next-to-next-to-leading-logarithmic (NNLL) accuracy for the wide class of observables known as event shapes. The key to this advance lies in the identification of the relation between critical NNLL analytic resummation ingredients and their parton-shower counterparts. Our analytic discussion is supplemented with numerical tests of the logarithmic accuracy of three shower variants for more than a dozen distinct event-shape observables in two final states. The NNLL terms are phenomenologically sizeable, as illustrated in comparisons to data.

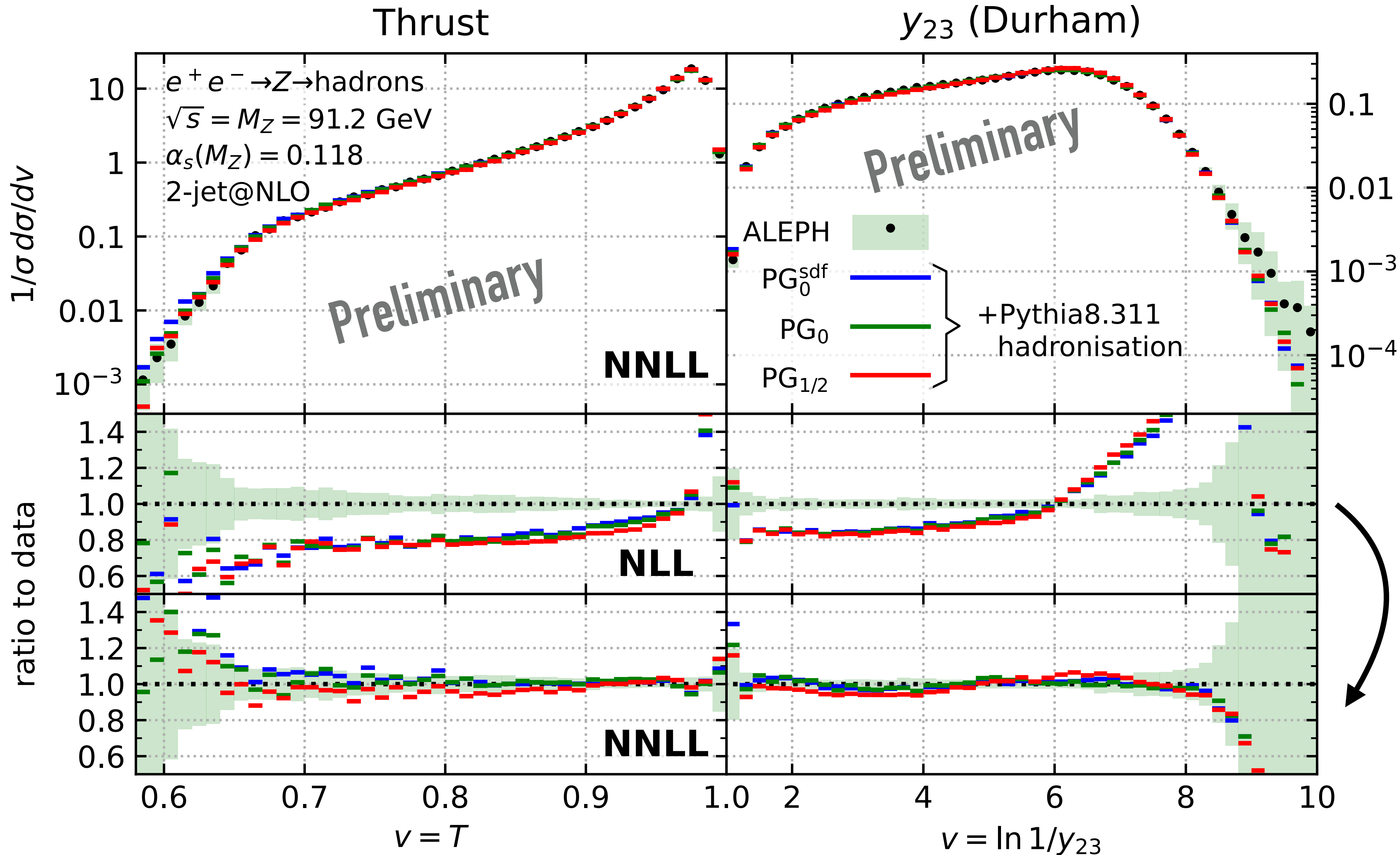
**COMING SOON!**

Dasgupta, El-Menoufi 2109.07496,  
+ van Beekveld, Helliwell, Monni 2307.15734,  
++ Karlberg 2402.05170

to get the integral  
the functional form of  $\Delta B_2(z)$

# NNLL showers vs NLL showers: pheno outlook

*The PanScales collaboration, to appear soon*



Agreement to **data** substantially better when using **NNLL** showers

# Conclusions

---

- **PanScales is first validated NLL shower**
  - All processes with **two colour legs** have been rigorously tested to be NLL for both global and non-global event shapes
  - benefits of **LL** → **NLL** include **reduced uncertainties** (reliable estimate)
  - NLO matching in place for some simple processes
- **Higher log accuracy is one of the next frontiers**
  - Double-soft (+ virtual) corrections: **NSL** accuracy for **non-global** event shapes, **NNDL** accuracy for subjet multiplicities.
  - Coming (very) soon: **NNLL** accuracy for **global event shapes** in  $e^+e^- \rightarrow j_1j_2$
- **Public code**
  - <https://gitlab.com/panscales/panscales-0.X>

*The PanScales collaboration,  
2312.13275*

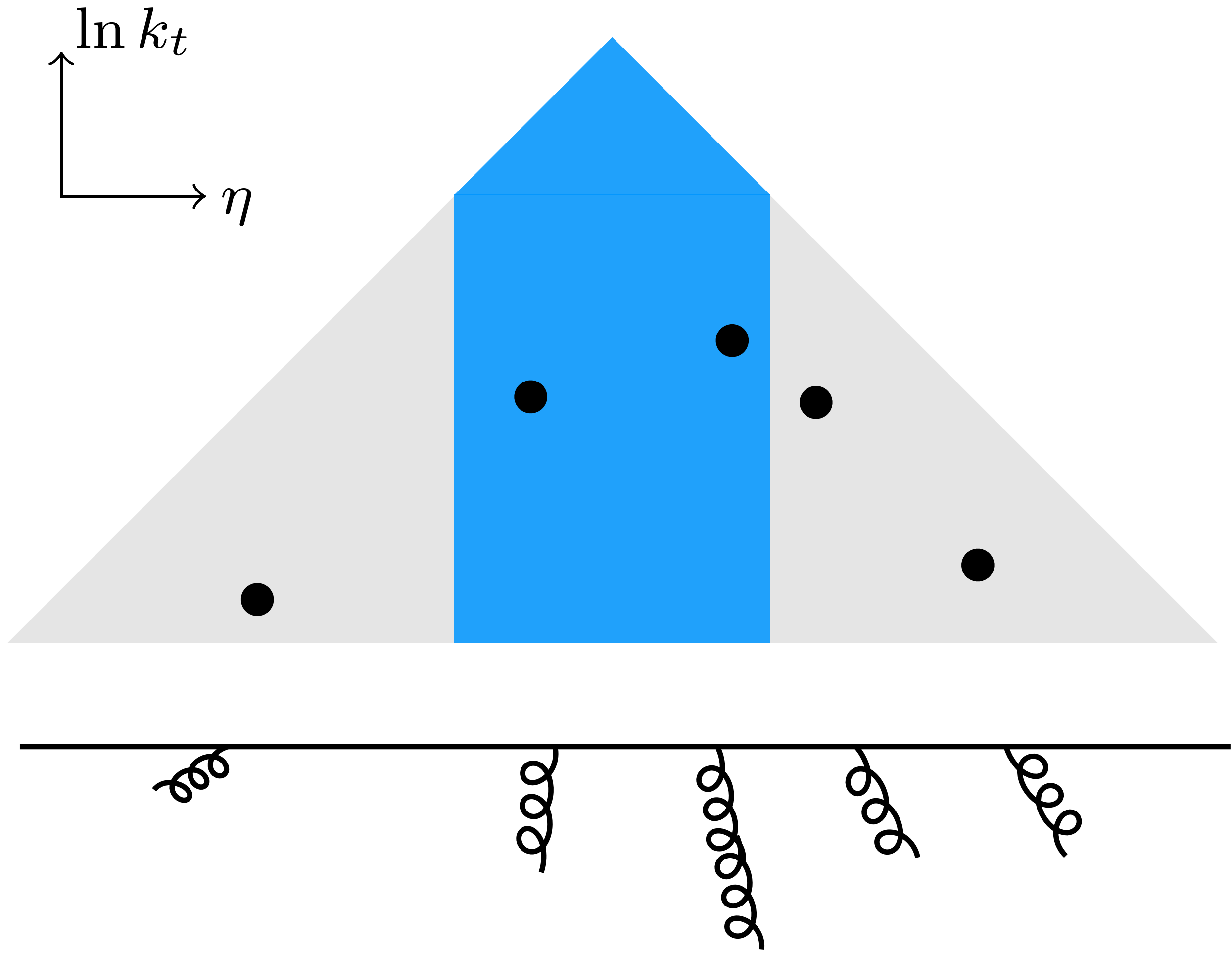
# Conclusions

---

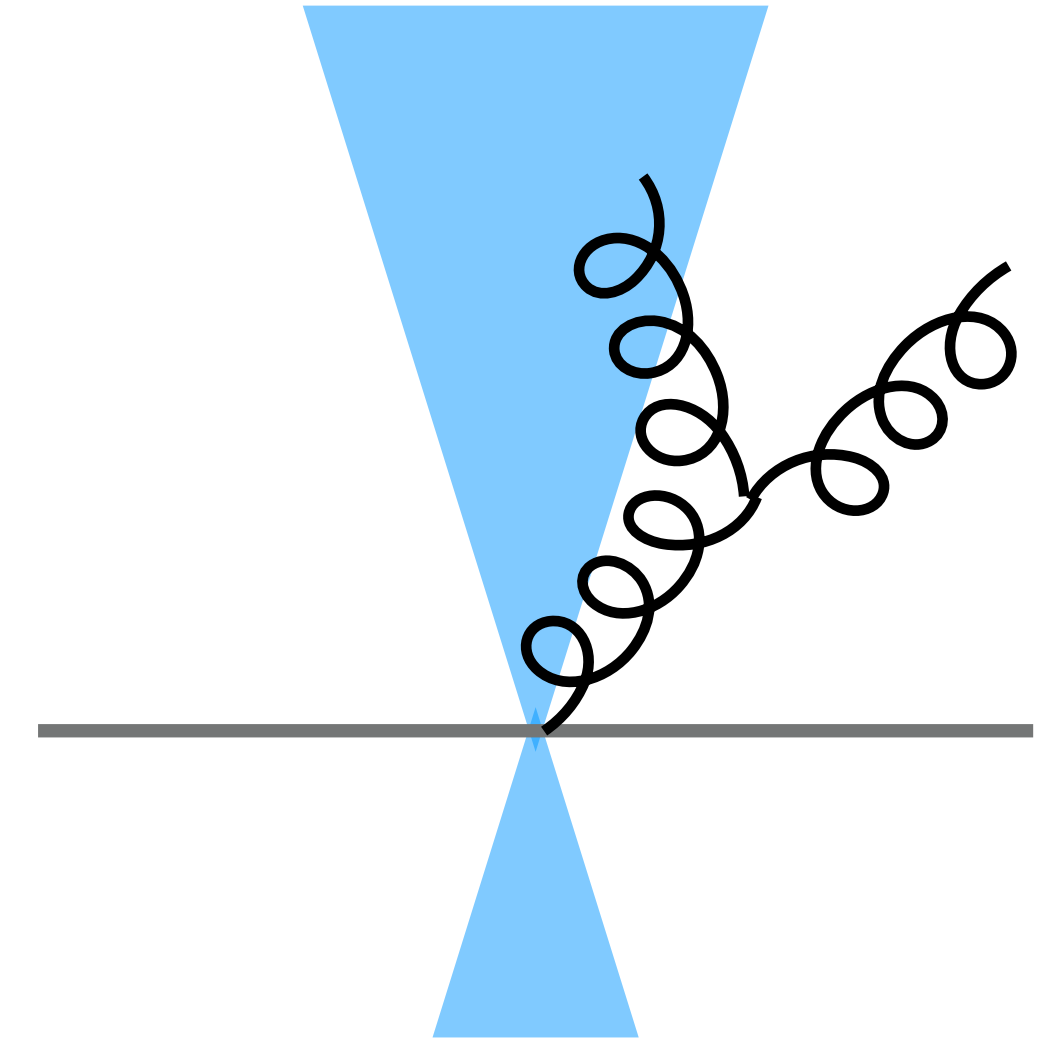
- **PanScales is first validated NLL shower**
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  - benefits of **LL** → **NLL** include **reduced uncertainties** (reliable estimate)
  - **NLO matching in place for some simple processes** **Current matching schemes typically preserve at best the LL... A lot of work to be done!!!**
- **Higher log accuracy is one of the next frontiers**
  - Double-soft (+ virtual) corrections: **NSL** accuracy for **non-global** event shapes, **NNDL** accuracy for **subject multiplicities**.
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*The PanScales collaboration,  
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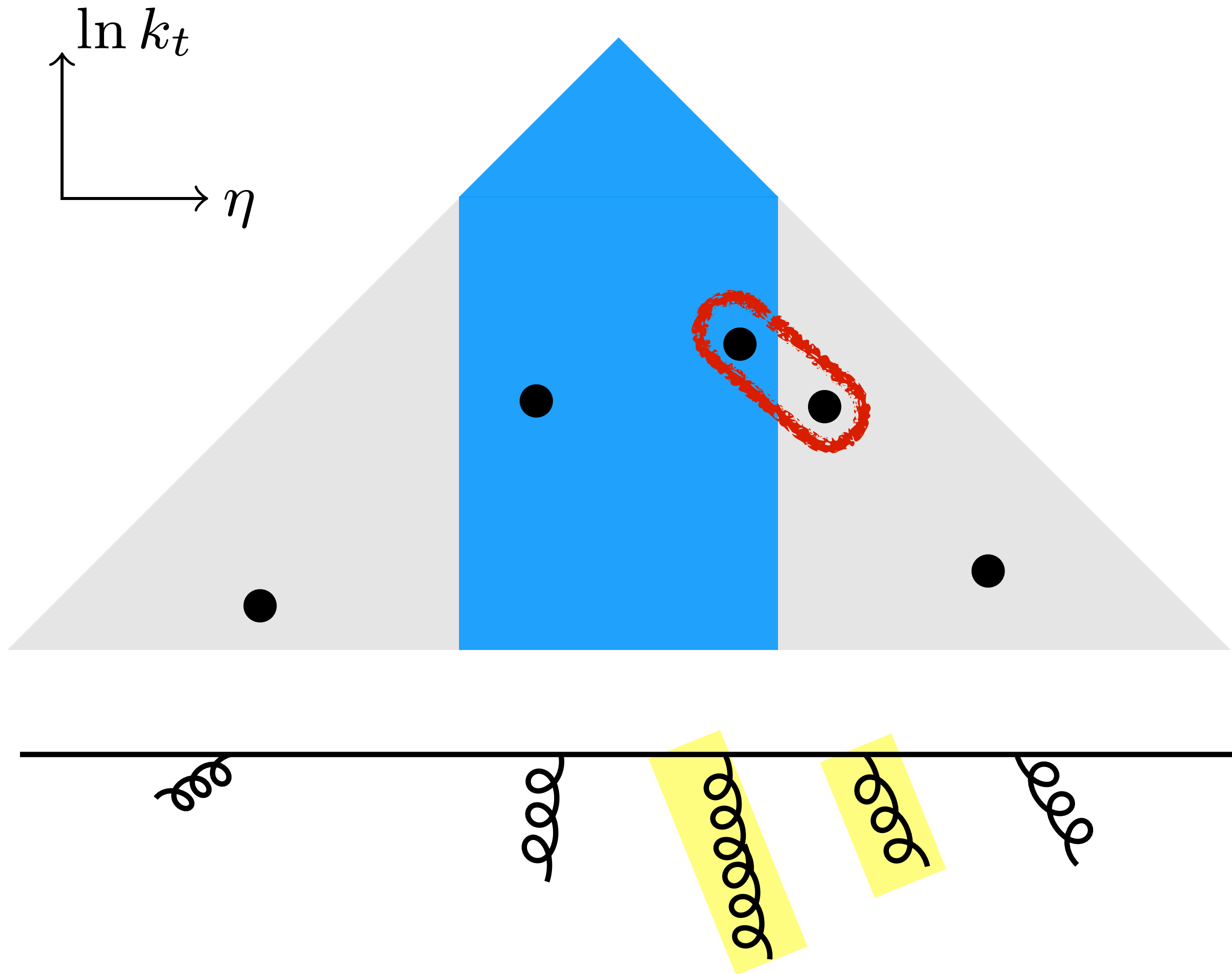
# NSL for the energy flow in a rapidity slice



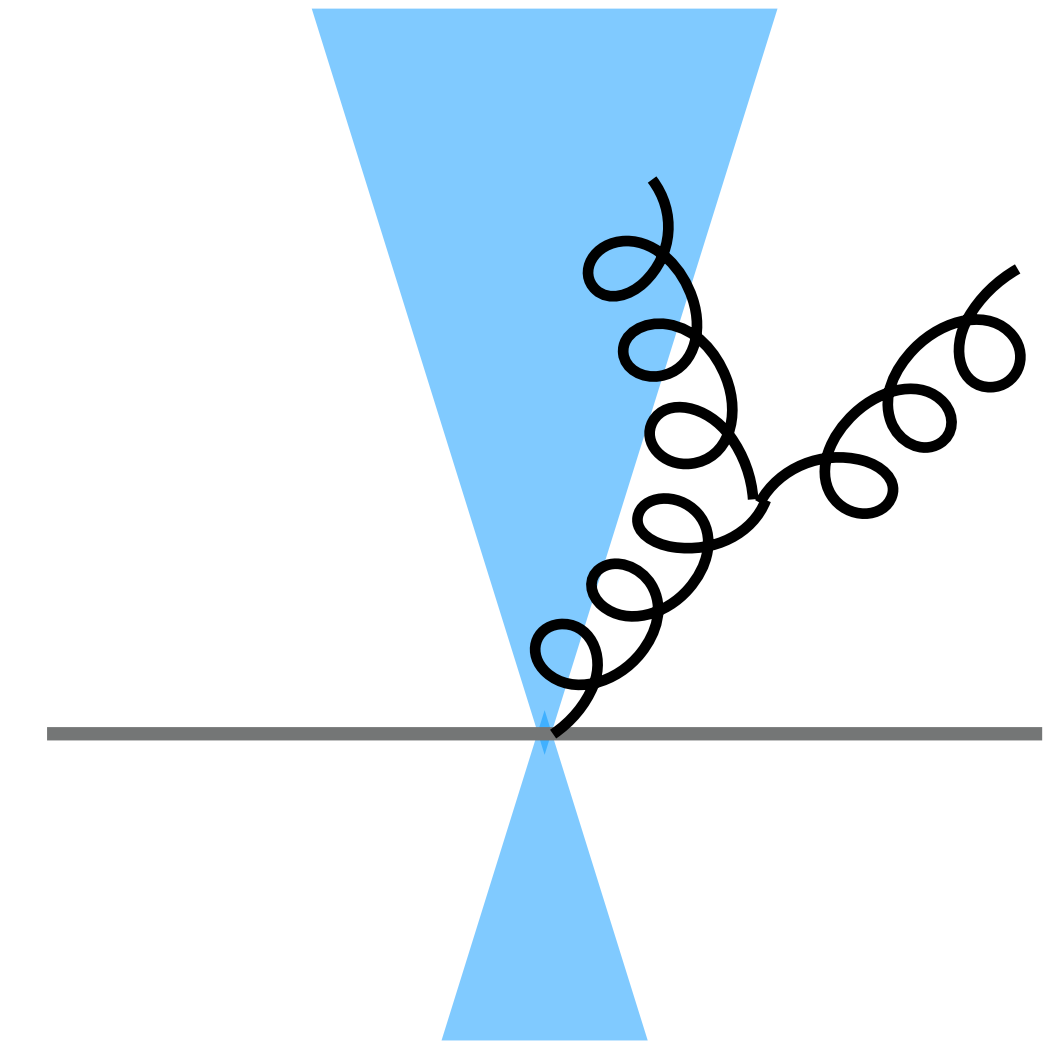
Non-global  
observable



# NSL for the energy flow in a rapidity slice



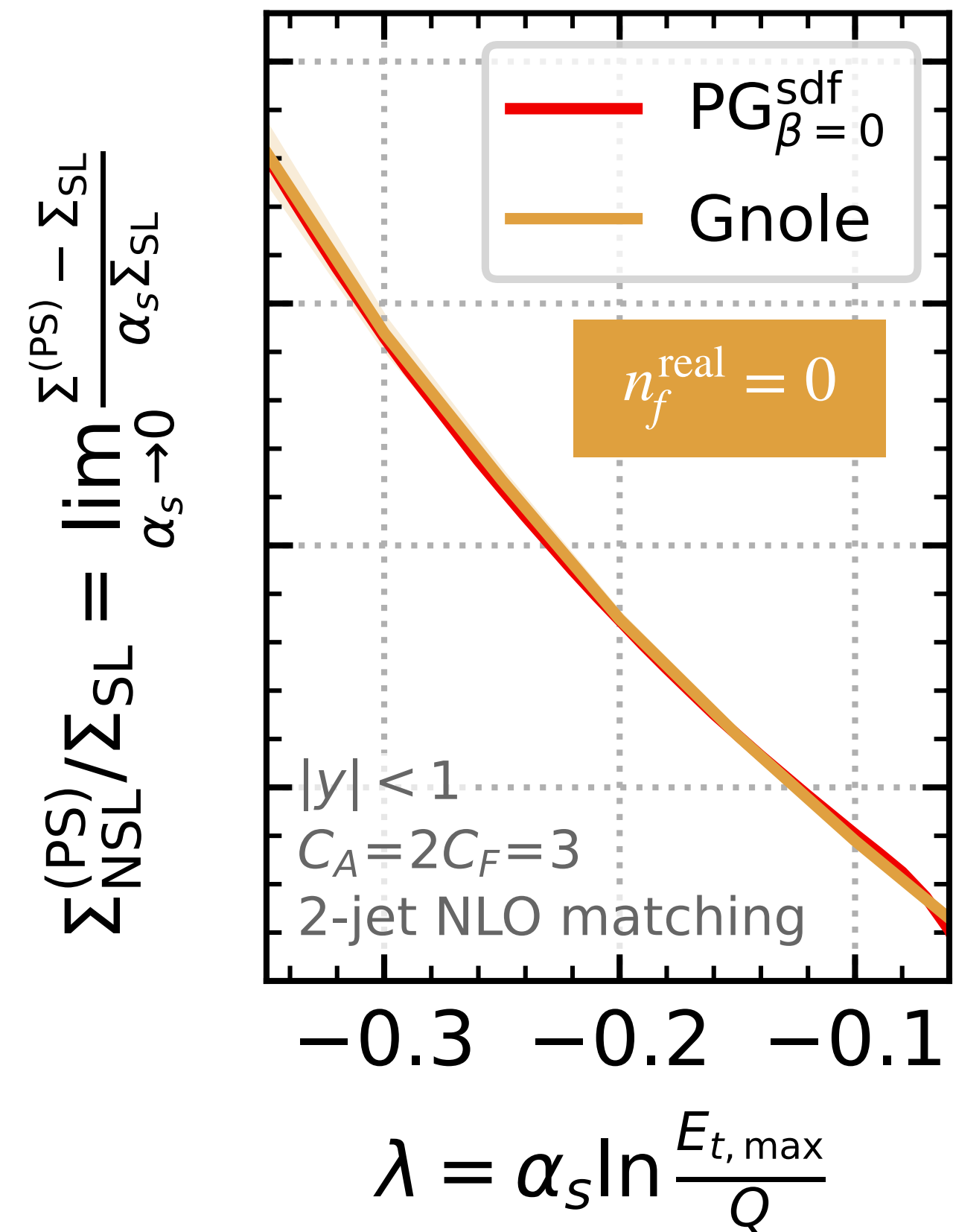
Non-global  
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- **NSL** ( $\alpha_s^n L^{n-1}$ ) analytic reference from Banfi, Dreyer, Monni, [2104.06416](#), [2111.02413](#) (“**Gnole**”)  
[NB: see also Becher, Schalch, Xu, [2307.02283](#)]

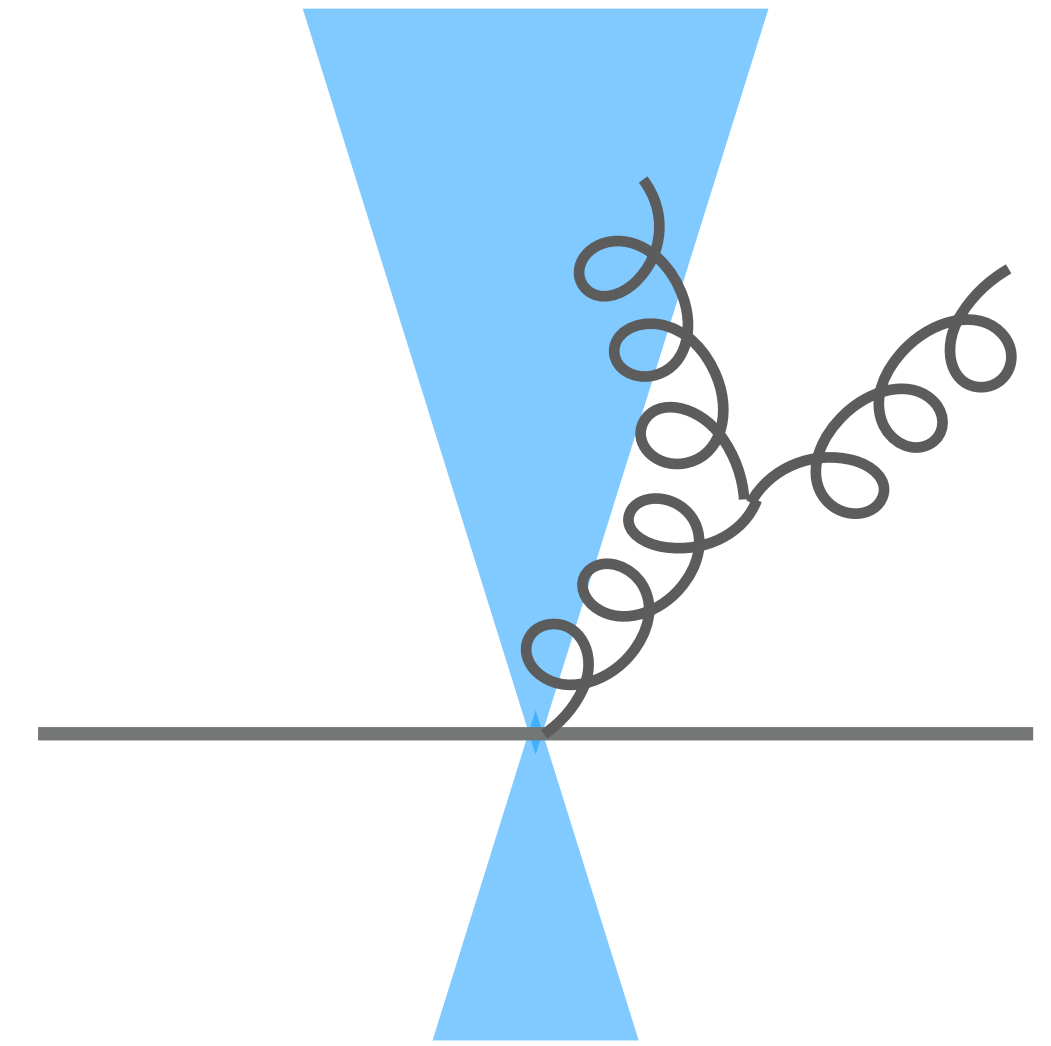


# NSL for the energy flow in a rapidity slice



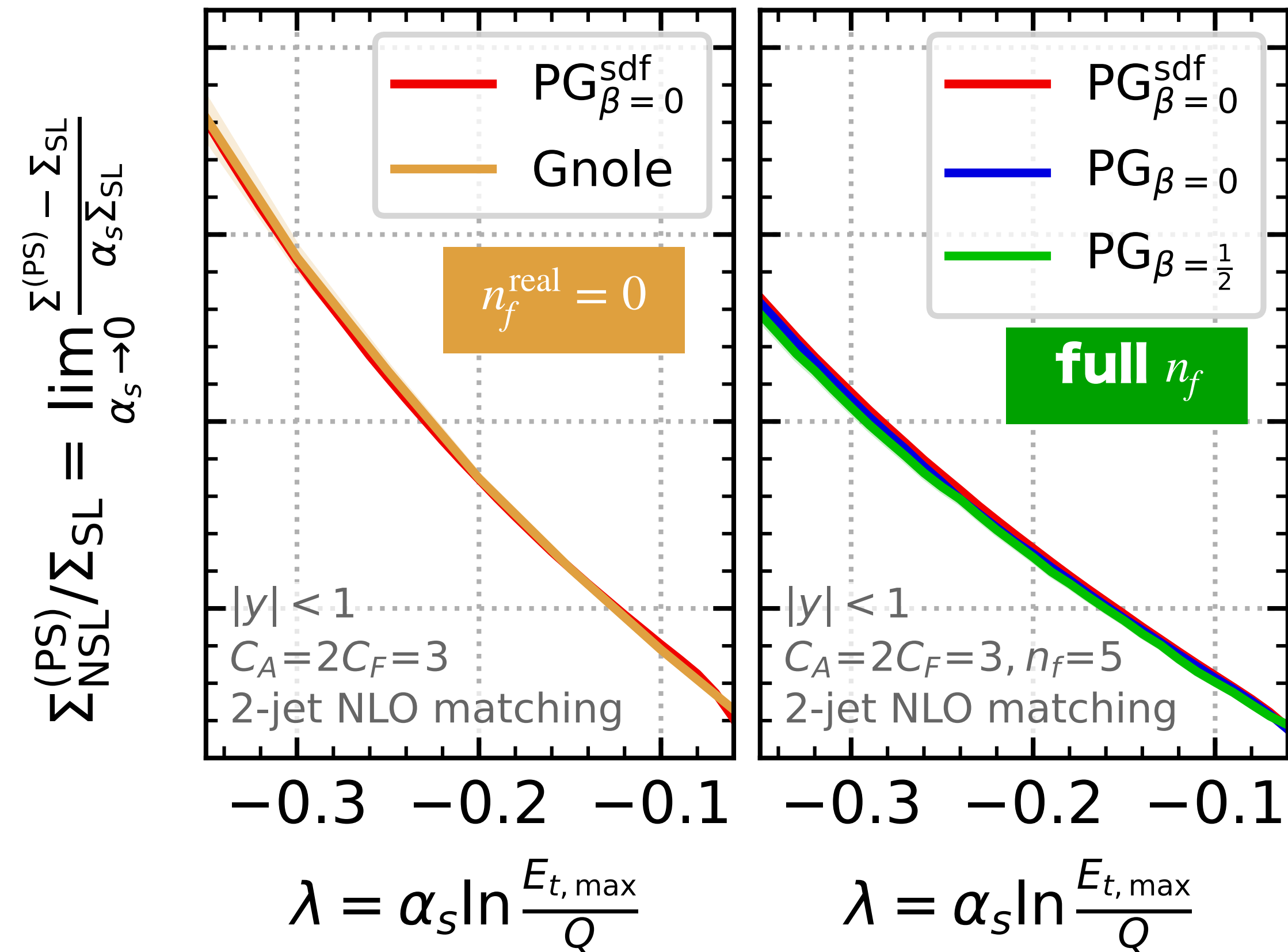
S.F.R., Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)

Non-global  
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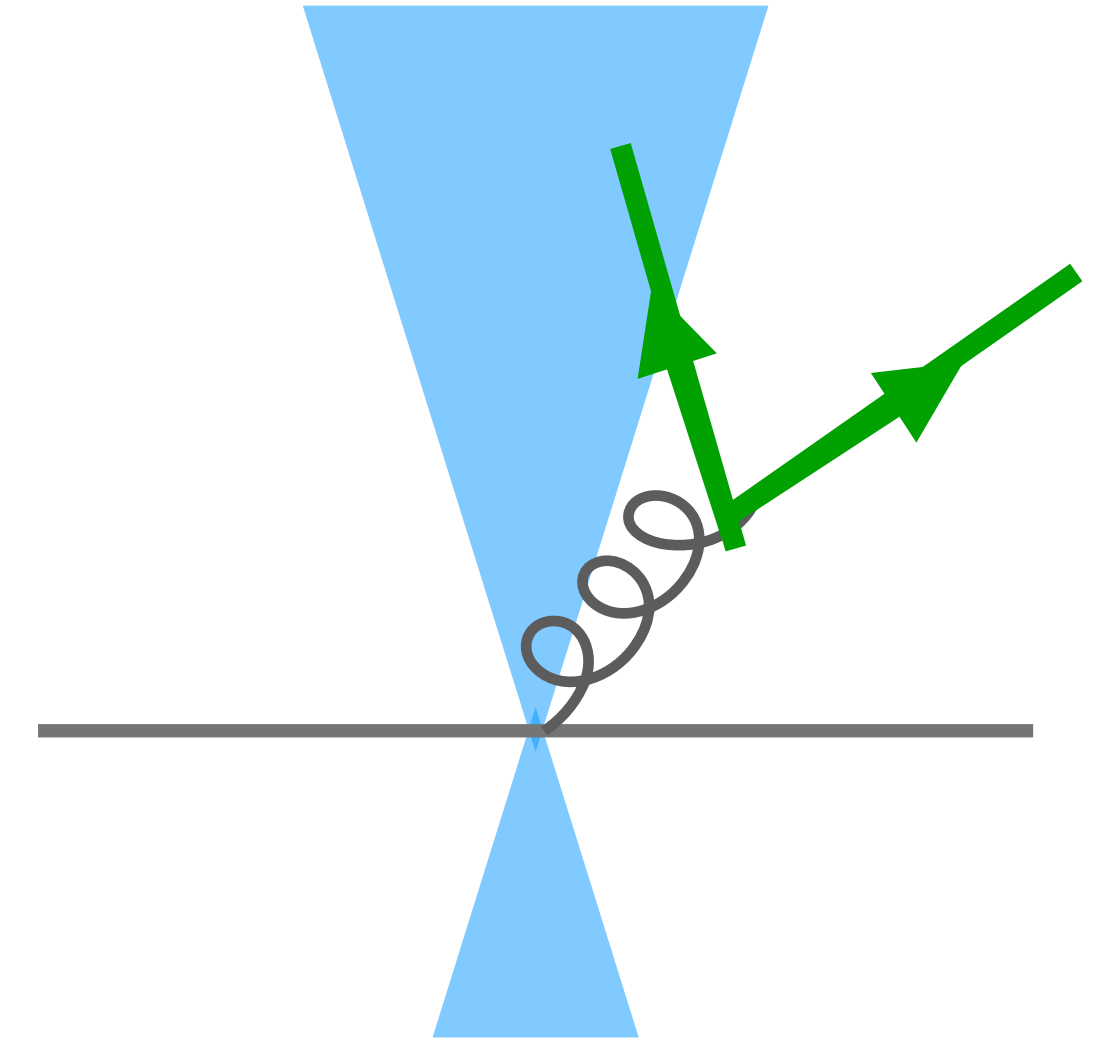
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# NSL for the energy flow in a rapidity slice



S.F.R., Hamilton, Karlberg, Salam,  
Scyboz, Soyez [2307.11142](#)

Non-global  
observable

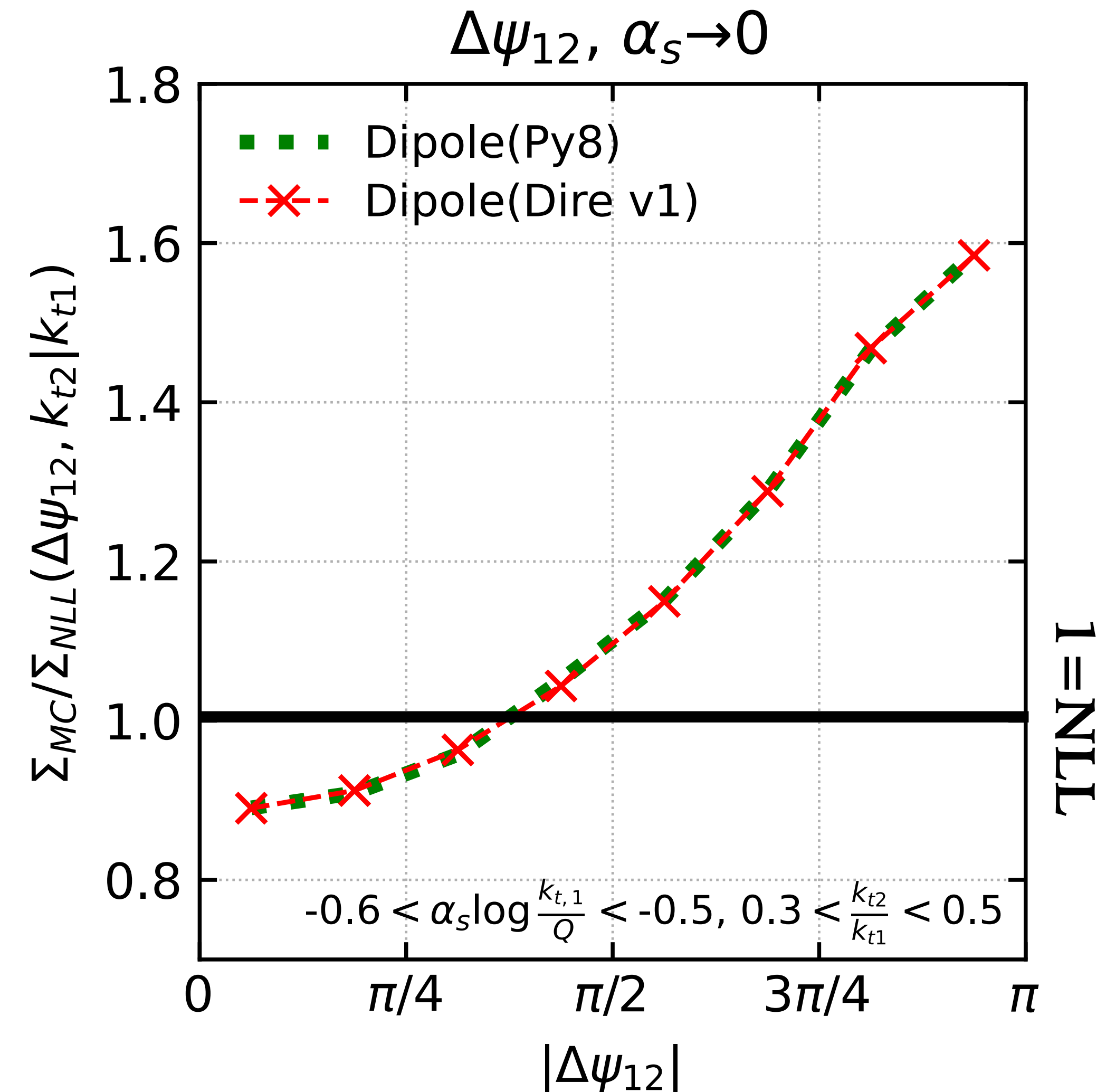


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 [NB: see also Becher, Schalch, Xu, [2307.02283](#)]
- First large- $N_c$  **full- $n_f$**  results for NSL non-global logs

# What is available in Shower Monte Carlo generators?

- Showers routinely used to interpret LHC (and LEP) data are **not NLL!**

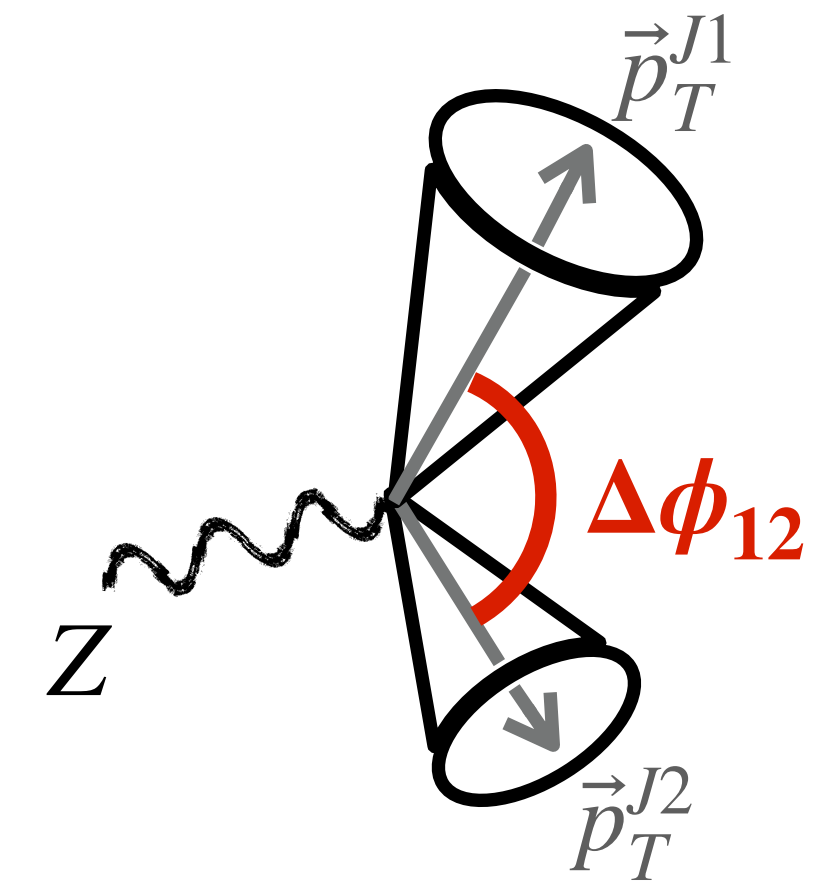
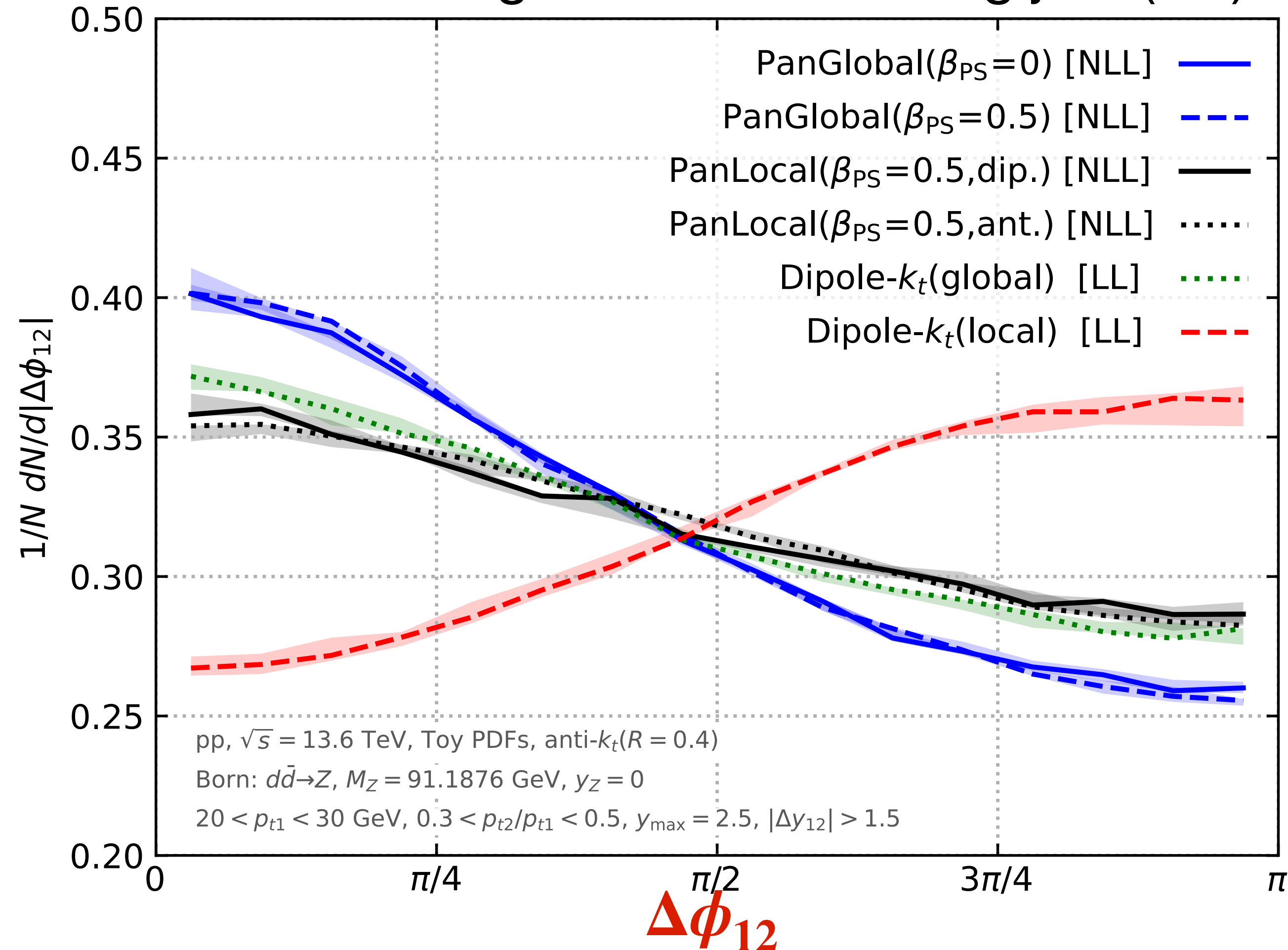
Dasgupta et al. [2002.11114](#)



# Exploratory phenomenology for Drell-Yan at the LHC

$$m_{\ell\ell} = 91.2 \text{ GeV}$$

Azimuthal angle between leading jets (DY)

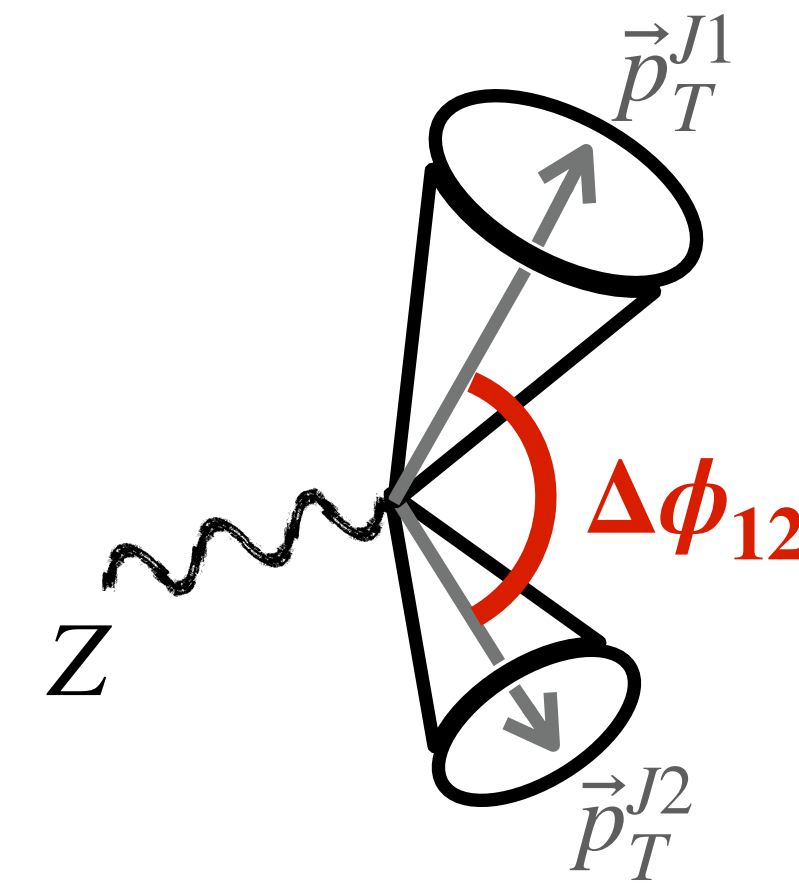
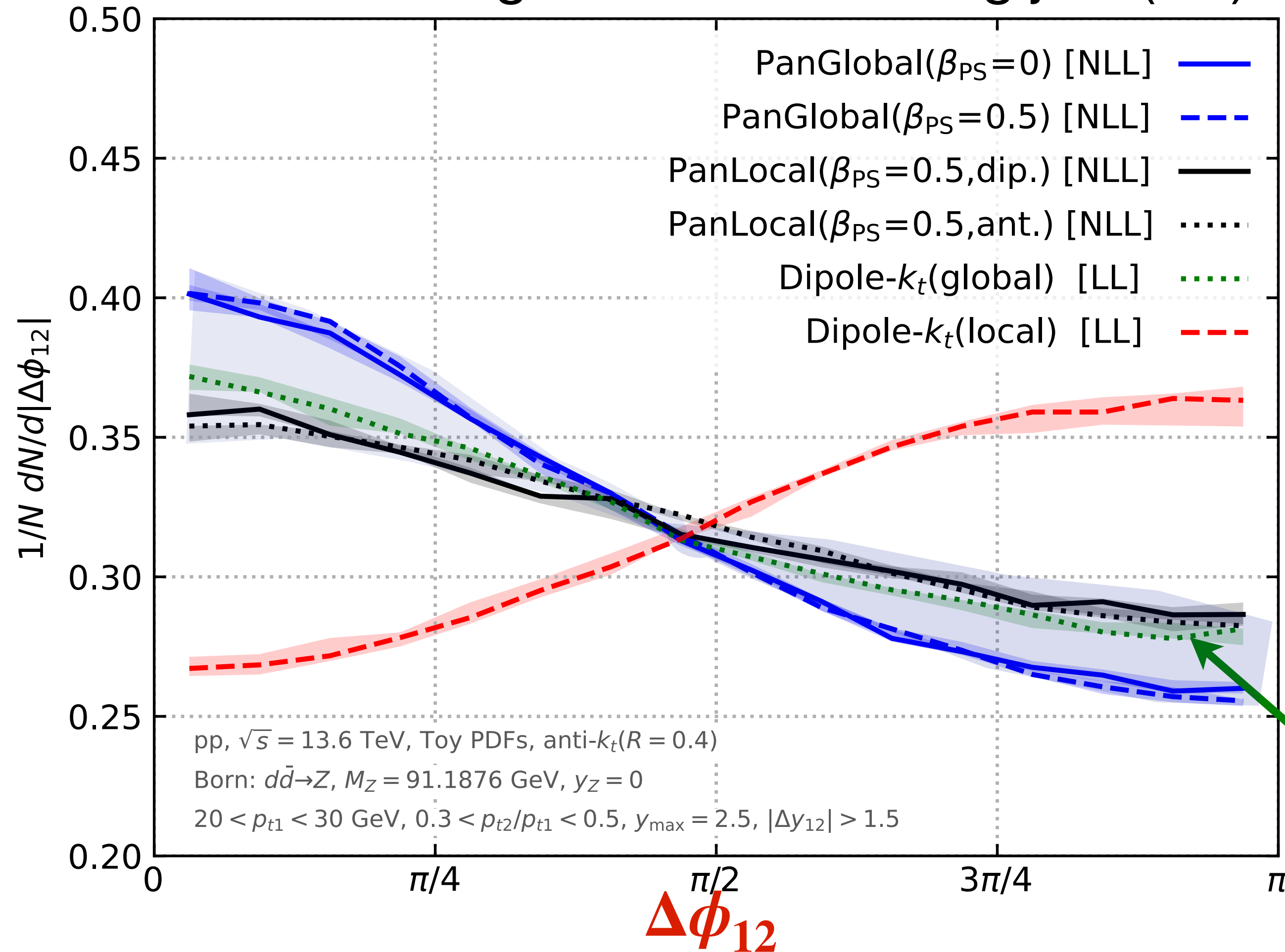


PanScales for  $pp \rightarrow$   
 colour singlet:  
[2207.09467](https://arxiv.org/abs/2207.09467), van  
 Beekveld, SFR,  
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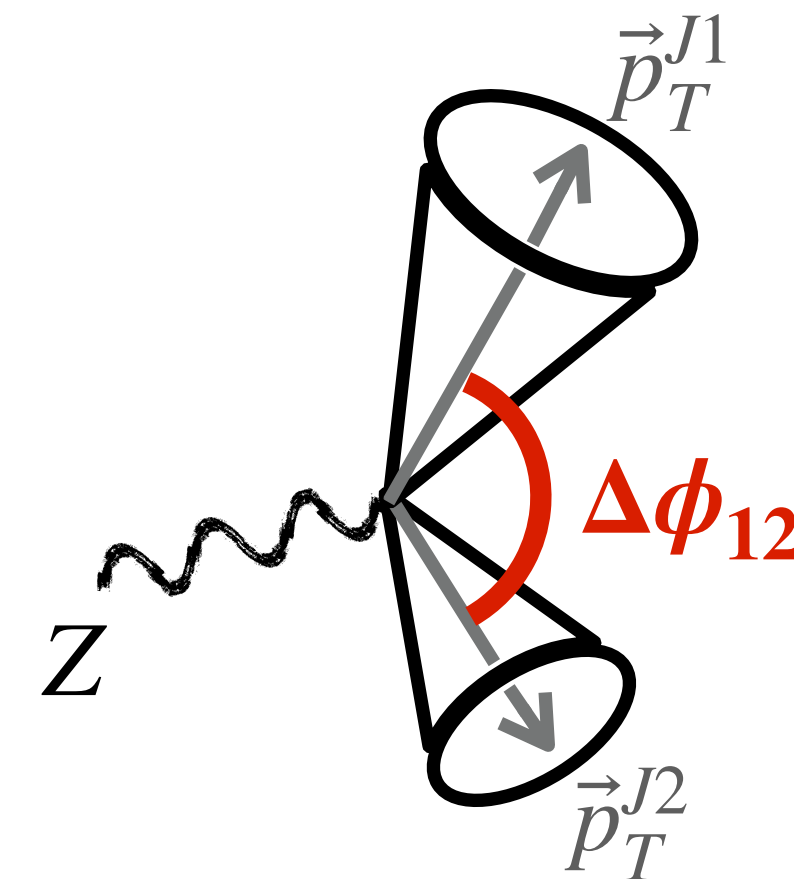
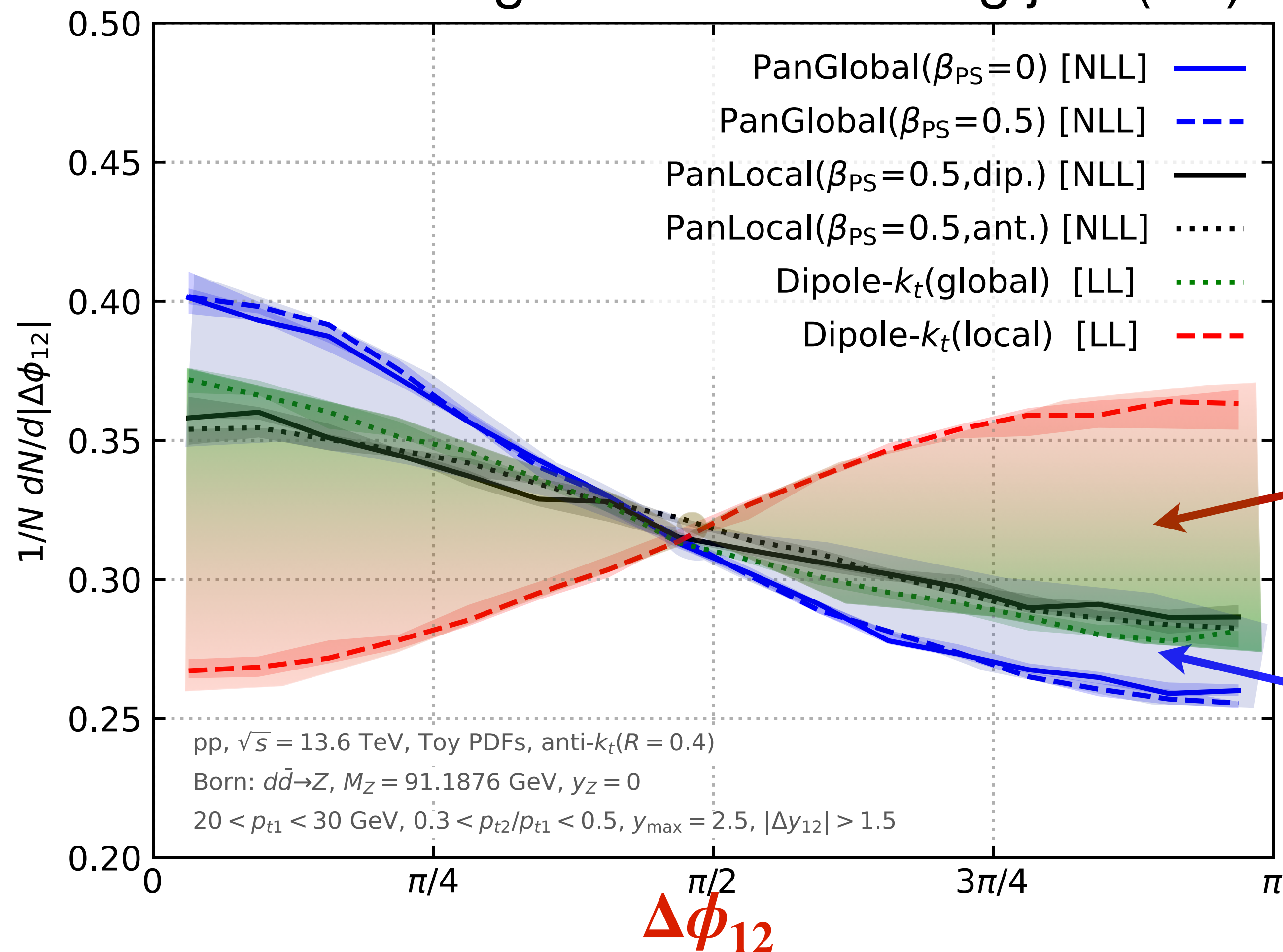
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 Verheyen:

This LL shower  
 lives within the  
 span of the  
 NLL showers

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 Hamilton, Salam  
 Soto Ontoso, Soyez,  
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**Scale  
 uncertainty  
 not  
 enough!**

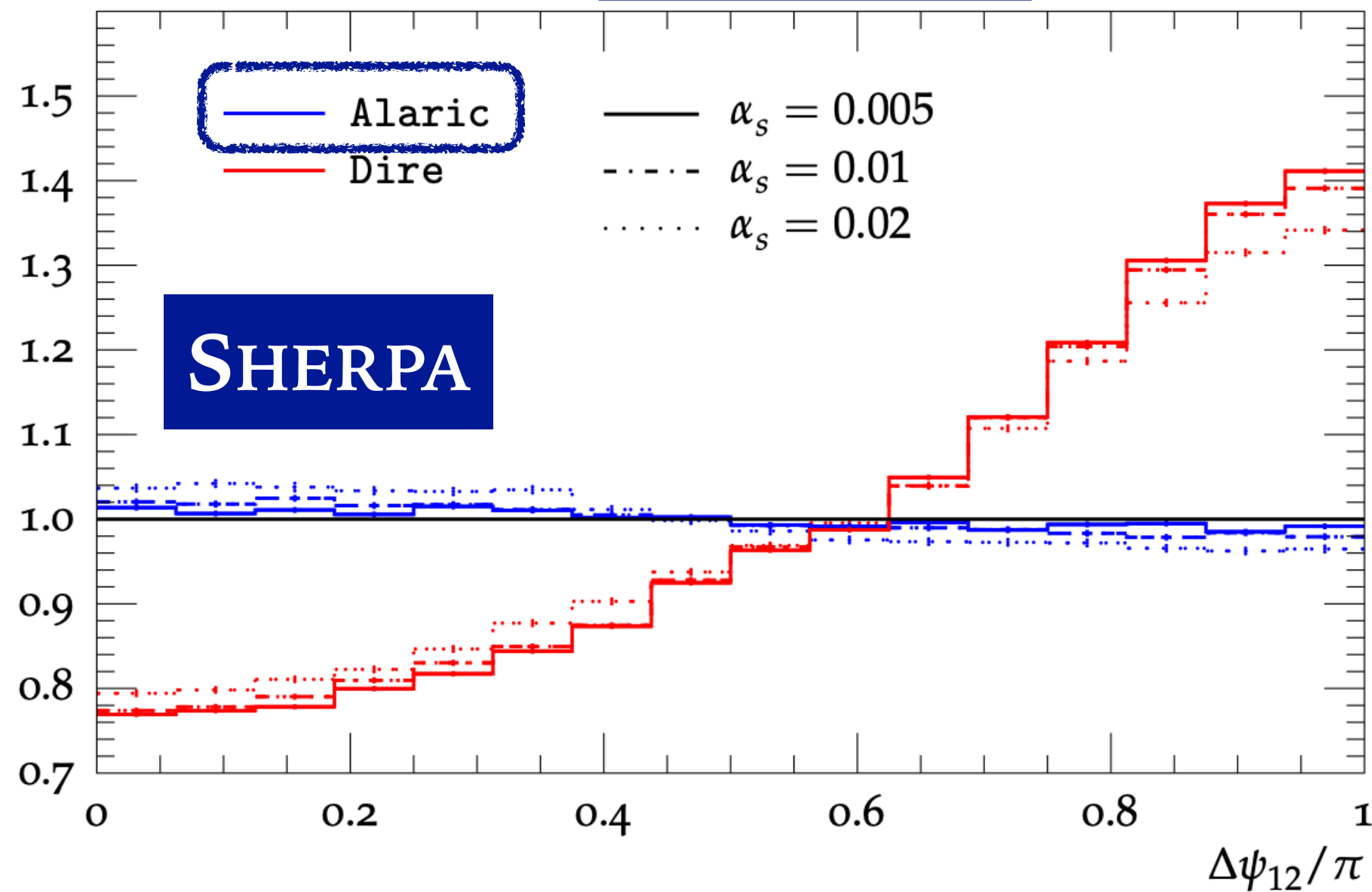
LL  
 showers

NLL  
 showers  
 have a lower  
 spread

# What can be available in Shower Monte Carlo generators?

- Showers routinely used to interpret LHC (and LEP) data are **not NLL**!
- **Many groups** are independently formulating new showers with **NLL accuracy for  $e^+e^-$**

Herren et al. [2208.06057](#)



**DEDUCTOR**  
Nagy&Soper,  
[2011.04777](#)

**CVOLVER**  
Forshaw et. al,  
[2003.06400](#)

Dasgupta et al. [2002.11114](#)

