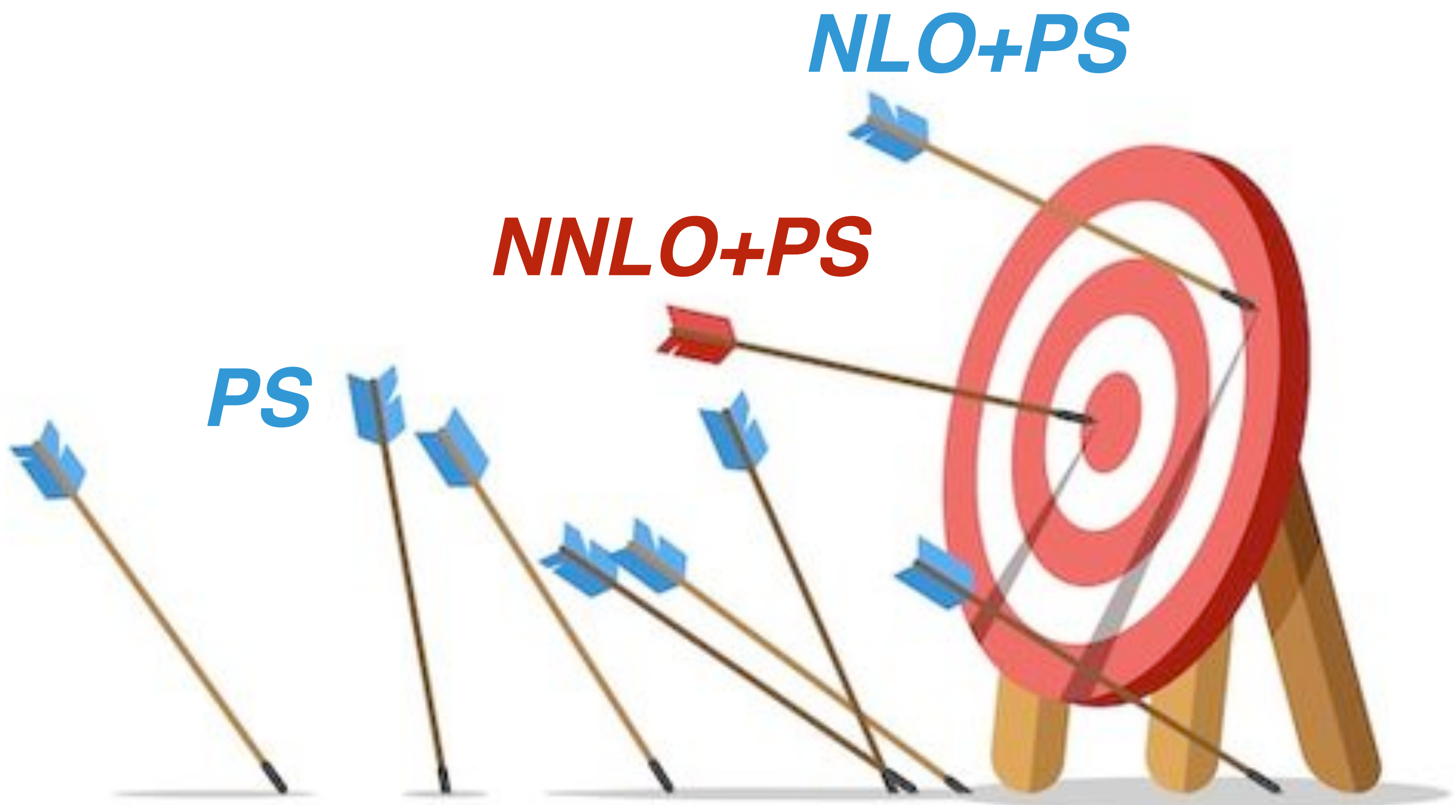
Pushing the NNLO+PS frontier towards new classes of processes with MiNNLO

Marius Wiesemann

Max-Planck-Institut für Physik



Loopfest XXII

Dallas (USA), May 20-22, 2024

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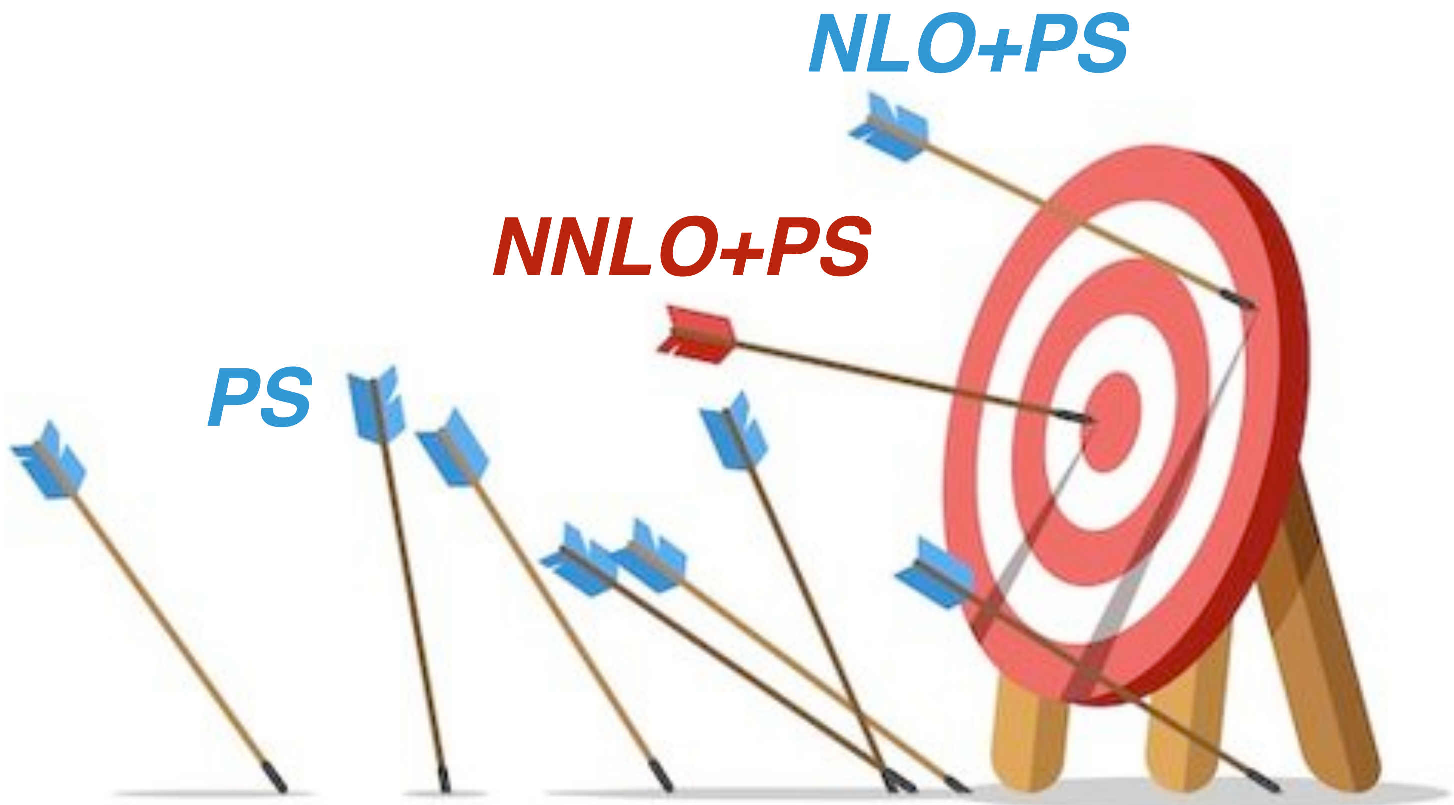
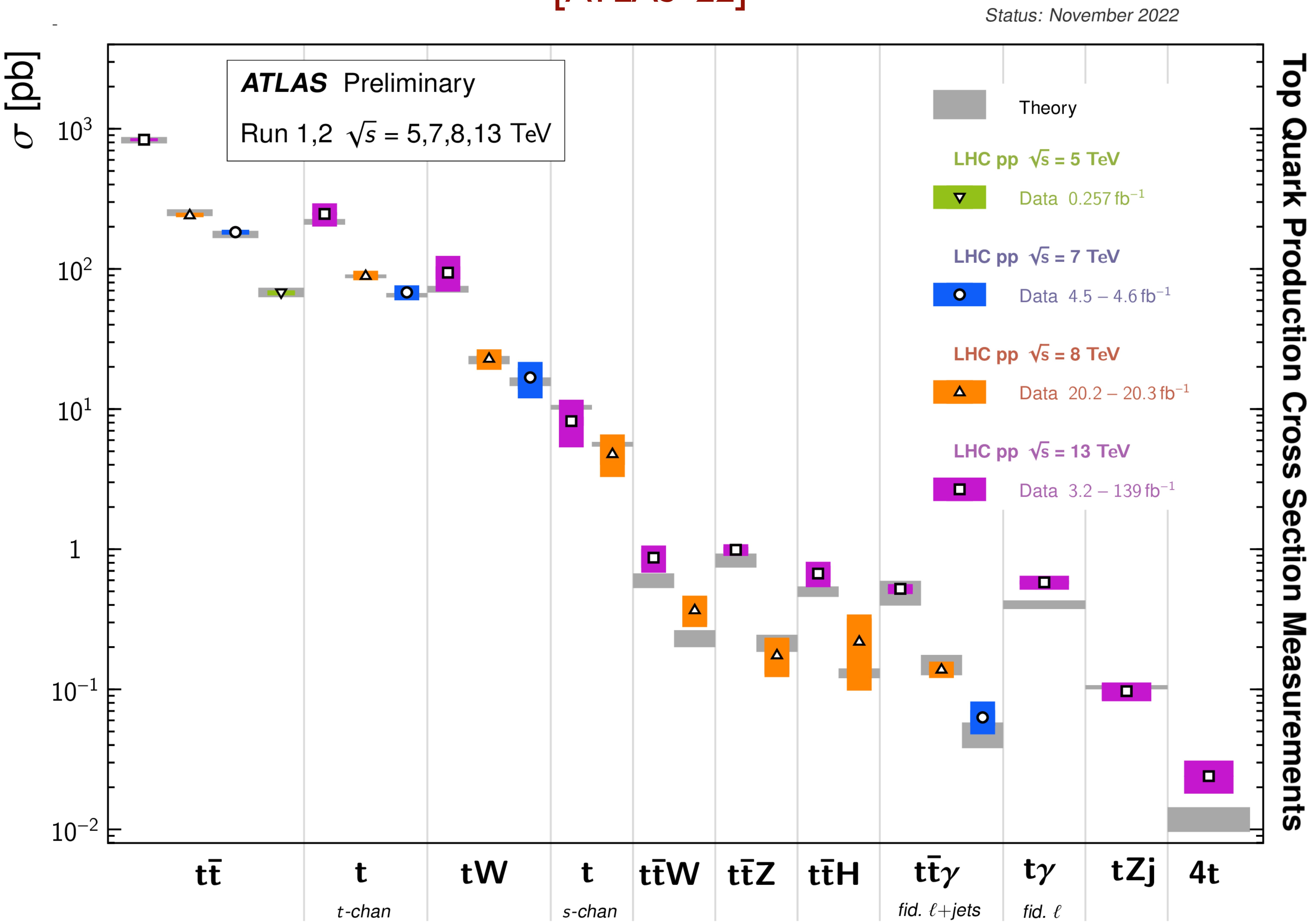


Loopfest XXII

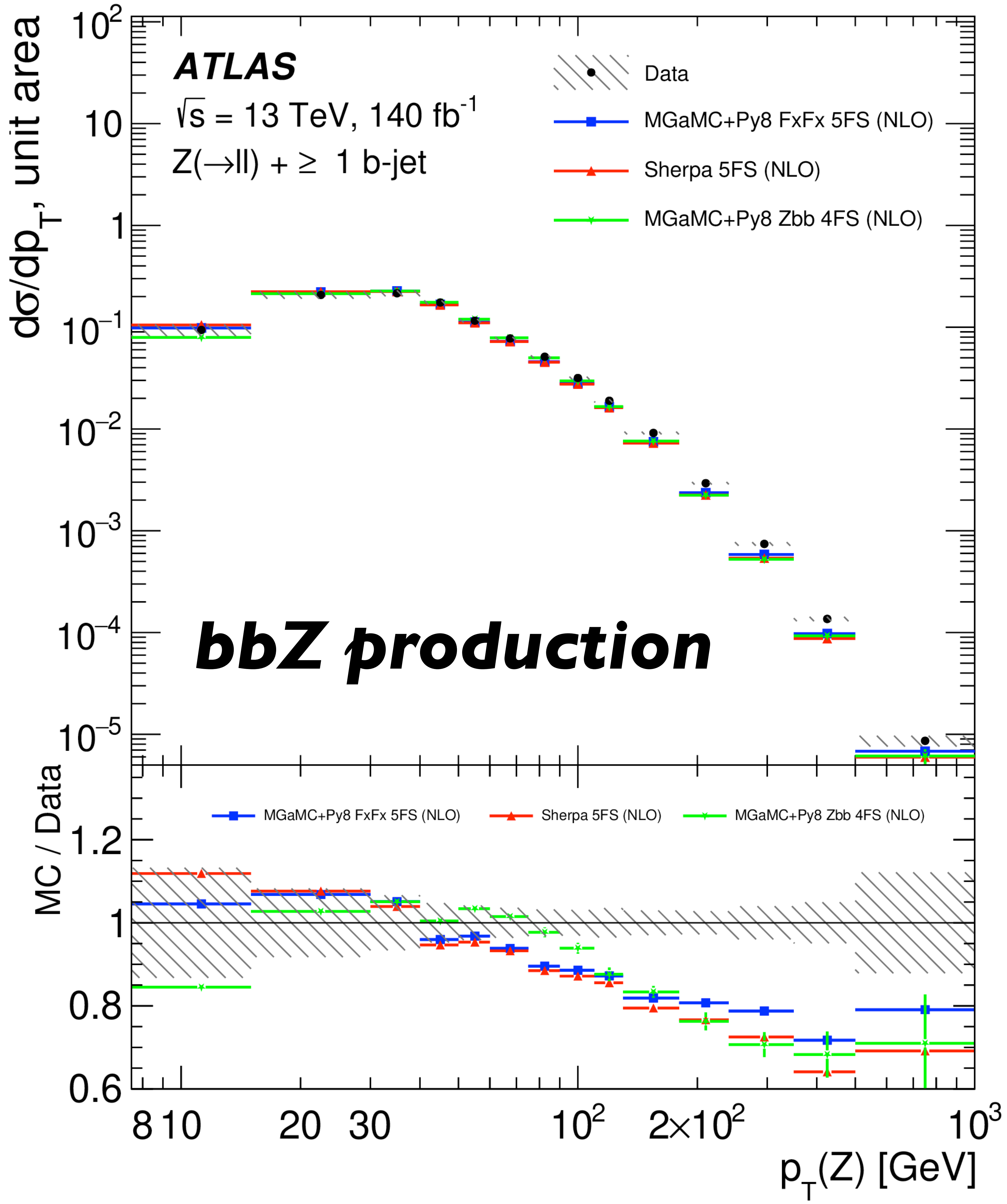
Dallas (USA), May 20-22, 2024

Heavy quark (+colour singlet) production

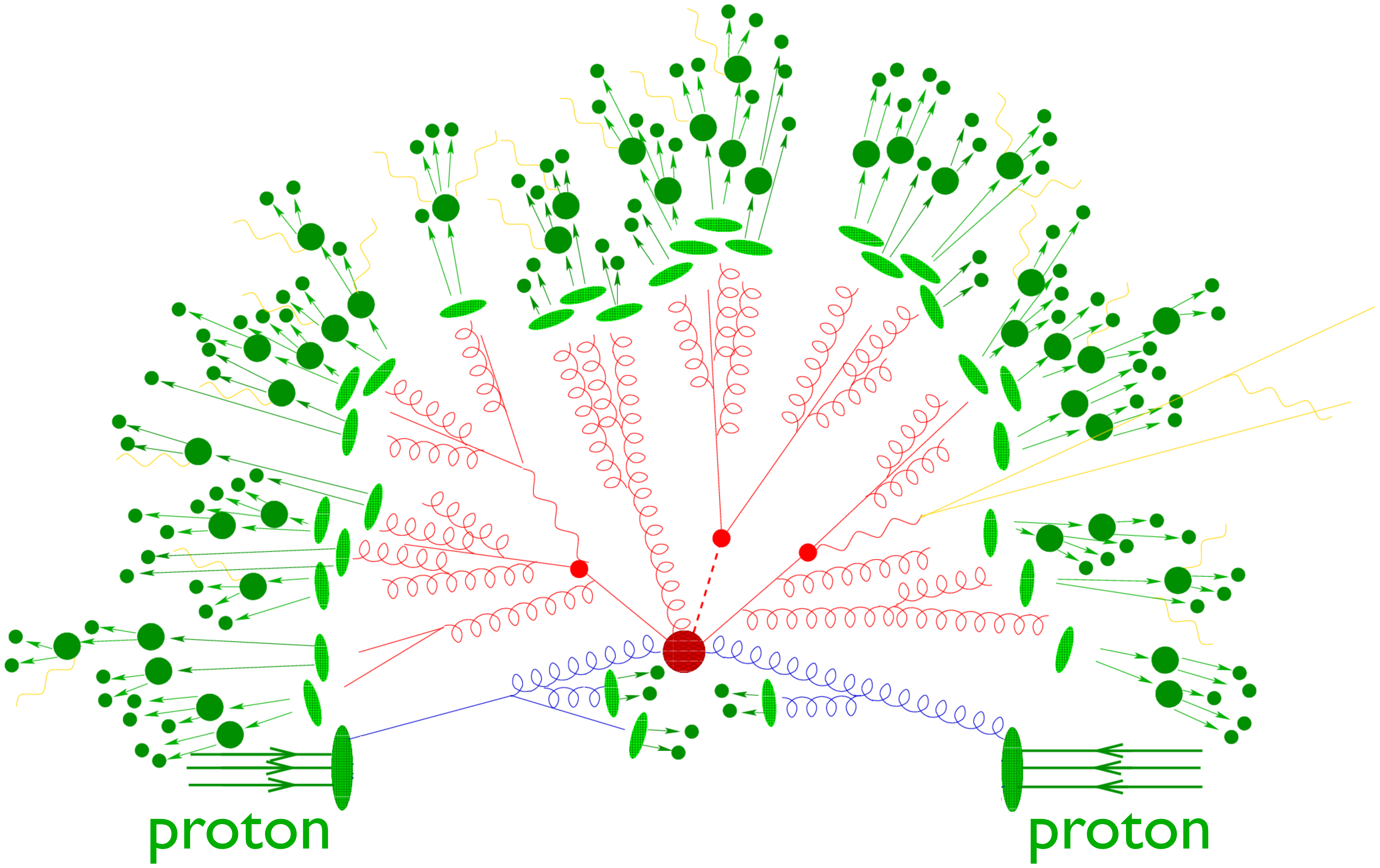
[ATLAS '22]



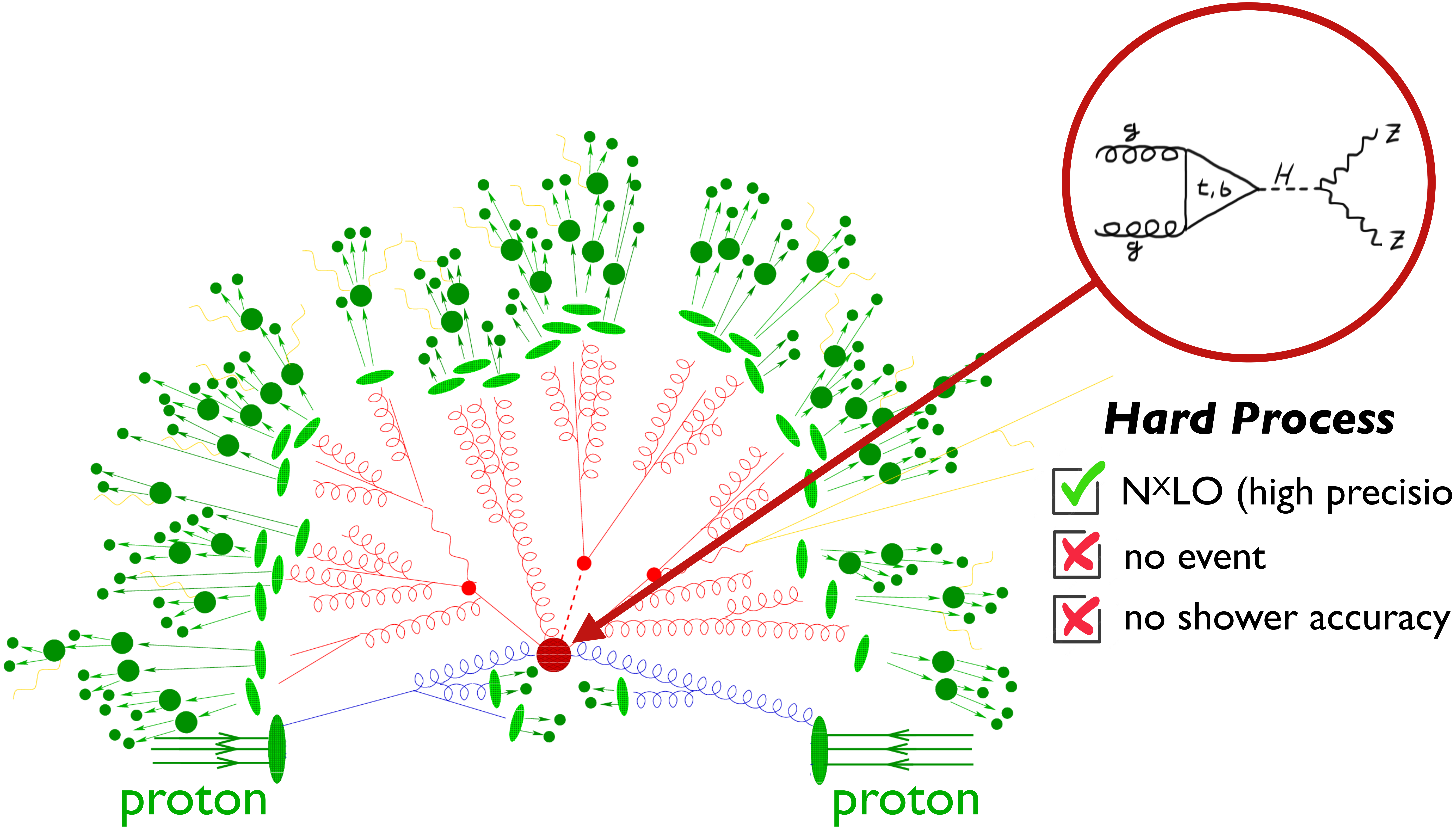
[ATLAS 2403.15093]



LHC event



LHC event



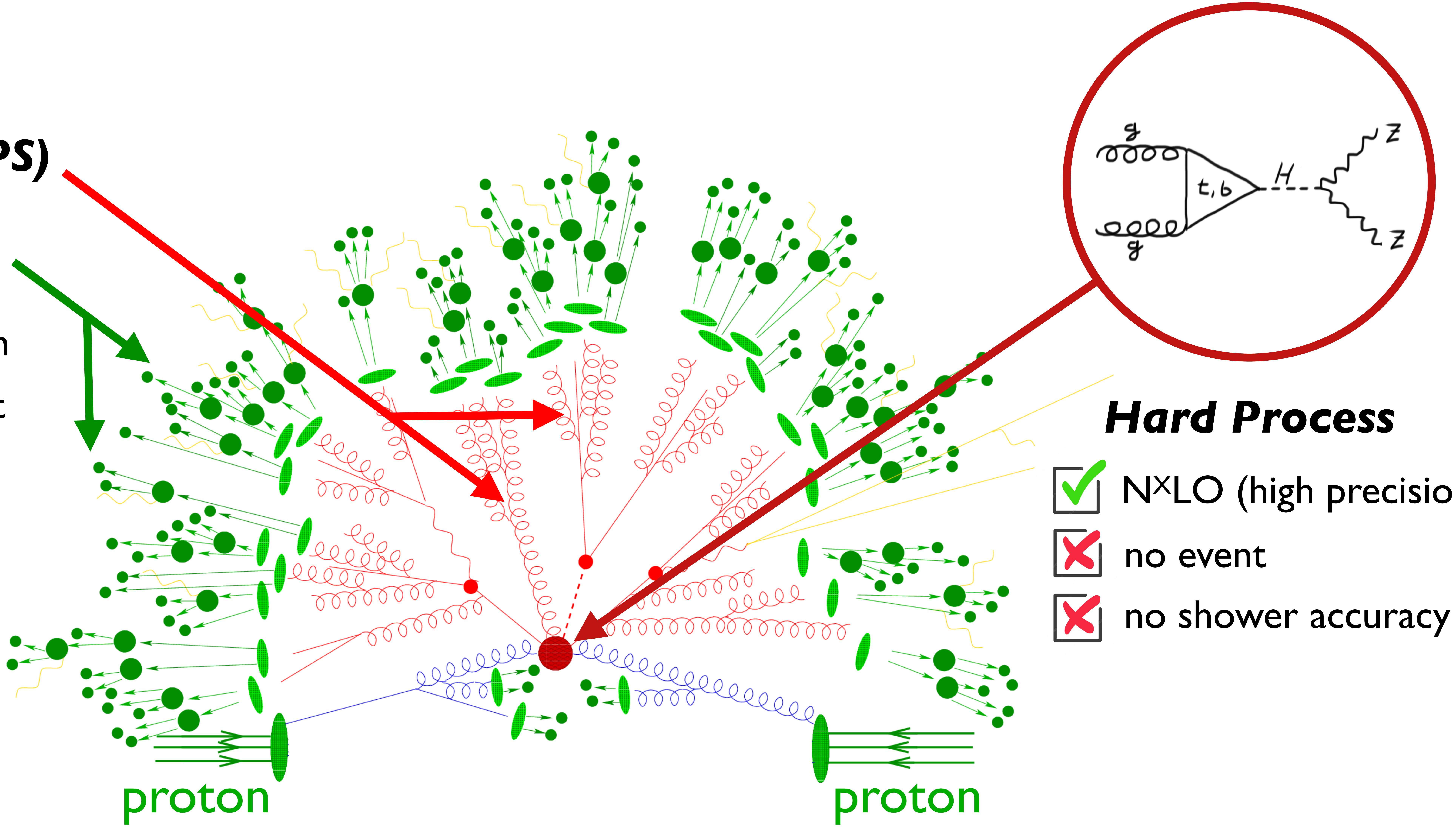
Hard Process

- ☑ N^{x}LO (high precision)
- ☒ no event
- ☒ no shower accuracy

LHC event

Parton Shower (PS)
+
Hadronization

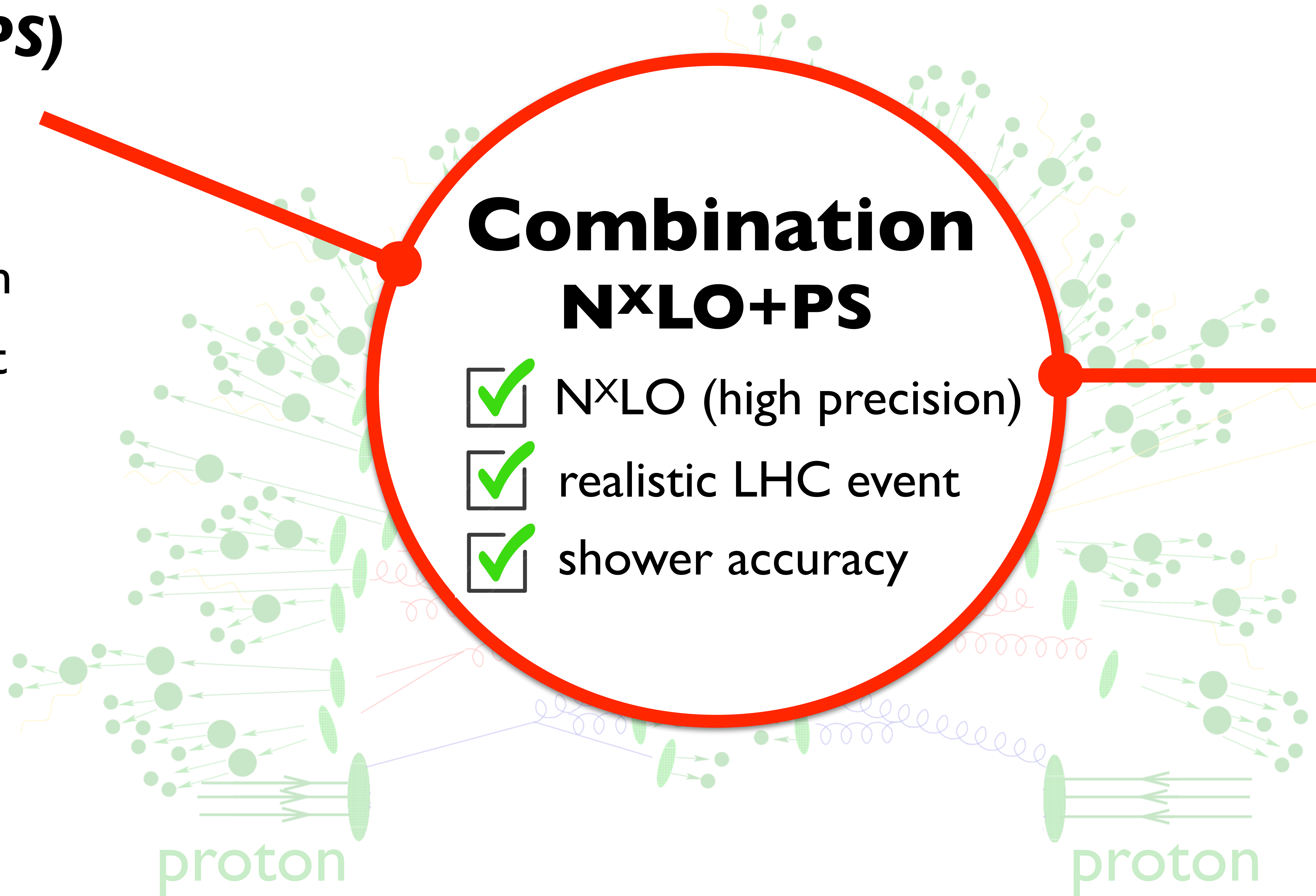
- ☒ no N^{X}LO precision
- ☑ realistic LHC event
- ☑ shower accuracy (low precision)



Hard Process

- ☑ N^{X}LO (high precision)
- ☒ no event
- ☒ no shower accuracy

LHC event



Parton Shower (PS)
+
Hadronization

- ☒ no N^{x}LO precision
- ☑ realistic LHC event
- ☑ shower accuracy (low precision)

Combination
N^{x}LO+PS

- ☑ N^{x}LO (high precision)
- ☑ realistic LHC event
- ☑ shower accuracy

Hard Process

- ☑ N^{x}LO (high precision)
- ☒ no event
- ☒ no shower accuracy

NNLO+PS: What do we want to achieve?

- NNLO accuracy for observables inclusive on radiation. $[d\sigma/dy_F]$
- NLO(LO) accuracy for $F+1(2)$ jet observables (in the hard region). $[d\sigma/dp_{T,j_1}]$
  - appropriate scale choice for each kinematics regime
- resummation from the Parton Shower (PS) $[\sigma(p_{T,j} < p_{T,\mathrm{veto}})]$
- preserve the PS accuracy (leading log - LL)

- possibly, no merging scale required.

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

NNLO+PS methods

NNLOPS: MiNLO+reweighting

[Hamilton, Nason, Oleari, Zanderighi '12, + Re '13], [Karlberg, Re, Zanderighi '14]

- LL accuracy (+ simple NLL terms) from PS
- no new unphysical scale (i.e. physically sound)
- numerically very intensive
- applied beyond 2→1 processes

Geneva

[Alioli, Bauer, Berggren, Tackmann, Walsh '15 + Zuberi '13]

- LL accuracy from PS (at most! no NNLL nonesense!)
- slicing cutoff (missing power corrections)
- numerical cancellations in slicing parameter
- applied beyond 2→1 processes

MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- LL accuracy (+ simple NLL terms) from PS
- no new unphysical scale (i.e. physically sound)
- numerically efficient
- applied beyond 2→1 and even beyond colour singlet

UNNLOPS

[Höche, Prestel '14 '15]

- extension of UNLOPS merging of event samples
- two-loop corrections entirely in 0-jet bin
- only applied to 2→1 processes

there was also some recent progress on NNLO+PS for sector showers [Campbell, Höche, Li, Preuss, Slands '21]

NNLO+PS timeline

MiNNLO$_{PS}$

$t\bar{t}$ $b\bar{b}$ $b\bar{b} \to H$

H $Z\gamma$ $\gamma\gamma$

Z W WW WZ $b\bar{b}Z$

ZZ

UNNLOPS

H Z

Geneva

Z

ZH $H \to b\bar{b}$ HH

WH $H \to gg$

H

$\gamma\gamma$ ZZ $W\gamma$

NNLOPS

H Z W WH ZH WW $H \to b\bar{b}$

NNLO+PS timeline

see also Aparna's talk yesterday

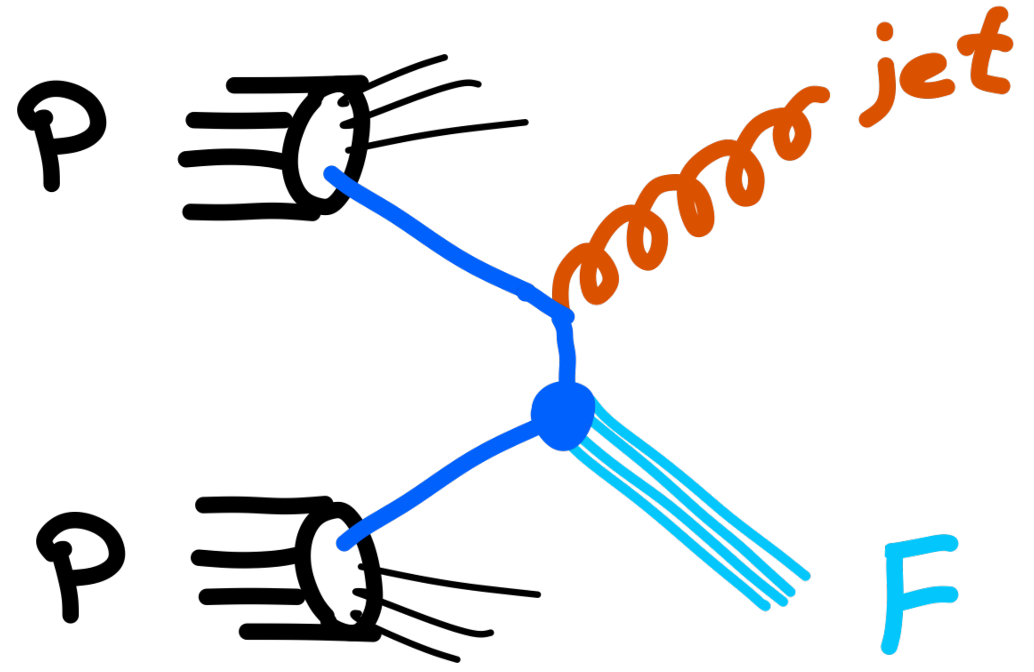
today's focus:

MiNNLO$_{PS}$: main idea

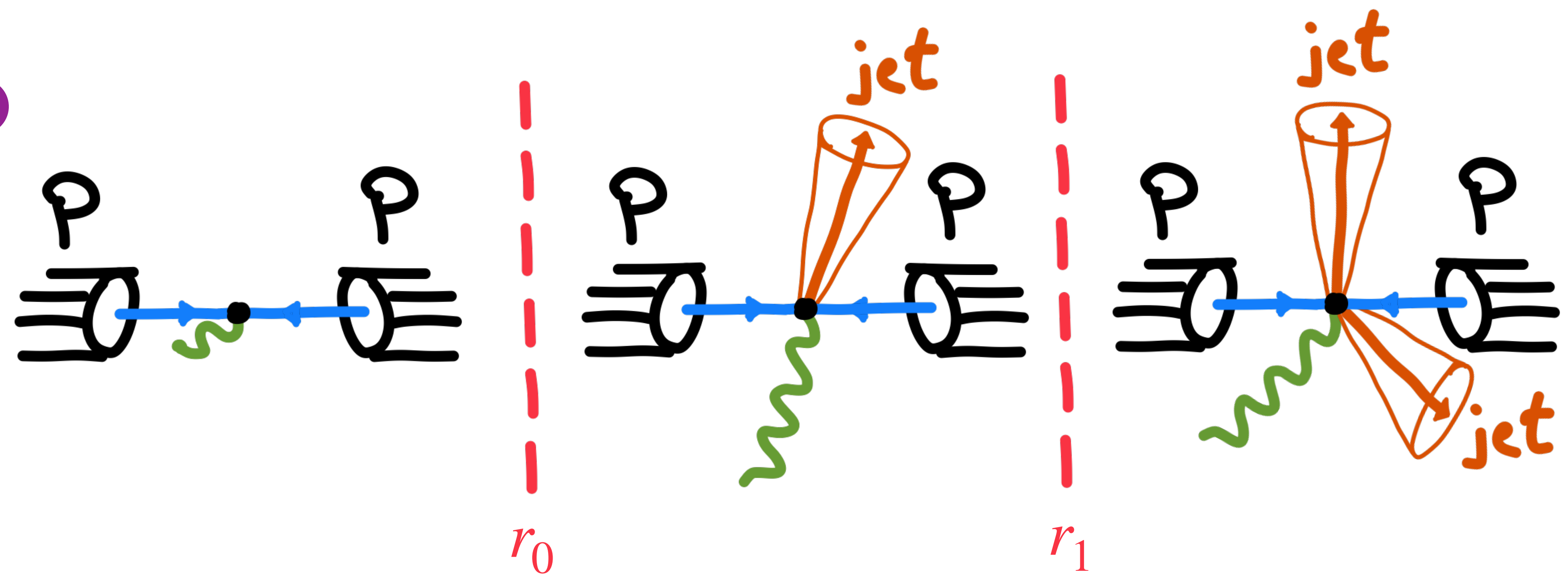
[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

N^{X}LO+Parton Shower (PS) for pp → F

N^{X-1}LO+Parton Shower (PS)
for pp → F + jet



all-order structure in
jet-resolution variable r_N



MiNNLO$_{PS}$: main idea

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- ✦ starting equation: $$\frac{\mathrm{d}\sigma_F^{\mathrm{res}}}{\mathrm{d}p_T\,\mathrm{d}\Phi_{\mathrm{B}}} = \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\mathscr{L}\right\} = e^{-S}\underbrace{\left\{S'\mathscr{L} + \mathscr{L}'\right\}}_{\equiv D}$$ $\mathscr{L} \sim H\,(C \otimes f)\,(C \otimes f)$ (symbolically)

- ✦ combine with $F+\mathrm{jet}$ fixed order $\mathrm{d}\sigma_{FJ}$:

$$\mathrm{d}\sigma^F = \mathrm{d}\sigma_F^{\mathrm{res}} + [\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}} - [\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}} = e^{-S}\left\{ D + \frac{[\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}}}{\underbrace{[e^{-S}]_{\mathrm{f.o.}}}_{1-S^{(1)}\ldots}} \underbrace{- \frac{[\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}}}{[e^{-S}]_{\mathrm{f.o.}}}}_{-D^{(1)}-D^{(2)}\ldots} \right\}$$

MiNNLO$_{\text{PS}}$: main idea

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- ✦ starting equation:

$$\frac{\mathrm{d}\sigma_F^{\mathrm{res}}}{\mathrm{d}p_T\,\mathrm{d}\Phi_{\mathrm{B}}} = \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\mathscr{L}\right\} = e^{-S}\underbrace{\left\{S'\mathscr{L}+\mathscr{L}'\right\}}_{\equiv D}$$

$$\mathscr{L} \sim H\,(C\otimes f)\,(C\otimes f)$$

(symbolically)

- ✦ combine with F + jet fixed order $\mathrm{d}\sigma_{FJ}$:

$$\mathrm{d}\sigma^F = \mathrm{d}\sigma_F^{\mathrm{res}} + [\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}} - [\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}} = e^{-S}\left\{D + \frac{[\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}}}{\underbrace{[e^{-S}]_{\mathrm{f.o.}}}_{1-S^{(1)}\ldots}} \underbrace{-\frac{[\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}}}{[e^{-S}]_{\mathrm{f.o.}}}}_{-D^{(1)}-D^{(2)}\ldots}\right\}$$

- ✦ expanded up to $\alpha_s^3(p_T)$ we have: (resummation scheme: $\mu_{\mathrm{R}} = \mu_{\mathrm{F}} \sim p_T$)

(very symbolic/simplified)

$$\mathrm{d}\sigma_F^{\mathrm{MiNNLO}} \sim e^{-S}\left\{\underbrace{\mathrm{d}\sigma_{FJ}^{(1)}\big(1+S^{(1)}\big)}_{\sim\alpha_s(p_T)} + \underbrace{\mathrm{d}\sigma_{FJ}^{(2)}}_{\sim\alpha_s^2(p_T)} + \underbrace{\big(D-D^{(1)}-D^{(2)}\big)}_{\geq\alpha_s^3(p_T)\ \longrightarrow\ D^{(3)}+\mathcal{O}(\alpha_s^4)} + \mathrm{regular}\right\}$$

MiNNLO$_{\text{PS}}$: main idea

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- ✦ starting equation:

$$\frac{\mathrm{d}\sigma_F^{\mathrm{res}}}{\mathrm{d}p_T\,\mathrm{d}\Phi_{\mathrm{B}}} = \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\mathscr{L}\right\} = e^{-S}\underbrace{\left\{S'\mathscr{L} + \mathscr{L}'\right\}}_{\equiv D}$$

$$\mathscr{L} \sim H\,(C \otimes f)\,(C \otimes f)$$

(symbolically)

- ✦ combine with F + jet fixed order $\mathrm{d}\sigma_{FJ}$:

$$\mathrm{d}\sigma^F = \mathrm{d}\sigma_F^{\mathrm{res}} + [\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}} - [\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}} = e^{-S}\left\{ D + \frac{[\mathrm{d}\sigma_{FJ}]_{\mathrm{f.o.}}}{\underbrace{[e^{-S}]_{\mathrm{f.o.}}}_{1-S^{(1)}\dots}} \underbrace{- \frac{[\mathrm{d}\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}}}{[e^{-S}]_{\mathrm{f.o.}}}}_{-D^{(1)}-D^{(2)}\dots} \right\}$$

- ✦ expanded up to $\alpha_s^3(p_T)$ we have: (resummation scheme: $\mu_{\mathrm{R}} = \mu_{\mathrm{F}} \sim p_T$)

MiNLO

$$\mathrm{d}\sigma_F^{\mathrm{MiNNLO}} \sim \boxed{e^{-S}\Big\{\underbrace{\mathrm{d}\sigma_{FJ}^{(1)}\big(1}_{\sim\alpha_s(p_T)} + \underbrace{S^{(1)}\big) + \mathrm{d}\sigma_{FJ}^{(2)}}_{\sim\alpha_s^2(p_T)}} + \underbrace{\left(D - D^{(1)} - D^{(2)}\right)}_{\sim\alpha_s^3(p_T)} + \text{regular}\Big\}$$

MiNNLO$_{PS}$: main idea

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- starting equation:

$$\frac{d\sigma_F^{\rm res}}{dp_T\, d\Phi_{\rm B}} = \frac{d}{dp_T}\left\{e^{-S}\mathscr{L}\right\} = e^{-S}\underbrace{\left\{S'\mathscr{L} + \mathscr{L}'\right\}}_{\equiv D} \qquad \mathscr{L} \sim H\,(C\otimes f)\,(C\otimes f)$$

(symbolically)

- combine with F + jet fixed order $d\sigma_{FJ}$:

$$d\sigma^F = d\sigma_F^{\rm res} + [d\sigma_{FJ}]_{\rm f.o.} - [d\sigma_F^{\rm res}]_{\rm f.o.} = e^{-S}\left\{D + \frac{[d\sigma_{FJ}]_{\rm f.o.}}{\underbrace{[e^{-S}]_{\rm f.o.}}_{1-S^{(1)}\ldots}} \underbrace{- \frac{[d\sigma_F^{\rm res}]_{\rm f.o.}}{[e^{-S}]_{\rm f.o.}}}_{-D^{(1)}-D^{(2)}\ldots}\right\}$$

- expanded up to $\alpha_s^3(p_T)$ we have: (resummation scheme: $\mu_{\rm R} = \mu_{\rm F} \sim p_T$)

MiNLO **NNLO correction**

$$d\sigma_F^{\rm MiNNLO} \sim \boxed{e^{-S}\Big\{\underbrace{d\sigma_{FJ}^{(1)}}_{\sim\alpha_s(p_T)}\big(1+\underbrace{S^{(1)}\big) + d\sigma_{FJ}^{(2)}}_{\sim\alpha_s^2(p_T)}} + \boxed{\underbrace{\left(D - D^{(1)} - D^{(2)}\right)}_{\sim\alpha_s^3(p_T)}} + \boxed{\text{regular}}\Big\}$$

beyond accuracy

MiNNLO$_{PS}$: master formula

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

✦ apply idea to POWHEG FJ calculation

$$d\sigma_{FJ} = d\Phi_{FJ}\,\tilde{B}^{FJ} \times \left\{ \Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \int d\Phi_{\mathrm{rad}} \Delta_{\mathrm{pwg}}(p_{T,\mathrm{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}$$

$$\tilde{B}^{FJ} \sim \left\{ d\sigma_{FJ}^{(1)} + d\sigma_{FJ}^{(2)} \right\}$$

MiNNLO$_{\mathit{PS}}$: master formula

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- ✦ NNLO+PS by turning POWHEG weight ($\tilde{B}$ function) NNLO accurate:

$$\mathrm{d}\sigma_F^{\mathrm{MiNNLO_{PS}}} = \mathrm{d}\Phi_{FJ}\,\tilde{B}^{\,\mathrm{MiNNLO_{PS}}} \times \left\{ \Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \int \mathrm{d}\Phi_{\mathrm{rad}} \Delta_{\mathrm{pwg}}(p_{T,\mathrm{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}$$

$$\tilde{B}^{\,\mathrm{MiNNLO_{PS}}} \sim e^{-S} \left\{ \mathrm{d}\sigma_{FJ}^{(1)} \left(1 + S^{(1)}\right) + \mathrm{d}\sigma_{FJ}^{(2)} + \left(D - D^{(1)} - D^{(2)}\right) \times F^{\mathrm{corr}} \right\}$$

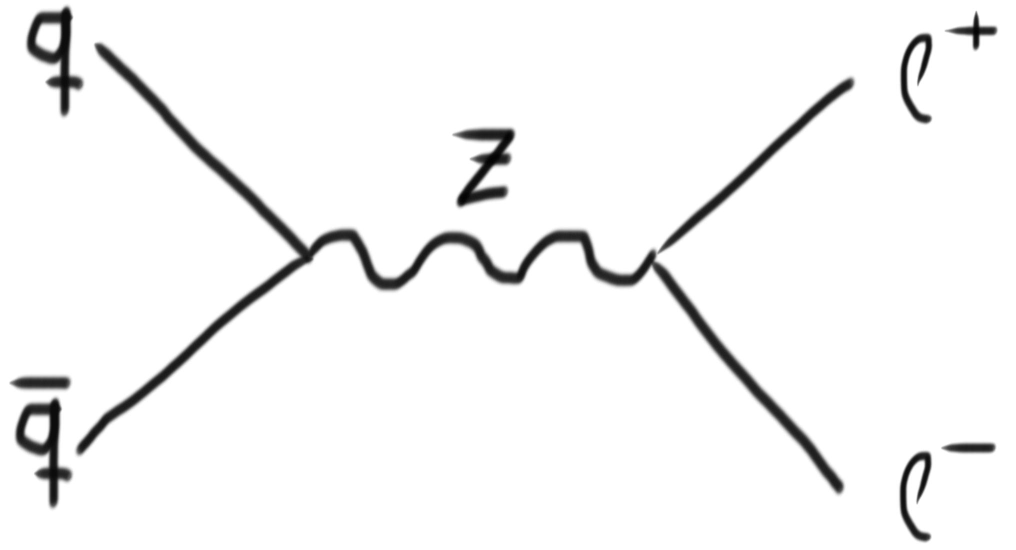
➙ *spreads NNLO corrections in the $F+\text{jet}$ phase space*

MiNNLO$_{\text{PS}}$: towards jet production

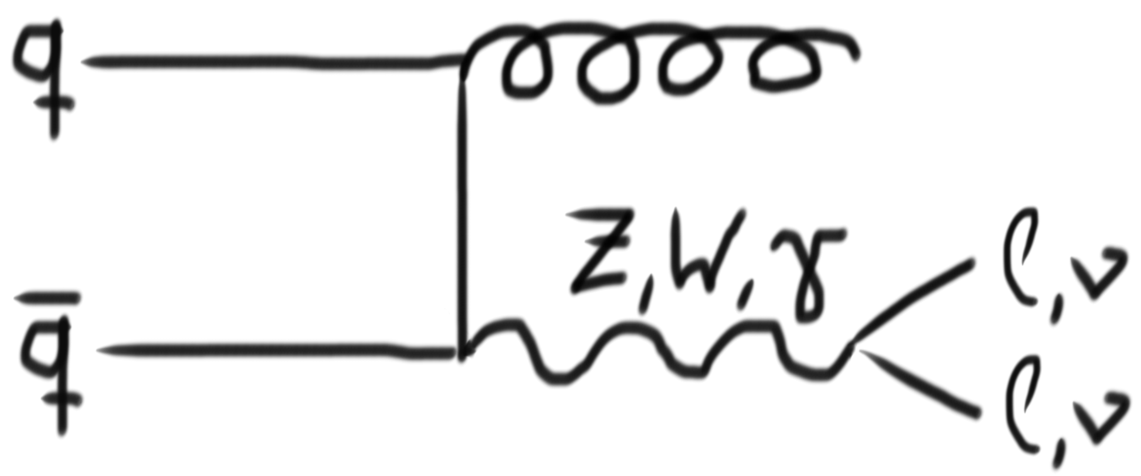
[Ebert, Rottoli, MW, Zanderighi, Zanoli '23]

✦ MiNNLO$_{\text{PS}}$ viable for any N-jet resolution variable (in principle), e.g. N-jettiness: $\boxed{p_T \to \tau_N}$

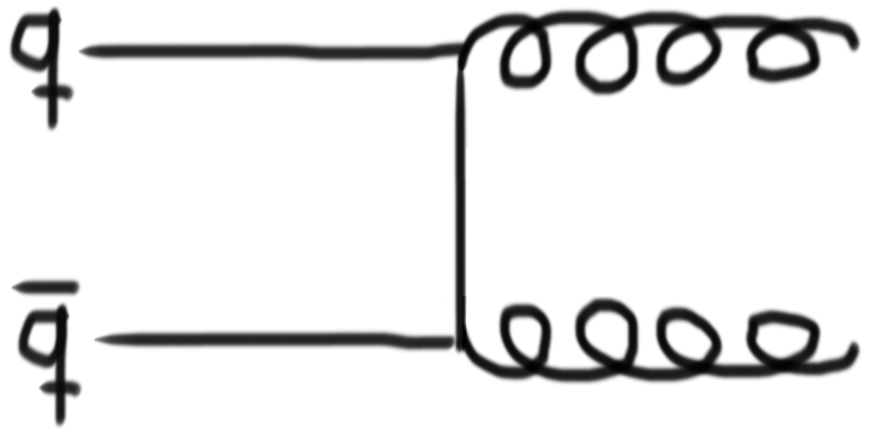
$$\tilde{B}^{\,\text{MiNNLO}_{\text{PS}}} \sim e^{-S(\tau_N)} \left\{ \mathrm{d}\sigma^{(1)}_{FJ}\big(1 + S^{(1)}(\tau_N)\big) + \mathrm{d}\sigma^{(2)}_{FJ} + \big(D(\tau_N) - D^{(1)}(\tau_N) - D^{(2)(\tau_N)}\big) \times F^{\text{corr}} \right\}$$



τ_0



τ_1



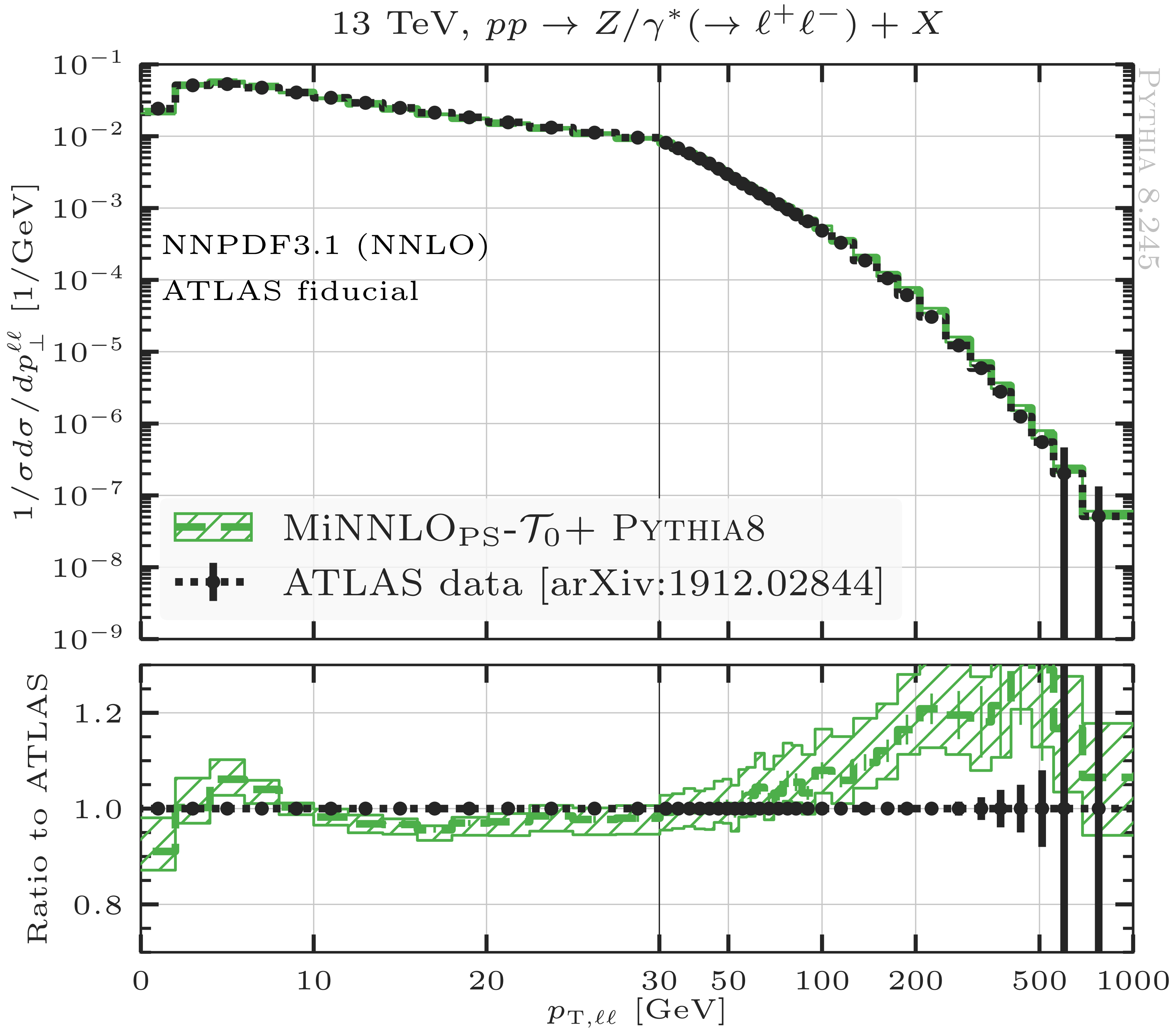
τ_2

see also Matthew's talk for recent developments in Geneva

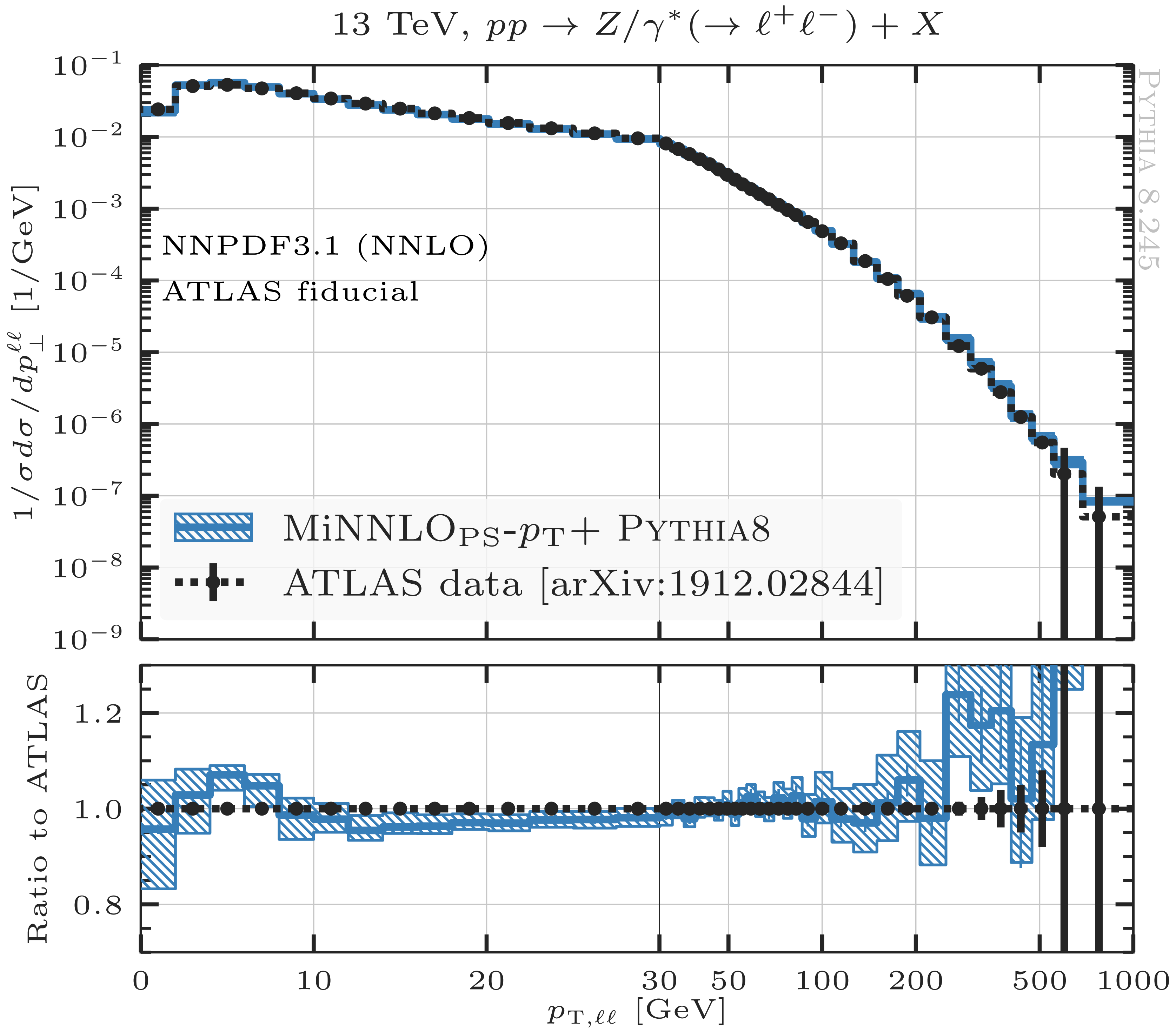
[Alioli et al. '23]

MiNNLO$_{PS}$: towards jet production

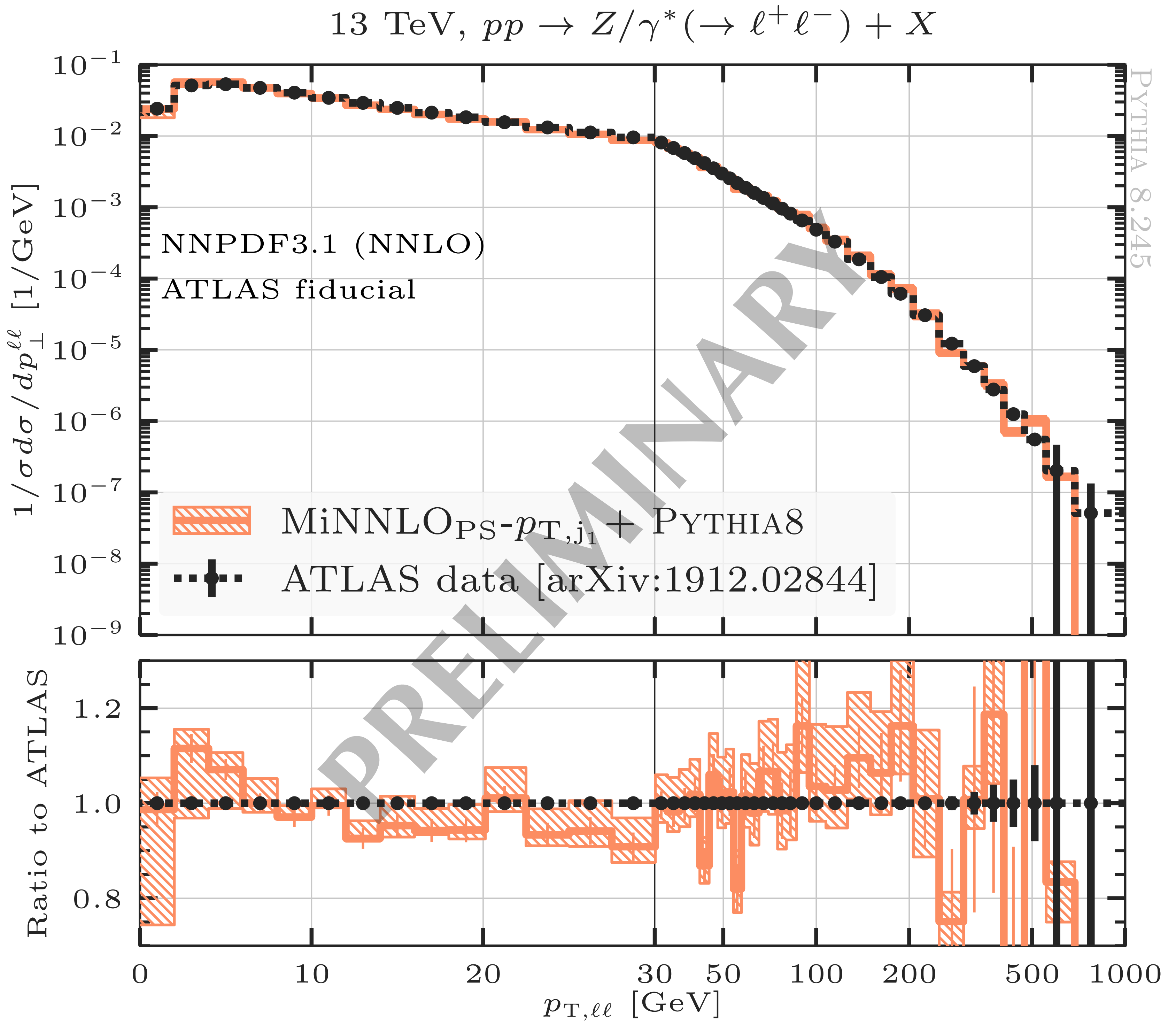
[Ebert, Rottoli, MW, Zanderighi, Zanoli '23]



τ_0



p_T



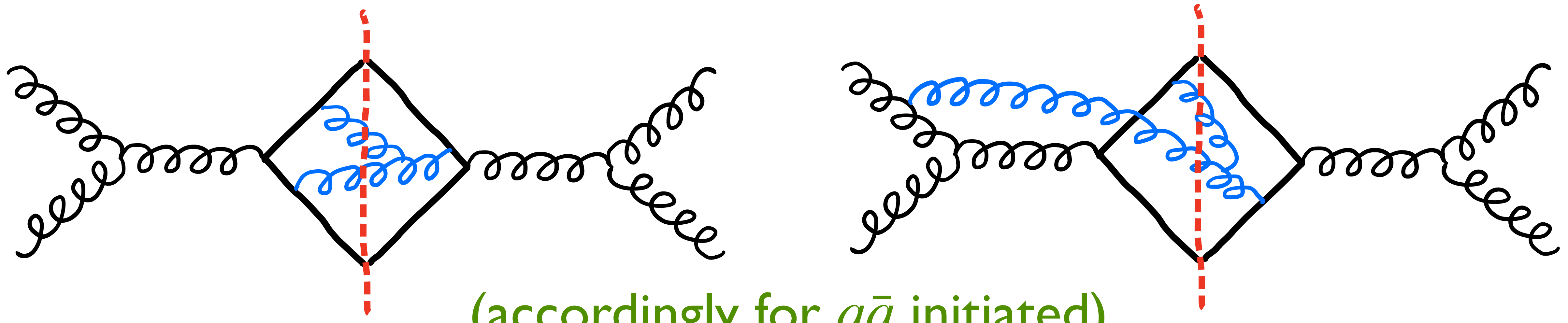
p_T^j

[from L. Rottoli's talk at Ringberg 2024]

MiNNLO$_{PS}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

- ✦ substantial complication due to final-state radiation and interferences



(accordingly for $q\bar{q}$ initiated)

- ✦ compare resummation formulas (very schematic):

colour singlet: $$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{e^{-S} \quad H \quad (C\otimes f)(C\otimes f)\right\}$$

heavy quark pair: $$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{e^{-S}\, \mathrm{Tr}(\mathbf{H\Delta})\,(C\otimes f)(C\otimes f)\right\}$$

[Catani, Grazzini, Torre '14]

Δ: operator/matrix in colour space that encodes soft emissions of $t\bar{t}$ and interferences

derived to NNLO in [Catani, Devoto, Grazzini, Mazzitelli, '23]

MiNNLO$_{PS}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) = \langle M|\boldsymbol{\Delta}|M\rangle, \quad \boldsymbol{\Delta} = \mathbf{V}^{\dagger}\,\mathbf{D}\,\mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\underset{\uparrow}{\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}}+\frac{\alpha_s^2(q)}{(2\pi)^2}\underset{\uparrow}{\boldsymbol{\Gamma}^{(2)}_{\mathbf{t}}}\right]\right\}$$

matrix in colour space

MiNNLO_PS: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

'B-type' correction to Sudakov

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) = \langle M|\boldsymbol{\Delta}|M\rangle, \quad \boldsymbol{\Delta} = \mathbf{V}^{\dagger}\,\mathbf{D}\,\mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}+\frac{\alpha_s^2(q)}{(2\pi)^2}\boldsymbol{\Gamma}^{(2)}_{\mathbf{t}}\right]\right\}$$

matrix in colour space

MiNNLO$_{PS}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) = \langle M|\,\boldsymbol{\Delta}\,|M\rangle,\qquad \boldsymbol{\Delta} = \mathbf{V}^\dagger\,\mathbf{D}\,\mathbf{V},\qquad \mathbf{V} = \exp\left\{-\int\frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}+\frac{\alpha_s^2(q)}{(2\pi)^2}\boldsymbol{\Gamma}^{(2)}_{\mathbf{t}}\right]\right\}$$

- ✦ approximations keeping NNLO and (N)LL
  - ❖ azimuthal average with $\left[\mathbf{D}\right]_\phi = 1$ ➙ modifies $H \to \overline{H}$ and $(C\otimes f) \to \overline{(C\otimes f)}$ at α_s^2

    see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]

MiNNLO$_{\text{PS}}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^F_{\text{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$$\mathrm{Tr}(\mathbf{H\Delta}) = \langle M|\mathbf{\Delta}|M\rangle, \quad \mathbf{\Delta} = \mathbf{V}^\dagger\,\mathbf{D}\,\mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}_{\mathbf{t}}^{(1)}+\frac{\alpha_s^2(q)}{(2\pi)^2}\mathbf{\Gamma}_{\mathbf{t}}^{(2)}\right]\right\}$$

✦ approximations keeping NNLO and (N)LL

- ❖ azimuthal average with $[\mathbf{D}]_\phi = 1$ ➙ modifies $H \to \overline{H}$ and $(C\otimes f) \to \overline{(C\otimes f)}$ at α_s^2

  see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]

- ❖ $\langle M|\mathbf{\Delta}|M\rangle \approx \underbrace{\langle M|M\rangle}_{=H}\,\frac{\langle M^{(0)}|\mathbf{\Delta}|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}$

absorb mistake at NNLO in $B^{(2)}$

MiNNLO$_{\text{PS}}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\dots\right]$$

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) = \langle M|\,\boldsymbol{\Delta}\,|M\rangle, \qquad \boldsymbol{\Delta} = \mathbf{V}^{\dagger}\,\mathbf{D}\,\mathbf{V}, \qquad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}_{\mathbf{t}}^{(1)}+\frac{\alpha_s^2(q)}{(2\pi)^2}\boldsymbol{\Gamma}_{\mathbf{t}}^{(2)}\right]\right\}$$

- approximations keeping NNLO and (N)LL
  - azimuthal average with $\left[\mathbf{D}\right]_\phi = 1$ ➙ modifies $H \to \overline{H}$ and $(C\otimes f) \to \overline{(C\otimes f)}$ at α_s^2
    see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]
  - $\langle M|\,\boldsymbol{\Delta}\,|M\rangle \approx \underbrace{\langle M|M\rangle}_{=H}\,\dfrac{\langle M^{(0)}|\,\boldsymbol{\Delta}\,|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}$
  - expand $\mathbf{V} = \underbrace{\exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}_{\mathbf{t}}^{(1)}\right\}}_{\equiv \mathbf{V}_{\mathrm{NLL}}}\times\left(1-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s^2(q)}{(2\pi)^2}\boldsymbol{\Gamma}_{\mathbf{t}}^{(2)}\right)+\mathcal{O}(\mathrm{N^3LL})$

absorb in $B^{(2)}$ coefficient

MiNNLO$_{PS}$: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

✦ using those approximations (exact up to NNLO & (N)LL) we have:

$$\tilde{B}^{(2)} = B^{(2)} + \frac{\langle M^{(0)}|\boldsymbol{\Gamma}^{(2)\dagger}+\boldsymbol{\Gamma}^{(2)}|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle} + \frac{2\,\mathrm{Re}\left\{\langle M^{(1)}|\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}|M^{(0)}\rangle\right\}}{\langle M^{(0)}|M^{(0)}\rangle} - \frac{2\,\langle M^{(0)}|\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}|M^{(0)}\rangle\,\mathrm{Re}\left\{\langle M^{(1)}|M^{(0)}\rangle\right\}}{\langle M^{(0)}|M^{(0)}\rangle^2}$$

and $$e^{-S}\langle M|\boldsymbol{\Delta}|M\rangle = e^{-\tilde{S}}\,\frac{\langle M^{(0)}|\mathbf{V}^{\dagger}_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}\,H + \mathcal{O}(\alpha_s^5)$$

(reminder: $\mathbf{V}_{\mathrm{NLL}} \equiv \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}\right\}$)

MiNNLOPS: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\dots\right]$$

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and $$e^{-S}\langle M|\boldsymbol{\Delta}|M\rangle = e^{-\tilde{S}}\underbrace{\frac{\langle M^{(0)}|\mathbf{V}^{\dagger}_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}}_{=\sum_i c_i\,\underbrace{e^{-\tilde{S}+S_i}}_{\equiv e^{\bar{S}_i}}} H + \mathcal{O}(\alpha_s^5)$$

use basis $|M^{(0)}\rangle$ where $\boldsymbol{\Gamma}^{(1)}$ diagonal

$$\bar{B}^{(1)} = B^{(1)} + \gamma_i$$

eigenvalues of $\mathbf{V}^{\dagger}_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}$ exponent

(reminder: $\mathbf{V}_{\mathrm{NLL}} \equiv \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}\right\}$)

MiNNLOPS: heavy quark production

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

$$\mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$${}^{2)}\log(M/q) + B^{(2)}\Big) + \ldots\Big]$$

MiNNLOPS for colour singlets

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

✦ starting equation:

$$\boxed{\frac{\mathrm{d}\sigma^{\mathrm{res}}_{F}}{\mathrm{d}p_T\,\mathrm{d}\Phi_{\mathrm{B}}} = \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\mathscr{L}\right\}} = e^{-S}\underbrace{\left\{S'\mathscr{L} + \mathscr{L}'\right\}}_{\equiv D}$$

$$\mathscr{L} \sim H\,(C\otimes f)\,(C\otimes f)$$

LL) we have:

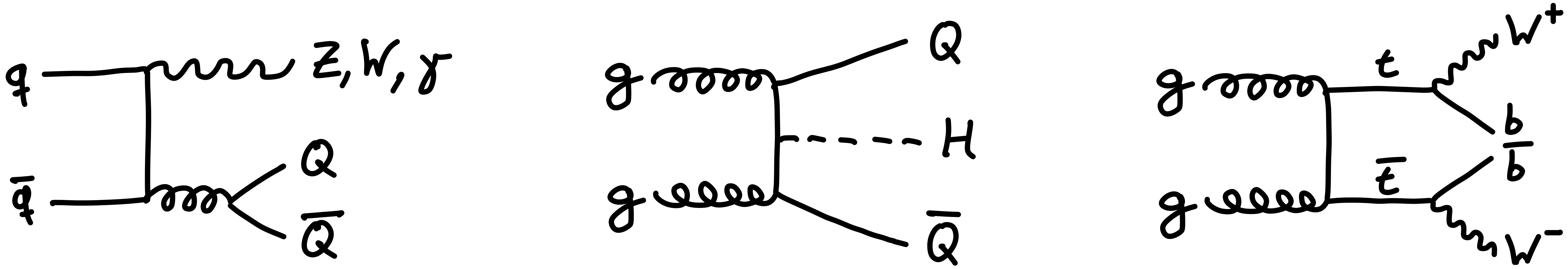
$$\frac{\mathbf{\Gamma}^{(1)\dagger} + \mathbf{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\,\mathrm{Re}\left\{\langle M^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle^{2}}$$

and $$e^{-S}\langle M\,|\,\mathbf{\Delta}\,|\,M\rangle = e^{-\tilde{S}}\underbrace{\frac{\langle M^{(0)}\,|\,\mathbf{V}^{\dagger}_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle}}_{=\sum_i c_i\,\underbrace{e^{-\tilde{S}+S_i}}_{\equiv e^{\bar{S}_i}}}\,H + \mathcal{O}(\alpha_s^5)$$

simplified to sum of terms with same structure as starting formula for colour singlet case

$$\boxed{\Rightarrow\ \mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{\sum_i e^{-\bar{S}_i}\,\underbrace{c_i\,\overline{H}\,\overline{(C\otimes f)}\,\overline{(C\otimes f)}}_{\equiv\overline{\mathscr{L}}_i}\right\} + \text{terms beyond NNLO \& (N)LL}}$$

MiNNLO heavy quark + colour singlet production



[Mazzitelli, Sotnikov, Wiesemann '24]

✦ same structure of singular/resummed cross section as $Q\bar{Q}$, but need to account for recoil:

colour singlet:
$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S} \quad H \quad (C\otimes f)(C\otimes f)\right\}$$

heavy quark pair + colour singlet:
$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\, \mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)(C\otimes f)\right\}$$

Soft function for Heavy quark production in ARbitrary Kinematics
[Devoto, Mazzitelli 'in preparation]

Results:

top-quark pair production ($\mathrm{t\bar{t}}$)

$t\bar{t}$ production

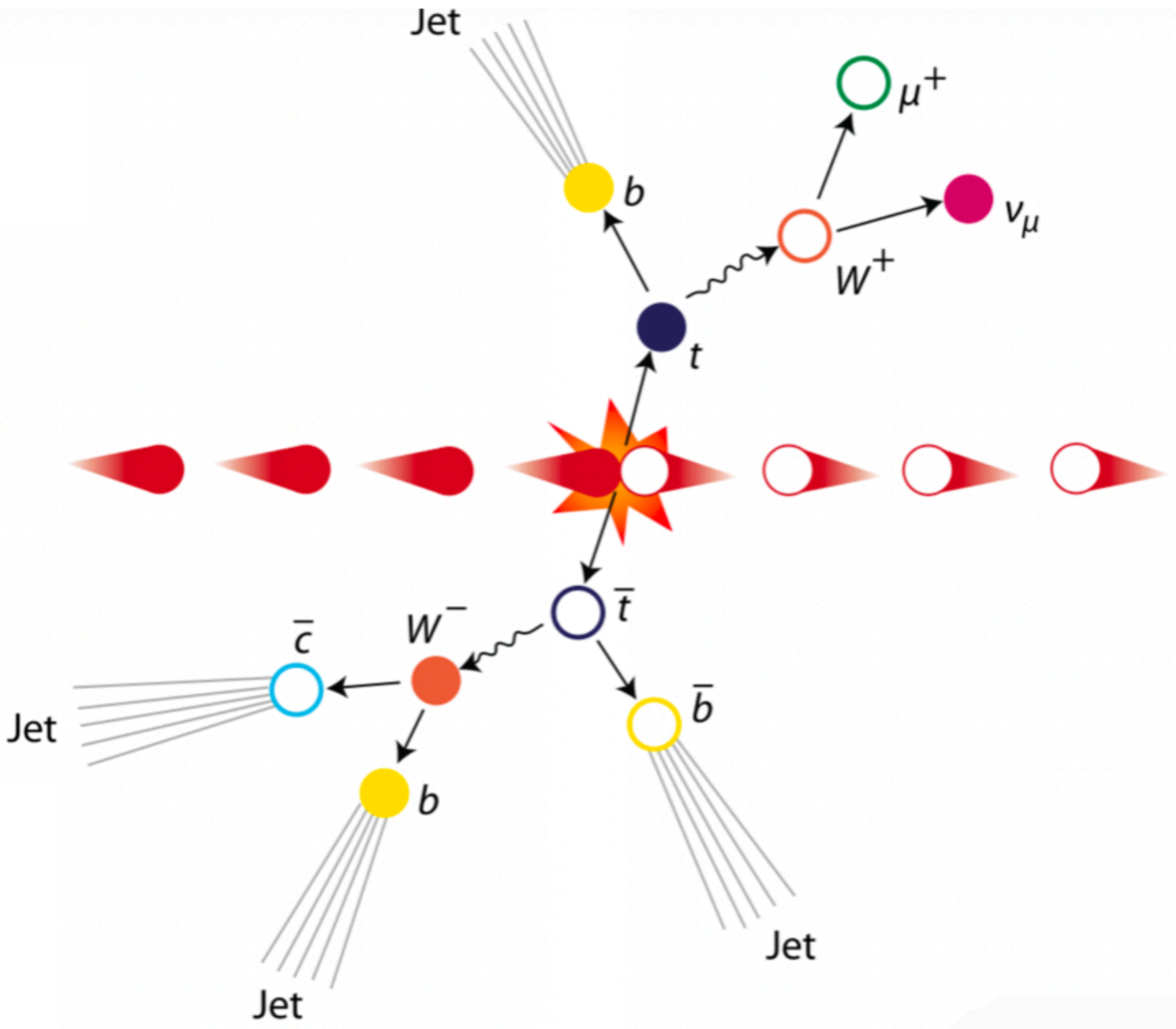
$$t\bar{t} \;\rightarrow\; b\bar{b}\, W^- W^+$$

Fully leptonic $W^+W^- \;\rightarrow\; l\bar{\nu}_l\,\bar{l}\nu_l$

Semi-leptonic $W^+W^- \;\rightarrow\; l\bar{\nu}_l\, q\bar{q}'$

Hadronic $W^+W^- \;\rightarrow\; q\bar{q}' q'\bar{q}$

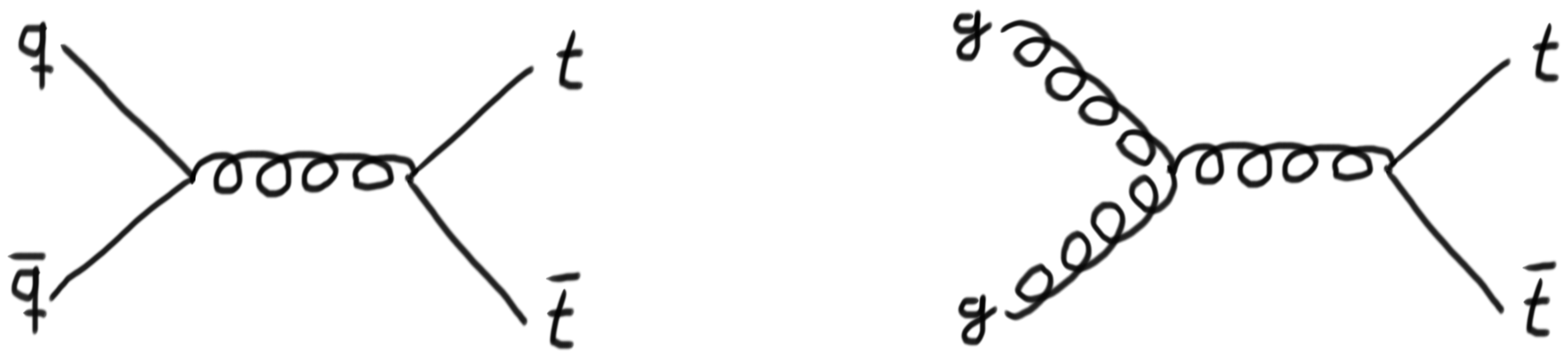
(where $q = \{u, c\}$ and $q' = \{d, s\}$)

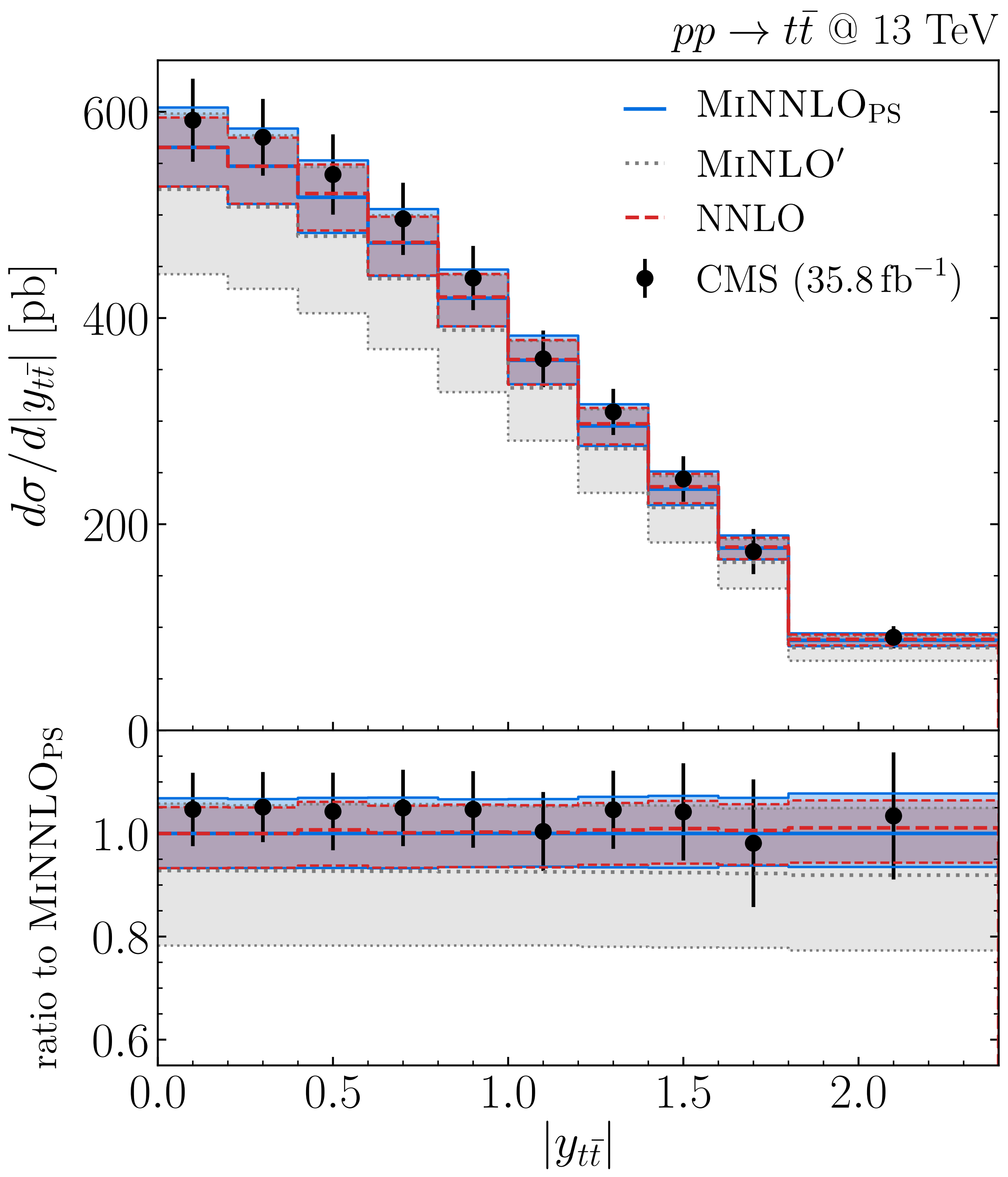


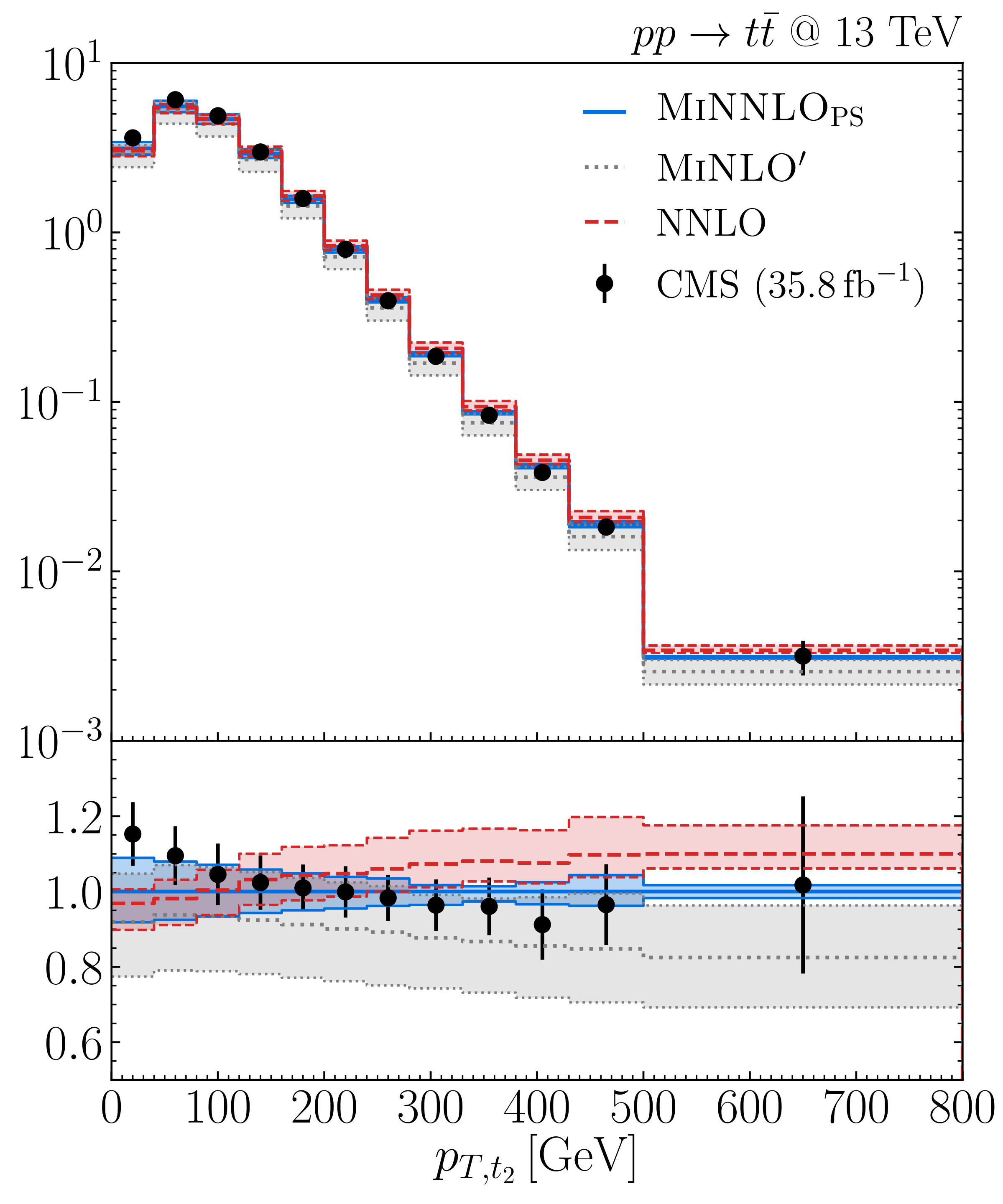
*approximated through a MadSpin-like approach using the full off-shell diagram at LO, keeping spin correlations

$t\bar{t}$ production

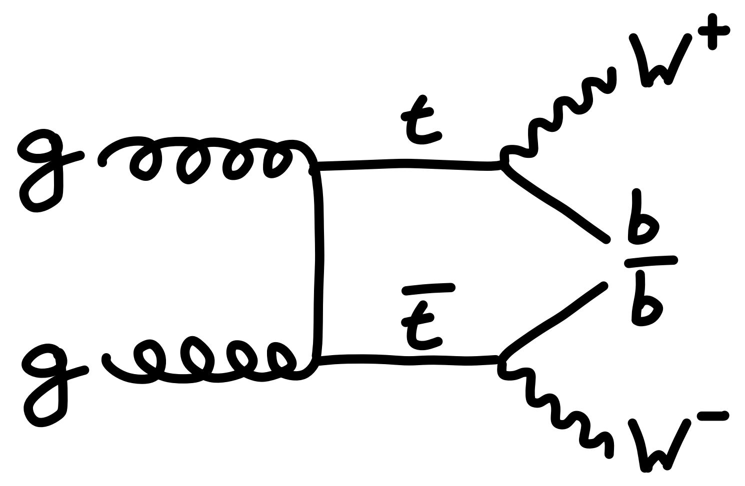
on-shell $t\bar{t}$ production

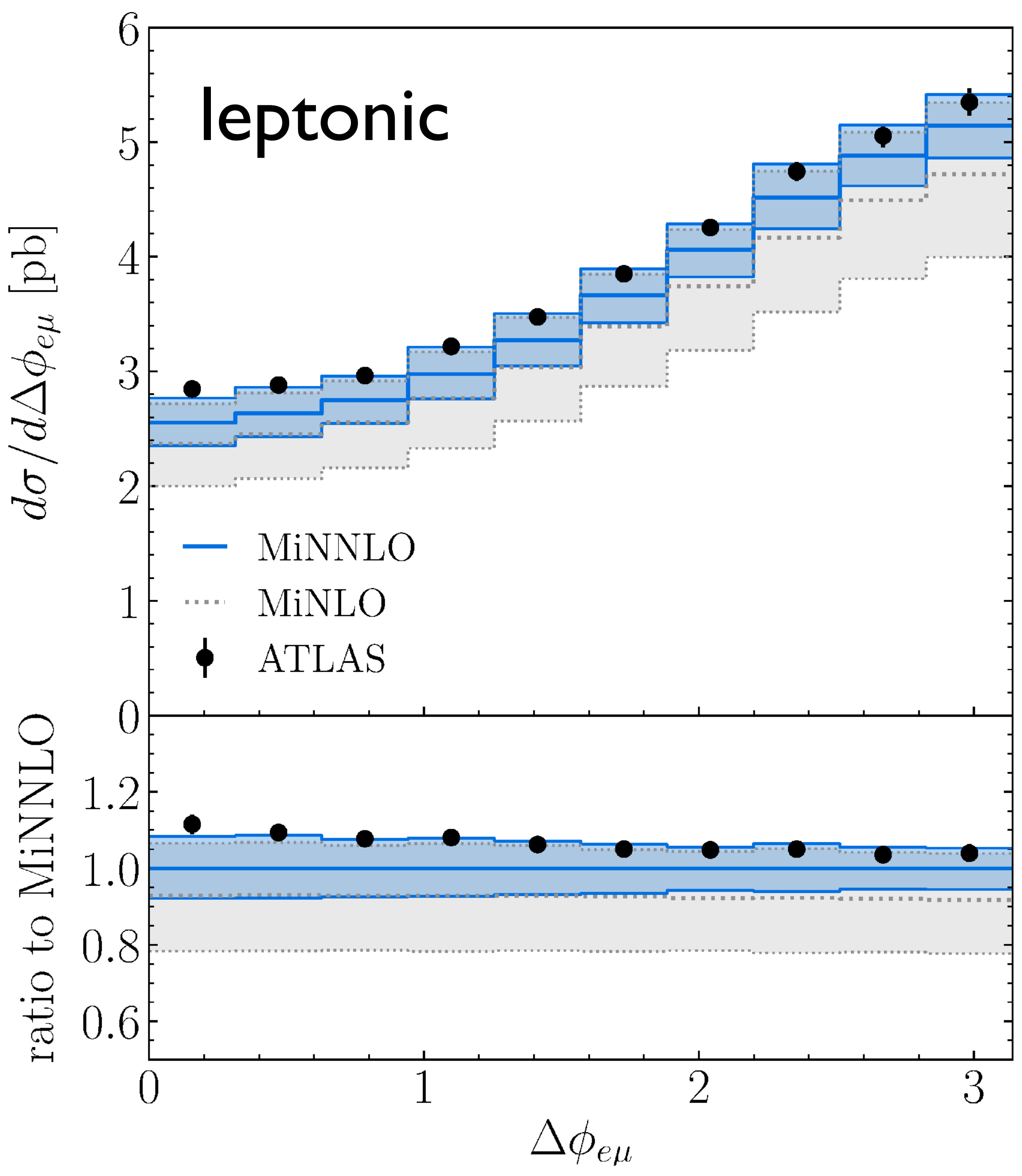


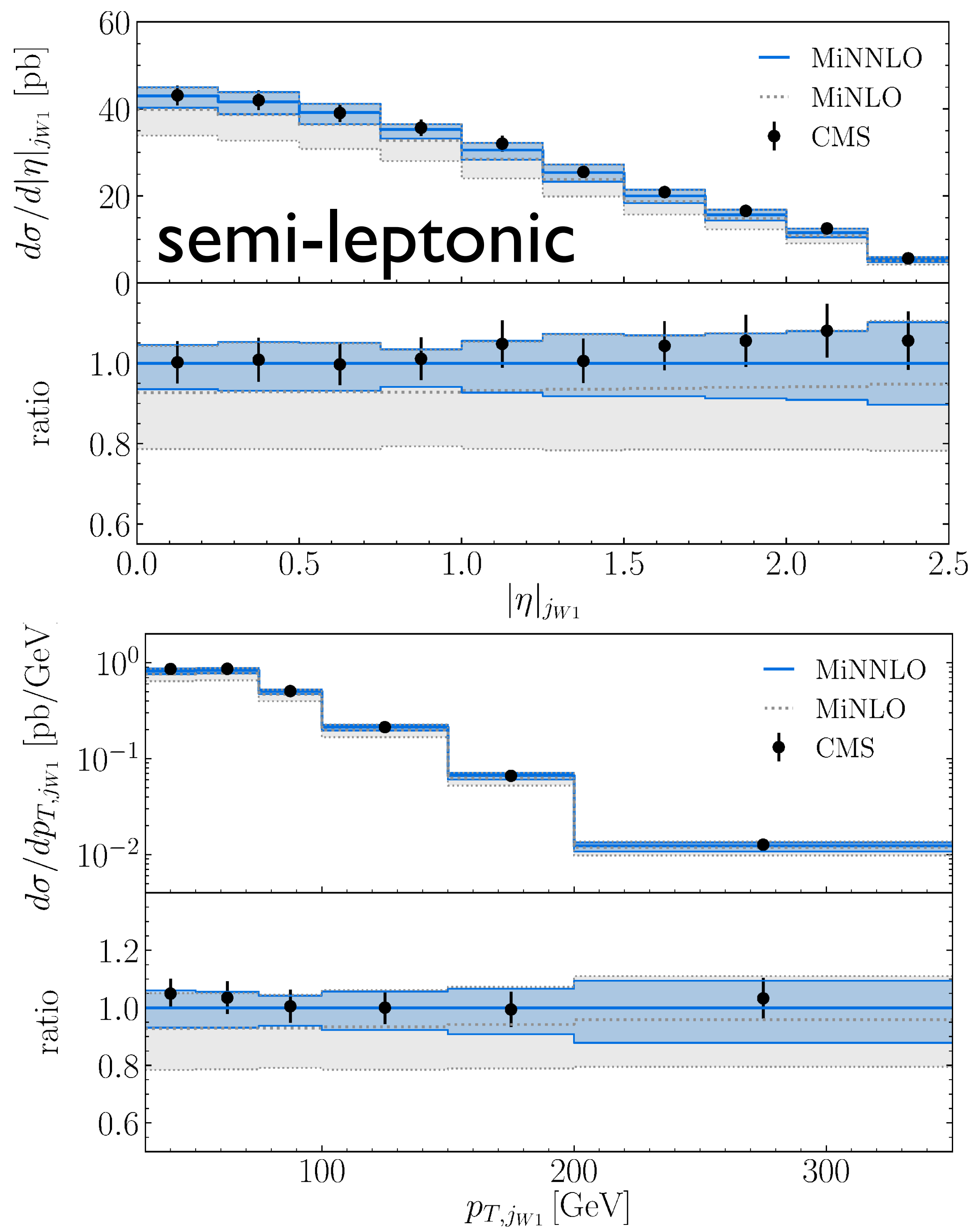




with off-shell top decays*







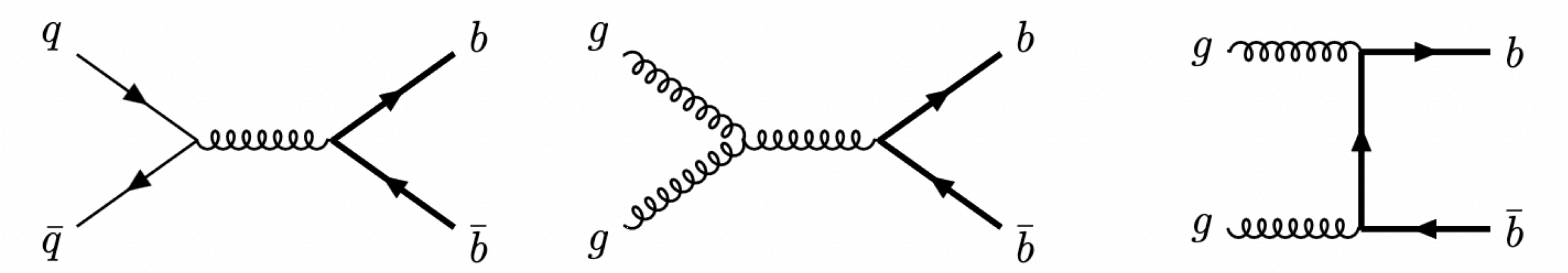
Results:

bottom-quark pair production ($b\bar{b}$)

(B-hadron and b-jet production)

$b\bar{b}$ production

[Mazzitelli, MW, Zanderighi, Ratti '23]



Validation against fixed order results from MATRIX

NLO	**MiNLO'**	**NNLO**	**MiNNLOps**
$348.5(3)^{+27\%}_{-24\%}\ \mu b$	$399.7(5)^{+22\%}_{-21\%}\ \mu b$	$435(2)^{+16\%}_{-15\%}\ \mu b$	$428.7(5)^{+13\%}_{-11\%}\ \mu b$

★ use four-flavour scheme (4FS) with massive bottom quarks

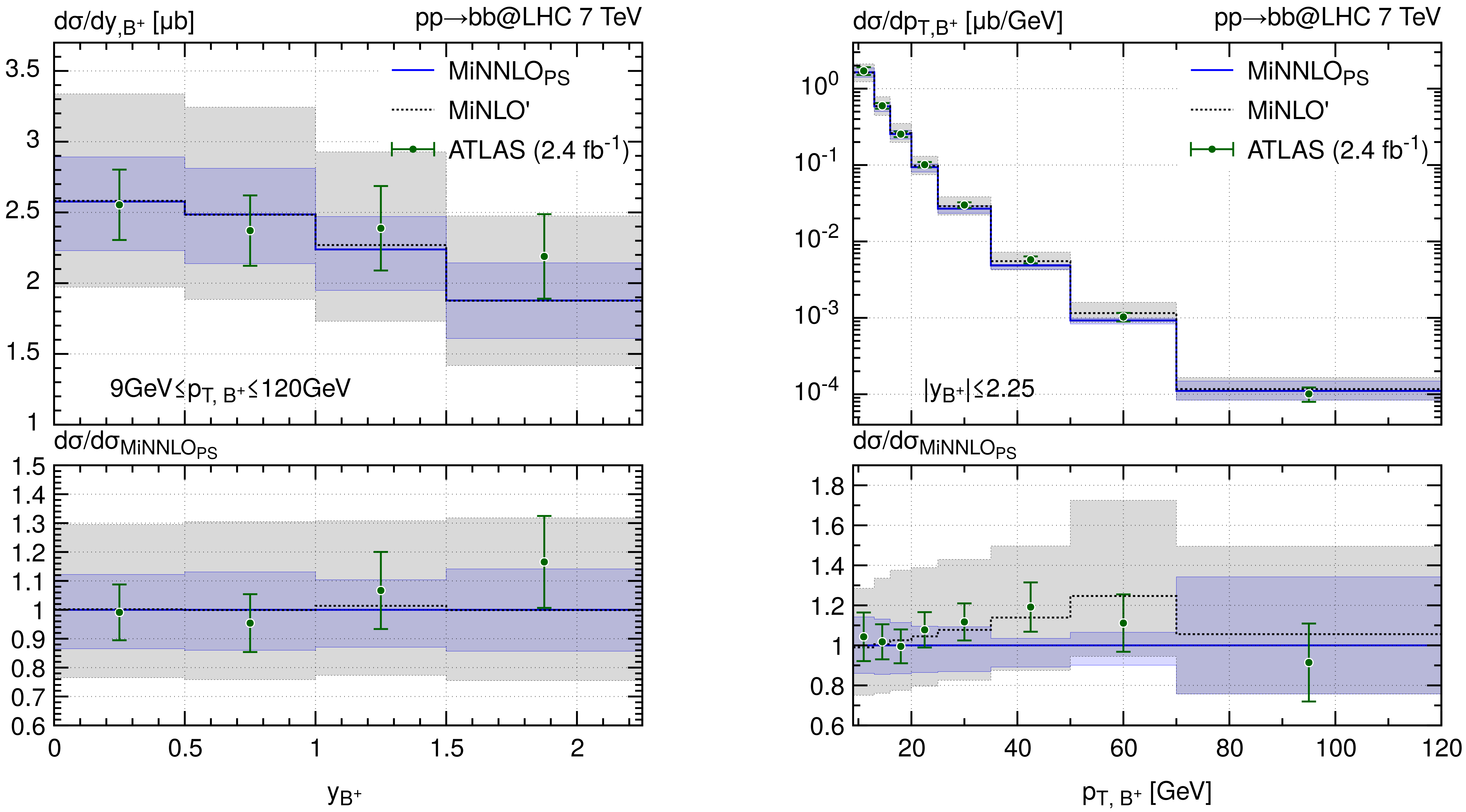
★ NNLO+PS matching important:

  ➝ realistic simulation of B-hadrons (through Pythia8)

  ➝ reliable at high bottom p_T through shower resummation

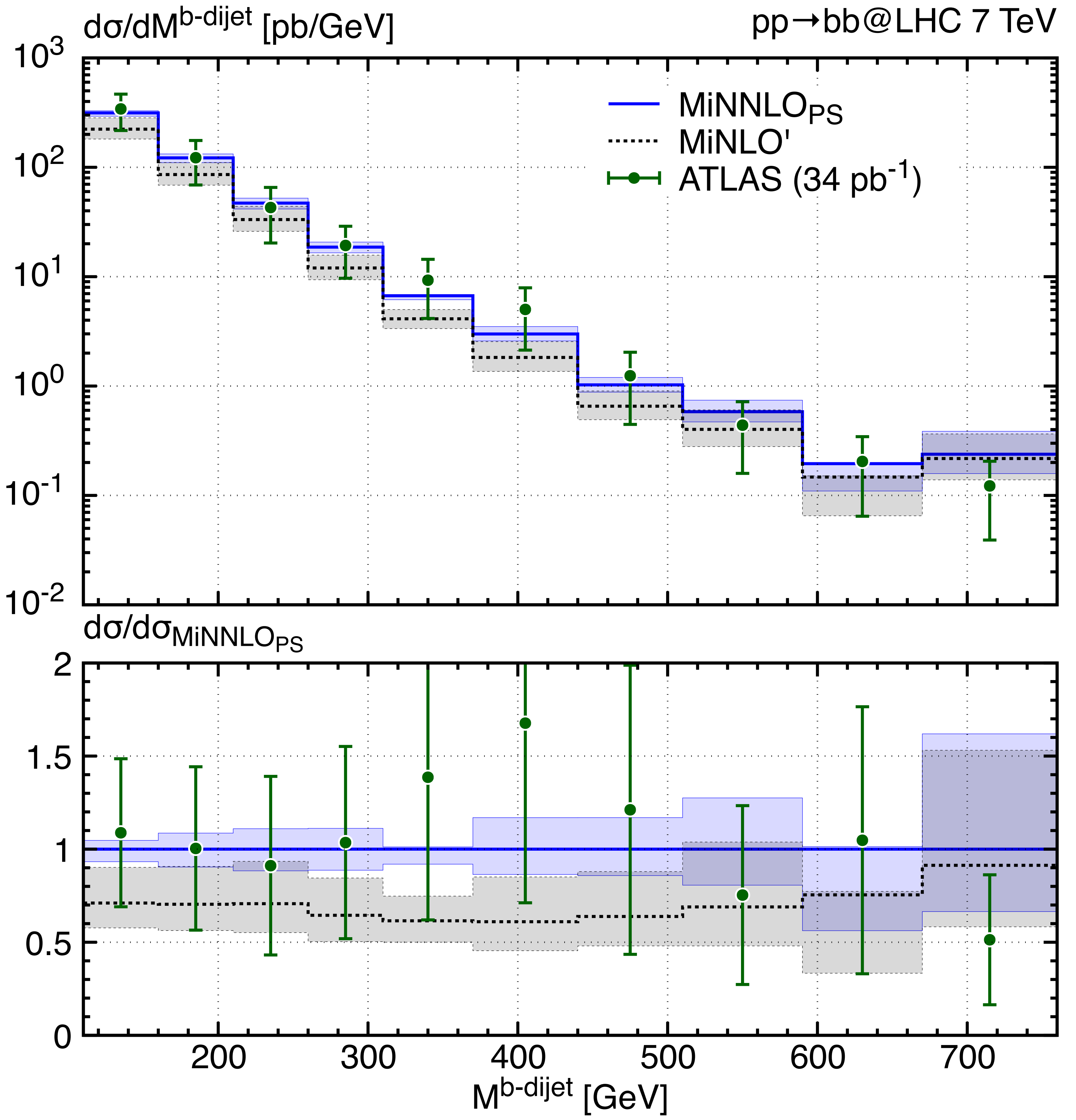
MiNNLO$_{PS}$: B-hadron production

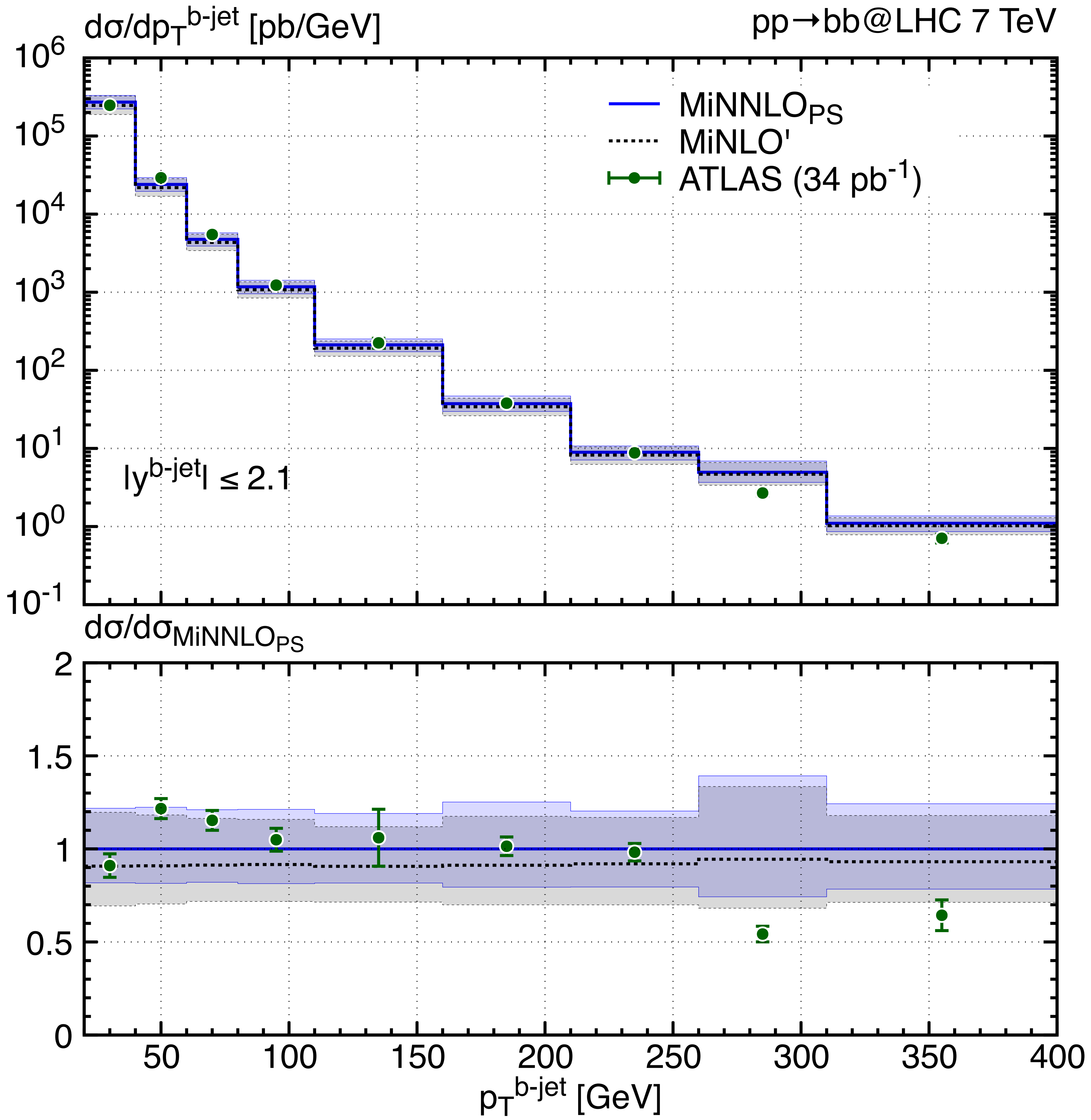
[Mazzitelli, MW, Zanderighi, Ratti '23]

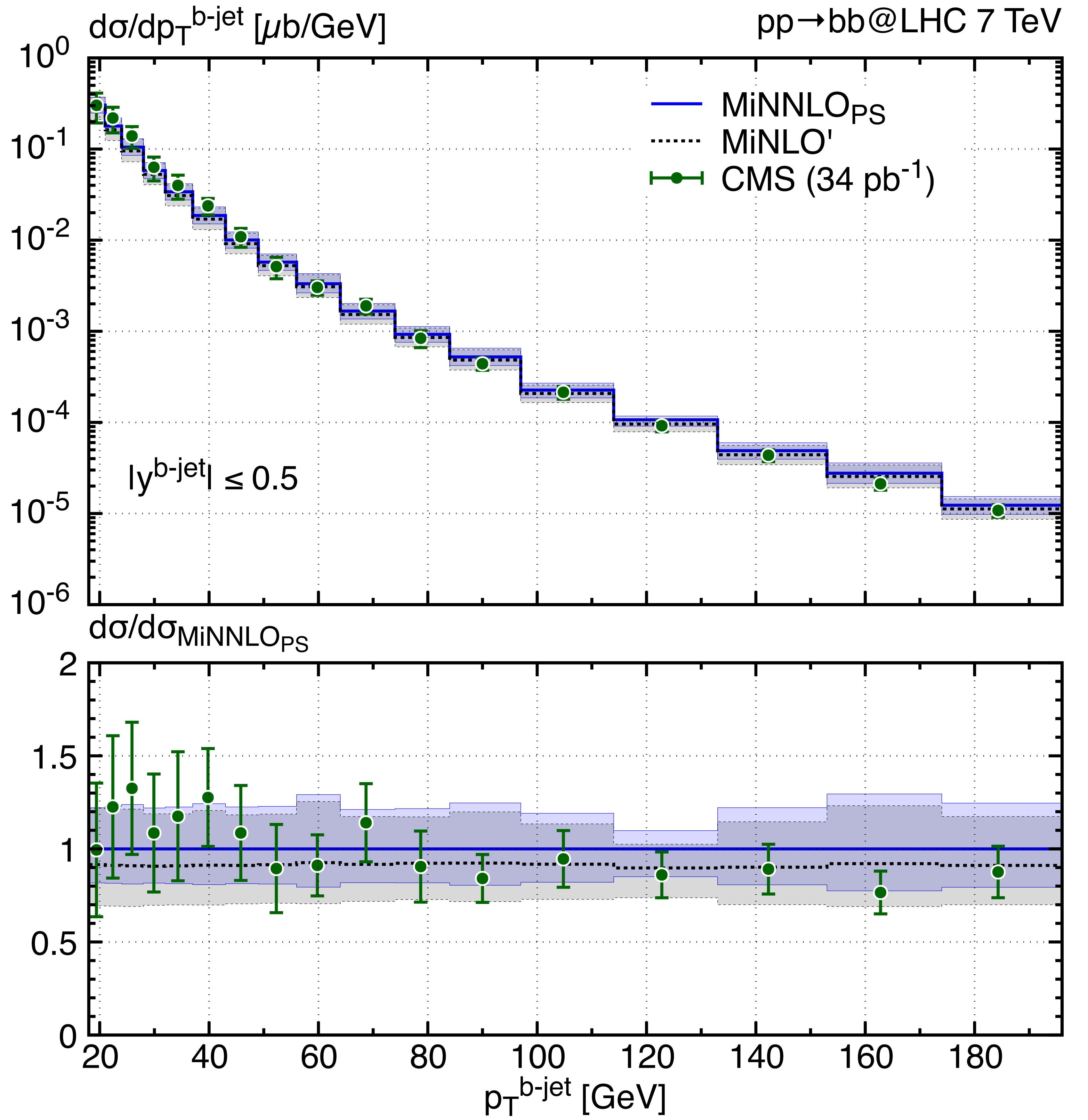


MiNNLO$_{PS}$: b-jet production

[Gauld, Mazzitelli, MW, Zanderighi, Ratti 'in preparation]





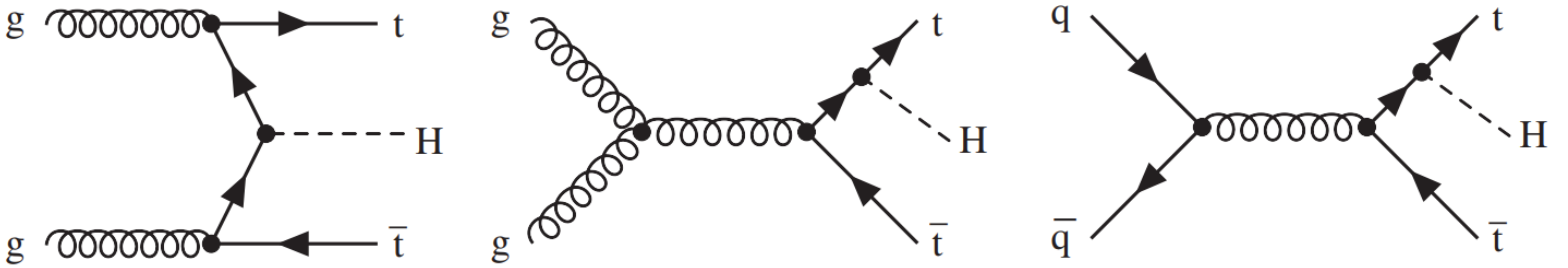


Results:

top-quark pair production in association with a Higgs boson ($t\bar{t}H$)

MiNNLO$_{PS}$: $t\bar{t}H$ production

[Mazzitelli, MW 'work in progress]

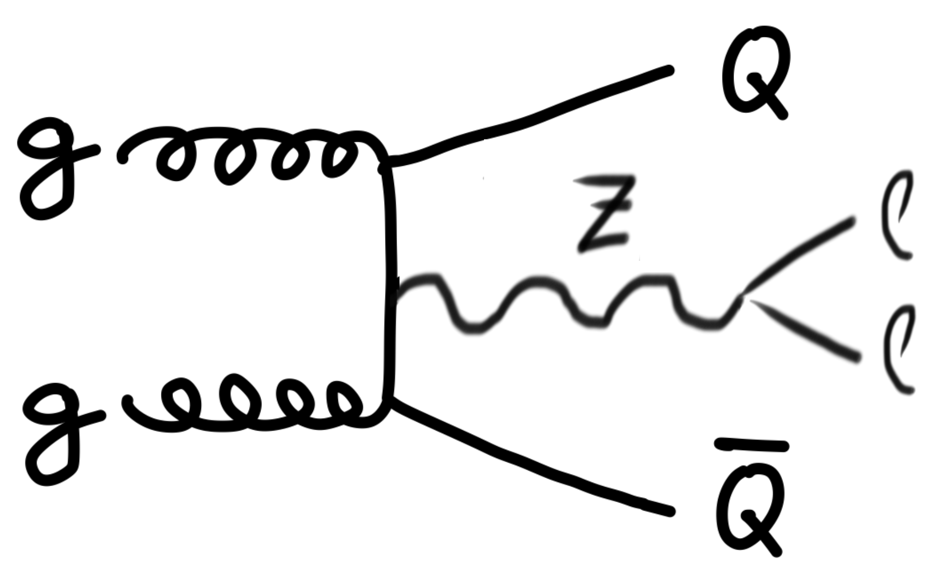


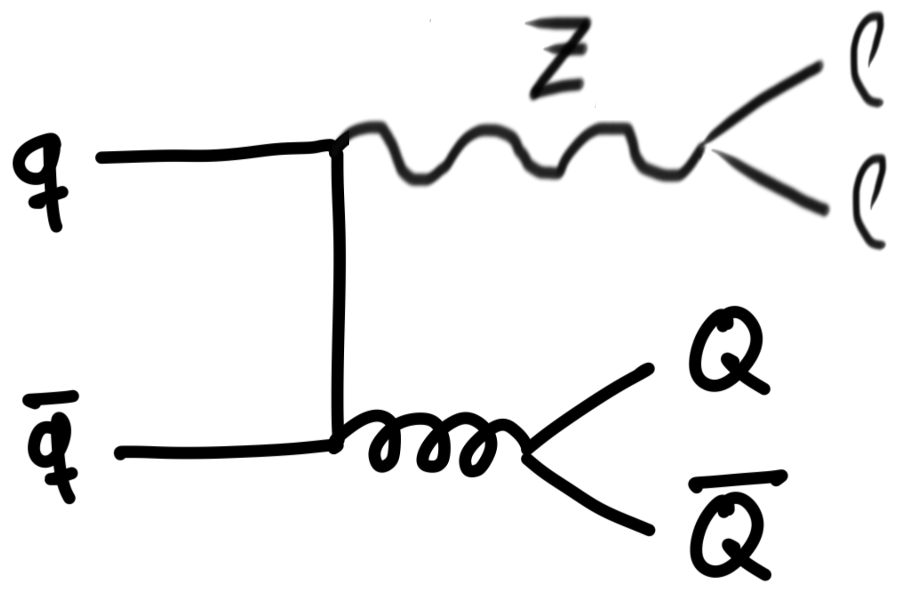
Results:

bottom-quark pair production in association with a Z boson ($b\bar{b}Z$)

MiNNLO$_{PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]





- ★ MiNNLO$_{PS}$ method general for all heavy-quark + colour singlet processes
- ★ bottom mass neither a large nor small scale: 4FS (massive bottom) and 5FS (massless bottom) viable
- ★ <u>complication</u>:
  Z couples to initial-state light quarks and final-state heavy quarks & coupling depends on quark falvour
- ★ 2-loop amplitude: most complicated ingredient & among most complicated 2-loop computed to date

MiNNLO$_{PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

Two-loop amplitude

- ★ complete calculation (five-point functions with massive b's) out of reach
- ★ we exploit small-mass expansion in m_b (massification procedure)

$1/\varepsilon$ poles in 5FS ⟷ $\log(m_b)$ in 4FS

$$\underbrace{2\text{Re}\langle R^{(0)} | R^{(2)} \rangle}_{\text{massive amplitude}} = \sum_{i=1}^{4} \underbrace{\kappa_i}_{\text{coefficients of massification}} \log^i(m_b/\mu_R) + \underbrace{2\text{Re}\langle R_0^{(0)} | R_0^{(2)} \rangle}_{\text{massless amplitude}} + \underbrace{\mathcal{O}(m_b/\mu)}_{\text{power corrections}}$$

MiNNLO$_{\mathrm{PS}}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

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massive amplitude

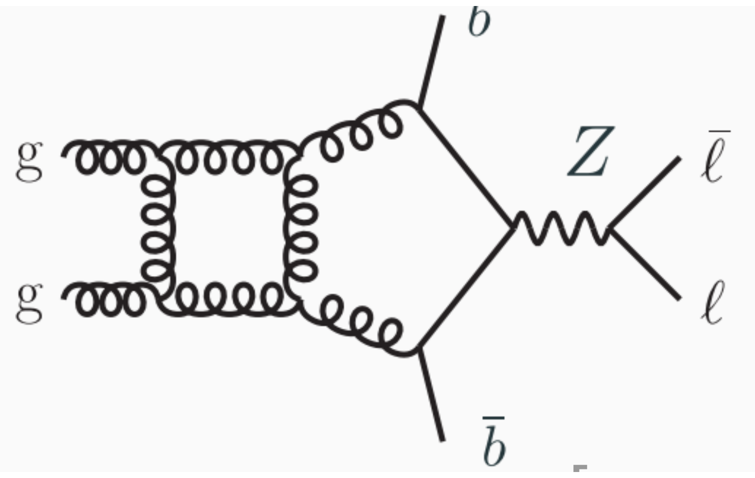
↑ coefficients of massification

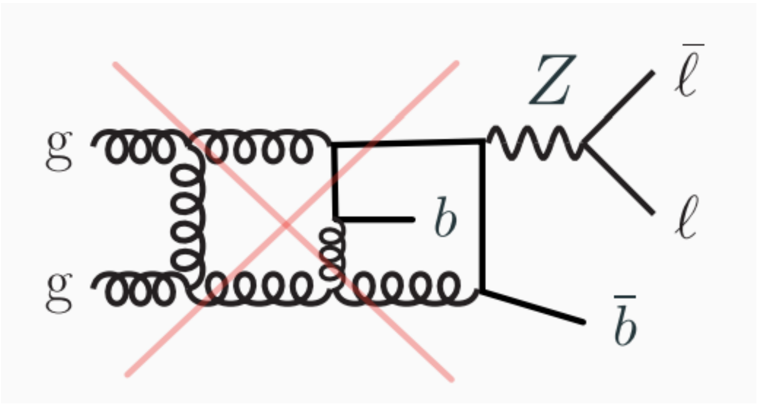
massless amplitude

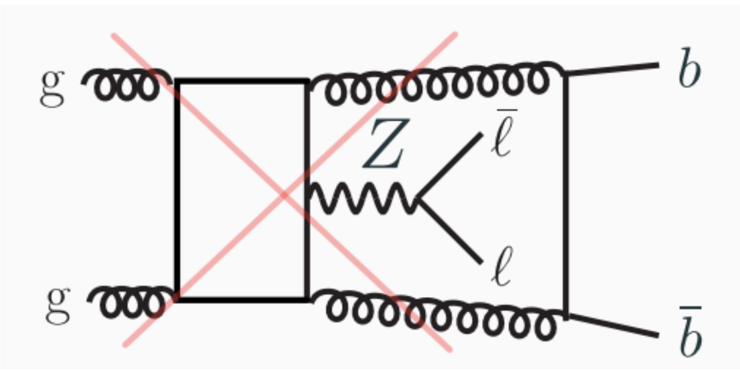
power corrections

- ★ logarithmic terms exact (massless loops: [Mitov, Moch '06], massive loops: [Wang, Xia, Yang, Ye '23])
- ★ infra-red safe mapping required from massive to massless momenta
- ★ massless two-loop in LC approx. & dropping Z coupling to closed quark loops (small at NLO)

(based on [Chicherin, Sotnikov, Zoia '2110.07541],
[Abreu, Cordero, Ita, Klinkert, Page, Sotnikov '2110.07541])







MiNNLO$_{\text{PS}}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

total cross section: $66\,\text{GeV} \leq m_{\ell^+\ell^-} \leq 116\,\text{GeV}$

	σ_{total} [pb]	ratio to NLO
NLO+PS ($m_{b\bar{b}\ell\ell}$)	$31.86(1)^{+16.3\%}_{-13.3\%}$	1.000
MiNLO′ ($m_{b\bar{b}\ell\ell}$)	$22.33(1)^{+28.2\%}_{-17.9\%}$	0.701
MiNNLO$_{\text{PS}}$ ($m_{b\bar{b}\ell\ell}$)	$50.58(4)^{+16.8\%}_{-12.2\%}$	1.587
NLO+PS ($H_T/2$)	$41.42(1)^{+19.2\%}_{-15.4\%}$	1.000
MiNNLO$_{\text{PS}}$ ($H_T/2$)	$58.60(5)^{+19.0\%}_{-13.2\%}$	1.414

MiNNLO$_{\text{PS}}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

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+60% NNLO correction !

MiNNLO$_{\text{PS}}$: $b\bar{b}Z$ production

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+60% NNLO correction !

+41% NNLO correction !

MiNNLO$_{PS}$: $b\bar{b}Z$ production

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→ MiNLO/multi-jet merging not suitable due to incomplete α_s^2 correction and large $\log(m_b)$ contribution in 2-loop (leading to miscancellation with $\log(m_b)$ from reals)
(only a problem for bottom quarks and processes with $Q \gg m_b$)

MiNNLO$_{\rm PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

Object	Selection
Dressed leptons	$p_{\rm T}$(leading) $> 35\,$GeV, $p_{\rm T}$(subleading) $> 25\,$GeV, $\lvert\eta\rvert < 2.4$
Z boson	$71 < m_{\ell\ell} < 111\,$GeV
Generator-level b jet	b hadron jet, $p_{\rm T} > 30\,$GeV, $\lvert\eta\rvert < 2.4$

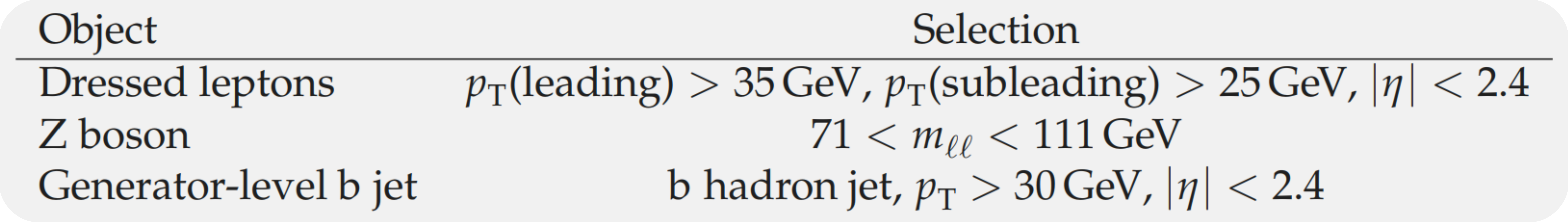
5FS MG5_aMC from CMS paper

$\sigma_{\rm fiducial}$ [pb]	$Z+\geq 1$ b-jet	$Z+\geq 2$ b-jets
NLO+PS (5FS)	7.03 ± 0.47	0.77 ± 0.07
NLO+PS (4FS)	4.08 ± 0.66	0.44 ± 0.08
MiNNLO$_{\rm PS}$ (4FS)	6.59 ± 0.86	0.77 ± 0.10
CMS	6.52 ± 0.43	0.65 ± 0.08

MiNNLO$_{\text{PS}}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

Comparison to CMS Z+b-jet analysis [CMS 2112.09659]



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5FS MG5_aMC
from CMS paper

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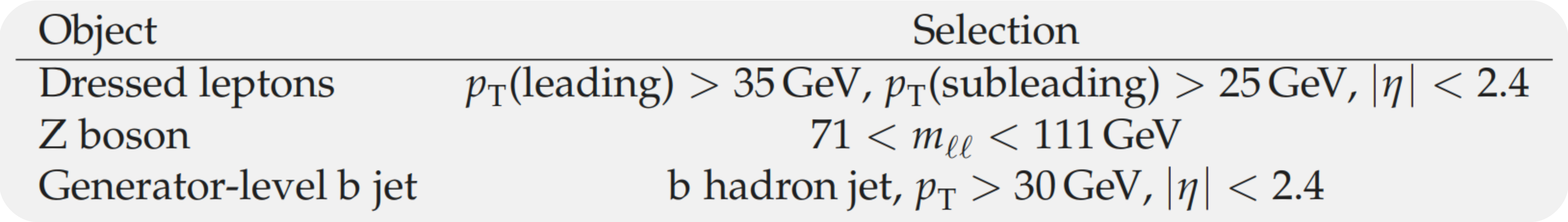
MiNNLO$_{\text{PS}}$ (4FS)	6.59 ± 0.86	0.77 ± 0.10
CMS	6.52 ± 0.43	0.65 ± 0.08

NNLO corrections make 4FS and 5FS compatible

MiNNLO$_{PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

Comparison to CMS Z+b-jet analysis [CMS 2112.09659]



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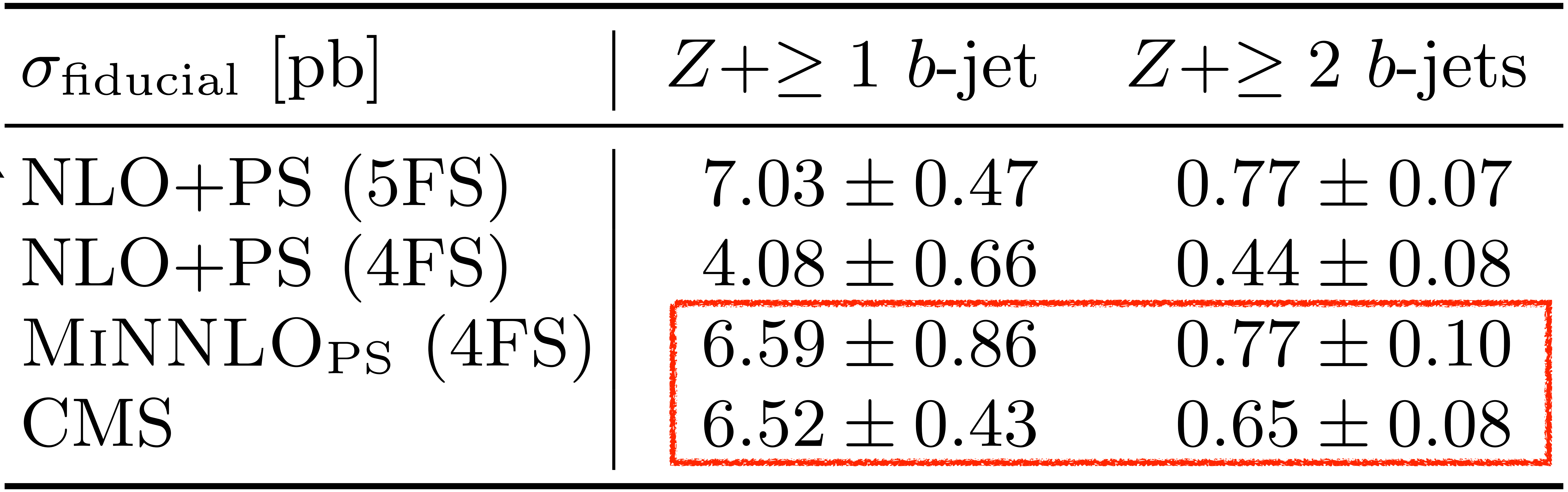
Generator-level b jet	b hadron jet, $p_T > 30\,\text{GeV}$, $\lvert\eta\rvert < 2.4$

5FS MG5_aMC from CMS paper



σ_{fiducial} [pb]	$Z+\geq 1$ b-jet	$Z+\geq 2$ b-jets
NLO+PS (5FS)	7.03 ± 0.47	0.77 ± 0.07
NLO+PS (4FS)	4.08 ± 0.66	0.44 ± 0.08
MiNNLO$_{\text{PS}}$ (4FS)	6.59 ± 0.86	0.77 ± 0.10
CMS	6.52 ± 0.43	0.65 ± 0.08

NNLO corrections lift tension of 4FS with data

MiNNLO$_{PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

Z+1b-jet distributions compared to CMS data [CMS 2112.09659]

MiNNLO$_{PS}$: $b\bar{b}Z$ production

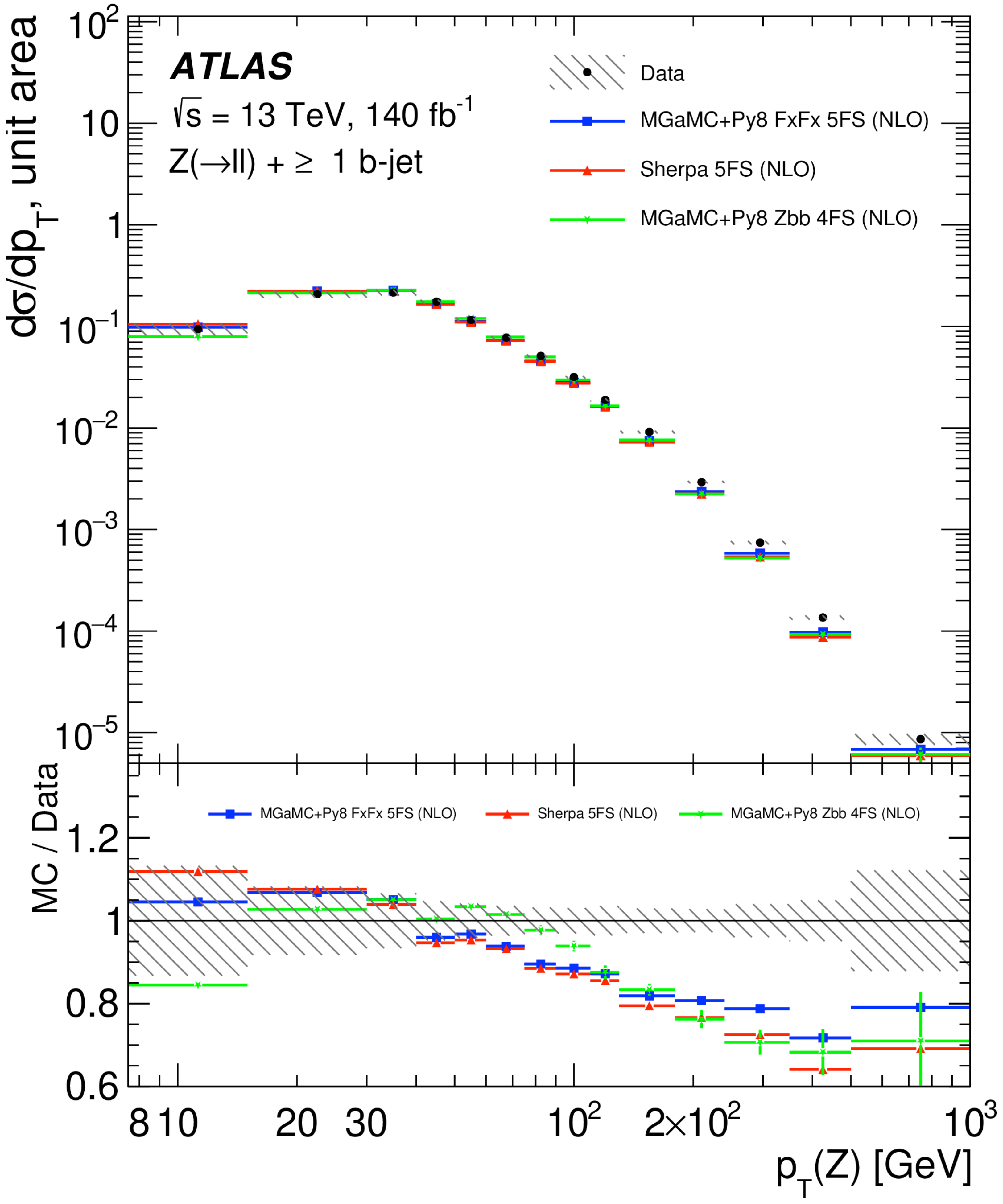
[Mazzitelli, Sotnikov, MW '24]

Z+2b-jet distributions compared to CMS data [CMS 2112.09659]

MiNNLO$_{PS}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

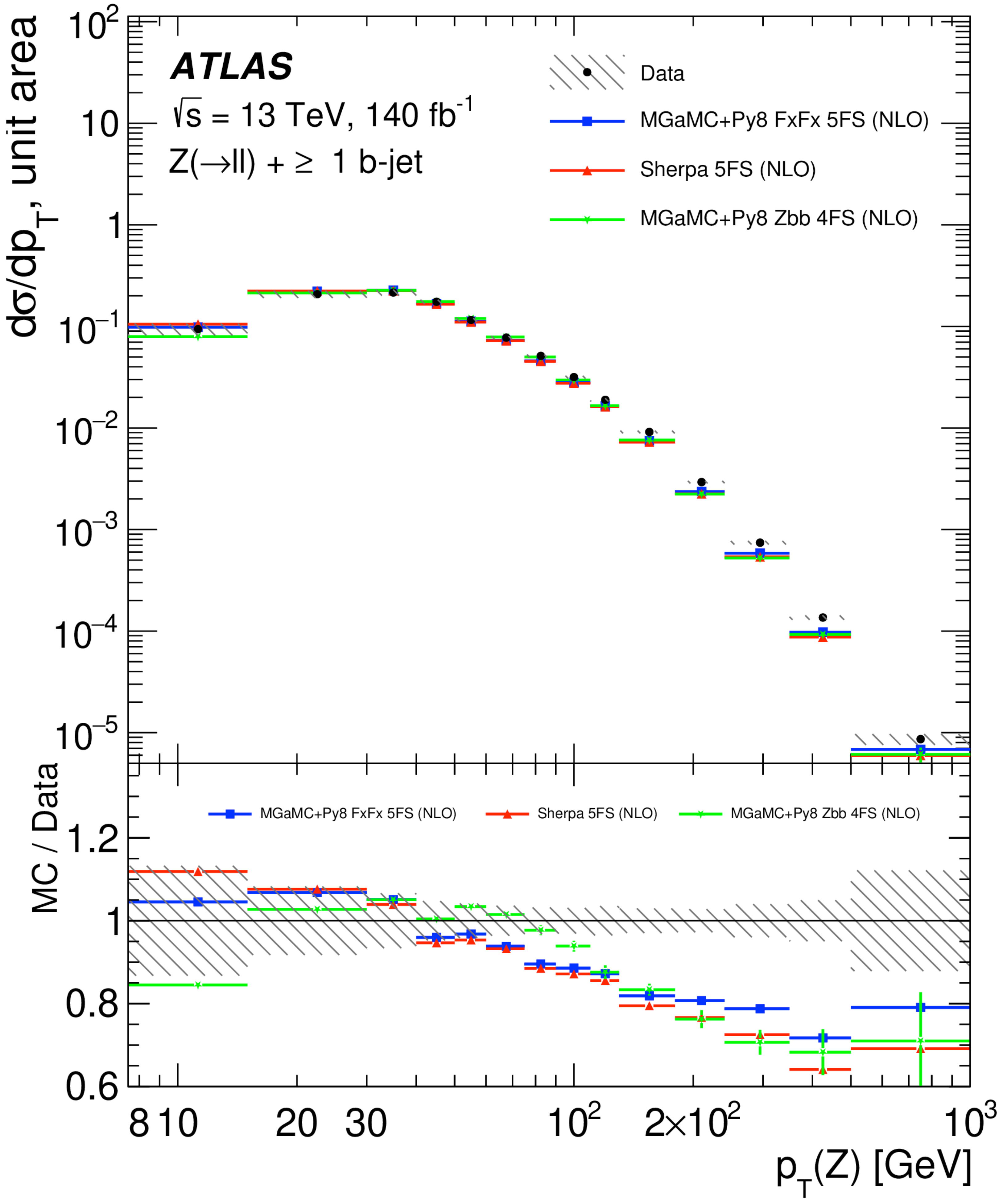
[ATLAS 2403.15093]



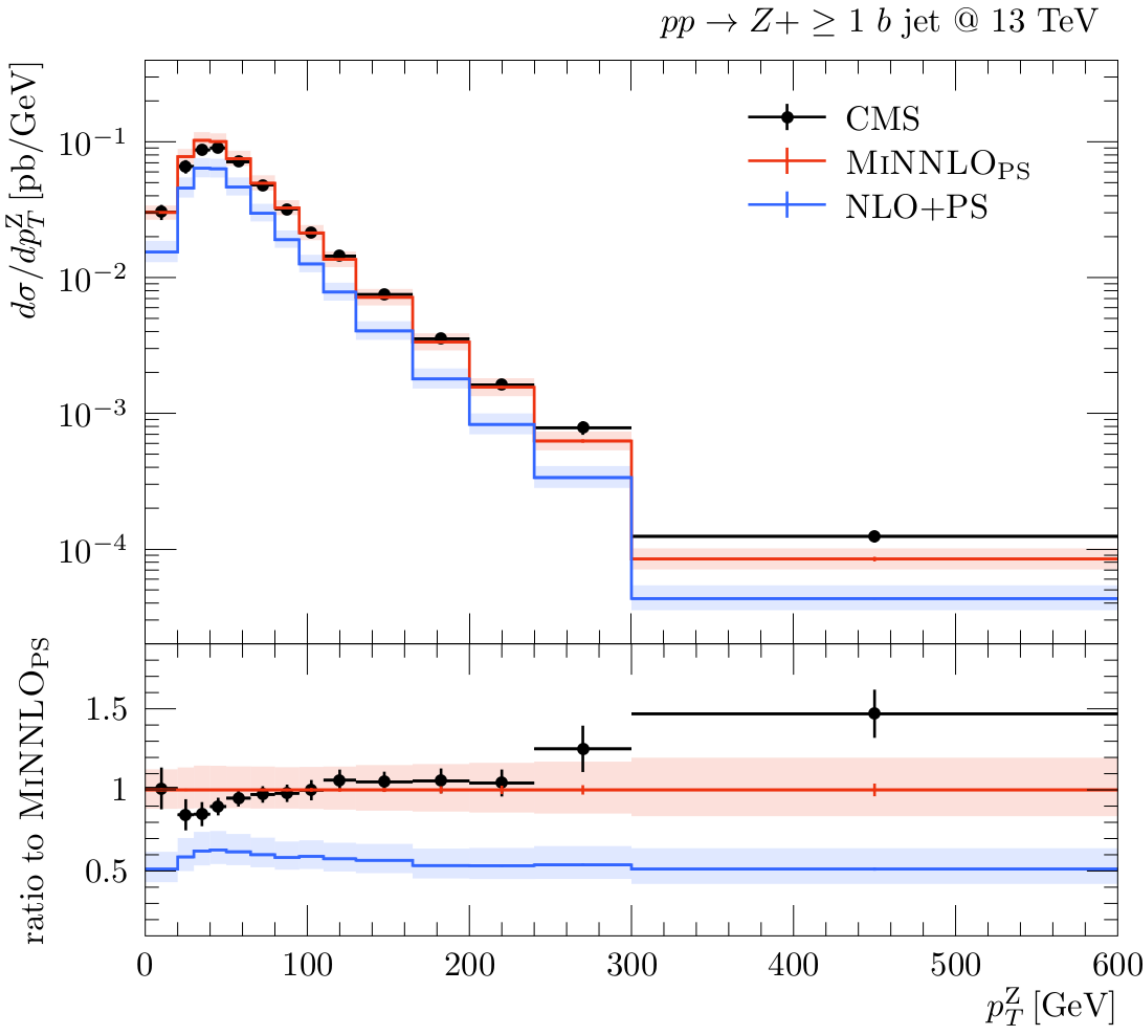
MiNNLO$_{\text{PS}}$: $b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

[ATLAS 2403.15093]



Z p$_T$ spectrum compared to CMS data [CMS 2112.09659]



Summary

★ NNLO+PS for 2 ➝ 2 available for colour singlet processes

★ First coloured processes at NNLO+PS: Heavy quark pair production ($t\bar{t}$ and $b\bar{b}$)

★ both NNLO corrections and matching to PS crucial, e.g. to describe B hadrons and b-jets

★ First results for QQ+colour singlet NNLO+PS ($b\bar{b}Z$ and preliminary results for $t\bar{t}H$ and $b\bar{b}H$)

Outlook

★ other interesting QQ+colour singlet processes: $t\bar{t}Z, t\bar{t}W, b\bar{b}W, c\bar{c}X \ldots$

★ new developments also enable off-shell $t\bar{t}$ with full top quark decays at NNLO+PS

★ NNLO+PS for processes with light jets possible (but highly non-trivial)
only 1-jettiness known (but no good observable); k_T^{ness} ? [Buonocore, Grazzini, Haag, Rottoli, Savoini '22]

★ NNLO+PS for 2 → 2

★ First coloured proces ion ($t\bar{t}$ and $b\bar{b}$)

★ both NNLO correcti e B hadrons and b-jets

★ First results for QQ+ ry results for $t\bar{t}H$ and $b\bar{b}H$)

★ other interesting QQ $X\ldots$

★ new developments al s at NNLO+PS

★ NNLO+PS for proce ivial)

only 1-jettiness know Grazzini, Haag, Rottoli, Savoini '22]

Stay tuned !

Back Up

MiNNLOPS for heavy quarks

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q) + B^{(1)}\right) + \frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q) + B^{(2)}\right) + \ldots\right]$$

'B-type' correction to Sudakov

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) = \langle M\,|\,\boldsymbol{\Delta}\,|\,M\rangle, \quad \boldsymbol{\Delta} = \mathbf{V}^{\dagger}\,\mathbf{D}\,\mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}} + \frac{\alpha_s^2(q)}{(2\pi)^2}\boldsymbol{\Gamma}^{(2)}_{\mathbf{t}}\right]\right\}$$

matrix in colour space

MiNNLO$_{\mathrm{PS}}$ for heavy quarks

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

$$\mathrm{d}\sigma^{F}_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$$\mathrm{Tr}(\mathbf{H\Delta}) = \langle M|\,\mathbf{\Delta}\,|M\rangle, \qquad \mathbf{\Delta} = \mathbf{V}^{\dagger}\,\mathbf{D}\,\mathbf{V}, \qquad \mathbf{V} = \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}^{(1)}_{\mathbf{t}}+\frac{\alpha_s^2(q)}{(2\pi)^2}\mathbf{\Gamma}^{(2)}_{\mathbf{t}}\right]\right\}$$

✦ approximations keeping NNLO and (N)LL

- azimuthal average with $\left[\mathbf{D}\right]_{\phi} = 1$ ➙ modifies $H \to \overline{H}$ and $(C\otimes f) \to \overline{(C\otimes f)}$ at α_s^2
  see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]
- $\langle M|\,\mathbf{\Delta}\,|M\rangle \approx \underbrace{\langle M|M\rangle}_{=H}\,\frac{\langle M^{(0)}|\,\mathbf{\Delta}\,|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}$ ⟵ re-absorb mistake at NNLO in $B^{(2)}$
- re-absorb in $B^{(2)}$ coefficient

  expand $\mathbf{V}$ $= \underbrace{\exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}^{(1)}_{\mathbf{t}}\right\}}_{\equiv\mathbf{V}_{\mathrm{NLL}}}\times\left(1-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s^2(q)}{(2\pi)^2}\mathbf{\Gamma}^{(2)}_{\mathbf{t}}\right)+\mathcal{O}(\mathrm{N^3LL})$

MiNNLOPS for heavy quarks

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

✦ using those approximations (exact up to NNLO & (N)LL) we have:

$$\tilde{B}^{(2)} = B^{(2)} + \frac{\langle M^{(0)}\,|\,\boldsymbol{\Gamma}^{(2)\dagger}+\boldsymbol{\Gamma}^{(2)}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle} + \frac{2\,\mathrm{Re}\left\{\langle M^{(1)}\,|\,\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle} - \frac{2\,\langle M^{(0)}\,|\,\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\,\mathrm{Re}\left\{\langle M^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle^2}$$

and $$e^{-S}\,\langle M\,|\,\boldsymbol{\Delta}\,|\,M\rangle = e^{-\tilde{S}}\,\frac{\langle M^{(0)}\,|\,\mathbf{V}^\dagger_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle}\,H + \mathcal{O}(\alpha_s^5)$$

(reminder: $\mathbf{V}_{\mathrm{NLL}} \equiv \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}\right\}$)

MiNNLO$_{\rm PS}$ for heavy quarks

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

$$\mathrm{d}\sigma^{F}_{\rm res} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{\mathrm{d}q^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\dots\right]$$

✦ using those approximations (exact up to NNLO & (N)LL) we have:

$$\tilde{B}^{(2)} = B^{(2)} + \frac{\langle M^{(0)}|\,\boldsymbol{\Gamma}^{(2)\dagger}+\boldsymbol{\Gamma}^{(2)}\,|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle} + \frac{2\,\mathrm{Re}\left\{\langle M^{(1)}|\,\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}\,|M^{(0)}\rangle\right\}}{\langle M^{(0)}|M^{(0)}\rangle} - \frac{2\,\langle M^{(0)}|\,\boldsymbol{\Gamma}^{(1)\dagger}+\boldsymbol{\Gamma}^{(1)}\,|M^{(0)}\rangle\,\mathrm{Re}\left\{\langle M^{(1)}|M^{(0)}\rangle\right\}}{\langle M^{(0)}|M^{(0)}\rangle^2}$$

and $$e^{-S}\langle M|\,\boldsymbol{\Delta}\,|M\rangle = e^{-\tilde{S}}\,\underbrace{\frac{\langle M^{(0)}|\,\mathbf{V}^{\dagger}_{\rm NLL}\mathbf{V}_{\rm NLL}\,|M^{(0)}\rangle}{\langle M^{(0)}|M^{(0)}\rangle}}\,H + \mathcal{O}(\alpha_s^5)$$

use basis $|M^{(0)}\rangle$ where $\boldsymbol{\Gamma}^{(1)}$ diagonal

$$= \sum_{i\in\text{colours}} c_i\,\underbrace{e^{-\tilde{S}+S_i}}_{\equiv\, e^{\bar{S}_i}}$$

eigenvalues of $\mathbf{V}^{\dagger}_{\rm NLL}\mathbf{V}_{\rm NLL}$ exponent

(reminder: $\mathbf{V}_{\rm NLL} \equiv \exp\left\{-\int\frac{\mathrm{d}q^2}{q^2}\frac{\alpha_s(q)}{2\pi}\boldsymbol{\Gamma}^{(1)}_{\mathbf{t}}\right\}$)

MiNNLOPS for heavy quarks

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

$$\mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$\ldots{}^{2)}\log(M/q) + B^{(2)}\big) + \ldots\Big]$$

MiNNLOPS for colour singlets

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- starting equation: $\boxed{\dfrac{\mathrm{d}\sigma^{\mathrm{res}}_F}{\mathrm{d}p_T\,\mathrm{d}\Phi_{\mathrm{B}}} = \dfrac{\mathrm{d}}{\mathrm{d}p_T}\left\{e^{-S}\mathscr{L}\right\}} = e^{-S}\underbrace{\left\{S'\mathscr{L} + \mathscr{L}'\right\}}_{\equiv D}$

$\mathscr{L} \sim H\,(C\otimes f)\,(C\otimes f)$

…LL) we have:

$$\ldots\boldsymbol{\Gamma}^{(1)\dagger} + \boldsymbol{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\,\mathrm{Re}\left\{\langle M^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle^2}$$

and $$e^{-S}\langle M\,|\,\boldsymbol{\Delta}\,|\,M\rangle = \underbrace{e^{-\tilde{S}}\frac{\langle M^{(0)}\,|\,\mathbf{V}^\dagger_{\mathrm{NLL}}\mathbf{V}_{\mathrm{NLL}}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle}} H + \mathcal{O}(\alpha_s^5)$$

$$= \sum_{i\in\mathrm{colours}} c_i\,\underbrace{e^{-\tilde{S}+S_i}}_{\equiv\, e^{\bar{S}_i}}$$

simplified to sum of terms with same structure as starting formula for colour singlet case

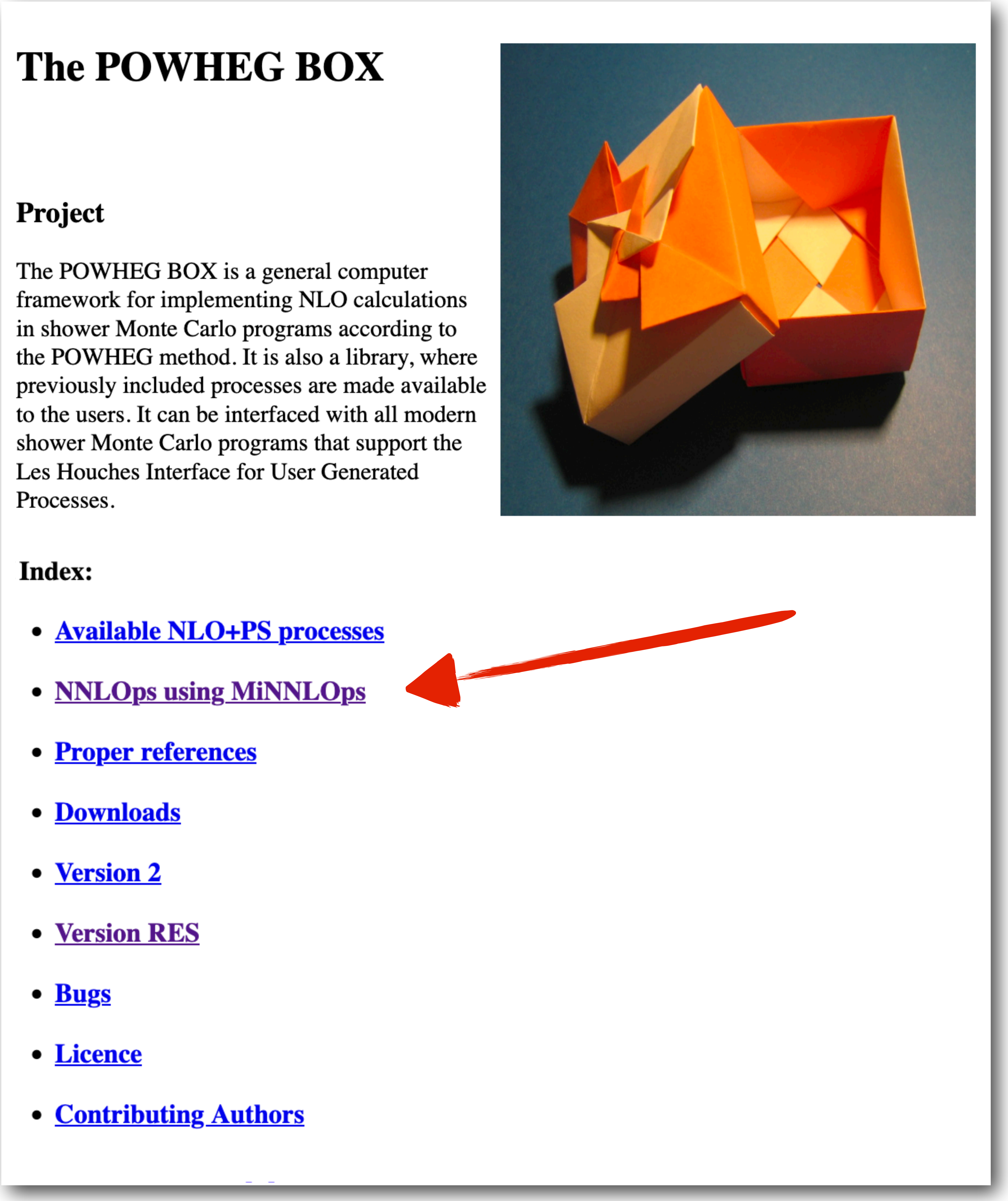
$$\boxed{\Rightarrow\ \mathrm{d}\sigma^F_{\mathrm{res}} \sim \frac{\mathrm{d}}{\mathrm{d}p_T}\left\{\sum_{i\in\mathrm{colours}} e^{-\bar{S}_i}\,\underbrace{c_i\,\overline{H}\,\overline{(C\otimes f)}\,\overline{(C\otimes f)}}_{\equiv\,\overline{\mathscr{L}}_i}\right\} + \text{terms beyond NNLO \& (N)LL}}$$

Setup for $t\bar{t}$ MiNNLOPS

[Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]

- scale setting:
  - overall factor in Born: $\alpha_s^2(m_{t\bar{t}}/2)$
  - MiNNLOPS scales: $\mu_R = \mu_F = \frac{m_{t\bar{t}}}{2} e^{-L}, \quad Q = \frac{m_{t\bar{t}}}{2}$
    (no direct correspondence to fixed-order ➙ differences within uncertainties expected)
  - 7-point scale variation
    (including scales in Sudakov ➙ slightly more conservative than in NNLO)
- new modified logarithm: $L = \begin{cases} \log\left(\frac{Q}{p_T}\right) & \text{for } p_T \leq Q/2 \\ 0 & \text{for } p_T \geq Q \end{cases}$
- showered with Pythia8, keeping top quarks stable
- comparison to data unfolded to inclusive phase space [CMS PRD 97 (2018) 112003]

MiNNLO$_{PS}$ generators public in POWHEG BOX



The POWHEG BOX

Project

The POWHEG BOX is a general computer framework for implementing NLO calculations in shower Monte Carlo programs according to the POWHEG method. It is also a library, where previously included processes are made available to the users. It can be interfaced with all modern shower Monte Carlo programs that support the Les Houches Interface for User Generated Processes.

Index:

- Available NLO+PS processes
- NNLOps using MiNNLOps
- Proper references
- Downloads
- Version 2
- Version RES
- Bugs
- Licence
- Contributing Authors

MiNNLO$_{PS}$ for $2 \to 1$ processes (H, Z, W) in POWHEG-BOX-V2

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

NEW

***Top-quark pair** generator now available [Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]*

MiNNLO$_{PS}$ has been extended to $2 \to 2$ colour-singlet processes (built in POWHEG-BOX-RES).

NEW

*First implementation of **Zγ** generator (both $Z \to \ell^+\ell^-$ and $Z \to \bar{\nu}\nu$ + aTGC @NNLO) [Lombardi, MW, Zanderighi '20, '21]*

NEW

*New approach to the existing **WW** generator [Lombardi, MW, Zanderighi '21]*

NEW

***ZZ** generator with incoherent combination of $\bar{q}q$ and gg channels [Buonocre, Koole, Lombardi, Rottoli, MW, Zanderighi '21]*

NEW

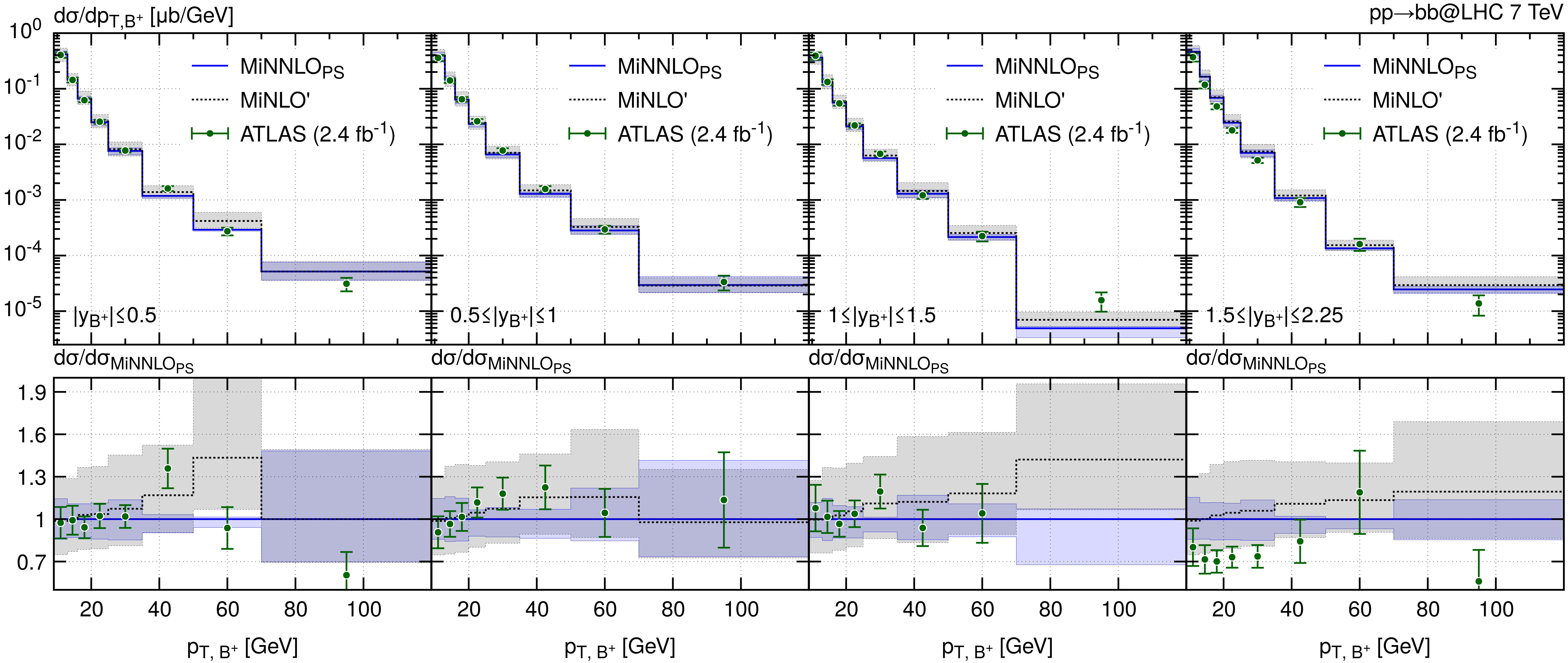
***VH** generator interfaced with **H→bb** decay (t.b.a.) [Zanoli, Chiesa, Re, MW, Zanderighi 'ongoing]*

NEW

More to come …

MiNNLO$_{PS}$: B-hadron production

[Mazzitelli, MW, Zanderighi, Ratti ’23]



MiNNLO$_{PS}$: B-hadron production

[Mazzitelli, MW, Zanderighi, Ratti ’23]

Setup of the calculation

- 13TeV collisions, $bb\ell\ell$ final state with ℓ=e,μ, m_b=4.92GeV, NNPDF31
- MiNNLO central scale setting: $\mu_R=\mu_F=m_{bb\ell\ell}\, e^{-L}$, $Q=m_{bb\ell\ell}/2$
  Born coupling central scale: $\mu_R^{(0)} = m_{bb\ell\ell}$
- Modified log $L = \log(Q/p_T)$ for $p_T<Q/2$, $L = 0$ for $p_T>Q$, interpolation in between
- Showering with Pythia8, using Monash tune
  Hadronization, multi-parton interactions and QED shower included
- OpenLoops for tree and one-loop amplitudes, including color- and spin-correlated
- Two-loop amplitudes from analytic results
    - Large expressions O(1Gb) → elaborate numerical stability checks and rescue system through higher precision
    - Evaluation of special functions through PentagonFunctions++

[Chicherin, Sotnikov, Zoia '21]