THE DOs AND DON'Ts OF GRAVITATIONAL WAVE DETECTION

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Work in progress with S. Ellis (U. Geneva)

$$\omega_g$$


ω_g
nHz
μHz
10 Hz
kHz
PULSAR TIMING
LIGO-VIRGO

$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)}$$ Causality

$$\omega_g$$

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$$\omega_0(T_*) = \omega(T_*)\frac{a(T_*)}{a_0} \gtrsim \boxed{100\ \mathrm{MHz}} \left(\frac{T_*}{10^{15}\mathrm{GeV}}\right)\left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

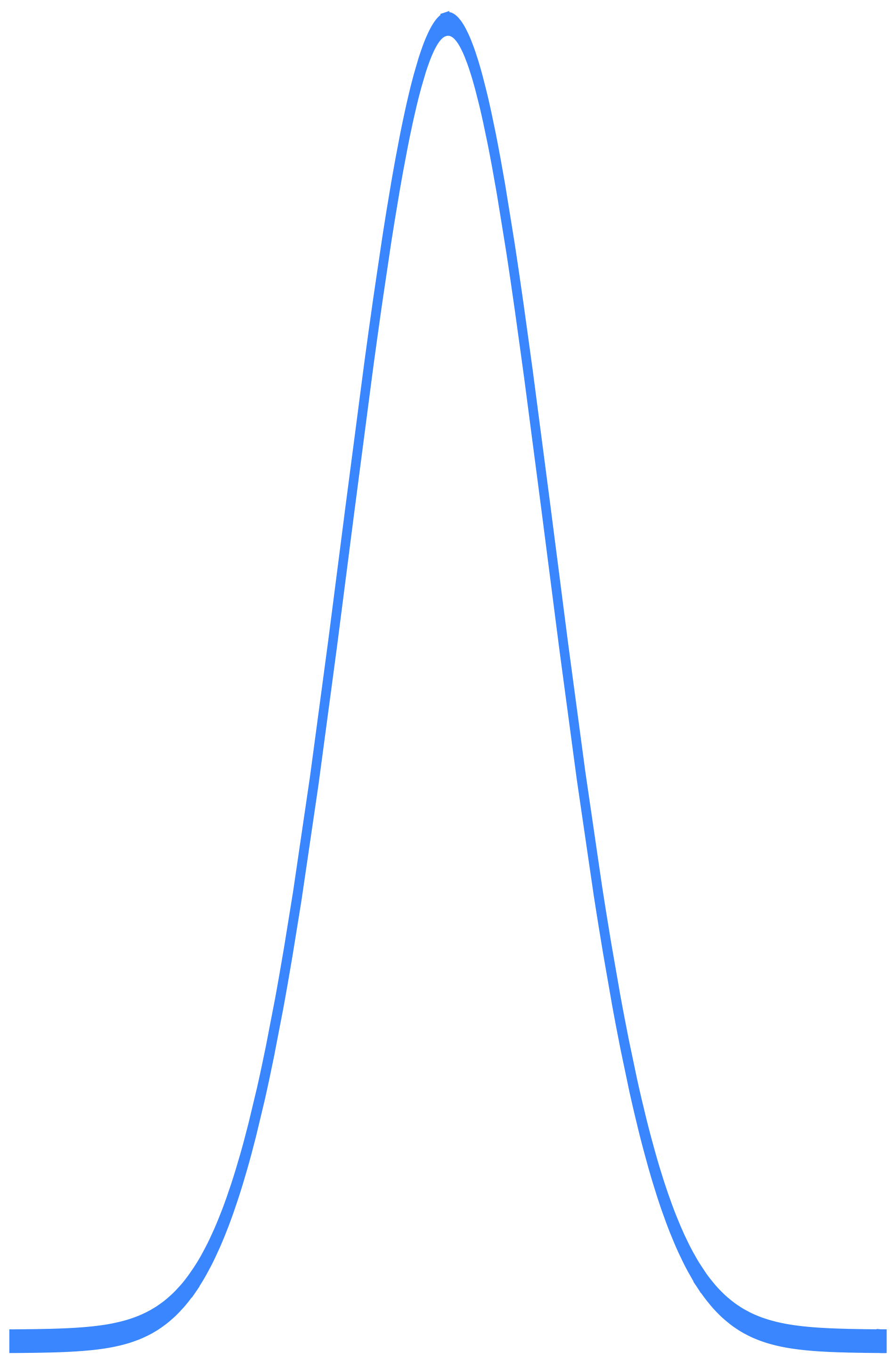

DETECTORS

DETECTOR

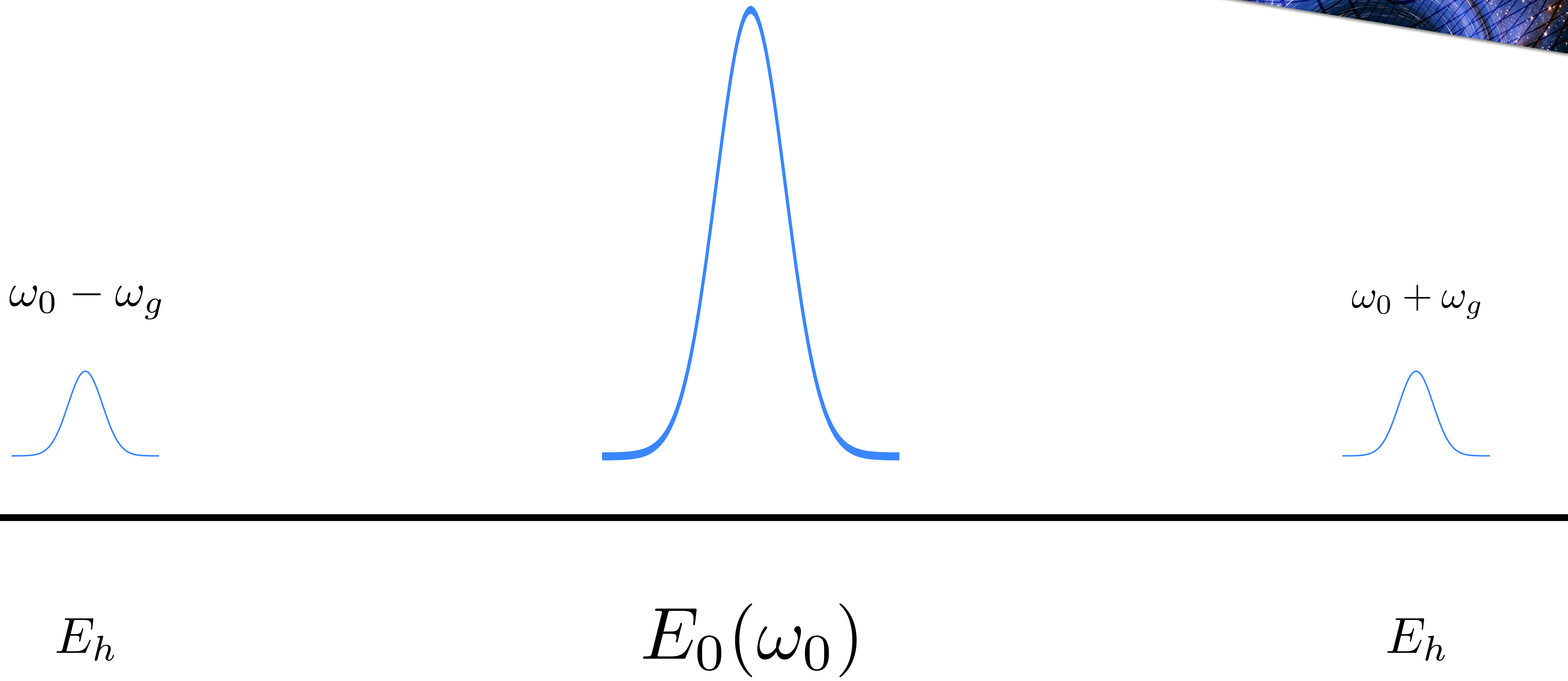
$$U_{\text{in}} \sim E_0^2 V_0$$

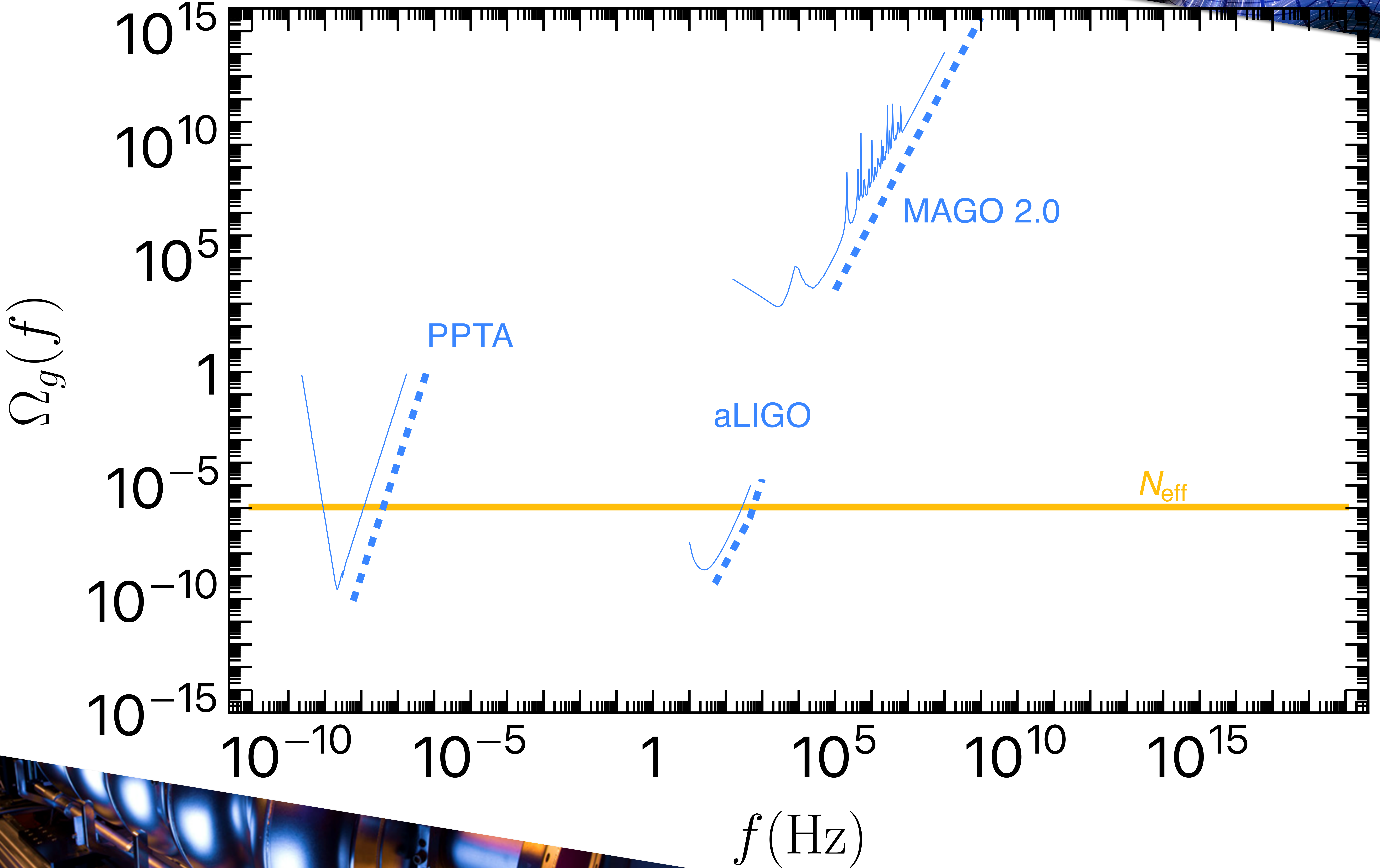
$$U_{\mathrm{in}} \sim E_0^2 V_0$$

$$\text{——} \; \mathcal{T}(\omega) \; \text{——}$$



$$E_0(\omega_0)$$


$\omega_0 - \omega_g$
$\omega_0 + \omega_g$
E_h
$E_0(\omega_0)$
E_h



$\Omega_g(f)$
10^{15}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
MAGO 2.0
PPTA
aLIGO
N_{eff}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
$f\,(\text{Hz})$


SIGNALS

$$U_{\mathrm{sig}} \sim U_{\mathrm{in}} \times \begin{cases} (h\mathcal{T})^2 \\ h\mathcal{T} \end{cases}$$

$$U_{\mathrm{in}} \sim E_0^2 V_0$$

CASE I: QUADRATIC SIGNALS

$$\langle E_h(t) E_0(t) \rangle \propto \langle \tilde{E}_h(\omega) \tilde{E}_0(\omega) \rangle = 0$$

$$\langle E_h(t) E_0(t) \rangle \propto \langle \tilde{E}_h(\omega) \tilde{E}_0(\omega) \rangle = 0$$

$\Delta\omega_d$

Detector Bandwidth

$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

In absence of a signal,
the detector is empty (classically)

In absence of a signal,
the detector is empty (classically)

$$P_{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

But we need at least one photon generated by the signal

QUADRATIC SIGNAL

$$P_{\text{sig}} \lesssim \underline{h^2 U_{\text{in}}} \omega_s \mathcal{T}^2(\omega_s)$$

Signal Energy

QUADRATIC SIGNAL

$$P_{\mathrm{sig}} \lesssim h^2 U_{\mathrm{in}} \underline{\omega_s} \mathcal{T}^2(\omega_s)$$

Maximum power
from Poynting's theorem

QUADRATIC SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm noise}} \simeq 1 \longrightarrow h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}\frac{1}{\mathcal{T}}}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\mathrm{int}}}{3H_0^2}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\rm int}}{3H_0^2}$$

$$U_{\rm in}^{\rm BBN} = 10^{12}\ J \left(\frac{\omega}{2\pi \times 80{\rm kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\rm int}}{3H_0^2}$$

$$U_{\rm in}^{\rm BBN} \simeq U_{\rm ITER}$$

$$U_{\rm in}^{\rm BBN} = 10^{12}\, J \left(\frac{\omega}{2\pi \times 80{\rm kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

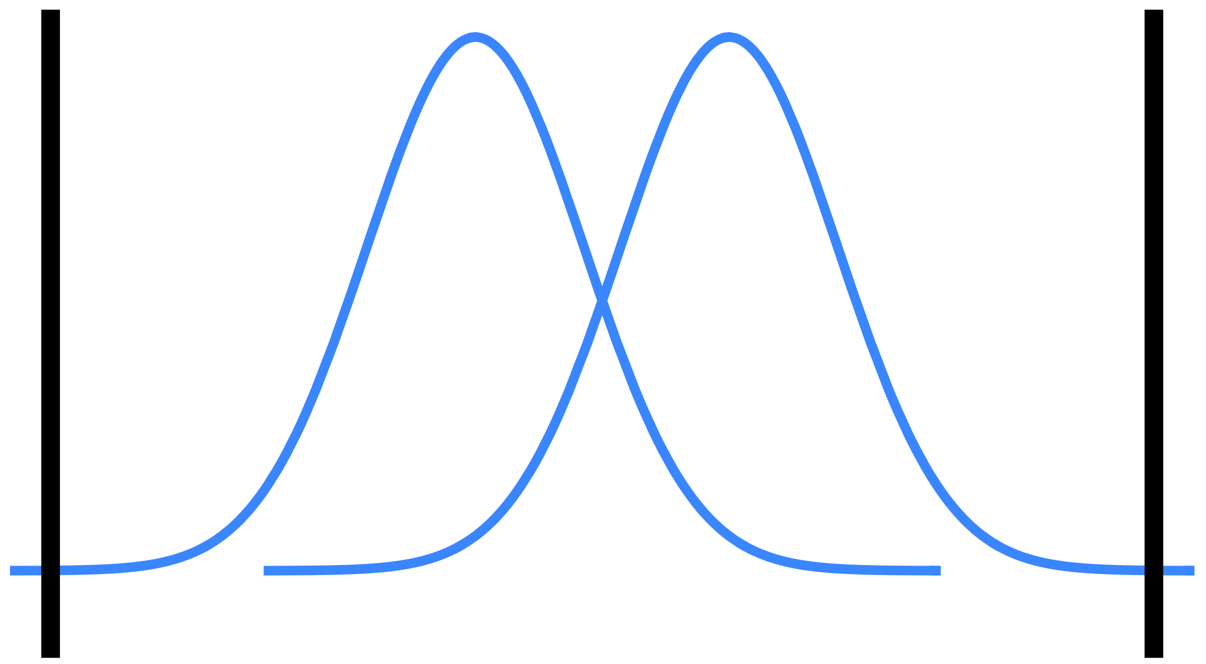
CASE II: LINEAR SIGNALS

$$\langle E_h(t)E_0(t)\rangle \propto \langle \tilde{E}_h(\omega)\tilde{E}_0\omega))\rangle \neq 0$$

CASE II: LINEAR SIGNALS

$\Delta\omega_d$

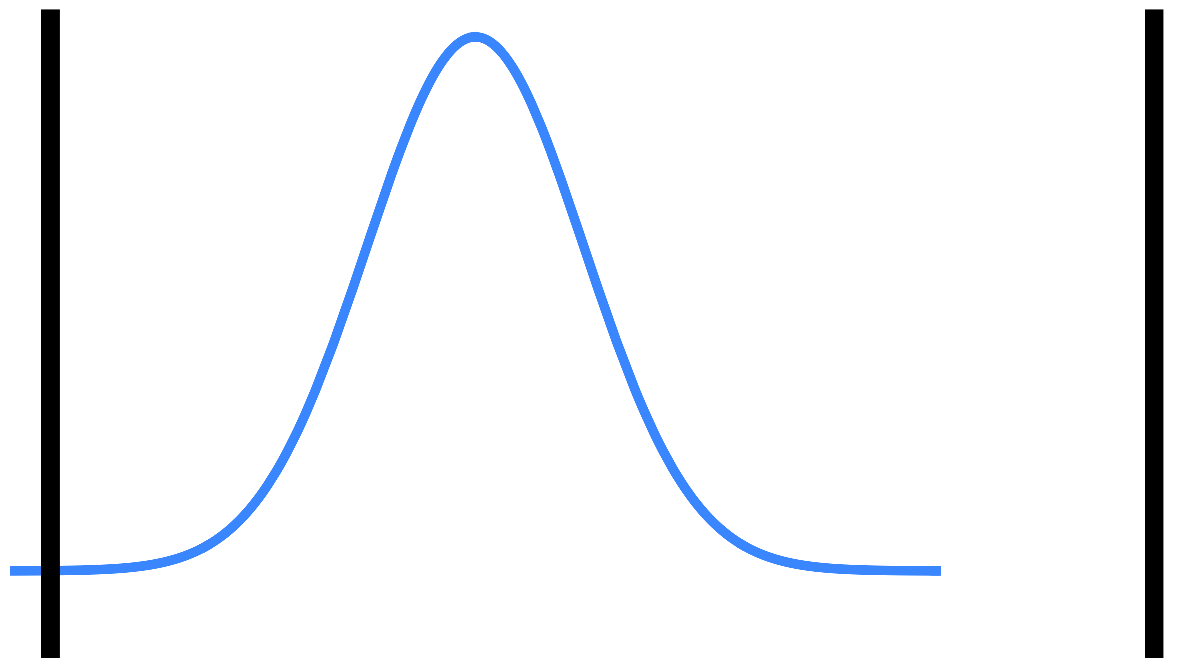
Detector Bandwidth



$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

CASE II: LINEAR SIGNALS

In absence of a signal,
the detector is not empty



$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}\left(1+\sqrt{\frac{P_{\text{in}}t_{\text{int}}}{2\pi\omega}}\right)$$

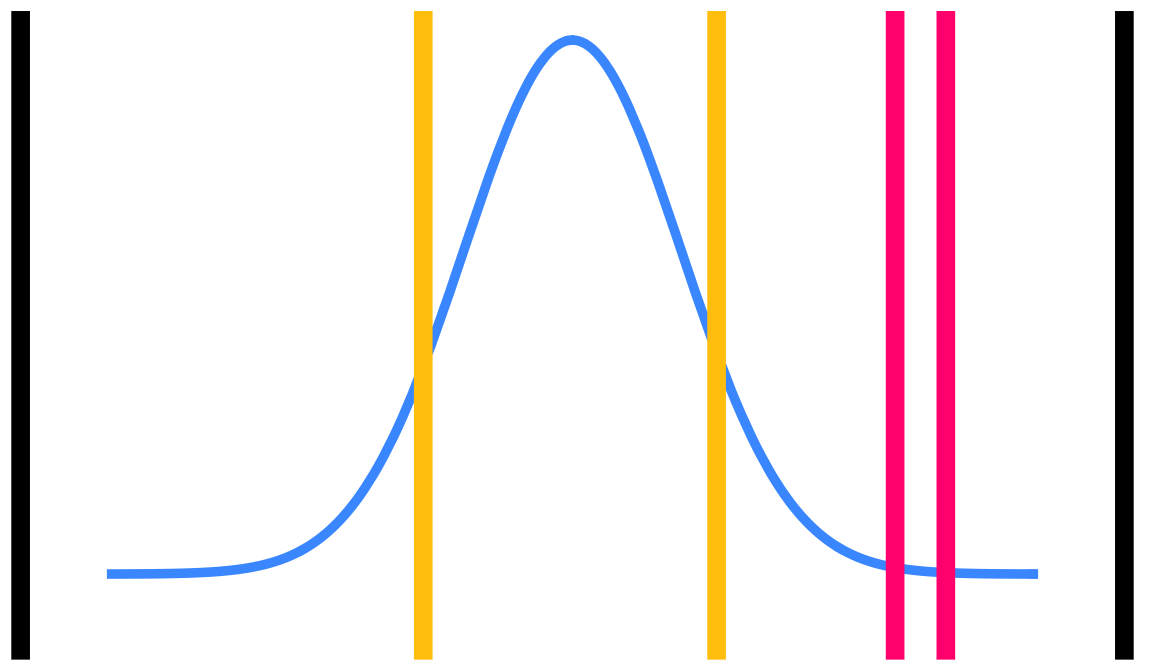
NARROW LINEAR SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm noise}} \simeq 1 \longrightarrow h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}} \frac{1}{\mathcal{T}}}$$

Same as quadratic!

BROAD SIGNAL

Detector Bandwidth $\Delta\omega_d$



t_{int}^{-1}

Resolution

Signal Width $\Delta\omega_s$

$$\Delta\omega \equiv \max[\min[\Delta\omega_d, \Delta\omega_s], t_{\text{int}}^{-1}]$$

BROAD SIGNAL

$$h_{\mathrm{min}}(\Delta f)\mathcal{T} \simeq \begin{cases} \sqrt{\dfrac{2\pi}{U_{\mathrm{in}}}} \left(\dfrac{\Delta\omega}{2\pi t_{\mathrm{int}}}\right)^{1/4} & \mathcal{O}(h^2) \\ \sqrt{\dfrac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}}} & \mathcal{O}(h) \end{cases}$$

$$\gtrsim \sqrt{\frac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}}}$$

Same as before

STATISTICS INTERLUDE

Stationary Gaussian Process with Zero Mean

$$\langle h(t) \rangle = 0$$

$$\langle h(t) h(t') \rangle = H(t' - t)$$

STOCHASTIC BACKGROUND

Stationary Gaussian Process with Zero Mean

$$\langle h(t) \rangle = 0$$

$$\langle h(t) h(t') \rangle = H(t' - t)$$

The signal is always quadratic

SNR

Single Detector

$$\mathrm{SNR} \simeq \left(\frac{S_h(\omega_s)}{S_n(\omega_s)} \right)^{1/2}$$

We get the same results as from the rough arguments in these slides

SNR

Single Detector

$$\mathrm{SNR} \simeq \left(\frac{S_h(\omega_s)}{S_n(\omega_s)} \right)^{1/2}$$

Two Detectors Optimal Filtering

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int d\omega \Gamma^2(\omega) \frac{S_h^2(\omega)}{S_n^2(\omega)} \right)^{1/4}$$

SNR

Single Detector

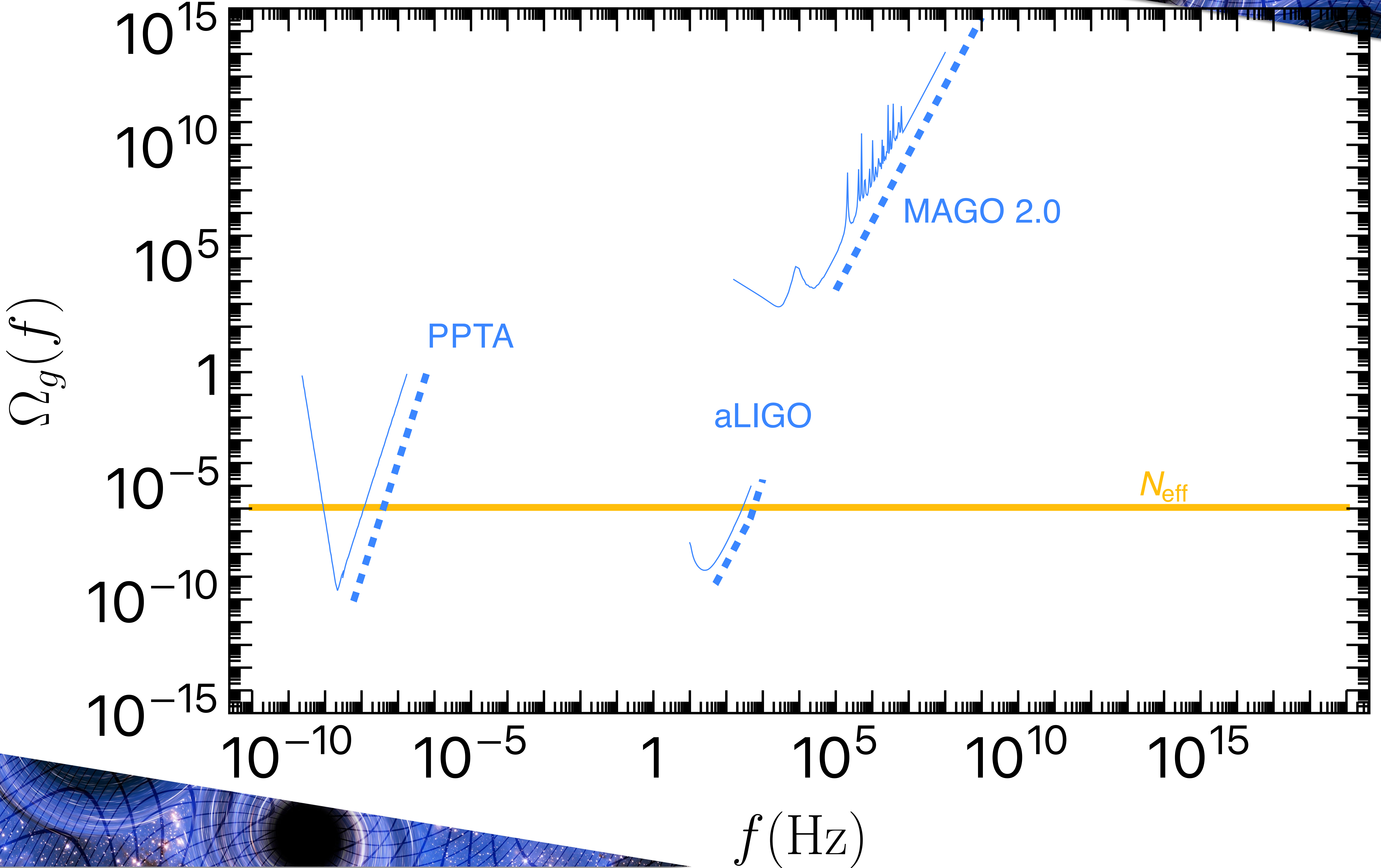
$$\mathrm{SNR} \simeq \left(\frac{S_h(\omega_s)}{S_n(\omega_s)} \right)^{1/2}$$

Two Detectors Optimal Filtering

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int d\omega \Gamma^2(\omega) \frac{S_h^2(\omega)}{S_n^2(\omega)} \right)^{1/4}$$

$$\Omega_g \to \Omega_g \times \frac{1}{\sqrt{\Delta\omega t_{\mathrm{int}}}}$$

If total energy density is not fixed



10^{15}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
$\Omega_g(f)$
MAGO 2.0
PPTA
aLIGO
$N_{\rm eff}$
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
$f\,(\mathrm{Hz})$


WHAT DID WE LEARN?

QUADRATIC vs LINEAR

$$h_{\mathrm{min}} \gtrsim \sqrt{\frac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}}} \frac{1}{\mathcal{T}}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\rm int}}{3H_0^2}$$

$$U_{\rm in}^{\rm BBN} = 10^{12}\, J \left(\frac{\omega}{2\pi \times 80\text{kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

QUADRATIC vs LINEAR

QUADRATIC

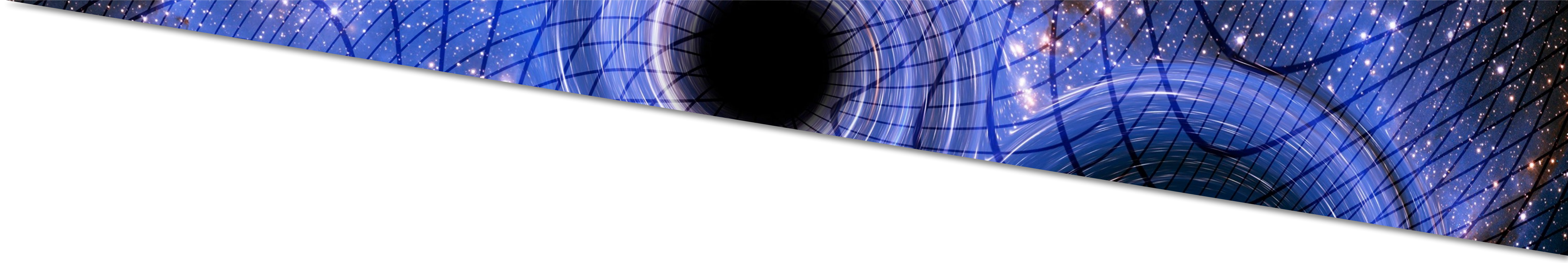
$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

LINEAR

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{\frac{P_{\text{in}} t_{\text{int}}}{2\pi\omega}}\right)$$

We can gain from quantum squeezing

THE MISSING PIECE



$$h_{\mathrm{min}} \gtrsim \sqrt{\frac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}} \boxed{\frac{1}{\tau}}}$$

???


TRANSFER FUNCTIONS

EM RESONATOR

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -C_g(\omega_0 \pm \omega_g)^2(\omega_g V^{1/3})^2 h E_0$$

EM RESONATOR

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -C_g(\omega_0 \pm \omega_g)^2(\omega_g V^{1/3})^2 h E_0$$

$$\frac{\omega_s^2\omega_1^2}{Q^2}U_{\rm sig} = h^2 U_{\rm in}(\omega_g V^{1/3})^4(\omega_0 \pm \omega_g)^4$$

EM RESONATOR

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -C_g(\omega_0 \pm \omega_g)^2(\omega_g V^{1/3})^2 h E_0$$

$$\frac{\omega_s^2\omega_1^2}{Q^2}U_{\rm sig} = h^2 U_{\rm in}(\omega_g V^{1/3})^4(\omega_0 \pm \omega_g)^4$$

$$\mathcal{T}^2 = (\omega_g V^{1/3})^4 \frac{(\omega_0 \pm \omega_g)^4}{\omega_s^2\omega_1^2} Q^2$$

EM RESONATOR

$$\mathcal{T}^2 \simeq \begin{cases} Q^2\,, & \mathrm{ADMX-like}: \omega_s = \omega_g = \omega_1,\ \omega_0 = 0\,, \\ Q^2\,\frac{\omega_g^4}{\omega_0^4}\,, & \mathrm{MAGO-like}: \omega_s = \omega_0 = \omega_1 \gg \omega_g\,. \end{cases}$$

EVERYTHING ELSE (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

The wave excites a mechanical mode

EVERYTHING ELSE
(LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -2\omega_1^2 V^{-1/3}\int d\omega'\,\tilde{u}_m(\omega'-\omega)\tilde{E}_0(\omega')$$

An EM resonator does the readout

EVERYTHING ELSE (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\mathcal{T}^2(\omega) = \frac{\omega_g^4 \omega_1^4}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2 \omega_1^2}{Q^2}\right)\left((\omega_m^2 - \omega_g^2)^2 + \frac{\omega_g^2 \omega_m^2}{Q_m^2}\right)}$$

EVERYTHING ELSE (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\mathcal{T}^2_{\mathrm{LIGO}} \simeq \frac{\omega_L^2}{\left(4\omega_g^2 + \frac{\omega_L^2}{Q^2}\right)} \simeq \frac{\omega_L^2 L_{\mathrm{eff}}^2}{(4\omega_g^2 L_{\mathrm{eff}}^2 + 1)}$$

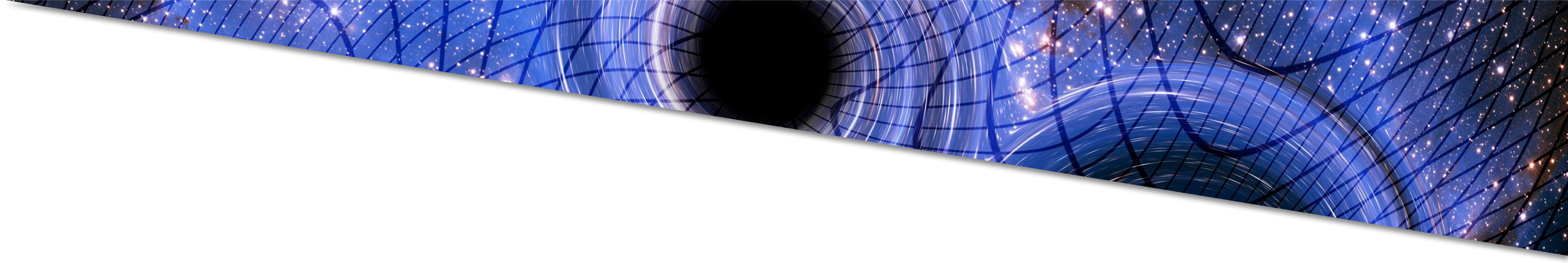
$$\omega_0 \simeq \omega_1 \simeq \omega_L \gg \omega_g \gg \omega_m$$

INTERFEROMETERS vs RESONATORS

$$\mathcal{T}_{\mathrm{LIGO}} \lesssim 10^{10}$$

$$\mathcal{T}_{\mathrm{res}} \simeq Q \lesssim 10^{12}$$

INTERFEROMETERS vs RESONATORS

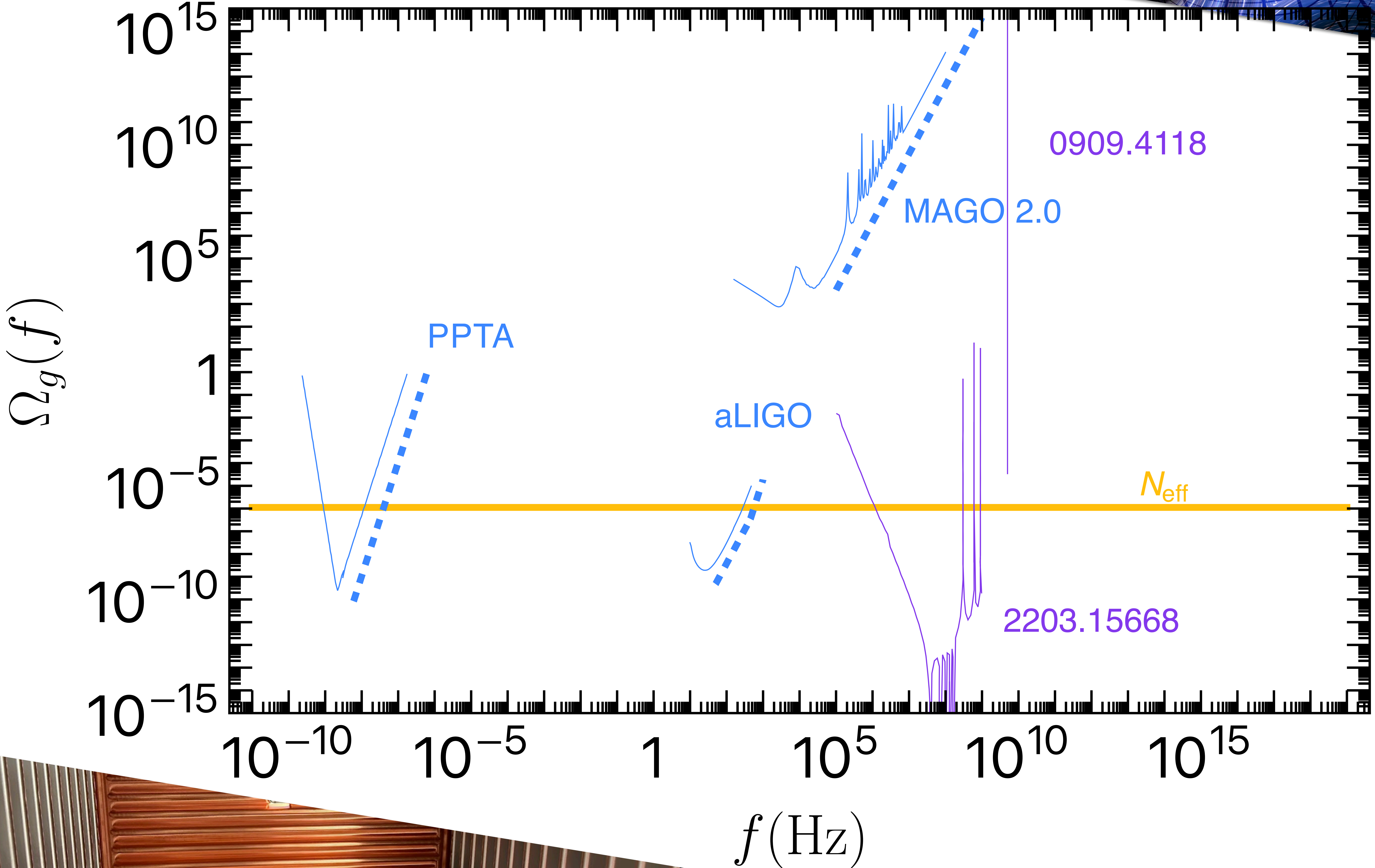


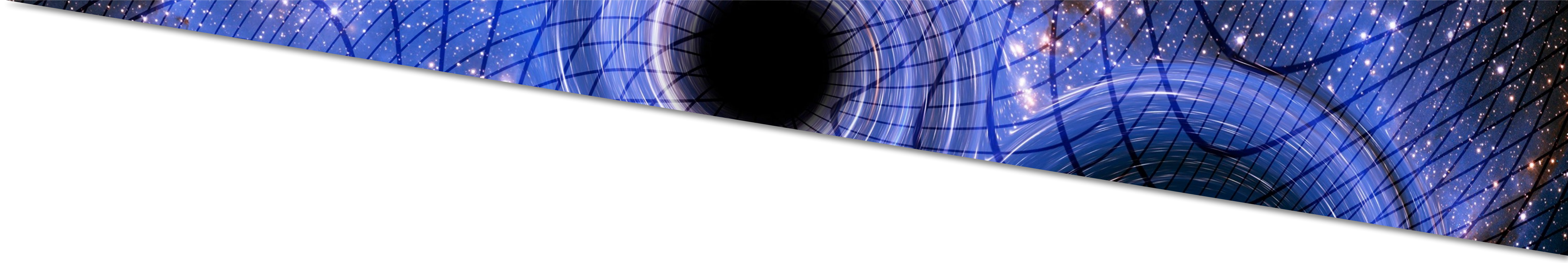
$$\Omega_g \sim \left(\mathcal{T}_{\mathrm{int}}\right)^{-2}$$

$$\Omega_g \sim \left(\mathcal{T}_{\mathrm{res}}\sqrt{\frac{\omega}{\Delta\omega}} \simeq \sqrt{Q}\right)^{-2}$$


SUSPICIOUS SENSITIVITIES

SOME SUSPICIOUS RESULTS

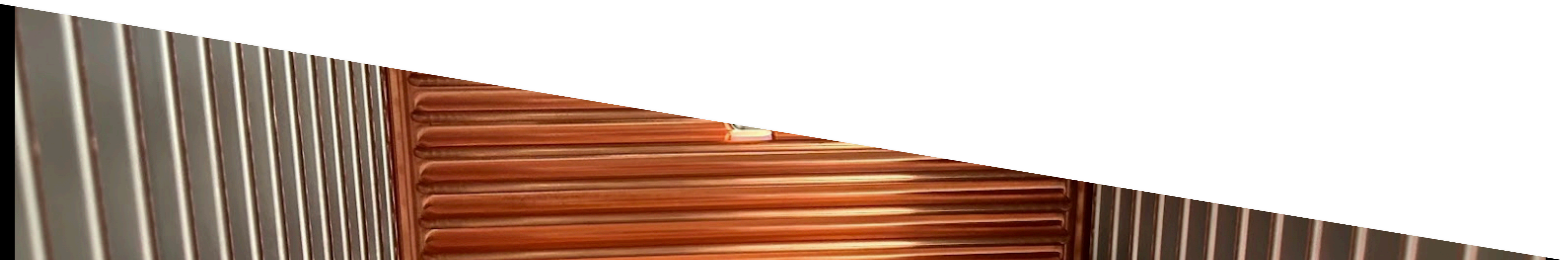


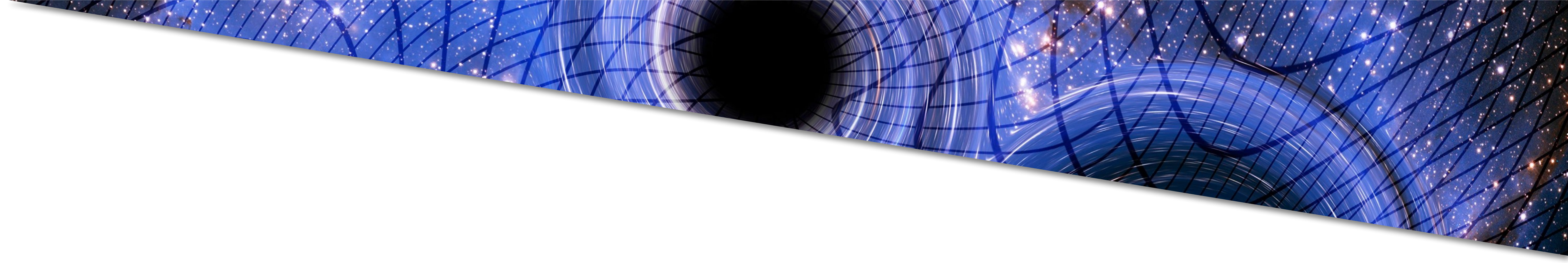


LI-BAKER

$$h_{\text{min}} \gtrsim 10^{-24} \qquad h_{\text{claim}} \gtrsim 10^{-30}$$

F. Li et al., Phys. Rev. D 80, 064013 (2009), 0909.4118



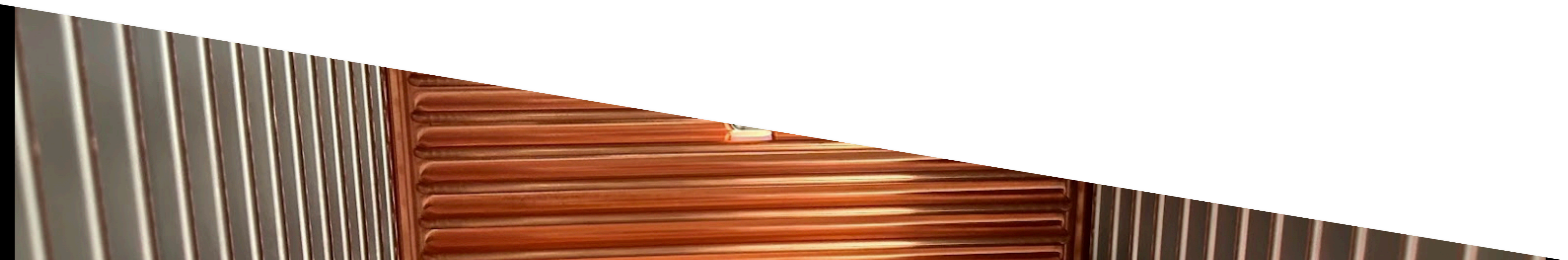


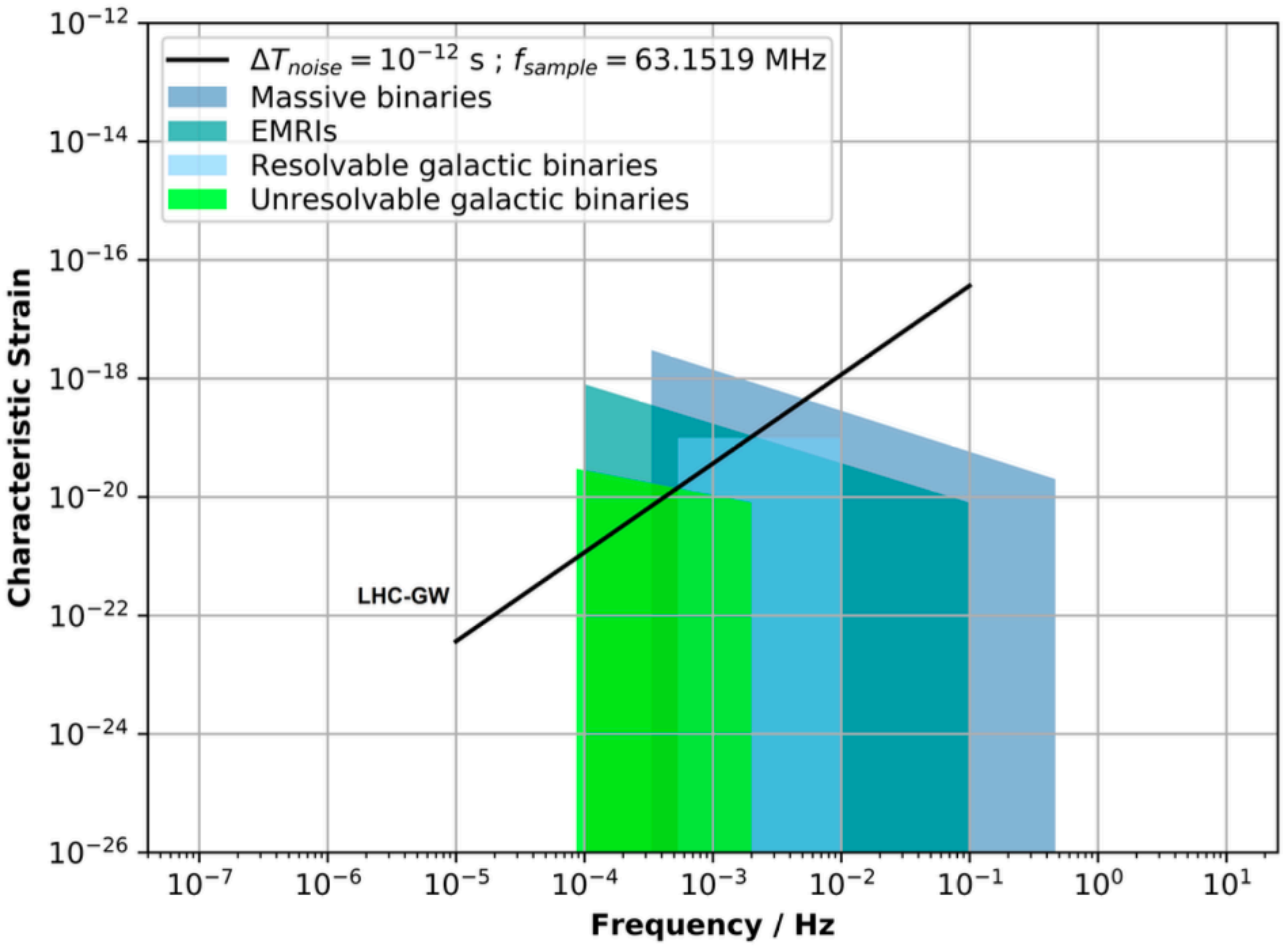
LINEAR CAVITIES

$$h_{\mathrm{min}} \gtrsim 10^{-20}$$

$$h_{\mathrm{claim}} \gtrsim 10^{-24} - 10^{-36}$$

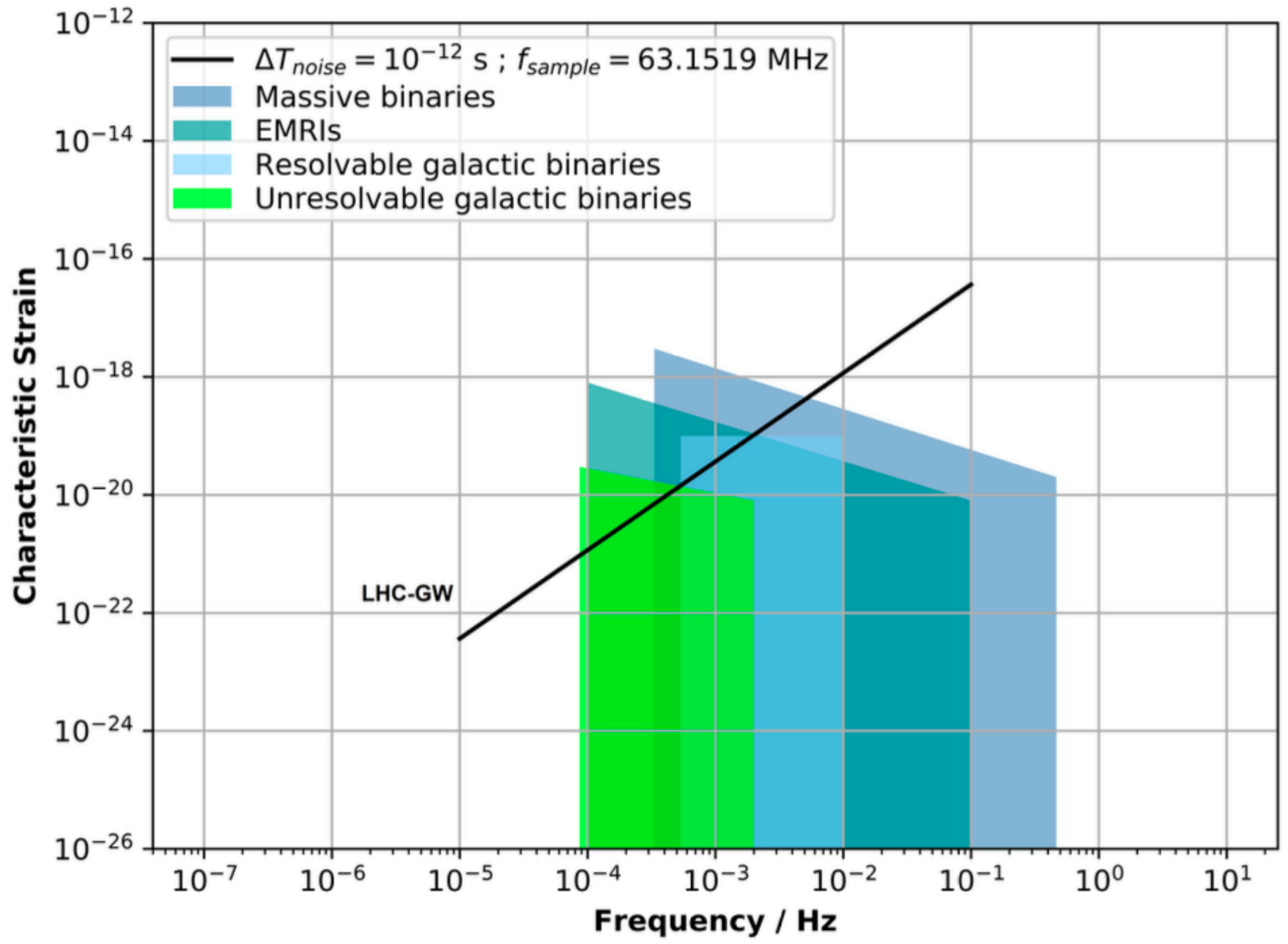
N. Herman, L. Lehoucq, and A. Fúzfa, Phys. Rev. D 108, 124009 (2023), 2203.15668





Rao, Bruggen, Lisle

Phys.Rev.D 102 (2020) 12, 122006, *Phys.Rev.D* 105 (2022) 6, 069903 (erratum)



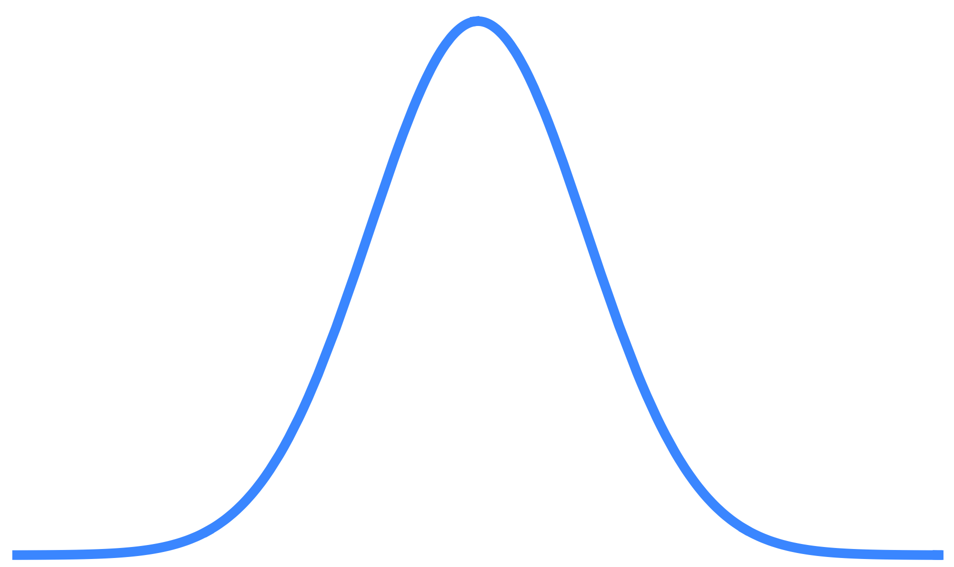
$$h \gtrsim 10^{-11} \qquad \mathcal{T}_{\rm LHC} \simeq \frac{\omega_g^2}{(10\ \mathrm{Hz})^2}$$

ACKNOWLEDGED IN
2301.08331


BEYOND THE QUANTUM LIMIT

NO QUALITATIVE CHANGE FOR QUADRATIC SIGNALS

$$P_{\mathrm{SQL}} \simeq \frac{2\pi\omega}{t_{\mathrm{int}}}$$



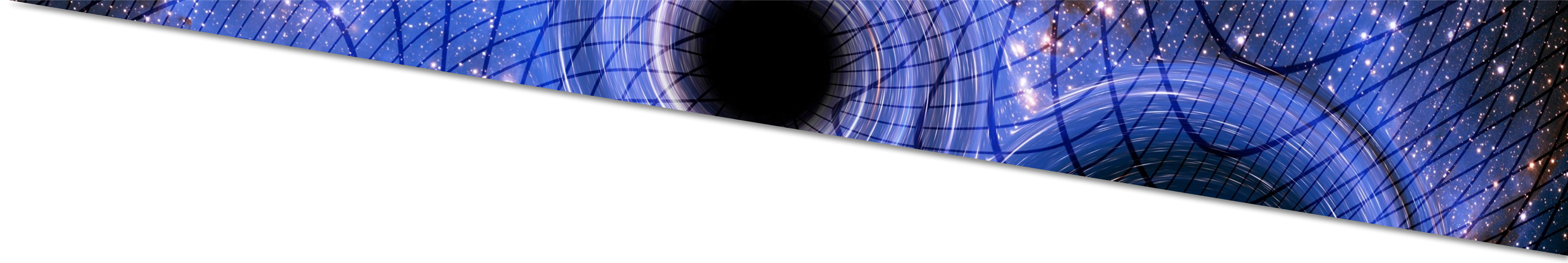
$$P_{\mathrm{min}} \simeq \frac{2\pi\omega}{t_{\mathrm{int}}}$$

LINEAR

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{\frac{P_{\text{in}} t_{\text{int}}}{2\pi\omega}}\right)$$

$$\simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{N_\gamma}\right)$$

SAME BUT QUANTUM (EM RESONATOR)



Perfect Resonator + GW

$$H_0 = \sum_n \omega_n a_n^\dagger(t) a_n(t) - h(\omega_g L)^2 C \sum_{m,n} \omega_m(\omega_m \pm \omega_g) a_n^\dagger a_m + \ldots + h(\omega_g L)^2 \omega_g B_0 (C_1 a_{n^*} + C_2 a_{n^*}^\dagger)$$

Measurement Port + Intrinsic Losses

$$H_R = \int d\omega \left\{ \omega b^\dagger(\omega) b(\omega) + g(\omega)[b(\omega) a^\dagger(t) - b^\dagger(\omega) a(t)] \right\}$$

SAME BUT QUANTUM (EM RESONATOR)

$$\dot{a}(t) = i[H_0, a(t)] - \frac{\kappa}{2}a(t) + \sqrt{\kappa_m}a_{\text{in}}^{m}(t) + \sqrt{\kappa_\ell}a_{\text{in}}^{\ell}(t)$$

$$\dot{a}^\dagger(t) = i[H_0, a^\dagger(t)] - \frac{\kappa}{2}a^\dagger(t) + \sqrt{\kappa_m}a_{\text{in}}^{m,\dagger}(t) + \sqrt{\kappa_\ell}a_{\text{in}}^{\ell,\dagger}(t)$$

SAME BUT QUANTUM (EM RESONATOR)

Intrinsic Losses

$$\dot{a}(t) = i[H_0, a(t)] - \frac{\kappa}{2}a(t) + \sqrt{\kappa_m}a_{\text{in}}^{m}(t) + \sqrt{\kappa_\ell}a_{\text{in}}^{\ell}(t)$$

$$\dot{a}^\dagger(t) = i[H_0, a^\dagger(t)] - \frac{\kappa}{2}a^\dagger(t) + \sqrt{\kappa_m}a_{\text{in}}^{m,\dagger}(t) + \sqrt{\kappa_\ell}a_{\text{in}}^{\ell,\dagger}(t)$$

Measurement Port

SQUEEZING

$$X = \frac{a + a^\dagger}{\sqrt{2}} \qquad Y = i\frac{a - a^\dagger}{\sqrt{2}}$$

SQUEEZING

$$S^{\mathrm{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\mathrm{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\mathrm{in}}_{Y_l Y_l}\right)$$

SQUEEZING

$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

Measurement Port

Intrinsic Losses

$$\frac{\kappa}{2} = \frac{\omega_n}{Q_n}$$

SQUEEZING

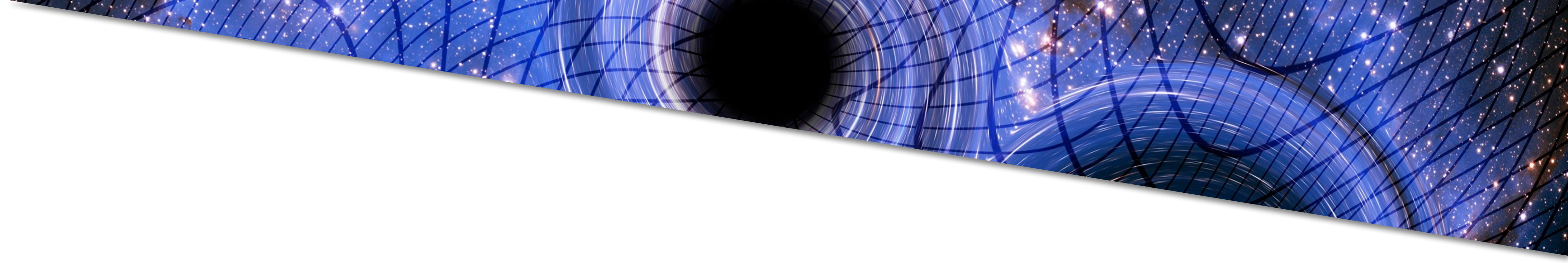
$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

SQL: $S^{\text{in}}_{Y_m Y_m} = S^{\text{in}}_{Y_l Y_l} = 1/2$

SQUEEZING

$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

SV: $$S^{\text{in}}_{Y_m Y_m} = \frac{e^{-2r}}{2} \ll S^{\text{in}}_{Y_l Y_l} = 1/2$$



MANY OTHER IDEAS

1. Quantum non-demolition measurements: for instance "speedometers" for interferometers, strongly suppress back action from the laser

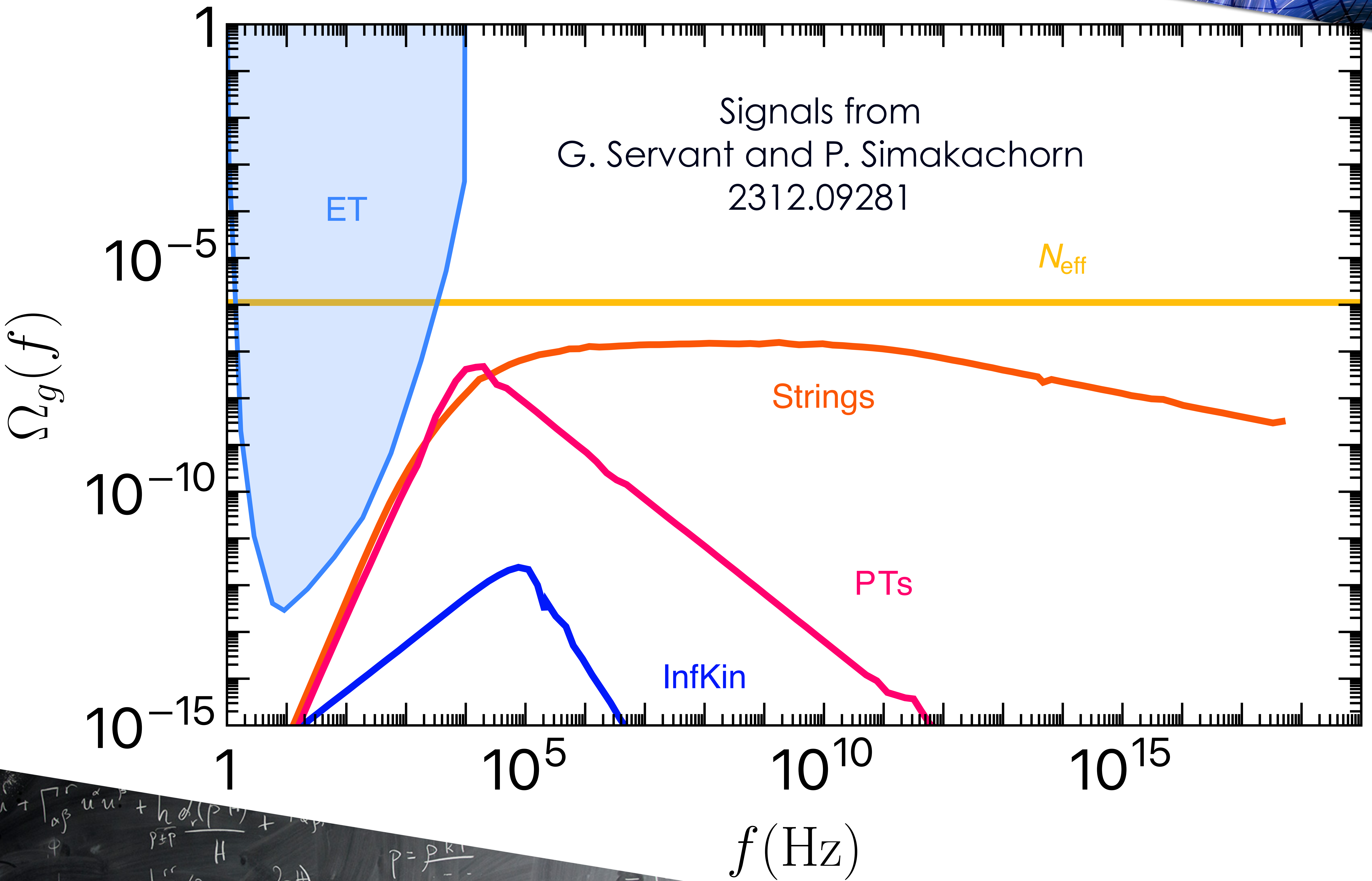
$$S_{hh} \simeq \frac{\kappa^2 + 4\Omega^2}{4U_{\rm in}\omega_L^2} + \frac{16U_{\rm in}\omega_L^2}{L^4 M_{\rm mirror}\Omega^4(\kappa^2+4\Omega^2)}$$

2. Entanglement of input photons to get an advantage that scales like N when combining N interferometers

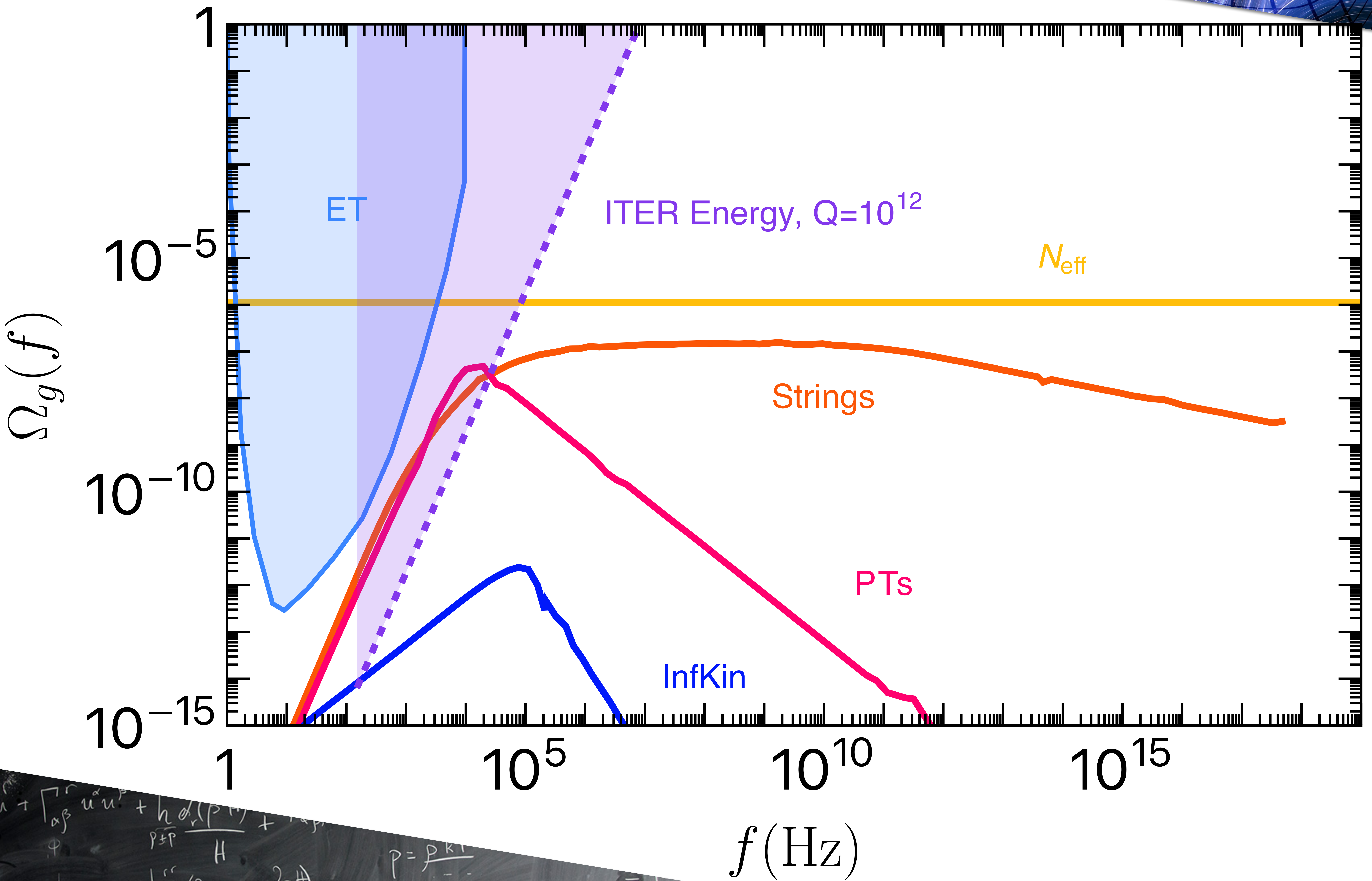
....


SIGNALS

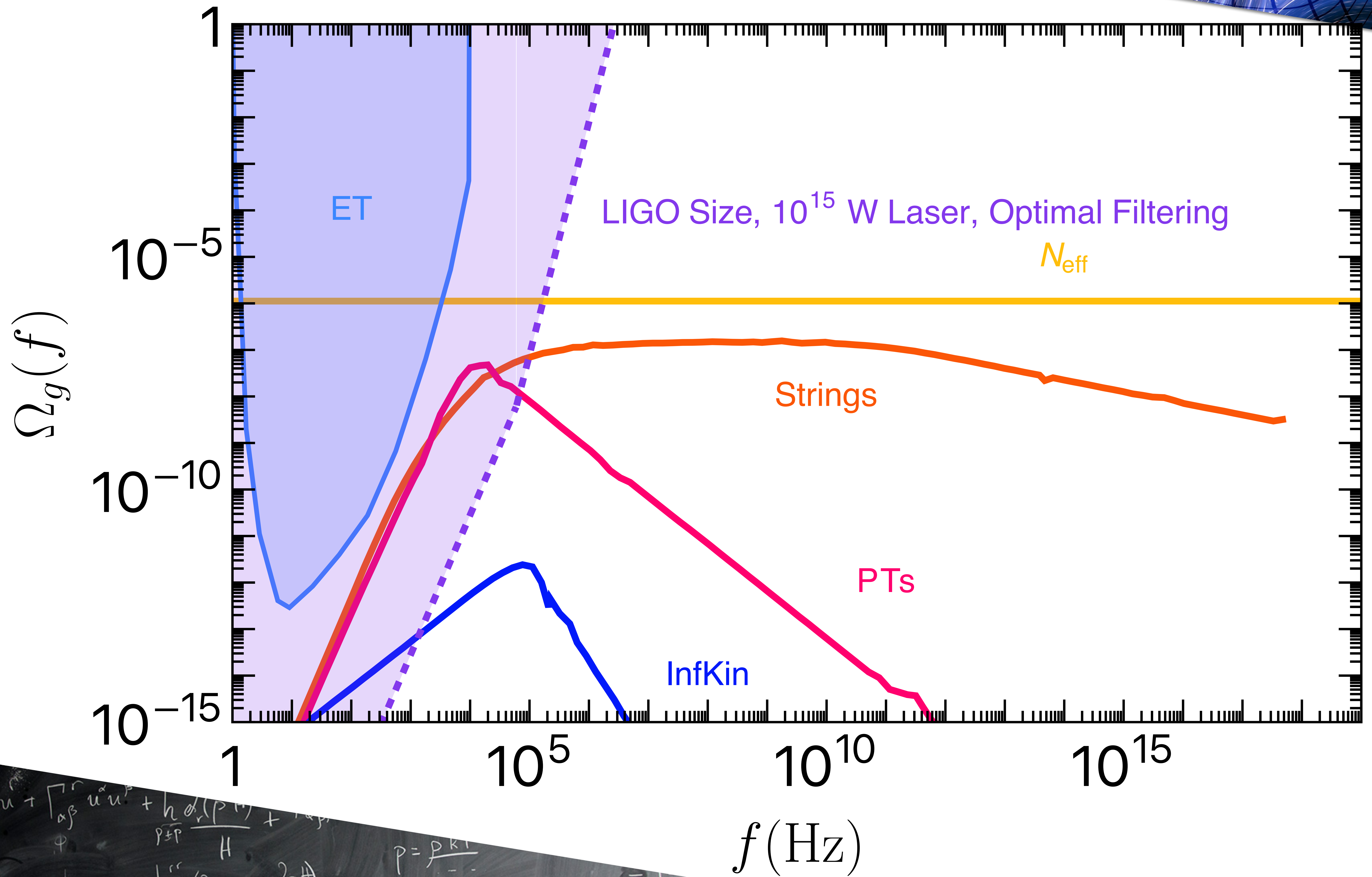
SOME LARGE SIGNALS



“CRAZY” RESONATOR

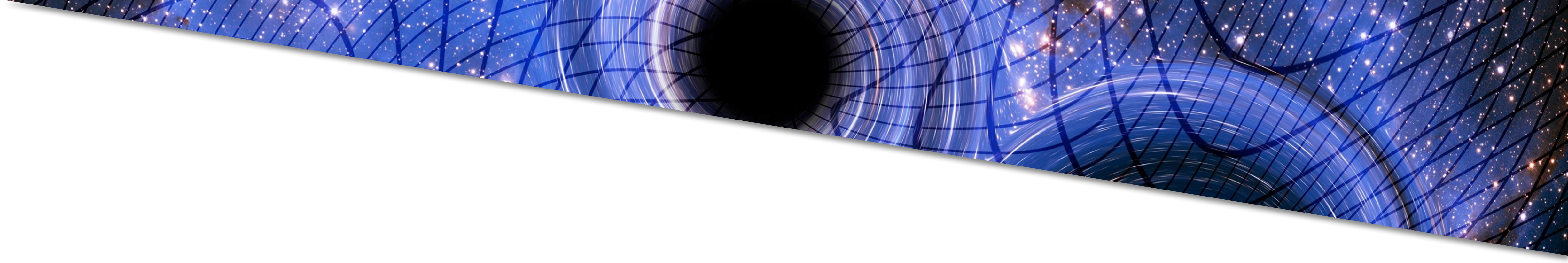


“CRAZY” LASER


ET
LIGO Size, 10^{15} W Laser, Optimal Filtering
$N_{\rm eff}$
Strings
PTs
InfKin
1
10^{-5}
10^{-10}
10^{-15}
1
10^5
10^{10}
10^{15}
$\Omega_g(f)$
$f\,(\mathrm{Hz})$

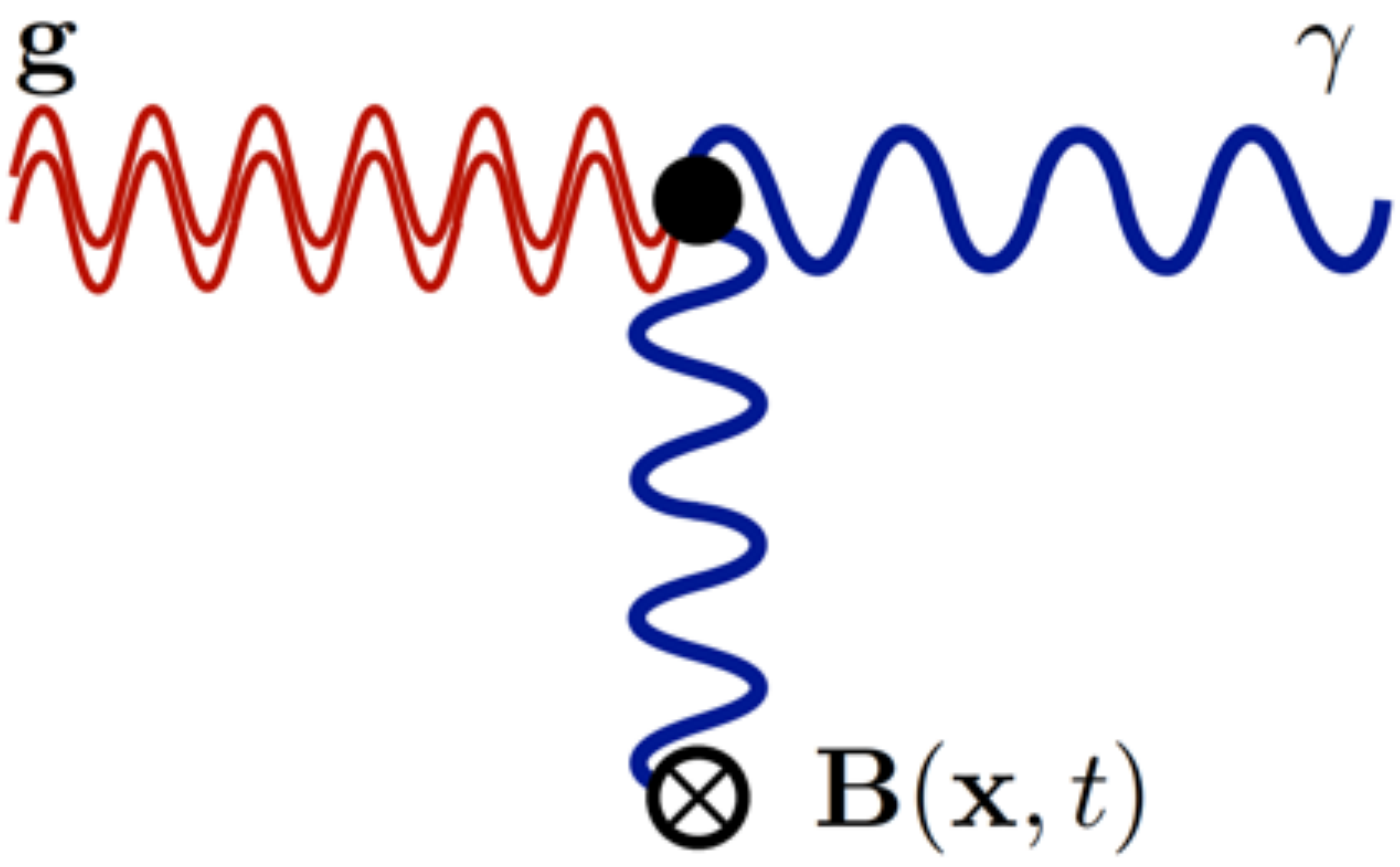
CONCLUSION

$$\Omega_g(\omega_g) \sim \omega_g^3 h^2$$

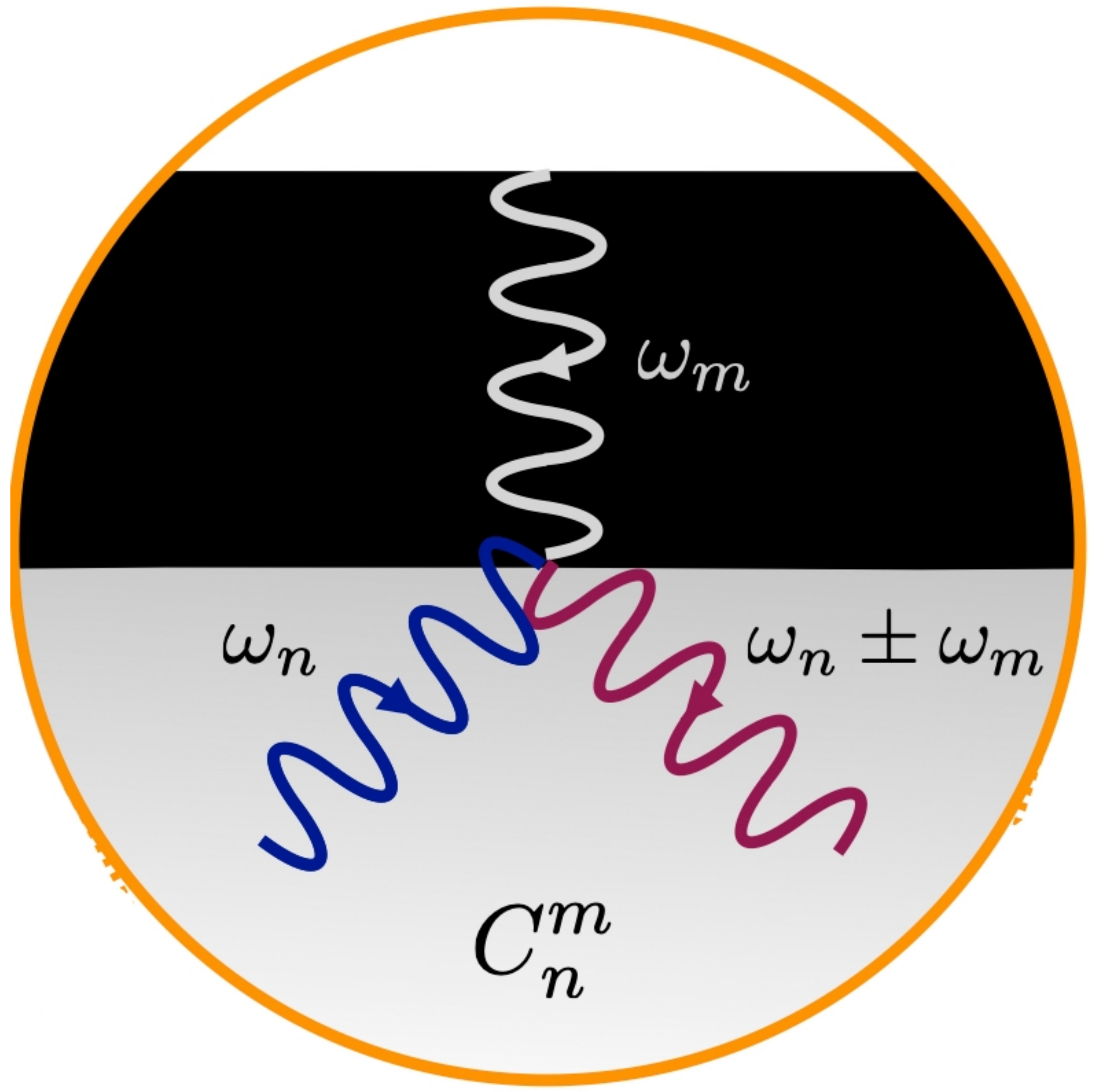


BACKUP

ELECTROMAGNETIC



MECHANICAL



SIGNAL

$$\langle U_{\mathrm{sig}} \rangle_h \sim \langle E_0(t) E_h(t) \rangle L_0^3 + \langle E_h^2(t) \rangle L_0^3$$
$$+ E_0^2 \langle L_0(t) L_h(t) \rangle L_0 + E_0^2 \langle L_h^2(t) \rangle L_0$$

CHOICE OF GAUGE

$$\langle U_{\rm sig} \rangle_h \sim \langle E_0(t) E_h(t) \rangle L_0^3 + \langle E_h^2(t) \rangle L_0^3$$

LIGO QUADRATIC

$$E_x(t) = -E_y(t) + \delta E_{0+h}(t),$$

$$\delta E_{0+h}(t) = E_0(t)\sin(\phi_0 + \delta\phi_h)\,,$$

$$\delta\phi_h = \boxed{(h\omega_L L)\frac{\sin\omega_g L}{\omega_g L}}\cos(\omega_g(t+L))\,,$$

LIGO QUADRATIC 2

$$\delta E_{0+h}(t) \simeq \delta E_0(t) + \boxed{\delta E_h(t)} = E_0(t)\sin(\phi_0) + hE_0' \frac{\omega_L}{\omega_g}\sin(\omega_g L)\boxed{e^{i(\omega_L \pm \omega_g)t}}$$

LIGO QUADRATIC 3

$$\delta E_{0+h}(t) \simeq \delta E_0(t) + \delta E_h(t) = E_0(t)\sin(\phi_0) + hE_0'\frac{\omega_L}{\omega_g}\sin(\omega_g L)e^{i(\omega_L \pm \omega_g)t}$$

$$\langle (E_x(t) + E_y(t))^2 \rangle \simeq \langle (\delta E_0(t) + \delta E_h(t))^2$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2}{3H_0^2 \Delta\omega} \gtrsim \boxed{\frac{\omega^3}{3H_0^2 U_{\rm in}} \frac{1}{\mathcal{T}^2(\omega)}}$$

SIGNALS (strings)

$$V = \frac{\lambda}{2}\left(|\phi|^2 - v^2\right)$$

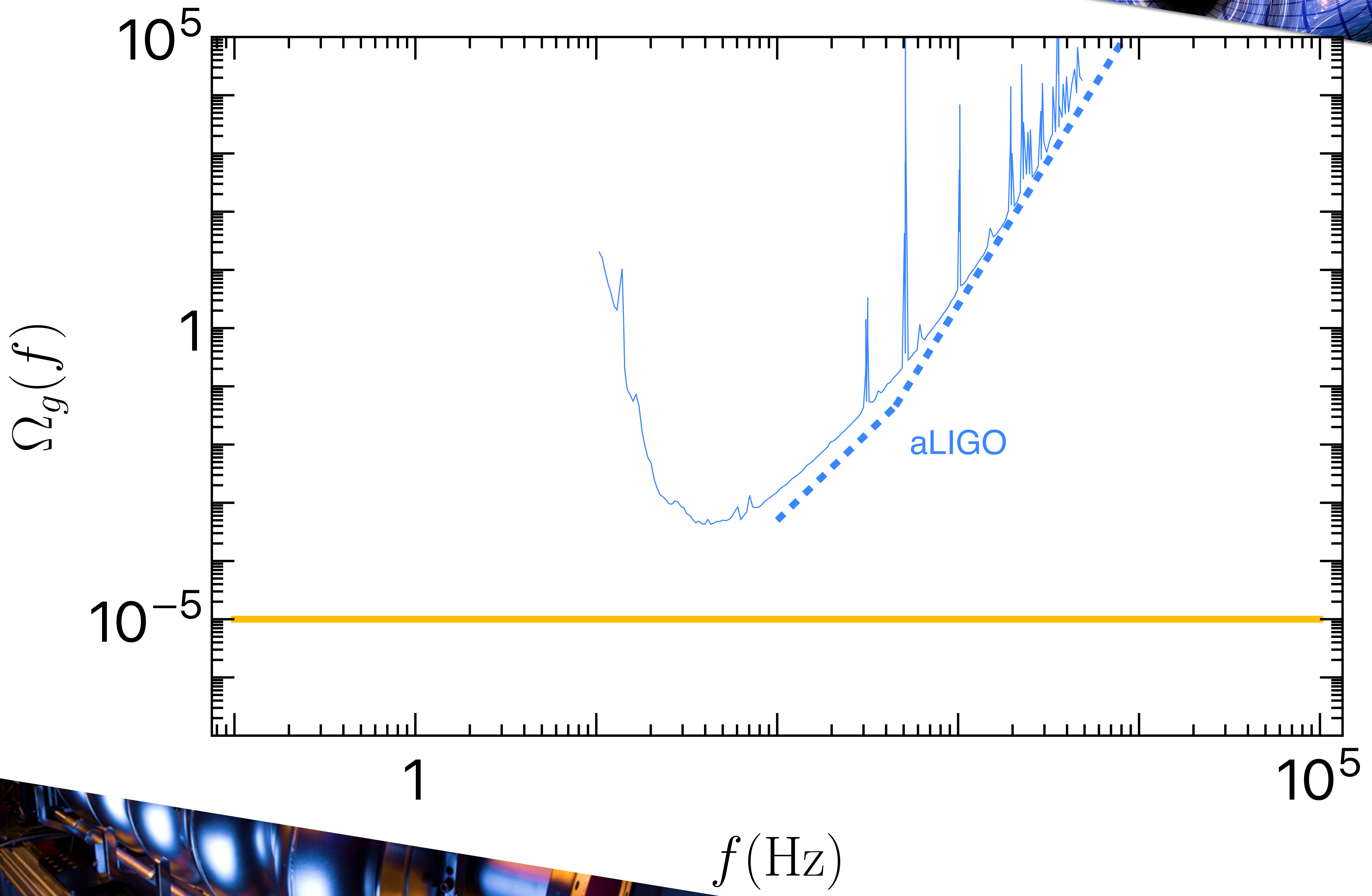
$$\mu \simeq v^2 \begin{cases} 1 \\ \log \frac{m_\phi}{H} \end{cases}$$

$$G_N \mu = 10^{-5}$$

SIGNALS

PT: $$\frac{\beta}{H} = 7 \quad \alpha = 10 \quad T \simeq 10^{10}\ \mathrm{GeV}$$

InfKin: $H_I \simeq 10^{16}\ \mathrm{GeV}$ + late time kination from QCD axion DM



10^5
1
10^{-5}
$\Omega_g(f)$
aLIGO
1
10^5
$f\,(\mathrm{Hz})$

IN-OUT FORMALISM

$$a_{\text{in}}(t) \equiv -\frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_0)} b(\omega, t_0)$$

$$a_{\text{out}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_1)} b(\omega, t_1)$$

BANDWIDTH SQUEEZING

$$Q_{\mathrm{cpl}} = Q_{\mathrm{int}}/(T/\omega_s)$$

$$\omega_s \to \omega_s e^{-2r}$$

$$h \to h e^{-r}$$

Any system in equilibrium if displaced by a small amount responds as a harmonic oscillator

$$\ddot{\delta}(t) + \frac{\dot{\delta}(t)}{\tau_s} + \omega_s^2 \delta(t)$$

Everything interacts with the gravitational wave: detector components, EM fields, us observers, …

$$\ddot{\delta}(t) + \frac{\dot{\delta}(t)}{\tau_s} + \omega_s^2 \delta(t) = \omega_g^2 f(\omega_g, t)$$

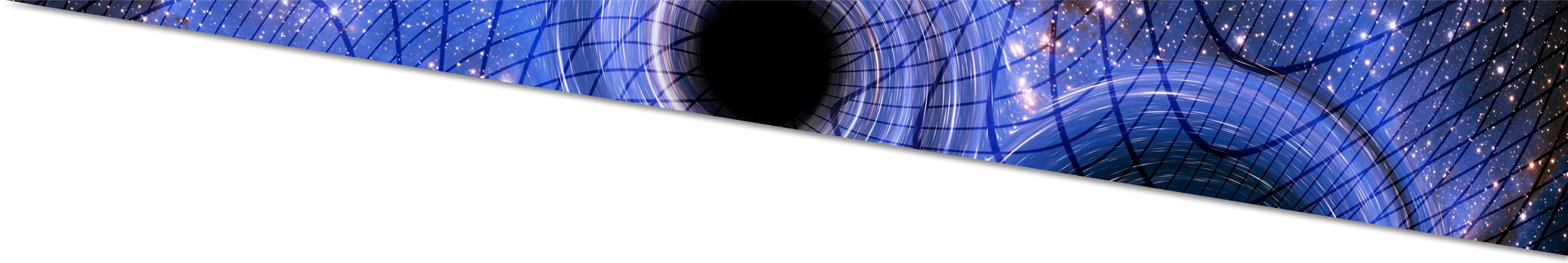
Everything interacts with the gravitational wave: detector components, EM fields, us observers, …

$$\tilde{\delta}(\omega) = \tilde{f}(\omega_g, \omega) \frac{\omega_g^2}{(\omega_s^2 - \omega^2) + i\tau_s \omega}$$

If $\omega_s \gg \omega_g$

we have a very suppressed response

$$\tilde{\delta}(\omega_g) \simeq \frac{\omega_g^2 \tilde{f}(\omega_g, \omega_g)}{\omega_s^2} \ll \tilde{f}(\omega_g, \omega_g)$$



If $\omega_s \gg \omega_g$

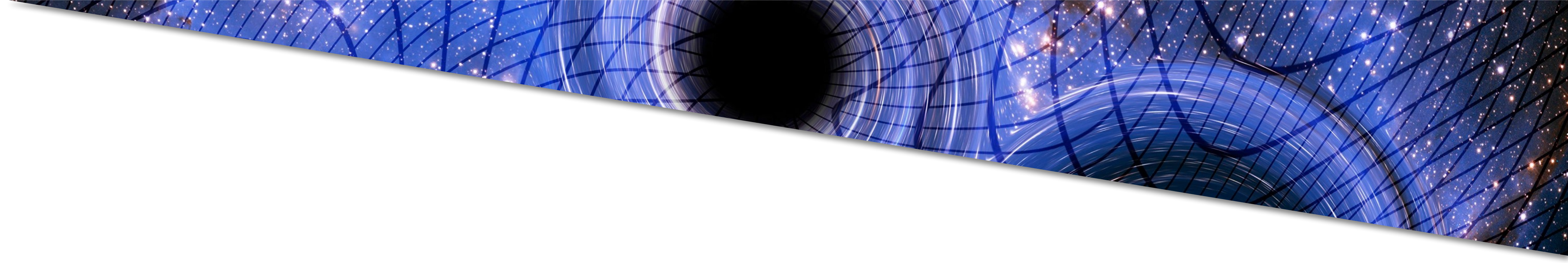
we have a **rigid** detector

$$\mathcal{T}(\omega_g) \simeq \frac{\omega_g^2}{\omega_s^2}$$

If $\omega_s \ll \omega_g$

we have a **flexible** detector

$$\mathcal{T}(\omega_g) \simeq 1$$

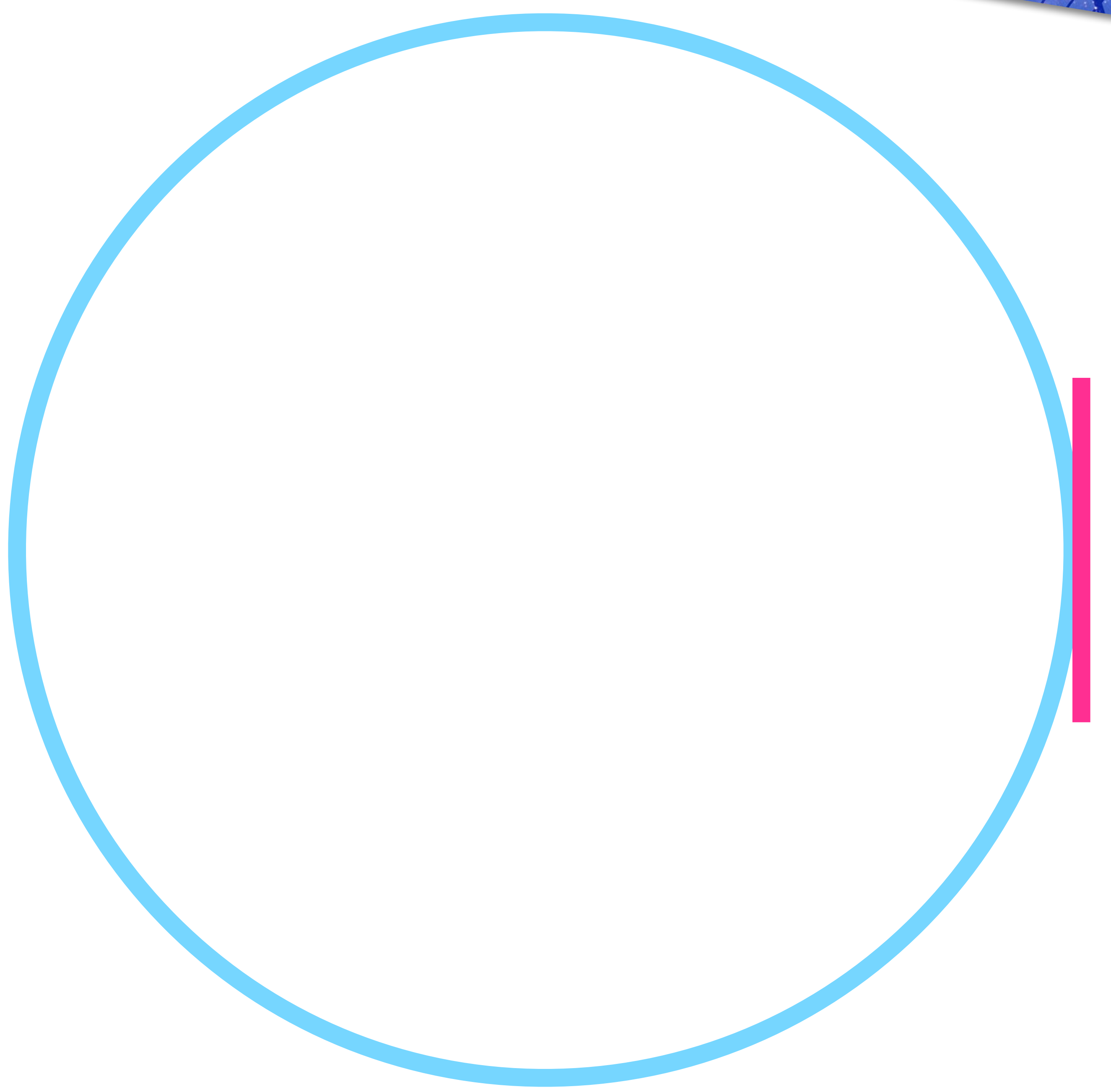


If $|\omega_s - \omega_g| \lesssim \tau_s^{-1} \ll \omega_g$

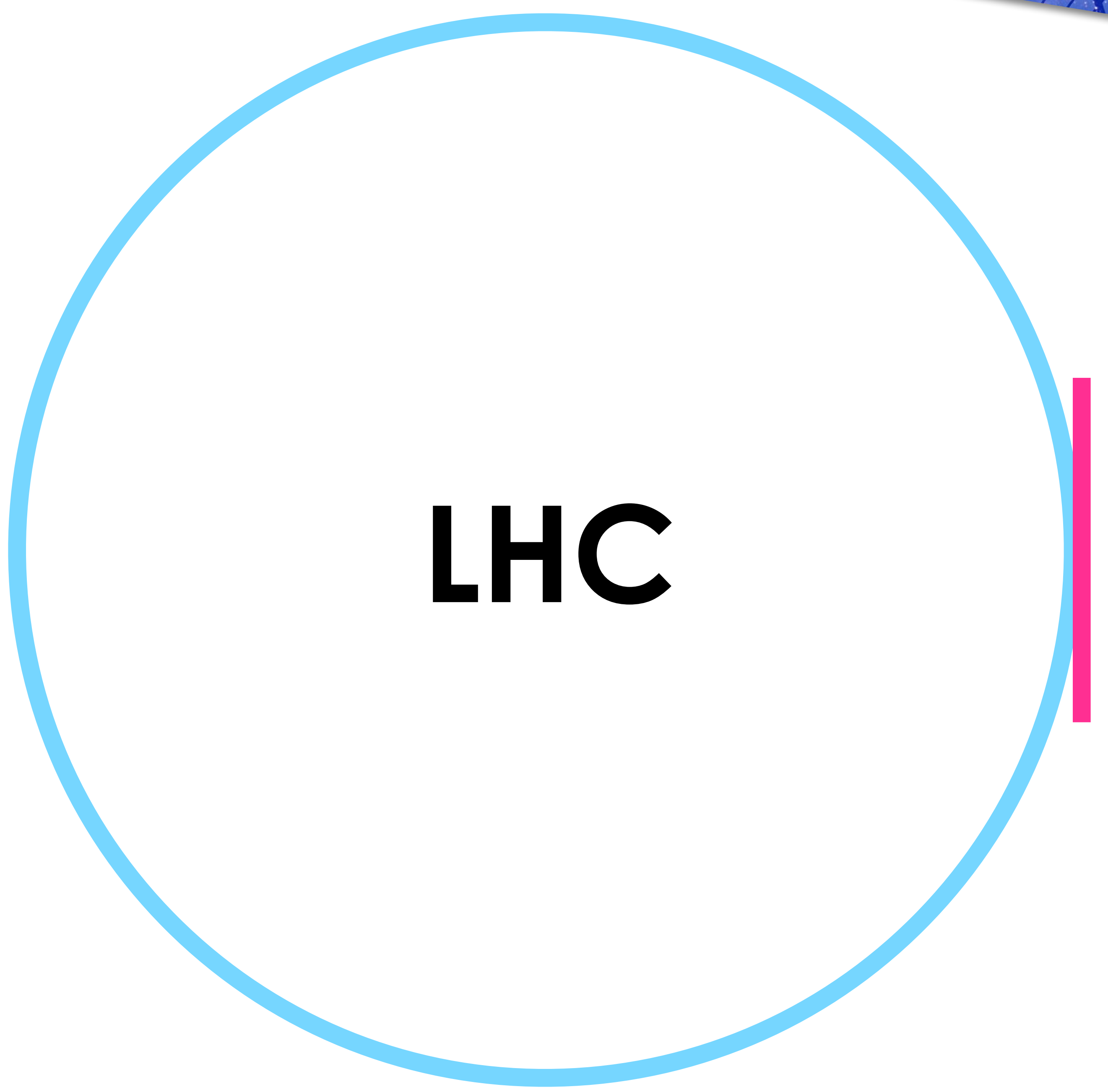
we have a **resonant** detector

$$\mathcal{T}(\omega_g) \simeq \omega_g \tau_s \gg 1$$

FIRST CASE STUDY

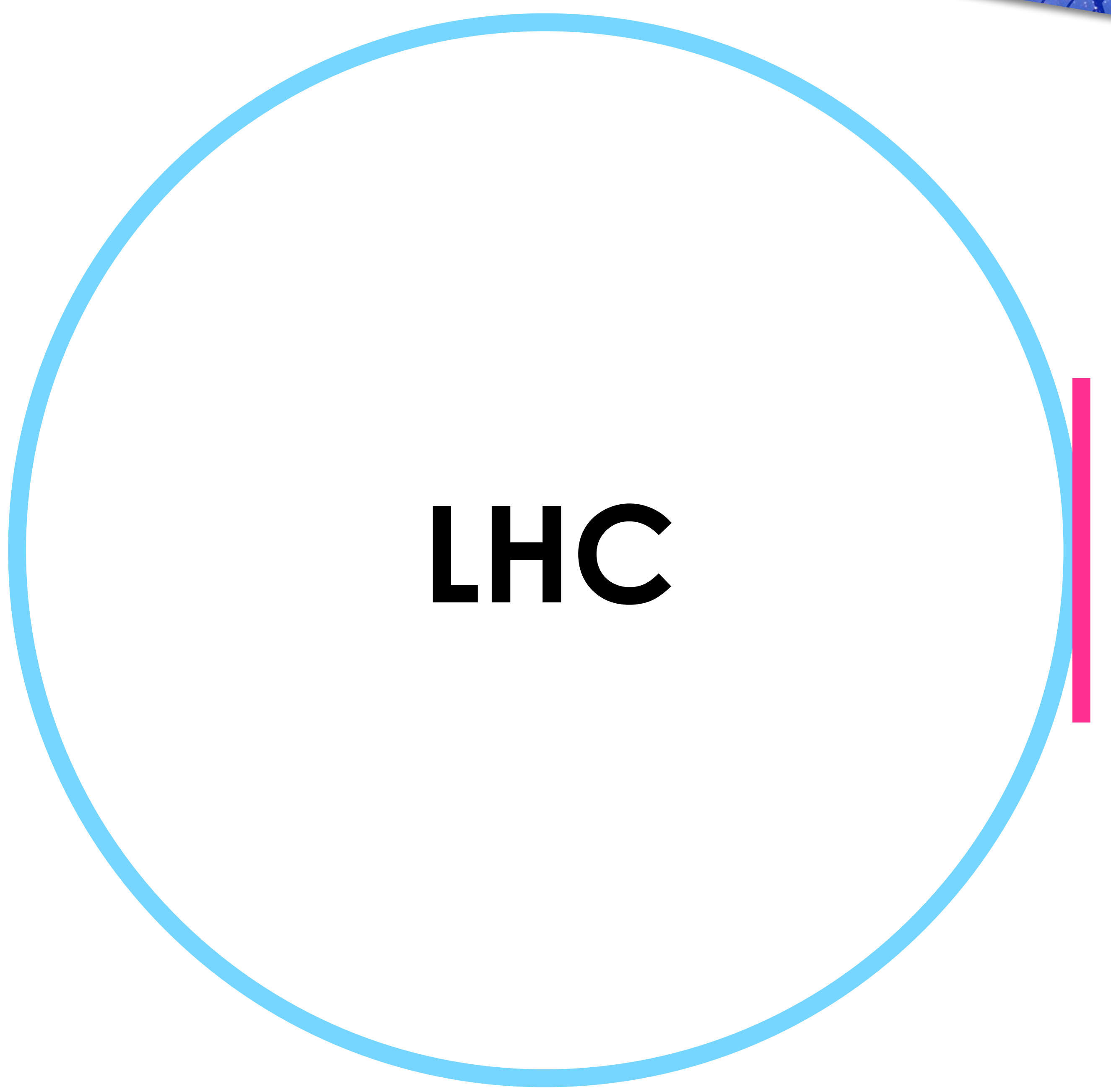


$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$



$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

$$\omega_l \approx \omega_0 \sqrt{\frac{\tilde{h} \alpha_C q V_{\mathrm{RF}}}{E}} \approx 10\ \mathrm{Hz}$$



$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \underline{\omega_g^2 f(\omega_g, t)}$$

Compute

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

DIMENSIONAL ANALYSIS

$$f(\omega_g, t) \simeq h \times L \times \cos(\omega_g t + \phi)$$

FIRST CASE STUDY

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

ON RESONANCE

$$f(\omega_g, t) \simeq h \times L \times \cos(\omega_g t + \phi)$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

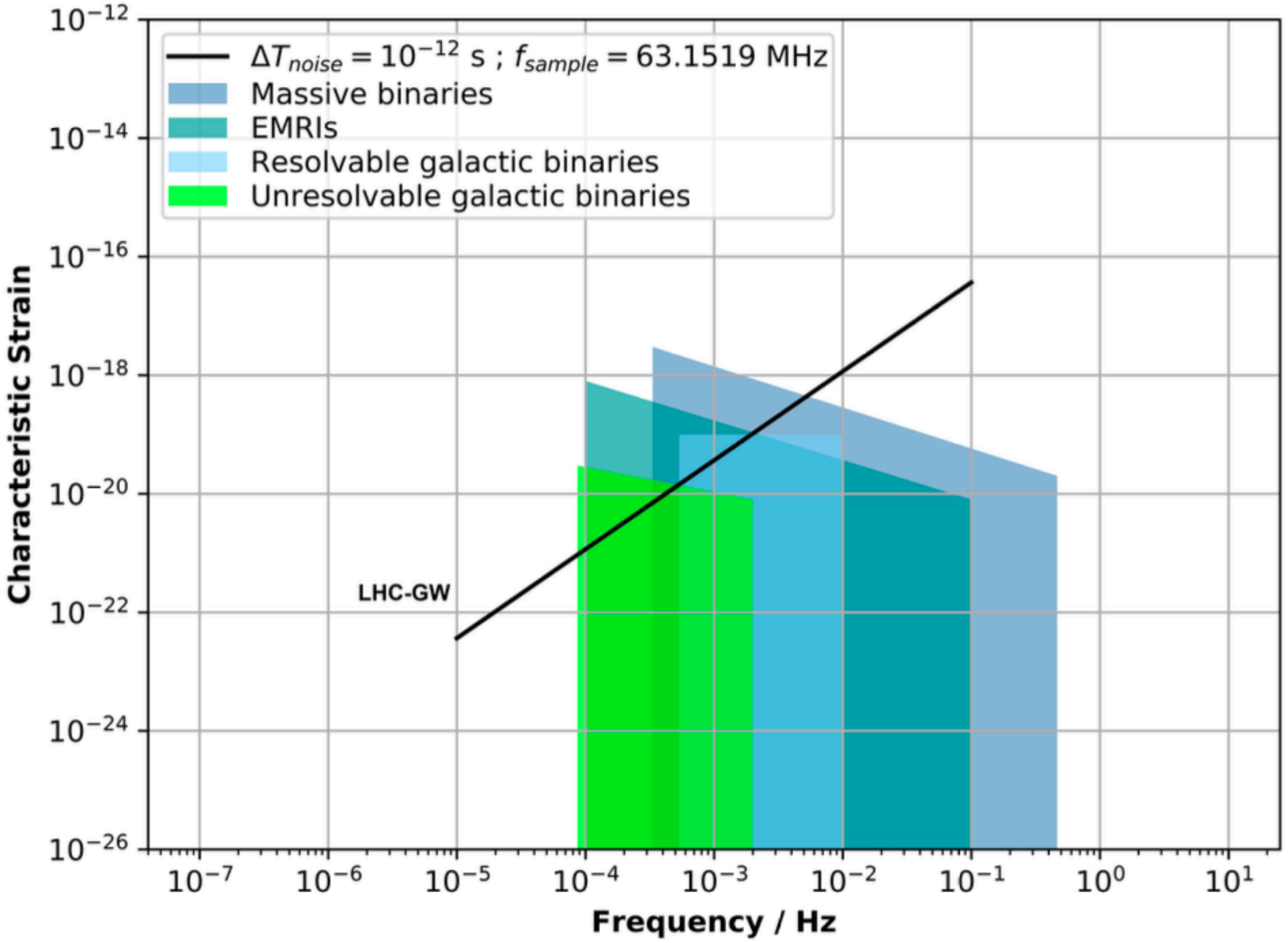
$$\boxed{h \gtrsim 10^{-11}}$$

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-7}$$

0.2 deg @ 400 MHz

$$h \gtrsim 10^{-11}$$

CLAIM FROM
2012.00529



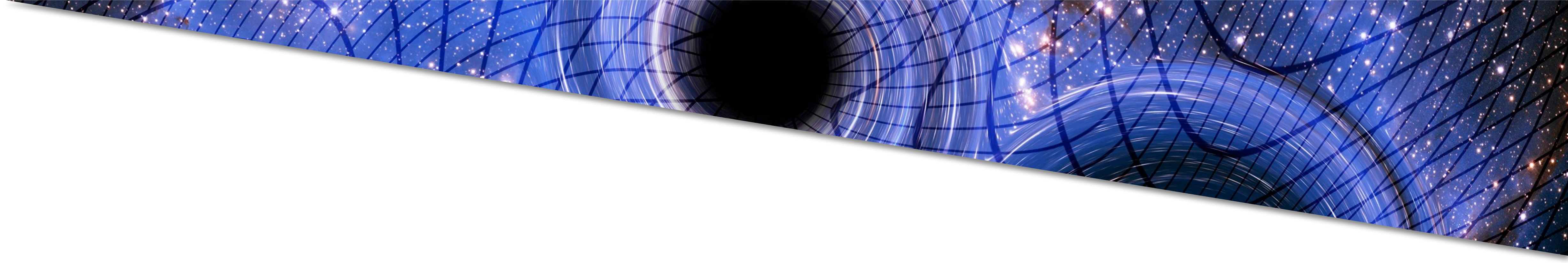
2012.00529

Neglected the transfer function of the LHC

$$\mathcal{T}(\omega_g) \simeq \frac{\omega_g^2}{\omega_s^2}$$

Mistake acknowledged in 2301.08331

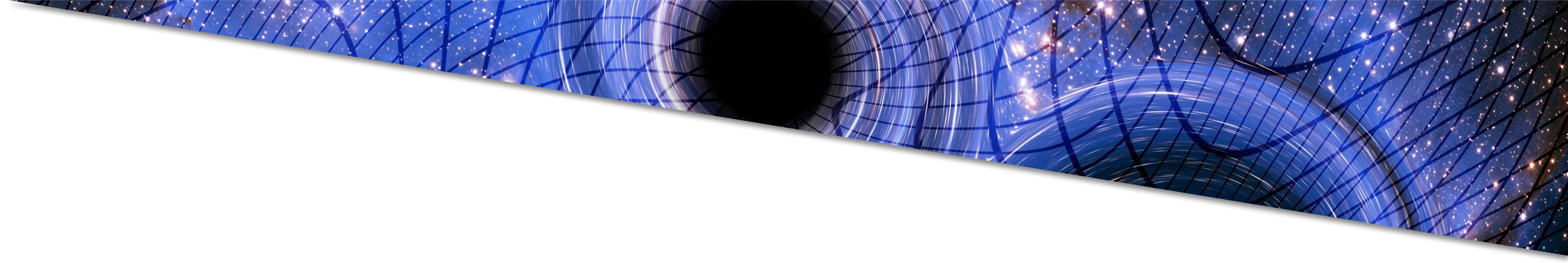
CASE I: QUADRATIC SIGNALS



$$S_n^{\min} \simeq \omega$$

CASE I: QUADRATIC SIGNALS

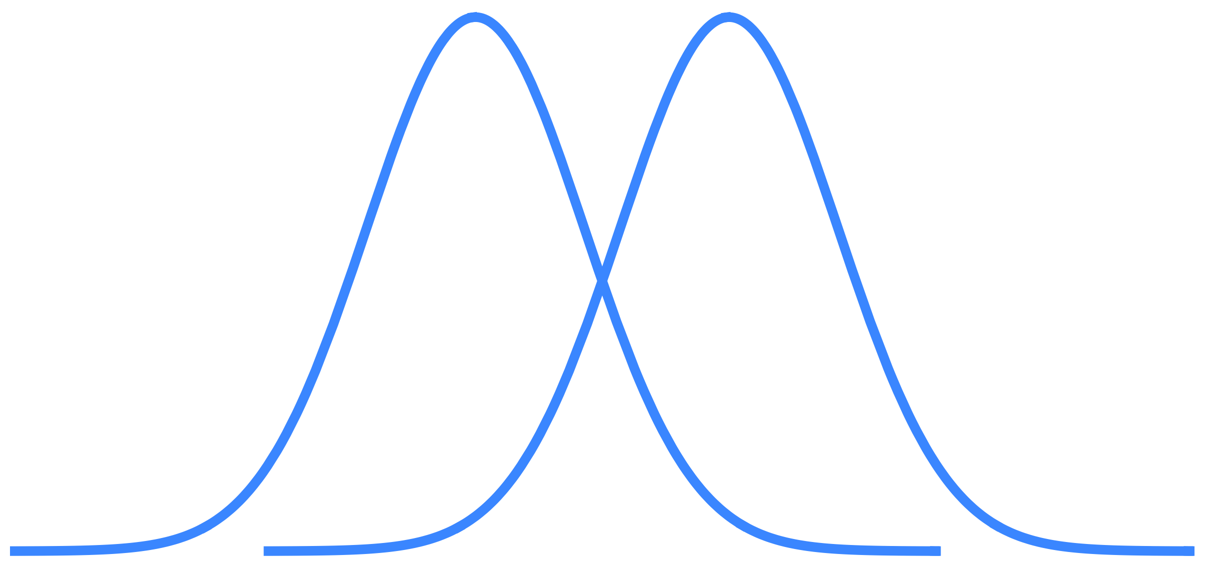
$$h_{\text{min}} \simeq \sqrt{\frac{2\pi}{U_{\text{in}}}} \left(\frac{\Delta\omega}{t_{\text{int}}} \right)^{1/4} \frac{1}{\mathcal{T}(\omega)} \gtrsim \sqrt{\frac{2\pi}{\underline{U_{\text{in}} t_{\text{int}}}}} \frac{1}{\mathcal{T}(\omega)}$$



CASE II: LINEAR SIGNALS

$$\langle E_h(t)E_0(t)\rangle \propto \langle \tilde{E}_h(\omega)\tilde{E}_0\omega)\rangle \neq 0$$

CASE II: LINEAR SIGNALS



CASE II: LINEAR SIGNALS

$$P_{\text{sig}} \simeq hU_{\text{in}}\Delta\omega\,\mathcal{T}^2(\omega)$$

(Almost) The same as before

CASE II: LINEAR SIGNALS

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}\left(1 + \frac{P_{\text{in}}}{\omega\Delta\omega}\right)$$

But much more noise

CASE II: LINEAR SIGNALS

$$h_{\rm min} \simeq \sqrt{\frac{2\pi}{U_{\rm in}}} \left(\frac{\Delta\omega}{t_{\rm int}}\right)^{1/4} \frac{1}{\mathcal{T}(\omega)} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}} \frac{1}{\mathcal{T}(\omega)}$$

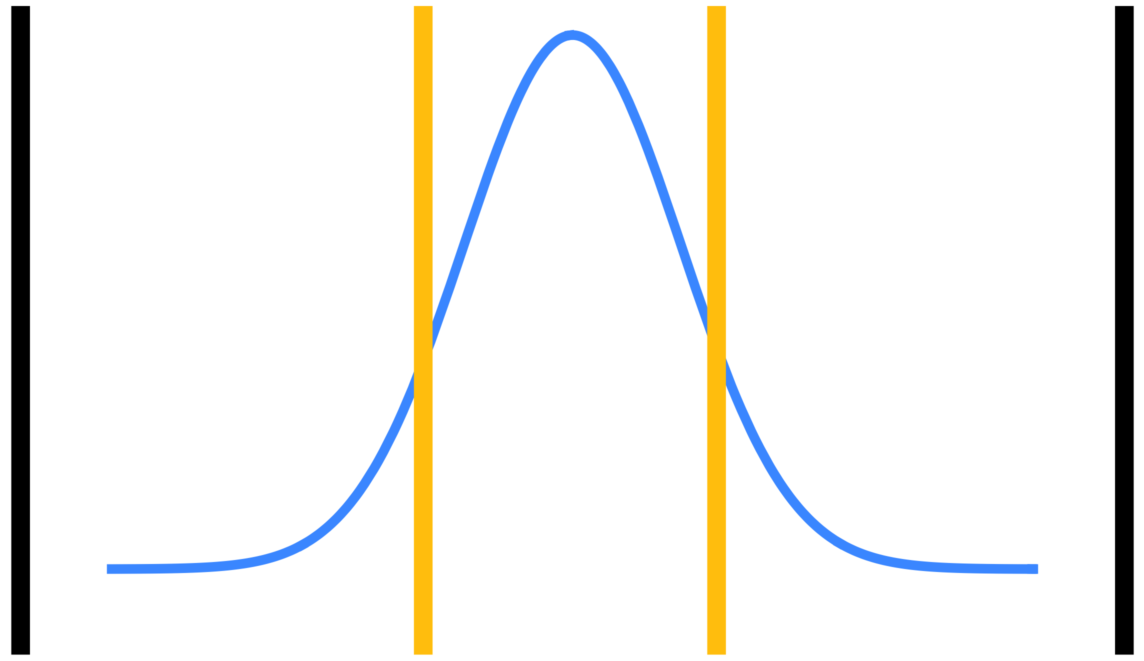
CASE II: LINEAR SIGNALS

$$h_{\min} \simeq \sqrt{\frac{2\pi}{U_{\rm in}}} \left(\frac{\Delta\omega}{t_{\rm int}}\right)^{1/4} \frac{1}{\mathcal{T}(\omega)} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}} \frac{1}{\mathcal{T}(\omega)}$$

Exactly the same as a quadratic signal!

$$\Delta\omega_d$$

Detector Bandwidth



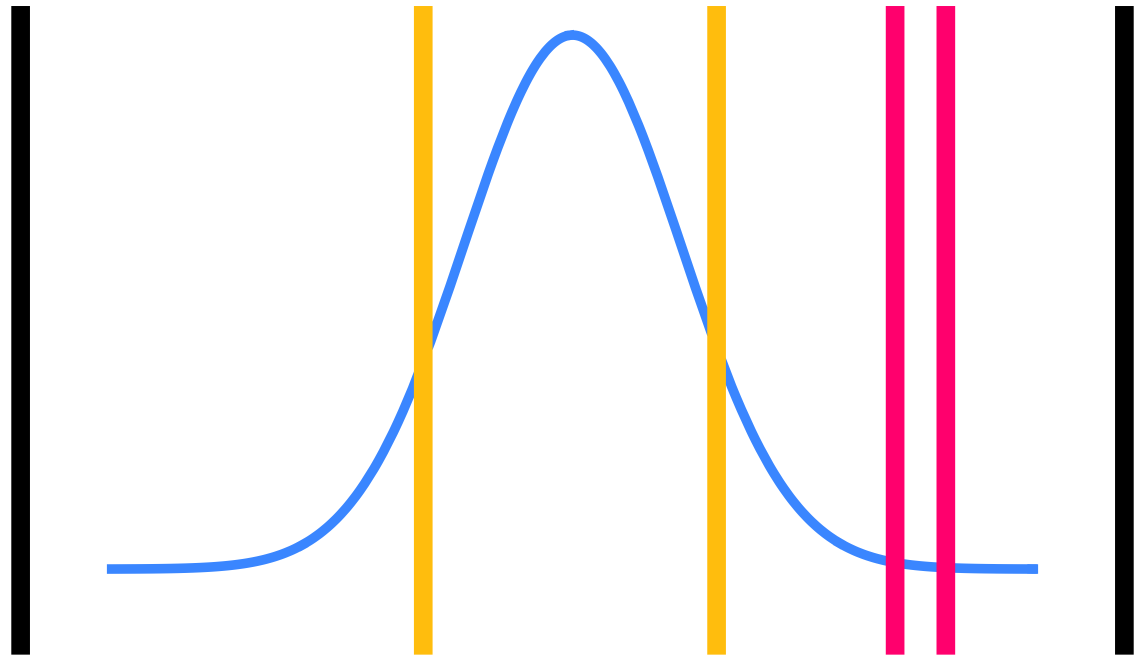
$$\min[\Delta\omega_d, \Delta\omega_s]$$

Signal Width

$$\Delta\omega_s$$

$\Delta\omega_d$

Detector Bandwidth



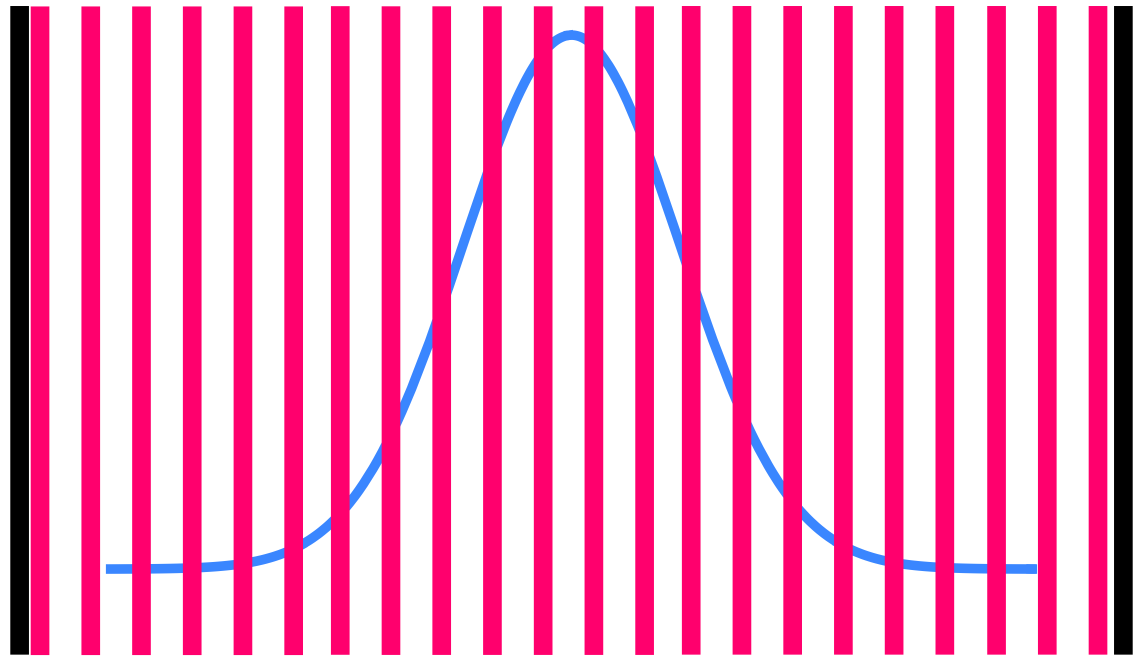
t_{int}^{-1}

Resolution

Signal Width

$\Delta\omega_s$

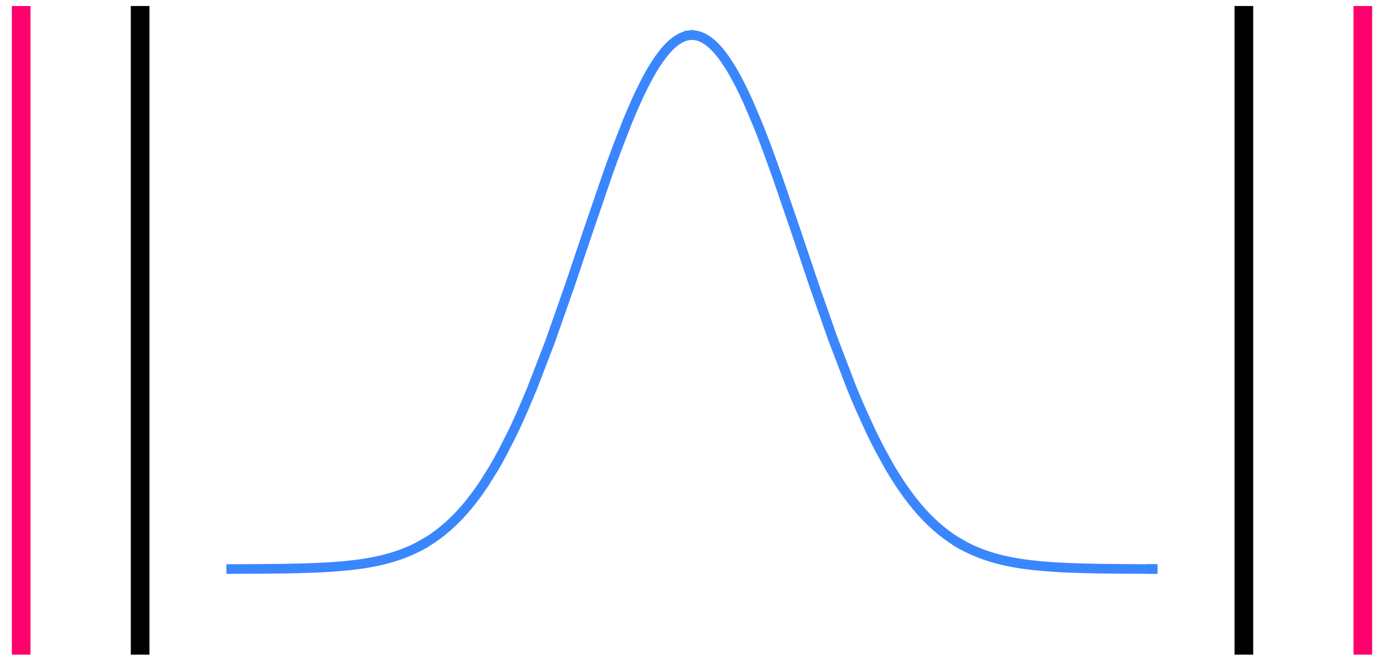
Binning



t_{int}^{-1}

Resolution

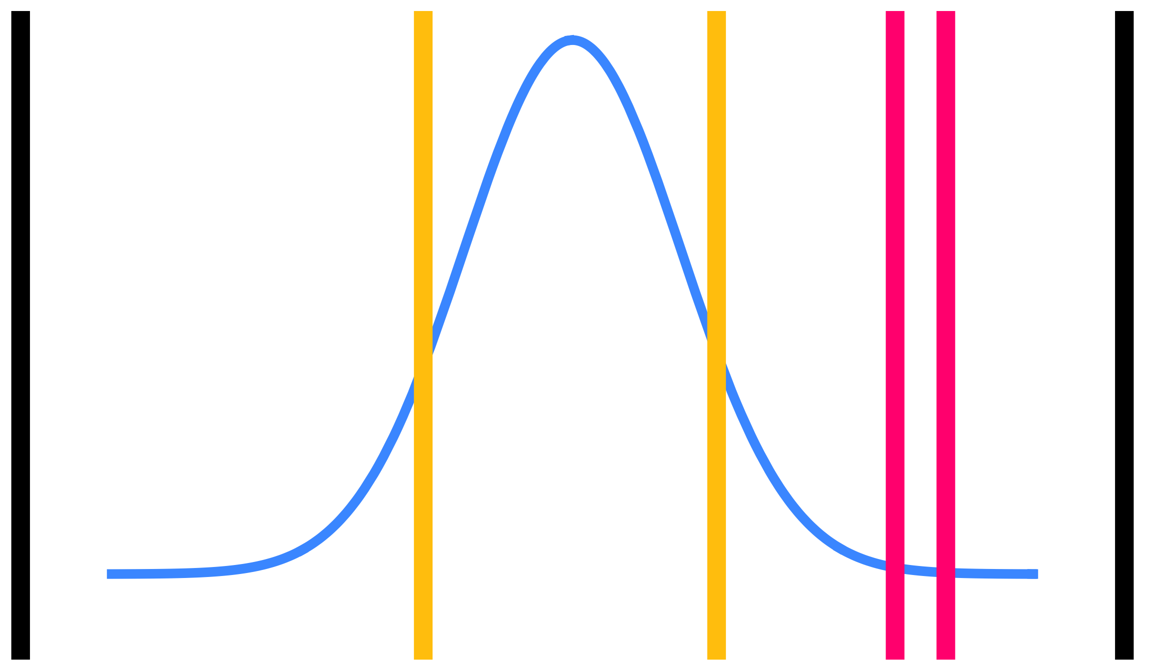
Poor resolution = 1 bin



t_{int}^{-1}

Resolution

Detector Bandwidth



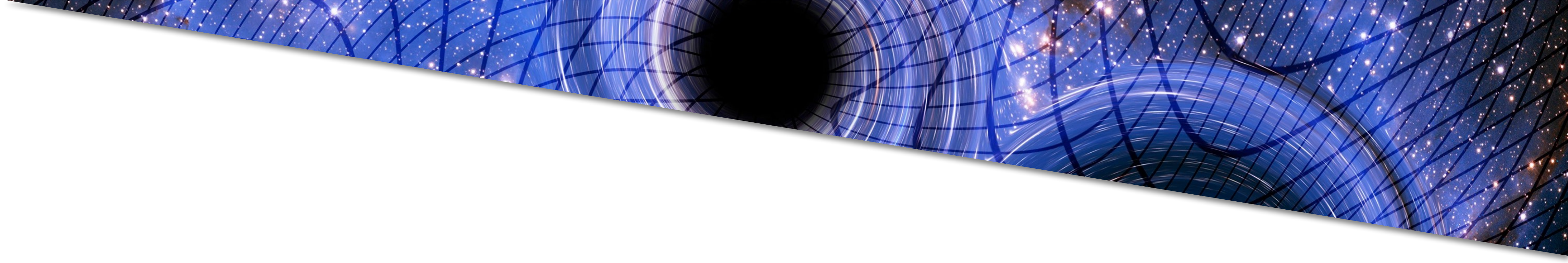
Resolution

Signal Width

$$\Delta\omega \equiv \max[\min[\Delta\omega_d, \Delta\omega_s], t_{\text{int}}^{-1}]$$

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int \frac{d\omega}{2\pi} \frac{S_{\mathrm{sig}}^2(\omega)}{S_n^2(\omega)} \right)^{1/2}$$

$$\simeq \sqrt{\frac{\Delta\omega}{2\pi t_{\mathrm{int}}}} \frac{P_{\mathrm{sig}}}{S_n}$$



$$P_{\mathrm{sig}} \simeq \left(\underline{h^2 U_{\mathrm{in}}}\,\Delta\omega\right)\,\mathcal{T}^2(\omega)$$

Signal Energy

$$P_{\mathrm{sig}} \simeq \left(h^2 U_{\mathrm{in}} \underline{\Delta\omega}\right) \mathcal{T}^2(\omega)$$

Relevant Bandwidth

LIKELY MISTAKE:
They imagined a linear experiment with one quantum of background

(N.B. PRELIMINARY CONCLUSION)