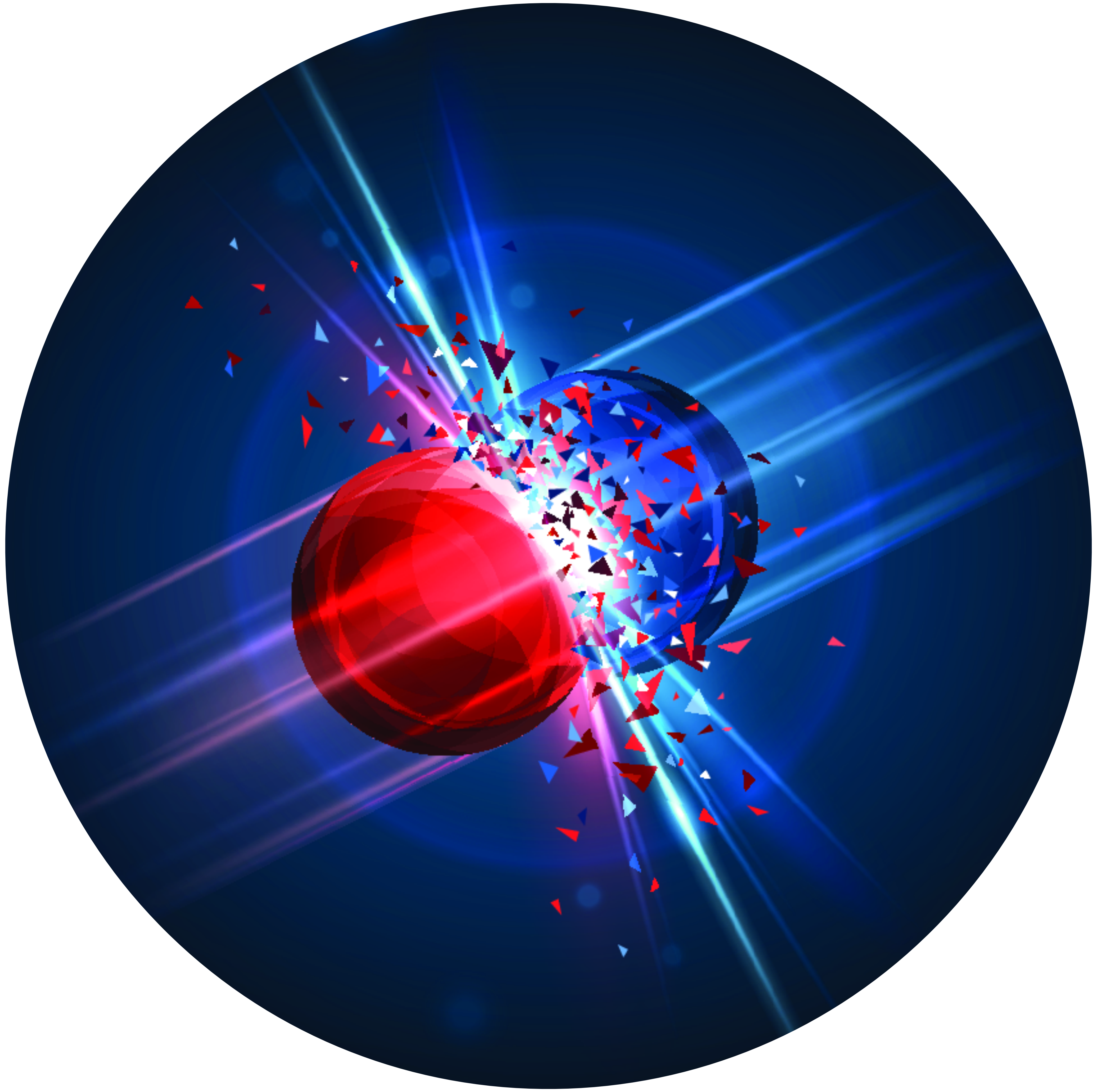
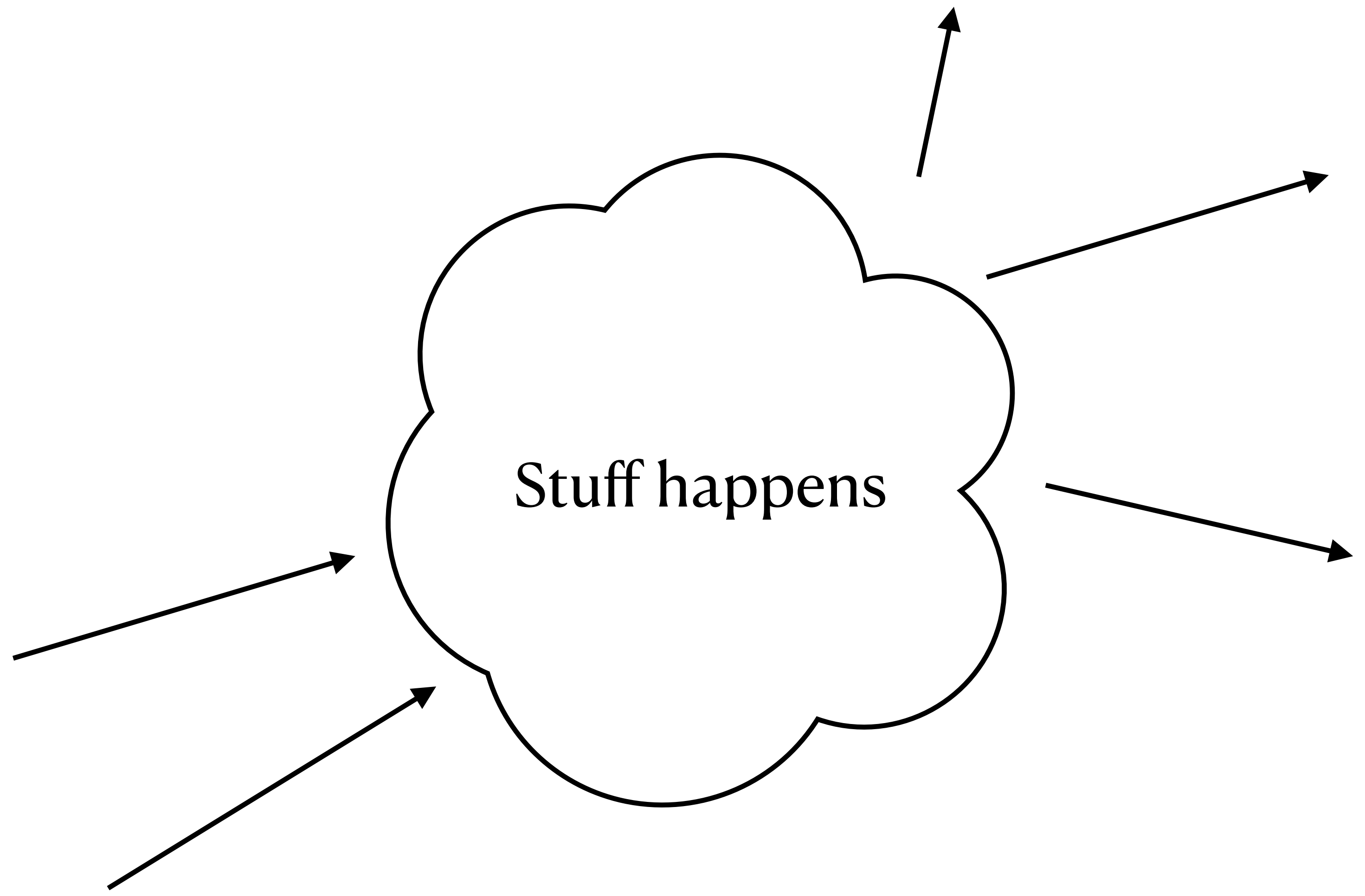


Geometry of Scattering Amplitudes

Andreas Helset, CERN

Scattering Amplitudes



Scattering matrix, "S-matrix"

$$\hat{S}: \quad |\psi_{\text{in}}\rangle \to |\psi_{\text{out}}\rangle = \hat{S}|\psi_{\text{in}}\rangle$$

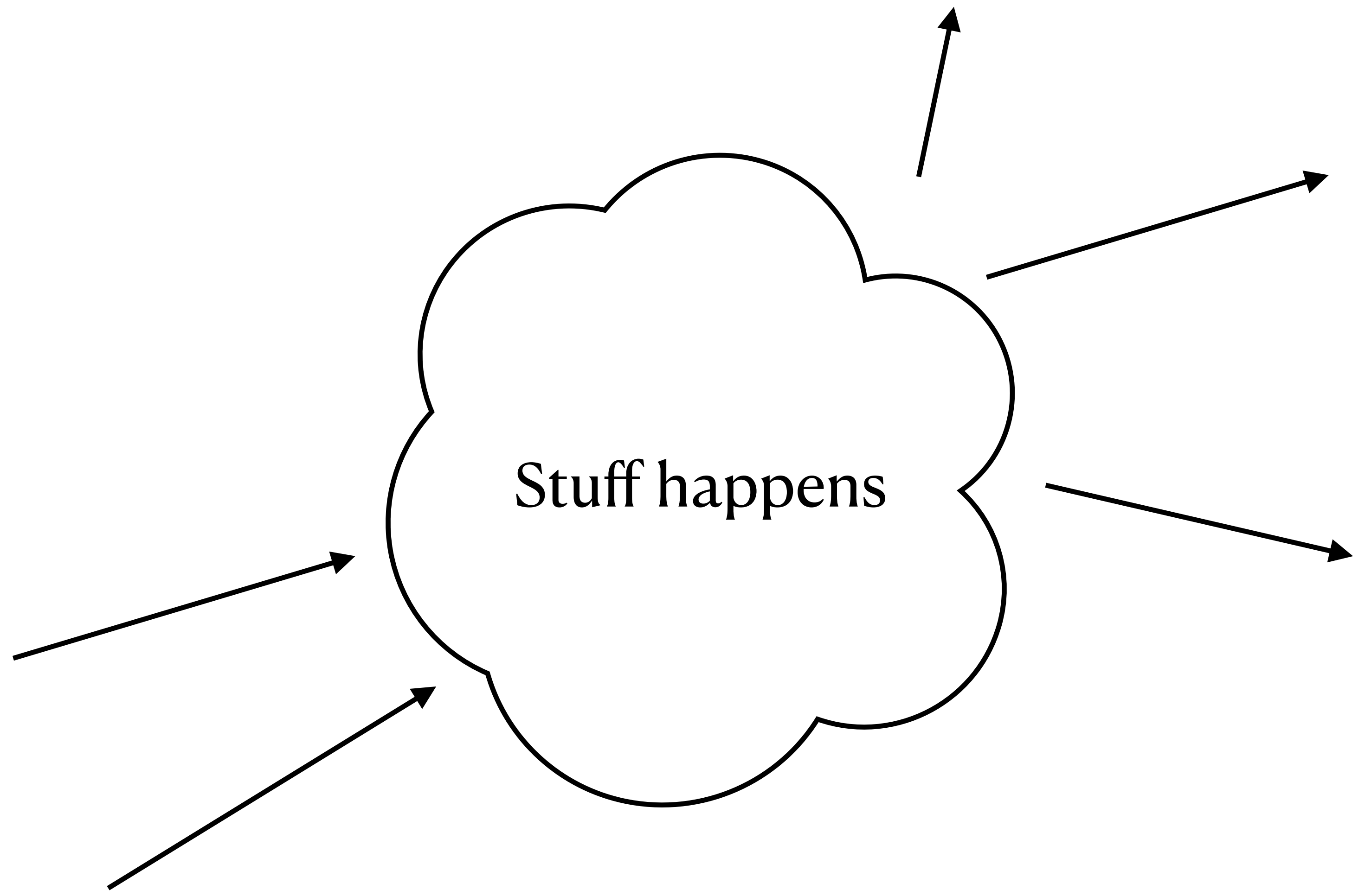
Key object for collider experiments

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{A}|^2$$

Unitarity

$$\hat{S}\hat{S}^\dagger = 1$$

Scattering Amplitudes



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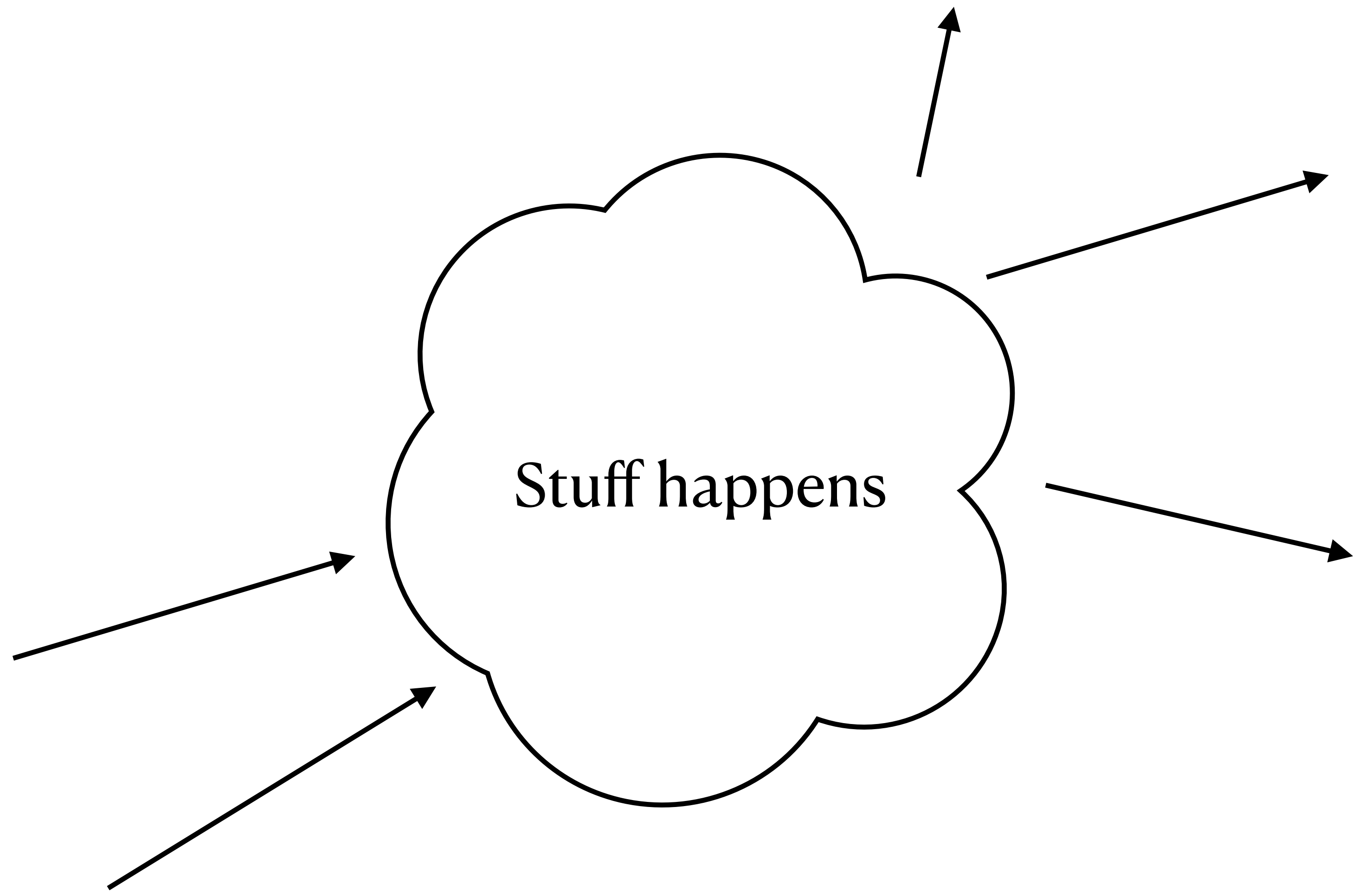
$$\mathcal{A}_3^s = x^2 \langle \mathbf{21} \rangle^{2s}$$

Arkani-Hamed, Huang, Huang ‘17

Scattering Amplitudes



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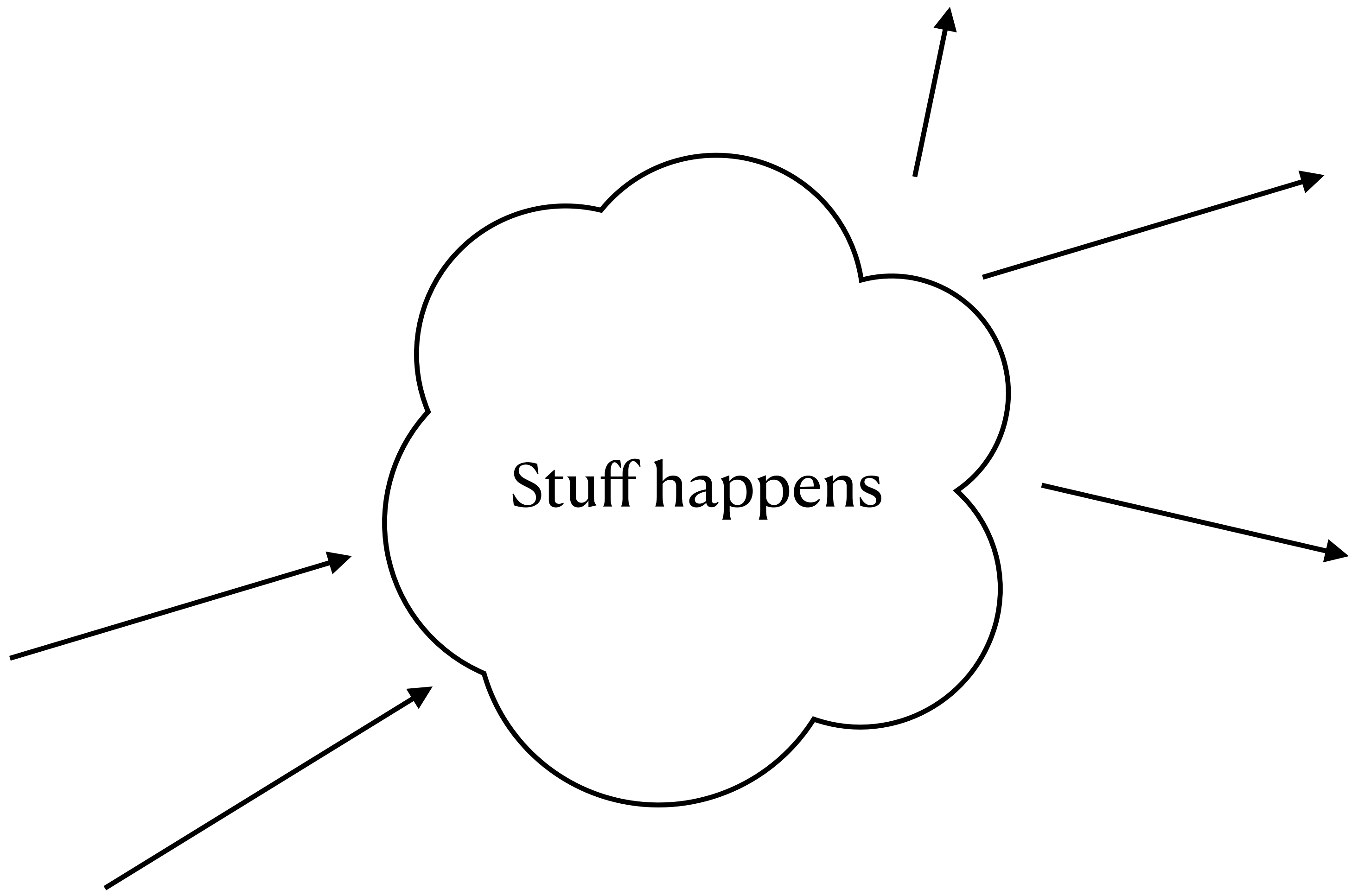
$$\lim_{q \to 0} \mathcal{A}_{n+1} = C \cdot \mathcal{A}_n$$

e.g. Adler '65

Scattering Amplitudes



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Hidden structure!

$$\mathcal{M}_4^{\text{GR}} = -is\mathcal{A}_4^{\text{YM}}(1,2,3,4)\mathcal{A}_4^{\text{YM}}(1,2,4,3)$$

Kawai, Lewellen, Tye ’86

Scattering Amplitudes

Surprising simplicity!

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}(\partial_\mu \phi^i \partial^\mu \phi^i) + \lambda_{3,ijk}\phi^k(\partial_\mu \phi^i \partial^\mu \phi^j) + \lambda_{4,ijkl}\phi^k \phi^l(\partial_\mu \phi^i \partial^\mu \phi^j) + \dots \\
&= \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j)
\end{aligned}
$$

Honerkamp, Meetz, … ‘70s

Scattering Amplitudes

Surprising simplicity!

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i \partial^\mu \phi^i) + \lambda_{3,ijk}\phi^k(\partial_\mu \phi^i \partial^\mu \phi^j) + \lambda_{4,ijkl}\phi^k \phi^l(\partial_\mu \phi^i \partial^\mu \phi^j) + \dots$$
$$= \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j)$$

Field redefinition $\phi^i \to \varphi^i(\phi)$

$$(\partial_\mu \phi)^i \to \left(\frac{\partial \varphi^i}{\partial \phi^j}\right)(\partial_\mu \phi)^j$$

$$g_{ij} \to \left(\frac{\partial \phi^k}{\partial \varphi^i}\right)\left(\frac{\partial \phi^l}{\partial \varphi^j}\right) g_{kl}$$

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Scattering Amplitudes

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Metric

Field redefinition $\phi^i \to \varphi^i(\phi)$

$$(\partial_\mu \phi)^i \to \left(\frac{\partial \varphi^i}{\partial \phi^j}\right)(\partial_\mu \phi)^j$$

$$g_{ij} \to \left(\frac{\partial \phi^k}{\partial \varphi^i}\right)\left(\frac{\partial \phi^l}{\partial \varphi^j}\right) g_{kl}$$

Christoffel symbol $\Gamma^i_{jk} = \frac{1}{2}g^{il}(g_{lk,j} + g_{jl,k} - g_{jk,l})$

Riemann curvature $R^i{}_{jkl} = \Gamma^i_{lj,k} + \Gamma^i_{kn}\Gamma^n_{lj} - (k \leftrightarrow l)$

Honerkamp, Meetz, ... '70s

Scattering Amplitudes

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Feynman diagrams

$$\mathcal{A}_4 = (\partial g)(p \cdot p')\frac{1}{s_{ij}}(\partial g)(p \cdot p') + \cdots + (\partial^2 g)(p_i \cdot p_j)$$

Scattering Amplitudes

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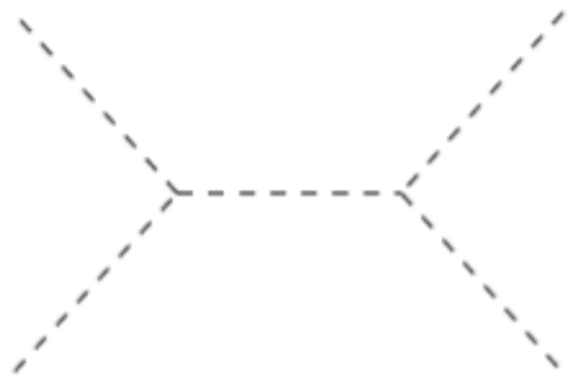

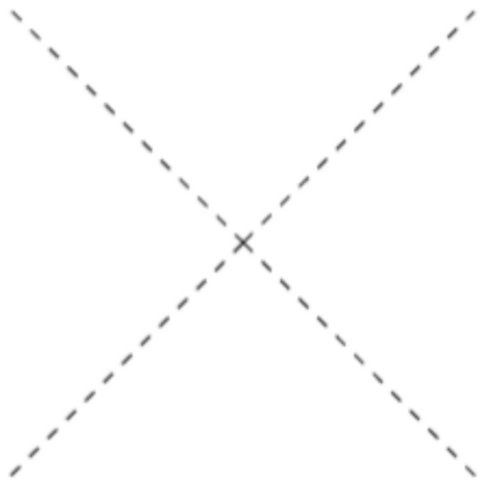

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Geometric amplitude

$$\mathcal{A}_4 = R_{ikjl}s_{34} + R_{ijkl}s_{24}$$

$$s_{ij} = (p_i + p_j)^2$$

Volkov '73

Scattering Amplitudes

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$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i \partial^\mu \phi^i) + \lambda_{3,ijk}\phi^k(\partial_\mu \phi^i \partial^\mu \phi^j) + \lambda_{4,ijkl}\phi^k \phi^l(\partial_\mu \phi^i \partial^\mu \phi^j) + \ldots$$
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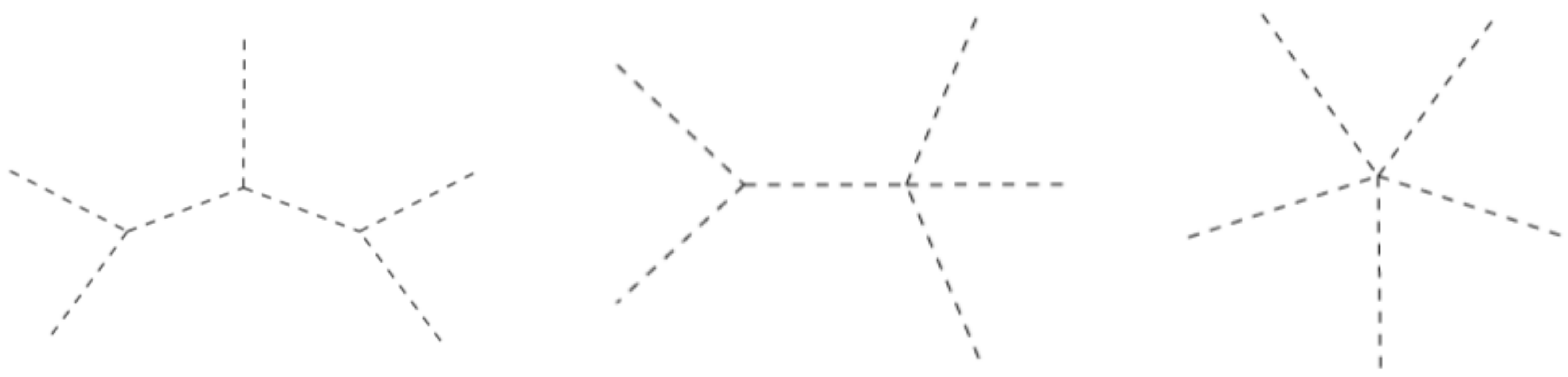

Geometric amplitude

$$\mathcal{A}_5 = \nabla_k R_{iljm} s_{45} + \nabla_l R_{ikjm} s_{35} + \nabla_l R_{ijkm} s_{25} + \nabla_m R_{ikjl} s_{34} + \nabla_m R_{ijkl}(s_{24} + s_{45})$$

Volkov ‘73

Scattering Amplitudes

Universal relations!

Geometric soft theorem

$$\lim_{q\to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$$

Cheung, AH, Parra-Martinez '21

Scattering Amplitudes

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Scattering Amplitudes

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$$\begin{aligned}\mathcal{A}_5 =& \nabla_k R_{iljm} s_{45} + \nabla_l R_{ikjm} s_{35} + \nabla_l R_{ijkm} s_{25} \\ &+ \nabla_m R_{ikjl} s_{34} + \nabla_m R_{ijkl}(s_{24} + s_{45})\end{aligned} \xrightarrow{p_5 \to 0} \nabla_m \mathcal{A}_4 = \nabla_m \left(R_{ikjl} s_{34} + R_{ijkl} s_{24}\right)$$

Cheung, AH, Parra-Martinez ‘21

Scattering Amplitudes

Universal relations!

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + \frac{1}{4}\lambda_{ijkl}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j)(\partial_\mu \phi^k \partial^\mu \phi^l)$$

Geometric soft theorem

$$\lim_{q \to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$$

$$\mathcal{A}_5^\lambda = \frac{1}{2} s_{12} s_{34} \nabla_{i_5} \lambda_{i_1 i_2 i_3 i_4} + \frac{1}{2} s_{13} s_{24} \nabla_{i_5} \lambda_{i_1 i_3 i_2 i_4} + \frac{1}{2} s_{14} s_{23} \nabla_{i_5} \lambda_{i_1 i_4 i_2 i_3} + \ldots$$

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Cheung, AH, Parra-Martinez '21

Scattering Amplitudes

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$$\Big\downarrow \; p_5 \to 0$$

$$\nabla_{i_5} \mathcal{A}_4^\lambda = \nabla_{i_5} \left(\frac{1}{2} s_{12} s_{34} \lambda_{i_1 i_2 i_3 i_4} + \frac{1}{2} s_{13} s_{24} \lambda_{i_1 i_3 i_2 i_4} + \frac{1}{2} s_{14} s_{23} \lambda_{i_1 i_4 i_2 i_3} \right)$$

Cheung, AH, Parra-Martinez '21

Scattering Amplitudes

Universal relations!

Geometric soft theorem (with a potential)

$$\lim_{q\to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n + \sum_{a=1}^{n} \frac{\nabla_i (m_a^2)_{i_a}{}^{j_a}}{(p_a+q)^2 - m_a^2} \left(1 + q \cdot \partial_{p_a}\right) \mathcal{A}_n$$

Cheung, AH, Parra-Martinez '21

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Stay on the mass shell

$$\nabla_i \left(p_b^2 - m_b^2\right) = -\nabla_i m_b^2$$

$$\sum_{a=1}^{n} \frac{\nabla_i (m_a^2)}{(p_a+q)^2 - m_a^2} (1 + q\cdot p_a) \left(p_b^2 - m_b^2\right) = +\nabla_i m_b^2$$

Cheung, AH, Parra-Martinez ‘21

Scattering Amplitudes

Universal relations!

On-shell recursion relations

Complex deformation

$$p_a \to p_a(1 - zc_a)$$

$$F_n(z) = \prod_{a=1}^{n}(1 - zc_a)$$

$$\mathcal{A}_n(0) = \frac{1}{2\pi i}\oint \frac{dz}{z}\frac{\mathcal{A}_n(z)}{F_n(z)}$$

Britto, Cachazo, Feng, Witten '05

Cheung et.al. '15

Cheung, AH, Parra-Martinez '21

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Factorization

Soft limit

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Soft limit

$$\sum_a \text{Res}_{z=1/c_a}\left(\frac{\mathcal{A}_n(z)}{zF_n(z)}\right) = \frac{\nabla_{i_a}\mathcal{A}_{n-1}(1/c_a)}{\prod_{b\neq a}(1 - c_b/c_a)}$$

Britto, Cachazo, Feng, Witten '05

Cheung et.al. '15

Cheung, AH, Parra-Martinez '21

Scattering Amplitudes

Hidden structure!

FIG. 1. Trivalent graph for four particles.

Yang-Mills theory

$$\mathcal{A}_4 = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

Bern, Carrasco, Johansson '08

Scattering Amplitudes

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Color factor

$$c_s = f_{a_1 a_2 b} f_{b a_3 a_4}$$

Jacobi identity

$$c_s + c_t + c_u = 0$$

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$$\begin{aligned} -2n_s = &\left[(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2)\epsilon_2^\mu - (1 \leftrightarrow 2)\right]\left[(\epsilon_3 \cdot \epsilon_4) p_3^\mu + 2(\epsilon_3 \cdot p_4)\epsilon_4^\mu - (3 \leftrightarrow 4)\right] \\ &+ s\left[(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3)\right] \end{aligned}$$

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Gravity

$$\mathcal{M}_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

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Nonlinear sigma model (symmetric coset)

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Bern et.al. '19

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Special Galileon amplitude

$$\mathcal{M}_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

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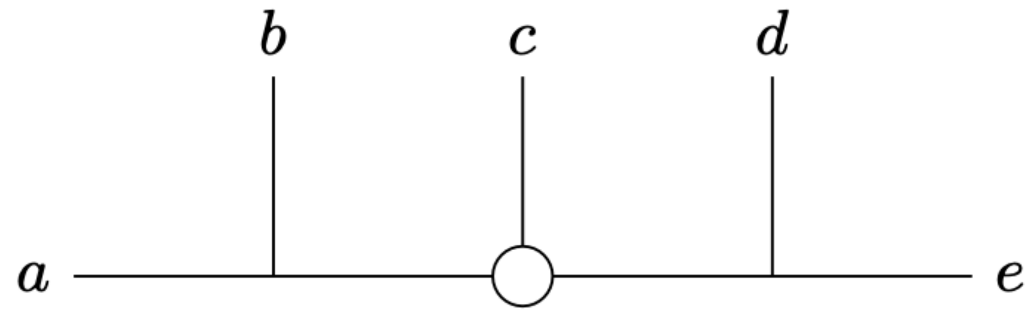

FIG. 2. Trivalent graph for five particles.

Nonlinear sigma model (symmetric coset)

$$\mathcal{A}_5 = \frac{n_1 c_1}{s_{12} s_{45}} + \ldots$$

Bern et.al. '19

Scattering Amplitudes

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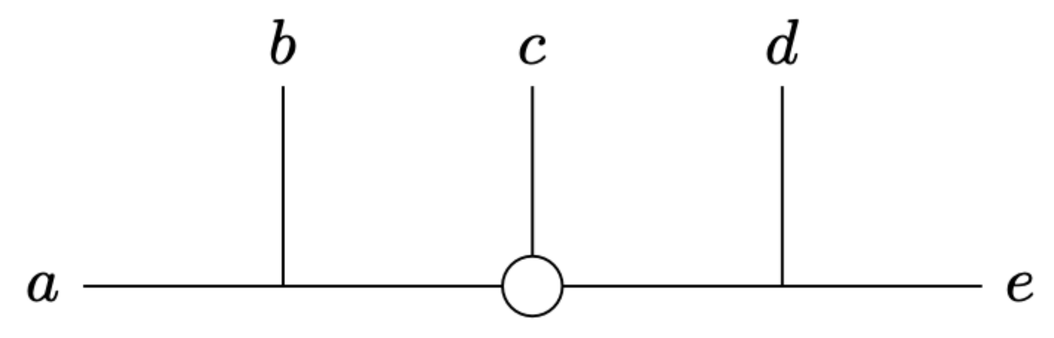

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Jacobi identity

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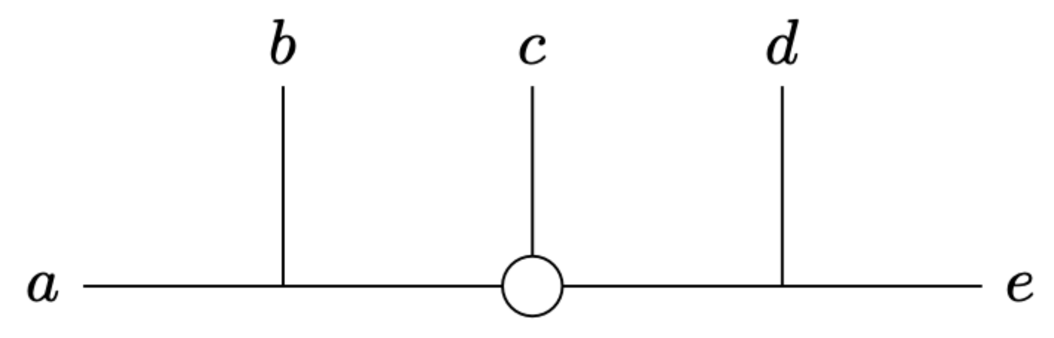

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Bern et.al. ‘19

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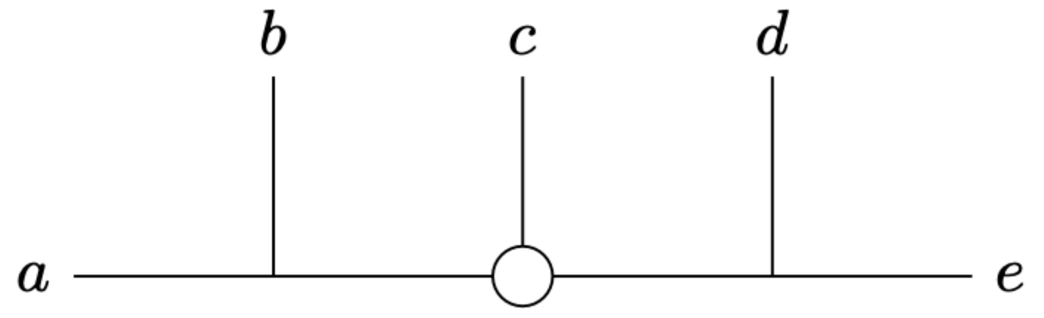

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Scattering Amplitudes

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Scattering Amplitudes

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Curvature factor

$$c_i = R_{abcd} \sim f_{abx} f^x{}_{cd}$$

Jacobi identity

$$c_s + c_t + c_u = 0$$

AH '24

FIG. 1. Trivalent graph for four particles.

Scattering Amplitudes

Hidden structure!

FIG. 1. Trivalent graph for four particles.

Geometric amplitude

$$\mathcal{A}_4 = R_{ikjl} s_{34} + R_{ijkl} s_{24}$$

Nonlinear sigma model (nonsymmetric coset)

$$\mathcal{A}_4 = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

Curvature factor

$$c_i = R_{abcd} \sim f_{abx} f^{x}{}_{cd}$$

$$n_s = s_{12}(s_{12} + 2 s_{23})$$

Jacobi identity

$$c_s + c_t + c_u = 0$$

Kinematic 'Jacobi identity'

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AH '24

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Special Galileon amplitude

$$\mathcal{M}_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Curvature factor

$$c_i = R_{abcd} \sim f_{abx} f^x{}_{cd}$$

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Curvature-kinematics duality

AH '24

Scattering Amplitudes

Hidden structure!

Geometric amplitude

$$\mathcal{A}_5 = \nabla_k R_{iljm} s_{45} + \nabla_l R_{ikjm} s_{35} + \nabla_l R_{ijkm} s_{25} + \nabla_m R_{ikjl} s_{34} + \nabla_m R_{ijkl}(s_{24} + s_{45})$$

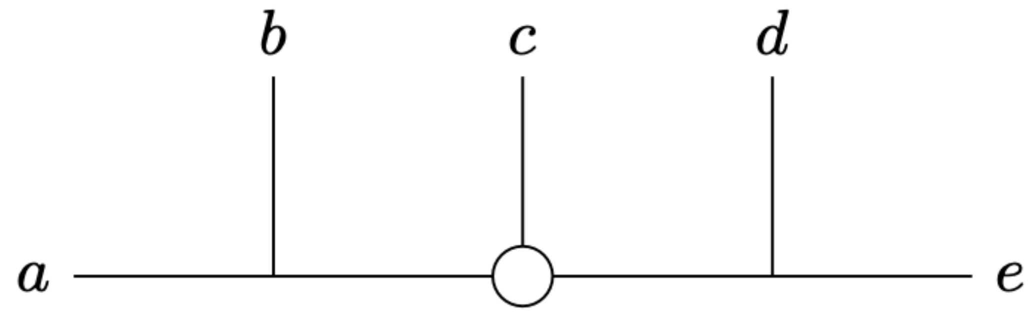

FIG. 2. Trivalent graph for five particles.

Scattering Amplitudes

Hidden structure!

Geometric amplitude

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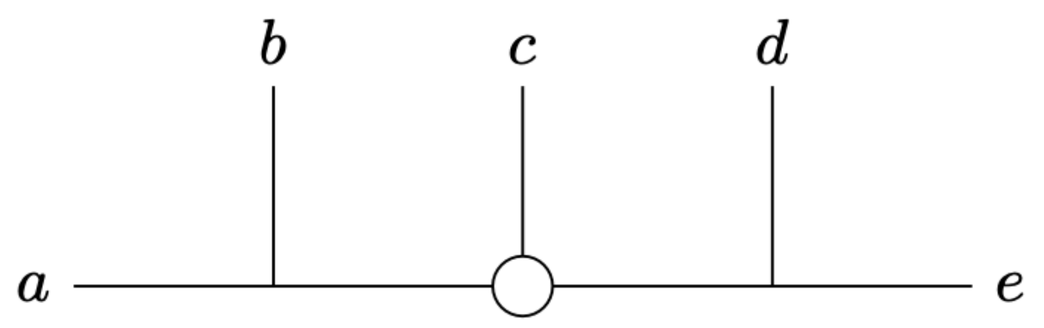


FIG. 2. Trivalent graph for five particles.

Nonlinear sigma model (nonsymmetric coset)

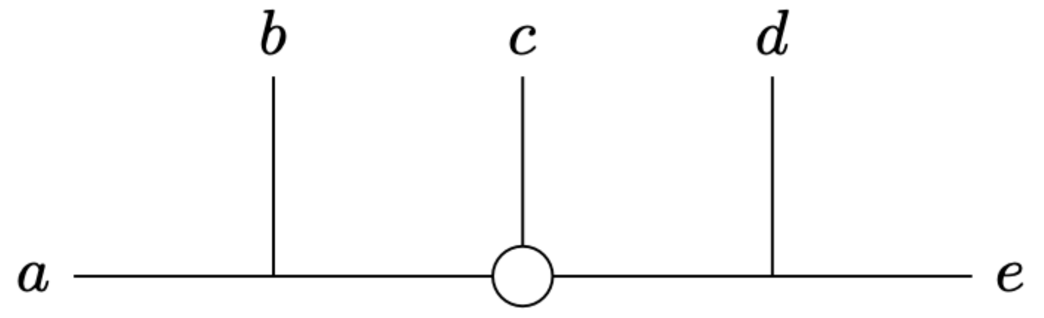
$$\mathcal{A}_5 = \frac{n_1 c_1}{s_{12} s_{45}} + \dots$$

Scattering Amplitudes

Hidden structure!

Geometric amplitude

$$\mathcal{A}_5 = \nabla_k R_{iljm} s_{45} + \nabla_l R_{ikjm} s_{35} + \nabla_l R_{ijkm} s_{25} + \nabla_m R_{ikjl} s_{34} + \nabla_m R_{ijkl}(s_{24} + s_{45})$$



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Nonlinear sigma model (nonsymmetric coset)

$$\mathcal{A}_5 = \frac{n_1 c_1}{s_{12} s_{45}} + \dots$$

Curvature factor

$$c_i = \nabla_c R_{abde} \neq f_{abx} f^x{}_{cy} f^y{}_{de}$$

Symmetry property

$$c(a, b, c, d, e) = +c(e, d, c, b, a)$$

AH '24

Scattering Amplitudes

Hidden structure!

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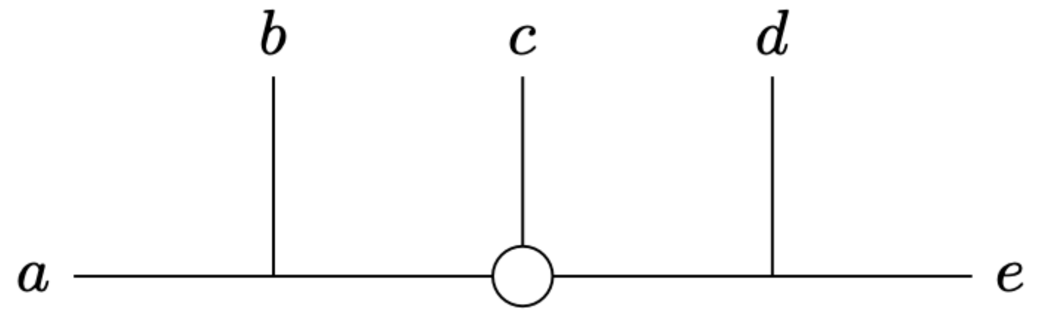

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General Galileon amplitude

$$\mathcal{M}_5 = \sum_{i \in \Gamma} \frac{n_i^2}{d_i} = \left(\sum_{a<b} s_{ab}^4\right) - \frac{1}{4}\left(\sum_{a<b} s_{ab}^2\right)^2$$

Curvature factor

$$c_i = \nabla_c R_{abde} \neq f_{abx} f^x{}_{cy} f^y{}_{de}$$

Symmetry property

$$c(a,b,c,d,e) = +c(e,d,c,b,a)$$

Scattering Amplitudes

Hidden structure!

Geometric amplitude

$$\mathcal{A}_5 = \nabla_k R_{iljm} s_{45} + \nabla_l R_{ikjm} s_{35} + \nabla_l R_{ijkm} s_{25} + \nabla_m R_{ikjl} s_{34} + \nabla_m R_{ijkl}(s_{24} + s_{45})$$

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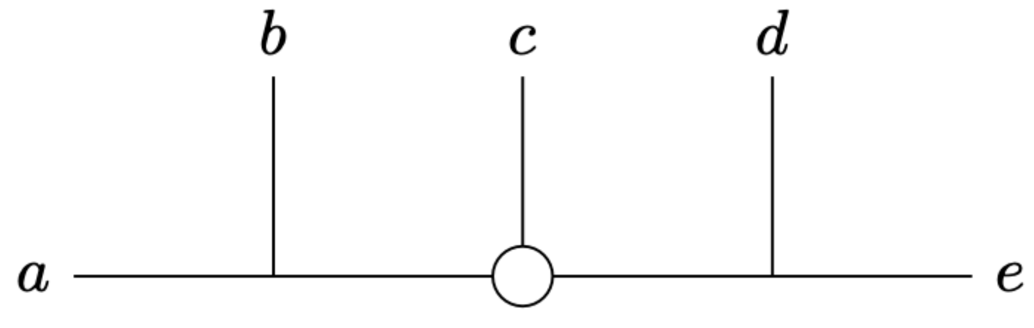

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Curvature-kinematics duality

AH '24

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

Assi, AH, Manohar, Pages, Shen '23

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

Field redefinition $\psi^p \to R^p{}_r(\phi)\psi^r$

$$k_{\bar{p}r} \to \left[(R^\dagger)^{-1} k R^{-1}\right]_{\bar{p}r}$$

$$\omega_{\bar{p}rI} \to \left[(R^\dagger)^{-1} \omega R^{-1}\right]_{\bar{p}rI} + \frac{1}{2}\left[(R^\dagger)^{-1} k (\partial_I R^{-1})\right]_{\bar{p}r} - \frac{1}{2}\left[(\partial_I (R^\dagger)^{-1}) k R^{-1}\right]_{\bar{p}r}$$

We need both terms to define the geometry

Assi, AH, Manohar, Pages, Shen '23

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

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We need both terms to define the geometry

'Riemann normal coordinates'

$$k_{\bar{p}r} = \delta_{\bar{p}r}$$
$$\omega_{\bar{p}ri}(v) = 0$$

Assi, AH, Manohar, Pages, Shen '23

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

Curvature for scalar-fermion field space

$$\bar{R}_{\bar{p}rij} = \omega_{\bar{p}rj,i} + \left[\omega^- \omega^+\right]_{\bar{p}irj} - (i \leftrightarrow j)$$

$$\omega^{\pm}_{\bar{p}ri} = \omega_{\bar{p}ri} \pm \frac{1}{2} k_{\bar{p}r,i}$$

Assi, AH, Manohar, Pages, Shen '23

Scattering Amplitudes

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$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

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Surprising structure!

$$\mathcal{A}_4 = (\bar{u} \not{p}_j u) \bar{R}_{\bar{p}rij}$$

$$\mathcal{A}_5 = (\bar{u} \not{p}_j u) \nabla_k \bar{R}_{\bar{p}rij} + (\bar{u} \not{p}_k u) \nabla_j \bar{R}_{\bar{p}rik}$$

Assi, AH, Manohar, Pages, Shen ‘23

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overset{\leftrightarrow}{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

Geometric soft theorem

$$\lim_{q \to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$$

Derda, AH, Parra-Martinez ‘24

Scattering Amplitudes

Geometry with fermions

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i \partial^\mu \phi^j) + i k_{\bar{p}r}(\phi) \bar{\psi}^{\bar{p}} \overleftrightarrow{\partial}_\mu \gamma^\mu \psi^r + i \omega_{\bar{p}ri}(\phi)(\partial_\mu \phi^i) \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r$$

Geometric soft theorem

$$\lim_{q \to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$$

$$\mathcal{A}_5 = (\bar{u} \not{p}_j u) \nabla_k \bar{R}_{\bar{p}rij} + (\bar{u} \not{p}_k u) \nabla_j \bar{R}_{\bar{p}rik}$$

$$\mathcal{A}_4 = (\bar{u} \not{p}_j u) \bar{R}_{\bar{p}rij}$$

Derda, AH, Parra-Martinez '24

Scattering Amplitudes

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Geometric soft theorem

$$\lim_{q \to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$$

$$p_k \to 0$$

$$\mathcal{A}_5 = (\bar{u}\not{p}_j u)\nabla_k \bar{R}_{\bar{p}rij} + (\bar{u}\not{p}_k u)\cancel{\nabla_j} \bar{R}_{\bar{p}rik} \longrightarrow \nabla_k \mathcal{A}_4 = \nabla_k \Big((\bar{u}\not{p}_j u)\bar{R}_{\bar{p}rij} \Big)$$

Derda, AH, Parra-Martinez '24

Scattering Amplitudes

Geometry with fermions

Geometric soft theorem (with potentials and Yukawa interactions)

$$\lim_{q\to 0}\mathcal{A}_{n+1} = \nabla_i\mathcal{A}_n + \sum_{a\in\{\text{scalars}\}}\frac{\nabla_i m_a^2}{(p_a+q)^2-m_a^2}(1+q\cdot\partial_{p_a})\mathcal{A}_n$$

$$+ \sum_{b\in\{\text{fermions}\}}\sum_{\text{spin}}\frac{\nabla_i M_b(\bar{u}u)}{(p_b+q)^2-m_b^2}(1+q\cdot\partial_{p_b})\mathcal{A}_n$$

$$+ \sum_{b\in\{\text{anti-fermions}\}}\sum_{\text{spin}}\frac{\nabla_i M_b(-\bar{v}v)}{(p_b+q)^2-m_b^2}(1+q\cdot\partial_{p_b})\mathcal{A}_n$$

Needed to stay on the mass shell

Derda, AH, Parra-Martinez '24

Scattering Amplitudes

Double soft theorem

Geometric double soft theorem (only scalars)

$$\lim_{q_1,q_2\to 0} \mathcal{A}_{n+2} = \nabla_{(i_1}\nabla_{i_2)}\mathcal{A}_n + \frac{1}{2}\sum_a \frac{p_a\cdot(q_1-q_2)}{p_a\cdot(q_1+q_2)} R_{i_1 i_2 i_a}{}^{j_a}\mathcal{A}_{n,\ldots j_a\ldots}$$

Derda, AH, Parra-Martinez '24

Scattering Amplitudes

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Geometric double soft theorem (scalars & fermions)

$$\lim_{q_1,q_2\to 0} \mathcal{A}_{n+2} = \nabla_{(i_1}\nabla_{i_2)}\mathcal{A}_n + \frac{1}{2}\sum_a \frac{p_a\cdot(q_1-q_2)}{p_a\cdot(q_1+q_2)} \bar{R}_{i_1 i_2 \alpha_a}{}^{\beta_a}\mathcal{A}_{n,\ldots\beta_a\ldots}$$

Derda, AH, Parra-Martinez ‘24

Scattering Amplitudes

Double soft theorem

Geometric double soft theorem (two soft scalars)

$$\lim_{q_1, q_2 \to 0} \mathcal{A}_{n+2} = \nabla_{(i_1} \nabla_{i_2)} \mathcal{A}_n + \frac{1}{2} \sum_a \frac{p_a \cdot (q_1 - q_2)}{p_a \cdot (q_1 + q_2)} \bar{R}_{i_1 i_2 \alpha_a}{}^{\beta_a} \mathcal{A}_{n, \ldots \beta_a \ldots}$$

Derda, AH, Parra-Martinez ‘24

Scattering Amplitudes

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$$\lim_{q_1,q_2\to 0}\mathcal{A}_{n+2}=\nabla_{(i_1}\nabla_{i_2)}\mathcal{A}_n+\frac{1}{2}\sum_a \frac{p_a\cdot(q_1-q_2)}{p_a\cdot(q_1+q_2)}\bar{R}_{i_1 i_2 \alpha_a}{}^{\beta_a}\mathcal{A}_{n,\ldots\beta_a\ldots}$$

Geometric double soft theorem (two soft fermions)

$$\lim_{q_1,q_2\to 0}\mathcal{A}_{n+2}=\frac{1}{2}\left\{\lim_{q_1\to 0},\lim_{q_2\to 0}\right\}\mathcal{A}_{n+2}+\frac{1}{2}\sum_a \frac{[q_1|p_a|q_2\rangle}{p_a\cdot(p_1+p_2)}\bar{R}_{\bar{r}_1 r_2 \alpha_a}{}^{\beta_a}\mathcal{A}_{n,\ldots\beta_a\ldots}$$

Derda, AH, Parra-Martinez '24

Scattering Amplitudes

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Simple replacement

$$p_a\cdot(q_1-q_2)\to[q_1|p_a|q_2\rangle$$

Derda, AH, Parra-Martinez '24

Chen et.al. '14

Standard Model Effective Field Theory

Encode heavy new physics in effective operators

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \dots$$

X^3

Q_G	${}^6Q_{G^3}$	$f^{\mathscr{A}\mathscr{B}\mathscr{C}} G_\mu^{\mathscr{A}\nu} G_\nu^{\mathscr{B}\rho} G_\rho^{\mathscr{C}\mu}$
Q_W	${}^6Q_{W^3}$	$\epsilon^{abc} W_\mu^{a\,\nu} W_\nu^{b\,\rho} W_\rho^{c\,\mu}$

H^6

Q_H	${}^6Q_{H^6}$	$(H^\dagger H)^3$

H^4D^2

$Q_{H\Box}$	${}^6Q_{H^4\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	${}^6Q_{H^4D^2}$	$(D^\mu H^\dagger H)\,(H^\dagger D_\mu H)$

X^2H^2

Q_{HG}	${}^6Q_{G^2H^2}$	$(H^\dagger H)\, G^{\mathscr{A}}_{\mu\nu} G^{\mathscr{A}\mu\nu}$
Q_{HW}	${}^6Q_{W^2H^2}$	$(H^\dagger H)\, W^a_{\mu\nu} W^{a\mu\nu}$
Q_{HB}	${}^6Q_{B^2H^2}$	$(H^\dagger H)\, B_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	${}^6Q_{WBH^2}$	$(H^\dagger \tau^a H)\, W^a_{\mu\nu} B^{\mu\nu}$

Table 3: Bosonic even-parity dimension-six operators in the SMEFT. The first column is the notation of Ref. [26], and the second column is the notation used in this paper.

Standard Model Effective Field Theory

Renormalization Group Equations

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \ldots$$

1-loop UV divergence from boson loop

$$\Delta S = \frac{1}{32\pi\epsilon}\int d^4x \left\{ \frac{1}{12}\mathrm{Tr}[Y_{\mu\nu}Y^{\mu\nu}] + \frac{1}{2}\mathrm{Tr}[X^2] \right\}$$

AH, Jenkins, Manohar ‘22

Standard Model Effective Field Theory

Renormalization Group Equations

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Curvature shows up

$$Y_{\mu\nu} = R^i{}_{jkl}(D_\mu Z)^k(D_\nu Z)^l + F^a_{\mu\nu}\nabla_j t^i_a$$

AH, Jenkins, Manohar '22

Standard Model Effective Field Theory

Renormalization Group Equations

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \dots$$

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AH, Jenkins, Manohar ‘22

SMEFT RGE to dimension 8

$$\begin{aligned}
{}^8\dot{C}^{(1)}_{H^6D^2} =& -96\ {}^6C_{H^6}{}^6C_{H^4\Box} - 12\ {}^6C_{H^6}{}^6C_{H^4D^2} + \left(352\lambda + 20g_1^2 + \frac{20}{3}g_2^2\right)\left({}^6C_{H^4\Box}\right)^2 \\
&+ \left(-23\lambda + \frac{1}{8}g_1^2 + \frac{161}{24}g_2^2\right)\left({}^6C_{H^4D^2}\right)^2 + (-64\lambda - 2g_1^2 + 12g_2^2)\,{}^6C_{H^4\Box}{}^6C_{H^4D^2} \\
&- 22g_2^2\ {}^6C_{H^4\Box}{}^6C_{W^2H^2} + 6g_1^2\ {}^6C_{H^4\Box}{}^6C_{B^2H^2} - \frac{32}{3}g_1g_2\ {}^6C_{H^4\Box}{}^6C_{WBH^2} \\
&+ 8g_2^2\ {}^6C_{H^4D^2}{}^6C_{W^2H^2} + 6g_1^2\ {}^6C_{H^4D^2}{}^6C_{B^2H^2} + \frac{43}{3}g_1g_2\ {}^6C_{H^4D^2}{}^6C_{WBH^2} \\
&+ 512\lambda\left({}^6C_{G^2H^2}\right)^2 + (192\lambda + 4g_2^2)\left({}^6C_{W^2H^2}\right)^2 + (64\lambda + 12g_1^2)\left({}^6C_{B^2H^2}\right)^2 \\
&+ (-3g_1^2 - 3g_2^2)\left({}^6C_{WBH^2}\right)^2 + \frac{80}{3}g_1g_2\ {}^6C_{W^2H^2}{}^6C_{WBH^2} + \frac{8}{3}g_1g_2\ {}^6C_{B^2H^2}{}^6C_{WBH^2} \\
&+ \left(68\lambda + \frac{1}{2}g_1^2 - \frac{31}{6}g_2^2\right){}^8C^{(1)}_{H^6D^2} + \left(-8\lambda + 7g_1^2 + \frac{17}{3}g_2^2\right){}^8C^{(2)}_{H^6D^2}\,, \qquad \text{(C.23)}
\end{aligned}$$

Agree with Chala et.al. ‘21 Das Bakshi et.al. ‘22

Standard Model Effective Field Theory

Renormalization Group Equations

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \dots$$

1-loop UV divergence from fermion loop

$$\Delta S = \frac{1}{32\pi^2\epsilon}\int d^4x \left\{\frac{1}{3}\mathrm{Tr}[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}] + \mathrm{Tr}[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2]\right\}$$

Assi, AH, Manohar, Pages, Shen '23

Standard Model Effective Field Theory

Renormalization Group Equations

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Curvature shows up

$$\mathcal{Y}_{\mu\nu} = R^p{}_{rij}(D_\mu\phi)^i(D_\nu\phi)^j + F^a_{\mu\nu}(\nabla t_a)^p{}_r$$

General covariant derivatives

$$\mathcal{D}_\mu M_{\bar{p}r} = D_\mu M_{\bar{p}r} - \bar{\Gamma}^{\bar{s}}_{\bar{p}I}(D_\mu\phi)^I M_{\bar{s}r} - \bar{\Gamma}^s_{rI}(D_\mu\phi)^I M_{\bar{p}s}$$

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SMEFT RGE to dimension 8

$$\begin{aligned}
{}^8\dot{C}^{(1)}_{H^6D^2} = &\left(2g_1^2\,{}^6C_{H^4D^2} + \frac{16}{3}g_1g_2\,{}^6C_{WBH^2}\right)\kappa_1 \\
&+\left(-\frac{32}{3}g_2^2\,{}^6C_{H^4\Box} + \frac{2}{3}g_2^2\,{}^6C_{H^4D^2} + 8g_1g_2\,{}^6C_{WBH^2}\right)\kappa_2 \\
&+\left(8\,{}^6C_{H^4\Box} + {}^6C_{H^4D^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) \\
&+2g_1^2\kappa_1^{(8)} + \frac{10}{3}g_2^2\kappa_2^{(8)} + 2g_2^2\kappa_3 + \frac{8}{3}g_2^2\kappa_4 + \left(4g_1^2 - \frac{10}{3}g_2^2\right)\kappa_5 + \frac{1}{2}(-g_1^2 + 2g_2^2)\kappa_6 \\
&+2\kappa_8 - 6\kappa_9^{(8)} - 10\kappa_{10}^{(8)} - 2\kappa_{11}^{(8)} - 6\kappa_{12} + 6\kappa_{13} + 6\kappa_{14} + 10\kappa_{15} + 6\kappa_{16} + 10\kappa_{17} \\
&-2\kappa_{18} - \kappa_{19} + 4\kappa_{20} + \frac{20}{3}g_1g_2\tau_2' + \frac{20}{3}g_2^2\tau_3 - 8g_2\tau_{11} - 12g_1\tau_{12} - 6g_2\tau_{14}\,, \qquad \text{(B.104)}
\end{aligned}$$

Assi, AH, Manohar, Pages, Shen '23

Agree with Chala et.al. '21 Das Bakshi et.al. '22

Scattering Amplitudes

Geometry of field space

Surprising simplicity!	$\mathcal{A}_4 = R_{ikjl} s_{34} + R_{ijkl} s_{24}$
Universal relations!	$\lim_{q \to 0} \mathcal{A}_{n+1} = \nabla_i \mathcal{A}_n$
Hidden structure!	Double copy: Galileon theory = (NLSM)^2
Practical calculations	RGE for Standard Model Effective Field Theory

Thank you!