EW physics at very high energies -- Splitting & Showering

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Topics:

- EW physics @ high energies
- Splitting functions, EW showering
- EWSB and Goldstone bosons
- Splittings in the broken phase: Ultra collinear
- EW evolution beyond the leading log

J.M. Chen, TH & B. Tweedie, arXiv:1611.00788;
TH, Y. Ma & K. Xie, arXiv:2007.14300; 2103.09844

EW Physics at Higher Energies

Some numerology:

$$(1).\ \frac{E}{v}: \quad G_F E_\beta^2 \sim \left(\frac{\text{MeV}}{M_W}\right)^2 \sim 10^{-8}, \quad \left(\frac{10\ \text{TeV}}{M_W}\right)^2 \sim 10^4\ !$$

$$\epsilon_L^\mu(p) \sim \frac{p^\mu}{M_W}$$ → sensitive to HE/UV physics.

$$(2).\ \frac{v}{E}: \quad \frac{v\ (250\ \text{GeV})}{10\ \text{TeV}} \approx \frac{\Lambda_{QCD}\ (300\ \text{MeV})}{10\ \text{GeV}}$$

At E>>v , $v/E,\ m_t/E,\ M_W/E \to 0!$

→ massless theory; EW symmetry restored!

v/E as power corrections:

- Like QCD: higher-twist terms Λ^2_{QCD}/Q^2.
- Unlike QCD: perturbatively defined!

Some numerology:

(3). $\frac{m_t}{10\ \text{TeV}} \sim \frac{m_b}{200\ \text{GeV}}$

The top quark at the a 10-TeV pCM Collider would be as “massless” as b-quark was at the Tevatron.

→ Top quark PDF? 6 active flavors?

Daswon, Ismail, I. Low (2014); TH, Sayre, Westhoff (2015).

At scale Q: $\frac{\alpha_s}{\pi} C_F \ln \frac{Q^2}{m_t^2} \sim \delta$

$$Q \approx m_t \cdot \exp(\frac{\pi\delta}{2\alpha_s C_F})$$

For $\delta = 20\% - 30\%,\ \alpha_s \sim 0.08,$

$Q = (25 - 110) m_t \Rightarrow (4 - 20)\ \text{TeV}.$

Some numerology:

(4). EW logarithms

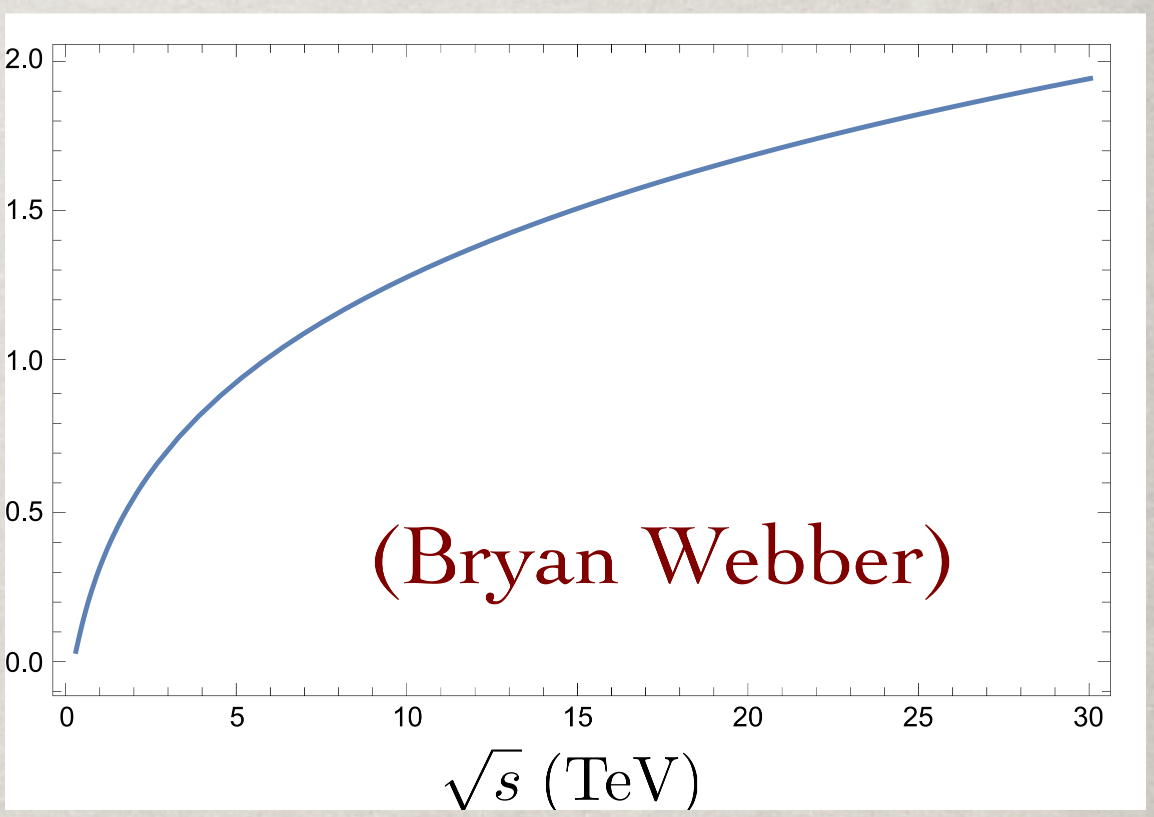
At scale Q : $\frac{\alpha_2}{\pi} C_w \ln^2 \frac{Q^2}{M_W^2} \sim \delta$

$$Q \approx M_W \cdot \exp(\frac{\pi\delta}{4\alpha_2 C_w})^{\frac{1}{2}}$$

J. Chiu, A. Manohar et al., 2005;
Manohar, Bauer et al. (SCET);
M. Chiesa et al., PRL (2013);
T. Becher et al., 1305.4202;
Bauer, Ferland, 1601.07190;

For $\delta = 50\%,\ \alpha_2 \sim 0.035,$

$Q \approx 30 M_W \Rightarrow 2.5$ TeV.



- Virtual Sudakov suppression;
- Real emission enhancement.

SU(2) versus SU(3):
Gauge boson splitting

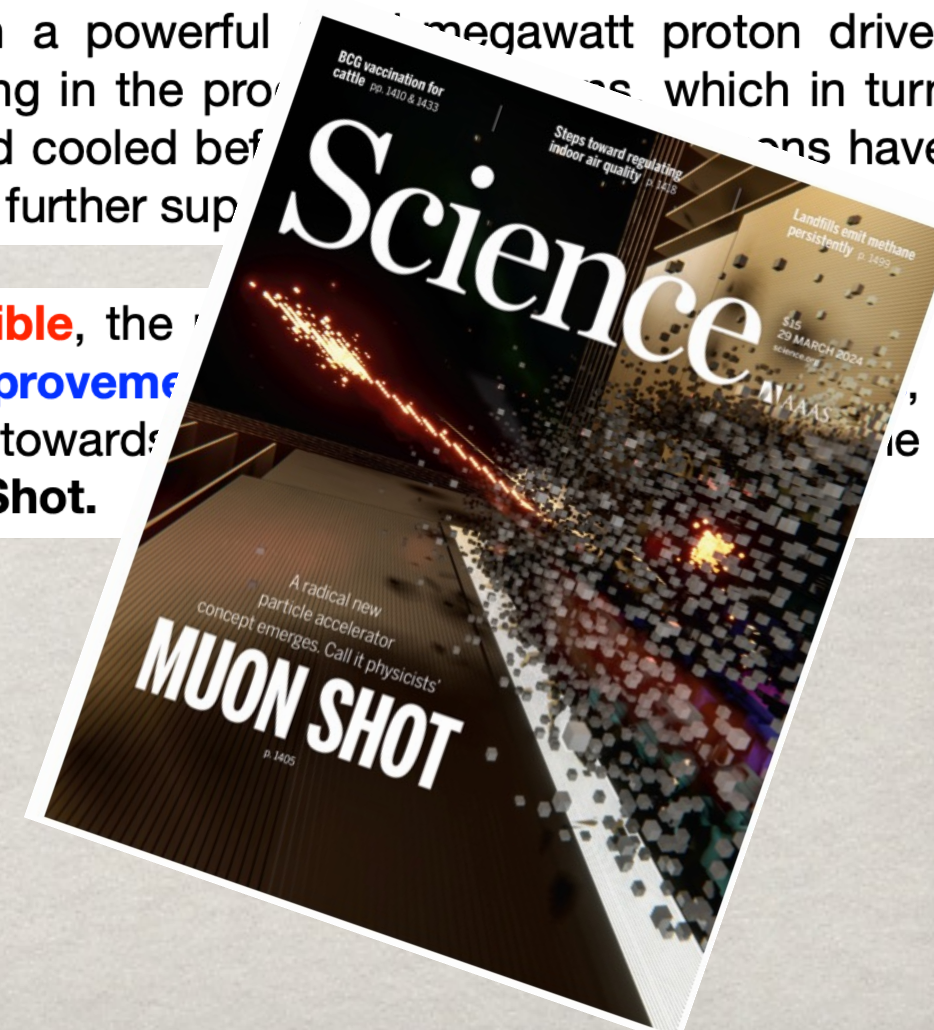
"Color factors" : $\frac{C_A}{C_F} = \frac{2N^2}{N^2-1} \Rightarrow (\frac{9}{4})_{N=3}$ and $(\frac{8}{3})_{N=2}$.

2.3 The Path to a 10 TeV pCM

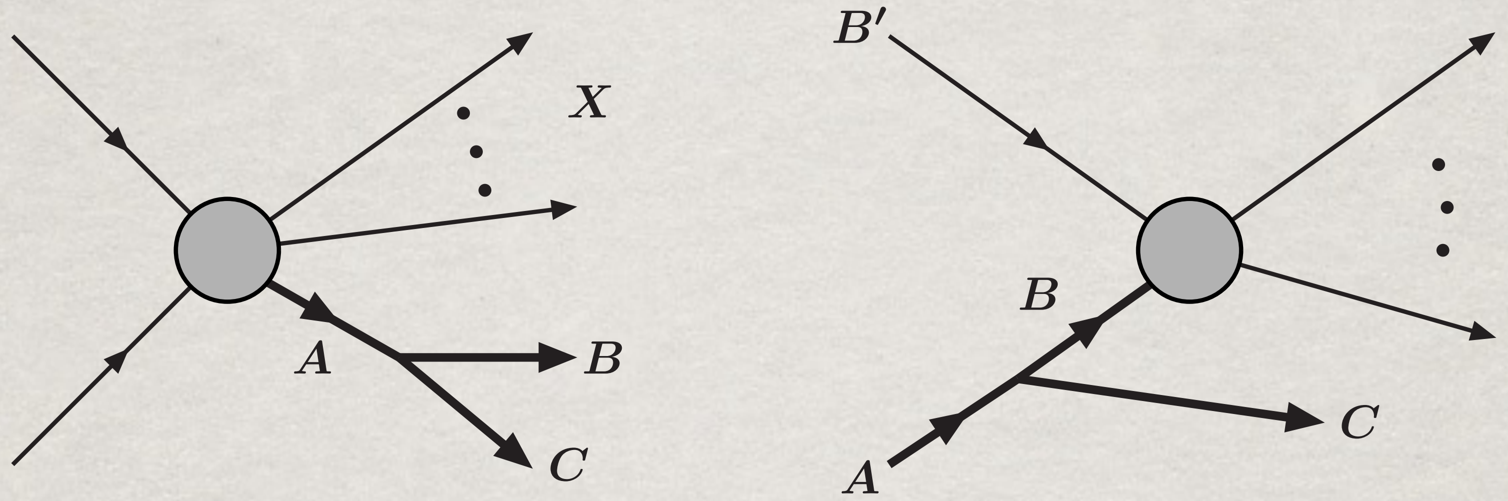
Realization of a future collider will require resources at a global scale and will be built through a world-wide collaborative effort where decisions will be taken collectively from the outset by the partners. This differs from current and past international projects in particle physics, where individual laboratories started projects that were later joined by other laboratories. The proposed program aligns with **the long-term ambition of hosting a major international collider facility in the US, leading the global effort** to understand the fundamental nature of the universe.

In particular, a muon collider presents an attractive option both for technological innovation and for bringing energy frontier colliders back to the US. The footprint of **a 10 TeV pCM muon collider is almost exactly the size of the Fermilab campus**. A muon collider would rely on a powerful ... megawatt proton driver
delivering very intense and short beam pulses to a target, resulting in the pro... which in turn
decay into muons. This cloud of muons needs to be captured and cooled bef... ons have
decayed. Once cooled into a beam, fast acceleration is required to further sup...

Although **we do not know if a muon collider is ultimately feasible**, the ...
Fermilab strengths and capabilities to **a series of proton beam improveme...** ,
each producing world-class science while performing critical R&D towards ... e
path is an unparalleled global facility on US soil. **This is our Muon Shot.**



Splitting: the dominant phenomena



$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A\to B+C}$$

$$E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathcal{P}_{A\to B+C}}{dz\,dk_T^2} \simeq \frac{1}{16\pi^2}\,\frac{z\bar{z}\,|\mathcal{M}^{(\text{split})}|^2}{(k_T^2+\bar{z}m_B^2+zm_C^2-z\bar{z}m_A^2)^2}$$

- On the dimensional ground: $|\mathcal{M}_{split}|^2 \sim k_T^2$ or m^2
- For the factorization formalism to be valid: infra-red safe & leading behavior

Ciafaloni et al., hep-ph/0004071, 0007096; J.M. Chen, TH & B. Tweedie, arXiv:1611.00788; C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562; 1808.08831; A. Manohar et al., 1803.06347.

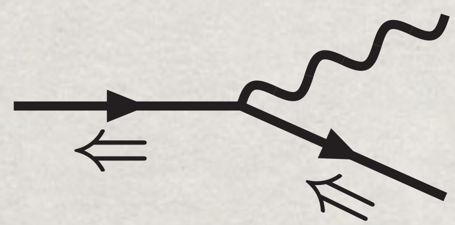
EW Splitting functions

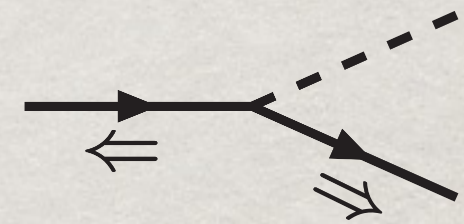
Start from the unbroken phase – all massless.

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{\phi} + \mathcal{L}_f + \mathcal{L}_{Yuk}$$

Chiral fermions: f_s, gauge bosons: $B, W^0, W^\pm$; $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$

e.g.: fermion splitting:





	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{1+\bar{z}^2}{z}\right)$		$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z}{2}\right)$
	$\to\ V_T\ f_s^{(\prime)}$	$[BW]_T^0\ f_s$	$H^{0(*)}\ f_{\text{-}s}$ or $\phi^\pm\ f'_{\text{-}s}$
$f_{s=L,R}$	$g_V^2 (Q_{f_s}^V)^2$	$g_1 g_2 Y_{f_s} T_{f_s}^3$	$y^2_{f_R^{(\prime)}}$

Ciafaloni et al., Hep-ph/0505047.

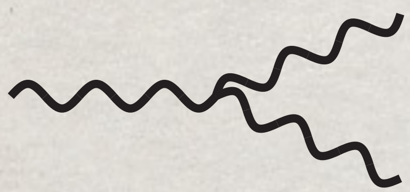
Infrared & collinear singularities (P_{gq})

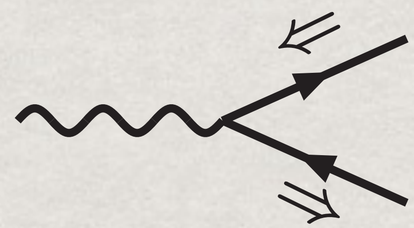
Collinear singularity, Chirality-flip, Yukawa

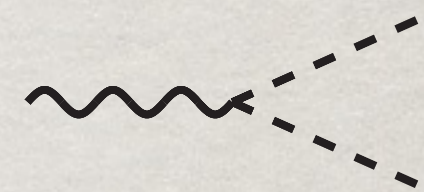
EW Splitting functions

SM in the unbroken phase

e.g.: Gauge boson splitting:







	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{(1-z\bar{z})^2}{z\bar{z}}\right)$	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z^2+\bar{z}^2}{2}\right)$	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(z\bar{z}\right)$	
	$\to\ W_T\, W_T$	$f_s\ \bar{f}^{(\prime)}_{\text{-}s}$	$\phi^+\ \phi^-$ or $H^0\ H^{0*}$	$\phi^+\ H^{0*}$ or $\phi^-\ H^0$
V_T	$2g_2^2\ (V{=}W^{0,\pm})$	$N_f g_V^2 (Q^V_{f_s})^2$	$\frac{1}{4}g_V^2$	$\frac{1}{2}g_2^2$
$[BW]^0_T$	0	$N_f g_1 g_2 Y_{f_s} T^3_{f_s}$	$\frac{1}{2}g_1 g_2 T^3_{\phi^+,H^0}$	0

Infrared & collinear (P_{gg})

Collinear (P_{qg})

Interference (BW^0) must be included!

Collinear (new)

EW gauge bosons from splitting@Fcc-hh

W radiation costs ~ 1/10

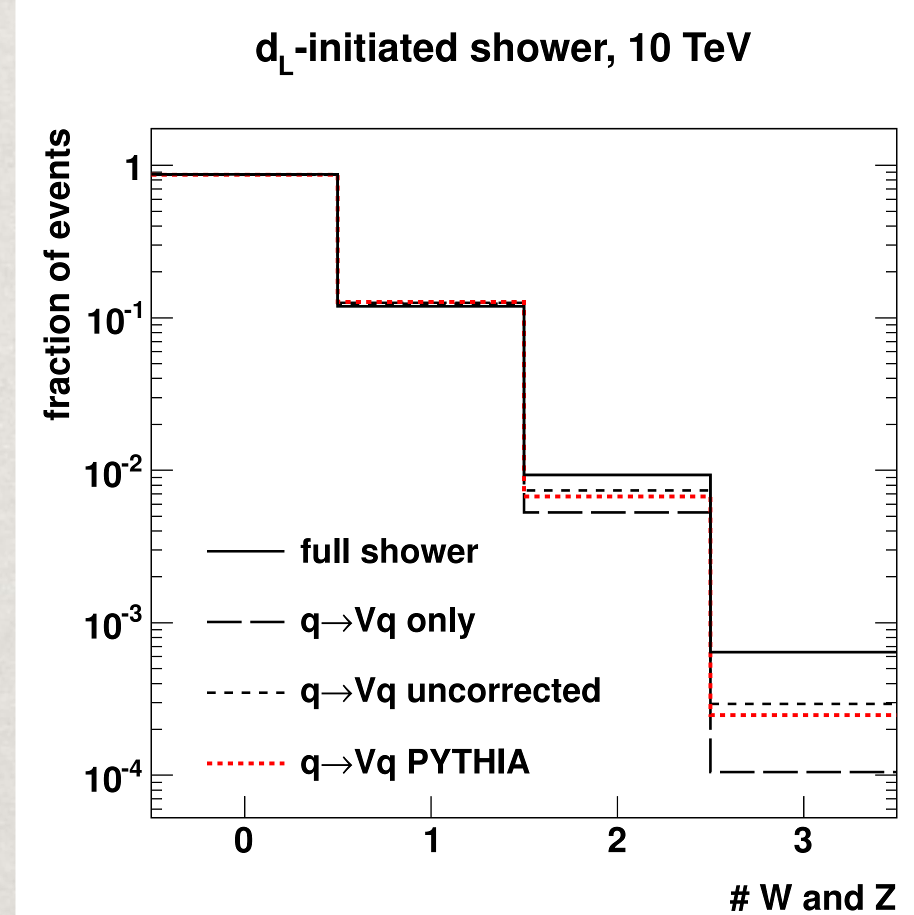
At 100 TeV: M. Mangano

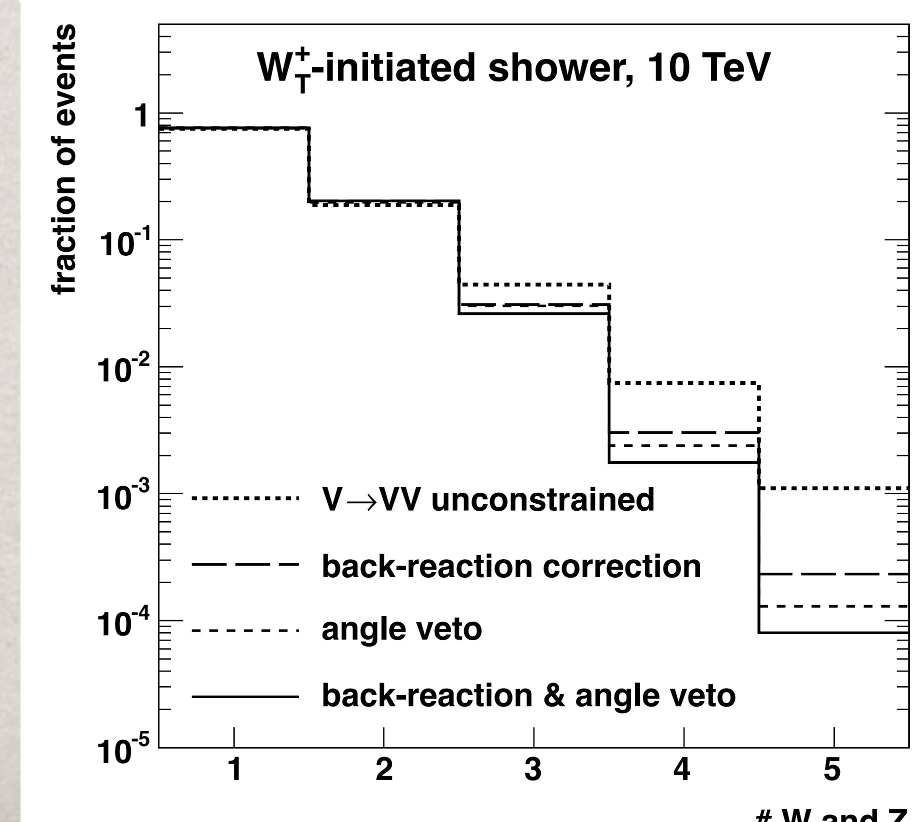
Diagramatic calculations

W W	σ=770 pb
W W W	σ=2 pb
W W Z	σ=1.6 pb
W W W W	σ=15 fb
W W W Z	σ=20 fb

....

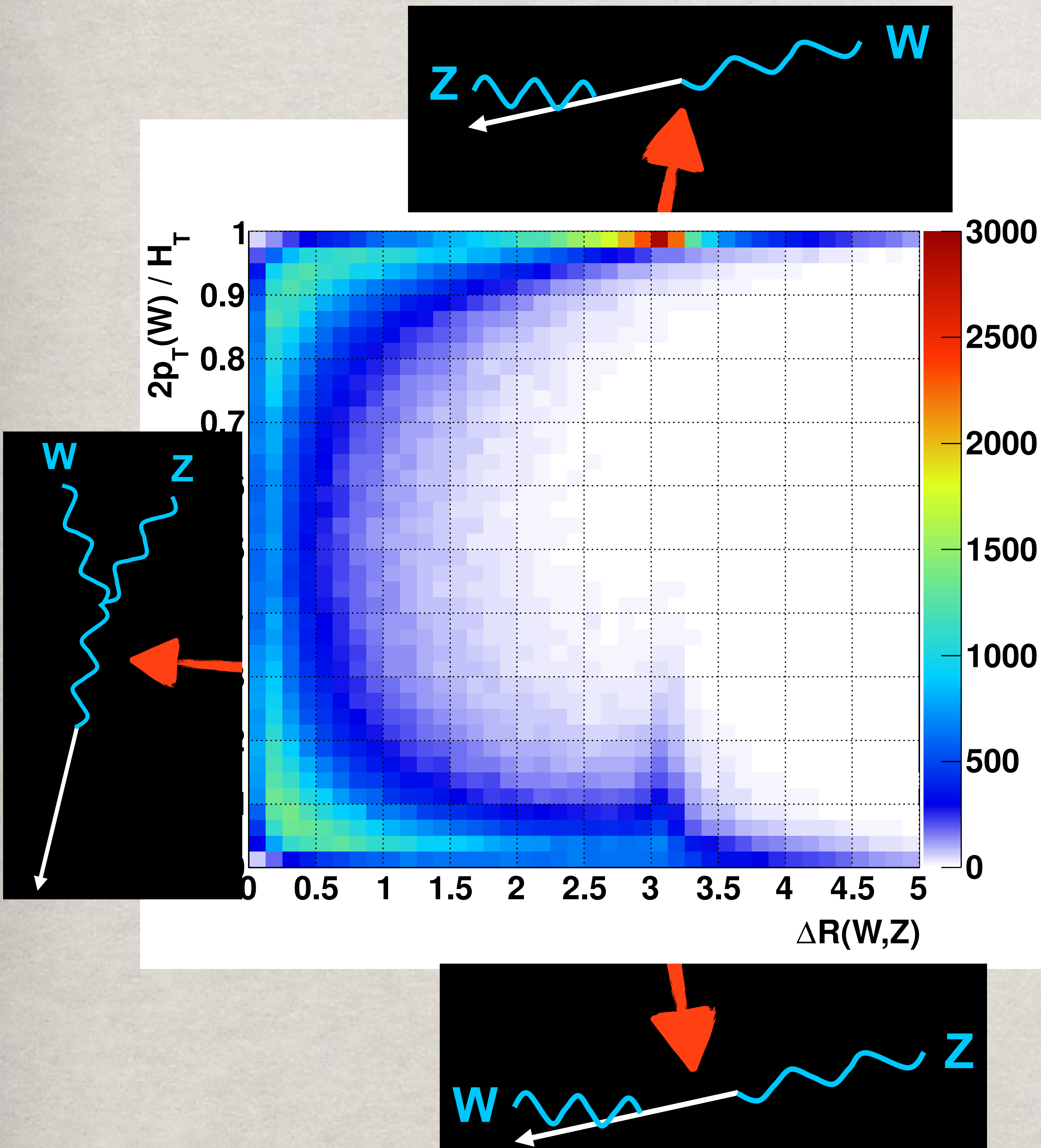
Each *W* costs you a factor of ~ 1/100 (EW coupling)

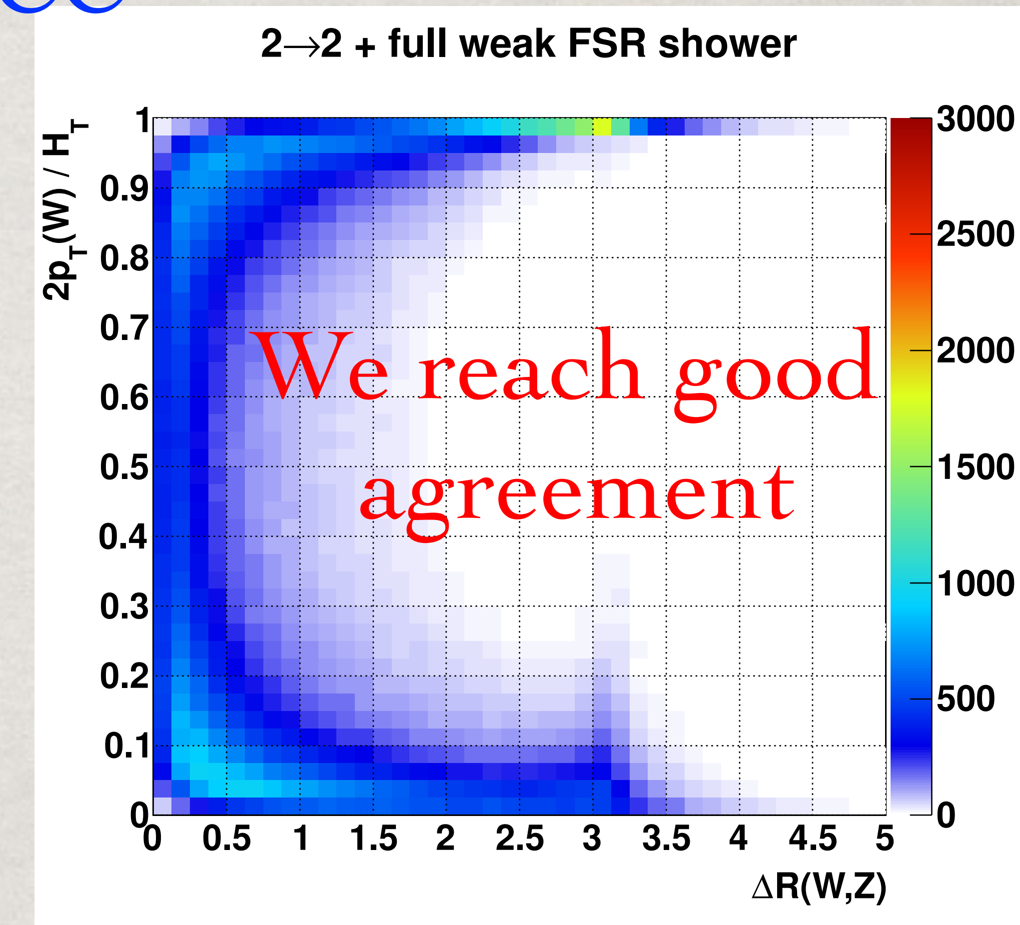


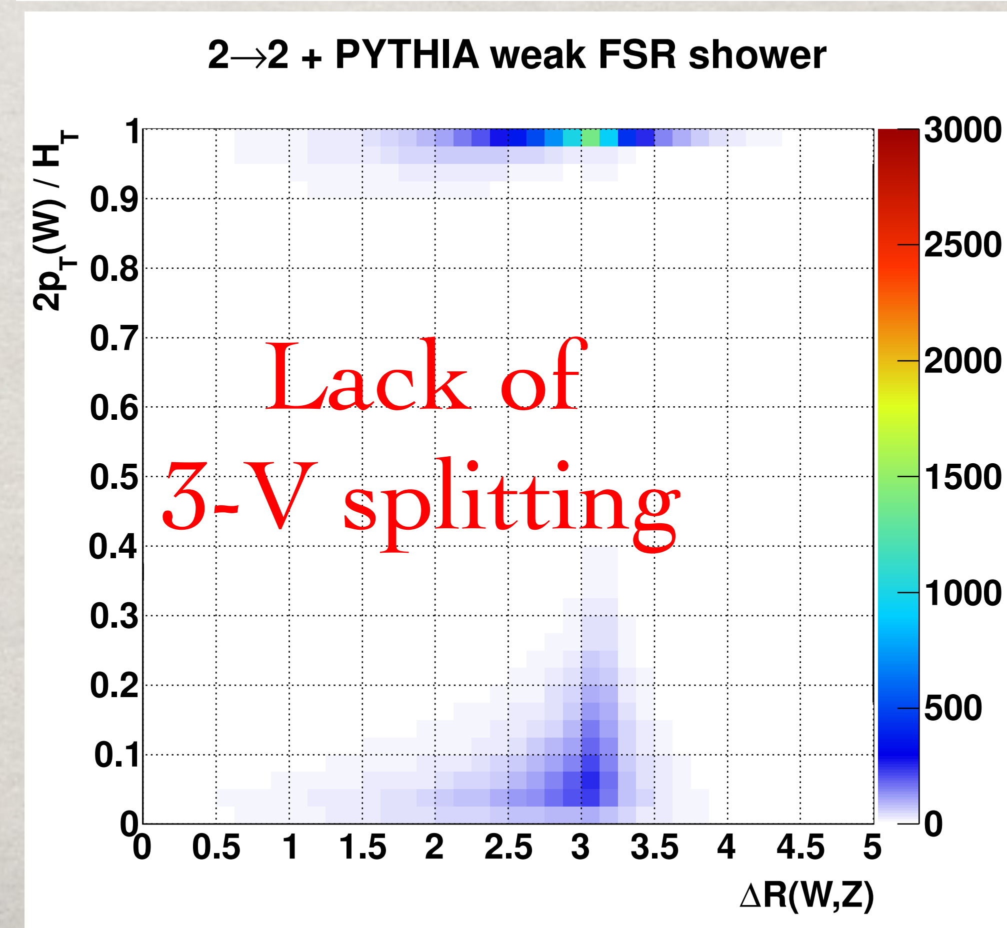


WZ+1 jet @ F_{CC}: 100 TeV

MadGraph 2→3 fixed order







P. Richardson et al., arXiv:2108.10817

EW Symmetry breaking & Goldstone-boson Equivalence Theorem (GET):

Lee, Quigg, Thacker (1977); Chanowitz, Gailard (1984);
Y.-P. Yao, C.-P. Yuan (1988); J. Bagger, C. Schmidt (1990) …

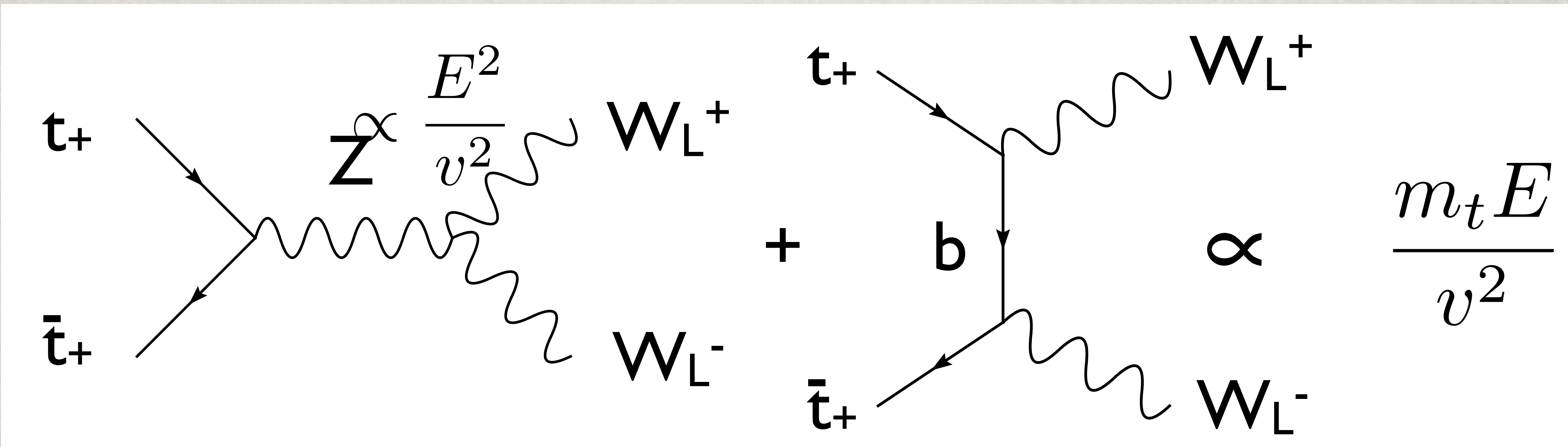
At high energies $E \gg M_W$, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!)

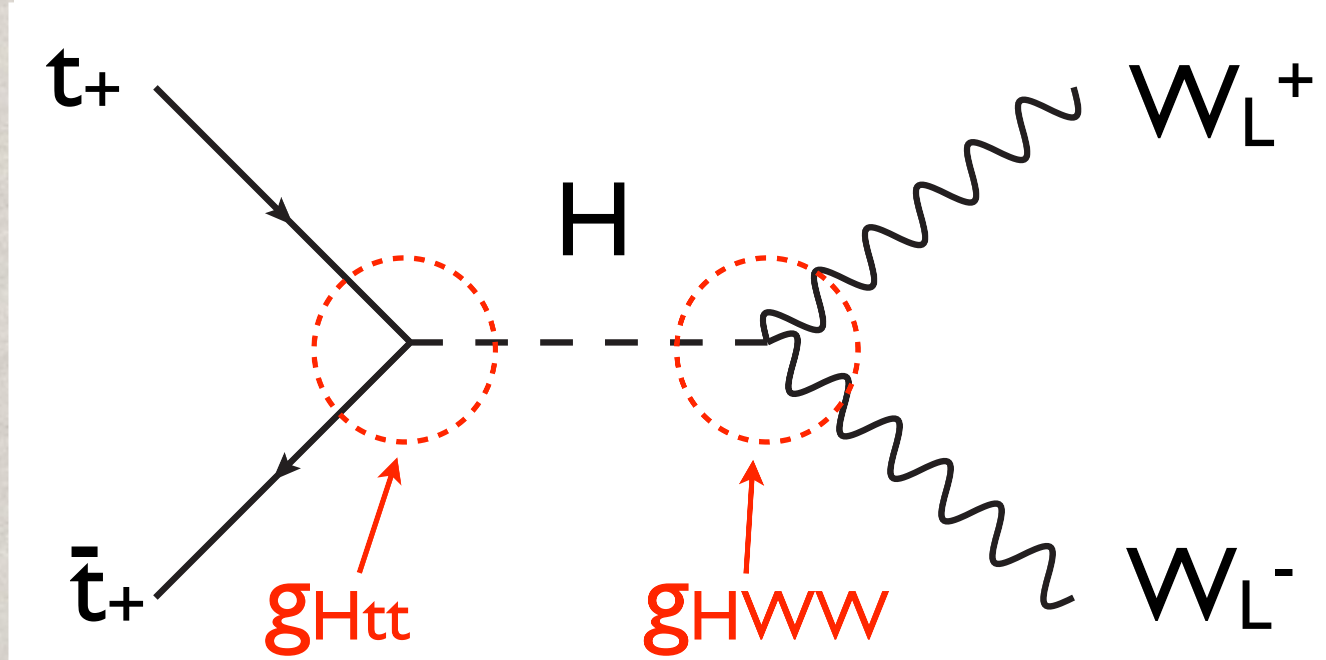
"Scalarization" to implement the Goldstone-boson Equivalence Theorem (GET):

$$\epsilon(k)^{\mu}_{L} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^{\mu}}{m_W} + O(M_W/E)$$

(a). Unitarity at higher energies:

$\epsilon(k)^{\mu}_{L} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W}$ bad high-energy behavior!





A "light Higgs" fixes it:

$$\propto \frac{m_t m_H}{v^2}$$

D. Dicus & V. Mathur (1973); Lee, Quigg, Thacker (1977).

(b). Puzzle of massless fermion radiation

V_L contributions dominant at high energies:

$$\epsilon(k)_L^\mu = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W}$$

Then, massless fermion splitting

$f \rightarrow f\, V_L$

would be zero, in accordance with GET for

$f \rightarrow f\, \phi \quad (y_f \rightarrow 0)$.

GET ignored the EWSB effects at the order M_W/E

(higher twist effects)

Corrections to GET
1st example: "Effective W-Approximation"

G. Kane, W. Repko, W. Rolnick (1984);
S. Dawson (1985); Chanowitz & Gailard (1984)

At colliding energies $E >> M_W$,

$$P_{q\to qV_T} = (g_V^2 + g_A^2)\,\frac{\alpha_2}{2\pi}\,\frac{1+(1-x)^2}{x}\ln\frac{Q^2}{\Lambda^2}$$

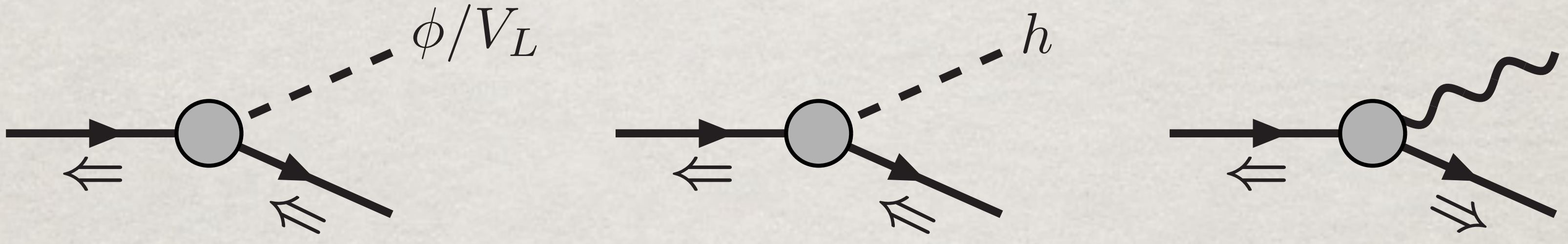
$$P_{q\to qV_L} = (g_V^2 + g_A^2)\,\frac{\alpha_2}{\pi}\,\frac{1-x}{x}$$

- $f \to f\,W_L$, $f\,Z_L$ do not vanish; no collinear-log!
  - Vector boson fusion observed at the LHC
    $WW, ZZ \to h$ & W^+W^+ scattering

EW Splitting functions in the broken phase

New fermion splitting: $\frac{v^2}{k_T^2}\frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$

V_L is of IR, h no IR



$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}\left(\frac{1}{z}\right)$	$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}$	$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}$
$\rightarrow\ V_L\ f_s^{(\prime)}\ (V \neq \gamma)$	$h\ f_s$	$V_T\ f_{\text{-}s}^{(\prime)}$

Chirality conserving:
Non-zero for massless f

Chirality flipping:
$\sim m_f$

The PDFs for W_L thus don't run at leading log

→ A broken gauge

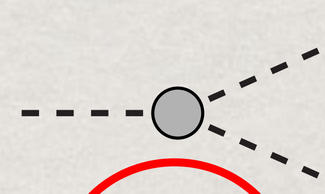
→ "Bjorken scaling" restored,

→ or higher-twist effects like Λ^2_{QCD}/Q^2.

Splitting in the Broken Gauge

New gauge boson splitting in 3-W_L

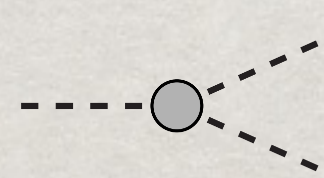
Vector boson V_L is of IR.

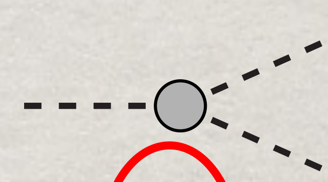


$$\frac{v^2}{k_T^2}\;\frac{dk_T^2}{k_T^2} \sim (1-\frac{v^2}{Q^2})$$

	$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}\left(\frac{1}{z\bar{z}}\right)$	
	$\to W_L^+\,W_L^-$	$Z_L\,W_L^\pm/Z_L$
$W_L^\pm$	0	$\frac{1}{16}g_2^4\left((\bar{z}-z)(2+z\bar{z})-t_W^2\bar{z}(1+\bar{z})\right)^2$
h	$\frac{1}{4}\left(g_2^2(1-z\bar{z})-\lambda_h z\bar{z}\right)^2$	$\frac{1}{8}\left(g_Z^2(1-z\bar{z})-\lambda_h z\bar{z}\right)^2$
Z_L	$\frac{1}{16}g_2^4\left((\bar{z}-z)(2+z\bar{z}-t_W^2 z\bar{z})\right)^2$	0
$[hZ_L]$	$\frac{i}{8}g_2^2\left(g_2^2(1-z\bar{z})-\lambda_h z\bar{z}\right)(\bar{z}-z)\left(2+z\bar{z}-t_W^2 z\bar{z}\right)$	0

h has no IR.





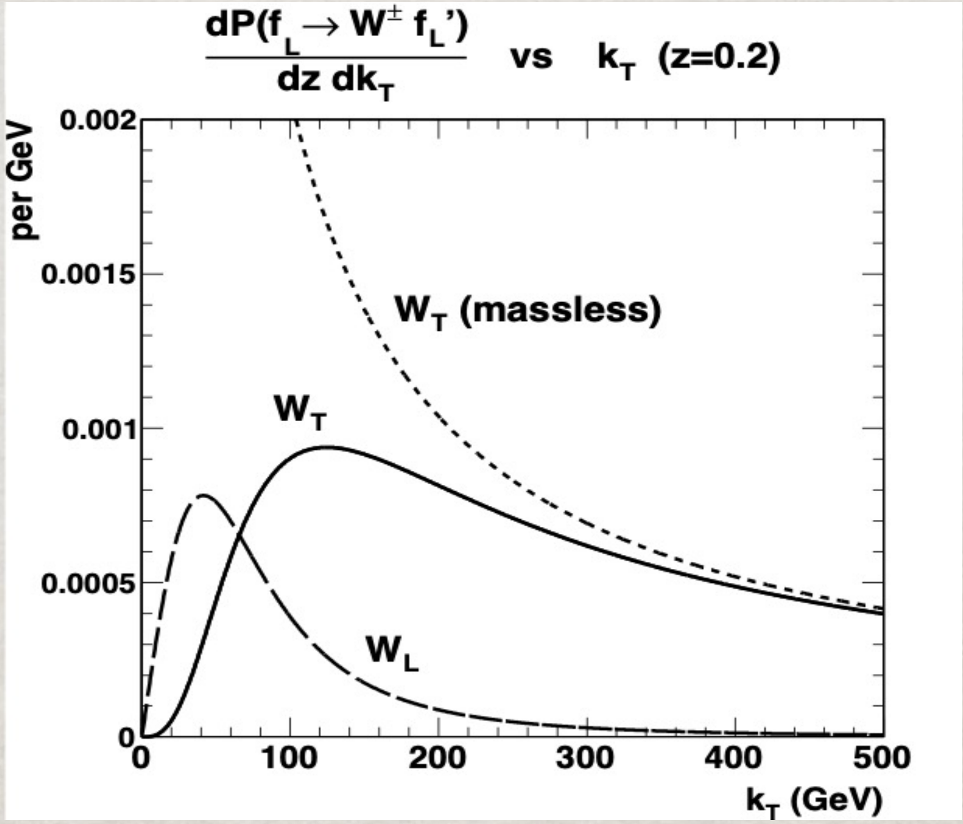
	$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}\left(\frac{1}{\bar{z}}\right)$	$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}$
	$\to h\,W_L^\pm/Z_L$	$h\,h$
$W_L^\pm$	$\frac{1}{4}z\left(g_2^2(1-z\bar{z})+\lambda_h\bar{z}\right)^2$	0
h	0	$\frac{9}{8}\lambda_h^2 z\bar{z}$
Z_L	$\frac{1}{4}z\left(g_Z^2(1-z\bar{z})+\lambda_h\bar{z}\right)^2$	0
$[hZ_L]$	0	0

"Ultra collinear behavior"

New characteristics with the mass:

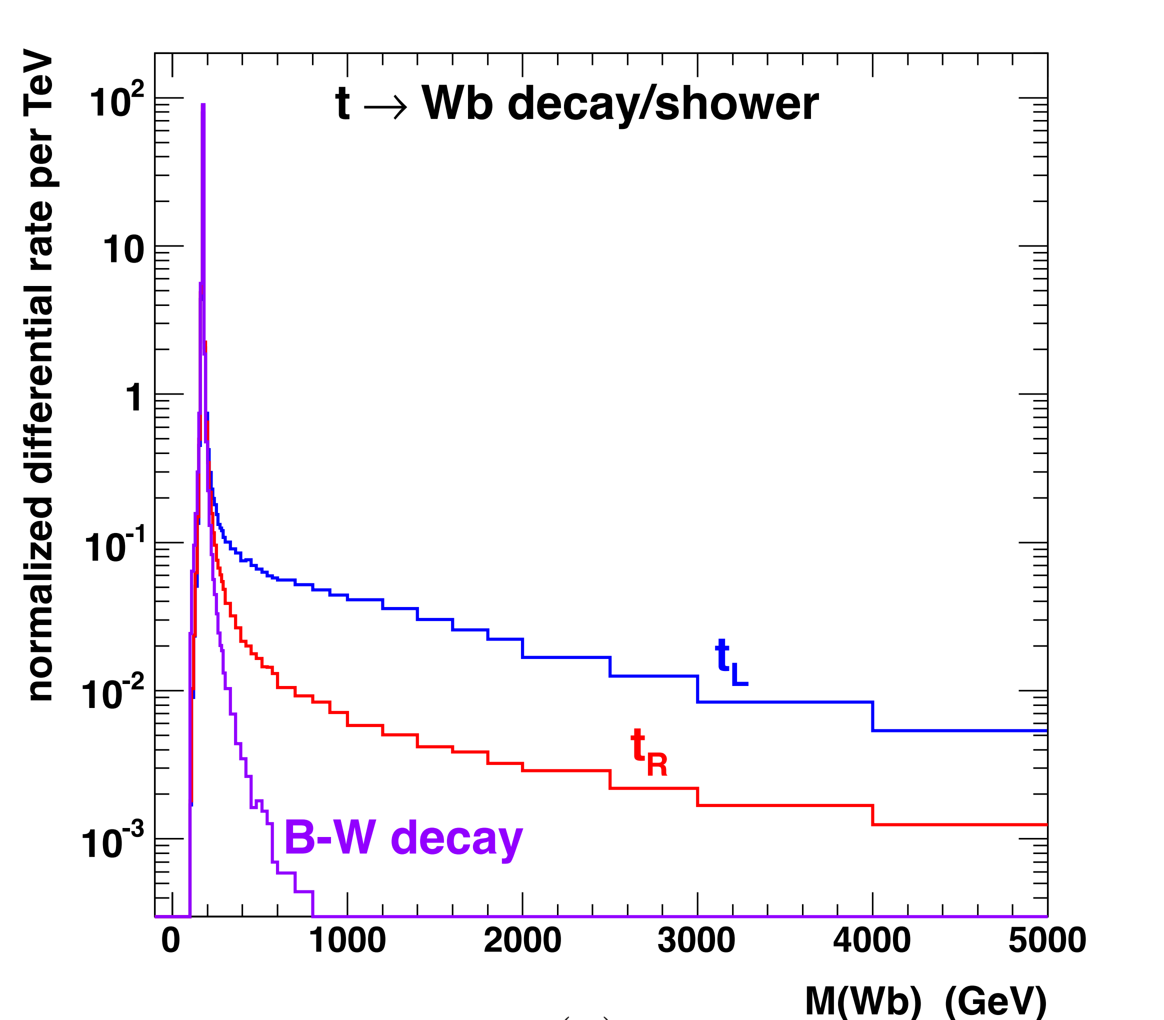
$k_T^2 > m_W^2$, it shuts off;
$k_T^2 < m_W^2$, flattens out!

$$\frac{v^2}{k_T^2}\,\frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$$

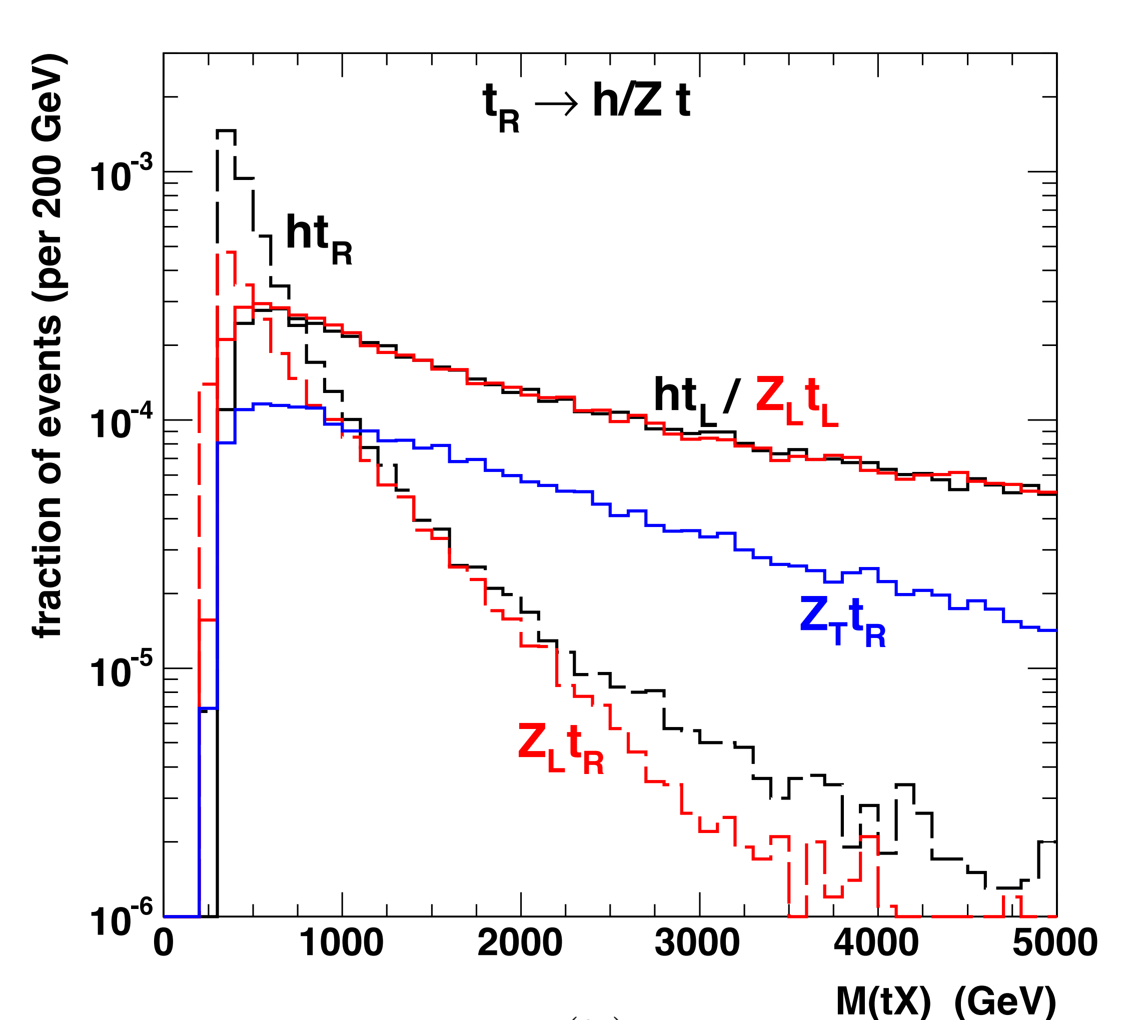


- Kinematic basis for "forward jet-tagging, central jet-vetoing" !
- The PDFs for W_L: no $\log(Q^2/M^2)$.

Top decay/showering (10 TeV):



(a)



(b)

J.M. Chen, TH & B. Tweedie, arXiv:1611.00788

Leading ultra-collinear: $t_R \to ht_R,\ Z_L t_R$

Yukawa: $\mathcal{P}(t_R \to ht_L) \simeq \mathcal{P}(t_R \to Z_L t_L) \approx 7.2 \times 10^{-3}$

U(1) gauge: $\mathcal{P}(t_R \to Z_T t_R) \approx 4.5 \times 10^{-3}$

In a nutshell: splitting probabilities

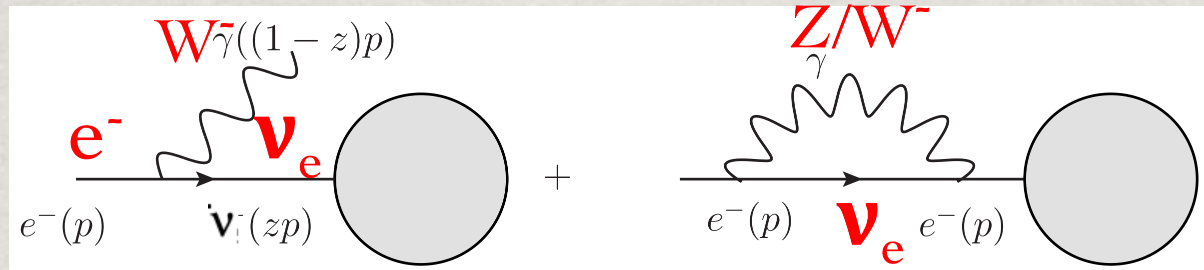
gauge couplings

Process	$\approx \mathcal{P}(E)$	$\mathcal{P}(1\ \text{TeV})$	$\mathcal{P}(10\ \text{TeV})$
$q \to V_T q^{(\prime)}$ (CL+IR)	$(3 \times 10^{-3}) \left[\log \frac{E}{m_W}\right]^2$	3%	7%
$q \to V_L q^{(\prime)}$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.8%	1.1%
$t_R \to W_L^+ b_L$ (CL)	$(8 \times 10^{-3}) \log \frac{E}{m_W}$	2%	4%
$t_R \to W_T^+ b_L$ (UC)	(6×10^{-3})	0.6%	0.6%
$V_T \to V_T V_T$ (CL+IR)	$(0.015) \left[\log \frac{E}{m_W}\right]^2$	8%	36%
$V_T \to V_L V_T$ (UC+IR)	$(0.014) \log \frac{E}{m_W}$	3%	7%
$V_T \to f\bar{f}$ (CL)	$(0.02) \log \frac{E}{m_W}$	5%	10%
$V_L \to V_T h$ (CL+IR)	$(2 \times 10^{-3}) \left[\log \frac{E}{m_W}\right]^2$	1%	4%
$V_L \to V_L h$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.4%	1%

- Non-Abelian gauge spliting larger than fermion splitting!
- Collinear splittings larger than perturbative radiation!

J.M. Chen, TH & B. Tweedie, arXiv:1611.00788

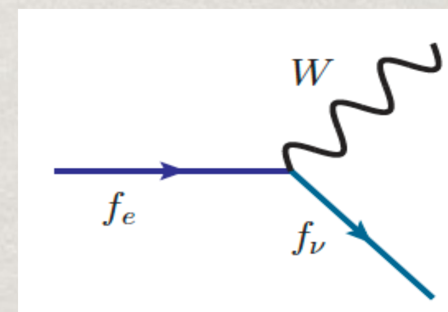
EW Evolution beyond Leading Log



Incomplete cancellation for non-inclusive process in SU(2):
SU(2) "color" (e,ν) distinguishable, unlike QCD!

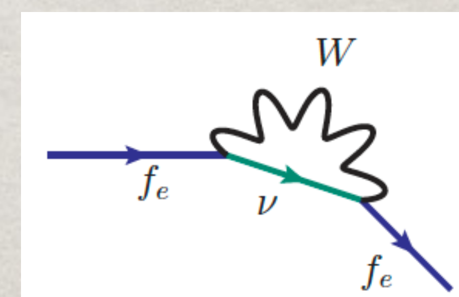
$$d\mathcal{P}_{\nu\leftarrow e} = d\mathcal{P}_{e\leftarrow\nu} \sim \frac{(T^{\pm})^2}{1-z} = \frac{(1/\sqrt{2})^2}{1-z}$$

$$d\mathcal{P}_{e\leftarrow e} = d\mathcal{P}_{\nu\leftarrow\nu} \sim \frac{(T^{3})^2}{1-z} = \frac{(1/2)^2}{1-z}$$



$$dV_{\nu\leftarrow e} = dV_{e\leftarrow\nu} = 0$$

$$dV_{e\leftarrow e} = dV_{\nu\leftarrow\nu} \sim -\int dz\, \frac{C_2(\mathbf{2})}{1-z} = -\int dz\, \frac{3/4}{1-z}$$



→ non-cancelled sub-leading $\log(Q^2/M_w^2)$
Bloch-Nordsieck theorem violation!

→ Need sufficiently inclusive processes & infrared safe-observables

Bauer et al., 1703.08562; Manohar et al., 1808.08831; TH, Ma, Xie, 2007.14300.

EW PDFs at a muon collider: "partons" dynamically generated

Initially, the "valence parton": $f_\ell(\xi, m_\ell^2) = \delta(1-\xi) + \ldots$

Leading order "sea parton": $\ell_R,\ \ell_L,\ \nu_L$ and $B, W^{\pm,3}$

Beyond leading order with DGLAP evolution:

$$\frac{\mathrm{d}f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$

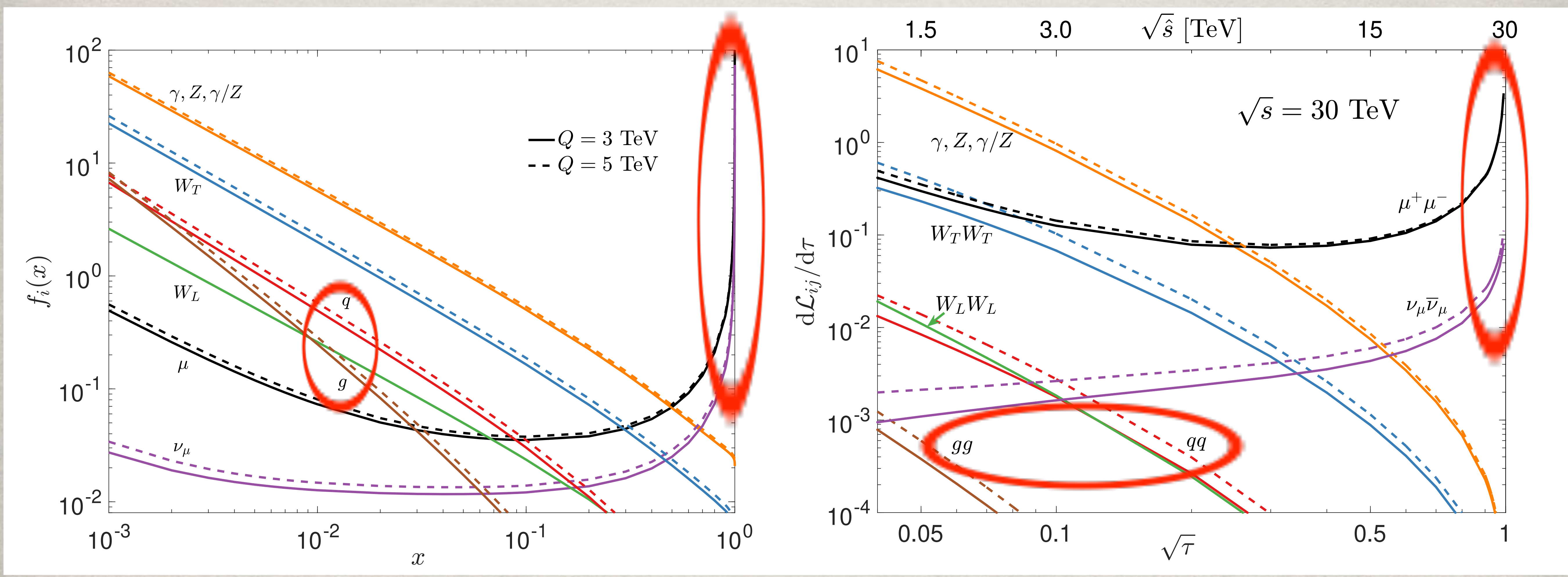
$$\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \frac{\mathrm{d}}{\mathrm{d}\log Q^2} \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

$$f_L = \sum_{i=e,\mu,\tau} (f_{\ell_i} + f_{\bar{\ell}_i}),\ f_U = \sum_{i=u,c} (f_{u_i} + f_{\bar{u}_i}),\ f_D = \sum_{i=d,s,b} (f_{d_i} + f_{\bar{d}_i})$$

Take into account two scales: $\mu_{QCD} \sim \Lambda_{QCD}$, $\mu_{EW} \sim v$

• EW PDFs at a muon collider:

"partons" dynamically generated $\frac{\mathrm{d}f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$



$\mu^\pm$: the valance. $\ell_R,\ \ell_L,\ \nu_L$ and $B, W^{\pm,3}$: LO sea.

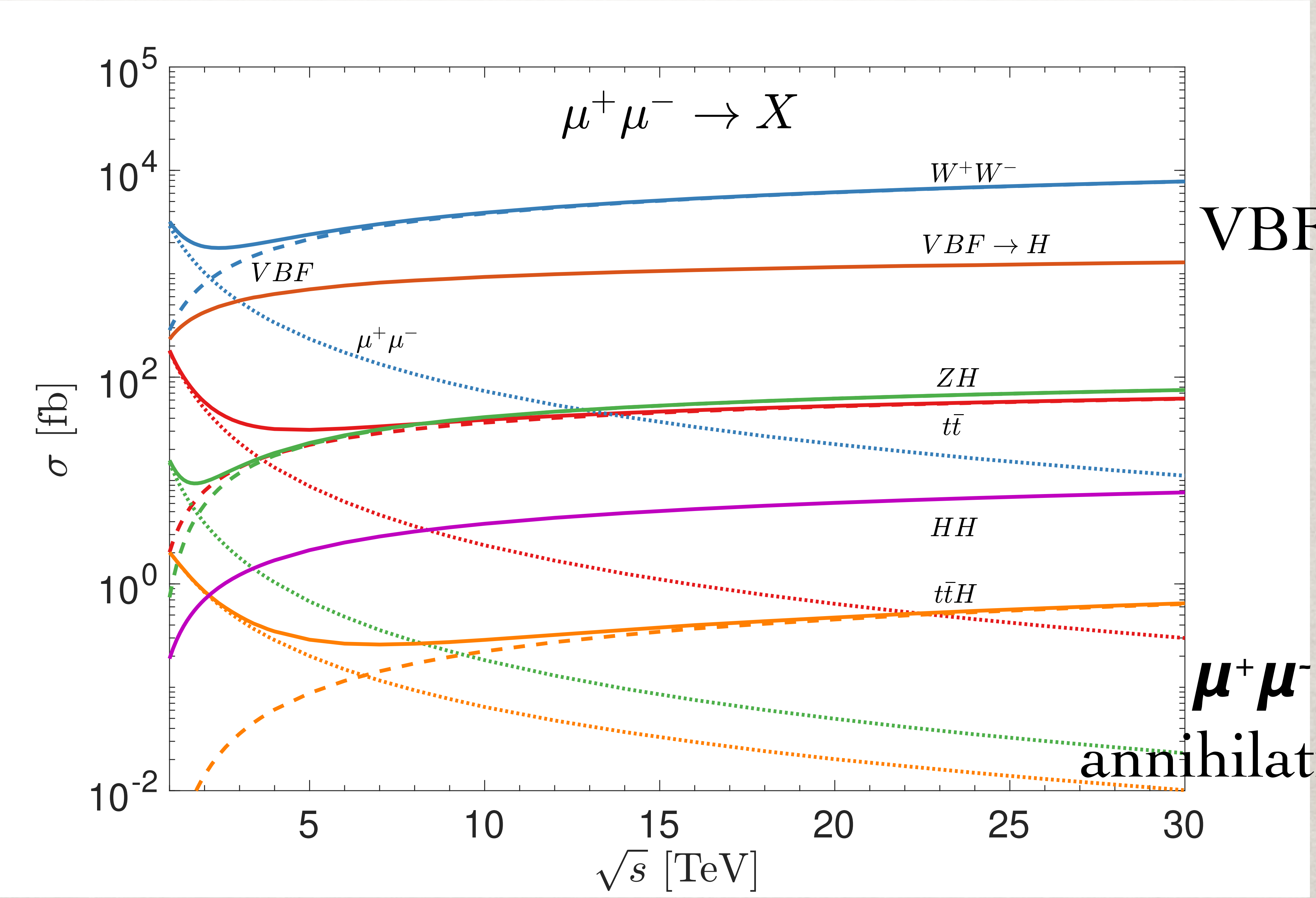
Quarks: NLO; gluons: NNLO.

TH, Yang Ma, Keping Xie, arXiv:2007.14300

- **"Semi-inclusive" processes**

Just like in hadronic collisions:

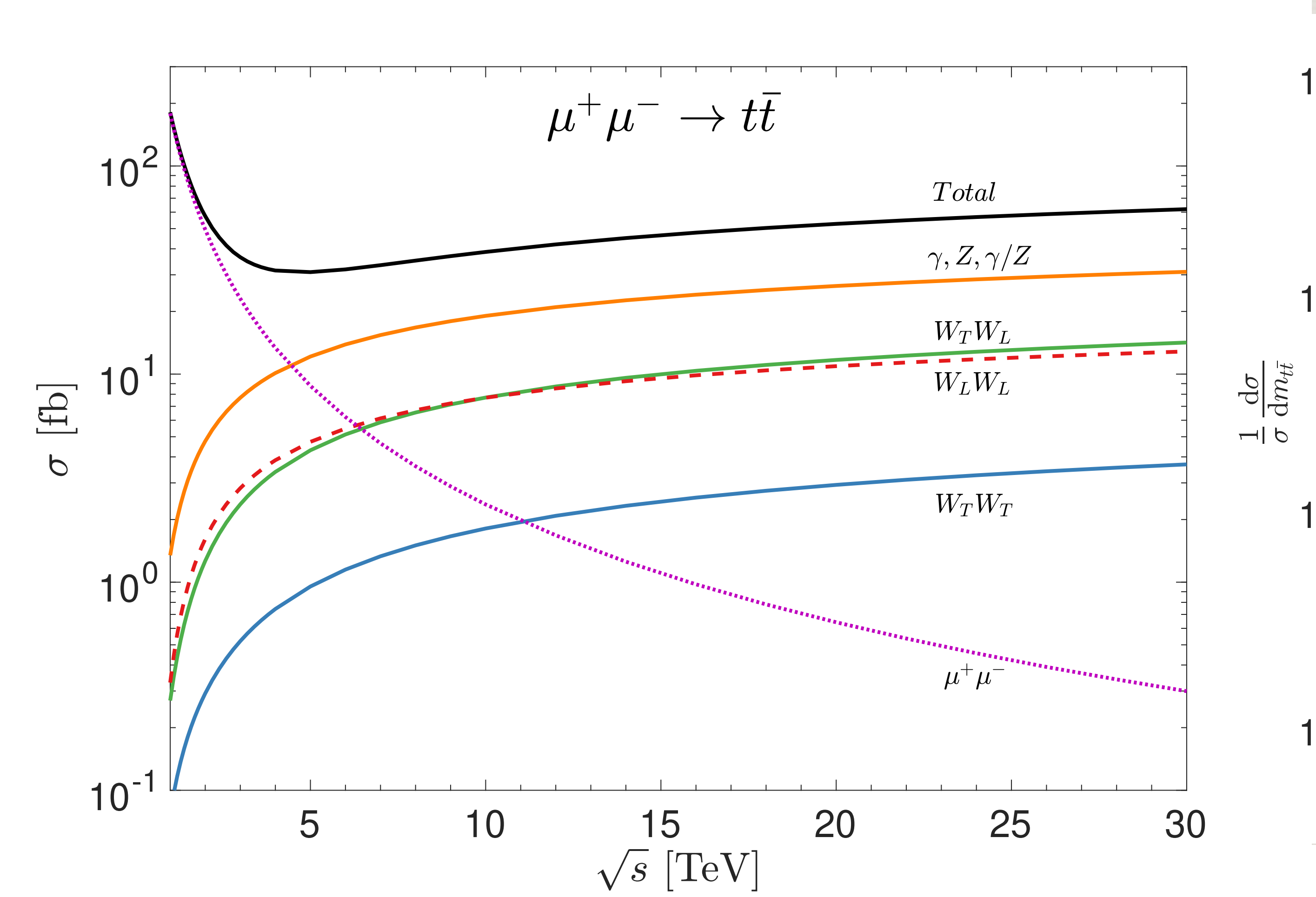
$\mu^+\mu^-$ → exclusive particles + remnants

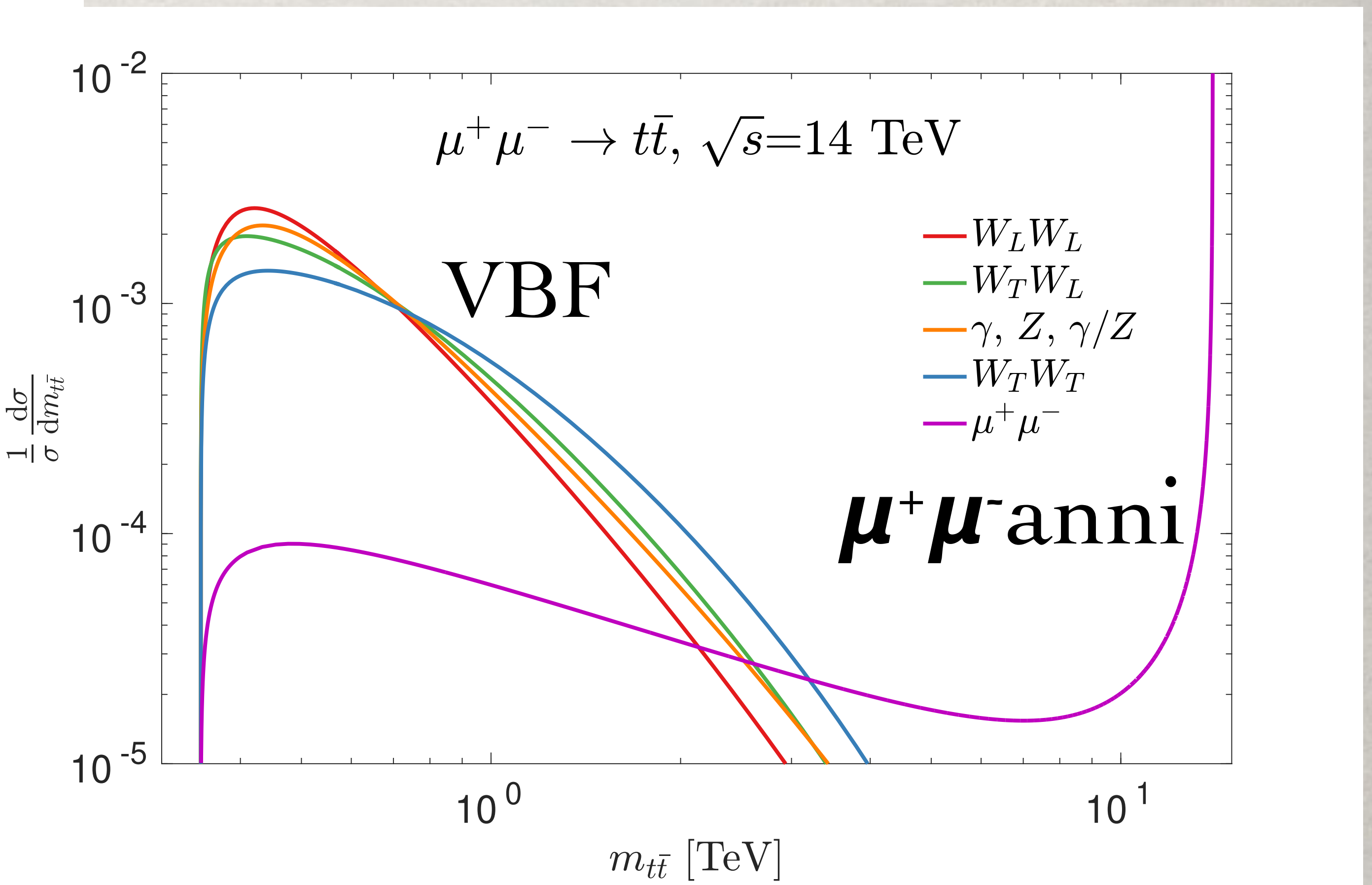


VBF

$\mu^+\mu^-$ annihilations

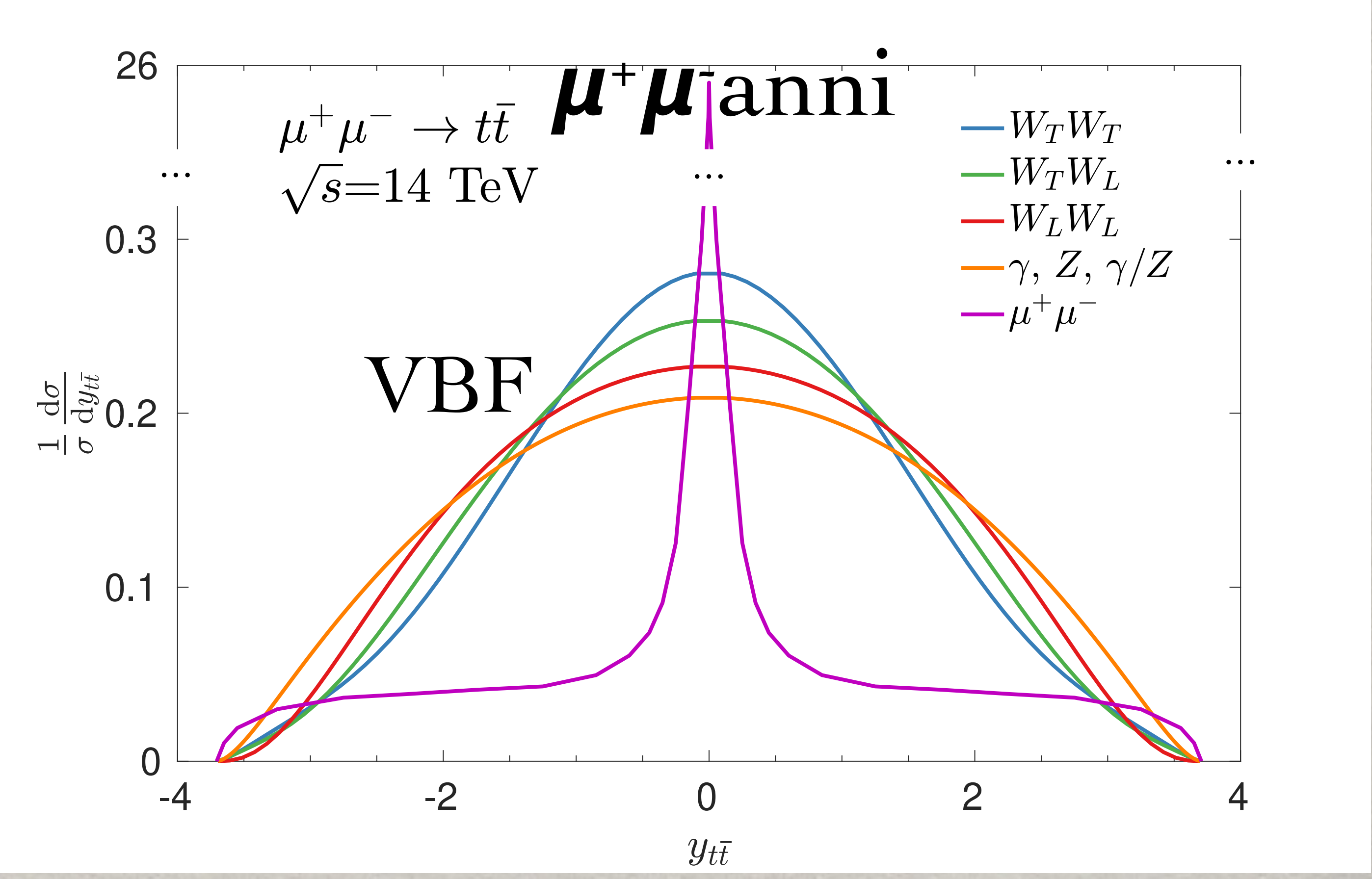
• separable sub-processes:





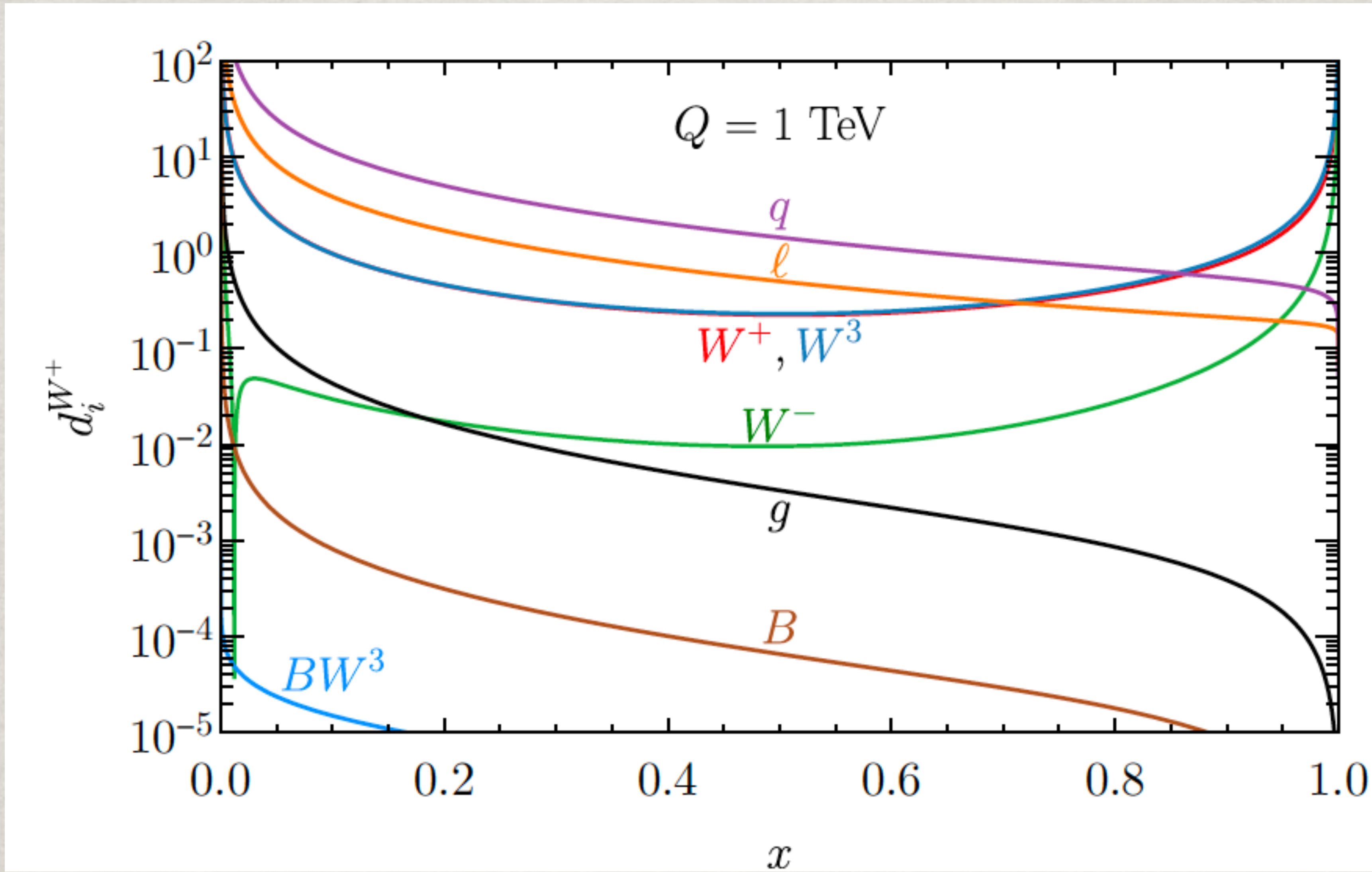
Partonic contributions

$\mu^+\mu^-$ Collider:
"Buy one, get one free"
Annihilation +VBF



"Leptonic showering"

finding a W^+ in the mother particle i (i.e., $i \to W$)



- With W/Z showers, all leptons/neutrino components exist!
- EW "jets": *e.g.*, a HE ν → an observable jet!

J.M. Chen, TH & B. Tweedie, arXiv:1611.00788;
TH, Ma, Xie, arXiv:2203.11129

Conclusions

- EW splitting/showering will become an increasingly important part at higher energies.
- It still has technical & conceptual challenges at higher energies.
- Be prepared:
  Very high-energy W, Z, h, t may serve as tools for the next discovery !

High-Energy Muon Collider

Collider benchmark points:

- The Higgs factory:

Barger, Berger, Gunion, Han
PRL *75*, 1995; PR 1997

$E_{cm} = m_H$; $L \sim 1\ \text{fb}^{-1}/\text{yr}$

$\Delta E_{cm} \sim 5$ MeV

Parameter	**Units**	**Higgs**
CoM Energy	TeV	0.126
Avg. Luminosity	$10^{34}\text{cm}^{-2}\text{s}^{-1}$	0.008
Beam Energy Spread	%	0.004
Higgs Production/10^7 sec		13'500
Circumference	km	0.3

- Multi-TeV colliders:

Lumi-scaling scheme: **σ** $L \sim$ const.

$$L \gtrsim \frac{5\ \text{years}}{\text{time}} \left(\frac{\sqrt{s_\mu}}{10\ \text{TeV}} \right)^2 2 \cdot 10^{35}\text{cm}^{-2}\text{s}^{-1}$$

1 ab^{-1} /yr

Current choices: 3, 10, 14(?) TeV @ FNAL

European Strategy, arXiv:1910.11775; arXiv:1901.06150; arXiv:2007.15684.