THE ROYAL SOCIETY

Gravitational Wave Background Creates Weyl Fermions!

Azadeh Malek-Nejad

King's College London

Setup

1) Quantum Fluctuations in Cosmology

2) Gravitational Particle Production

3) GW-infuced Fermion Production

4) Outlook

w/ Joachim Kopp

arXiv:2405.09723
arXiv:2406.01534

Quantum Fluctuations in Cosmology

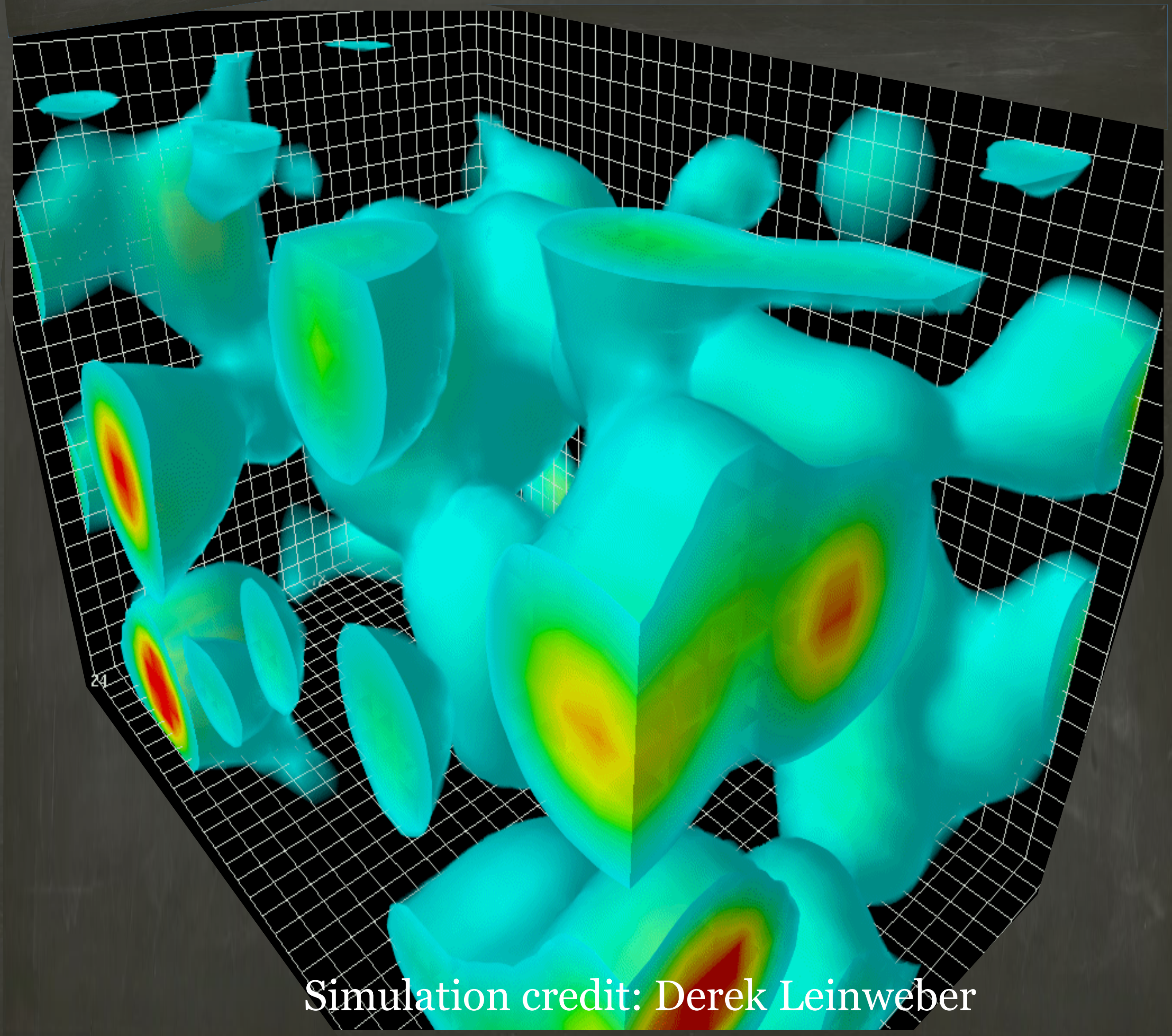
$$\hbar \neq 0$$

Quantum Vacuum $\hbar \neq 0$

Due to Uncertainty Principle

$$\Delta x \, \Delta p \geq \hbar/2$$

quantum vacuum is NOT nothing!



Simulation credit: Derek Leinweber

Quantum Vacuum $\hbar \neq 0$

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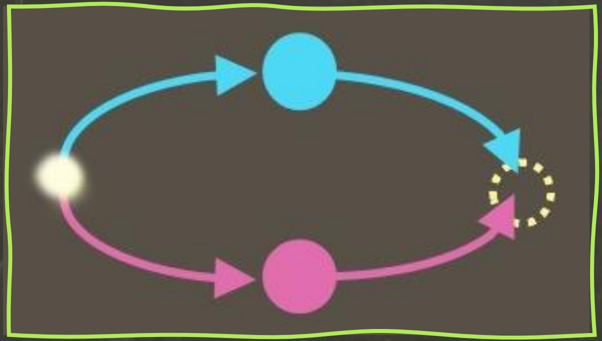
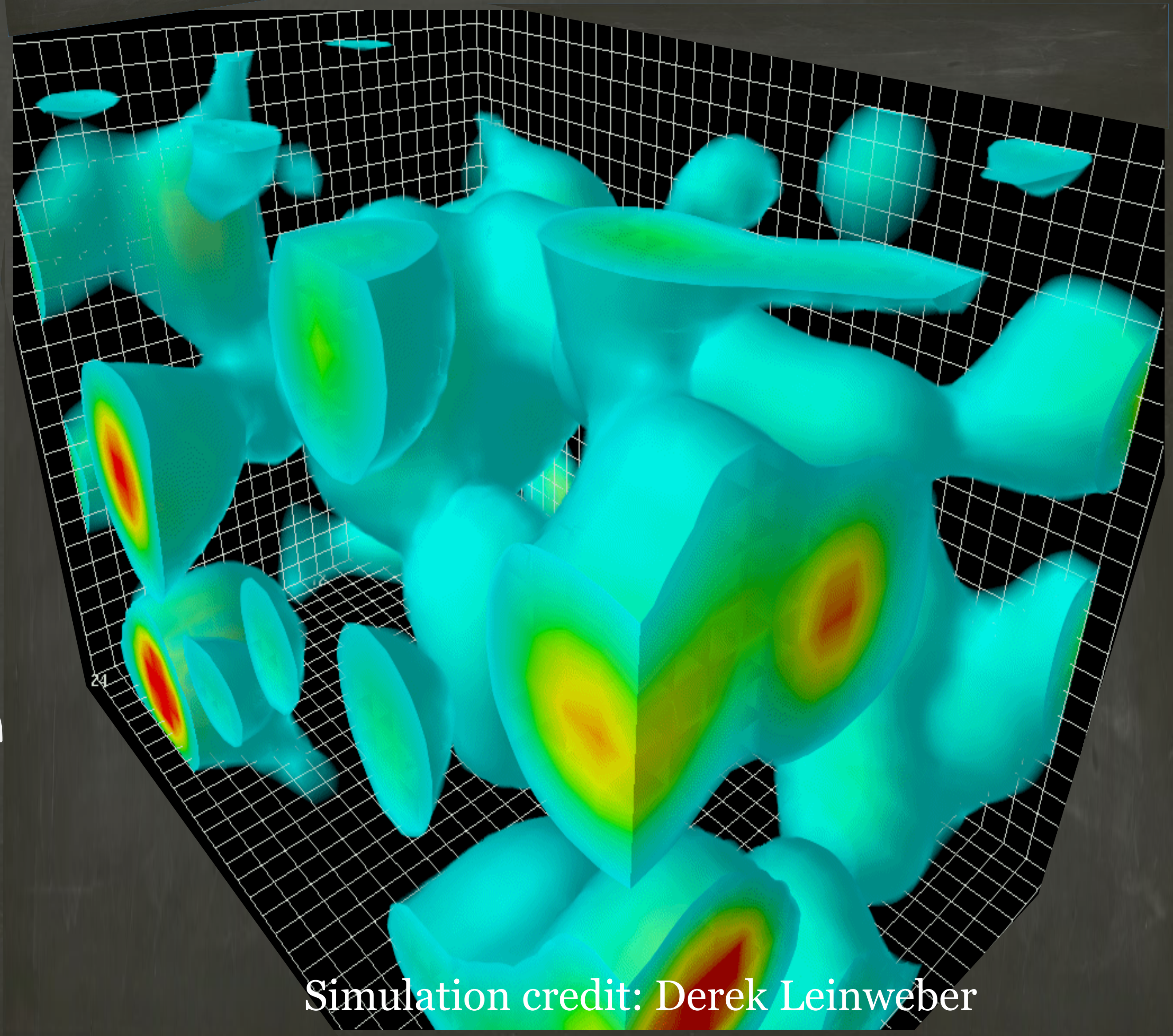
But, a vast ocean made of

Virtual particles

vacuum  vacuum



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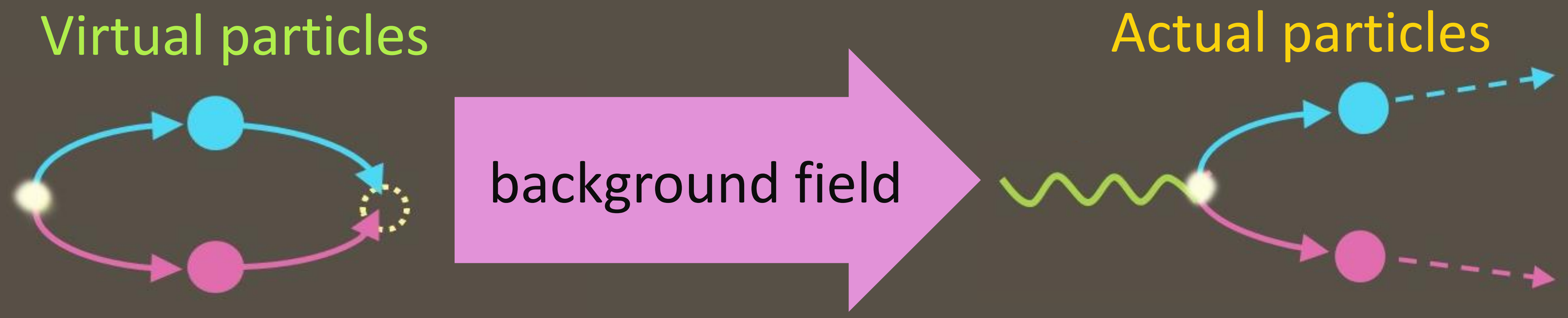
Quantum Vacuum → Particle Production

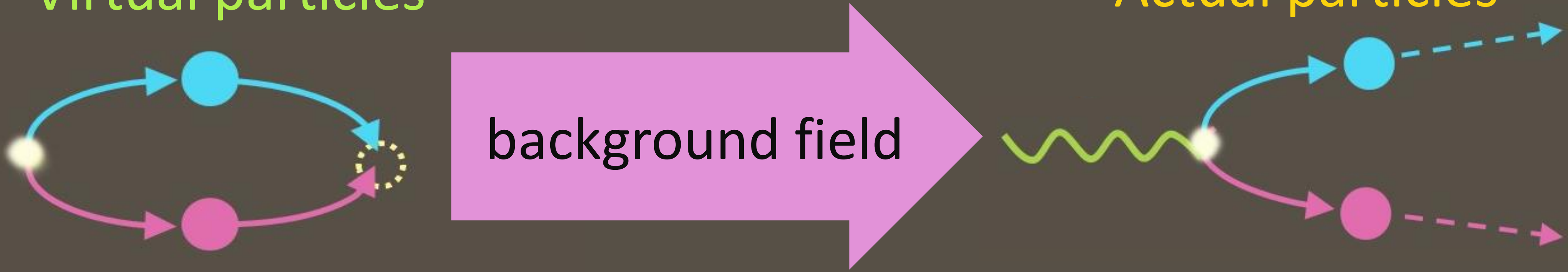


Background field can upgrade them into actual particles!

Quantum Vacuum — Particle Production

Virtual particles

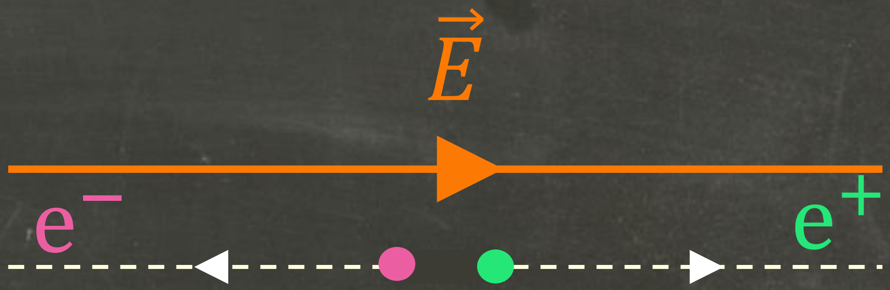
Actual particles



Background field can upgrade them into actual particles!

Examples of such BG fields:

1) Electric Field *Schwinger effect*



Work of the Lorentz force over Comptom wavelength

$$eE\,\lambda_{comp} = mc^2$$

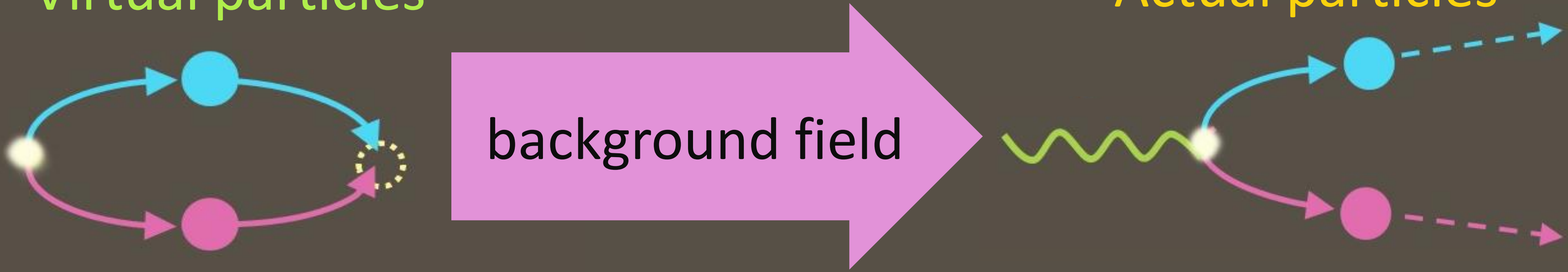
$$E = \frac{m^2c^3}{e\hbar} = 10^{18}\,V/m$$

J. Schwinger (1951)

Quantum Vacuum → Particle Production

Virtual particles

Actual particles



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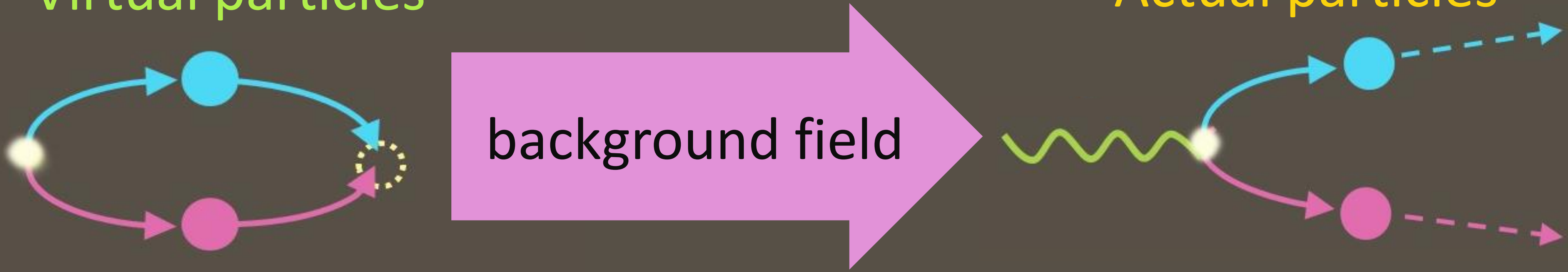
J. Schwinger (1951)

Schwinger Effect in Early Universe

- K. Lozanov, **A. M**, E. Komatsu **2017**
- **A. M.**, E. Komatsu **2018**
- L. Mirzagholi, **A. M**, K. Lozanov **2019**
- V. Domcke et al 2019
- E. Komatsu **2022** (Nature Review)

Quantum Vacuum — Particle Production

Virtual particles — Actual particles



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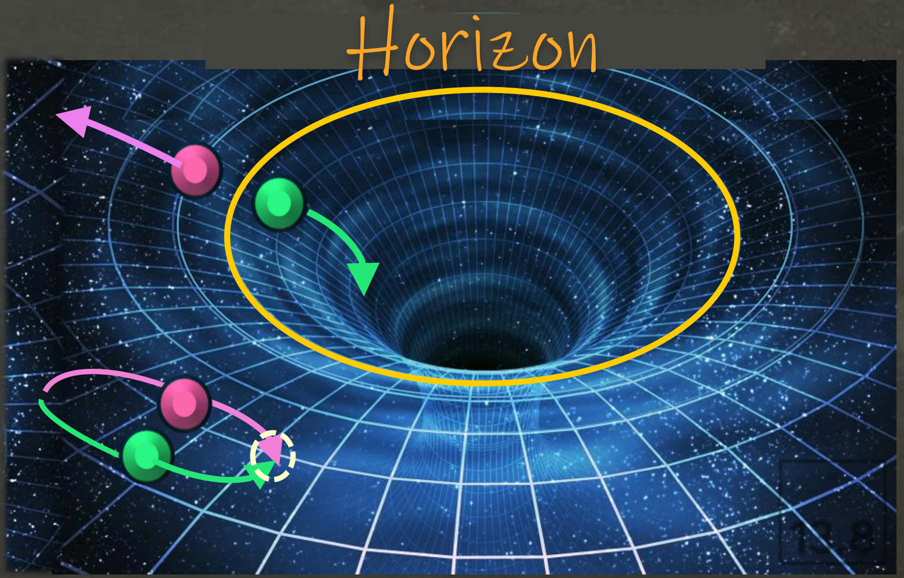
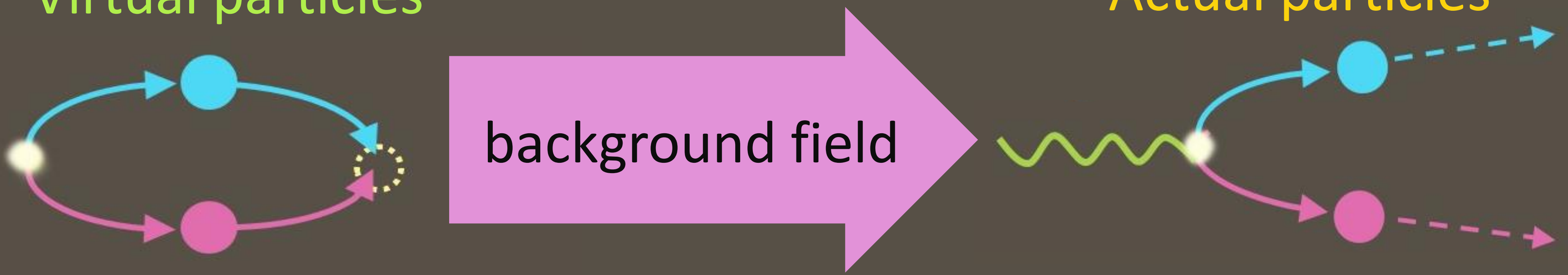
2) Gravitational *Hawking radiation*



Quantum Vacuum   Particle Production

Virtual particles   Actual particles



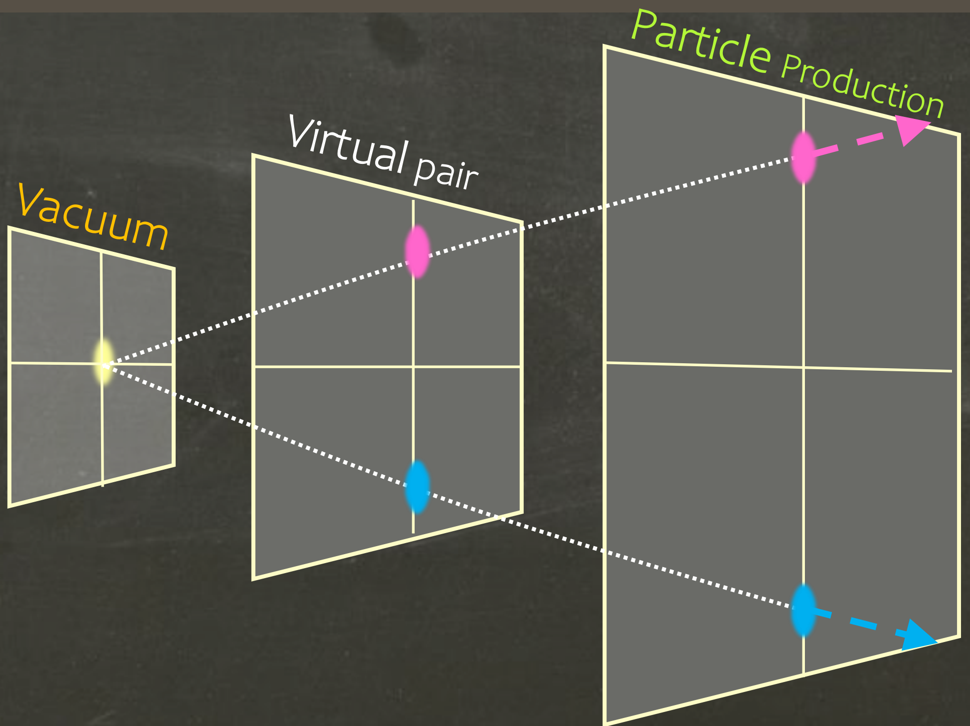
Background field can upgrade them into actual particles!

Examples of such BG fields:

1) Electric Field *Schwinger effect*

2) Gravitational *Hawking radiation*

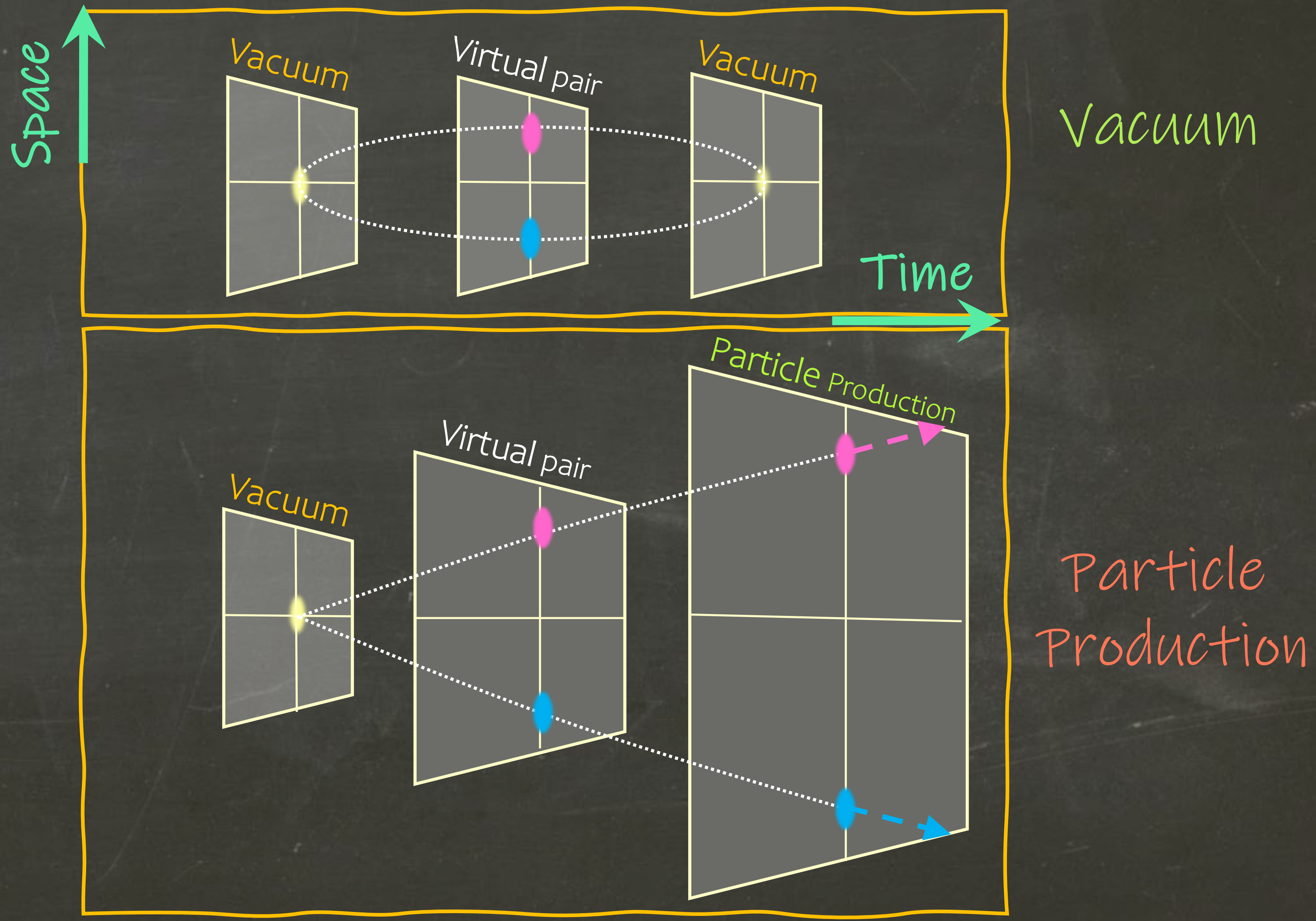
or expansion of the Universe!



Expanding Universe Produces Particles!

Flat Space:

Expanding space:



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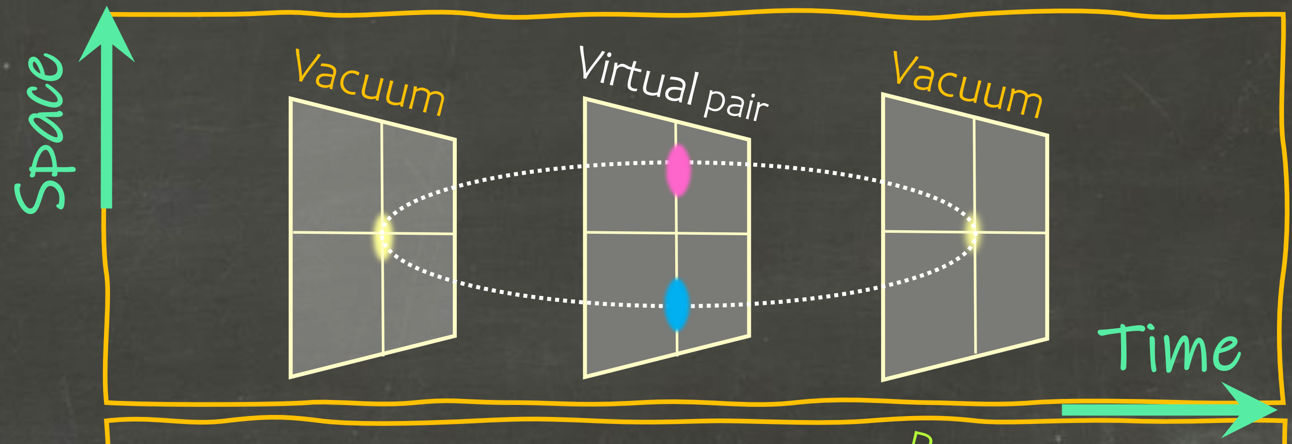
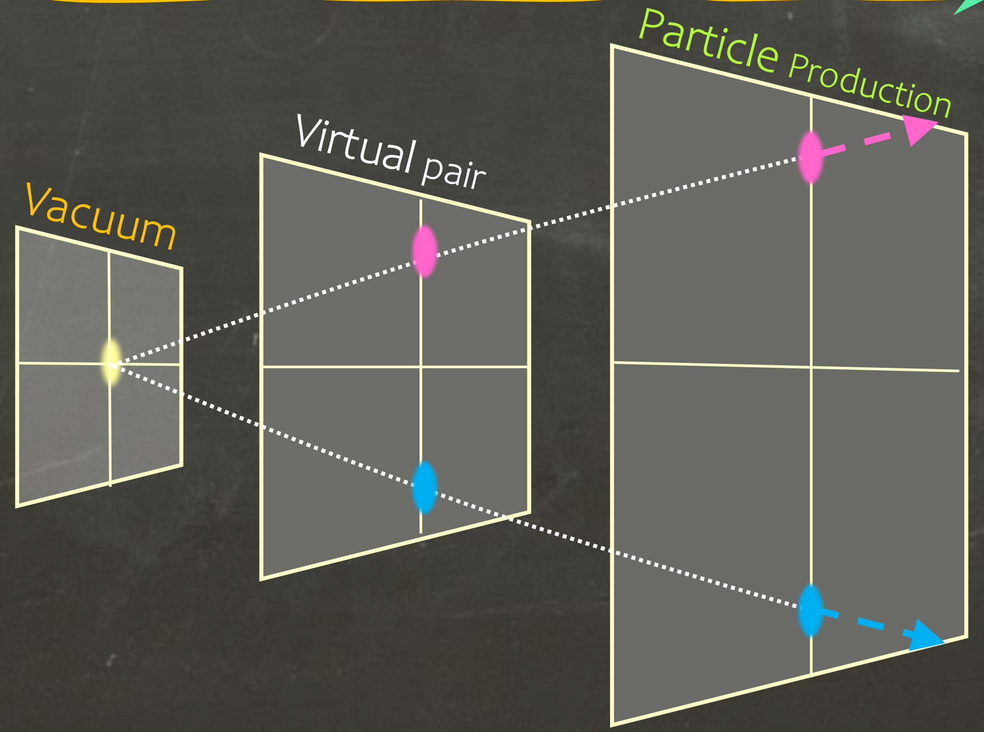


Vacuum

Expanding space:



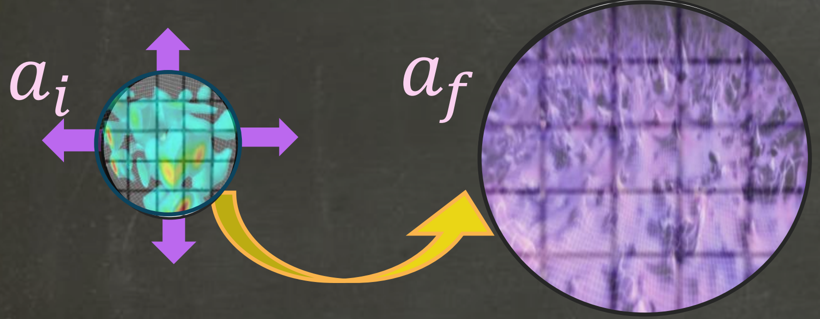
Particle Production

Edwin Schrödinger
(1939)

Shocked by his discovery, Schrödinger found it an alarming phenomenon!

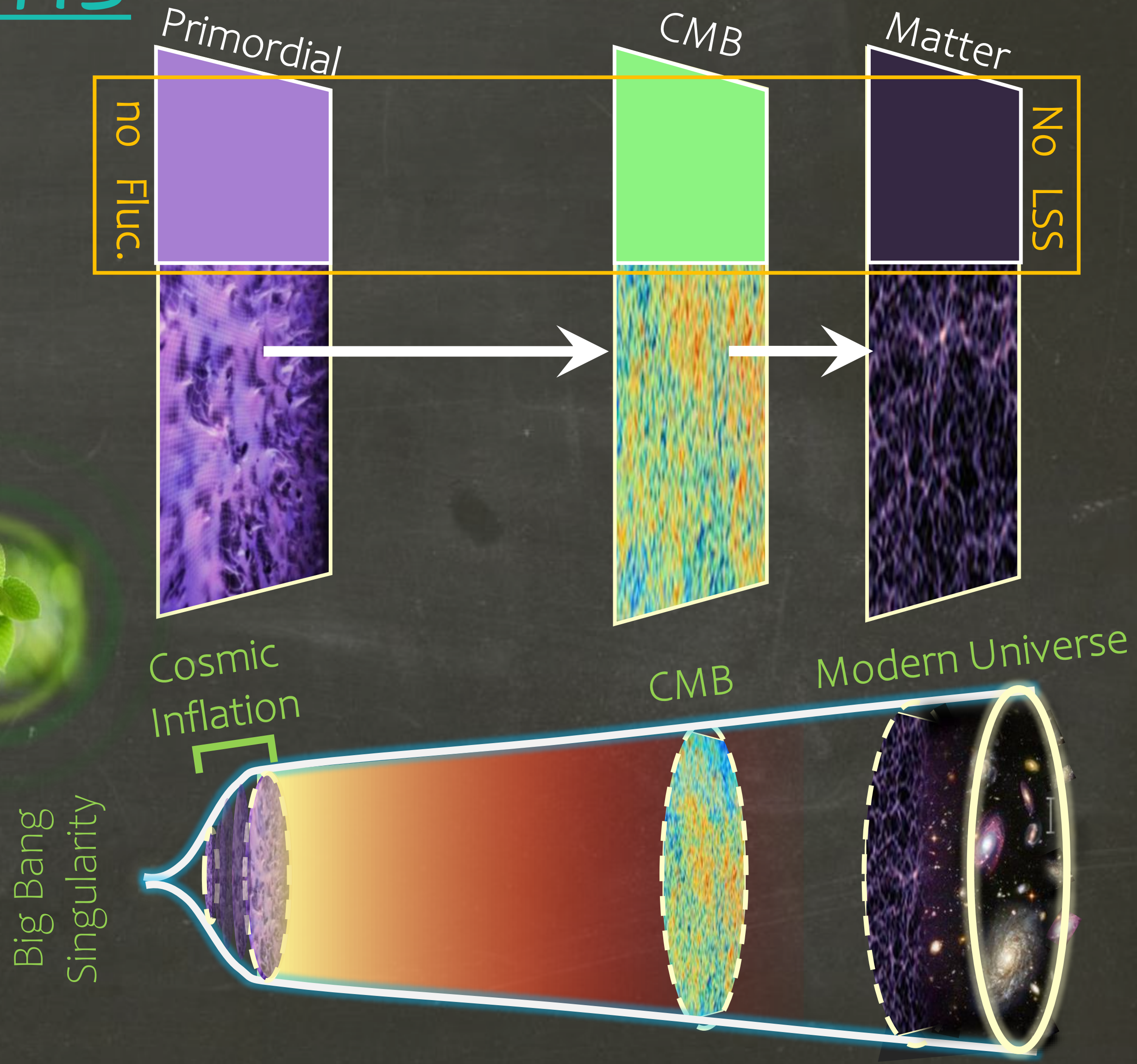
Cosmic Perturbations

Exponential expansion turns initial quantum vacuum fluctuations into



actual cosmic perturbations!

We are the product of quantum fluctuations in the very early universe!



Gravitational Particle Production

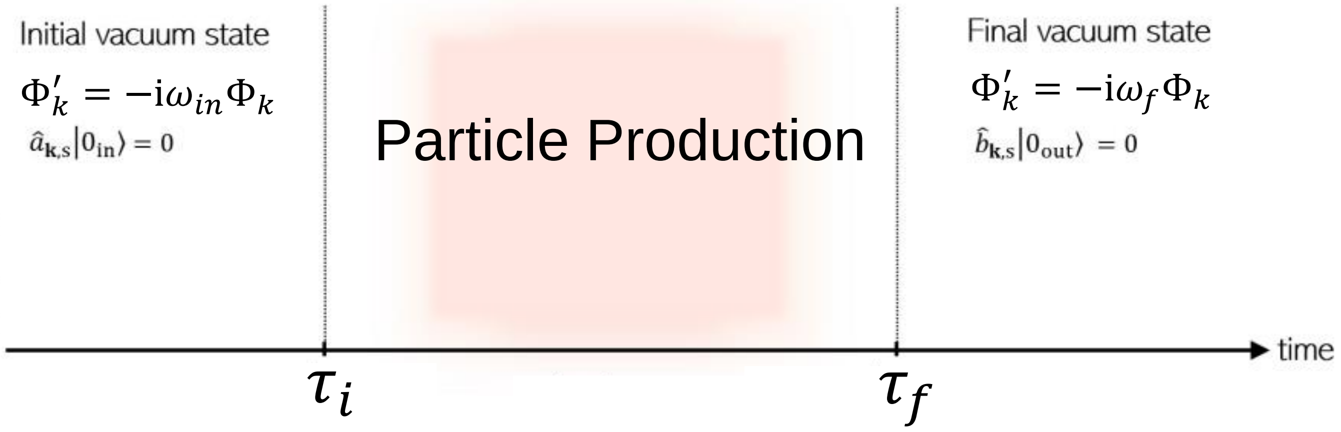
Scalar Field in Expanding Universe

Consider scalar field $\mathcal{L} = \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} m^2 \Phi^2 + \frac{1}{2} \xi \Phi^2 R$.

In cosmological background $ds^2 = -dt^2 + a^2(t)\, \delta_{ij}\, dx_i dx_j$,

The field equation of scalar field is $\Phi_k{}'' + \omega^2(\tau)\Phi_k = 0$

Effective frequency $\omega_k^2(\tau) = k^2 + a^2(\tau) m^2 + \left(\frac{1}{6} - \xi\right) a^2(\tau) R(\tau)$



Scalar field can feel the expansion of Universe

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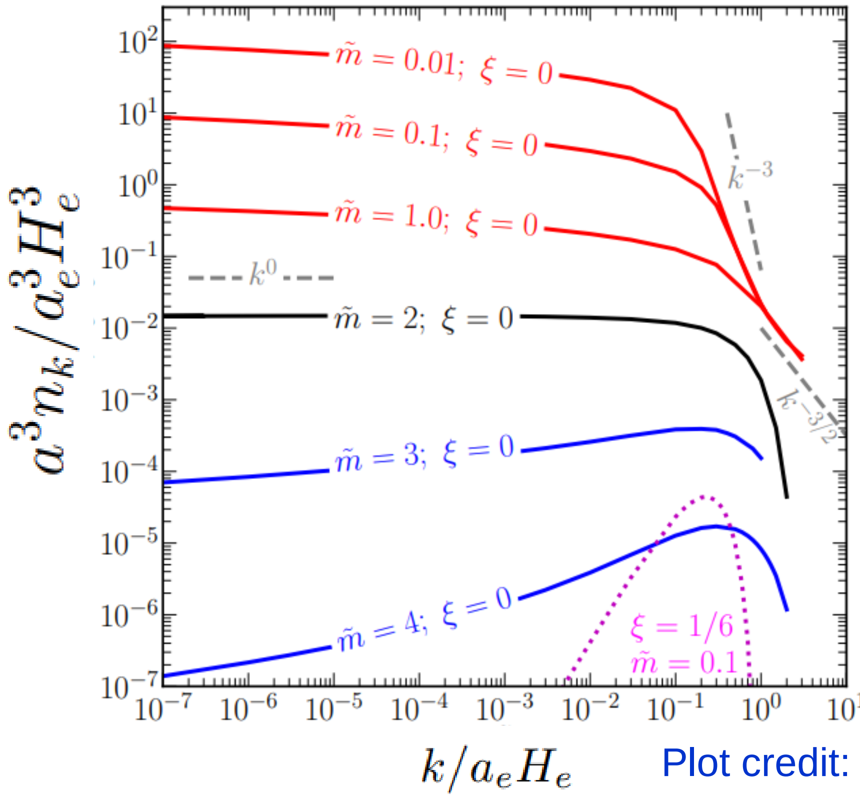

Plot credit: Kolb & Long 2023

$$\Phi_k'' + \omega^2(\tau)\Phi_k = 0$$

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Scalar field can feel the expansion of Universe

An example of **Cosmological Gravitational Particle Production (CGPP)**

Fermions in Expanding Universe

Consider spin $\frac{1}{2}$ massless fermions $\mathcal{L}_\psi = i\psi_D^\dagger \gamma^\mu \mathcal{D}_\mu \psi_D,$

Spinor covariant derivative $\mathcal{D}_\mu = \nabla_\mu - \omega_\mu$ Spin connection

In cosmological background $ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx_i dx_j,$

The field equation of massless fermion is $\left(\gamma^0(\partial_0 + \frac{3}{2}H) + \frac{1}{a}\gamma^i\partial_i\right)\psi_D = 0.$

Effect of gravity

The effect of the cosmological background can be absorbed in the canonically renormalized field $\Psi \equiv a^{3/2}\psi_D.$

Lives in flat space

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No CGPP for massless fermions! (conformal symmetry)

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Lives in flat space

How to Create Fermions in Expanding Universe?

Breaking the conformal symmetry of Weyl fermions by interactions, e.g.

- Couple your Weyl fermion with
  - Inflaton field,
  - Standard Model,
  - Dark sector coupled to thermal bath
- make the fermion massive to produce them gravitationally!

Cosmological Gravitational Particle Production (CGPP)

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Cosmological Gravitational Particle Production (CGPP)

Is that the best Gravity can do to produce fermions!?

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Cosmological Gravitational Particle Production (CGPP)

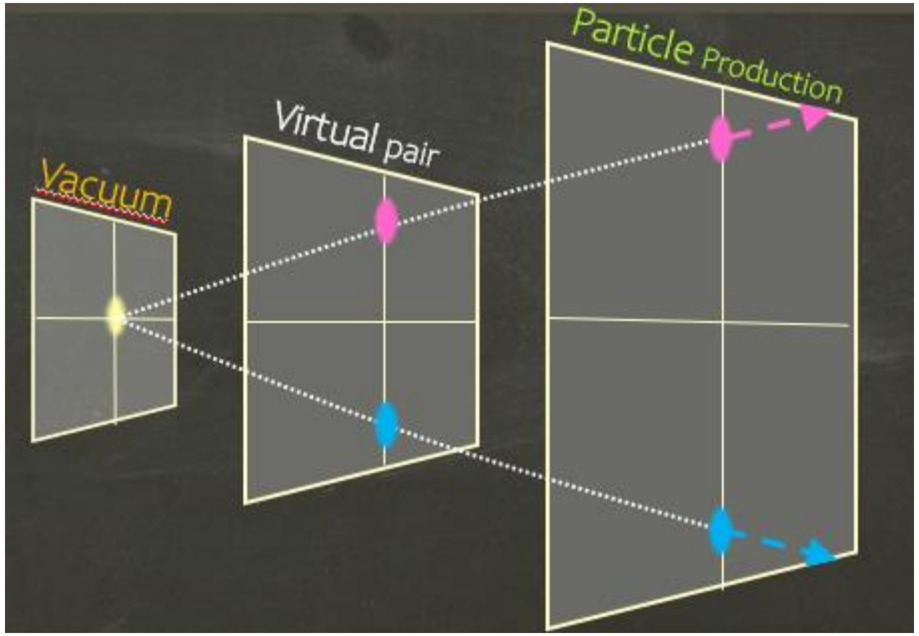
No!

Is that the best Gravity can do to produce fermions!?

Gravitational Particle Production Mechanism

	Production Mechanism	Underlying Physics	Conditions
I)	Cosmological Gravitational Particle Production (CGPP)	Cosmic expansion	super-massive fields M> 10^{13} GeV

Kolb & Long 2017



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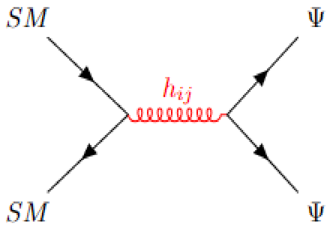
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}\ GeV$	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022

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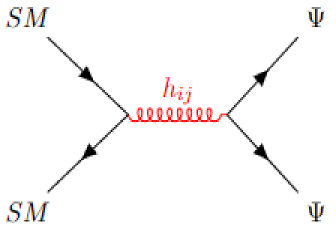
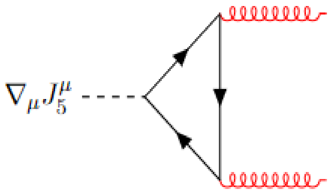
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}$ GeV	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation $h_L \neq h_R$ Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016

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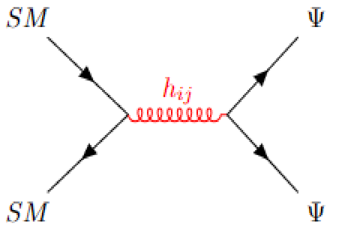
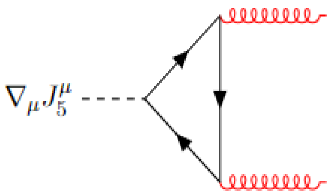
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}\ GeV$	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation $h_L \neq h_R$ Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016

What does Unpolarized Gravitational Waves do!?

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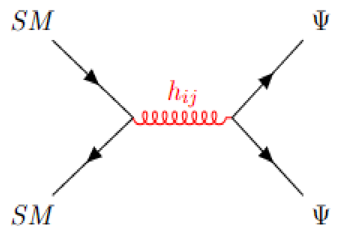
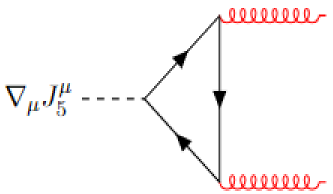
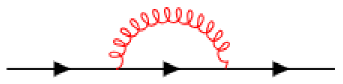
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation $h_L \neq h_R$ Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016
IV)	GW-Induced Freeze-In		GWs Background	**A.M.** & Kopp 2024

Gravitational Wave-induced Freeze-in

Based on
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arXiv:2405.09723
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Gravitational Wave-induced Freeze-in

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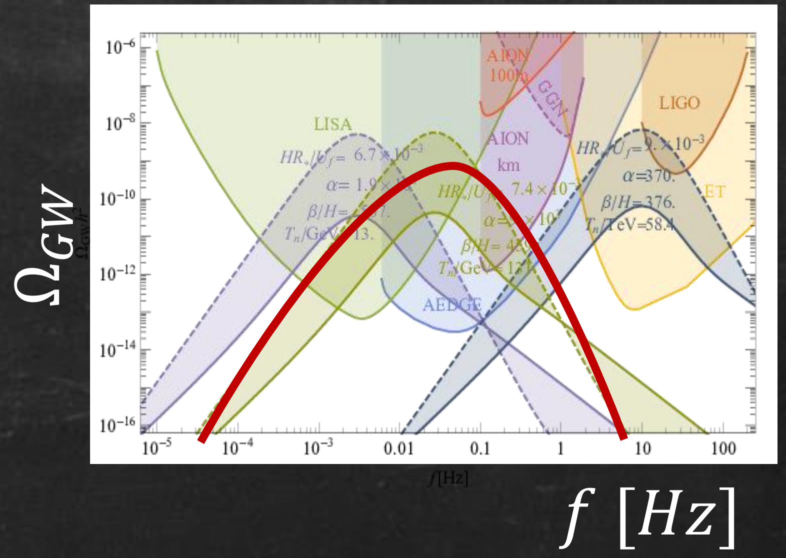
In a nutshell it is the production of Weyl fermions by a stochastic background of GWs as Ω_{GW}

Plot credit: Ellis et. Al. 2020



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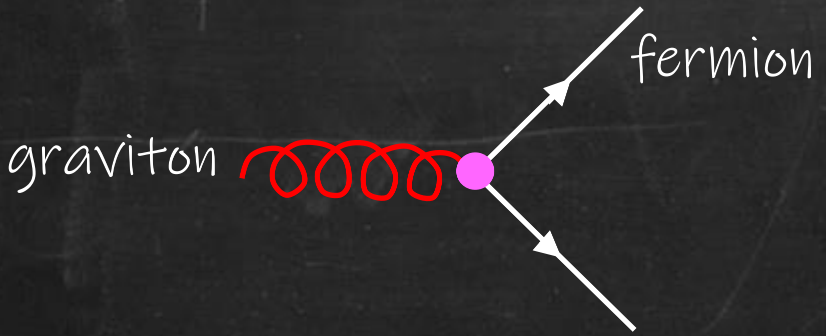

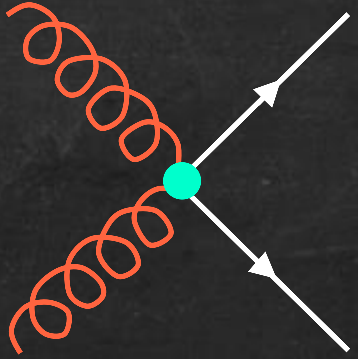

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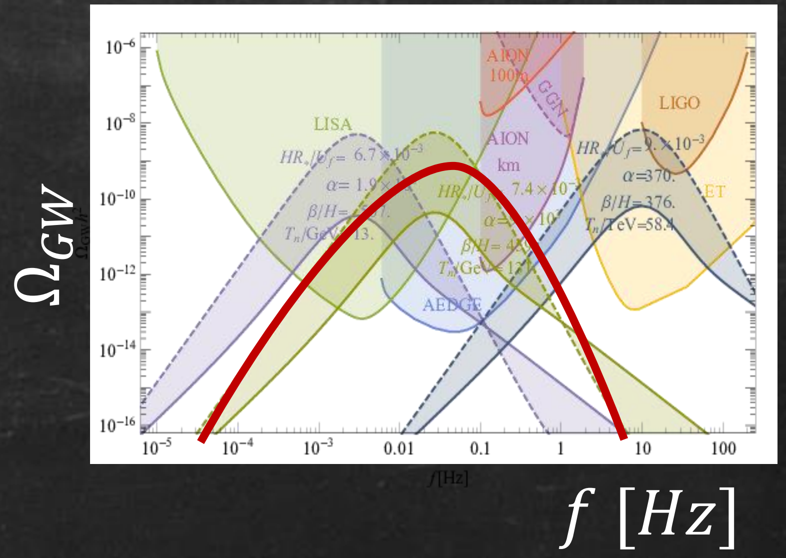

The graviton–fermion Interaction

Cosmological background with Gravitational Waves

transverse-traceless

$$ds^2 = -dt^2 + a^2(t)\,\hat{g}_{ij}\,dx_i dx_j,$$

$$\hat{g}_{ij} = \left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{jk} + \dots\right)$$

Consider spin $\frac{1}{2}$ Weyl fermions

$$\mathcal{L}_{\psi} = i\psi_D^{\dagger}\gamma^{\mu}\mathcal{D}_{\mu}\psi_D,$$

Free massless fermions can be written as $\Psi \equiv a^{3/2}\psi,$

Does not feel the expansion of Universe

The graviton–fermion Interaction

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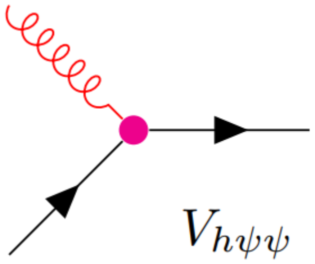
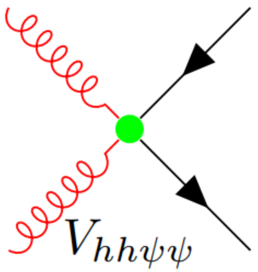
Cubic vertex



$$\mathcal{L}^{(2)}_{\rm int} = -\frac{i}{16a^3} \mathrm{e}^\mu{}_\alpha h_{ij}\partial_\mu h_{ik}\bar{\Psi}_D \Gamma^{\alpha jk}\Psi_D,$$

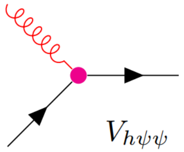
Quartic vertex



The graviton–fermion Interaction

We use In-In formalism to compute the energy density of Weyl fermions

$$\mathcal{L}^{(1)}_{\rm int} = -\frac{i}{2a^4} h_{ij} \bar{\mathbf{\Psi}}_D \gamma^i \overleftrightarrow{\partial}_j \mathbf{\Psi}_D .$$



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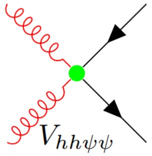

Expectation value of an arbitrary operator in In-In formalism

$$\langle Q(t)\rangle = \left\langle \bar{\mathrm{T}} \exp\left[i \int_{t_i^-}^{t} dt''\, H_{\rm int}(t'') \right] Q_I(t)\, \mathrm{T} \exp\left[-i \int_{t_i^+}^{t} dt'\, H_{\rm int}(t') \right] \right\rangle ,$$

Interaction Hamiltonian

$$H_{\rm int}(t) = -\int d^3x\, a^3(t)\, \mathcal{L}_{\rm int}(t, \mathbf{x}),$$

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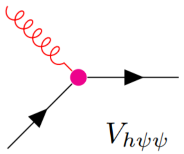

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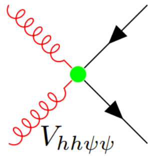

Energy density of Weyl fermions

$$\rho_\psi(\tau, \mathbf{x}) = T_{\mu\nu} n^\mu n^\nu = \frac{i}{a^4} \mathbf{\Psi}^\dagger \overleftrightarrow{\partial}_\tau \mathbf{\Psi} - \mathcal{L}_\psi,$$

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$$\mathcal{L}_{\text{int}}^{(2)} = -\frac{i}{16a^3} \mathbf{e}^{\mu}{}_{\alpha} h_{ij} \partial_\mu h_{ik} \bar{\boldsymbol{\Psi}}_D \Gamma^{\alpha jk} \boldsymbol{\Psi}_D ,$$

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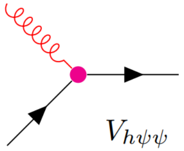
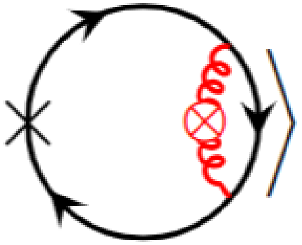
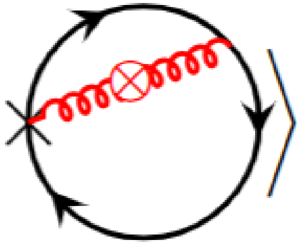
Diagrammatically we have



$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle$,

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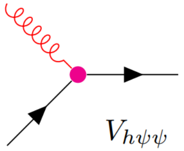
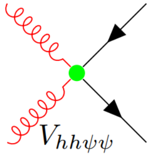
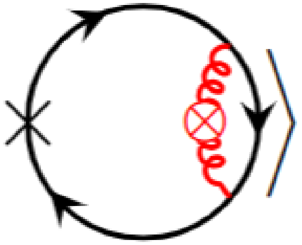
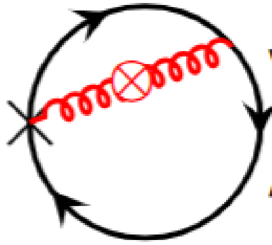
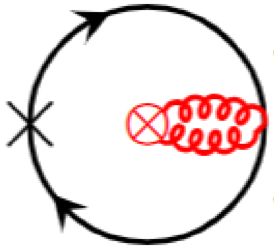
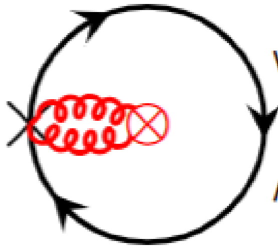




$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle$,

$\varrho_3 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_4 = \Big\langle \rho_\psi^{(2)}$  $\Big\rangle$.

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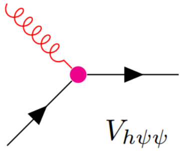
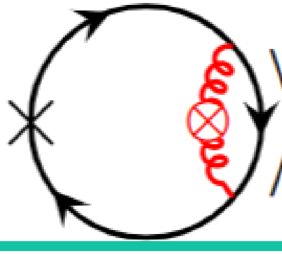
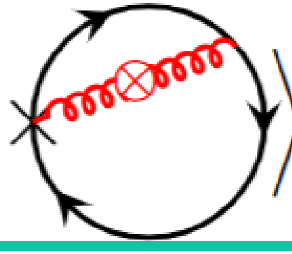


$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle$,

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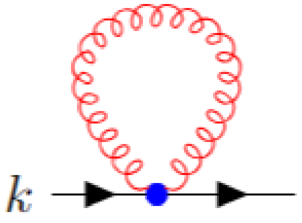
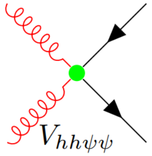
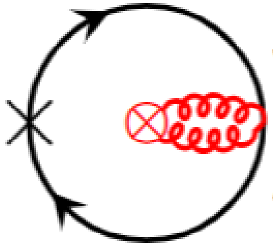
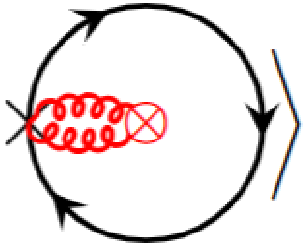




$\varrho_3 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_4 = \Big\langle \rho_\psi^{(2)}$  $\Big\rangle$.

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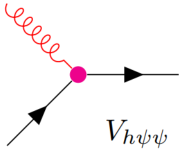
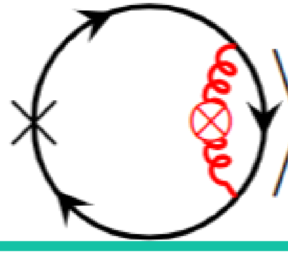
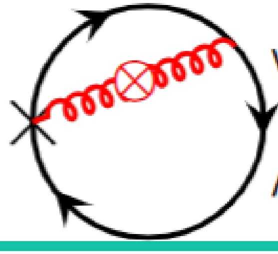


$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle$,

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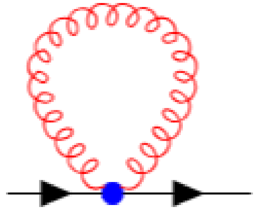

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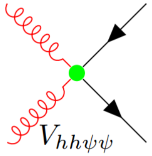
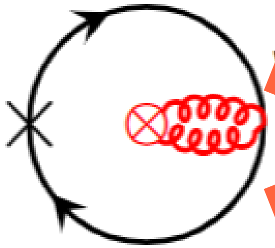
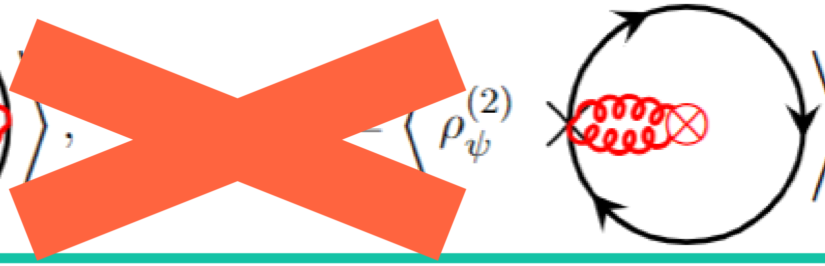


$\varrho_3 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$, $\Big\langle \rho_\psi^{(2)}$  $\Big\rangle$.

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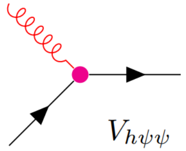

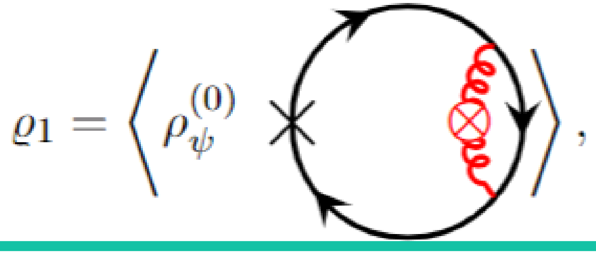

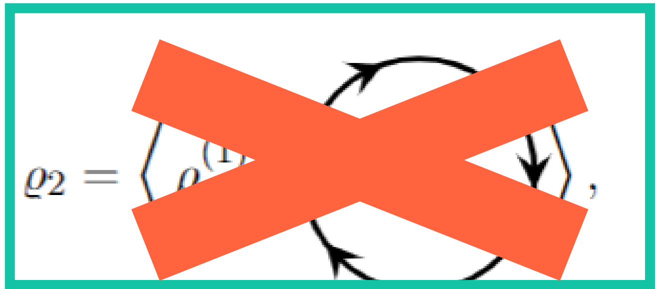

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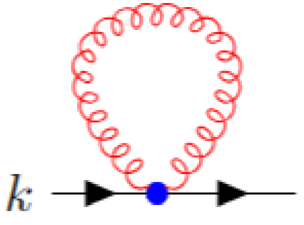

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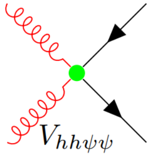

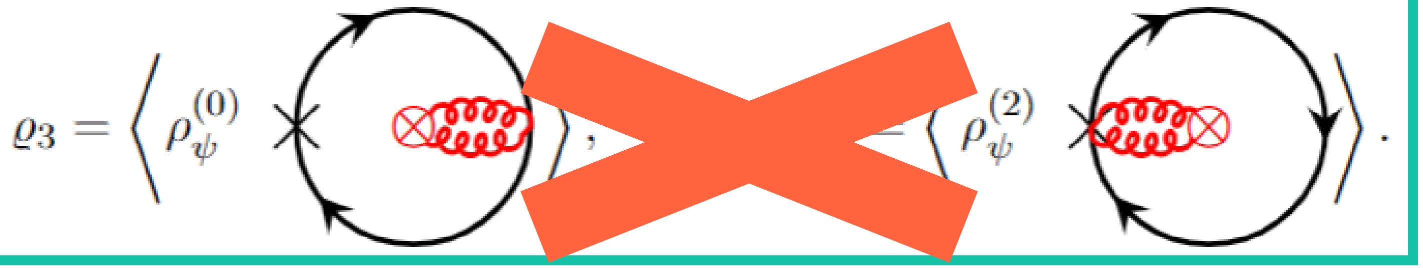

GW-induced Freeze-in

The energy density of Weyl fermions

Unequal time power spectrum of GWs

$$\langle\rho_\psi(\tau)\rangle = \frac{1}{4\pi}\frac{1}{a^4(\tau)}\int_{\tau_i^+}^{\tau} d\tau' \int_{\tau_i^-}^{\tau} d\tau'' \int q^2 dq \,\langle \mathrm{h}_q(\tau'')\mathrm{h}_q^*(\tau')\rangle \int k^4 dk \int d\theta \,\sin^3\theta \, e^{i(k+\omega)(\tau'-\tau'')}$$
$$\times (k+\omega)\left[2 - \frac{\sin^2\theta((\omega+k)^2+q^2)}{2\omega(\omega+k-q\cos\theta)}\right] + c.c.\,.$$

It acts like radiation $\quad \langle P_\psi(\tau)\rangle = \frac{1}{3}\langle\rho_\psi(\tau)\rangle \propto \frac{1}{a^4(\tau)}.$

It depends on the degree of temporal coherency of GWs background

$$\langle \mathrm{h}_q^*(\tau'')\mathrm{h}_q(\tau')\rangle = \gamma_q(|\tau'-\tau''|)\sqrt{\langle|\mathrm{h}_q(\tau')|^2\rangle\,\langle|\mathrm{h}_q(\tau'')|^2\rangle},$$

Fully incoherent

$$\gamma_q(|\tau'-\tau''|) = \Delta\eta\,\delta(\tau'-\tau''),$$

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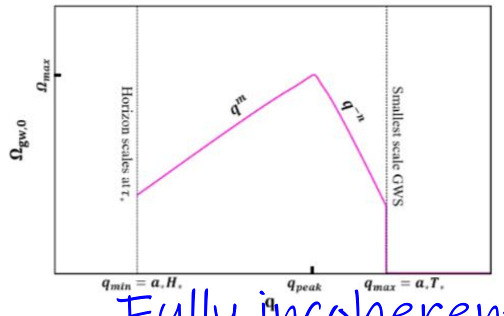
Broken Power-law Spectrum



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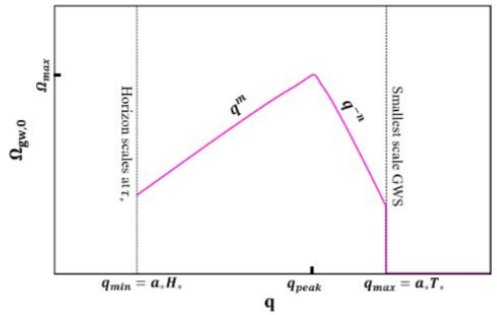

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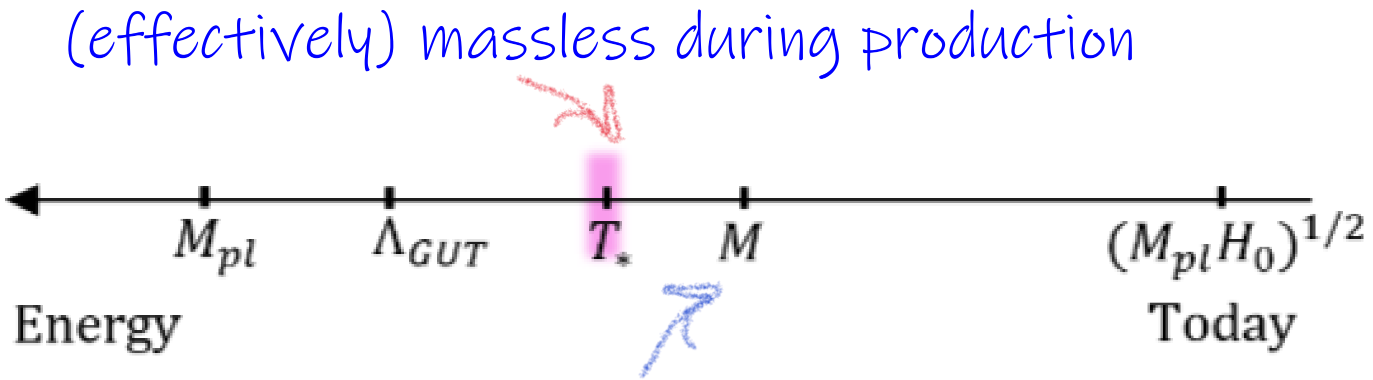
Final result $\quad \langle \rho_\psi(\tau) \rangle = \left(\frac{q_{\mathrm{peak}}}{a(\tau)} \right)^4 \left(\frac{H_0}{\mathcal{H}_*} \right)^2 z_*^2 \, C \qquad \Omega_{\mathrm{peak}},$

GW-induced Freeze-in & Dark Matter

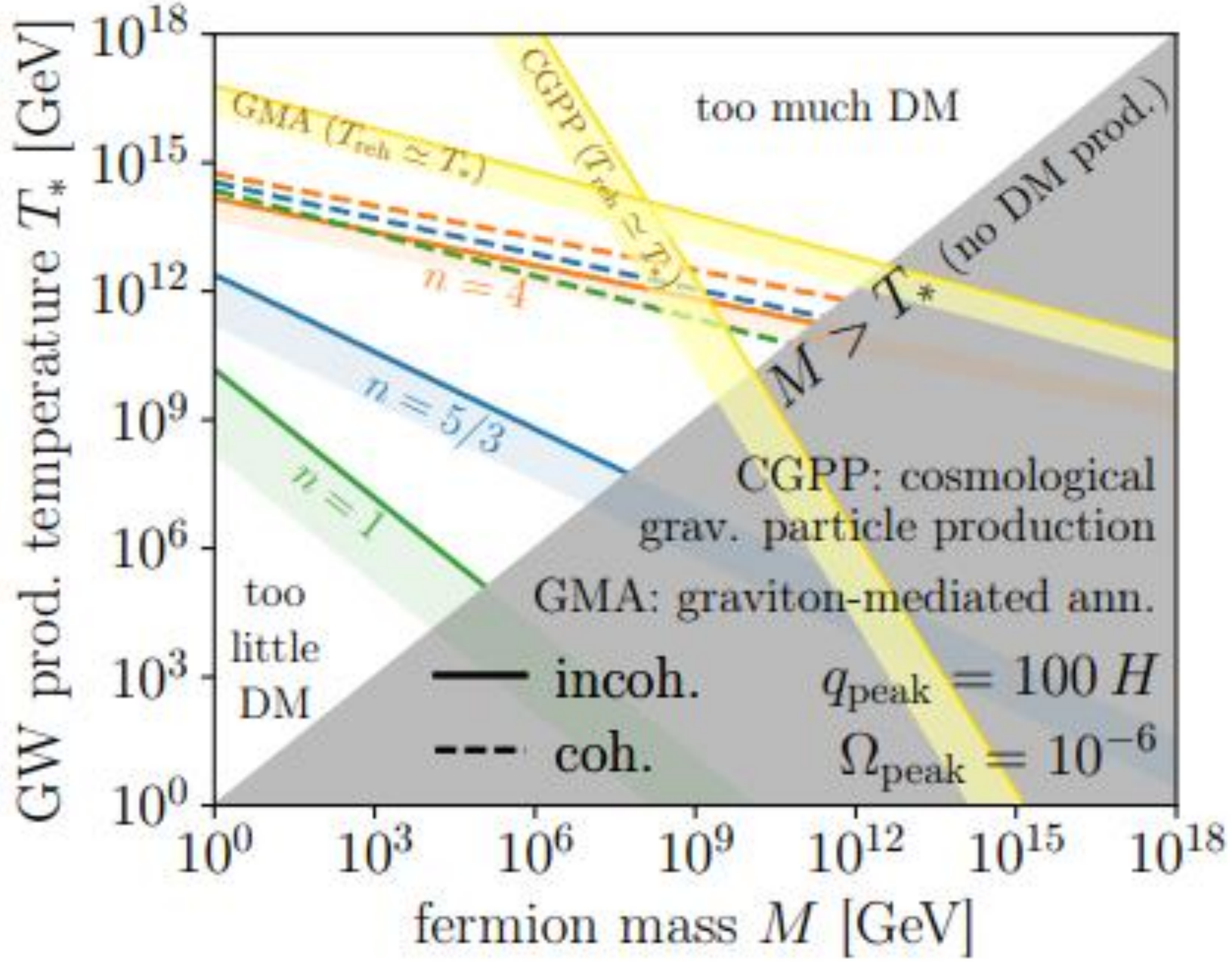


Fermion eventually becomes massive with mass M

$$\Omega_{\psi,0} \simeq 0.36 \times \mathcal{C}\left(\frac{M}{T_*}\right)\left(\frac{q_{\rm peak}/\mathcal{H}_*}{100}\right)^4\left(\frac{g_*(\tau_*)}{106.75}\right)^{4/3}\left(\frac{T_*}{3\times 10^{11}\,{\rm GeV}}\right)^5\left(\frac{\Omega_{\rm peak}}{10^{-6}}\right).$$

Parameter space of GW-induced freeze-in of fermion

A.M. & Kopp 2024



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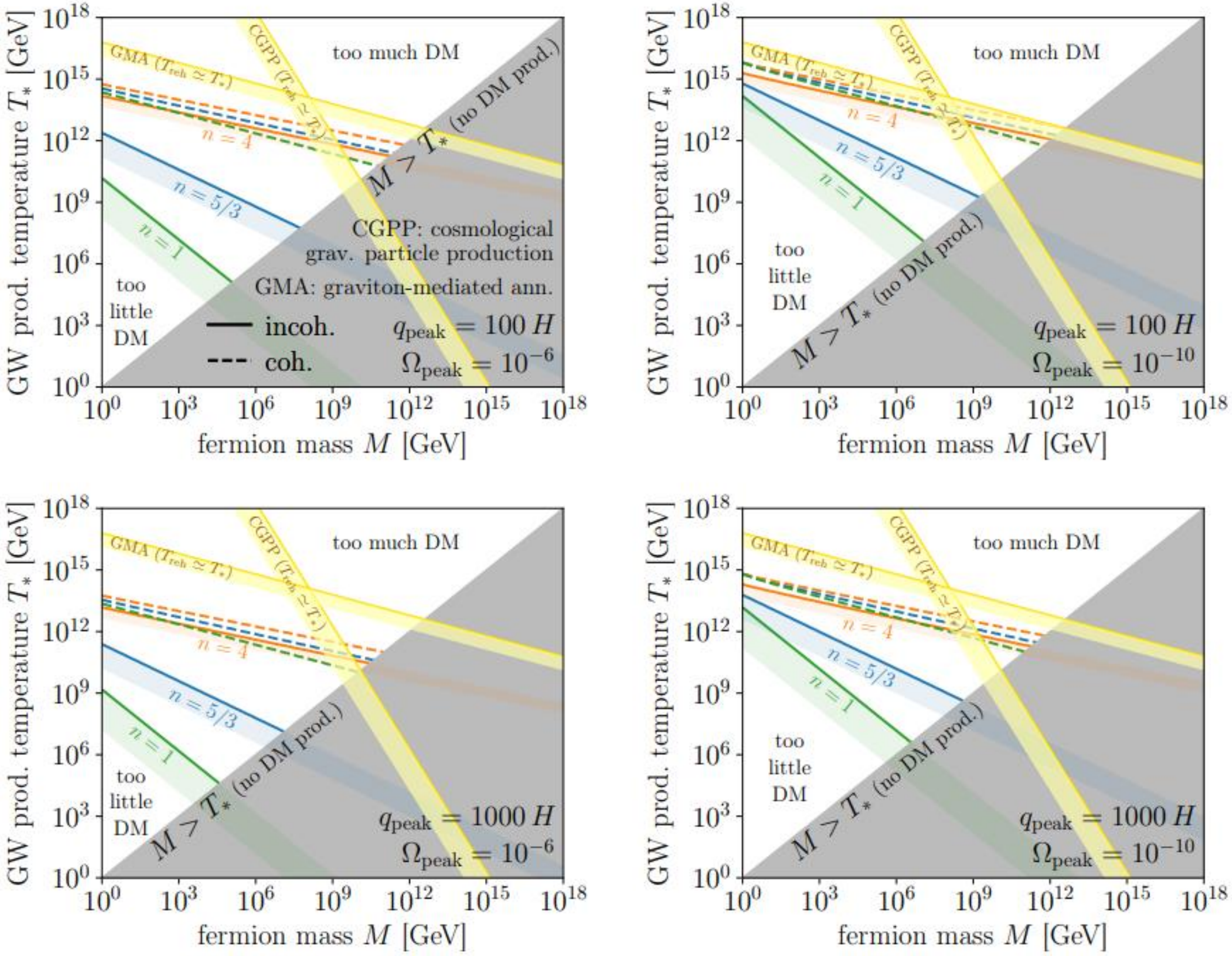
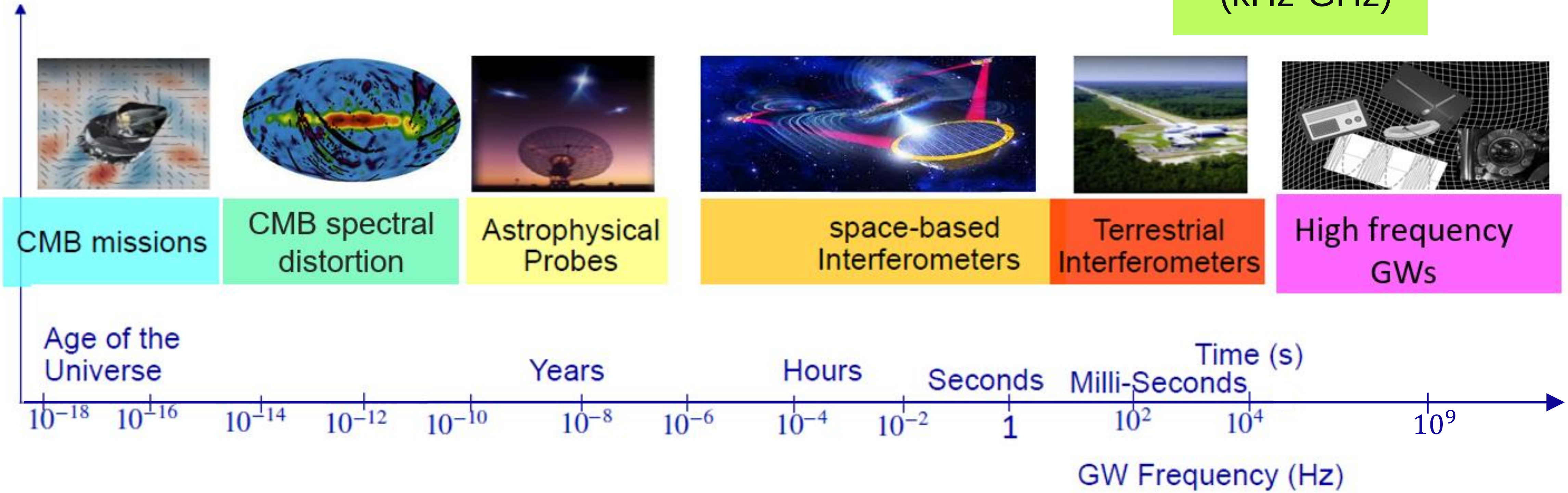


Gravitational Waves Spectrum

GW-induced freeze-in mechanism requires a GWs spectrum with peak frequency

$\mathbf{f_{peak}} \in$ (kHz-GHz)



Age of the Universe = Billions of Years

Summary

Gravity and Quantum Effects in Cosmology can still surprise us:
We discussed an effect that is zero at tree level and non-zero at 1-loop in cosmic perturbations!

Cosmic Perturbations (like GWs) naturally break the conformal symmetry of Weyl Fermions

It leads to a new mechanism for production of dark fermions in early universe, i.e. GW-induced freeze-in of fermionic dark matter.

Questions?!

Image Credit: James Web Space Telescope (a giant cosmic question mark)

Scalar number density for minimal coupling

$$\frac{a^3 n_k}{a_e^3 H_e^3} \approx \begin{cases} \frac{1}{8\pi^2}\tilde{m}^{-1}\tilde{k}^0 & 0 < \tilde{k} < \tilde{m}^{1/3} \\ \frac{1}{8\pi^2}\tilde{k}^{-3} & \tilde{m}^{1/3} < \tilde{k} < \frac{m_\varphi \kappa}{H_e} \\ C\tilde{k}^{-3/2} & \frac{m_\varphi \kappa}{H_e} < \tilde{k} < \tilde{a}_{\mathrm{RH}} \frac{m_\varphi \kappa}{H_e} \end{cases},$$