Non-invertible Naturalness and Quantum Flavodynamics

Seth Koren

(they/them or whatever)

University of Notre Dame

Leptons: 2211.07639 with Clay Córdova, Sungwoo Hong, Kantaro Ohmori

Quarks: 2402.12453 with Clay & Sungwoo

Related ideas in my

SM proton stability: 2204.01741

Discrete B-L cosmology: 2204.01750

SM flavor 2-group: 2212.13193 with Clay

SM 1-form symmetry: 2406 with Adam Martin,

240X with Sam Homiller, many many with Sungwoo, …

Dirac (1938) Naturalness → ’t Hooft (1980) Technical Naturalness →

Clay, Kantaro, Seth, Sungwoo (2022-2024): Non-invertible Naturalness

A spurion for a non-invertible symmetry can be *generated by instantons* in a UV theory.

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Infrared symmetry analysis points you to a Dirac natural model! More powerful than learning a UV model need not be destabilized toward the IR

Find a Dirac natural origin for a technically natural parameter:

$$y_\nu \sim y_\tau \exp\left(-S_{\text{inst}}\right)$$

Find a Dirac natural origin for an unnatural parameter:

$$y_b \sim y_t \exp\left(-S_{\text{inst}}\right)$$

$$\Rightarrow \bar{\theta} \simeq 0$$

Why generalized global symmetries?

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0-form symmetry
charged local
operators
e.g. particles

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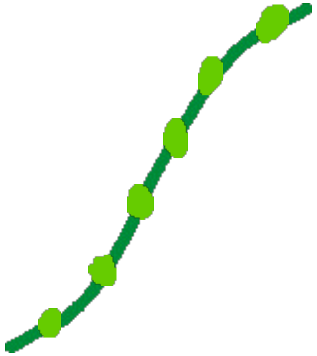
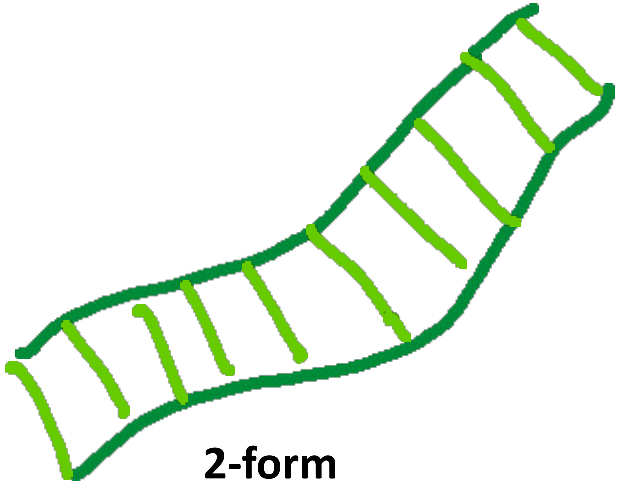
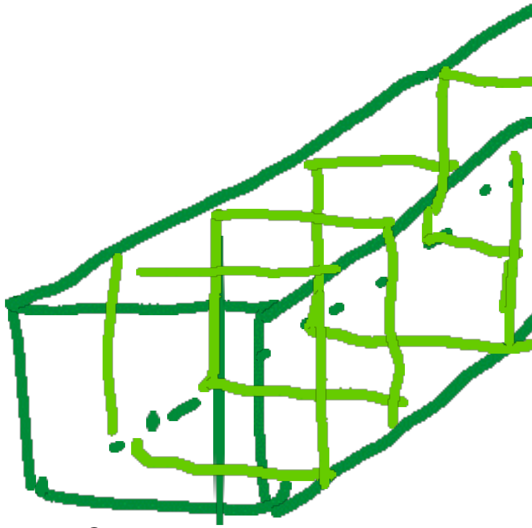


1-form
line operators
e.g. Wilson line



2-form
surface operators
e.g. cosmic string



3-form
volume operators
e.g. domain wall

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^{\mu\nu} = 0$$

Generally $\partial_\mu J^{\mu_1 \mu_2 \cdots \mu_{p+1}} = 0$ antisymmetric

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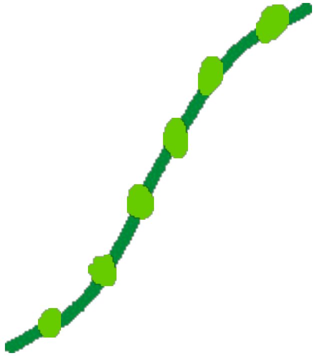

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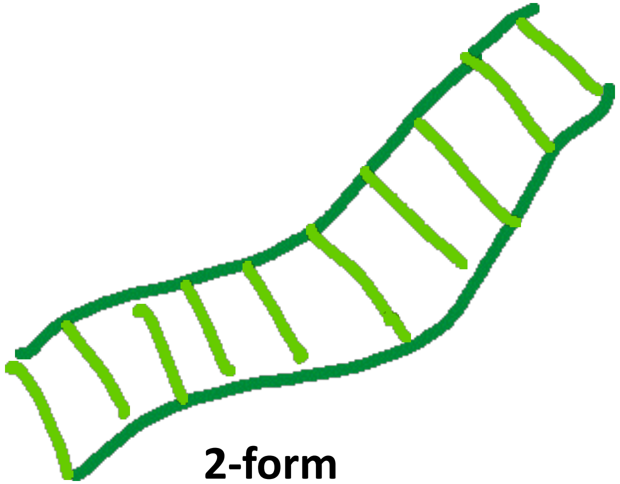

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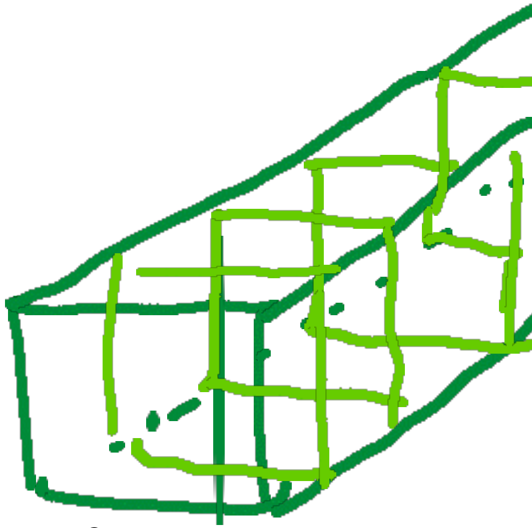

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to Lagrangian e.g. $\delta\mathscr{L} = MNN$

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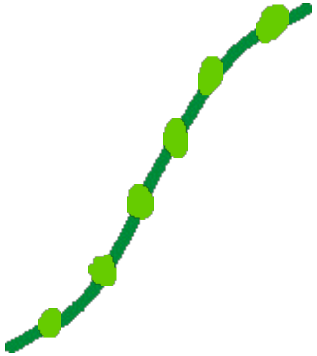

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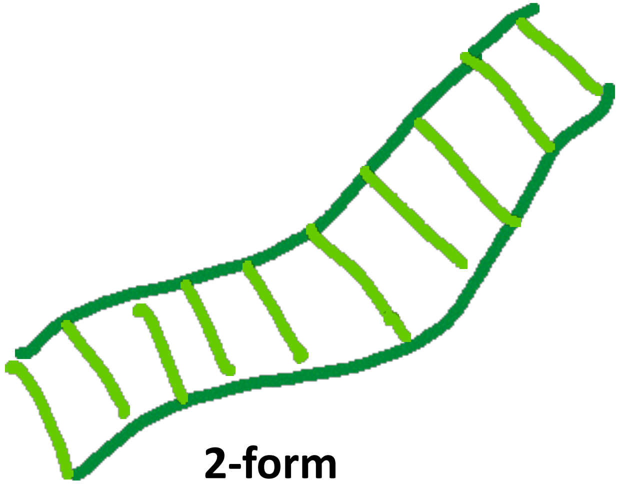

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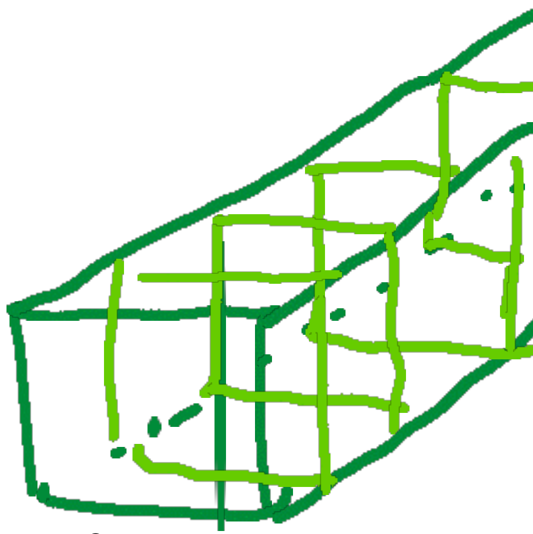

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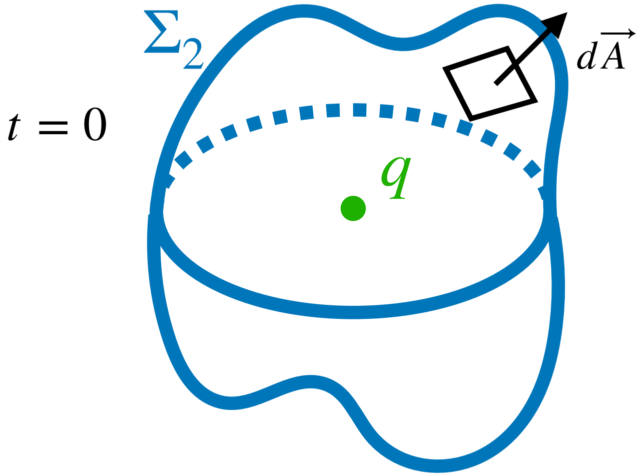
Break by adding charged operator to Lagrangian e.g. $\delta\mathscr{L} = MNN$

Break only with the appearance of new dynamical degrees of freedom!

Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.



$$q = \int_{\Sigma_2} \vec{E} \cdot d\vec{A}$$

Generalized Global Symmetry of Electromagnetism

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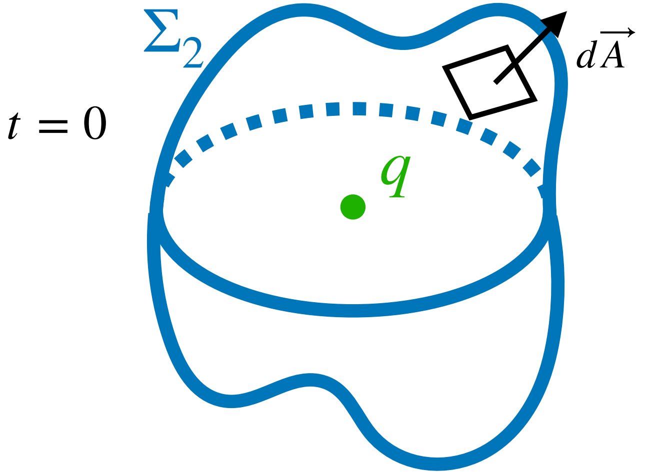
In pure electromagnetism, the photon field strength is conserved $J_E^{\mu\nu} \sim \dfrac{1}{e^2} F^{\mu\nu}, \quad \partial_\mu J_E^{\mu\nu} = 0$

Gauss' law computes a Noether charge for an electric 1-form symmetry!



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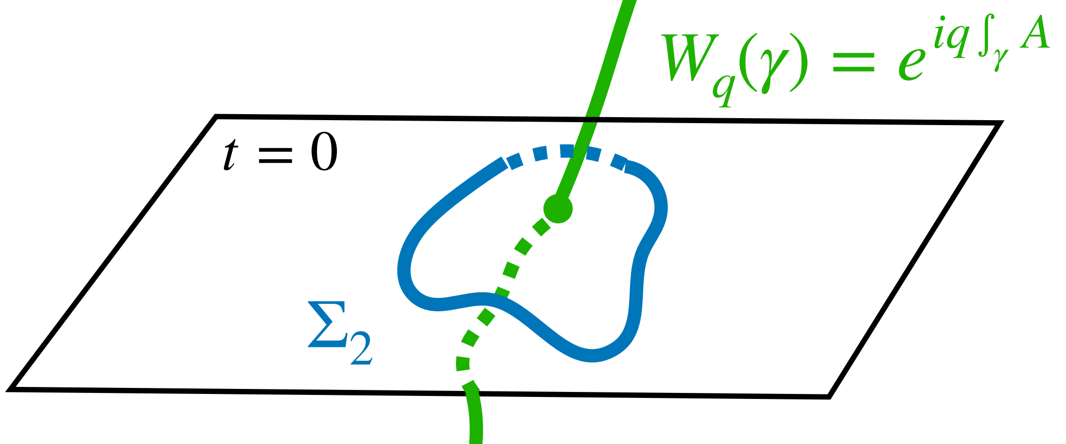
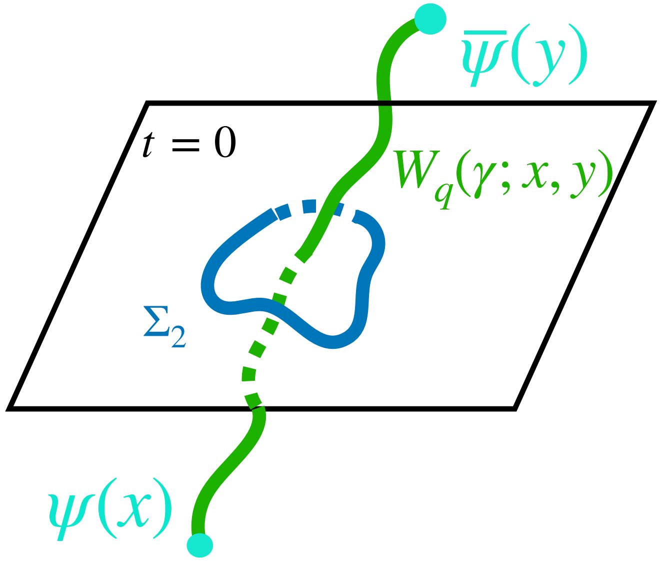


$$q = \int_{\Sigma_2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} dS^{\rho\sigma}$$

Emergent 1-form symmetry

The 1-form symmetry is emergent in the low-energy, long-distance theory $E \ll m_e$.

Once we see the dynamical electron, then Wilson lines can ‘end’.



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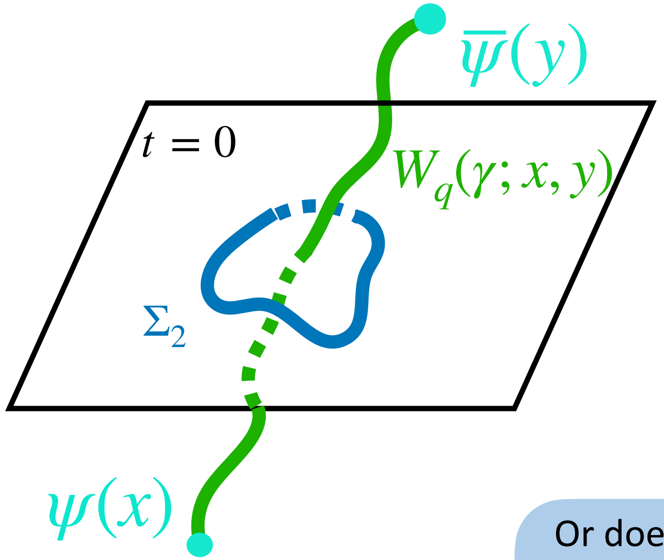

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Or does a discrete 1-form symmetry remain? Test at LHC! SK & A. Martin

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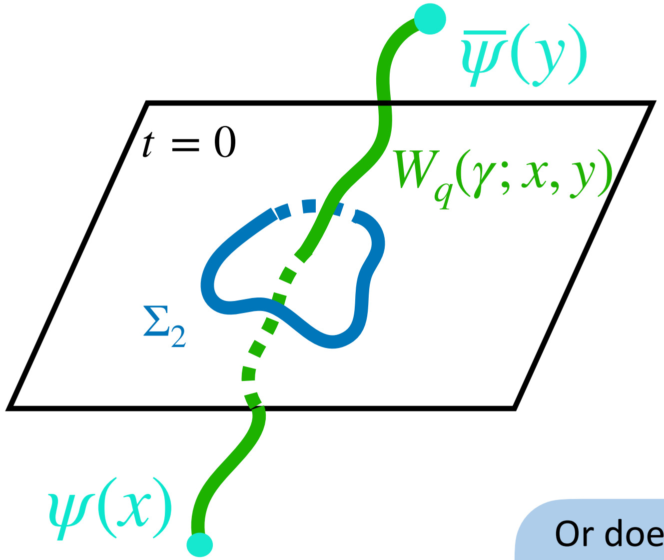

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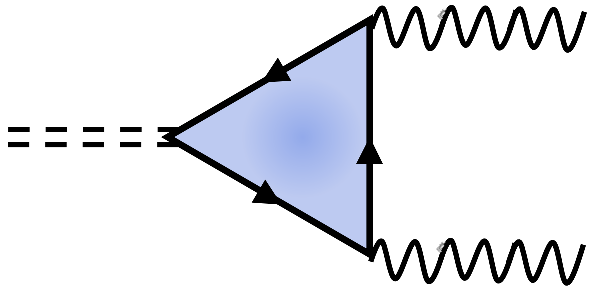
Mutatis mutandis a magnetic one-form symmetry for a theory H with 't Hooft lines classified by $\pi_1(H)$

Quick review: Instantons and Anomalies

Good classical zero-form global symmetry $U(1)_X$ can be anomalous in quantum theory with G gauge group

$$\partial_\mu J^\mu_X = 0 \quad \longrightarrow \quad \partial_\mu J^\mu_X = \frac{\mathscr{A}_X}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$$



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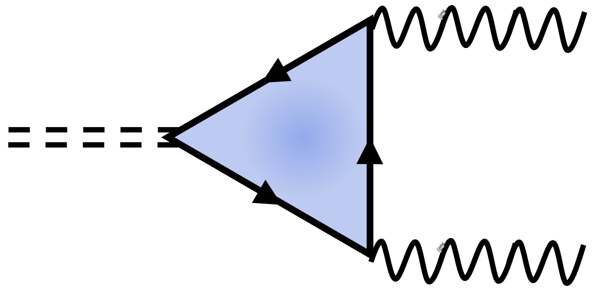

What field configurations can 'saturate' this anomaly and actually violate $U(1)_X$?

$$\int_{\mathbb{R}^4} F^{\mu\nu}\tilde{F}_{\mu\nu} \propto \int_{\partial\mathbb{R}^4 \simeq S^3} \hat{n}_\mu J^\mu_{\rm CS}$$ = number of times A_μ 'winds' around infinity

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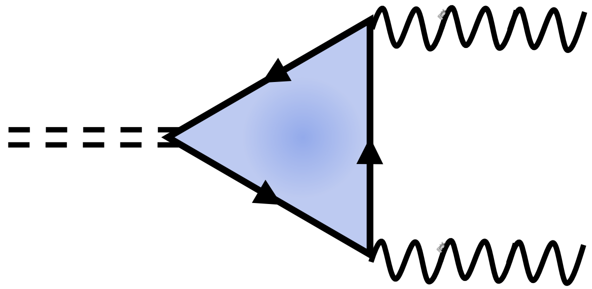

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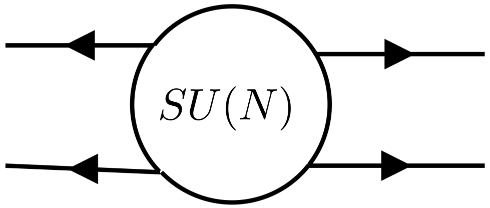
Instantons lead to effective operators known as 't Hooft vertices which violate G-anomalous symmetries

Multiplicity of ψ_i legs given by Dirac index I_{ψ_i}



Violates anomalous $U(1)_X$ by $\mathscr{A}_X = \sum_{\psi_i} q_{\psi_i} I_{\psi_i}$

Unsaturated Anomalies - Missing Instantons

We said instantons are the field configurations which can saturate the anomaly

$$\partial_\mu J^\mu_X = \frac{\mathscr{A}}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

But what about when they don't?

E.g. famously $\pi_3\big(U(1)\big) = 1$ and *there are no Abelian instantons in* $\mathbb{R}^4$, so $\int_{\mathbb{R}^4} F\tilde{F} = 0$

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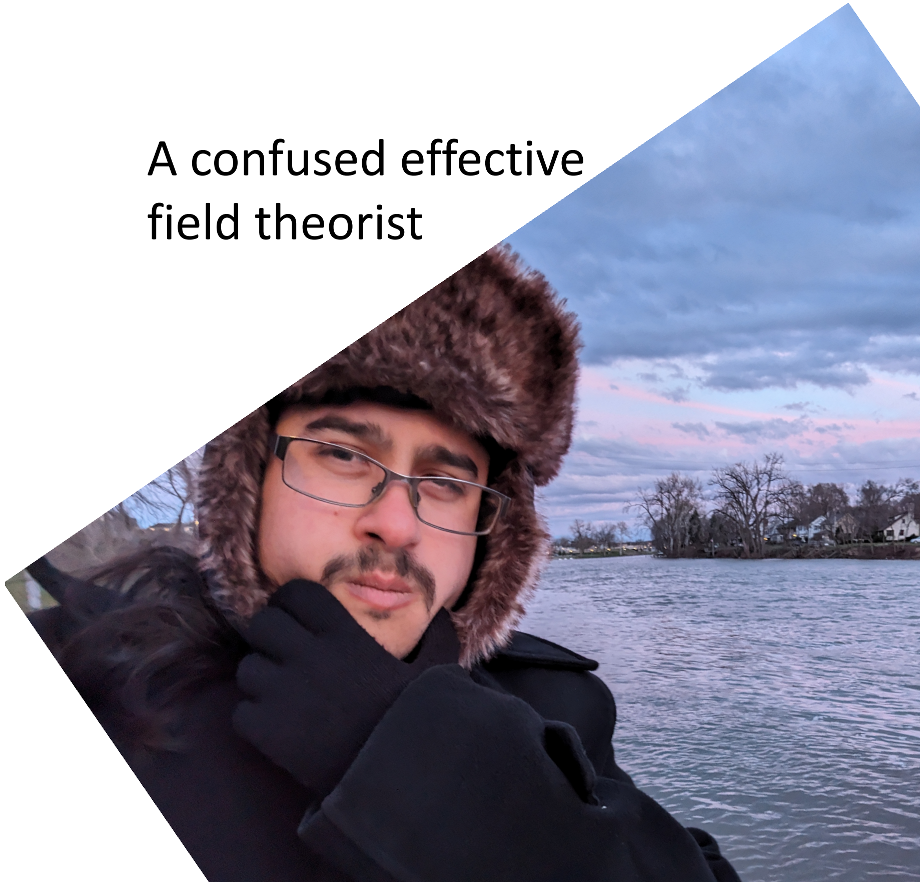
Old lesson: X is anomalous but S-matrix preserves X anyway

EFT philosophy: If there is ever a zero, there should be a symmetry!



A confused effective field theorist

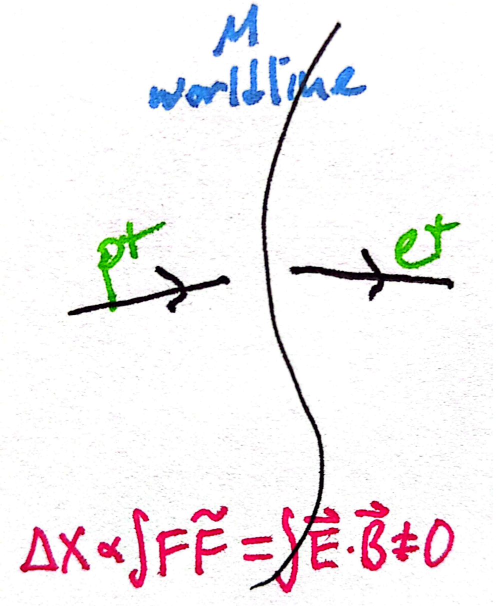
EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous there must remain a subtle sort of symmetry that demands the S-matrix preserves X

A hint: X *can* be violated around magnetic monopoles

c.f. Callan-Rubakov

Dirac ’31
Callan, Rubakov ‘80s
Ongoing…



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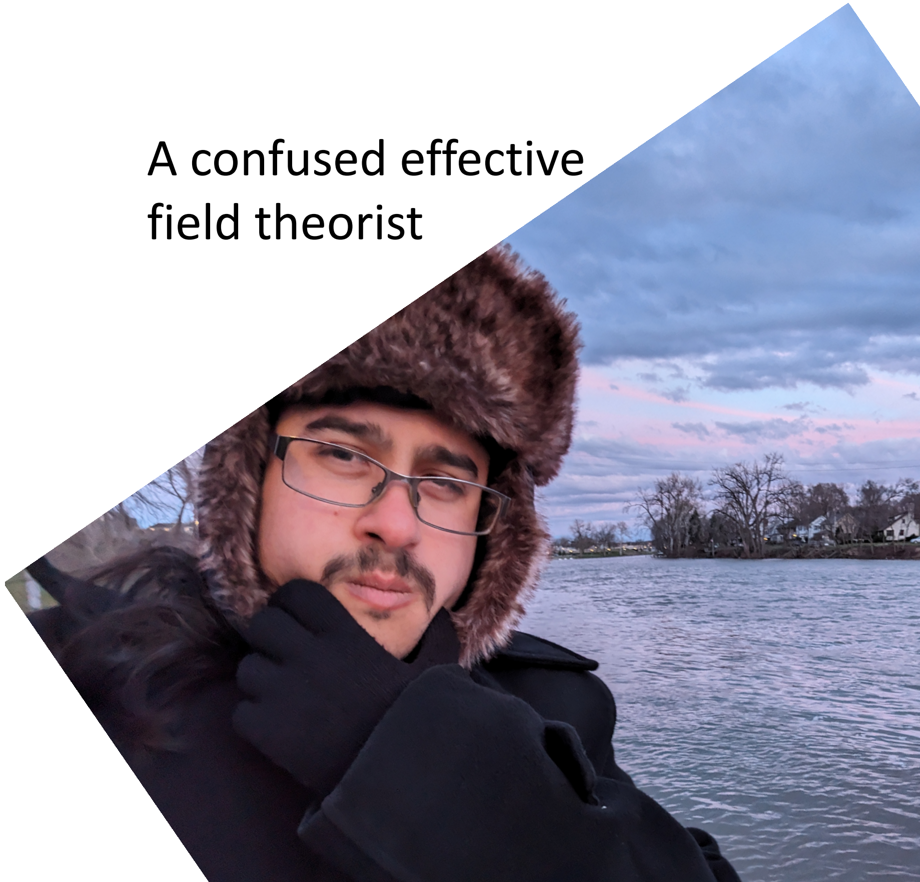


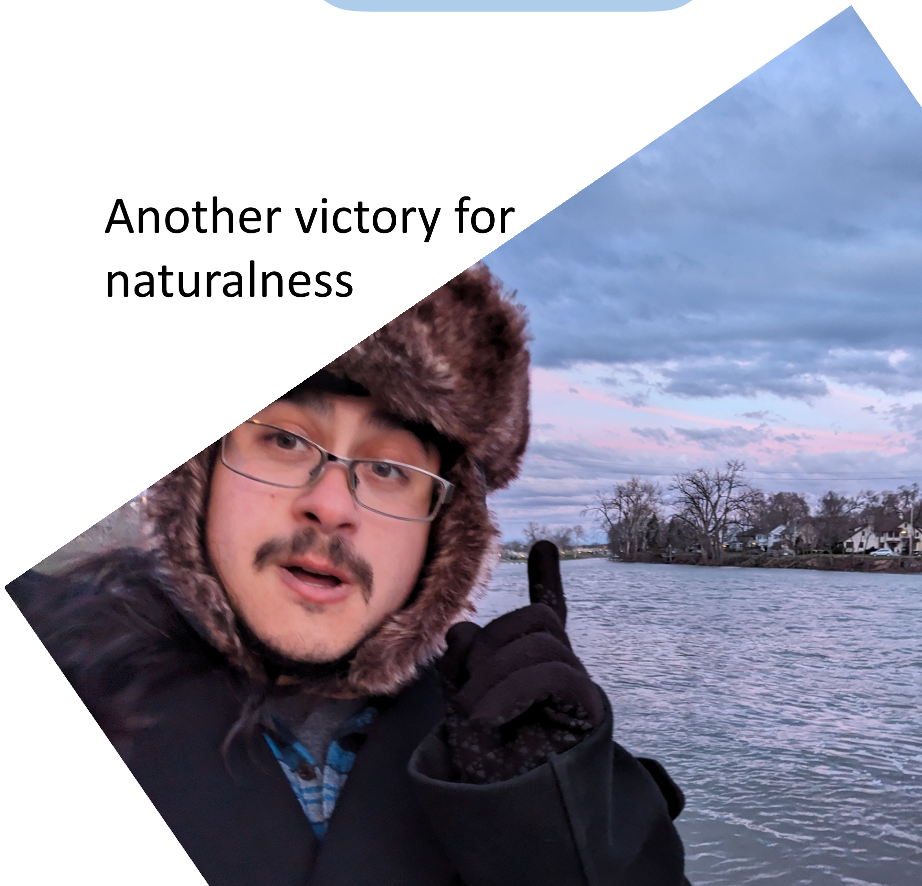
There's a subtler notion of symmetry!

X not fully broken, but converted to a non-invertible symmetry! This must act both on local fields and on 't Hooft lines.

Choi, Lam, Shao
2205.05086
Córdova, Ohmori
2205.06243

$$\psi(x) \to \psi(x) e^{i\alpha} \qquad e^{i\oint_\gamma A_m} \to e^{i\oint_\gamma A_m + i\alpha \oint_\gamma A}$$

Another victory for naturalness



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Non-invertible symmetry must break when there are dynamical monopoles that break the magnetic one-form symmetry

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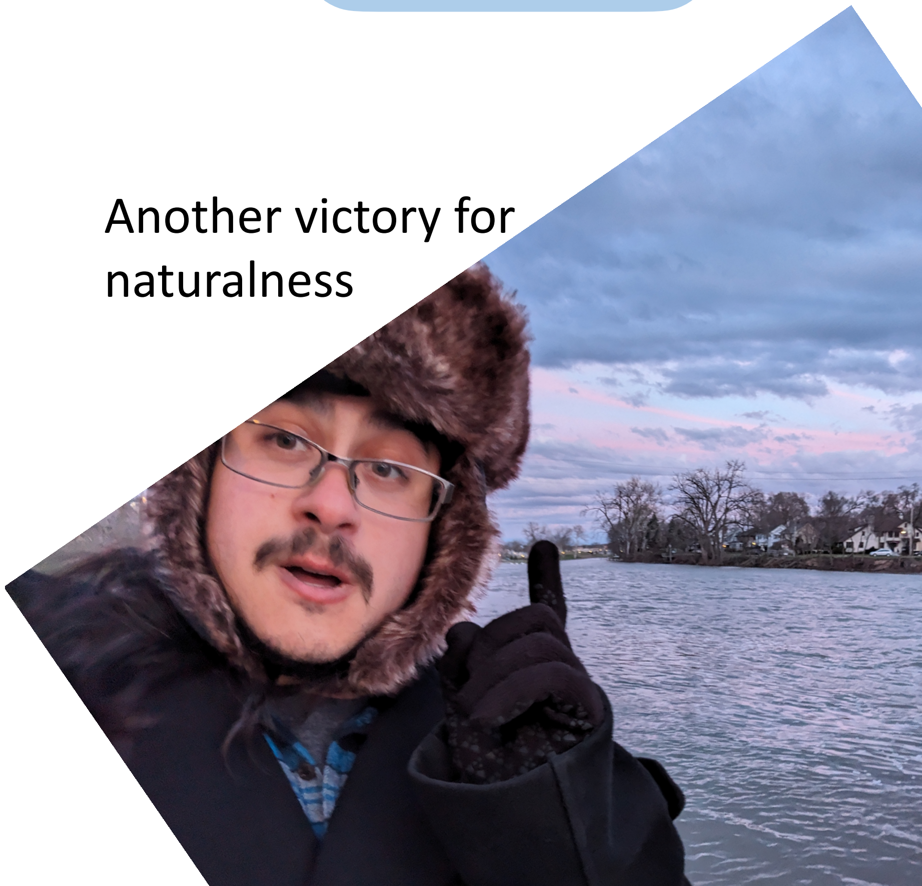
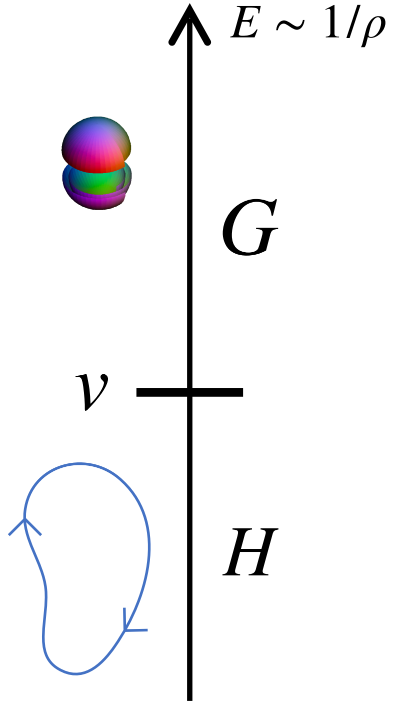


Model-building strategy



A classical global symmetry X protects some operator $\mathcal{O}$ and has an H anomaly

$$\partial_\mu J^\mu_X = \frac{\mathcal{A}}{8\pi^2} H^{\mu\nu}\tilde{H}_{\mu\nu}$$

But some values of $\int_{\mathcal{M}} H\tilde{H}$ not realized for $\mathcal{M} = \mathbb{R}^4$, so X is not violated in S-matrix of the H theory

Non-invertible X symmetry tells us $\mathcal{O}$ could be generated by instantons in the theory $G \supset H$ which has G/H-monopoles

The Standard Model

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$-$	$-$	$-$
$SU(2)_L$	$\mathbf{2}$	$-$	$-$	$\mathbf{2}$	$-$	$\mathbf{2}$
$U(1)_Y$	$+1$	-4	$+2$	-3	$+6$	-3

- Beautiful, yet incomplete.
- Simple way to go beyond is to gauge some of the approximate symmetries of the SM
- E.g. the SU(5) approximate symmetry is broken by $g_1 \neq g_2 \neq g_3$ (and $y_u \neq y_d \neq y_e$)
- Here we'll play with the $U(3)^5$ approximate symmetries of the SM fermions

SM flavor symmetries actually in 2-group Córdova & SK '23 Annalen der Physik

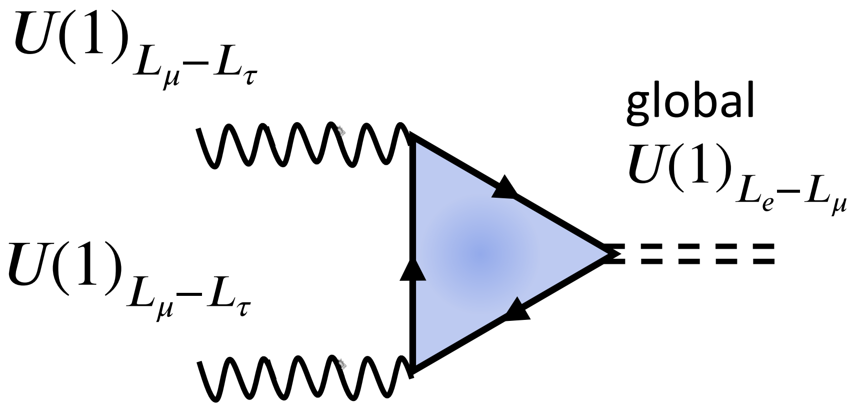
Nonperturbative Quantum Lepton Flavodynamics

Neutrino Masses from Generalized Symmetry Breaking

arXiv:2211.07639, Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

Now let's go beyond and gauge $U(1)_{L_\mu - L_\tau}$

There's a new ABJ anomaly diagram to consider



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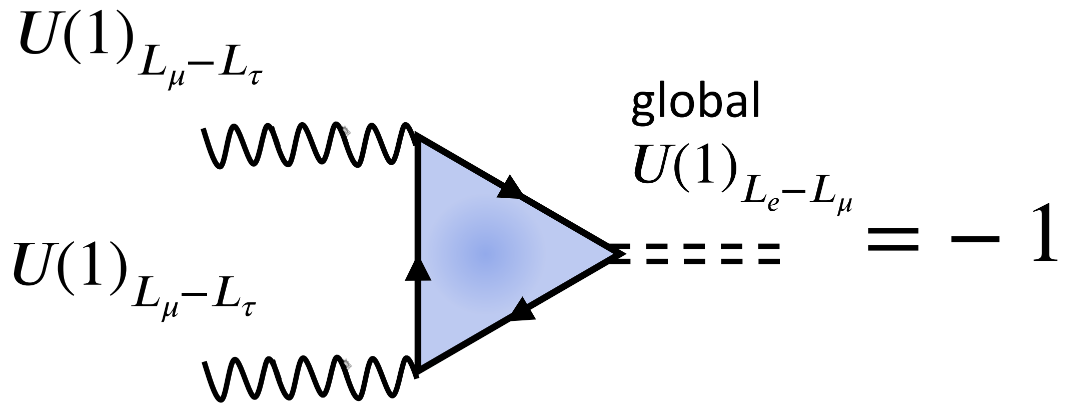


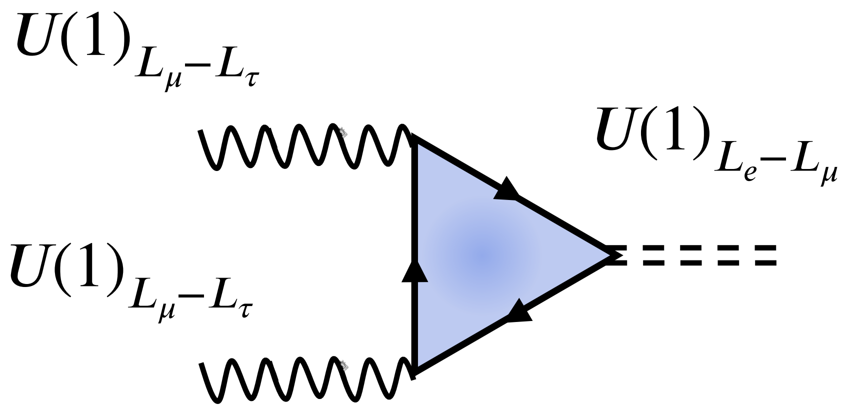
So the global $U(1)_{L_e - L_\mu}$ current is no longer conserved

$$\partial_\mu J^\mu_{L_e - L_\mu} = \frac{-1}{8\pi^2} F_{L_\mu - L_\tau} \tilde{F}_{L_\mu - L_\tau}$$

But the $U(1)_{L_\mu - L_\tau}$ gauge theory cannot saturate this anomaly

Non-invertible symmetry!

Beyond with $Z'_{L_\mu - L_\tau}$



$= -1$

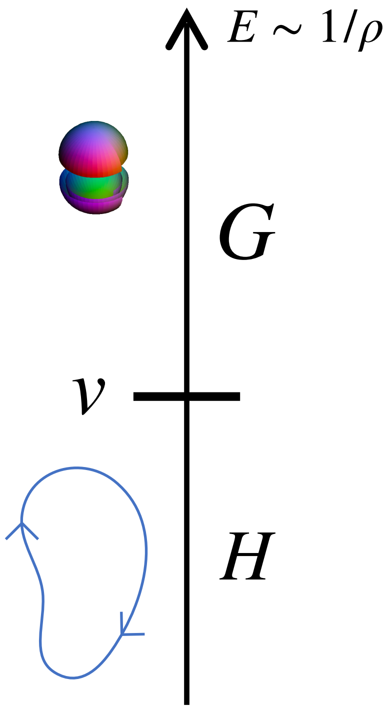
Non-invertible symmetry protects neutrino masses, focus on $\mathbb{Z}_3^L \subset U(1)_{L_e - L_\mu}$

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $\left(\tilde{H}L\right)^2$

$$L = (L_e - L_\mu) - (L_\mu - L_\tau) \quad \text{(mod 3)}$$

Model-building strategy



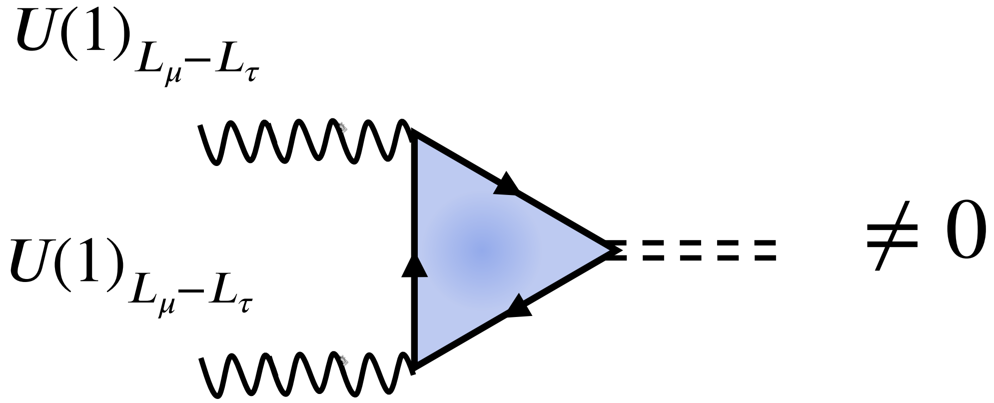
A classical global symmetry $X = \mathbb{Z}_3^L$ protects the operators $\mathcal{O}_{ij} = (\tilde{H}L_i)(\tilde{H}L_j)$ and has an $H = U(1)_{L_\mu - L_\tau}$ anomaly

But while $\int_{\mathscr{M}} H\tilde{H} \in \mathbb{Z}$ generally, $\int_{\mathbb{R}^4} H\tilde{H} = 0$

X is a non-invertible symmetry!

In a theory $G \supset H$ with lepton flavor monopoles, $\mathcal{O}_{ij}$ could be classically absent and generated only by G-instantons.

Beyond with $Z'_{L_\mu - L_\tau}$ and N!



Non-invertible symmetry protects neutrino masses
either with or without right-handed neutrinos

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $\left(\tilde{H}L\right)^2$

	L_i	$\bar{e}_i$	N_i
$\mathbb{Z}_3^{\tilde{L}+N}$	$+1$	-1	$+1$

Disallows HLN

Dirac masses:

Far UV: Write charged lepton yukawas

$$\mathscr{L} \sim y_\tau H \mathbf{L}\bar{\mathbf{e}}$$

	$SU(3)_H$	$U(1)_{\mu-\tau}$	$U(1)_L$	$U(1)_N$
$\mathbf{L}$	3	$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$+1$	0
$\bar{\mathbf{e}}$	$\bar{3}$	$\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	0
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Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{L}\mathbf{N}$

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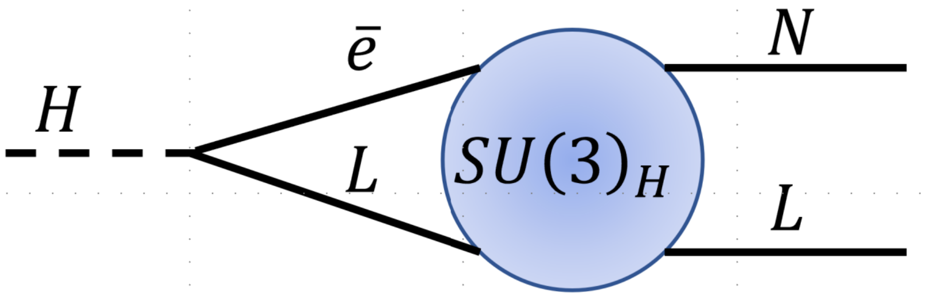
Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{LN}$

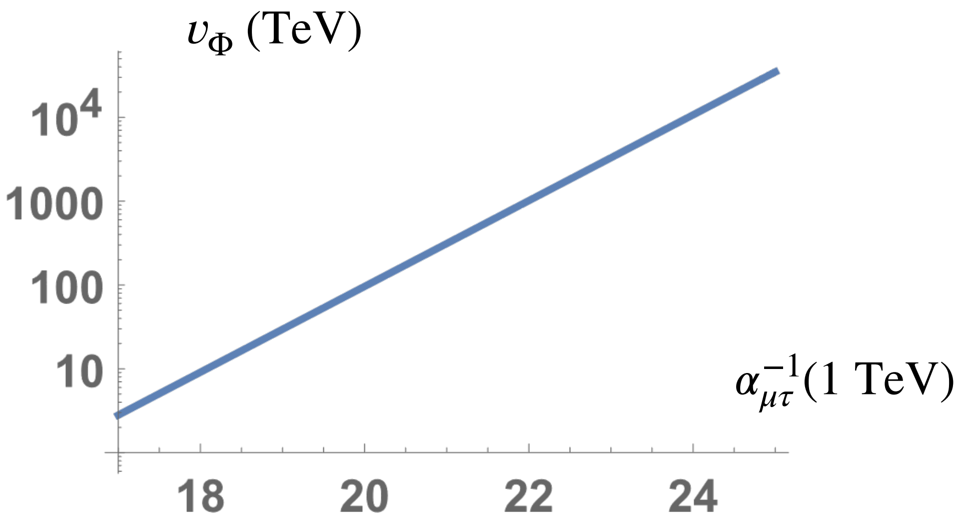


$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{LN}$$

Economical and predictive

Given the discovery of such a Z' for $U(1)_{L_\mu - L_\tau}$, learn the scale at which $SU(3)_H \to U(1)_{L_\mu - L_\tau}$

$$v_\Phi^2 \sim M_{Z'}^2 \left(\frac{m_\nu}{m_\tau}\right)^{3/2} \exp \frac{3\pi}{4\alpha_{\mu\tau}(M_{Z'}^2)}$$



Texture from Higgses implementing $SU(3)_H \to U(1)_{L_\mu - L_\tau} \to \emptyset$

c.f. [Alonso-Álvarez, Cline '21]

Nonperturbative Quantum Quark Flavodynamics

Non-Invertible Peccei-Quinn Symmetry and the Massless Quark Solution to the Strong CP Problem

arXiv:2402.12453, Clay Córdova, Sungwoo Hong, SK

Quark Weak CP and Strong CP Violation

The 'strong CP angle' $\bar{\theta} = \arg e^{-i\theta} \det\left(y_u y_d\right)$ is constrained to $\bar{\theta} \lesssim 10^{-10}$!

Even worse, we also have the 'weak CP angle' $\tilde{J} = \mathrm{Im}\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right)$

oft parameterized by m_i, θ_{ij}, and the phase $\delta_{\mathrm{CKM}} \sim 1.14$

A small value of $\bar{\theta}$ is not technically natural $\Rightarrow$ the strong CP problem.

Upon RG evolution, $\delta\bar{\theta} \propto c\delta_{\mathrm{CKM}}$

Ellis & Gaillard '79

Peccei-Quinn for Strong CP

Now consider a Peccei-Quinn symmetry protecting the up quark mass

$$U(1)_{\mathrm{PQ}}: \quad \bar{u} \to \bar{u} e^{i\alpha} \quad \Rightarrow \quad \tilde{H} Q \bar{u} \text{ charged so } y_u = 0$$

If the PQ symmetry is good, $y_u \to 0$, and so $\det y_u \to 0$ and there's no strong CP violation

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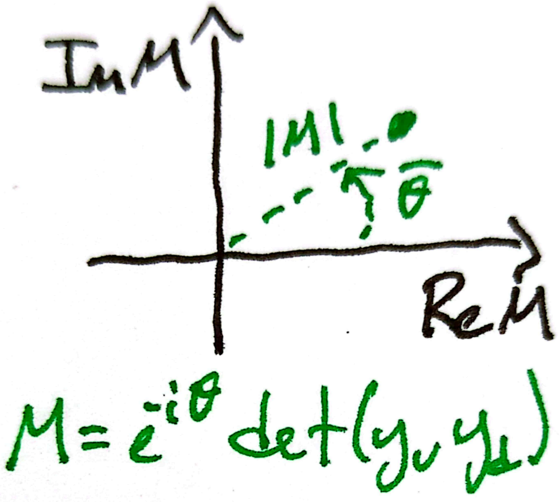
Easier to parameterize in 'Cartesian coordinates' for complex parameter $M \in \mathbb{C}$



Def $M = e^{-i\theta} \det\left(y_u y_d\right)$, so $\bar{\theta} = \arg M$

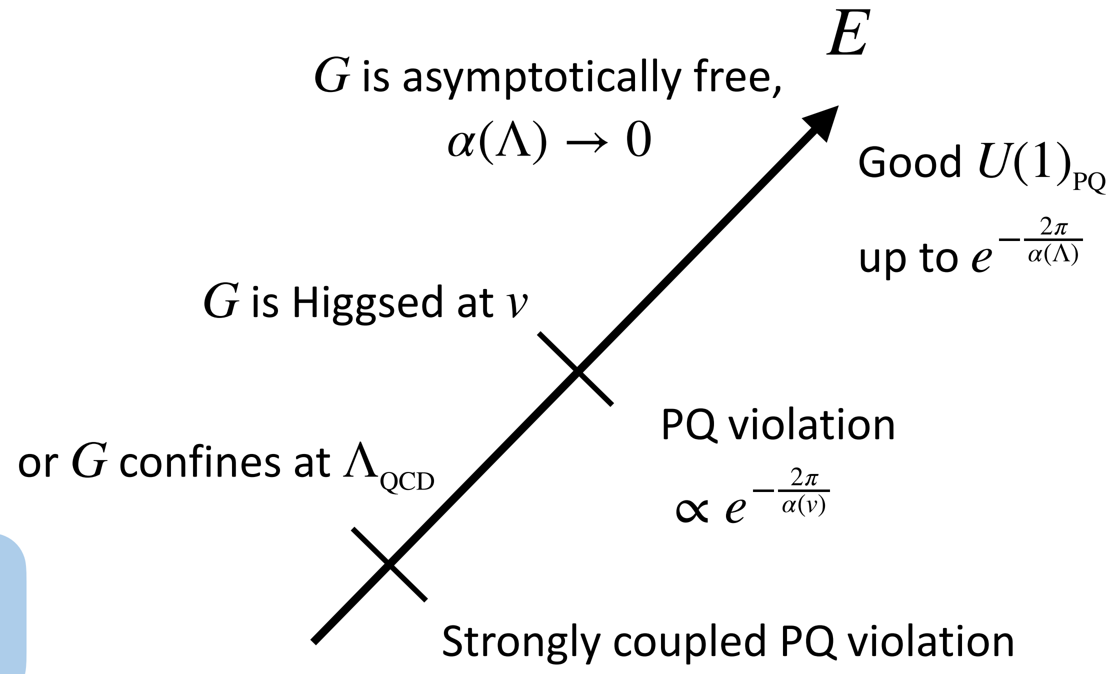
Transforms as CP : $\mathrm{Im}(M) \to -\mathrm{Im}(M)$

Peccei-Quinn Violation

Massless up quark?! Not in the IR.
A PQ symmetry which begins good is violated by instantons at low energies

UV $y_u = 0$ is then violated by QCD instantons to generate mass, automatically $M \in \mathbb{R}_+$

Georgi-McArthur '81
Kaplan-Manohar '86
Choi, Kim, Sze '88



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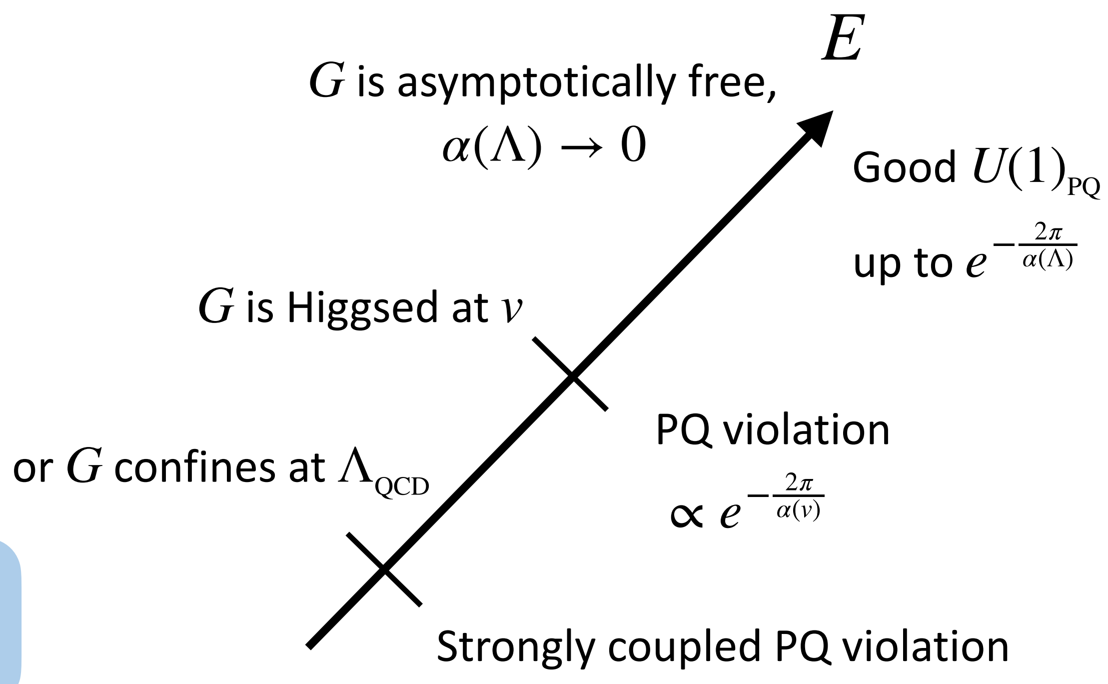

Heroic efforts by lattice physicists tell us the SM does not bear out the massless up quark solution

Flavour Lattice Averaging Group 2019

Could there be any UV model where instantons revive this solution?

Quark Flavor Z'

In lepton sector we had anomaly-free $U(1)_{L_i - L_j}$

Likewise here we can think about gauging e.g. $U(1)_{B_i - B_j}$

Structurally parallel, just broken more by the larger Yukawas

But in fact in the quark sector we can gauge flavored baryon number in a slightly more subtle way because $N_c = 3 = N_g$!

Quark Flavor Z’

One such combination is $U(1)_{B_1+B_2-2B_3}$ which is anomaly-free

Gauge group can be $\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3}\right)/\mathbb{Z}_3$ where the quotient refers to identifying certain discrete transformations in either factor

These non-trivial possibilities modify the topological data in a crucial way

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At intermediate scales you can realize $\left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ along similar lines

These non-trivial possibilities modify the topological data in a crucial way

	$SU(3)_c$	$SU(3)_H$
$\mathbf{Q}$	3	3
$\bar{\mathbf{u}}$	$\bar{3}$	$\bar{3}$
$\bar{\mathbf{d}}$	$\bar{3}$	$\bar{3}$

Non-invertible symmetry

With the $\mathbb{Z}_3$ global structure, there are color and flavor instantons with fractional instanton numbers

E.g. fractional part of color instanton $\mathcal{N}_C = \frac{1}{8\pi^2}\int_M \mathrm{Tr}\left(F_C \wedge F_C\right) = \frac{1}{3}\int_M \omega \wedge \omega \mod 1$

The diagonal quotient locks the fractional parts together $\mathcal{N}_C = \mathcal{N}_H \mod 1$

Integer instanton -> broken, fractional -> non-inv

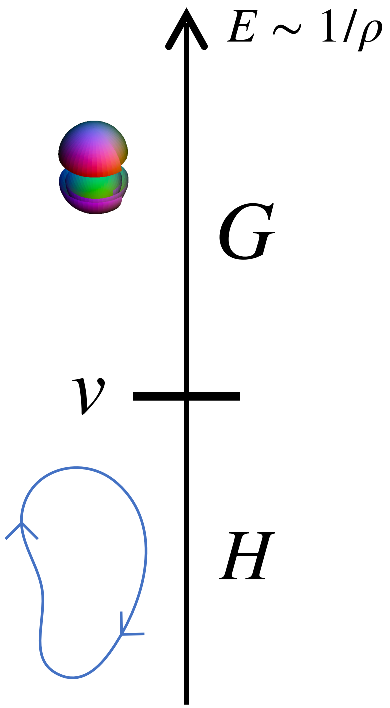
$\mathcal{A}_f = \sum_{\psi_i} q^f_{\psi_i} I_{\psi_i}$ where $I_{\psi_i} = n_{\psi_i} T_{\psi_i} \mathcal{N}_C + n_{\psi_i} T_{\psi_i} \mathcal{N}_H$

Non-invertible $\mathbb{Z}_3$

	Q_i	$\bar{u}_i$	$\bar{d}_i$
$\mathbb{Z}_3^{\tilde{B}+d}$	$+1$	-1	$+1$

y_d is a spurion for this non-invertible symmetry!

Model-building strategy



A classical global symmetry $X = \mathbb{Z}_3^{\tilde{B}+d}$ protects the operators $\mathcal{O}_{ij} = HQ_i\bar{d}_j$ and has an $H = \left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ anomaly

But while $\int_{\mathcal{M}} H\tilde{H} \in \mathbb{Z}/3$ generally, $\int_{\mathbb{R}^4} H\tilde{H} \in \mathbb{Z}$

X is a non-invertible symmetry!

In a theory $G \supset H$ with quark color-flavor monopoles, $\mathcal{O}_{ij}$ could be classically absent and generated only by G-instantons

Color-flavor unification!

This all points to a beautiful $SU(9)$ unified theory in which the colors and flavors of the quarks are placed together into the fundamental

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

$$\mathscr{L}_0 = y_t \tilde{H} \mathbf{Q} \bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

Again start with good $U(1)_{\text{PQ}}$ and no strong CP violation, then

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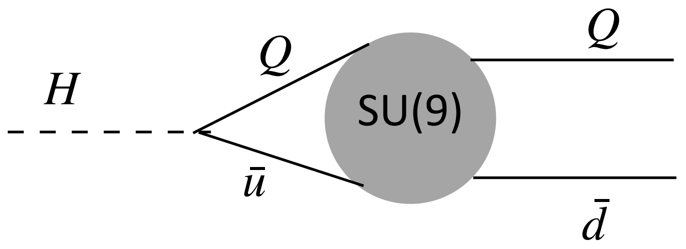
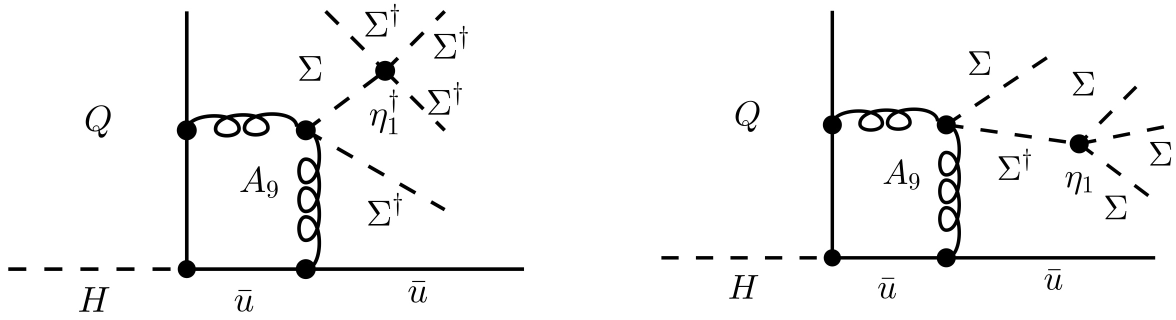


$$\mathscr{L}(\Lambda_9) \sim y_t H \mathbf{Q}\bar{\mathbf{u}} + y_t^\star e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda_9)}} H \mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

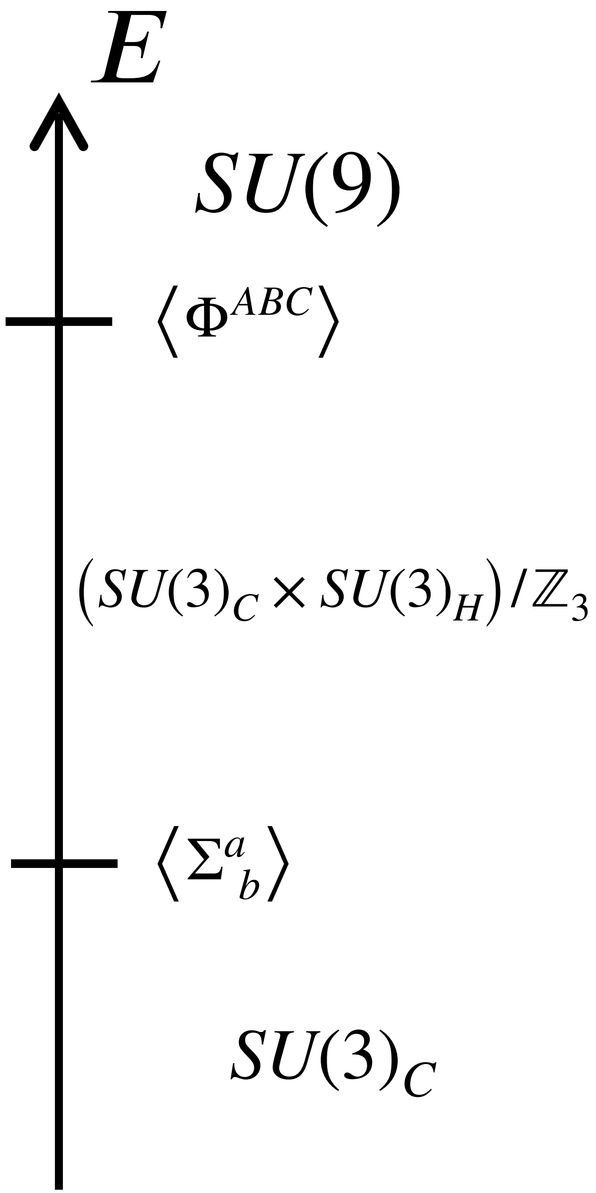
Generating CKM

Idea: Communicating flavor- and CP-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate hermitian yukawas. Then $\bar{\theta} = \arg\det e^{-i\theta} y_u y_d = 0$

$$V(\Sigma) = \eta_1 \mathrm{Tr}\left(\Sigma^4\right) + \text{ h.c. } + \ldots$$



$$(y_u)^a{}_b \sim y_t \left(\mathbb{1}^a{}_b + \frac{\alpha_9}{(4\pi)} \frac{\eta_1^\dagger (\Sigma^{\dagger 4})^a{}_b}{\Lambda_9^4} + \frac{\alpha_9}{(4\pi)} \frac{\eta_1 (\Sigma^4)^a{}_b}{\Lambda_9^4} + \ldots \right)$$



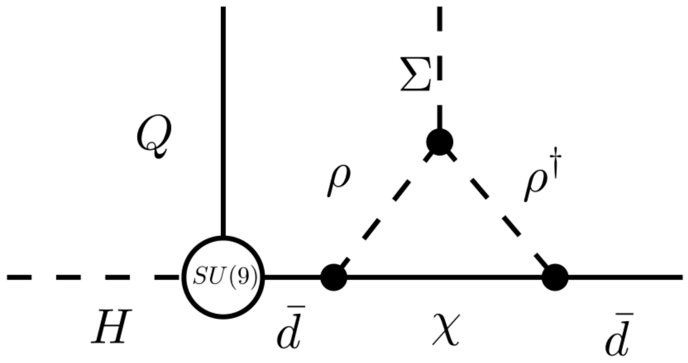
Generating CKM

Another wrinkle: Want to generate

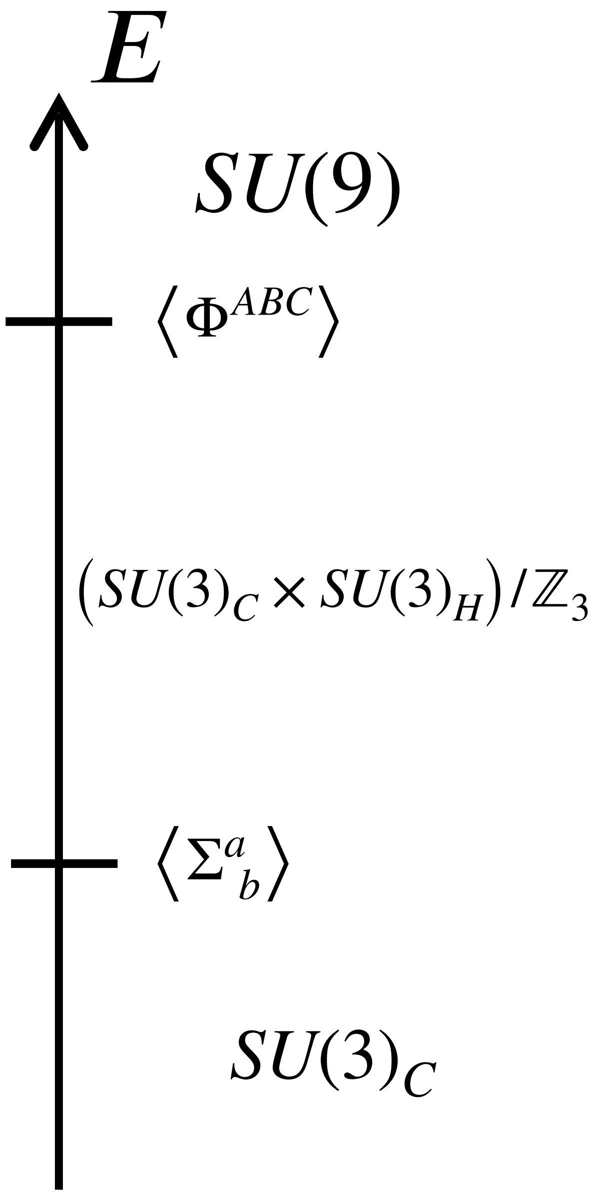
$$\delta_{CKM} \propto \arg\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right) \neq 0$$

Must treat $\bar{u}, \bar{d}$ differently so they don't commute in flavor space.

Add states with 'downphilic' interactions e.g. ρ a scalar fundamental, χ a singlet fermion



$$(y_d)^a{}_b += \frac{|\lambda_d|^2}{(4\pi)^2}\left(a\Sigma^a{}_b + a^\dagger\Sigma^{\dagger a}{}_b\right)$$



Seth’s Conclusions

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> Already we have located new unified theories of the SM fermions with instanton effects which can solve SM naturalness issues! Both technically natural, and not.

Seth’s Conclusions

At least any place nonperturbative effects might be phenomenologically relevant, I expect paradigm of generalized global symmetries will offer better understanding

Already we have located new unified theories of the SM fermions with instanton effects which can solve SM naturalness issues! Both technically natural, and not.

As particle physicists we are not yet done learning about the role of symmetries!

A primate pleased they newly uncovered some simple, reductionist BSM models

Backup slides

Rants and other things I didn't have time for

Wrong conclusion

- Incorrect takeaway: “They used these fancy new symmetry ideas but in the end the UV model could be explained in terms of instantons. We’ve known about that stuff since the 80s. So who cares about generalized symmetries?”

- Correct takeaway: “These intriguing instanton effects have been sitting this close to the SM for decades and nobody saw it?! What can generalized symmetries tell me about my favorite BSM model??

Massless quark wins on quality

Both axion and massless quark solutions rely on good quality Peccei-Quinn symmetries, but only the former has a quality 'problem' because its required quality is ridiculously unnatural

Worse issue for the axion because

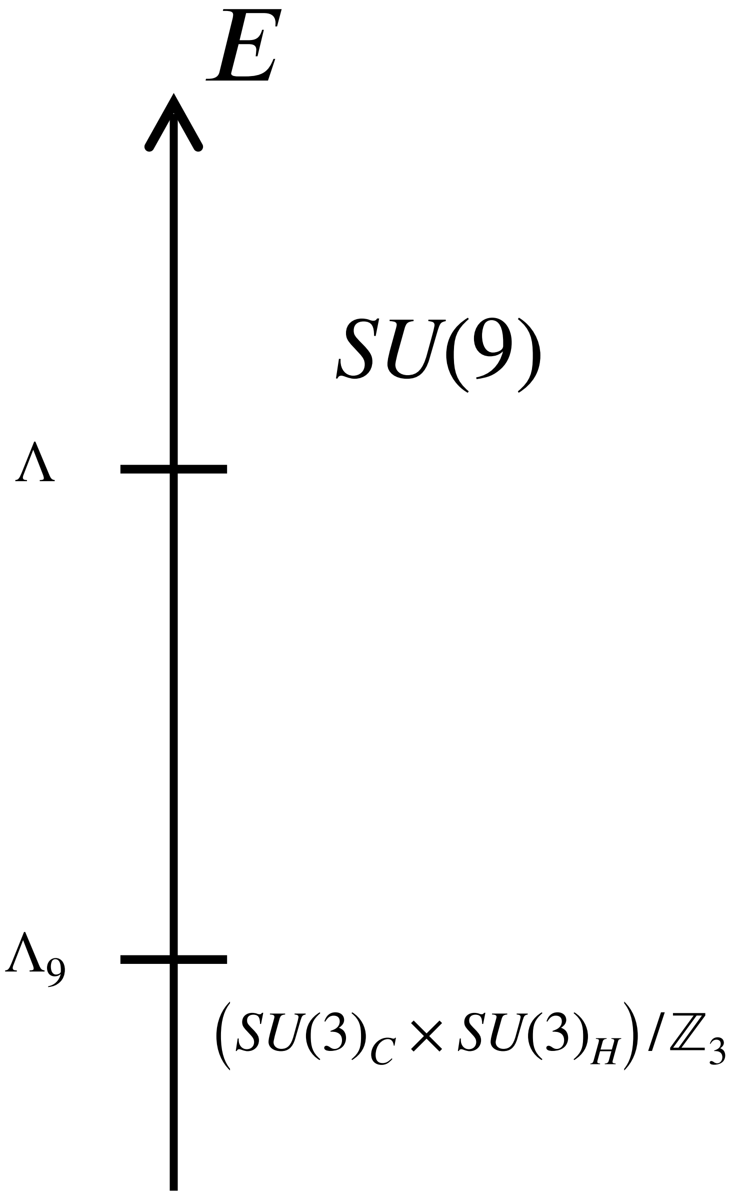
- With PQ-charged scalar ϕ can have all sorts of PQ-violating ops e.g. $\mathscr{L} \supset c_n M_{\rm Pl}^{4-n} \phi^n$
- We have strong astrophysical bounds on $\langle \phi \rangle = f_a \gtrsim 10^8$ GeV
- The *potential* $V_{\rm grav} \sim f_a^4 \left(f_a / M_{\rm pl} \right)^{n-4}$ cannot overpower $V_{\rm inst} \sim \Lambda_{\rm QCD}^4$

Whereas we can sustain some extra additive contribution to $M = e^{-i\theta} \det \left(y_u y_d \right)$

as long as its magnitude is small, as $\bar{\theta} \sim$ Im(M)/Re(M)

$\mathscr{L} \supset c_\Sigma \tilde{H} Q \Sigma \bar{d} / M_{\rm Pl}$ can have some random phase and $O(1)$ coupling as long as $\langle \Sigma \rangle / M_{\rm pl} \lesssim \bar{\theta}$, implying $\langle \Sigma \rangle \lesssim 10^8$ GeV. Quark flavor physics is not too far away!

Strong CP in more detail



We begin in the far UV with a good $U(1)_{\text{PQ}}$

$$\mathscr{L}_0 = y_t \tilde{H} \mathbf{Q} \bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

And so of course $M = e^{-i\theta} \det \left(y_u y_d\right) = 0$

We flow down in energies and begin to generate

$$\mathscr{L}(\Lambda) \sim y_t H \mathbf{Q} \bar{\mathbf{u}} + y_t^\star e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda)}} H \mathbf{Q} \bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

With exactly the right phase to ensure

$$\bar{\theta} = \arg e^{-i\theta_9} \det y_u y_d = -\theta_9 + \arg |y_t|^2 e^{i\theta_9} = 0$$

Further at the matching scale

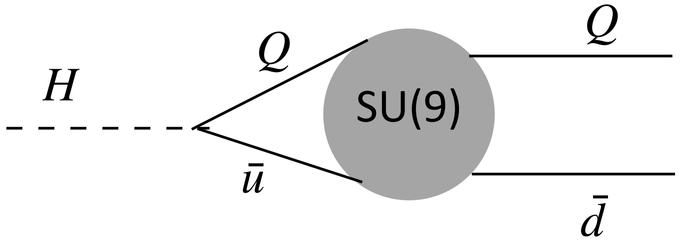
$$\mathscr{L}(\Lambda_9) \sim y_t H \mathbf{Q} \bar{\mathbf{u}} + y_t^\star e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_s(\Lambda_9)}} H \mathbf{Q} \bar{\mathbf{d}} + \text{ h.c.} + \frac{i3\theta_9}{32\pi^2} \left(G\tilde{G} + K\tilde{K}\right)$$

And the matching accounts for the yukawas now being 3x3 matrices

$$\bar{\theta} = -3\theta_9 + \arg \det |y_t|^2 e^{i\theta_9} = 0$$

Need a better estimate of instanton effects

$y_b/y_t \sim 1/40$ is not so small that we can ignore the polynomial prefactor

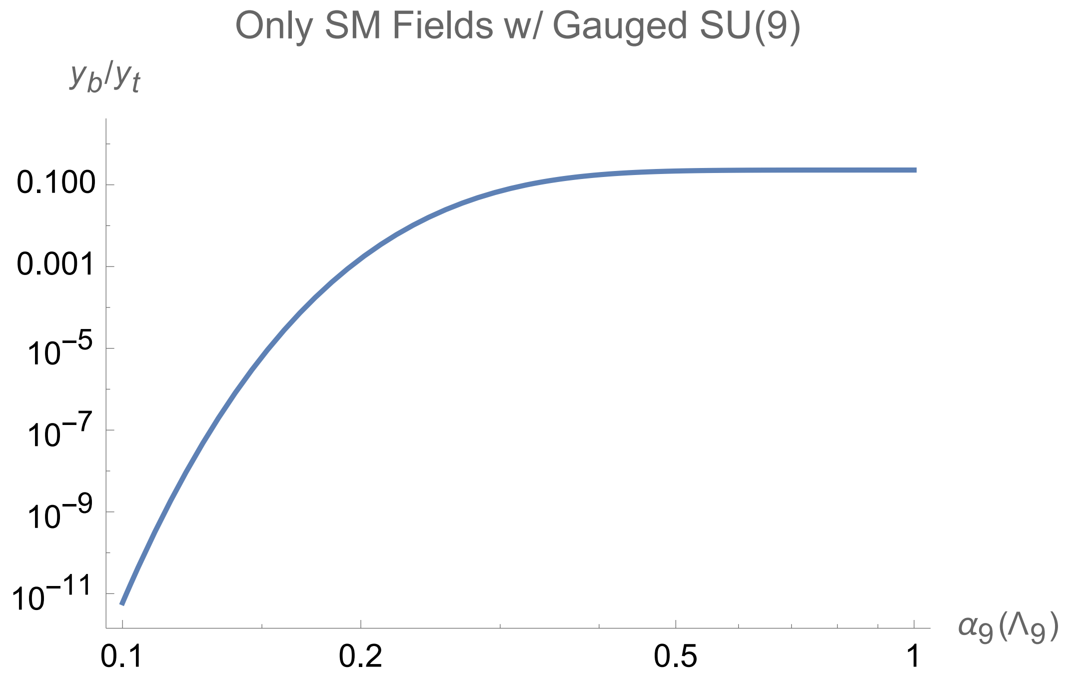


$$\sim \int_{1-inst} \mathscr{D}A\mathscr{D}\phi_i\mathscr{D}\psi_i\ H\mathbf{Q}\bar{\mathbf{d}}\ e^{-S_{gauge}-\int \mathscr{L}_{int}}$$

Thankfully ’t Hooft taught us how to do this in 1976. Must integrate over all the zero-modes of the 1-inst solution.

$$A_\mu(x) = \frac{2}{g}\frac{\rho^2}{(x-x_0)^2}\frac{\eta_{a\mu\nu}(x-x_0)^\nu J^a}{(x-x_0)^2+\rho^2}$$

As well as quadratic fluctuations for any charged scalar fields, and solve for the charged fermion zero-modes



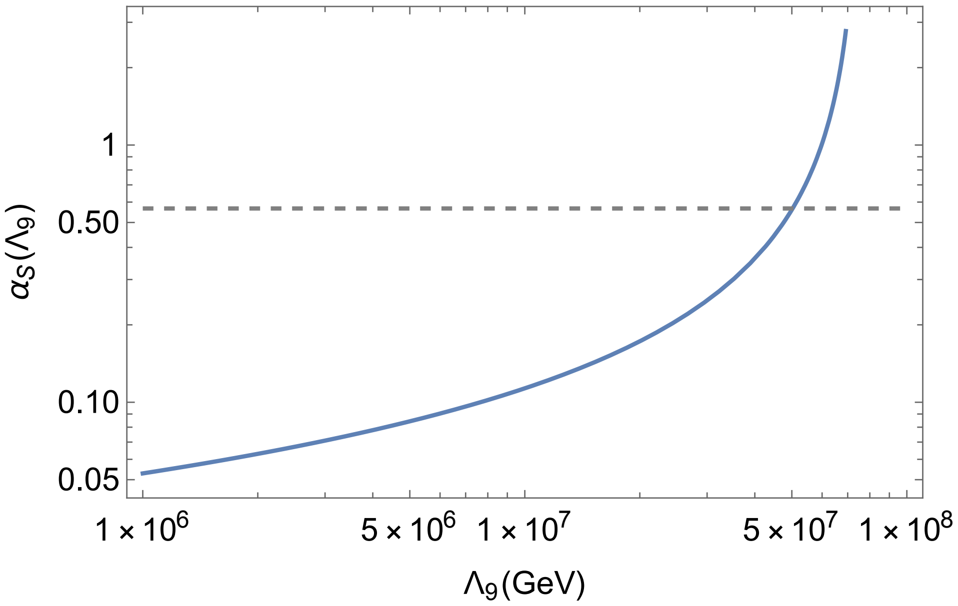
Run, run, run

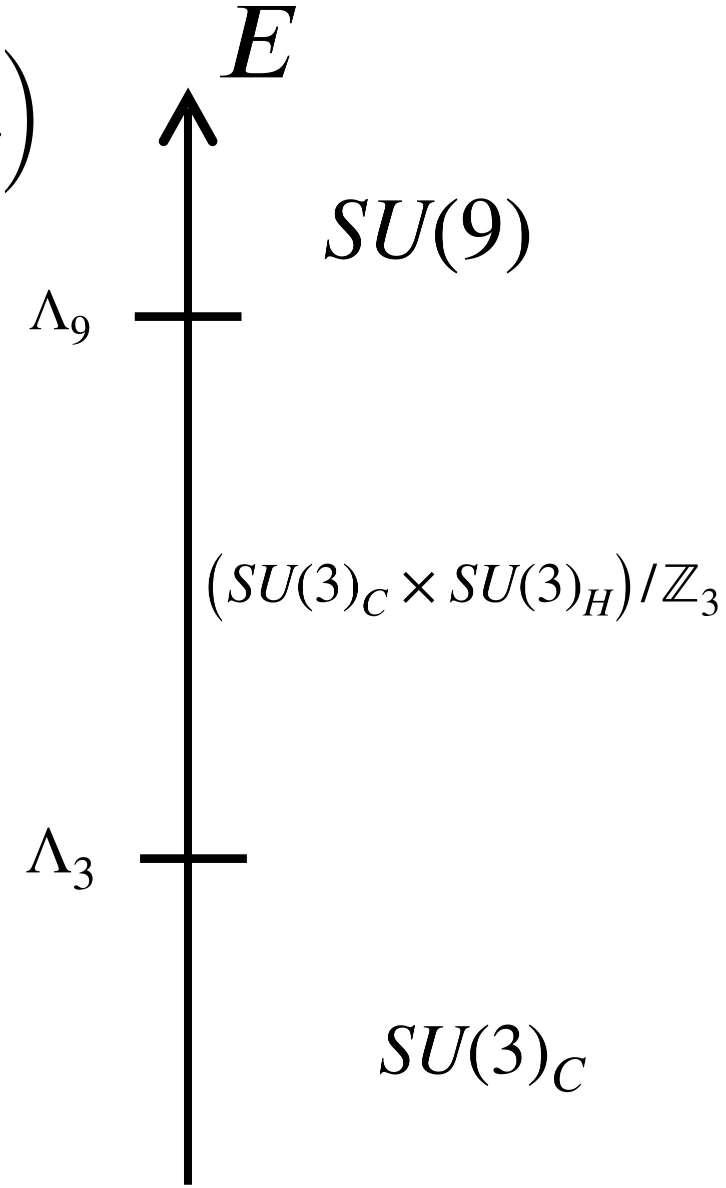
$$\beta_3 = \left(\frac{11}{3}N_c - \frac{4}{3}n_f I_f - \frac{1}{3}n_s I_s - \frac{1}{6}n_r I_r\right)$$

With the many gluons of $SU(9)$ the UV theory is easily asymptotically free

But strong coupling must grow in intermediate phase

Can be easily achieved if some of the colored scalars get masses a bit below Λ_9





Color-flavor embedding

This 'special embedding' $SU(9) \to \left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ has a non-trivial 'index of embedding': the fundamental 9 branches to the $(3{,}3)$ so the Dynkin index changes non-trivially $k = \mu_{IR}/\mu_{UV} = 3$

See Csaki, Murayama '98 for good discussion

So the $SU(9)$ theory has 'extra' instantons that the IR theory does not: a fermion has $k = 3$ times as many zero-modes in the $SU(3)_C$ instanton background, so we must interpret this as a 3-instanton of the $SU(9)$ theory

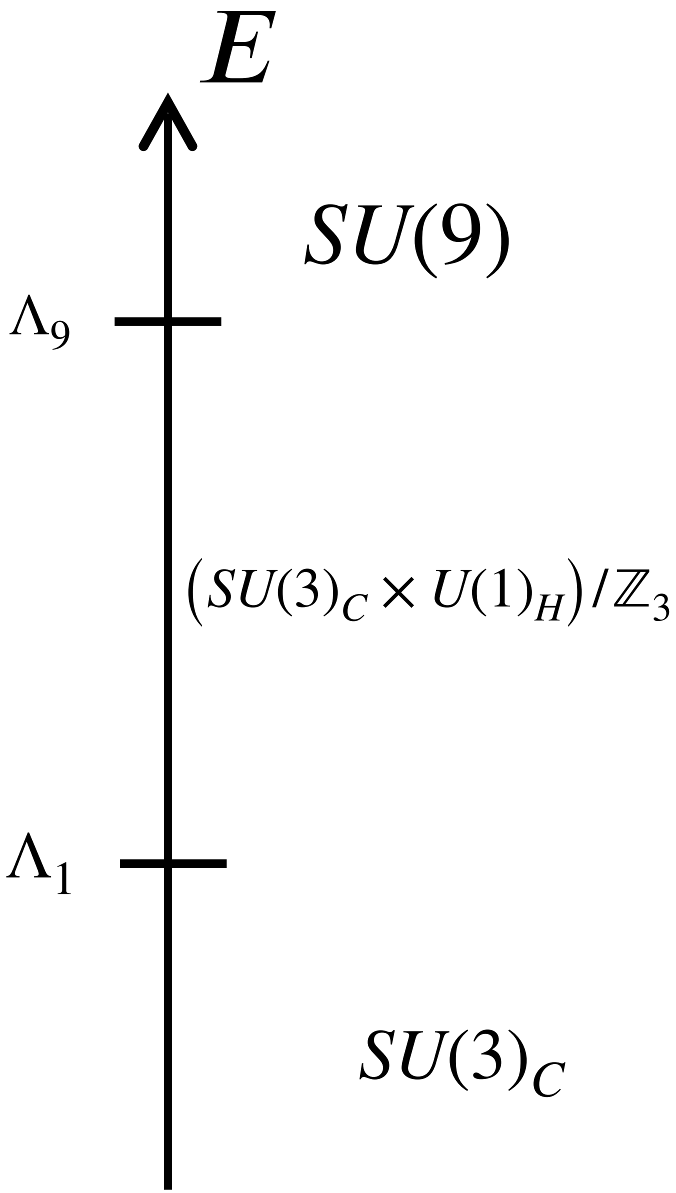
Matching the instanton actions implies a non-trivial matching of the gauge coupling across Λ_9, as $e^{-\frac{8\pi^2}{\alpha_{IR}}} = e^{-k\frac{8\pi^2}{\alpha_{UV}}}$, so $\alpha_9(\Lambda_9) = 3\alpha_C(\Lambda_9)$

Abelian Z’ also promising

In the case of $\left(SU(3)_C \times U(1)_H\right)/\mathbb{Z}_3$ the new gauge boson (and new non-invertible symmetry) can observationally appear much sooner

And furthermore maybe don’t need as large Higgs representations to do breaking, so less suppression of instanton density

But need to more clearly understand generating flavor texture in this scheme



Fractional instanton analysis on $S^2 \times S^2$

See Anber, Hong, Son 2109.03245

Turn on general allowed magnetic fluxes (background 2-form fields for the magnetic 1-form symmetry) and calculate instanton numbers

$$Q_c = \frac{N_c - 1}{N_c} \int_{M_4 = M_2 \times \Sigma_2} \frac{w_2 \wedge w_2}{2} = \frac{N_c - 1}{N_c} \oint_{M_2} w_2 \oint_{\Sigma_2} w_2 = m_1 m_2 \left(1 - \frac{1}{N_c} \right)$$

$$Q_H = \frac{1}{8\pi^2} \int H_2 \wedge H_2 = s_1 s_2$$

Then compute Dirac indices of fermions

$$I_{\psi_i} = n_{\psi_i} T_{\psi_i} Q_c + \mathrm{dim}_{\psi_i} n_{\psi_i} q_{\psi_i}^2 Q_H$$

And find anomaly coefficients for each $U(1)_{\text{global}}[CH]^2$

Why multiple generations?

One sort of answer from physics effects you can only get with multiple generations

- CP violation in CKM *would have been* a great answer if this were responsible for electroweak baryogenesis, but alas. Kuzmin, Rubakov, Shaposhnikov ’85
- SM has anomaly-free $\mathbb{Z}_{2N_g}^{B+L}$ so proton not destabilized for $N_g > 1$ but lifetime very long anyway. Explained in SK ’22

To me this question motivates thinking about BSM effects you can have only because $N_g > 1$, and especially interesting things that can happen for $N_g = 3$

Related forthcoming SK & S. Homiller: Only for $N_g = 3$ can one write a Totally Anti-Symmetric Triplet Yukawa (TASTY) model of flavor: $y_t \epsilon_{ijk} H^i Q^j \bar{u}^k$

Why multiple generations?

Anomaly cancellation in ultraviolet embeddings of the SM?

- In 6d, ‘global’ $SU(2)_L$ anomaly (Kiritsis ’86 following Witten ’82) requires multiple generations (Dobrescu & Poppitz ’01)
  - This is not true! Global anomalies actually not classified by homotopy groups $\pi_n(G)$ but by *bordism* as we have gradually understood is captured by the Atiyah-Patodi-Singer η-invariant (summarized in Witten & Yonekura, ’19) and in particular there is no 6d SU(2) anomaly (Mannier & Moore, ’18; Davighi & Lohitsiri ’20; Lee & Tachikawa ’20)

- Electroweak $SU(3)_L \times U(1)_N$ embedding can have anomalies cancel across three non-universal generations (Singer, Valle, Schechter ’80, emphasized in ’92 by Foot, Hernandez, Pisano, Pleitez; Frampton) (but could also do universal embedding)

Big idea: (B)SM physics with multiple generations differs even in the flavor-singlet sector

Is the proton stable in the Standard Model? Yes, there is an exact anomaly-free $\mathbb{Z}_{2N_g}^{B+L}$ symmetry.

Mixed anomalies gauge^2 x global of SM

	$U(1)_{\text{Baryons}}$	$U(1)_{\text{Leptons}}$
$SU(2)_L^2$	N_{g}	N_{g}
$U(1)_Y^2$	$-18N_{\text{g}}$	$-18N_{\text{g}}$

SK 2204.01741

Remarkably this can be extended flavorfully and embedded in the gauge symmetry of $SU(12) \times SU(2)_L \times SU(2)_R$

	Fields	$\mathrm{U}(1)_X$
Quarks	$q_i,\ u_i,\ d_i$	m
Electrons and taus	$\ell_{1,3},\ e_{1,3},\ \nu_{1,3}$	$n - 3m$
Muons	$\ell_2,\ e_2,\ \nu_2$	$-2n - 3m$

Davighi, Greljo, Thomsen 2202.05275

Maybe eventually some things to say about the big question of why we have multiple generations?

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The Standard Model

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$-$	$-$	$-$
$SU(2)_L$	$\mathbf{2}$	$-$	$-$	$\mathbf{2}$	$-$	$\mathbf{2}$
$U(1)_Y$	$+1$	-4	$+2$	-3	$+6$	-3

What are its generalized global symmetries?

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$-$	$-$	$-$
$SU(2)_L$	$\mathbf{2}$	$-$	$-$	$\mathbf{2}$	$-$	$\mathbf{2}$
$U(1)_Y$	$+1$	-4	$+2$	-3	$+6$	-3

Zero-form symmetries:

Start with large classical flavor symmetry $\left(U(N_g)^{(0)}\right)^5$

$$\mathscr{L} = y_u^{ij}\tilde{H}Q_i\bar{u}_j + y_d^{ij}HQ_i\bar{d}_j + y_e^{ij}HL_i\bar{e}_j$$

Left-over classical $U(1)_B \times U(1)_L \to U(1)_{B-L} \times \mathbb{Z}_{N_g}^L$ broken by electroweak instantons

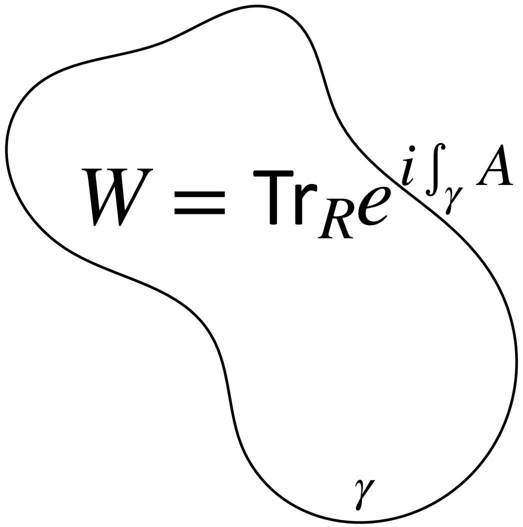
This last factor, since we have $N_g > 1$, is responsible for SM proton stability

SK '22; Wang, Wan, You '22

The SM with massless neutrinos has exact $U(1)_{L_\mu - L_\tau} \times U(1)_{L_e - L_\mu}$ but we know from oscillations that these are not symmetries of the real world

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$-$	$-$	$-$
$SU(2)_L$	$\mathbf{2}$	$-$	$-$	$\mathbf{2}$	$-$	$\mathbf{2}$
$U(1)_Y$	$+1$	-4	$+2$	-3	$+6$	-3

An aside on SM one-form symmetries



Hypercharge magnetic one-form symmetry: $U(1)^{(1)}_m$

Electric one-form symmetry? We don't know!

See D. Tong '17

Certain center transformations do not act on any of the SM fields, e.g. consider $\mathbb{Z}_2 \subset SU(2)_L \times U(1)_Y$ under which $\psi \mapsto \psi \left((-1) I_L\right) e^{\pi i Y}$

So the *global structure* of the SM gauge group is

$G_{SM_q} \equiv \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\mathbb{Z}_q$ with q=1,2,3,6

Which has electric one-form symmetry $\mathbb{Z}^{(1)}_{6/q}$

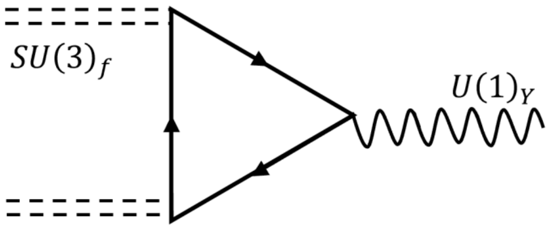
Global structure, fractionally-charged particles, and SMEFT
SK & A. Martin coming

See also recent discussion in axion theories by Reece; Choi, Forslund, Lam, Shao; Cordova, Hong, Wang

Approximate higher structure:

The non-abelian parts $SU(3)^5$ are intertwined with the magnetic one-form symmetry $U(1)_m^{(1)}$ in the form of a 2-group

Córdova & SK ’22



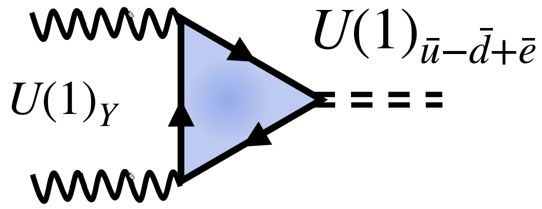
Flavor^2	$U(1)_Y$
$SU(3)^2_Q$	$+1 \cdot 2 \cdot N_c$
$SU(3)^2_u$	$-4 \cdot N_c$
$SU(3)^2_d$	$+2 \cdot N_c$
$SU(3)^2_L$	$-3 \cdot 2$
$SU(3)^2_e$	$+6$

One	None		
Two	$\{L, Q\}$	$\{L, \bar{d}\}$	$\{L, \bar{e}\}$
Three	$\{\bar{u}, \bar{d}, \bar{e}\}$	$\{\bar{u}, \bar{e}, Q\}$	$\{\bar{u}, \bar{d}, Q\}$
Four	None		
Five	$\{Q, \bar{u}, \bar{d}, L, \bar{e}\}$		

Zero-form symmetries so intertwined must be broken by the scale of magnetic one-form symmetry breaking, which (at zero yukawa) tells you the possible unified multiplets

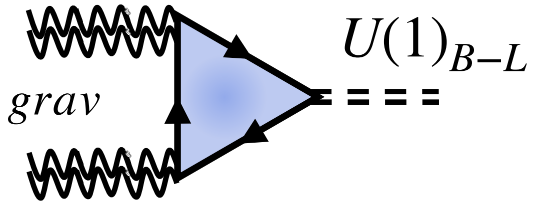
Finite yukawas are ‘spurions’ of 2-group symmetry-breaking and can perturb away from this structure if they control the mass of some vector-like fermions

Non-invertible symmetries:



Approximate $U(1)_{\bar{u}-\bar{d}+\bar{e}}$ is non-invertible due to a mixed anomaly with hypercharge, $U(1)_Y^2 U(1)_{\bar{u}-\bar{d}+\bar{e}} = 72N_g$

No BSM model-building use yet, but Shao, Lam, Choi ’22 use this for a ‘symmetry-based’ derivation of $\pi^0 \to \gamma\gamma$



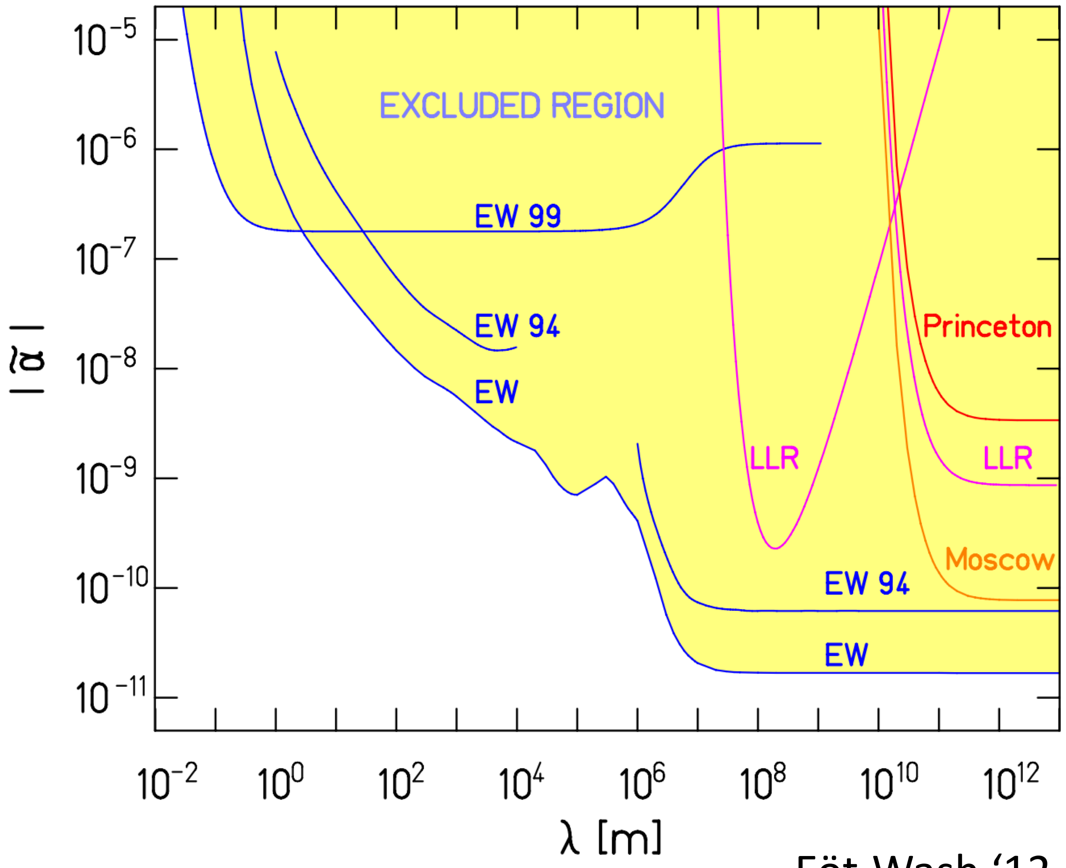
$U(1)_{B-L}$ has a mixed gravitational anomaly without exactly N_g right-handed neutrinos, and Putrov, Wang ’23 showed this also leads to a non-invertible symmetry! Could be used for ‘gravitational leptogenesis’ Alexander, Peskin, Sheikh-Jabbari ’06.

One further IR 'ambiguity'

Given the SM matter content, it's an empirical question whether $U(1)_{B-L}$ is actually a global symmetry or perhaps a weakly coupled gauge symmetry

A $\mathbb{Z}_N$ subgroup may be gauged and unbroken: this "B-L BF theory" is an extension of the SM with 0 new dof

Comes with magnetic two-form symmetry $\mathbb{Z}_N^{(2)}$



Eöt-Wash '12

Remarkably little work on this. I suggested for $N = 2N_g$ these cosmic strings could resolve the cosmological lithium problem.