Shadow Matter

CERN

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Gauge Theories and Gravity without Constraints

The quantum field theory of the world produces the classical theories that we discovered first (GR, EM, Newton's laws)

We guess the QFT by, *e.g.*, picking the Hamiltonian (and d.o.f.) judiciously

Here I will explore EM and GR without standard constraints on the initial states and see some new effects

Pre(r)amble

Classical Equations of Motion

$$S = \int dt L(q, \dot{q})$$

$$\frac{\delta S}{\delta q} = 0$$

Hamiltonian can be build from: $p \equiv \frac{\delta L}{\delta \dot{q}}$

Ehrenfest

$$\left[\hat{q}_i, \hat{p}^j\right] = i\delta_i^j$$

for $H = \dfrac{p^2}{2m} + V(q)$

$$\partial_t q_i = i\left[H, q_i\right] = \frac{\partial H}{\partial p^i} = \frac{p_i}{m}$$

$$\partial_t p^i = i\left[H, p^i\right] = -\frac{\partial H}{\partial q_i} = -\frac{\partial V(q)}{\partial q_i}$$

(up to commutators)

Note: If there is a q_k without a p_k, we lose an e.o.m.

Non-dynamical d.o.f.

Simple example: $$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + V(\hat{q}_2 - \hat{q}_1)$$

$$Q = \frac{q_1 + q_2}{2} \qquad q = q_2 - q_1$$

$$P = p_1 + p_2 \qquad p = \frac{p_2 - p_1}{2}$$

$$\to H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + V(q)$$

$$[P, H] = 0 \qquad \hat{P}\,|\,P\rangle = P\,|\,P\rangle$$

Think of eigenstates of P as super-selection sectors

Can choose

$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = 0$$

$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = P\,|\,\psi\rangle_{\text{phys}}$$

$$\langle\psi\,|\,\hat{P}\,|\,\psi\rangle_{\text{phys}} = P$$

Non-dynamical d.o.f.

Next example: $$H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + V(q) + \frac{P}{M}\overline{V}(q)$$

Still, $[P, H] = 0$

Again, can choose

$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = 0$$

$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = P\,|\,\psi\rangle_{\text{phys}}$$

$$\langle\psi\,|\,\hat{P}\,|\,\psi\rangle_{\text{phys}} = P$$

Measurement allows us to tell what super-selection sector we are in

EM

EM

classical: $$S = \int d^4x \mathscr{L} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu + \cdots \right]$$

Gauss' Law $$\frac{\delta S}{\delta A_0} = \underbrace{\partial_\mu F^{\mu 0}}_{\nabla \cdot E} - J^0 = 0$$

Ampere's Law $$\frac{\delta S}{\delta A_i} = \underbrace{\partial_\mu F^{\mu i}}_{\dot{E} + \nabla \times B} - J^i = 0$$

conjugate momenta:

$$\pi^i \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_i} = \partial^0 A^i - \partial^i A^0 \equiv -E^i \qquad \pi^0 \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_0} = 0$$

EM

Hamiltonian: $$H = \int d^3x \mathscr{H}$$

$$\mathscr{H} = \frac{1}{2}\mathbf{E}^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 - A_0(\nabla \cdot E - J^0) + \mathbf{A} \cdot \mathbf{J} + \cdots$$

Make this quantum — choose A_0 — gauge freedom makes it plausible that this choice doesn't affect dynamics.

Looks like a coupling constant. Simple choice: $A_0 = 0$ (Weyl gauge)

Commutators canonical: $$\left[E^i(\mathbf{x}), A_j(\mathbf{y})\right] = i\,\delta^i_j\,\delta(\mathbf{x} - \mathbf{y})$$

EM

From the Schrödinger Eq:

$$\partial_t \langle \mathbf{A} \rangle = i\langle [H, \mathbf{A}] \rangle = \langle \mathbf{E} \rangle$$

$$\partial_t \langle \mathbf{E} \rangle = i\langle [H, \mathbf{E}] \rangle = \langle \nabla \times \mathbf{B} + \mathbf{J} \rangle$$ Ampere's Law

Gauss' Law? $G \equiv \nabla \cdot \mathbf{E} - J^0$

Note: $[H, G] = 0$ (G's conjugate doesn't appear!)

Can require: $G\,|\,\psi\rangle_{\text{phys}} = 0$

EM

Ampere's Law $$\partial_t \langle \mathbf{E} \rangle = i \langle \left[H, \mathbf{E} \right] \rangle = \langle \nabla \times \mathbf{B} + \mathbf{J} \rangle$$

$$G \equiv \nabla \cdot \mathbf{E} - J^0$$

Gauss' Law — Can require:

$$G \, | \, \psi \rangle_{\text{phys}} = 0$$

EM

Ampere's Law $\partial_t \langle \mathbf{E} \rangle = i \langle \left[H, \mathbf{E} \right] \rangle = \langle \nabla \times \mathbf{B} + \mathbf{J} \rangle$

Gauss' Law — Can require:

$$G \equiv \nabla \cdot \mathbf{E} - J^0$$

$$G \,|\, \psi \rangle_{\text{phys}} = 0$$

Could instead require: $G \,|\, \psi \rangle_{\text{phys}} = \rho_{\text{sh}}(\mathbf{x}) \,|\, \psi \rangle_{\text{phys}}$

Or even: $\langle \psi \,|\, G \,|\, \psi \rangle = 0 \;\text{ or }\; \rho_{\text{sh}}(\mathbf{x})$

These appears as normal QED, potentially with a static charge density b.g.

$$\langle \nabla \cdot \mathbf{E} - J^0 - \rho_{\text{sh}}(\mathbf{x}) \rangle = 0$$

equivalent physics to infinite mass charge distribution

Gravity (toy)

Gravity: minisuperspace

zero-mode only (FRW): $ds^2 = -N(t)^2 dt^2 + a(t)^2 d\mathbf{x}^2$

classical: $S = \int d^4x \sqrt{-g} \left(M_{\rm pl}^2 R + \underbrace{\mathscr{L}_{\rm matter}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots} \right) + S_{GHY}$

Gravity: minisuperspace

zero-mode only (FRW): $ds^2 = -N(t)^2 dt^2 + a(t)^2 d\mathbf{x}^2$

classical: $S = \int d^4x \sqrt{-g} \, (\, M_{\rm pl}^2 R \; + \underbrace{\mathscr{L}_{\rm matter}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots}) \; + \; S_{GHY}$

$$\frac{\delta S}{\delta N} = a^3 \left(6M_{\rm pl}^2 \frac{\dot{a}^2}{N^2 a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$ 1st Friedmann Eq

$$\frac{\delta S}{\delta a} = 3Na^2 \left(4M_{\rm pl}^2 \left(\frac{\ddot{a}}{N^2 a} + \frac{\dot{a}^2}{2N^2 a^2} - \frac{\dot{a}\dot{N}}{N^3 a} \right) + \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$ 2nd Friedmann Eq

$$\frac{\delta S}{\delta \phi} = -\frac{a^3}{N} \left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\dot{N}\dot{\phi}}{N} + N^2 \frac{\partial V(\phi)}{\partial \phi} \right)$$ matter EOM

Gravity: minisuperspace

classical:

$$S = \int d^4x \sqrt{-g} \left(M_{\rm pl}^2 R + \underbrace{\mathscr{L}_{\rm m}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots} \right) + S_{GHY}$$

conjugate momenta:

$$\pi_a \equiv \frac{\delta \mathscr{L}}{\delta \dot{a}} = -6M_{\rm pl}^2 \frac{a\dot{a}}{N} \qquad \pi_N \equiv \frac{\delta \mathscr{L}}{\delta \dot{N}} = 0 \qquad \pi_\phi = \frac{\delta \mathscr{L}}{\delta \dot{\phi}} = \frac{a^3}{N}\dot{\phi}$$

Gravity: minisuperspace

classical: $$S = \int d^4x\sqrt{-g}\,(\ M_{\rm pl}^2 R \quad + \quad \underbrace{\mathscr{L}_{\rm m}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\cdots}\ ) \ + \ S_{GHY}$$

conjugate momenta:

$$\pi_a \equiv \frac{\delta\mathscr{L}}{\delta\dot a} = -\,6M_{\rm pl}^2\,\frac{a\dot a}{N} \qquad \pi_N \equiv \frac{\delta\mathscr{L}}{\delta\dot N} = 0 \qquad \pi_\phi = \frac{\delta\mathscr{L}}{\delta\dot\phi} = \frac{a^3}{N}\dot\phi$$

Hamiltonian:

$$H = \Big[\pi\dot a + \pi_\phi\dot\phi - \mathscr{L}\Big]_{\dot a=\cdots,\dot\phi=\cdots} = -\,\frac{N}{24M_{pl}^2 a}\pi^2 + \frac{N}{2a^3}\pi_\phi^2 + Na^3V(\phi)$$

$$= N\tilde H(\pi_a, a, \pi_\phi, \phi)$$

Gravity: minisuperspace

Schrödinger eq:

$$i\frac{d}{dt}|\psi\rangle = N(t)\,\tilde{H}\,|\psi\rangle \quad \rightarrow i\frac{d}{N(t)dt}|\psi\rangle = \tilde{H}\,|\psi\rangle$$

N maintains time reparameterization invariance.

N is a parameter — Can pick it. Simple choice: $N = 1$

Gravity: minisuperspace

Schrödinger eq:

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N is a parameter — Can pick it. Simple choice: $N = 1$

$$\partial_t\langle a\rangle = i\langle[H,a]\rangle \to \langle\pi_a\rangle = \langle -6M_{\rm pl}a\dot{a}\rangle$$

(choosing operator ordering wisely)

$$\partial_t\langle\pi_a\rangle = i\langle\left[H,\pi_a\right]\rangle \to$$

(replacing π_a's with $\dot{a}$'s, assuming classical states)

$$\longrightarrow \frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{M_{\rm pl}^2}p$$

2nd Friedmann Eq

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Gravity: minisuperspace

1st Friedmann Eq? From classical:

$$0 = \frac{\delta S}{\delta N} = \frac{\delta \int \pi_a \dot{a} + \pi_\phi \dot{\phi} - N\tilde{H}}{\delta N} \longrightarrow \tilde{H} = 0$$

The ‘Hamiltonian Constraint’ *is* the 1st Friedmann Eq

Standard treatment (like Gauss): $\tilde{H}\,|\,\psi\rangle_{\rm phys} = 0$ (Wheeler-deWitt)

Gravity: minisuperspace

1st Friedmann Eq? From classical:

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The 'Hamiltonian Constraint' *is* the 1st Friedmann Eq

Standard treatment (like Gauss): $\tilde{H}\,|\,\psi\rangle_{\text{phys}} = 0$ (Wheeler-deWitt)

??? No Schrödinger equation ???

The "problem of time" in quantum gravity

Simple fix: $\langle\psi\,|\,H\,|\,\psi\rangle = 0$

Gravity: minisuperspace

1st Friedmann Eq? From classical:

$$0 = \frac{\delta S}{\delta N} = \frac{\delta \int \pi_a \dot{a} + \pi_\phi \dot{\phi} - N\tilde{H}}{\delta N} \longrightarrow \tilde{H} = 0$$

Simple fix: $\langle \psi | \tilde{H} | \psi \rangle = 0$

For classical states: $\tilde{H} = a^3 \left(3M_{\rm pl}^2 \left(\frac{\dot{a}}{a} \right)^2 - \rho \right) = 0$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Gravity: minisuperspace

Could choose: $\langle\psi|\tilde{H}|\psi\rangle = \mathbb{C}$

$\mathbb{C}$ is constant as $\left[\tilde{H},\tilde{H}\right] = 0$

For classical states: $\tilde{H} = a^3\left(3M_{\rm pl}^2\left(\frac{\dot{a}}{a}\right)^2 - \rho\right) = \mathbb{C}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm pl}^2}\left(\rho + \frac{\mathbb{C}}{a^3}\right)$$

behaves as another matter component

Interlude

Point 1: The equations that are modified (Gauss, 1st Friedmann) are constraints on initial conditions.

Point 2: The Schrödinger equation will evolve any state you give it. These are simply a broader set of initial conditions.

Point 3: These are initial conditions — Inflation will redshift all of this stuff away. For rest of talk, assume inflation didn't happen.

GR

GR

$$S = \int d^4x \, \underbrace{(\sqrt{-g}\, M_{\rm pl}^2 R \; + \; \sqrt{-g}\, \mathscr{L}_{\rm matter}(\phi))}_{\mathscr{L}_g} \; + \; S_{GHY}$$

classical:

$$\frac{\delta S}{\delta g_{\mu\nu}} = \sqrt{-g} \; \left(M_{\rm pl}^2 G^{\mu\nu} - T^{\mu\nu} \right) = 0$$

conjugates:

$$\frac{\delta \mathscr{L}_g}{\delta \dot{g}_{ij}} \equiv \pi^{ij} \qquad\qquad \frac{\delta \mathscr{L}_g}{\delta \dot{g}_{0\mu}} = 0$$

GR

$$S = \int d^4x \, \underbrace{(\sqrt{-g}\, M_{\rm pl}^2 R \;+\; \sqrt{-g}\, \mathscr{L}_{\rm matter}(\phi))}_{\mathscr{L}_g} \;+\; S_{GHY}$$

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conjugates:

$$\frac{\delta \mathscr{L}_g}{\delta \dot{g}_{ij}} \equiv \pi^{ij} \qquad\qquad \frac{\delta \mathscr{L}_g}{\delta \dot{g}_{0\mu}} = 0$$

construct Hamiltonian in easy gauge: $g_{0\mu} = -\,\delta_{0\mu}$ $\gamma_{ij} \equiv g_{ij}$

(synchronous)

$$\left[\gamma_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right] = i\, \delta^k_{(i}\delta^l_{j)}\, \delta(\mathbf{x} - \mathbf{y})$$

$$\mathscr{H} = \frac{1}{\sqrt{\gamma}}(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2) - \sqrt{\gamma}\; {}^{(3)}R + \cdots$$

GR

EOM: $$\partial_t \langle \pi^{ij} \rangle \sim - \left\langle \frac{\delta H}{\delta \gamma_{ij}} \right\rangle \longrightarrow G^{ij} = 8\pi G T^{ij} \qquad \left(\frac{\delta S}{\delta g_{ij}} = 0 \right)$$

(under rug: coherent states, operator ordering)

Remaining equations constraints — impose on the initial states

$\langle \mathscr{H} \rangle = 0$ Hamiltonian $\longrightarrow M_{\rm pl}^{-2} \sqrt{-g}\, G^{00} - \sqrt{-g}\, T^{00} = 0$

$\langle \chi^i \rangle = 0$ Momentum $\longrightarrow M_{\rm pl}^{-2} \sqrt{-g}\, G^{0i} - \sqrt{-g}\, T^{0i} = 0$

GR

EOM: $$\partial_t \langle \pi^{ij} \rangle \sim - \left\langle \frac{\delta H}{\delta \gamma_{ij}} \right\rangle \longrightarrow G^{ij} = 8\pi G T^{ij} \qquad \left(\frac{\delta S}{\delta g_{ij}} = 0 \right)$$

(under rug: coherent states, operator ordering)

Remaining equations constraints — impose on the initial states

$$\langle \mathscr{H} \rangle = 0 \qquad \text{Hamiltonian} \qquad \longrightarrow \quad M_{\rm pl}^{-2}\sqrt{-g}\; G^{00} - \sqrt{-g}\; T^{00} = 0$$

$$\langle \chi^i \rangle = 0 \qquad \text{Momentum} \qquad \longrightarrow \quad M_{\rm pl}^{-2}\sqrt{-g}\; G^{0i} - \sqrt{-g}\; T^{0i} = 0$$

$$\langle \mathscr{H} \rangle = \mathbb{H} \quad \longrightarrow \quad M_{\rm pl}^{-2}\sqrt{-g}\; G^{00} - \sqrt{-g}\; T^{00} = \mathbb{H}$$

$$\langle \chi^i \rangle = \mathbb{P}^i \quad \longrightarrow \quad M_{\rm pl}^{-2}\sqrt{-g}\; G^{0i} - \sqrt{-g}\; T^{0i} = \mathbb{P}^i$$

GR

$$G^{00} = 8\pi G\left(T^{00} + \frac{\mathbb{H}}{\sqrt{-g}}\right)$$

$$G^{0i} = 8\pi G\left(T^{0i} + \frac{\mathbb{P}^i}{\sqrt{-g}}\right)$$

$$G^{ij} = 8\pi G T^{ij}$$

$\longrightarrow$

$$G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\rm sh})$$

$$T^{\mu\nu}_{\rm sh} \equiv \frac{1}{\sqrt{-g}}\begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{H}, \mathbb{P}^i$ are constrained functions

(identity) $\nabla_\mu G^{\mu\nu} = 0$

(EOM) $\nabla_\mu T^{\mu\nu} = 0$

$$\longrightarrow \nabla_\mu T^{\mu\nu}_{\rm sh} = 0 \longrightarrow$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij}\mathbb{P}^j) = 0$$

GR

$$G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\rm sh})$$

What do these source terms do?

$$T^{\mu\nu}_{\rm sh} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

look at limits

GR

limit: $\mathbb{P}^i = 0 \qquad \rightarrow \partial_0 \mathbb{H} = 0$

$$G^{00} = 8\pi G\,(T^{00} + \frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}})$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

$$G^{0i} = 8\pi G\, T^{0i}$$

$$G^{ij} = 8\pi G\, T^{ij}$$

in synchronous gauge, write as:

$$T^{\mu\nu}_{\rm sh} = \rho_{\rm sh} u^\mu u^\nu$$

$$u^\mu = \{1, 0, 0, 0\} \qquad \rho_{\rm sh} = \mathbb{H}(\mathbf{x})/\sqrt{-g}$$

how does this behave? look at dust:

GR

how does this behave? look at dust:

$$T^{\mu\nu}_{\text{dust}} = \rho u^\mu u^\nu \qquad u^\mu = \{1, 0, 0, 0\}$$

$$\Gamma^0_{00} = \Gamma^i_{00} = \Gamma^0_{0i} = \Gamma^0_{i0} = 0$$ in synchronous gauge

GR

how does this behave? look at dust:

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$$\Gamma^0_{00} = \Gamma^i_{00} = \Gamma^0_{0i} = \Gamma^0_{i0} = 0$$ in synchronous gauge

thus: $$0 = \sqrt{-g}\,\nabla_\mu T^{\mu\nu}_{\text{dust}} = \sqrt{-g}(\partial_\mu(\rho u^\mu u^\nu) + \Gamma^\mu_{\mu\lambda}\rho u^\lambda u^\nu + \Gamma^\nu_{\mu\lambda}\rho u^\mu u^\lambda)$$

GR

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$$= \sqrt{-g}\; u^\nu \partial_0 \rho + \sqrt{-g}\; \Gamma^\mu_{\mu 0}\rho u^\nu + \sqrt{-g}\; \cancel{\Gamma^\nu_{00}}\rho$$

GR

how does this behave? look at dust:

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$$= \sqrt{-g}\; u^\nu \partial_0 \rho + \sqrt{-g}\;\Gamma^\mu_{\mu 0}\rho u^\nu + \sqrt{-g}\;\cancel{\Gamma^\nu_{00}}\rho$$

$$= \sqrt{-g}\; u^\nu \partial_0 \rho + u^\nu(\partial_0\sqrt{-g})\rho$$ using $\partial_\lambda \sqrt{-g} = \sqrt{-g}\;\Gamma^\mu_{\mu\lambda}$

GR

how does this behave? look at dust:

$$T^{\mu\nu}_{\rm dust} = \rho u^\mu u^\nu \qquad\qquad u^\mu = \{1,0,0,0\}$$

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$$0 = u^\nu \partial_0(\rho\sqrt{-g}) \longrightarrow \rho = \mathbb{H}(\mathbf{x})/\sqrt{-g}$$

GR

how does this behave? look at dust:

$$T^{\mu\nu}_{\rm dust} = \rho u^\mu u^\nu \qquad u^\mu = \{1,0,0,0\}$$

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$$= \sqrt{-g}\; u^\nu \partial_0 \rho + u^\nu(\partial_0\sqrt{-g})\rho$$ using $\partial_\lambda\sqrt{-g} = \sqrt{-g}\;\Gamma^\mu_{\mu\lambda}$

$$0 = u^\nu \partial_0(\rho\sqrt{-g}) \longrightarrow \rho = \mathbb{H}(\mathbf{x})/\sqrt{-g}$$

This is the general form of pressure-less dust!

GR

limit: $\mathbb{P}^i = 0$

if $\mathbb{H}(\mathbf{x}) > 0$ everywhere, then can do a spatial coordinate redefinition:

$$\frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}} \longrightarrow \frac{\overline{\mathbb{H}}}{\sqrt{-g}}$$

GR

limit: $\mathbb{P}^i = 0$

if $\mathbb{H}(\mathbf{x}) > 0$ everywhere, then can do a spatial coordinate redefinition:

$$\frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}} \longrightarrow \frac{\overline{\mathbb{H}}}{\sqrt{-g}}$$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad h = h_{ij}\delta^{ij}$$

at linear order $\quad T^{00}_{\rm sh} = (\bar{\rho}_{\rm sh} + \delta\rho_{\rm sh}) = \dfrac{\overline{\mathbb{H}}}{\sqrt{-g}} \simeq \dfrac{\overline{\mathbb{H}}}{a^3}(1 - h/2)$

or $\dot{\delta} = -\dot{h}/2$

GR

limit: $\mathbb{H}(\mathbf{x}) = \overline{\mathbb{H}} + \delta\mathbb{H}(\mathbf{x})$ $\mathbb{P}^i \equiv \delta\mathbb{P}^i$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j$$

Perturbative expansion around homogeneity

GR

limit: $\mathbb{H}(\mathbf{x}) = \overline{\mathbb{H}} + \delta\mathbb{H}(\mathbf{x})$ $\mathbb{P}^i \equiv \delta\mathbb{P}^i$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j$$

Perturbative expansion around homogeneity

constraints: $\partial_0\mathbb{H} + \partial_i\mathbb{P}^i = 0$, $\partial_0(g_{ij}\mathbb{P}^j) = 0$

at linear order: $\partial_0(a^2\,\delta\mathbb{P}^j) = 0$

$\delta\mathbb{P}^i \sim a^{-2}$ and $\delta\mathbb{P}^i/\sqrt{-g} \sim a^{-5}$

Redshift quickly away outside horizon

GR

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

general: $\mathbb{H}(x)$ $\mathbb{P}^i(x)$

$$T^{\mu\nu}_{\mathrm{sh}} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

GR

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$$T^{\mu\nu}_{\rm sh} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Can construct: $T^{\mu\nu}_{\rm sh} = \rho_{\rm sh}\, u^\mu u^\nu + q^\mu u^\nu + u^\mu q^\nu$

$$q^\mu = (0, \mathbb{P}^1, \mathbb{P}^2, \mathbb{P}^3)/\sqrt{-g} \qquad \nabla_\mu(\rho_{\rm sh} u^\mu) = -\nabla_\mu q^\mu$$

q^μ produces a ‘heat flux’ that differs from normal particle dynamics

GR

Crazy things — $\mathbb{H}(x)$ could be negative in some places (mimicking negative mass particles) and violating the NEC

In the early universe, if $4\, q^\mu q_\mu > \rho_{\rm sh} > 0$, shadow matter still violates the NEC

$$\rho_{\rm sh} = \mathbb{H}(\mathbf{x})/\sqrt{-g} \qquad\qquad q^\mu = (0, \mathbb{P}^1, \mathbb{P}^2, \mathbb{P}^3)/\sqrt{-g}$$

Possibilities of wormholes or bouncing cosmology

GR + EM

covariant derivatives

$$\nabla_\mu F^{\mu\nu} = (J^\nu + J^\nu_{\rm sh}) \qquad \nabla_\mu J^\mu_{\rm sh} = 0$$

$$\sqrt{-g}\,J^\mu_{\rm sh} \equiv \{\mathbb{J}(\mathbf{x}), 0, 0, 0\}$$

time independent

$$J^\mu_{\rm sh} = \rho^{ch}_{\rm sh} v^\mu \qquad v^\mu = \{1, 0, 0, 0\}$$

synchronous gauge

shadow charge density follows geodesics and does not respond to electromagnetic fields directly

GR + EM

additional modification to Einstein's equations

$$T^{\mu\nu} = \mathscr{E}^{\mu\nu} + T^{\mu\nu}_{matter} \equiv F^{\mu\lambda}F^{\nu}_{\lambda} - \frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma} + T^{\mu\nu}_{matter}$$

$$\begin{aligned}\nabla_\mu T^{\mu\nu} &= \nabla_\mu \mathscr{E}^{\mu\nu} + \nabla_\mu T^{\mu\nu}_{matter} \\ &= F^{\nu}{}_{\lambda}(J^\lambda + J^\lambda_{\text{sh}}) - F^{\nu}{}_{\lambda}J^\lambda\end{aligned}$$

modified constraints

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij}\mathbb{P}^j) = -F_{i0}\,\mathbb{J}$$

Shadow Matter

Loosening the initial conditions of GR allows for source terms that could explain why we think we see dark matter

New source terms for EM produce a charged component of the fake dark matter. Could effect the CMB, BBN, galactic dynamics, and direct detection. Challenging pheno (plasma dynamics)

New source terms could violate NEC with no microscopic instabilities. New phenomena possible

If Shadow Matter is most or all of dark matter, it is in conflict with inflation. Worth exploring new ideas for initial conditions.

Thank you!