On-shell techniques for the standard-model effective theory

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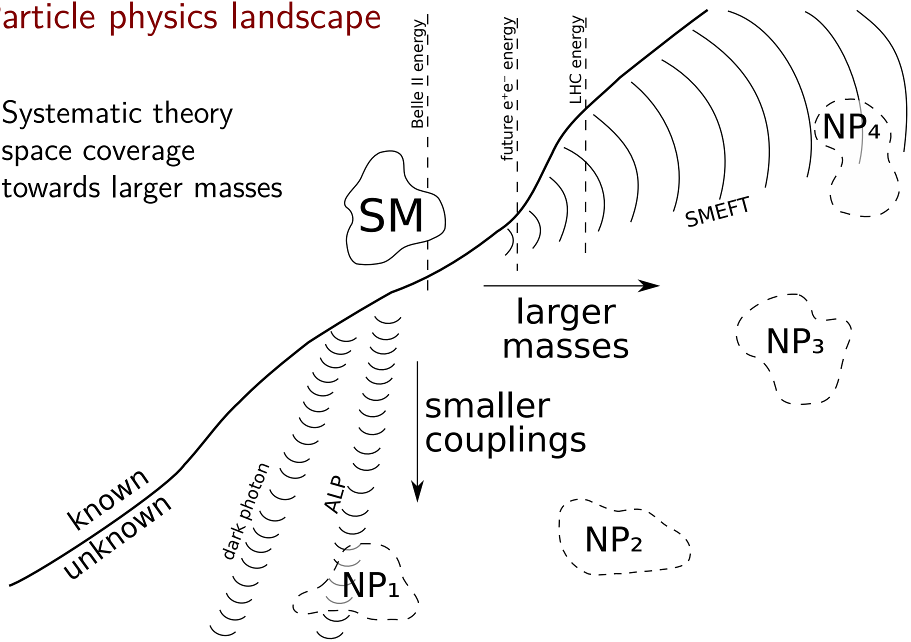
EFT matching from analyticity and unitarity
SciPost Phys. 16 (2024) 071, [2308.00035]
with Stefano De Angelis

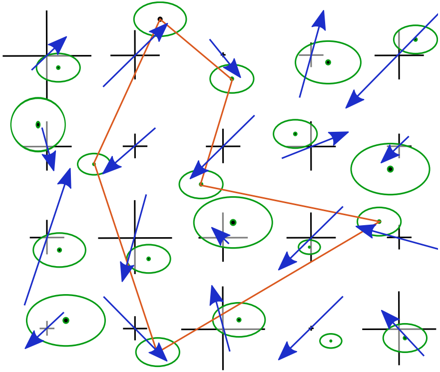
Crossroads between theory and pheno – CERN – 26 June 2024

Particle physics landscape

Systematic theory space coverage towards larger masses



Isolating subtle patterns of new physics



array of sensitive observables

- precise SM&EFT predictions
- precise measurements

$\rightarrow$ correlate deviations

Building LHC's legacy

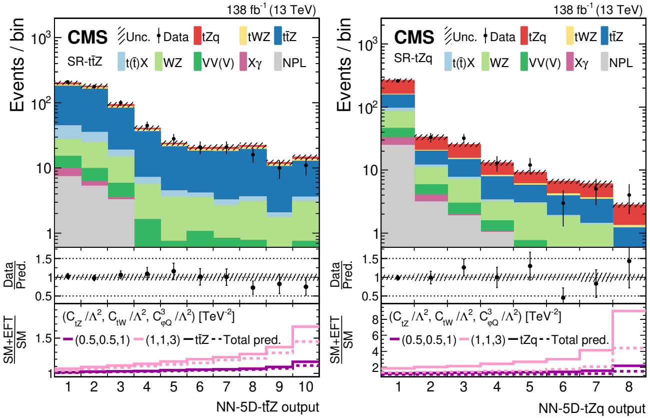
- Improved sensitivity (multidimensionally)
- Global picture (limited assumptions)
- Precise interpretation (better EFT predictions)
- Models' connection (matching, charting)
- New understanding (on-shell techniques)

SMEFT progresses

ML optimisation for SMEFT

$tZ + X$ process in the three-lepton signal region

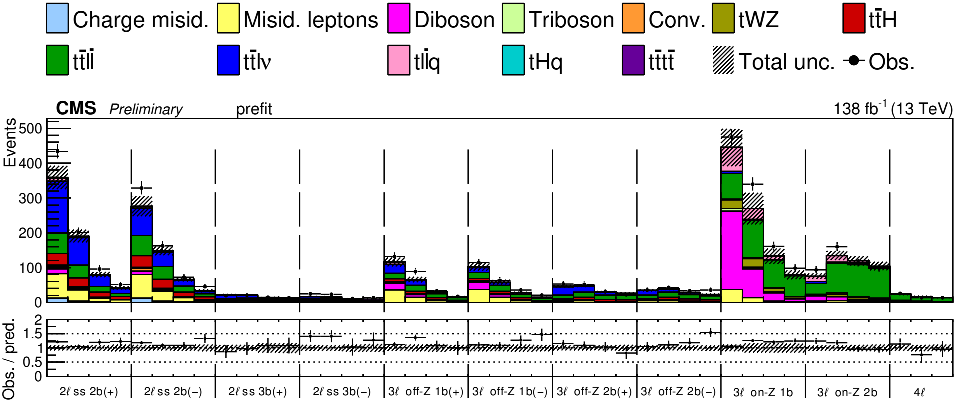
1. discriminate $t\bar{t}Z$, tZj signals and backgrounds
2. train SM vs. $(c_{tZ}, c_{tW}, c^3_{\phi q})$ from reweighted samples



Beyond signal and background processes

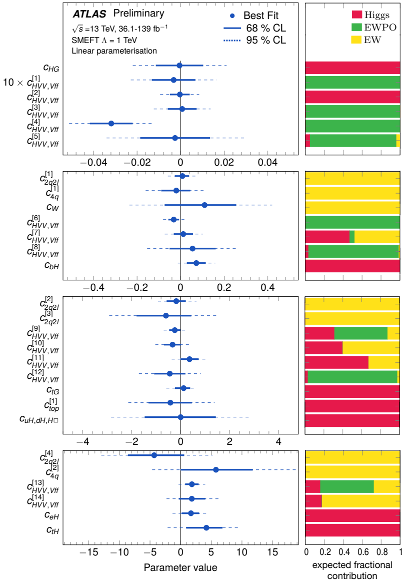
- leptons+b's+jets final state, p_T bins, 178 data points
- contains tth, ttZ, ttW, tZq, tHq, diboson, etc.
- 26 top operator contributions from reweighting
- towards publication of 26D likelihood

[Valsecchi LHCP23]



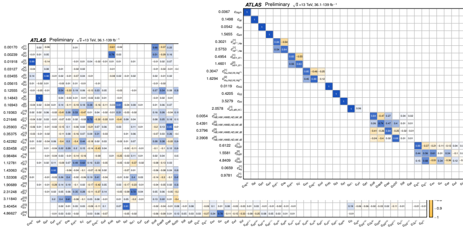
ATLAS Higgs+diboson+EWPO combination

[ATLAS '22]



- Higgs '21 STXS combination
- diboson $WW, WZ, 4\ell, Zjj$
- Z pole from LEP+SLC

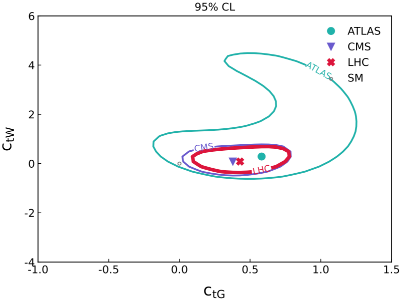
- principal component analysis removing flat directions
- fit results for 22 eigen-vectors
- lin results

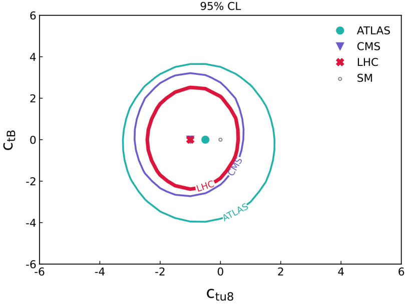


ATLAS+CMS top combination

[Skopven RFI23]

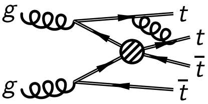
- full likelihoods:
  - $4t$ ($n\ell$ ATLAS), $4t$ ($n\ell$ CMS),
  - $tt\gamma$ (1ℓ CMS), $tt\gamma$ (2ℓ CMS),
  - ttZ ($n\ell$ ATLAS)
- 700^+ bin, $\sim$20 processes
- 8 operators, lin & quad, also in tth and ttW
- uncorrelated systematics

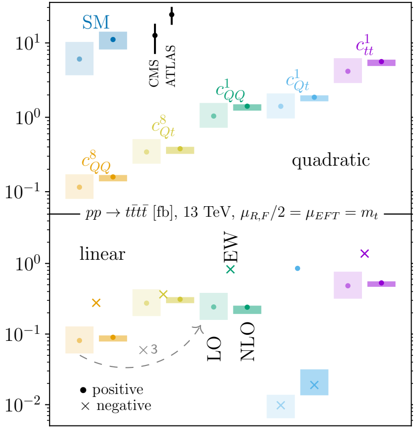




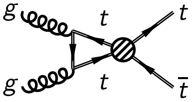
SMEFT at one loop

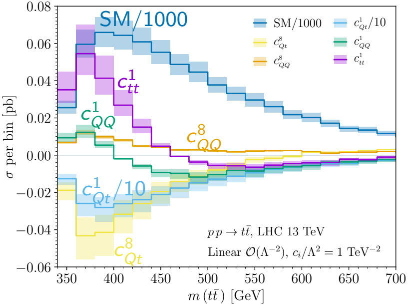
Better accuracy and uncertainties





New sensitivities ... and degeneracies

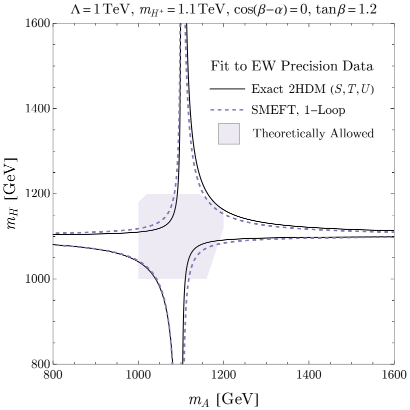


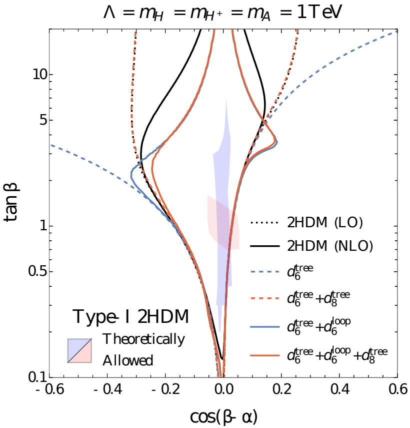


... to be resolved with more differential measurements

Matching at one loop: the 2HDM

[Das Bakshi, Dawson, Fontes, Homiller '24]





- EWPO constraints arising first at one-loop
  - mild impact so far; more important with new Z pole?
- more accurate large-tan β description
  - from Yukawa operators; probed with new Higgs measurements

On-shell techniques

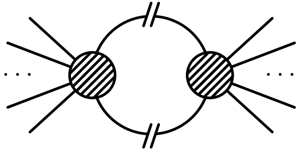
Analyticity and unitarity of on-shell amplitudes

bypass unphysical fields, operators, Lagrangians

avoid gauge and field redefinition redundancies

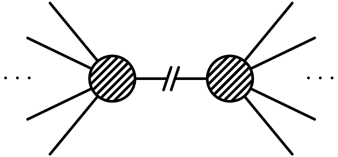
loops cut into lower loops

$+$ rational terms



trees cut into smaller trees

$+$ contact terms



recursive construction from the simplest amplitudes

or more direct extraction of various quantities

SMEFT applications

- operator enumeration

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Henning, Melia '19], [Falkowski '19], [GD, Machado '19]
[Li, Ren, et al. '20, '20], [Accettulli Huber, De Angelis '21], [Harlander, Kempkens, Schaaf '23]

- kinematics characterisation

[Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20]
[Balkin et al. '21], [Dong, Ma, Shu, Zheng '21, '22], [De Angelis '22]
[Bradshaw, Chang, Chen, Liu, Luty '22, '23], [Liu, Ma, Shadmi, Waterbury '23]

- anomalous dimensions

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20], [Jiang et al. '20], [Elias Miró et al. '20, '21]
[Baratella et al. '20, '20, '21], [Accettulli Huber, De Angelis '21], [Delle Rose et al. '22], [Baratella '22]
[Machado, Renner, Sutherland '22], [Chala '23]

- matching to UV models

[Delle Rose, von Harling, Pomarol '22]
[De Angelis, GD '23]

Operator enumeration

Massless three points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h > 0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h < 0 \end{cases}$$

up to a constant coefficient

$$\begin{aligned} f^+f^+s \quad & [12] \\ v^+v^+s \quad & [12]^2 \\ f^+f^-v^+ \quad & [13]^2/[12] \\ v^+v^+v^- \quad & [12]^3/[23][31] \\ t^+t^+t^- \quad & \left([12]^3/[23][31]\right)^2 \end{aligned}$$

$$[g] = 1 - |h|$$

$$h \equiv \sum h_i$$

Massless higher-point contact terms

Multiple independent structures for given helicities

non-vanishing Lorentz invariants $\left(s_{ij} \equiv 2\, p_i \cdot p_j,\ \epsilon_{ijkl} \equiv \epsilon_{\mu\nu\rho\sigma} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\right)$

solving
- · little-group covariance
- · momentum conservation
- · Schouten identity $[12][34] - [13][24] + [14][23] = 0$

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solving · little-group covariance

· momentum conservation

· Schouten identity $[12][34] - [13][24] + [14][23] = 0$

e.g. GR-SM-EFT up to dim-8: [GD, Machado '19]

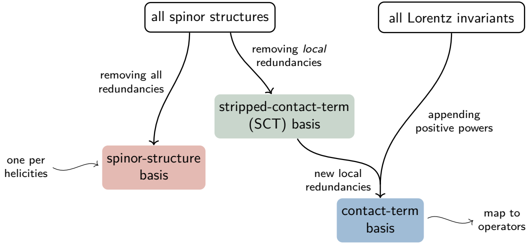
$$\begin{aligned}
t^+t^+t^+t^+ &: \quad [12]^4[34]^4 + [13]^4[24]^4 + [14]^4[23]^4 \\
t^+t^+v^+v^+ &: \quad [12]^4[34]^2, [12]^2[13][14][24][23] \\
t^+v^+f^+f^- &: \quad [12]^2[13][124\rangle \qquad \times\text{polynomial}(s_{ij}, \epsilon_{ijkl}) \\
t^+f^+f^+f^+f^+ &: \quad [12][13][14][15] \\
\ldots & \qquad \ldots
\end{aligned}$$

also from Hilbert series: [Ruhdorfer et al. '19]

Divide and conquer

Separate spinors and Lorentz invariants [GD, Kitahara, Machado, Shadmi, Weiss '20]

e.g. $[12][34] \times s_{12}$



Exclude seemingly non-local relation

✗ $[12][34] = -[13][24]\ s_{12}/s_{13}$ at $d = 6$

✓ $[12][34]\ s_{13} = -[13][24]\ s_{12}$ at $d = 8$

Counting from Hilbert series [Bradshaw, Chang, Chen, Liu, Luty '22, '23]

e.g. $\mathrm{H}_{f^+f^+f^+f^+}(d) = \dfrac{2d^6 - d^8}{(1-d^2)^2}$

Kinematics characterisation

Massive little-group-covariant spinors

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} = q^i\rangle[q^i + k^i\rangle[k^i = i^J\rangle[i_J \qquad \text{with } k_i^2 = 0 = q_i^2,\ J=1,2 \quad 2k^i\cdot q^i = m_i^2$$

Spin s from $2s$ symmetrized spin $1/2$

left implicit, e.g. $\langle 1^I 3^J\rangle[2^K 3^{J'}] + (J \leftrightarrow J')$ written as $\langle\mathbf{13}\rangle[\mathbf{23}]$

Leading high-energy limit is just *unbolding*

Three-point examples:

ffs $[\mathbf{12}],\ \langle\mathbf{12}\rangle$

vvs $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

ssv $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

ffv $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

... counting by spin irreps addition

Four-point example: *ffZh*

- Twelve independent structures: [GD, Kitahara, Shadmi, Weiss '19]

$$\mathcal{M}(\mathbf{1}_f, \mathbf{2}_f, \mathbf{3}_Z, \mathbf{4}_h) \ni \begin{matrix} [\mathbf{13}]\langle\mathbf{23}\rangle \\ \langle\mathbf{13}\rangle[\mathbf{23}] \end{matrix} \quad \begin{matrix} [\mathbf{13}][\mathbf{23}] \\ \langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ [\mathbf{12}]\langle\mathbf{3}(\mathbf{1}\pm\mathbf{2})\mathbf{3}] \\ \langle\mathbf{12}\rangle\langle\mathbf{3}(\mathbf{1}\pm\mathbf{2})\mathbf{3}] \end{matrix} \quad \begin{matrix} [\mathbf{312}\rangle[\mathbf{13}] \\ \langle\mathbf{321}]\langle\mathbf{23}\rangle \\ [\mathbf{321}\rangle[\mathbf{23}] \\ \langle\mathbf{312}]\langle\mathbf{13}\rangle \end{matrix} \quad \times\ \text{poly}(s_{ij})$$

- Counted by Hilbert series: [Gráf, Henning, Lu, Melia, Murayama '22] [Bradshaw, Chang, Chen, Liu, Luty '22, '23]

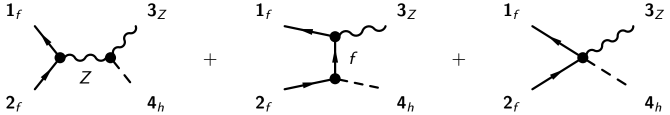
$$\mathrm{H}_{ffZh}(d) = \frac{2d^5 + 6d^6 + 4d^7}{(1-d^2)^2}$$

→ fully characterised kinematics, beyond lowest operator dim.

EW symmetry from perturbative unitarity

[GD, Kitahara, Shadmi, Weiss '19]

$f\, f\, Z\, h \sim$



factorisable s, t, u channels — contact terms

$$\xrightarrow[\text{energy}]{\text{high}} \begin{cases} \dfrac{[12]}{m_Z}\left(c^{\text{left}}_{ffZ} - c^{\text{right}}_{ffZ}\right)\left(c^{\text{right}}_{ffh} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \\ \dfrac{\langle 12\rangle}{m_Z}\left(c^{\text{left}}_{ffZ} - c^{\text{right}}_{ffZ}\right)\left(c^{\text{left}}_{ffh} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \end{cases}$$

as for the SM in the '70

[Llewellyn-Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

Matching to UV models

EFT matching from cuts?

[Delle Rose, von Harling, Pomarol '22] examined, on-shell, the *magic zeros* found by [Arkani-Hamed, Harigaya '21] in $(g-2)_\mu$

finding that the rational terms of loops do not contribute

Lessons from positivity

- Unitarity:

$\mathcal{A}^\dagger(+i\epsilon) \overset{\text{CPT}}{=} \mathcal{A}(-i\epsilon)$

sum over intermediate state X

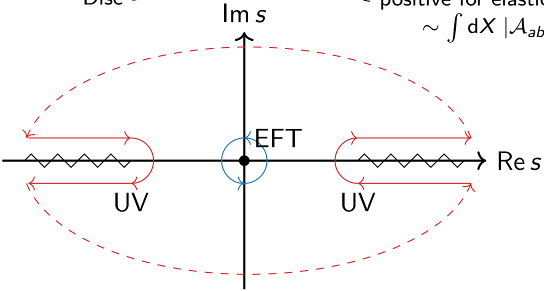
$$(\mathcal{A} - \mathcal{A}^\dagger)/i = \mathcal{A} \cdot \mathcal{A}^\dagger$$

Disc

positive for elastic+forward
$\sim \int \mathrm{d}X \, |\mathcal{A}_{ab\to X}|^2$

- Analyticity:

controlled at 4 points



vanishes by Froissart for $n \geq 2$

$$\underset{s=0}{\mathrm{Res}} \frac{\mathcal{A}^{\mathrm{EFT}}_{ab\to ab}(s)}{s^{n+1}} = \frac{1}{2\pi} \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} \left[\mathrm{Disc}\, \mathcal{A}^{\mathrm{UV}}_{ab\to ab} + (-1)^n \, \mathrm{Disc}\, \mathcal{A}^{\mathrm{UV}}_{a\bar{b}\to a\bar{b}} \right] + C_\infty$$

≥ 0 $\quad$ ≥ 0

so $\quad c_n \geq 0 \quad$ for n even ≥ 2

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

Lessons from positivity

- Unitarity:

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sum over intermediate state X

$$(\mathcal{A} - \mathcal{A}^\dagger)/i = \mathcal{A} \cdot \mathcal{A}^\dagger$$

Disc

ward

- Analyt

contro

at 4 po

UV

Operator coefficients derive from cuts in four-point (elastic forward) amplitudes.

vanishes by Froissart for $n \geq 2$

$$\operatorname*{Res}_{s=0} \frac{\mathcal{A}^{\text{EFT}}_{ab\to ab}(s)}{s^{n+1}} = \frac{1}{2\pi}\int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} \left[\text{Disc}\,\mathcal{A}^{\text{UV}}_{ab\to ab} + (-1)^n\,\text{Disc}\,\mathcal{A}^{\text{UV}}_{a\bar{b}\to a\bar{b}}\right] + C_\infty$$

≥ 0 ≥ 0

so $c_n \geq 0$ for n even ≥ 2

[Caron-Huot, Wilhelm '16]

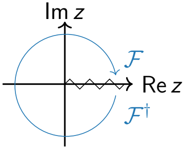
Lessons from anomalous dimensions

- In a massless theory, any $(\log \mu^2)$ comes with a $(-\log s_I)$

- A dilation $z^{D/2}$ with $D \equiv \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$ captures all Mandelstam logs in a single $(-\log z)$ and disregards logs of s_I/s_J ratios

momentum influx

- Form factors $\mathcal{F} \equiv {}_{\text{out}}\langle p_1, \ldots p_m | \mathcal{O}(q) | 0 \rangle_{\text{in}}$ have all $s_I \equiv (\sum_{i \in I} p_i^\mu)^2$ Mandelstams positive



- Dilated form factors $\hat{\mathcal{F}}(z) \equiv z^{D/2} \mathcal{F}$ only have singularities at positive z's

at $\sum_k \alpha_k M_k^2 / \sum_I \alpha_I s_I$ in Feynman parameterisation

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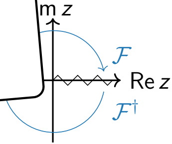
- A dilation $z^{D/2}$ with $D \equiv \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$ captures all Mandelstam logs in a single ... s

Dilated form factors have a known analytic structure at any multiplicity.



- Form ... $s_I \equiv$ ... positive

- Dilated form factors $\hat{\mathcal{F}}(z) \equiv z^{D/2}\mathcal{F}$ only have singularities at positive z's

at $\sum_k \alpha_k M_k^2 / \sum_I \alpha_I s_I$ in Feynman parameterisation

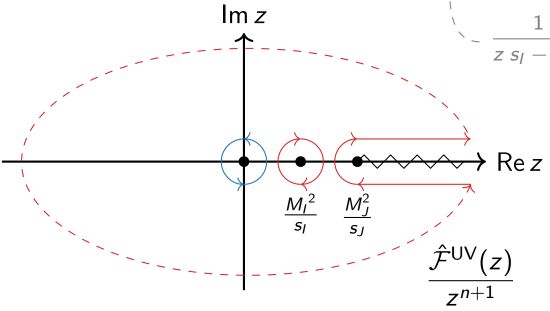
[De Angelis, GD '23]

Dispersive matching for massless EFTs

- equate $\mathcal{F}^{\text{EFT}}$ and $\mathcal{F}^{\text{UV}}$ order by order in the zero-momentum expansion
- dilate (with $z^{D/2}$) and enforce $\underset{z=0}{\text{Res}} \frac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = \underset{z=0}{\text{Res}} \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$
- EFT: $\underset{z=0}{\text{Res}} \frac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = c_n \,\text{poly}_n(s_I)$ with $\mathcal{F}^{\text{EFT}}_{\text{tree}} = \sum_k c_k \,\text{poly}_k(s_I)$
- UV: $\underset{z=0}{\text{Res}} \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \oint_{z=0} dz \, \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \left[\sum \text{Res} + \int \text{Disc} + \int_\infty\right] \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$

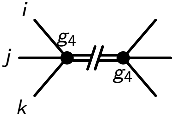
$\log(M_J^2 - z\, s_J)$

$\frac{1}{z\, s_I - M_I^2}$



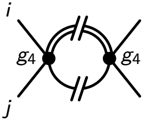
EFT matching from just cuts!

Simple toy $\Phi\phi^3$ example



$$: \underset{z=M^2/s_{ijk}}{\text{Res}} \frac{|\mathcal{A}(\phi\phi\phi \to \Phi)|^2}{z s_{ijk} - M^2} \frac{1}{z^{n+1}}$$

$$= \frac{g_4^2}{M^2} \left(\frac{s_{ijk}}{M^2}\right)^n$$



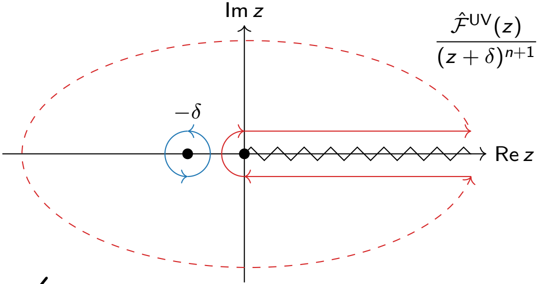
$$: \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \int \text{dLIPS} \, |\mathcal{A}(\phi\phi \to \phi\Phi)|^2$$

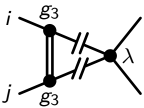
$$= \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \frac{1}{8\pi} \left(1 - \frac{M^2}{z s_{ij}}\right) g_4^2$$

$$= \frac{g_4^2}{16\pi^2 n(n+1)} \left(\frac{s_{ij}}{M^2}\right)^n \qquad \text{for } n > 0$$

- use amplitudes instead of form factors in practice
- all EFT orders obtained at once
- nothing to know about, or compute in, the EFT
- fewer legs and loops

Massless cut subtlety





$$: \int_0^\infty \frac{dz}{z^{n+1}} \int \mathrm{dLIPS}_d \frac{\lambda g_3^2}{(l-p_i)^2 - M^2}$$

$$= \frac{\lambda g_3^2 M^{d-6}}{8(4\pi)^{\frac{d}{2}-2}} \left(\frac{s_{ij}}{M^2}\right)^n \frac{(-1)^{n+1} n! \csc\frac{\pi d}{2}}{\Gamma[\frac{d}{2}+n]}$$

$$= \frac{\lambda g_3^2}{16\pi^2 M^2} \frac{(-1)^n}{n+1} \left(\frac{s_{ij}}{M^2}\right)^n \left(\frac{1}{\bar{\epsilon}} + H_{n+1} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\epsilon)\right)$$

NB: phase-space integrals with more uncut propagators become complicated (unlike the hard region expansion, $s_l \ll l^2 \sim M^2$, at low EFT orders)

The power of dimreg

- Loop contributions in the massless EFT lead to scaleless $\int_0^\infty \mathrm{d}z\; z^{\alpha+\epsilon} = 0$.

  $\longrightarrow$ The tree-level EFT amplitude is extracted.

- The soft region ($l^2 \sim s_I \ll M^2$) of UV loops is similarly scaleless.

  $\longrightarrow$ The hard region ($s_I \ll l^2 \sim M^2$) of UV loops is extracted.

- UV loop contributions at infinity lead to scaleless $\lim_{|z|\to\infty} |z|^{\alpha+\epsilon} = 0$.

  $\longrightarrow$ The boundary is tree-level exact.

Rare boundary terms

multiplicity → m sum of coupling dim. → $[c_{\text{EFT}}]$

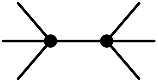
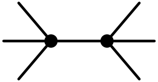
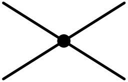
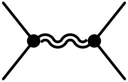
- no matching information unless $n \geq \min(4 - m - [c_{\text{EFT}}])/2$
- no boundary term unless $n \leq \max(4 - m - [c_{\text{UV}}])/2$

  $\rightarrow \min[c_{\text{UV}}] \leq 4 - m - 2n \leq \max[c_{\text{EFT}}]$

- but $\min[c_{\text{UV}}] = 0$ for a renormalisable theory
  and $\max[c_{\text{EFT}}] = 0$ for four- and higher-point contact-term operators

  $\rightarrow$ then boundary only for $[c_{\text{UV}}] = 0 = [c_{\text{EFT}}]$ and $n = (4 - m)/2$

e.g.

n	EFT	UV
-1		
0		

On-shell techniques for the standard-model effective theory

SMEFT can isolate subtle patterns of heavy new physics,
and encode LHC's legacy.

On-shell additions to the theory toolbox
ease computation and gain new understanding.

Dispersive EFT matching, from cuts.

simpler building blocks

all EFT orders accessible at once

not knowledge required about the EFT

... back to *magic zeros*, massive EFT generalisation,
other types of EFTs, multi-loop, positivity beyond four points, ...