Hamiltonian Truncation and Effective Field Theory

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University of Geneva

based on work with Tim Cohen, Rachel Houtz, Markus Luty and Dorian Wenzel
arXiv: 2110.08273 and work in progress

Crossroads Between Theory and Phenomenology

Landscape of quantum field theory

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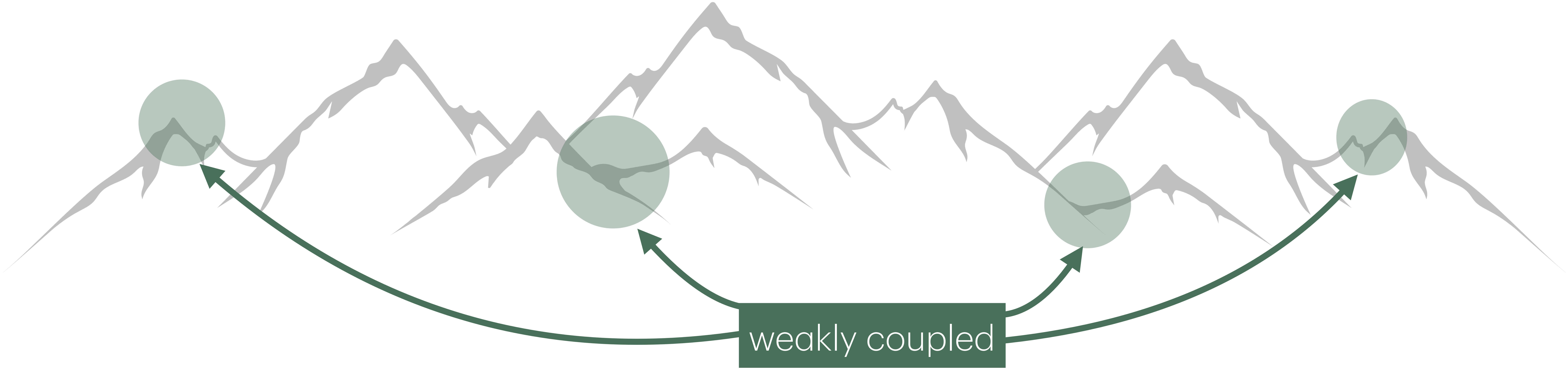
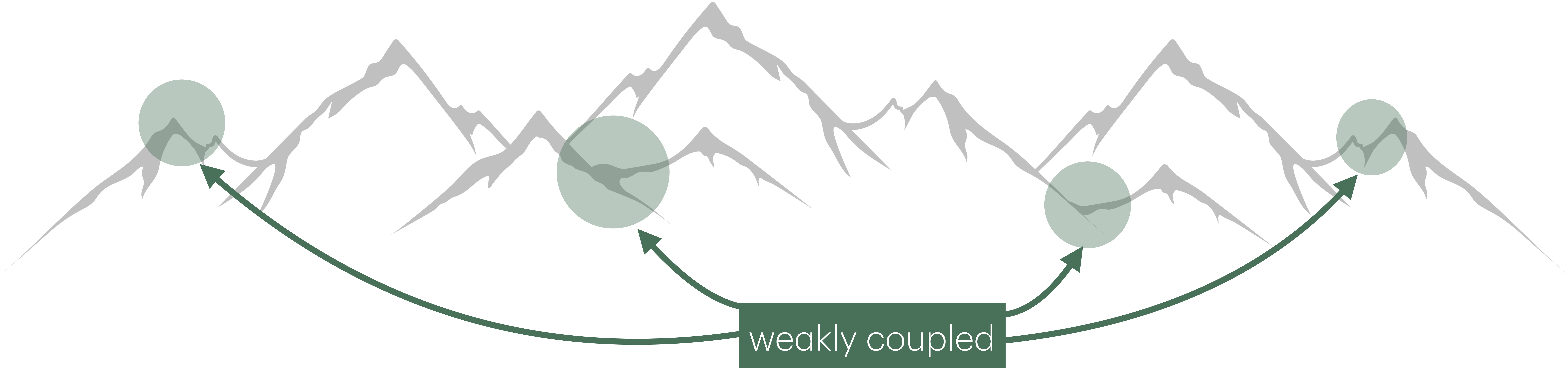


Landscape of quantum field theory

- QFT is nature's language
- vast landscape of possible QFTs
- powerful tools for weak coupling
- need **new equipment** to explore whole landscape



QFT at strong coupling

weak coupling perturbation theory

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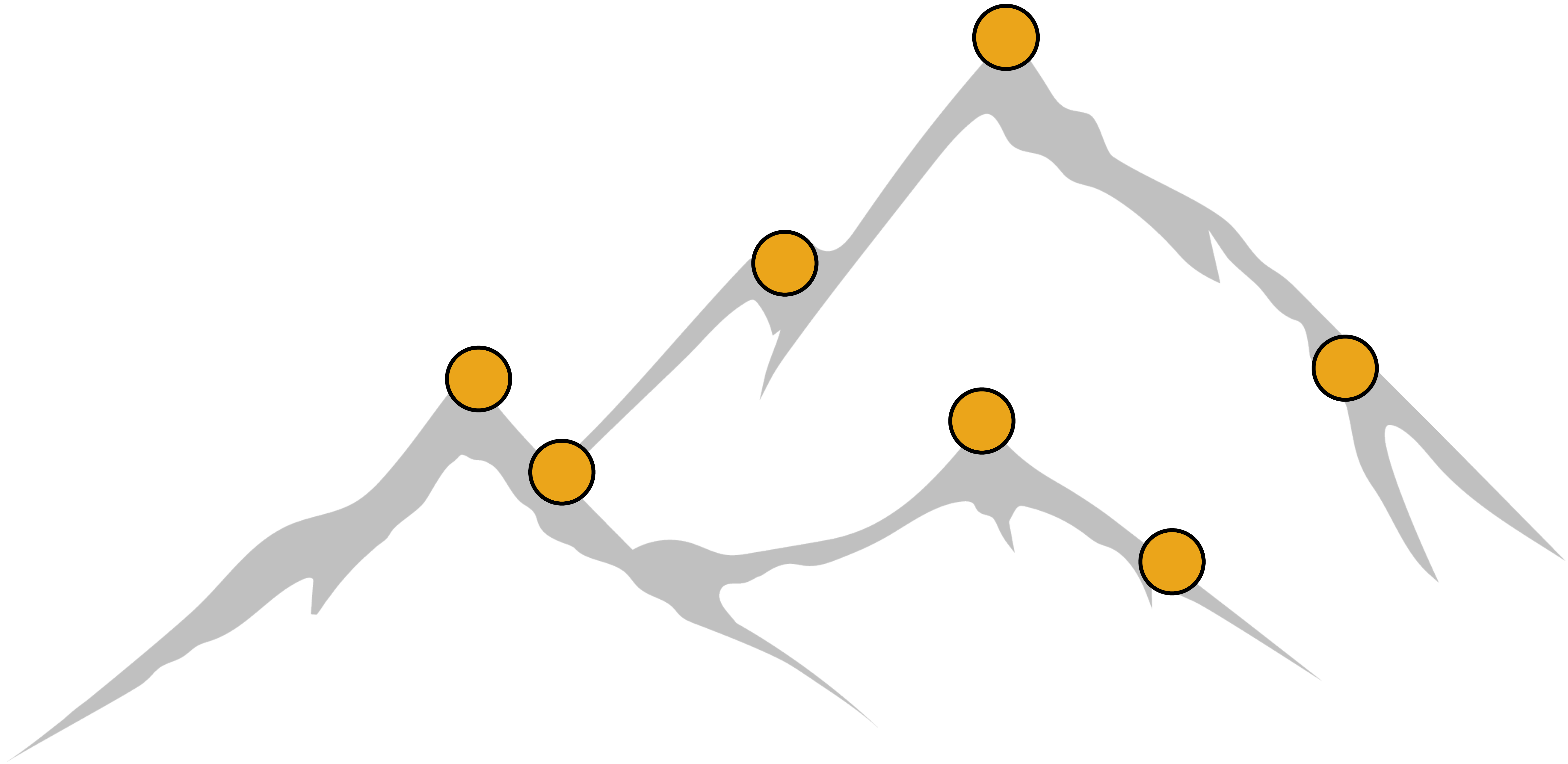

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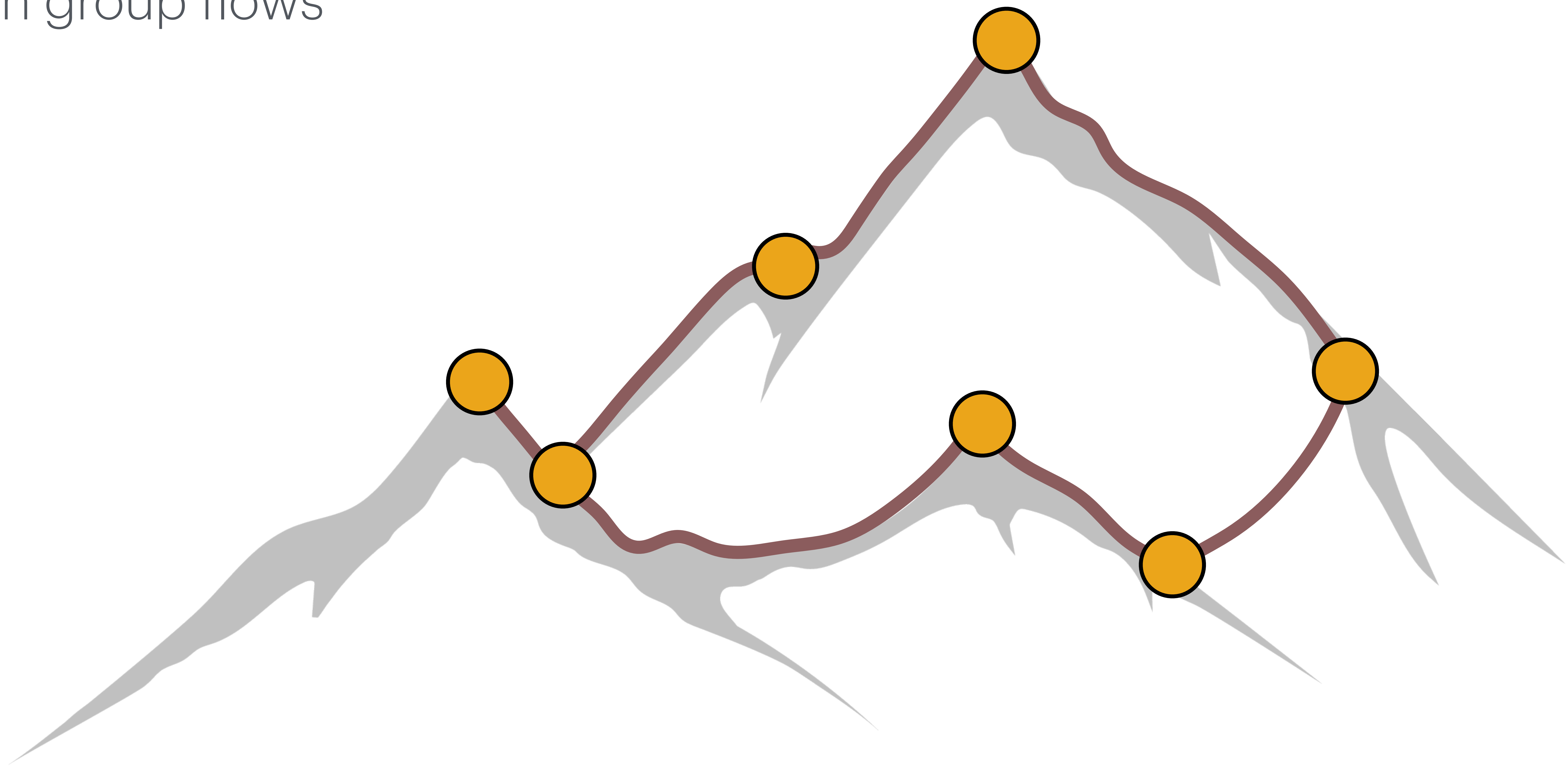

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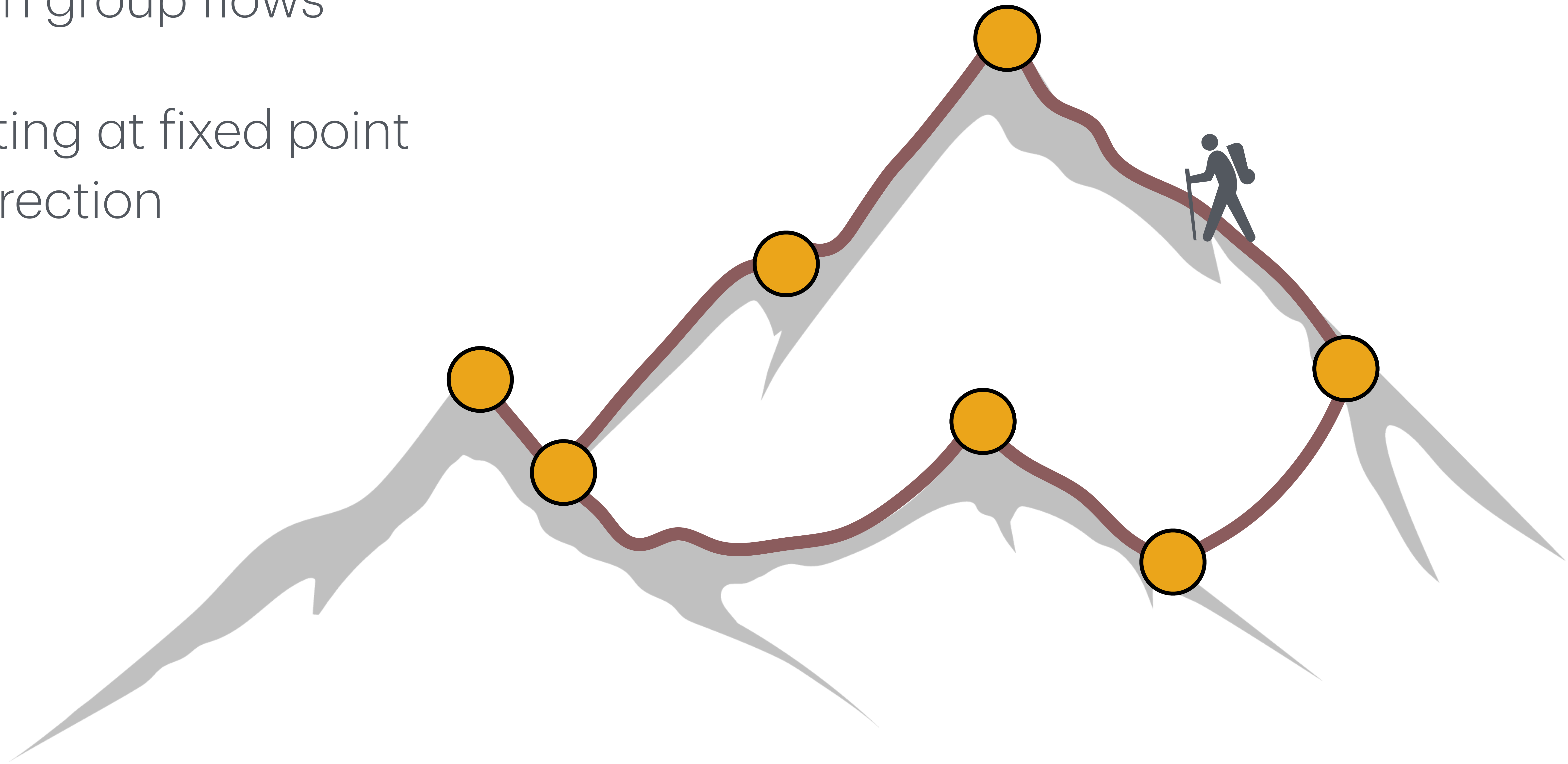

QFT = fixed points + flows

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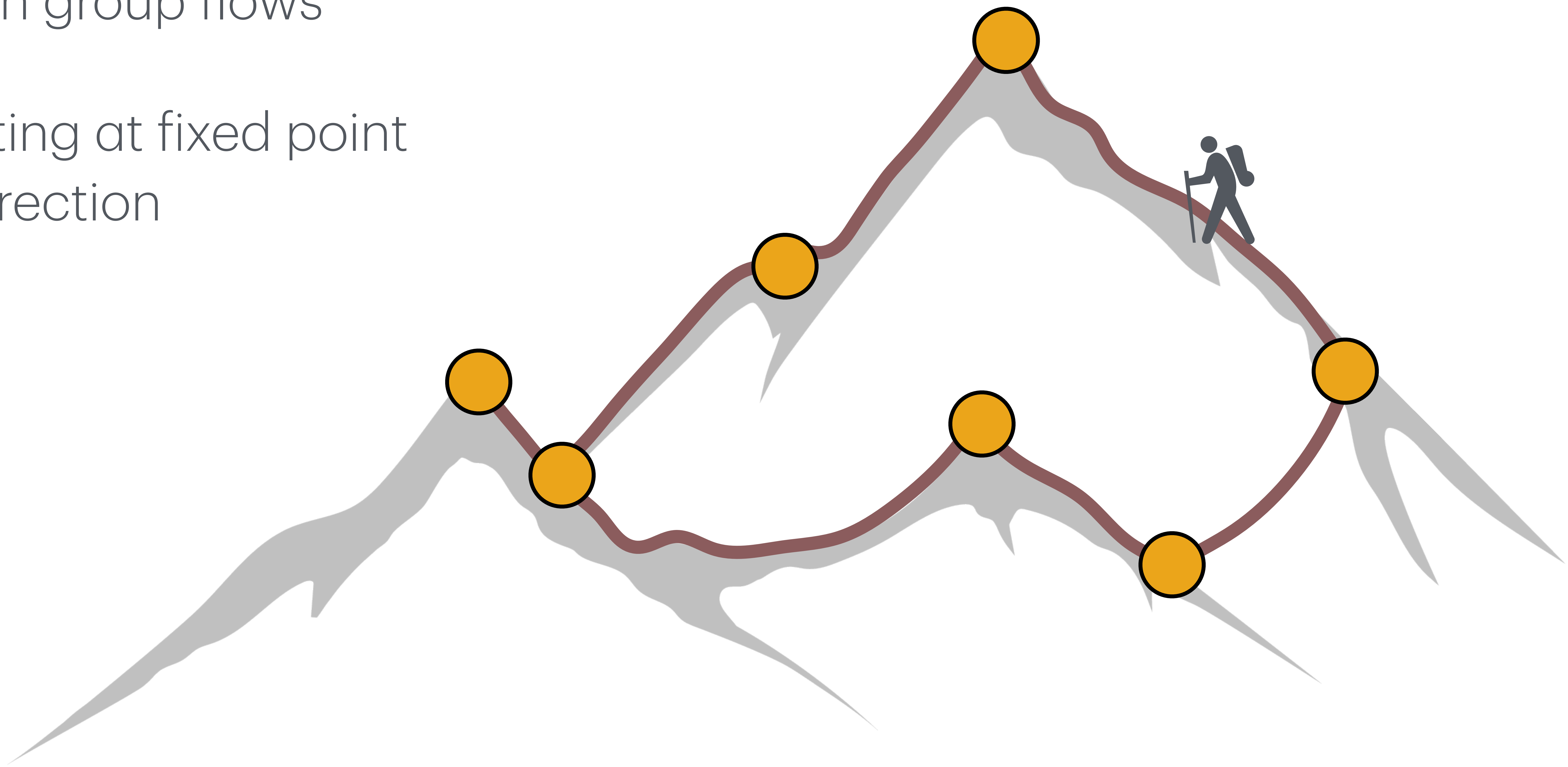
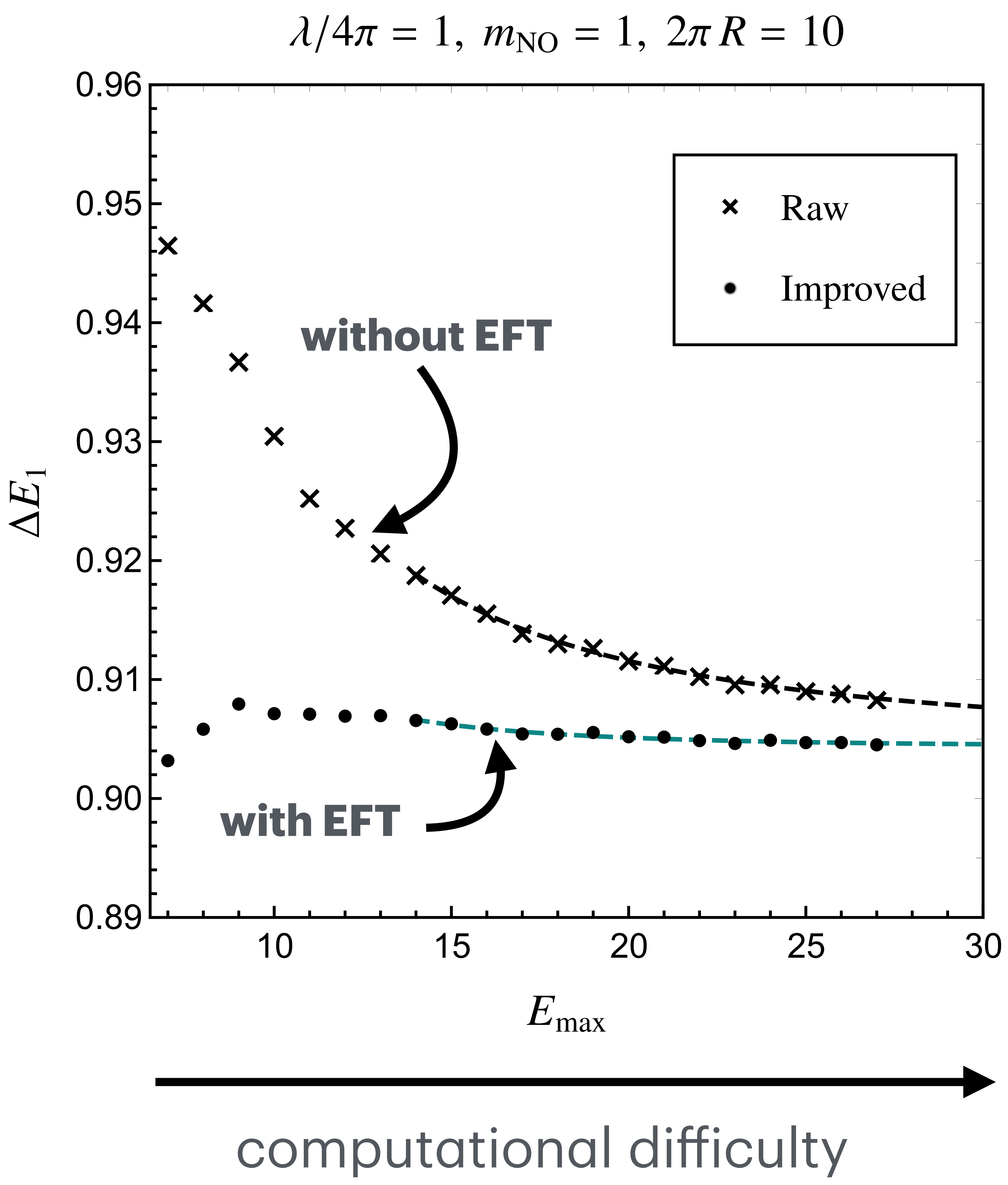
- can think of **any** QFT as starting at fixed point and flowing in a particular direction
- captures intuition
    - universality
    - relevant vs. irrelevant



Hamiltonian truncation solidifies this conceptual picture

Punchline

- Hamiltonian truncation = non-perturbative method for computing observables in strongly coupled QFTs
- effective field theory = powerful tool for compensating for ignorance
- effective field theory techniques **drastically improve** convergence in Hamiltonian truncation calculations



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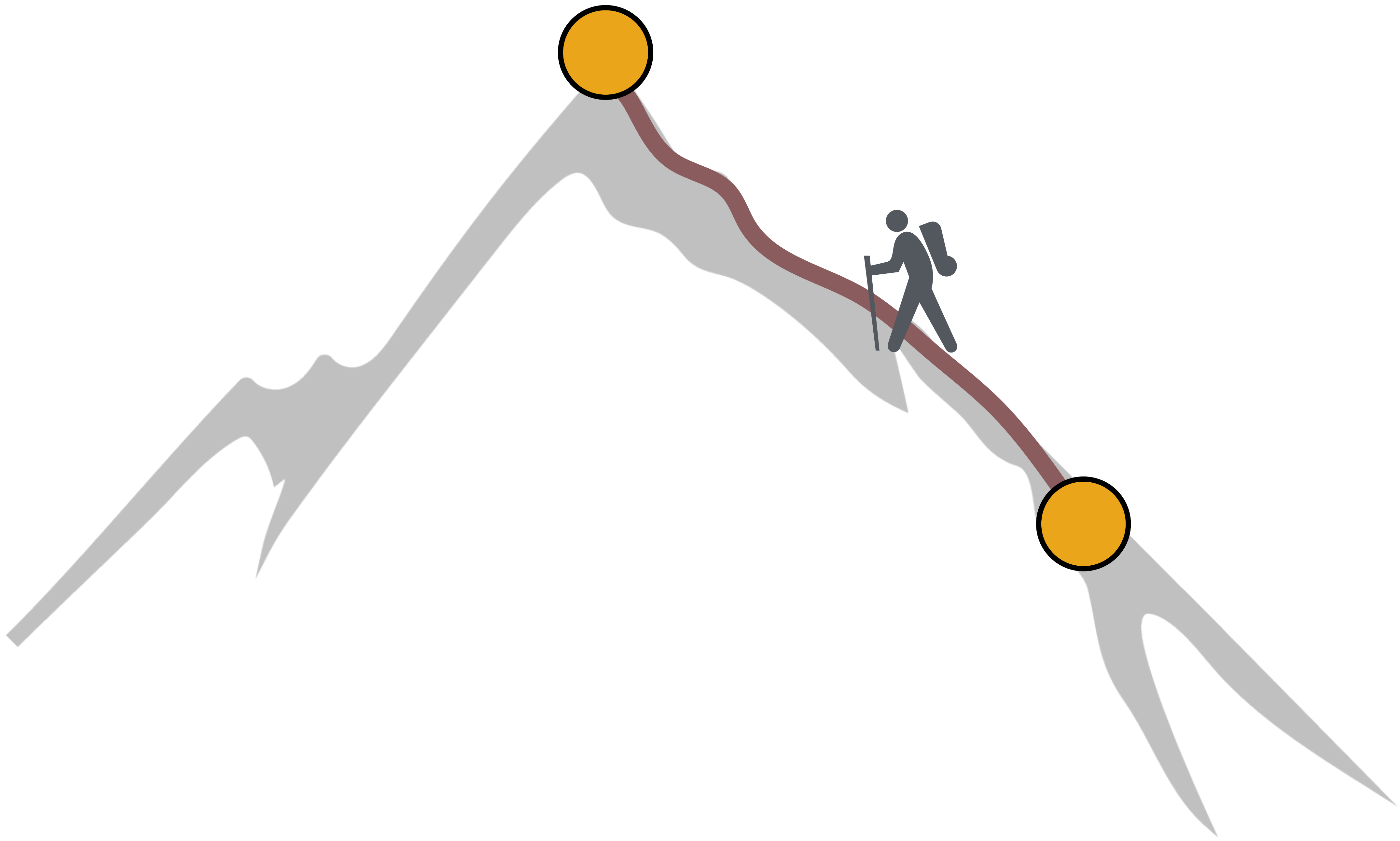

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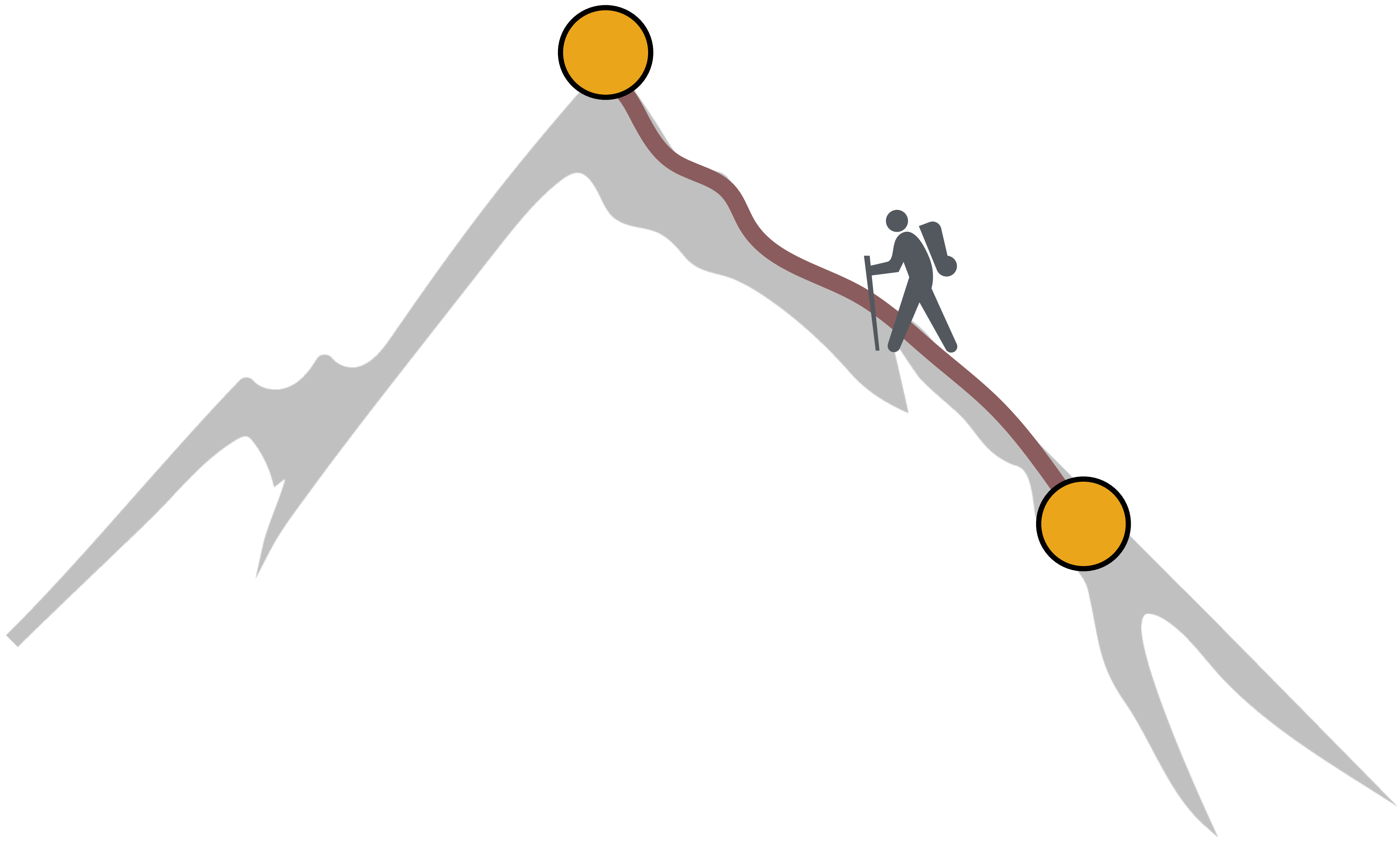
$$H = H_0 + V$$



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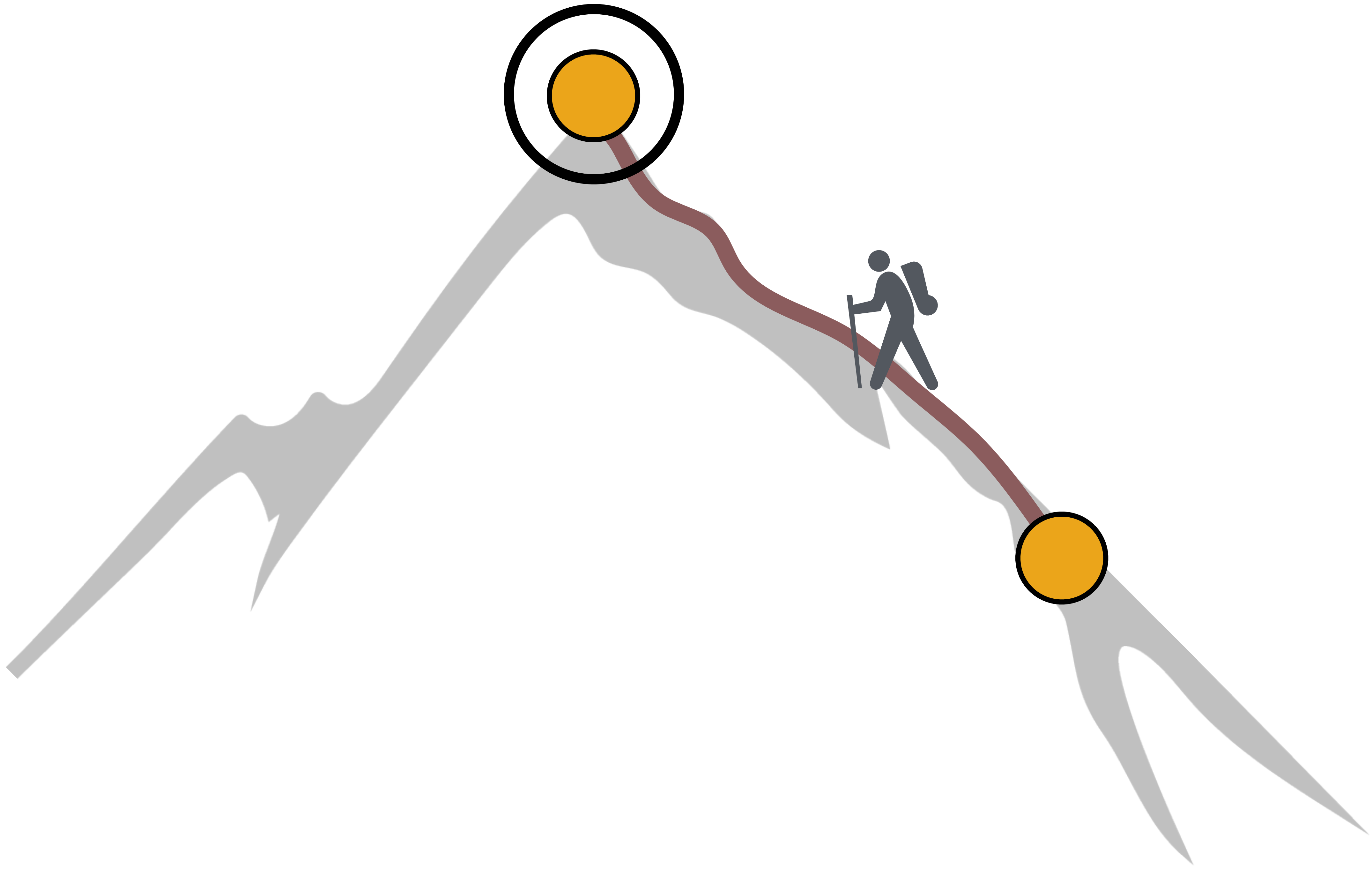

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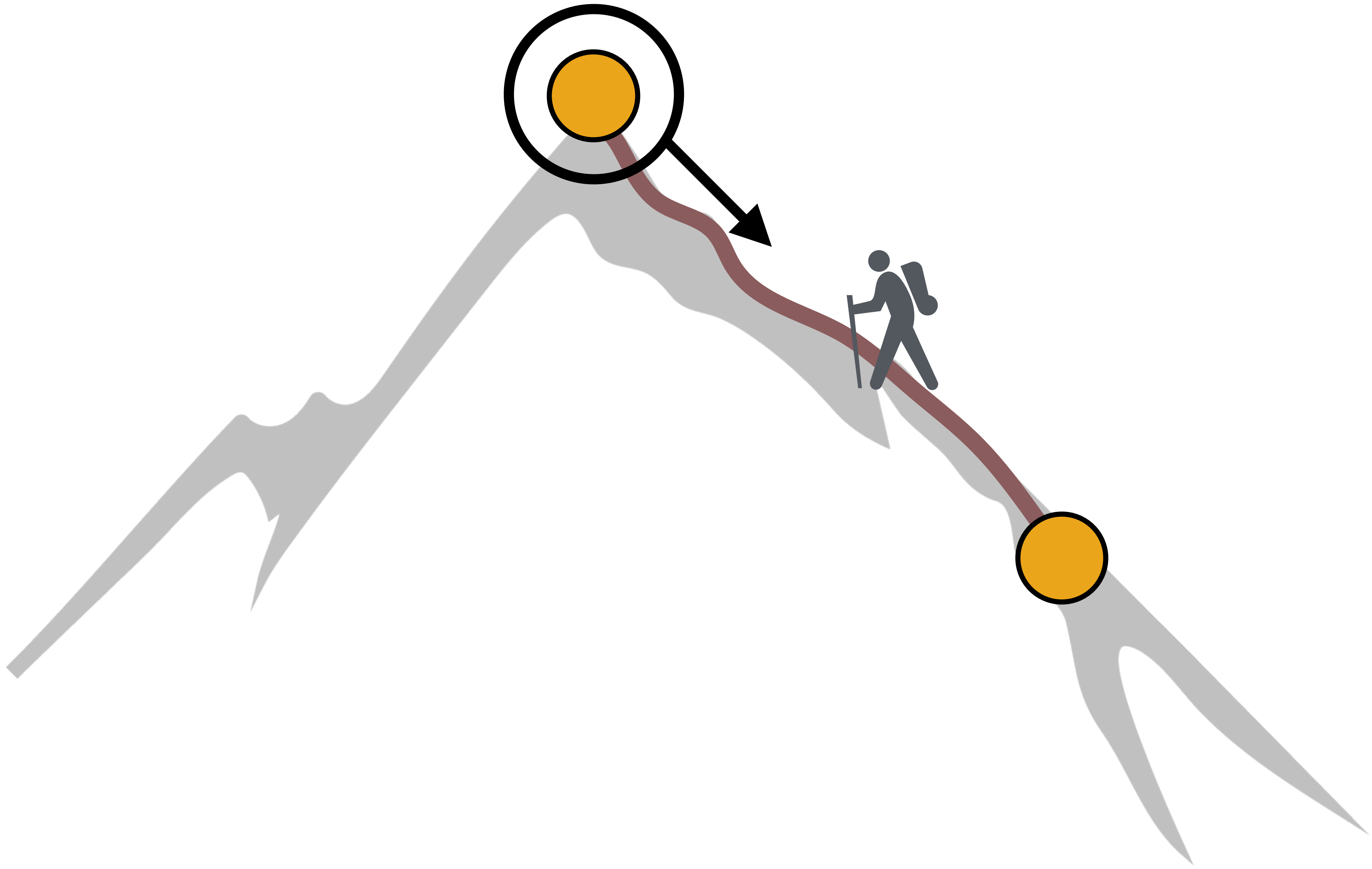

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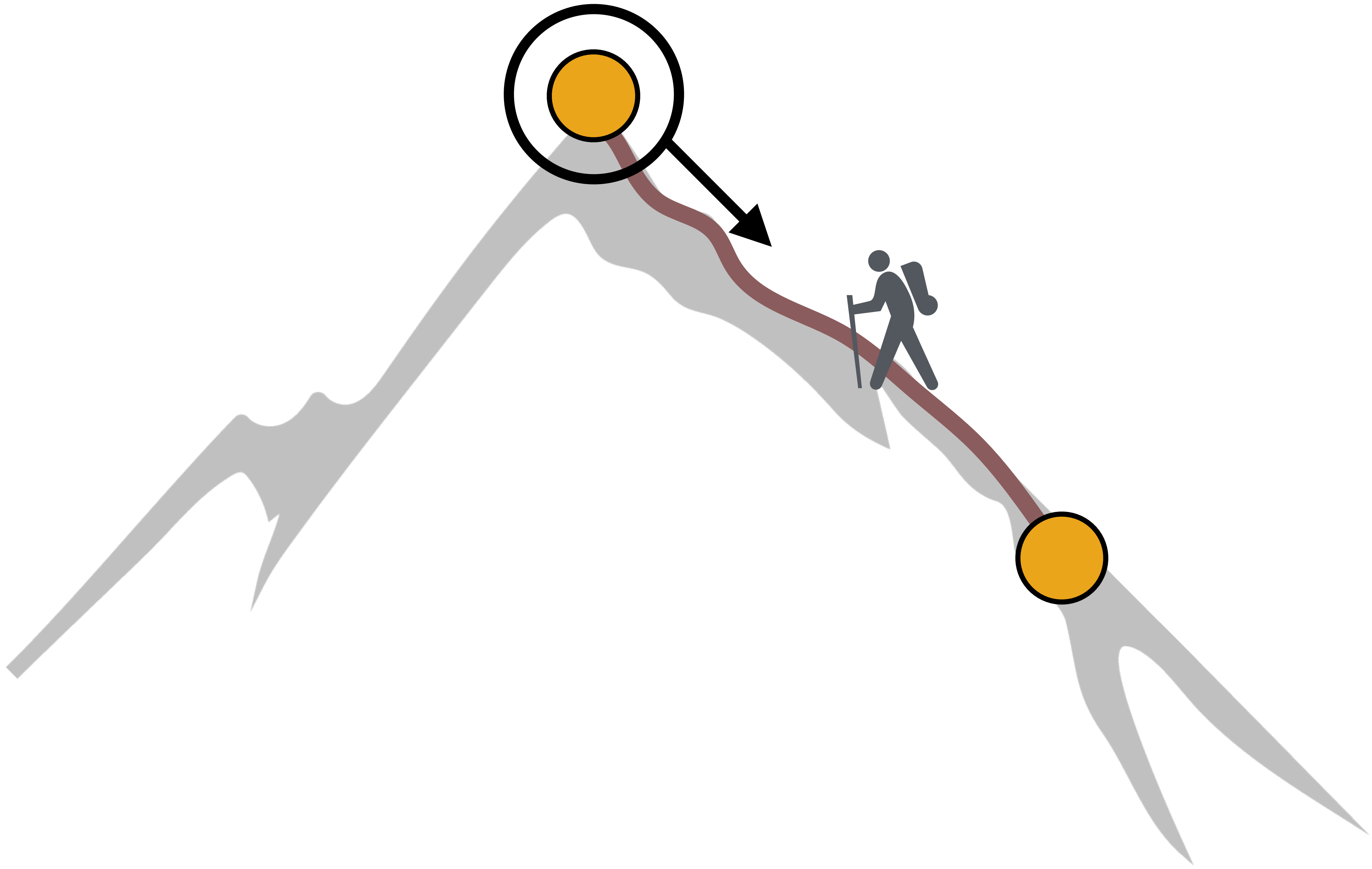

can't diagonalize full H → truncate!

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- lots of approaches: DLCQ (Pauli et al ’85), TCSA (Yurov, ’90), Massive Fock Space (Brooks et al ’84, Rychkov et al ’14), LCT (Katz et al ’16), RCMPS (Tilloy ’21), ...

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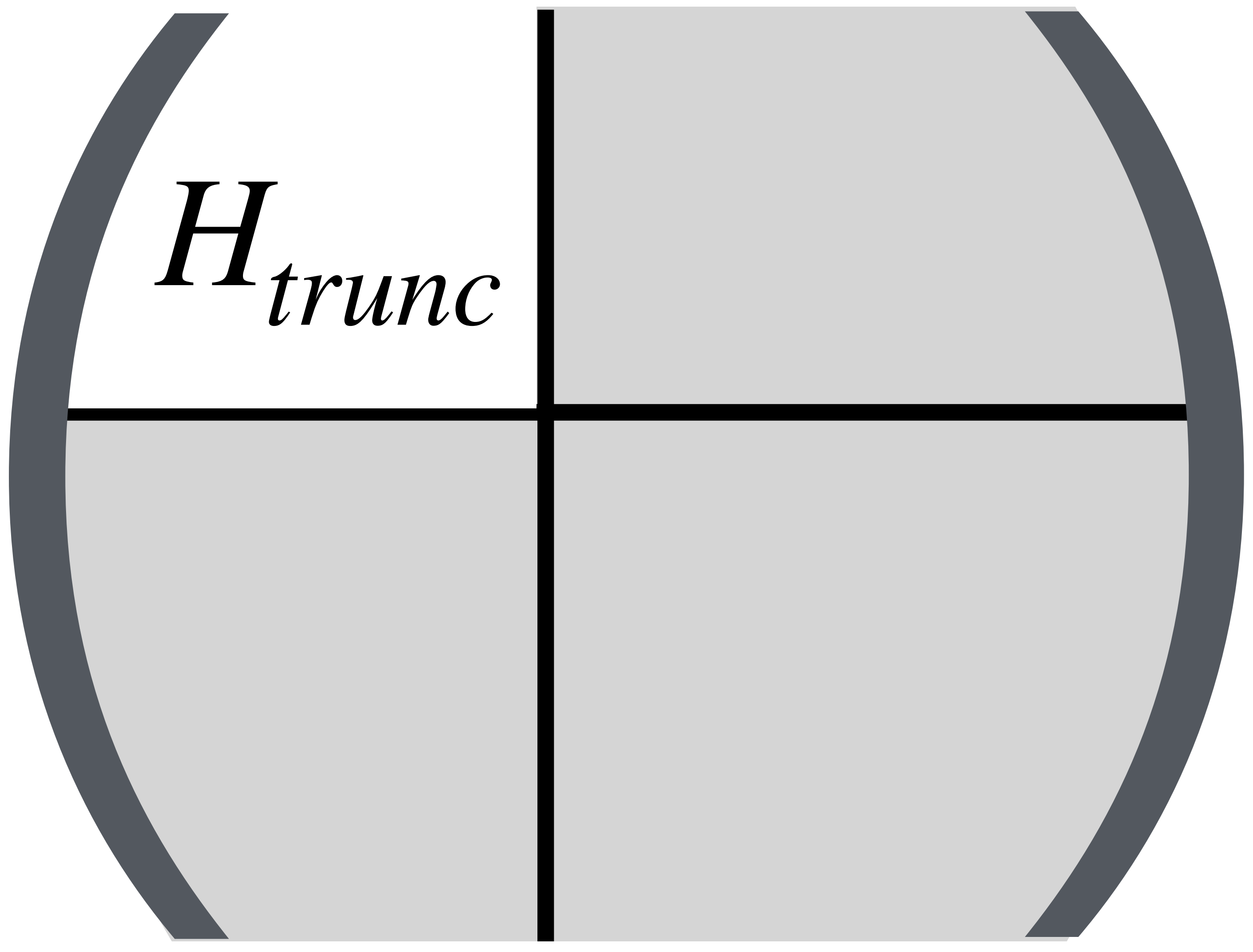
$H =$ 

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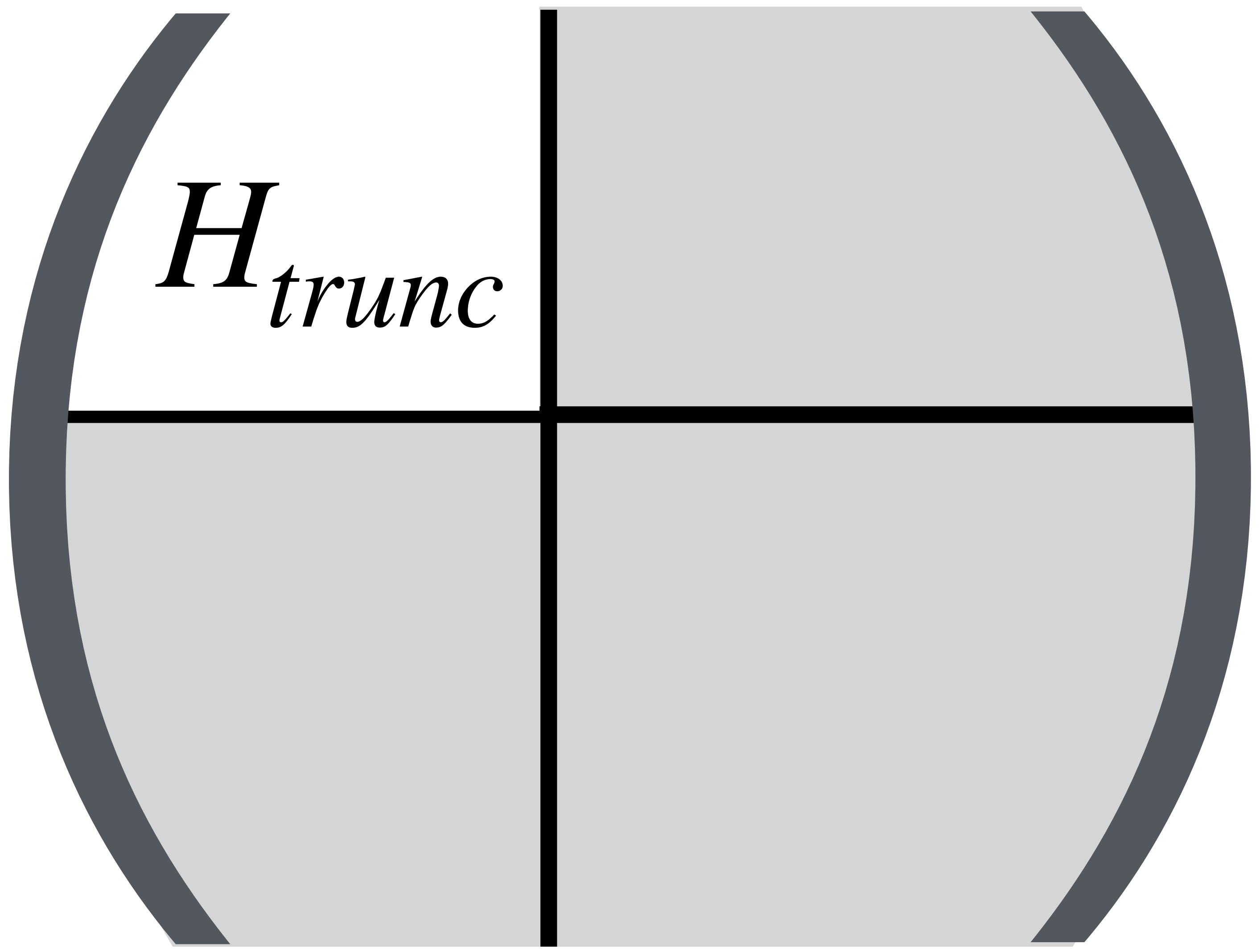
$H =$ 

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- diagonalize ➡ eigenvalues: $\frac{3}{2} + \frac{21}{4}\lambda \pm \sqrt{1+\frac{9}{4}\lambda(11\lambda + 4)}$

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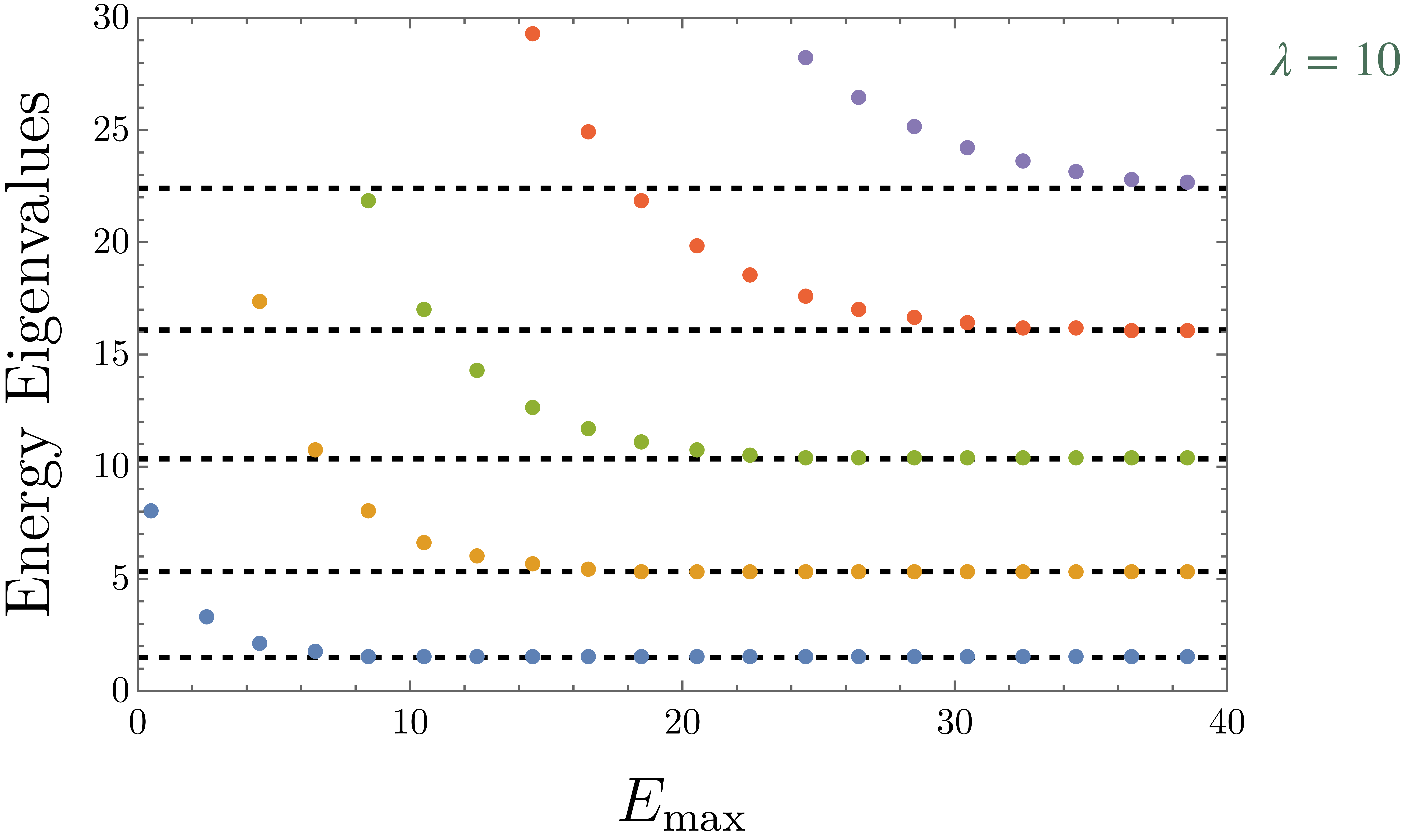

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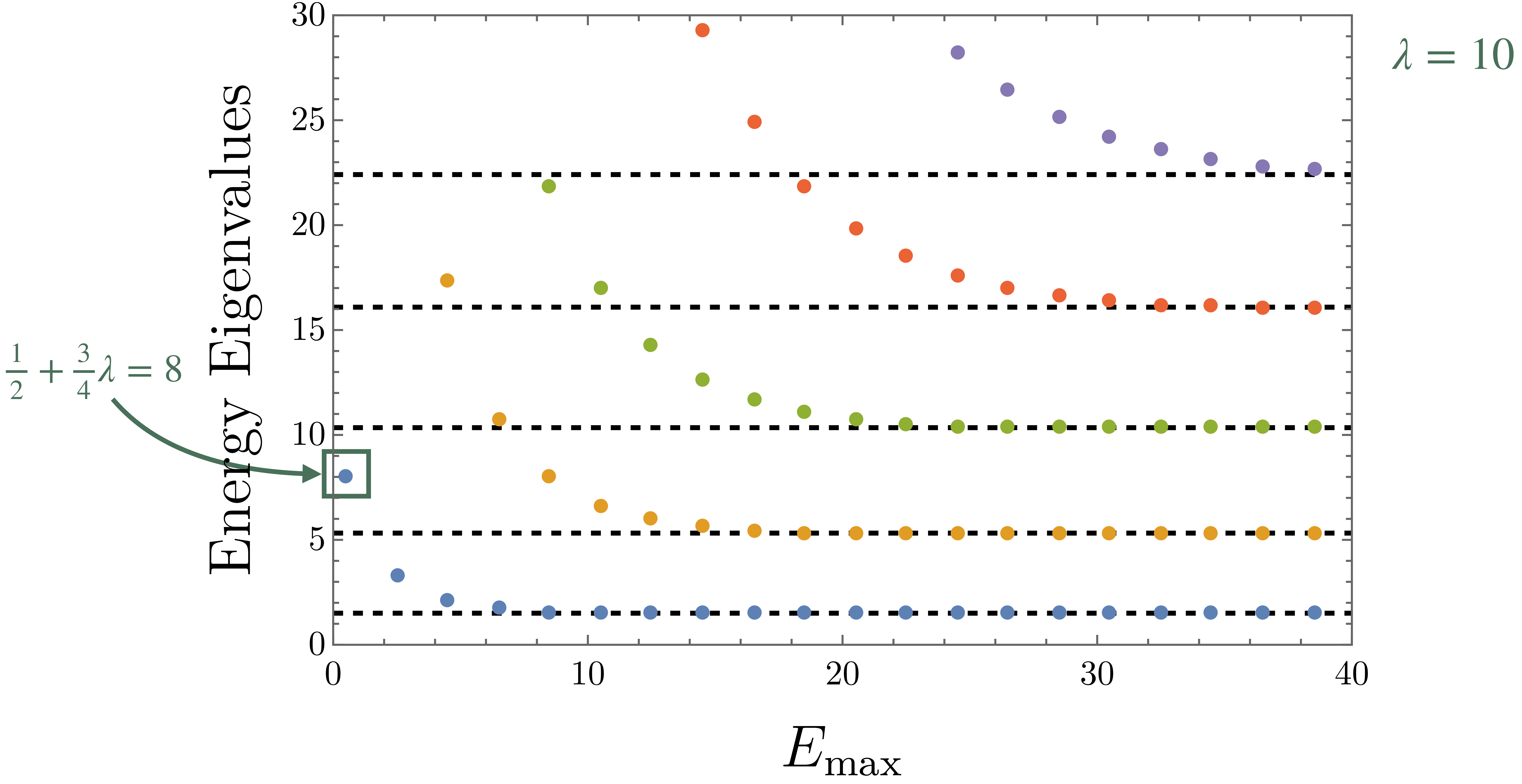

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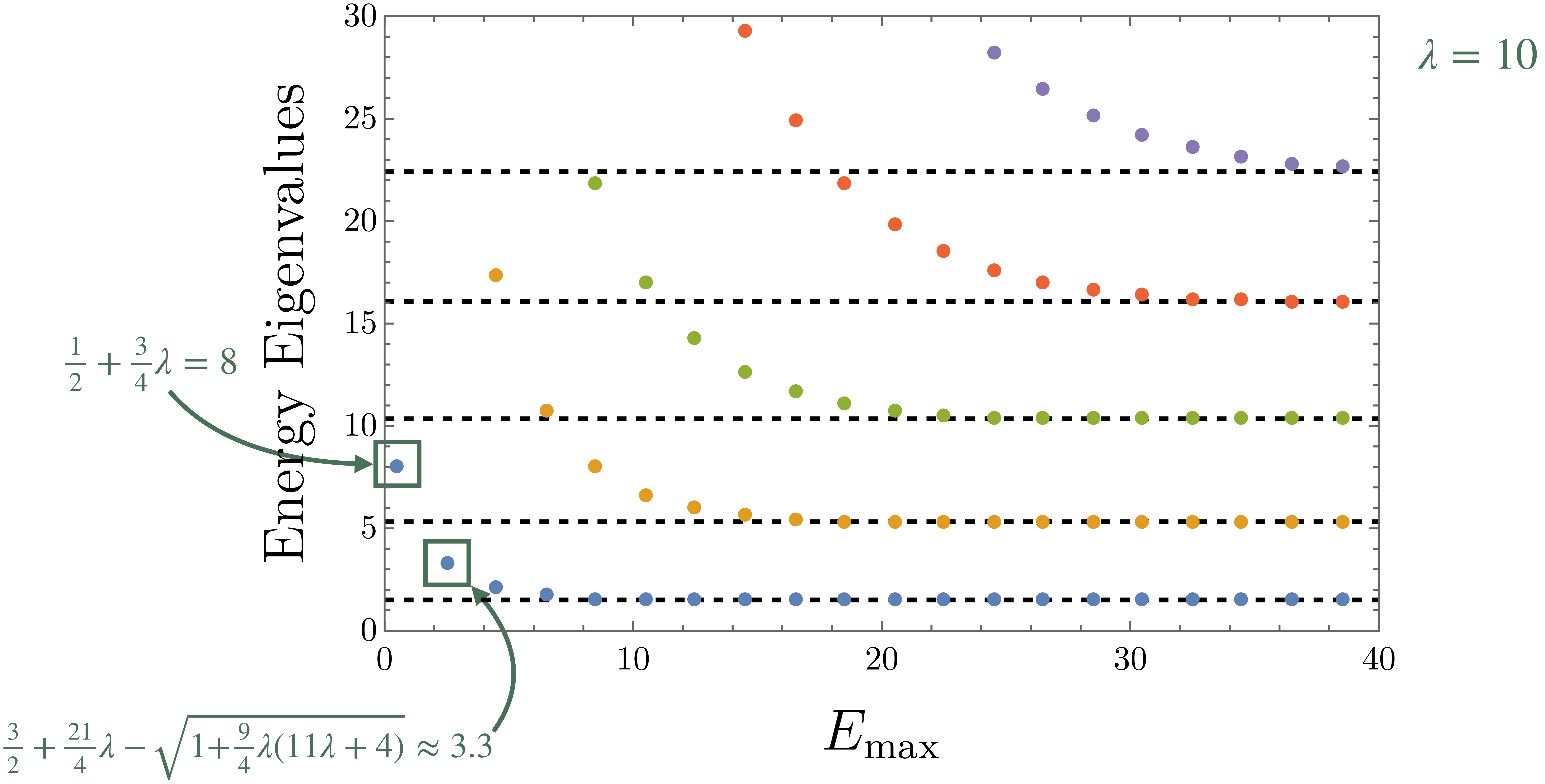

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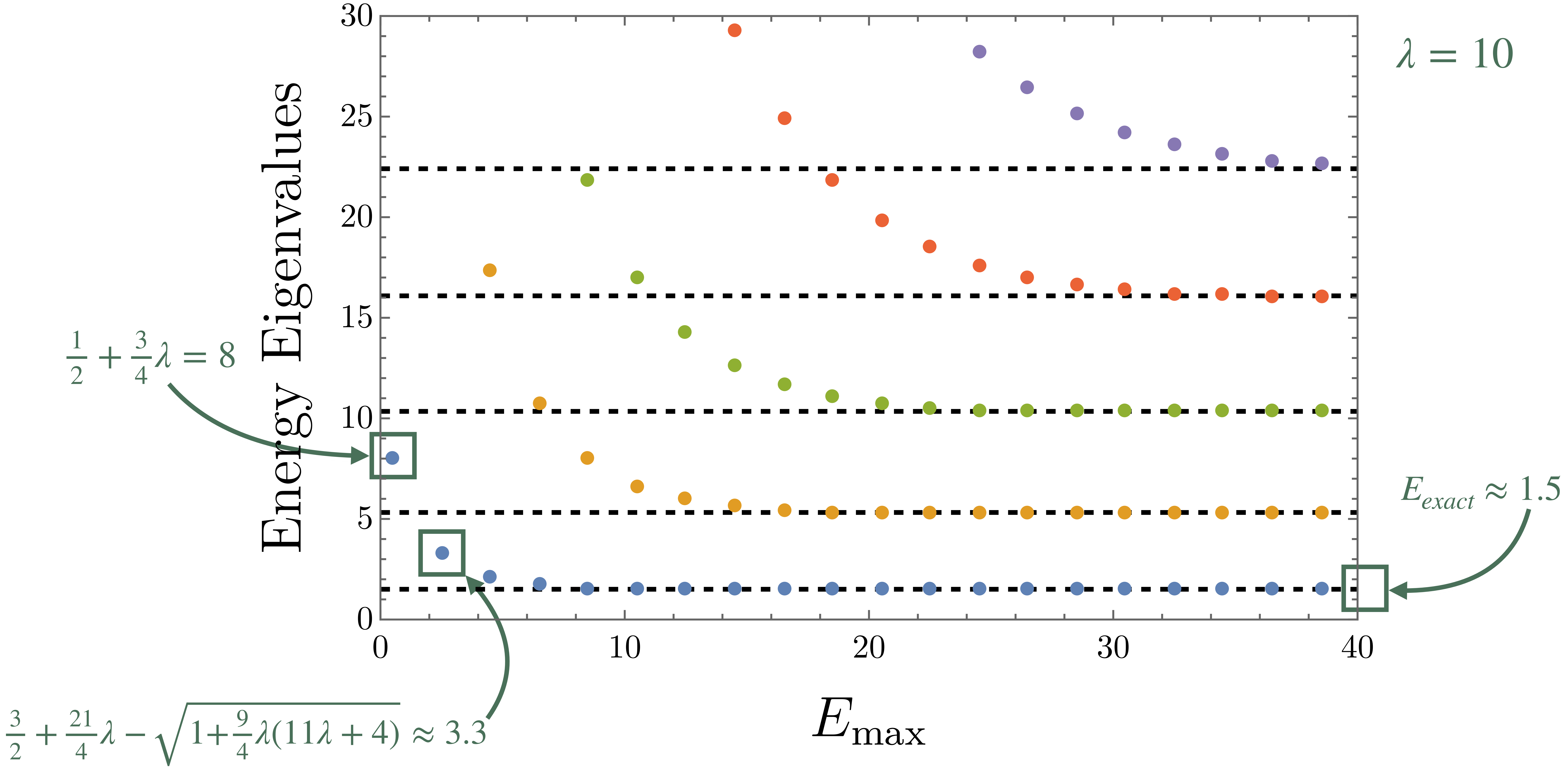

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- direct access to **dynamics** ($i\partial_t \Psi = H\Psi$)

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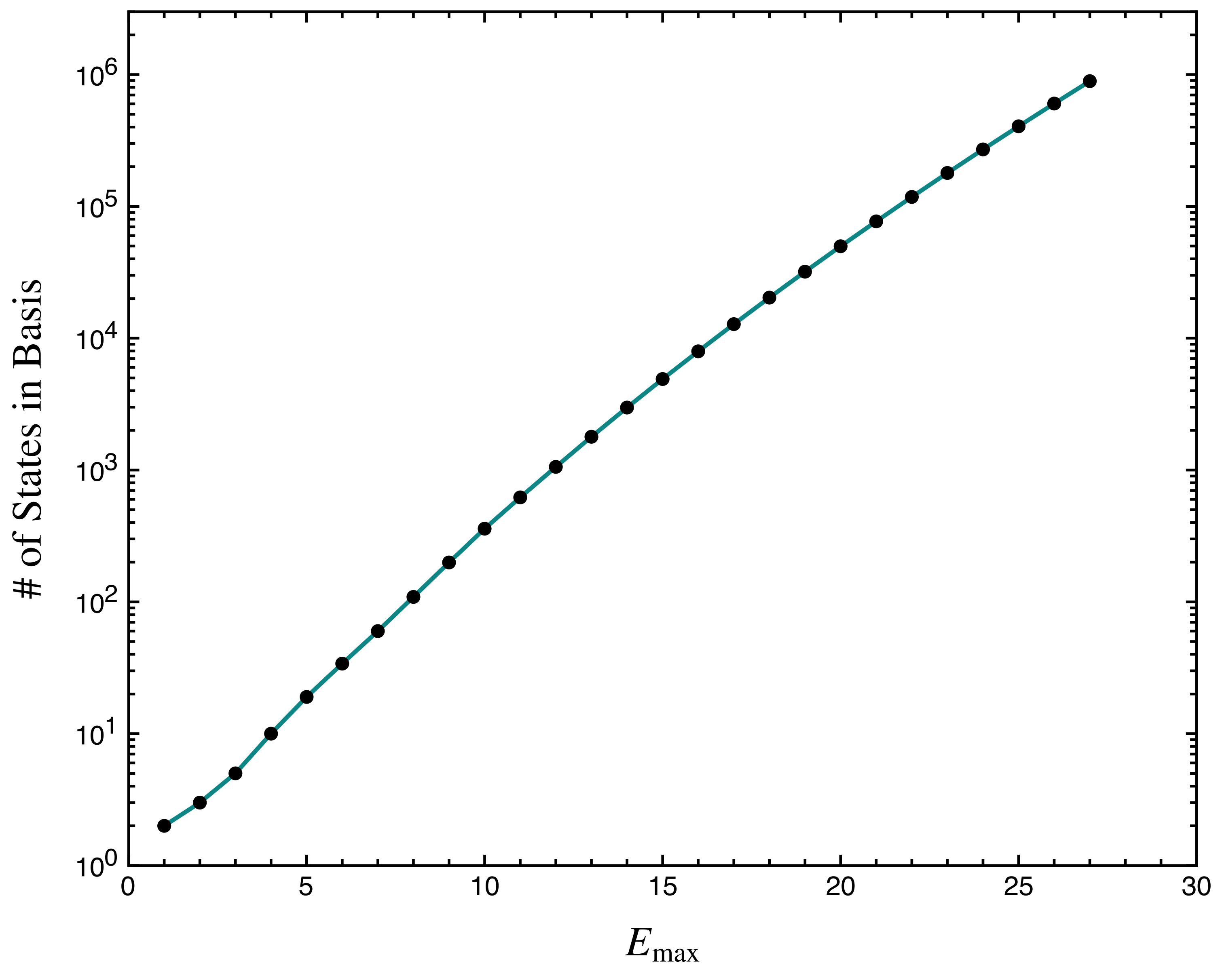

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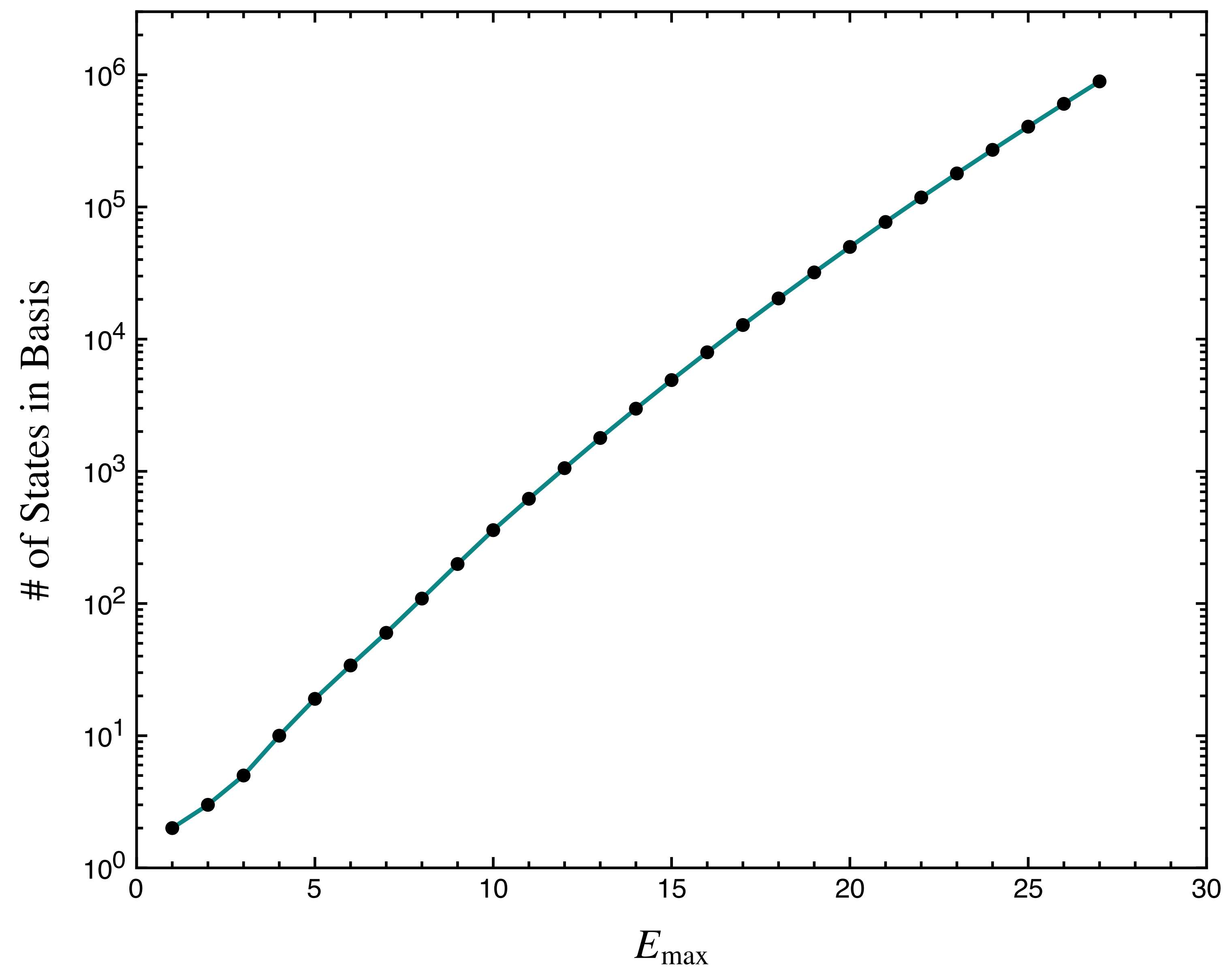

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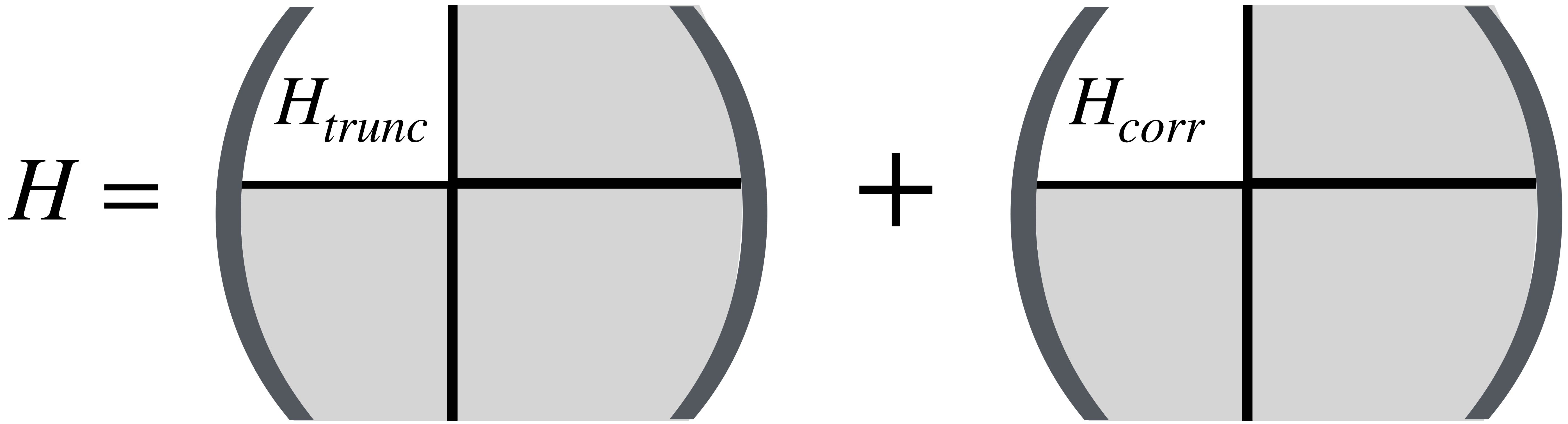
- add corrections to account for effects from states outside truncated Hilbert space ("integrate out")



- similar approaches: Feverati et al '06, Hogervorst et al '14, Elias-Miro et al '17, ...

Effective field theory

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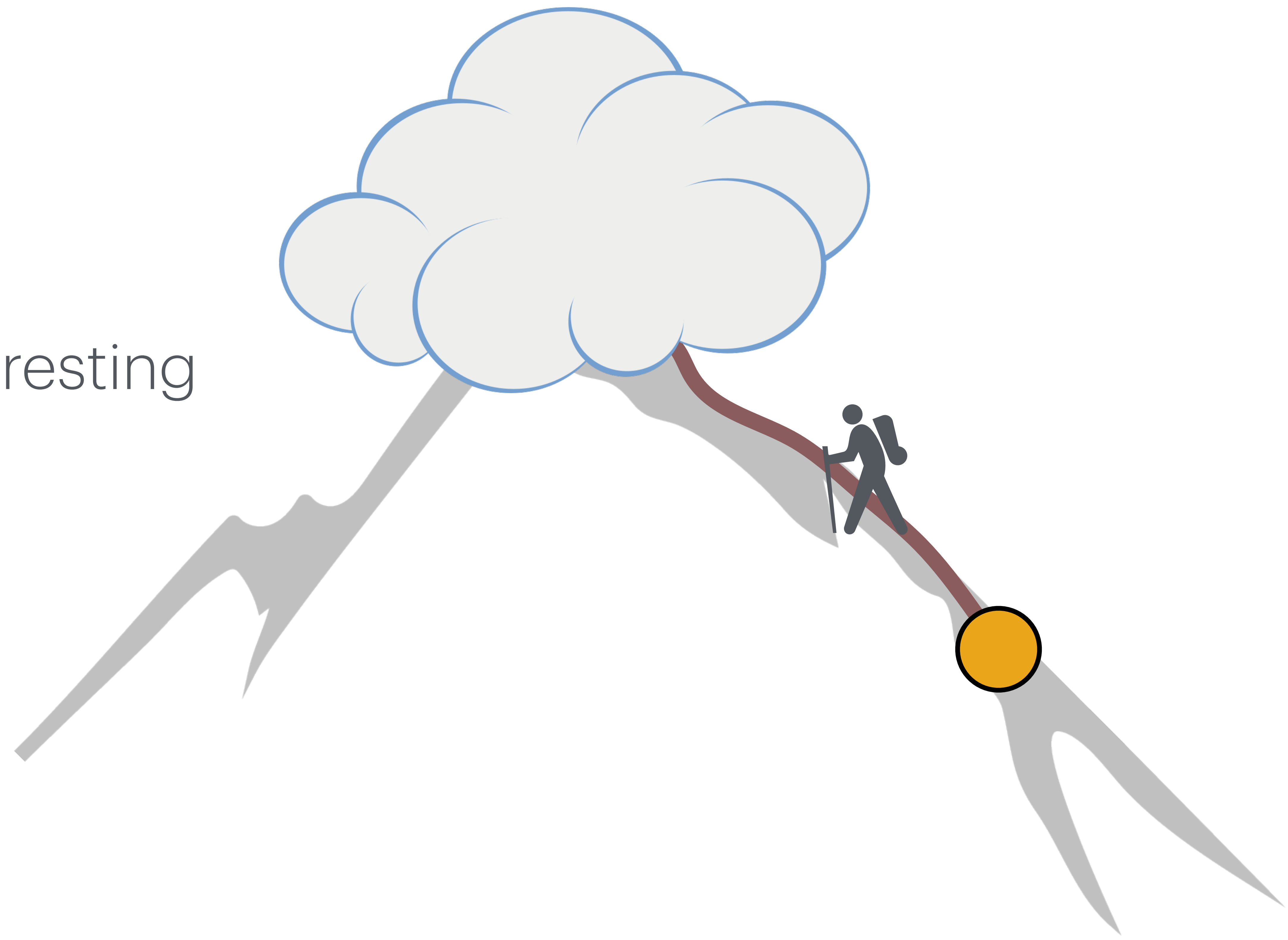

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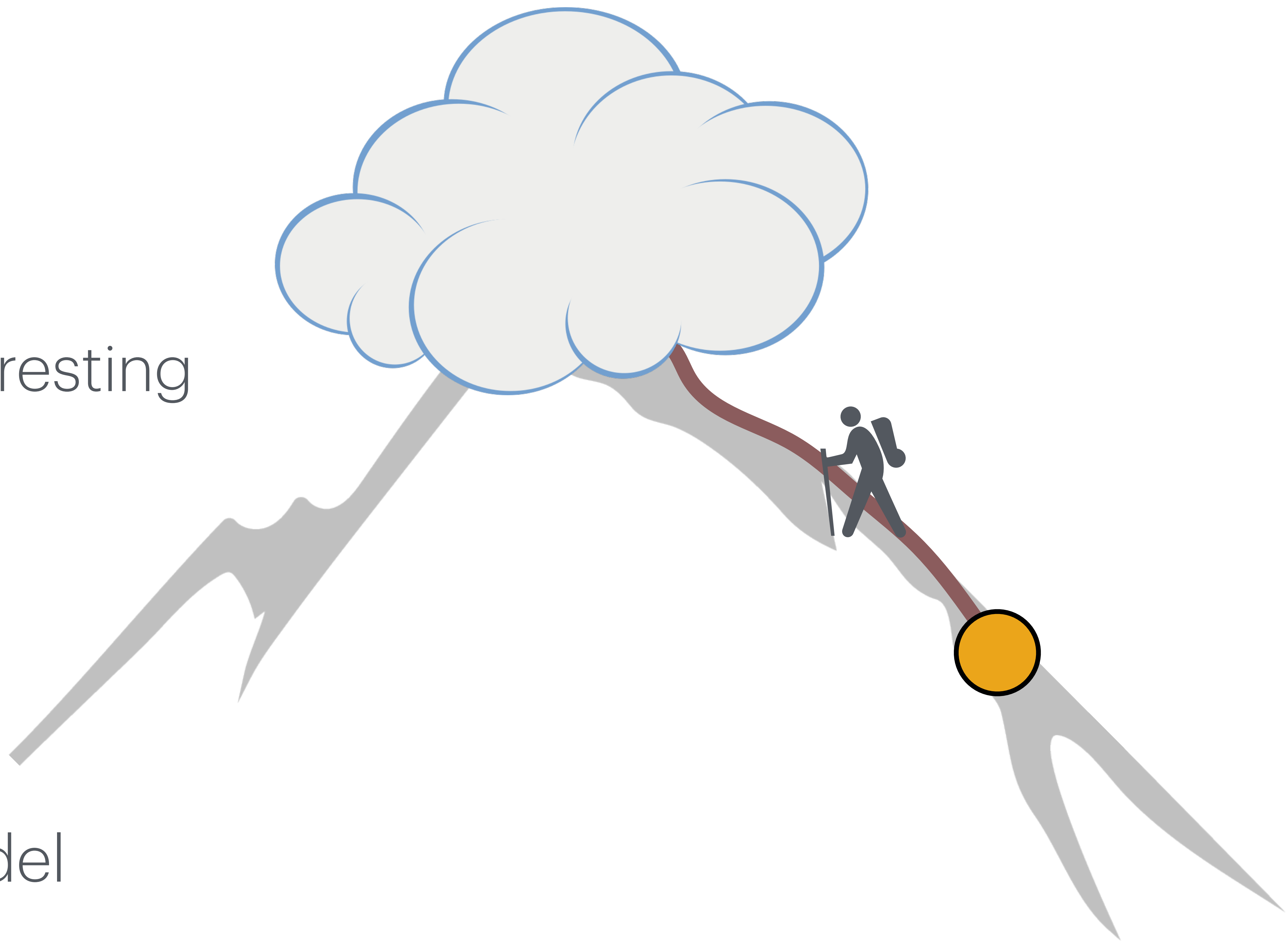

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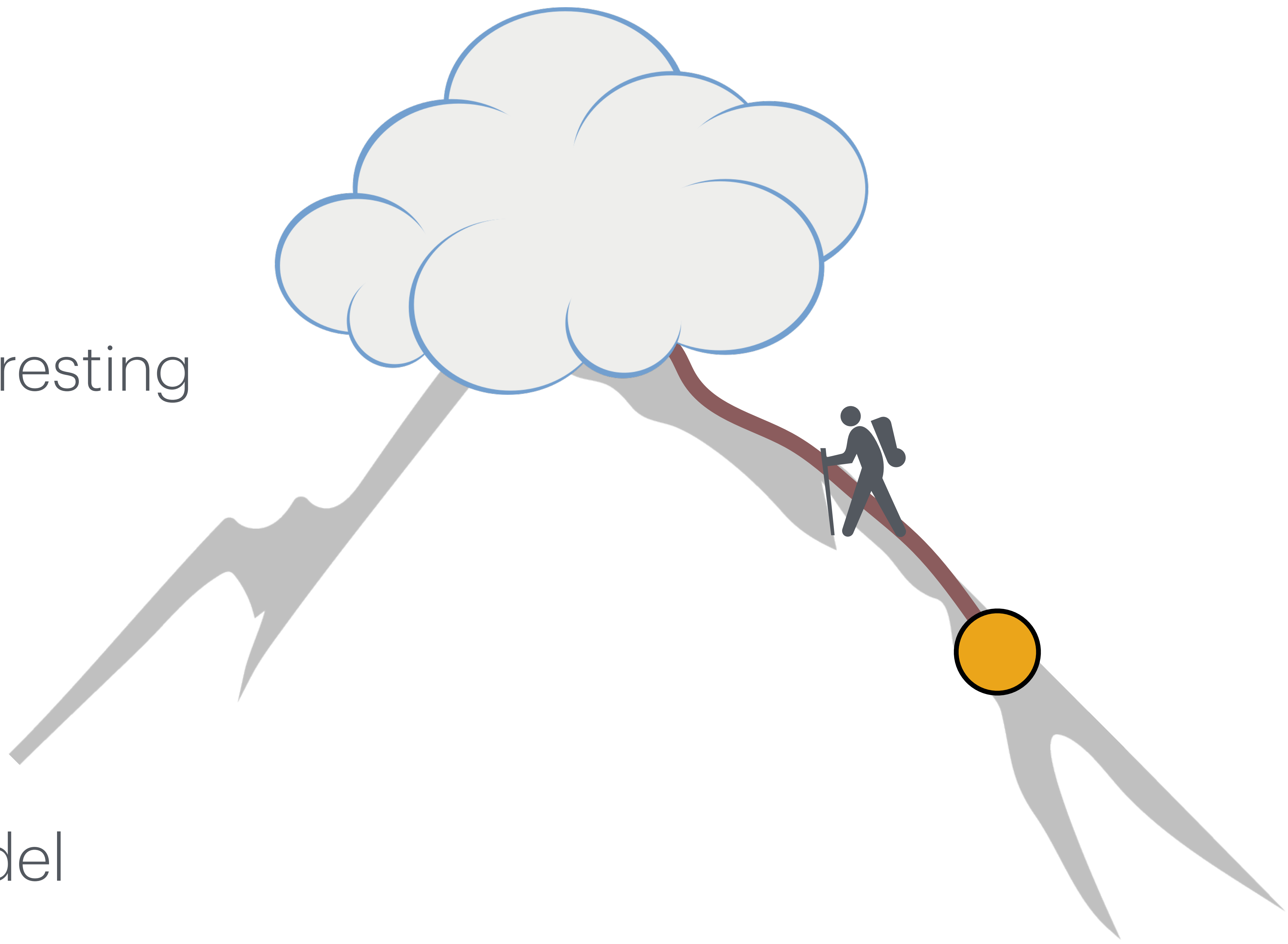

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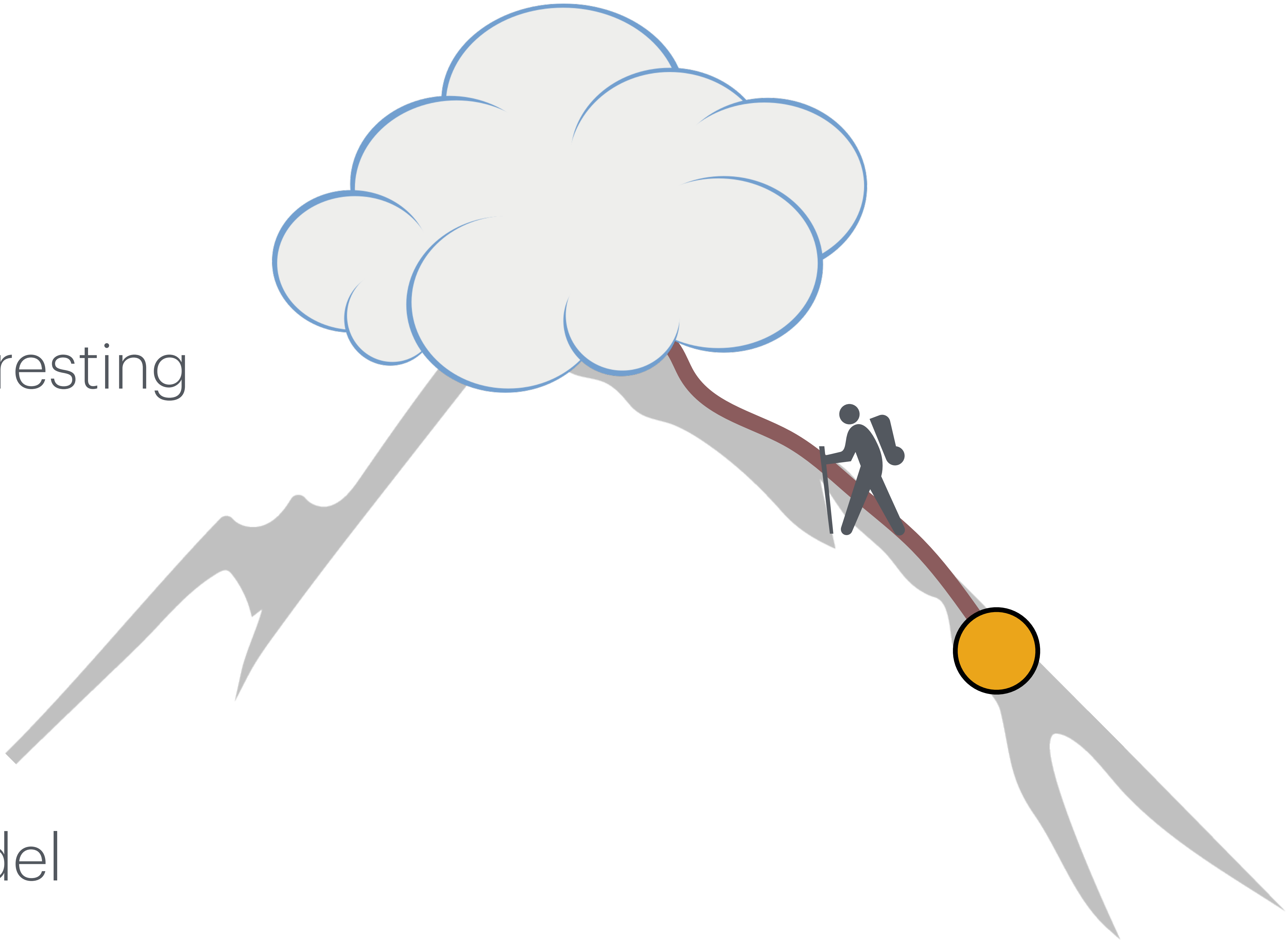

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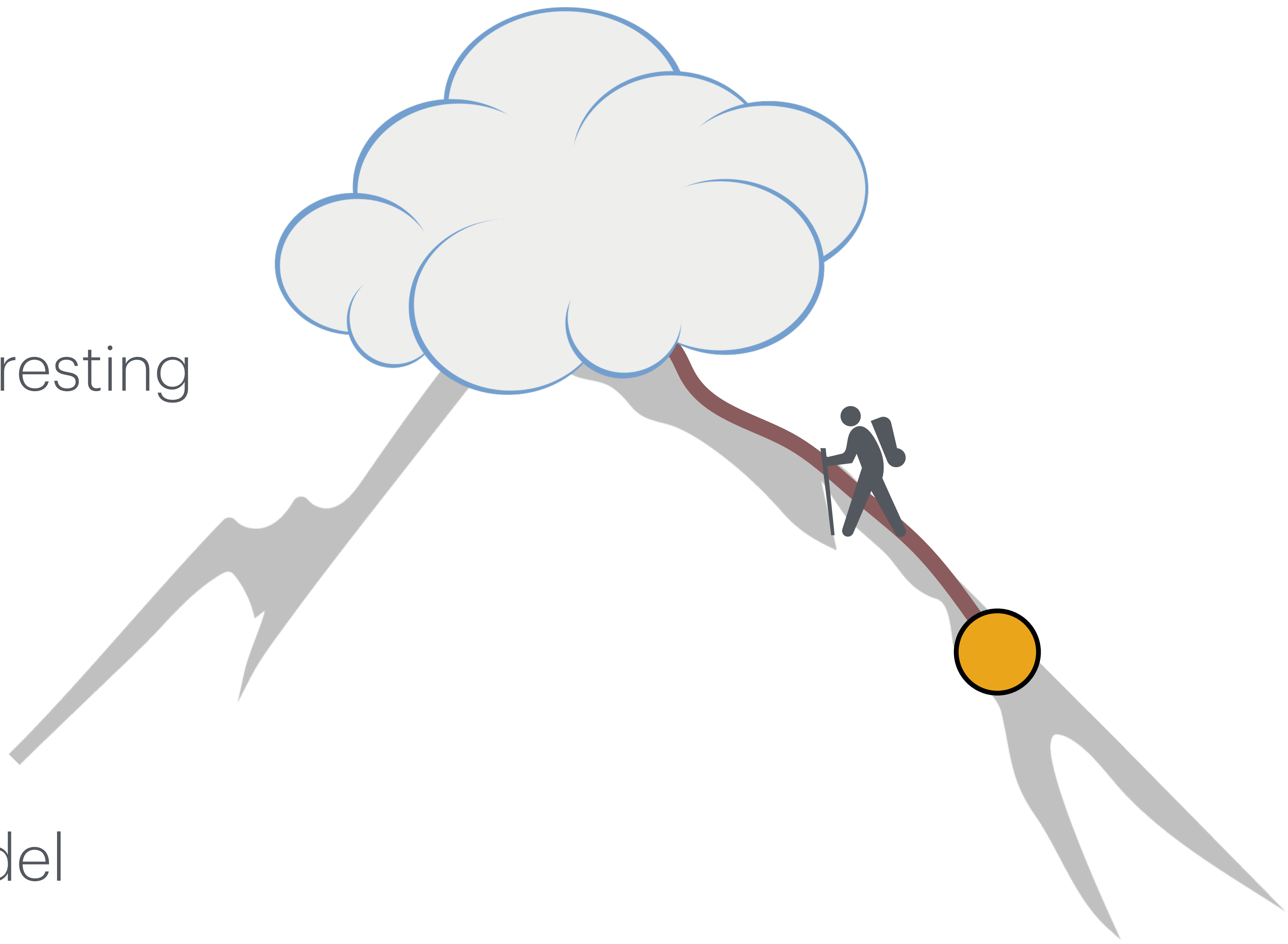

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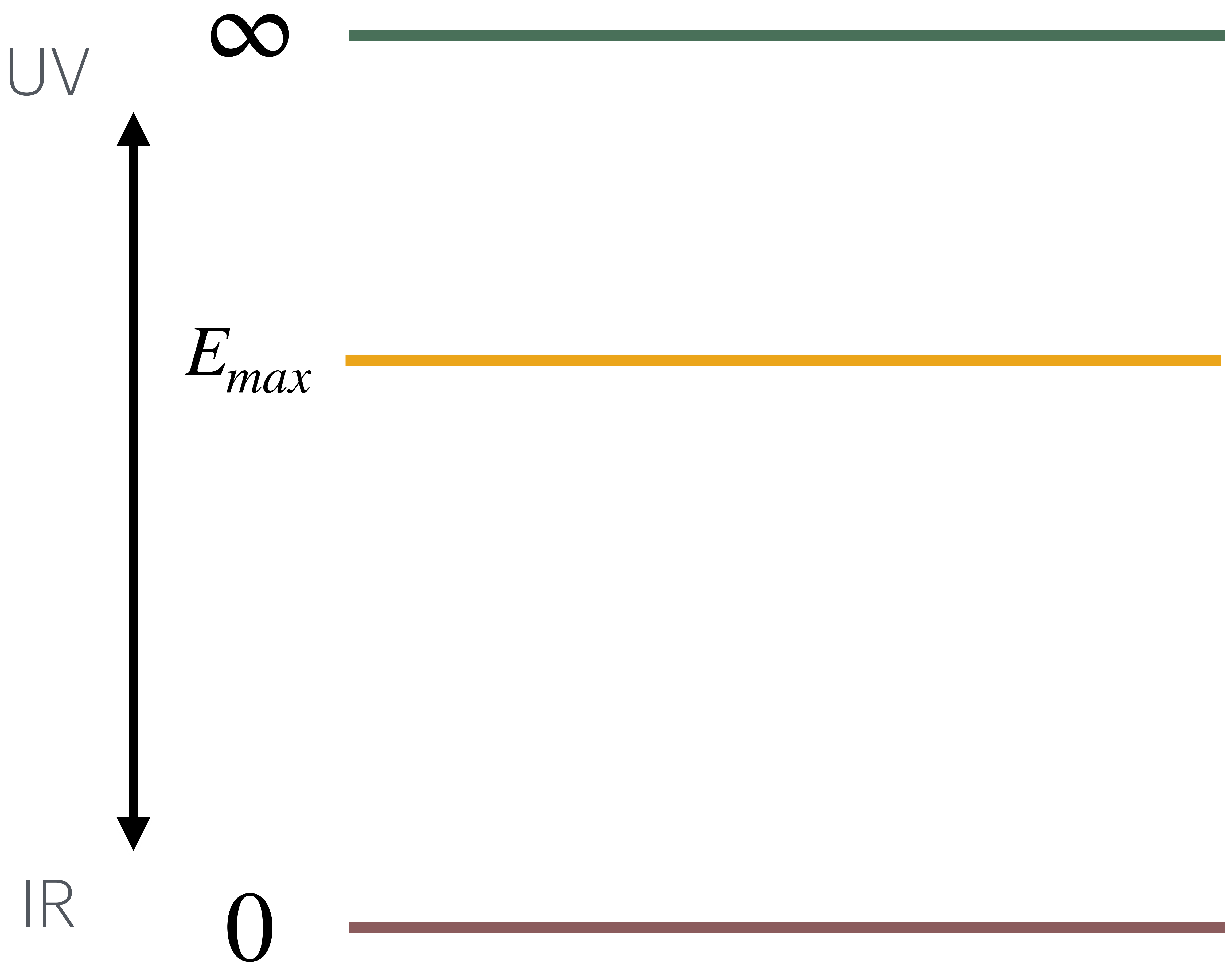
- consistent power counting
- written in terms of local operators

Effective field theory for truncation



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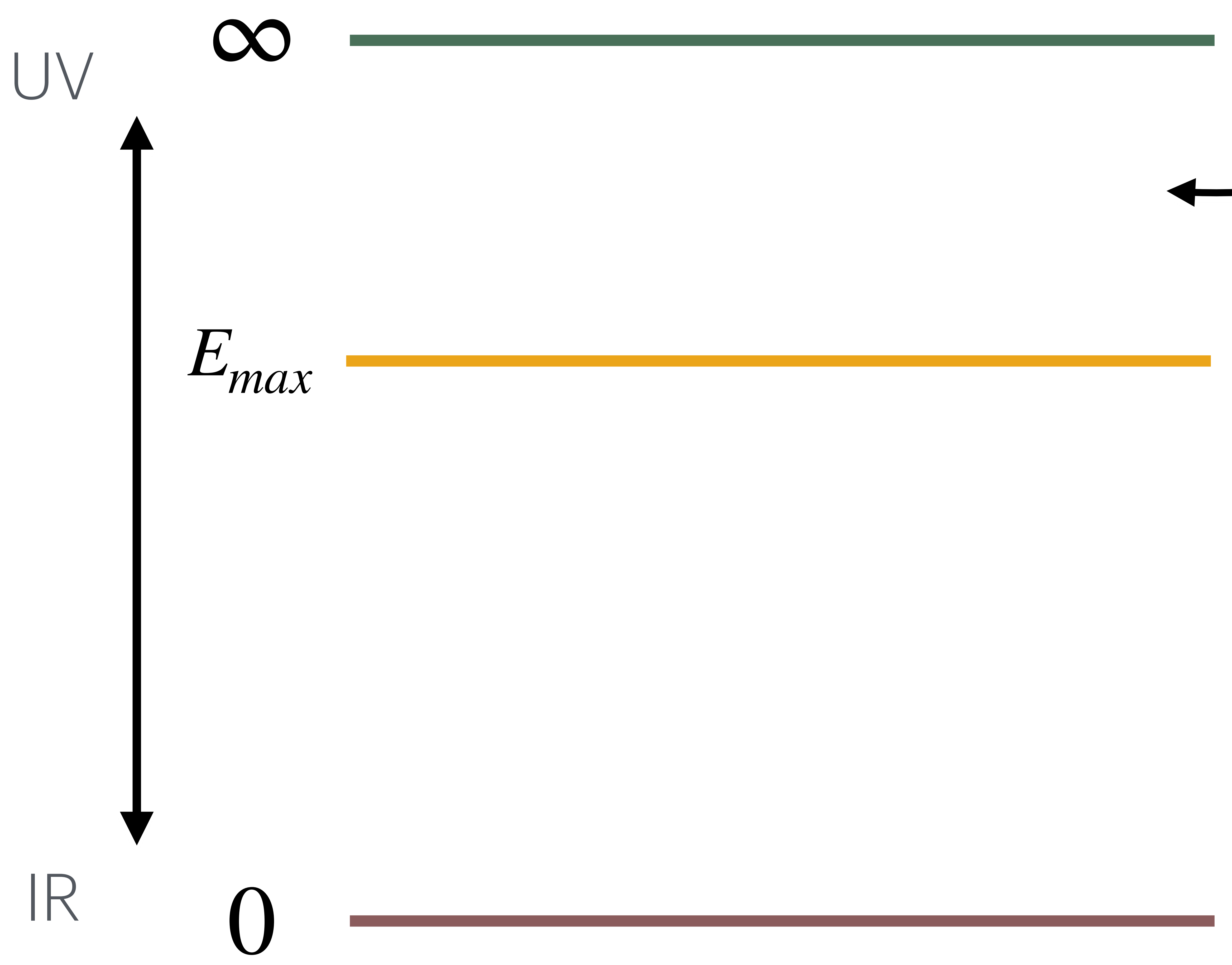

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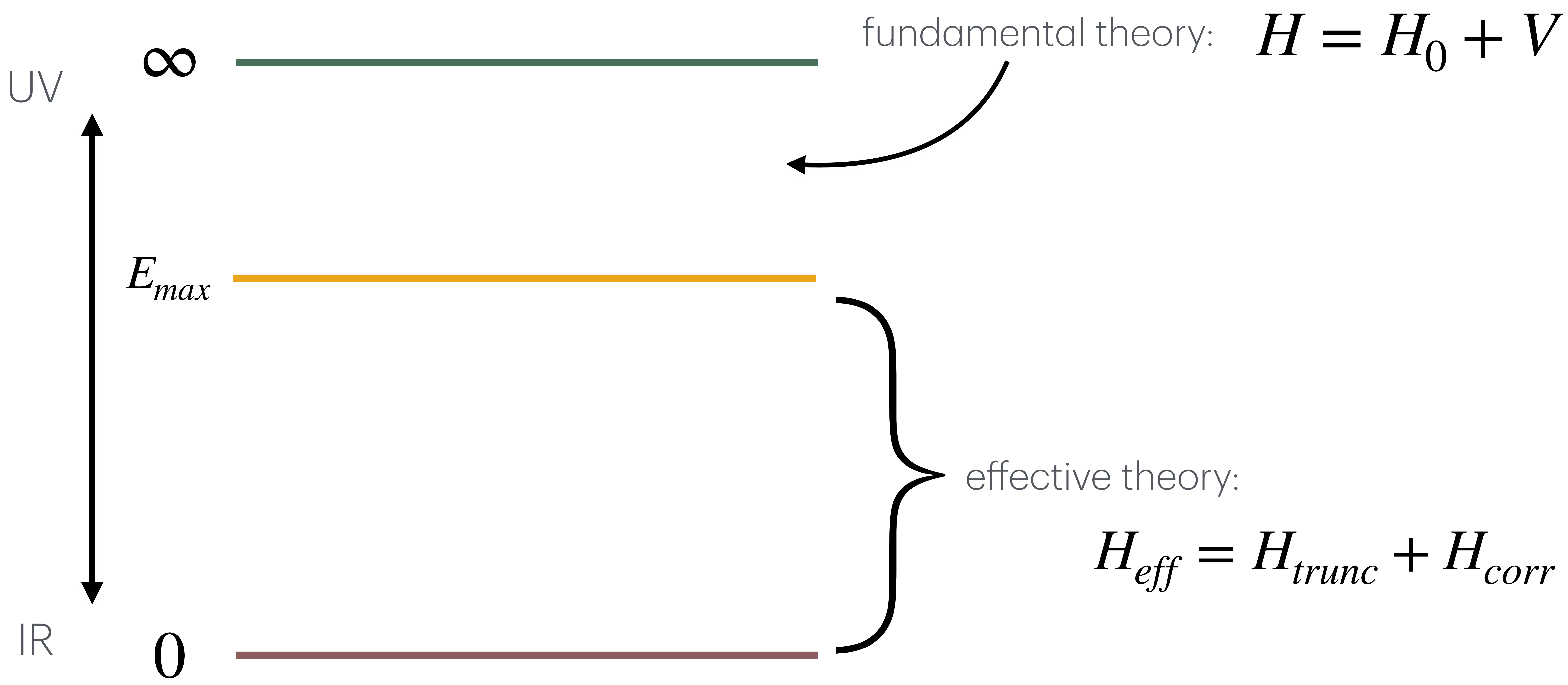

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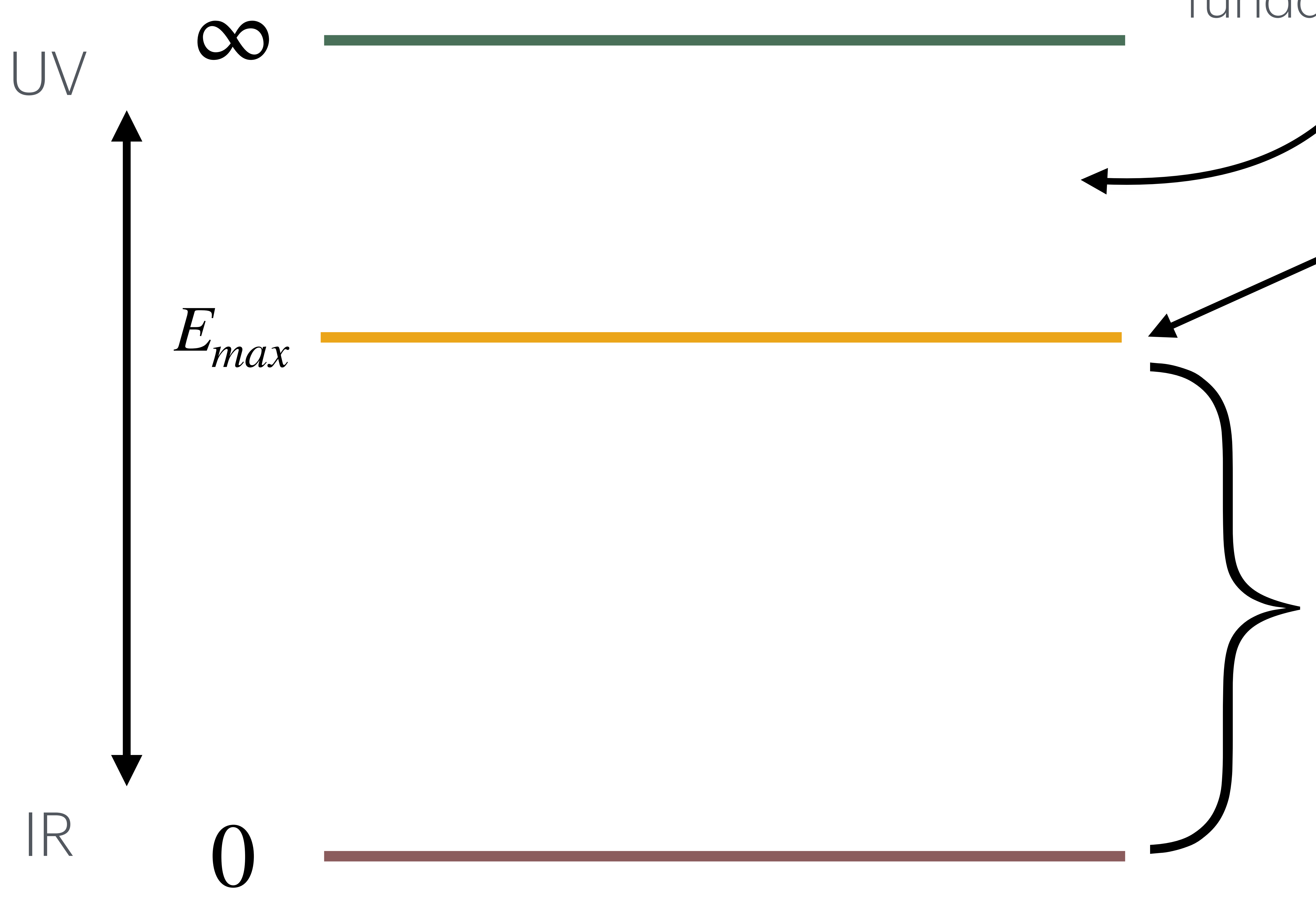

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scale of "new physics": $\Lambda_{UV} = E_{max}$

effective theory: $H_{eff} = H_{trunc} + H_{corr}$

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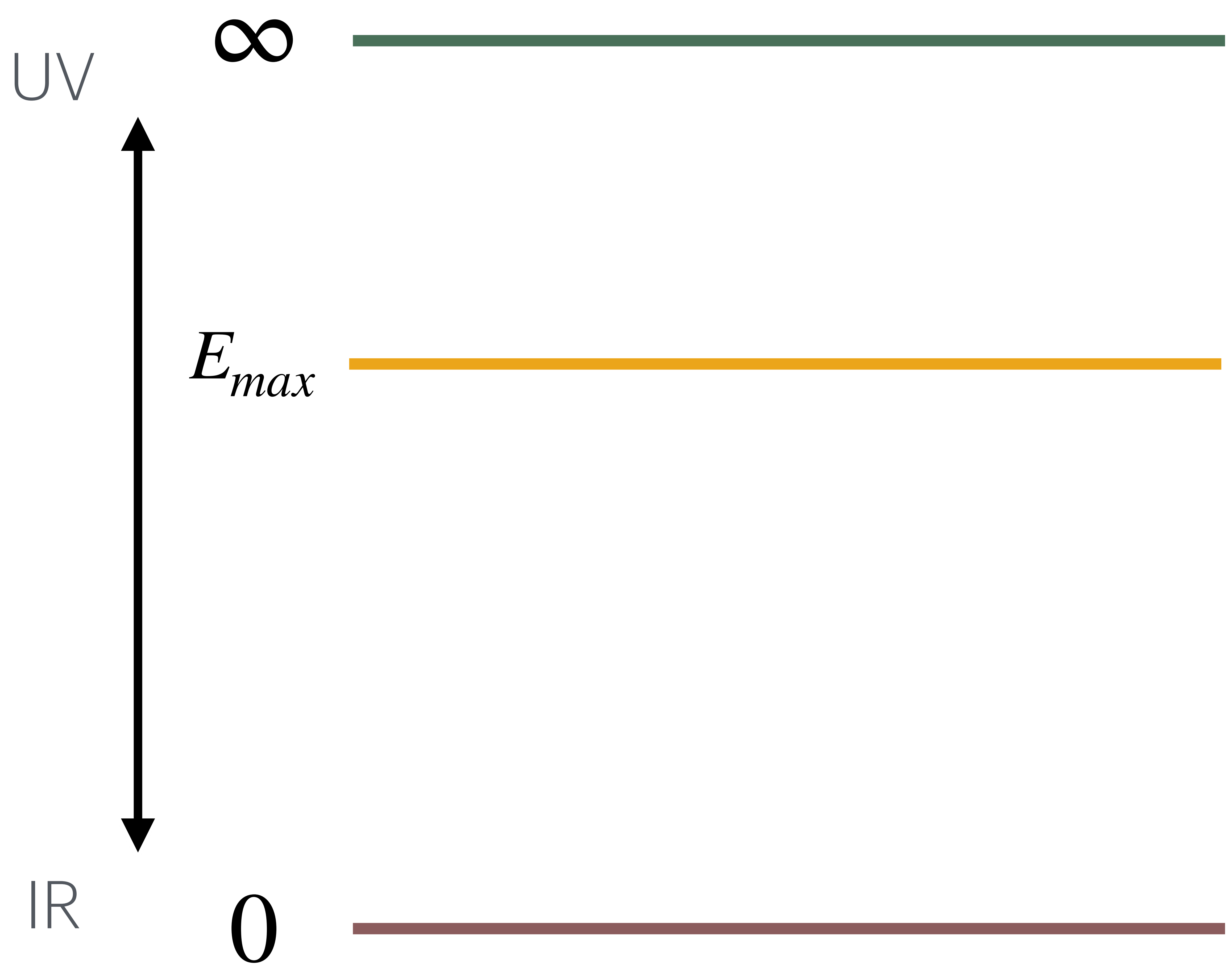

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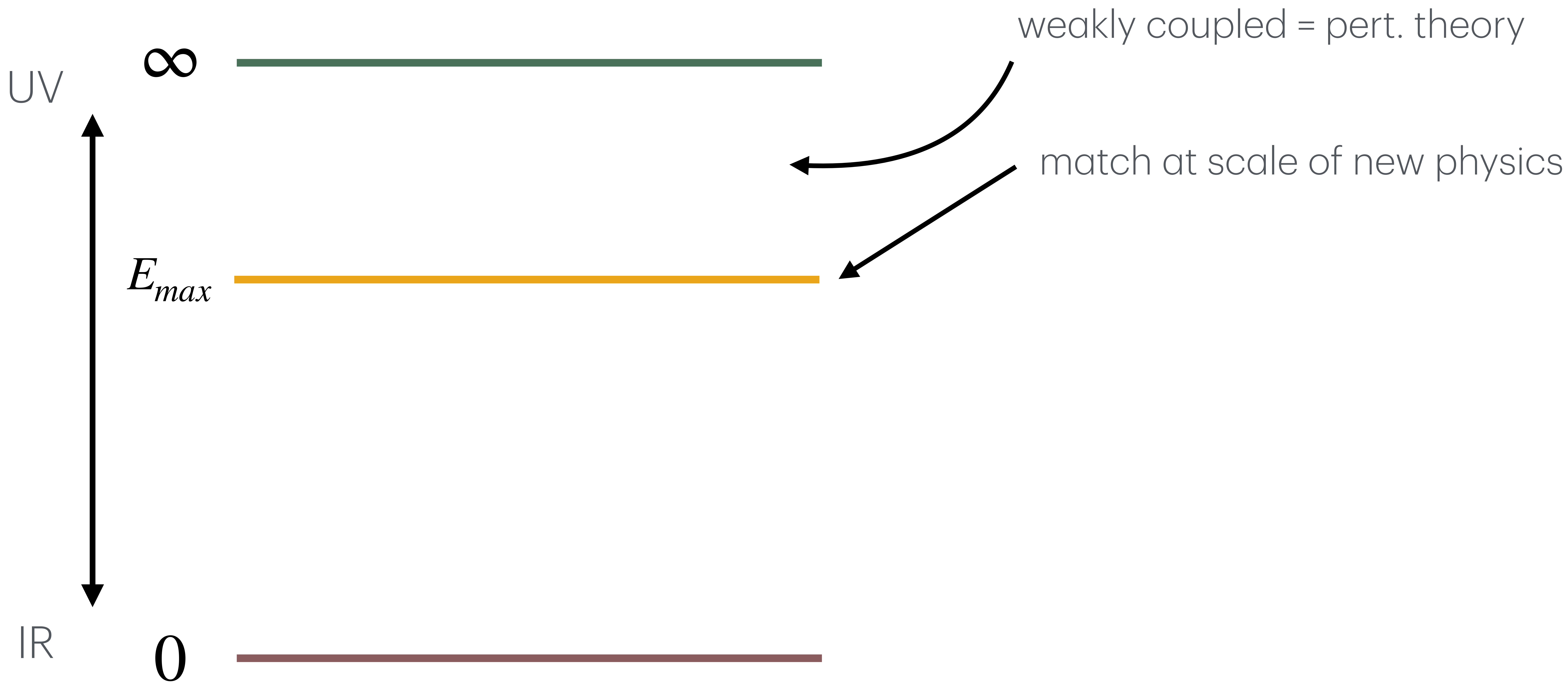

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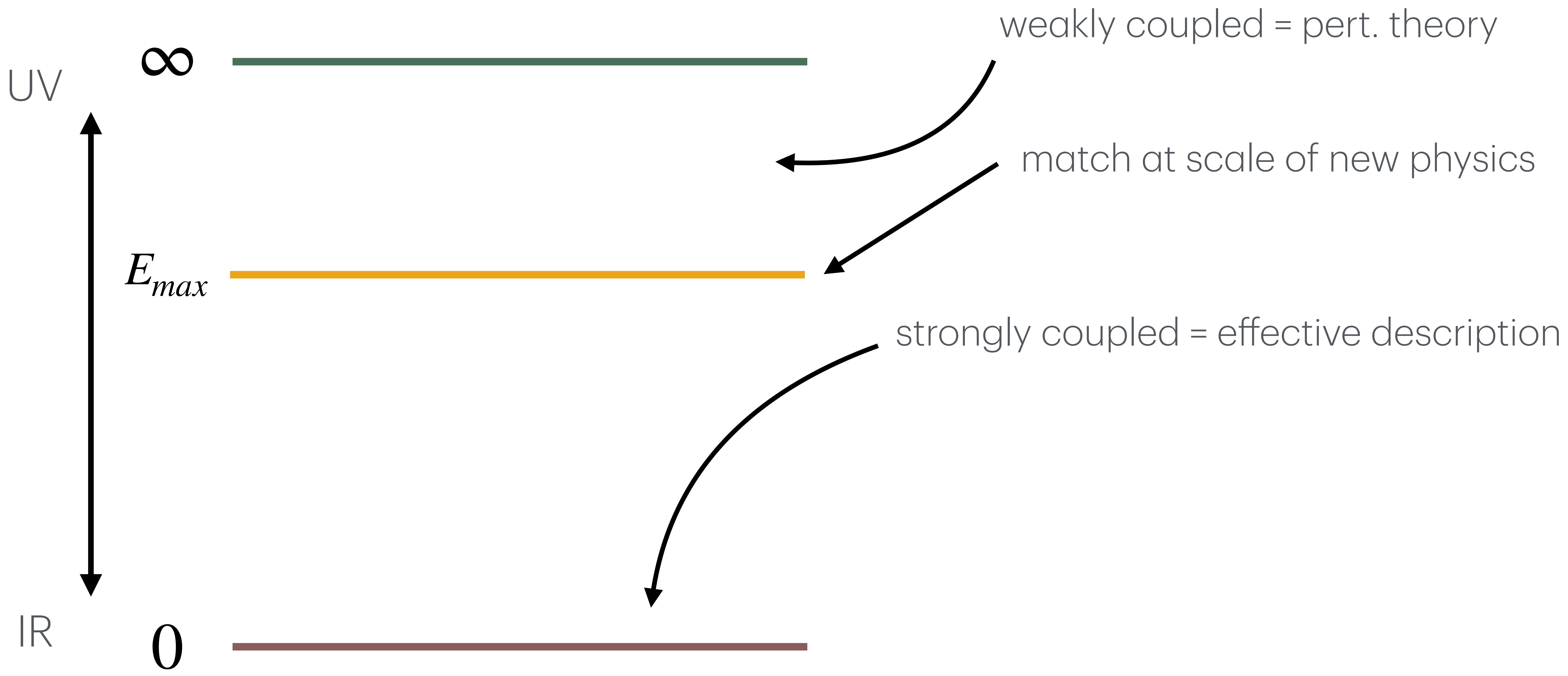

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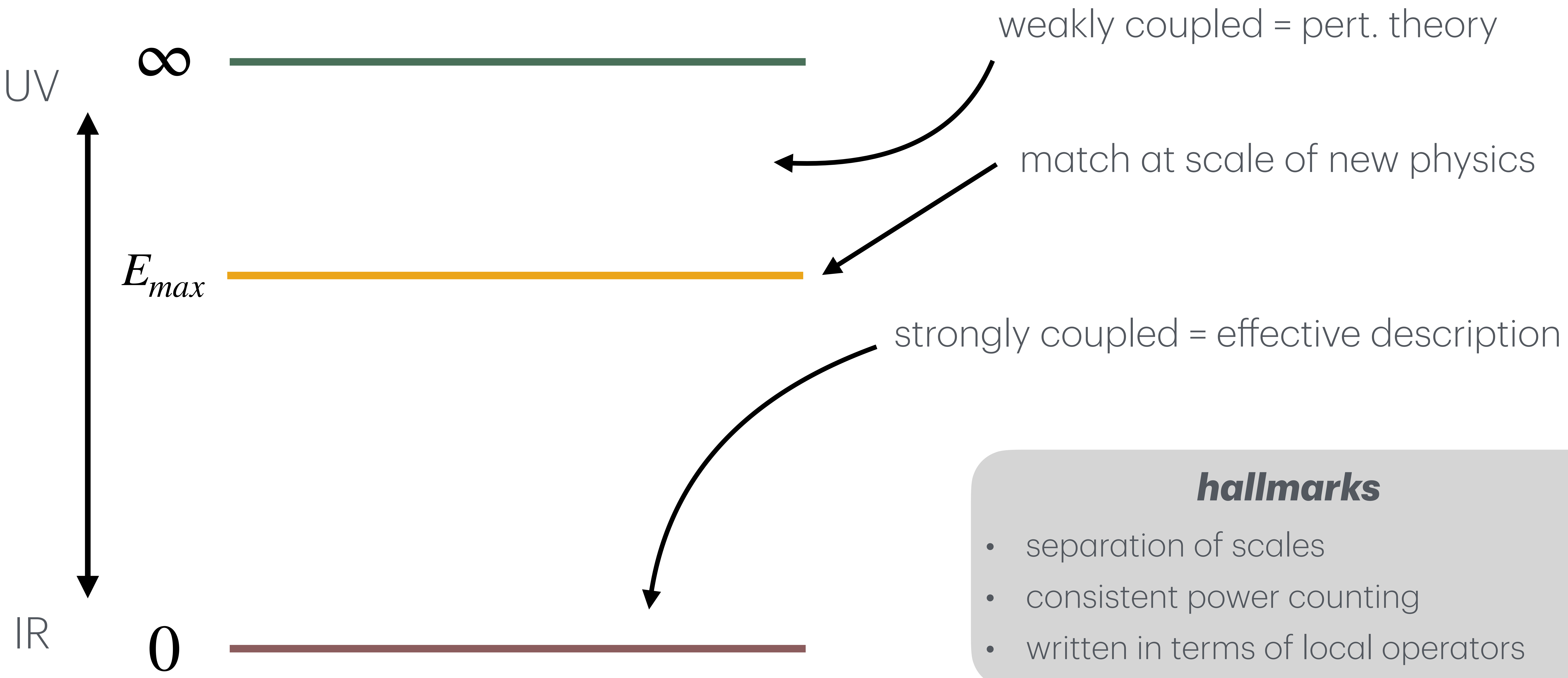

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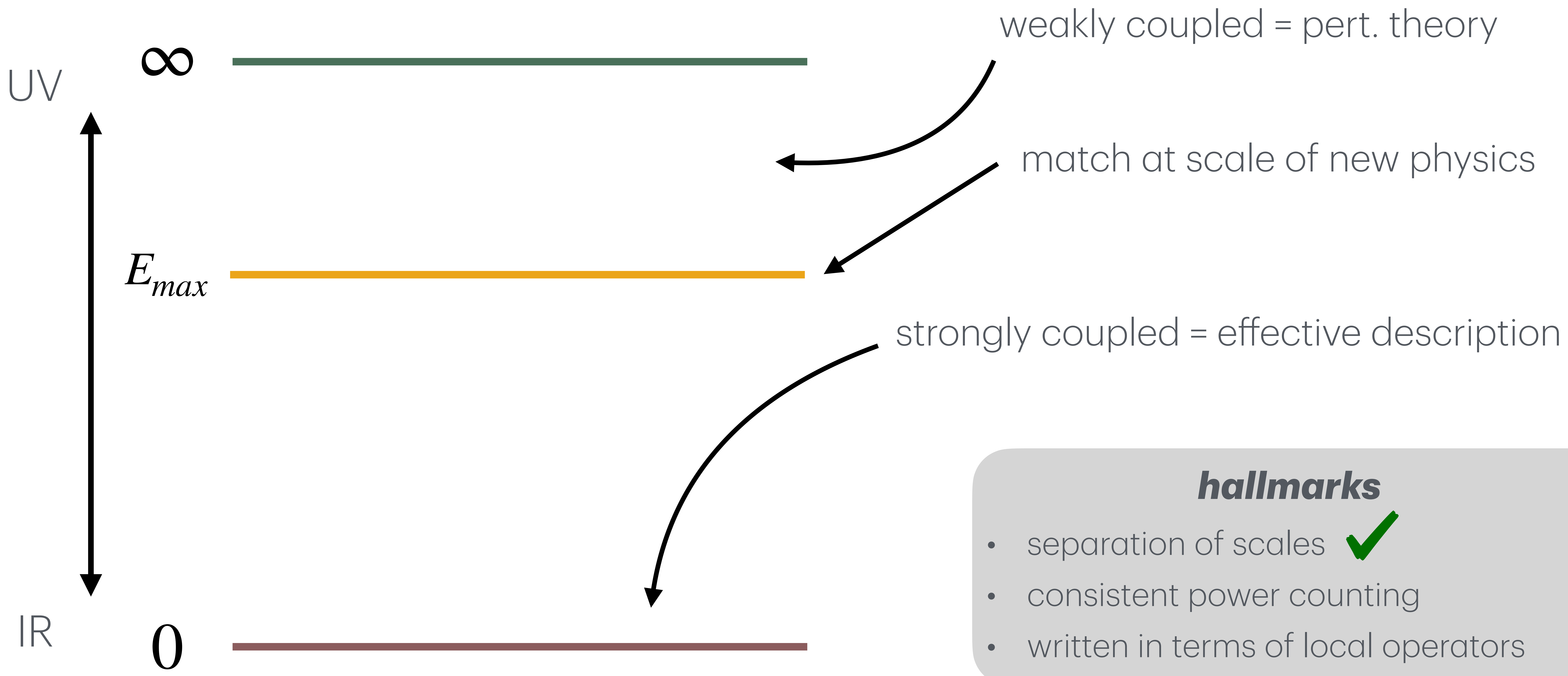

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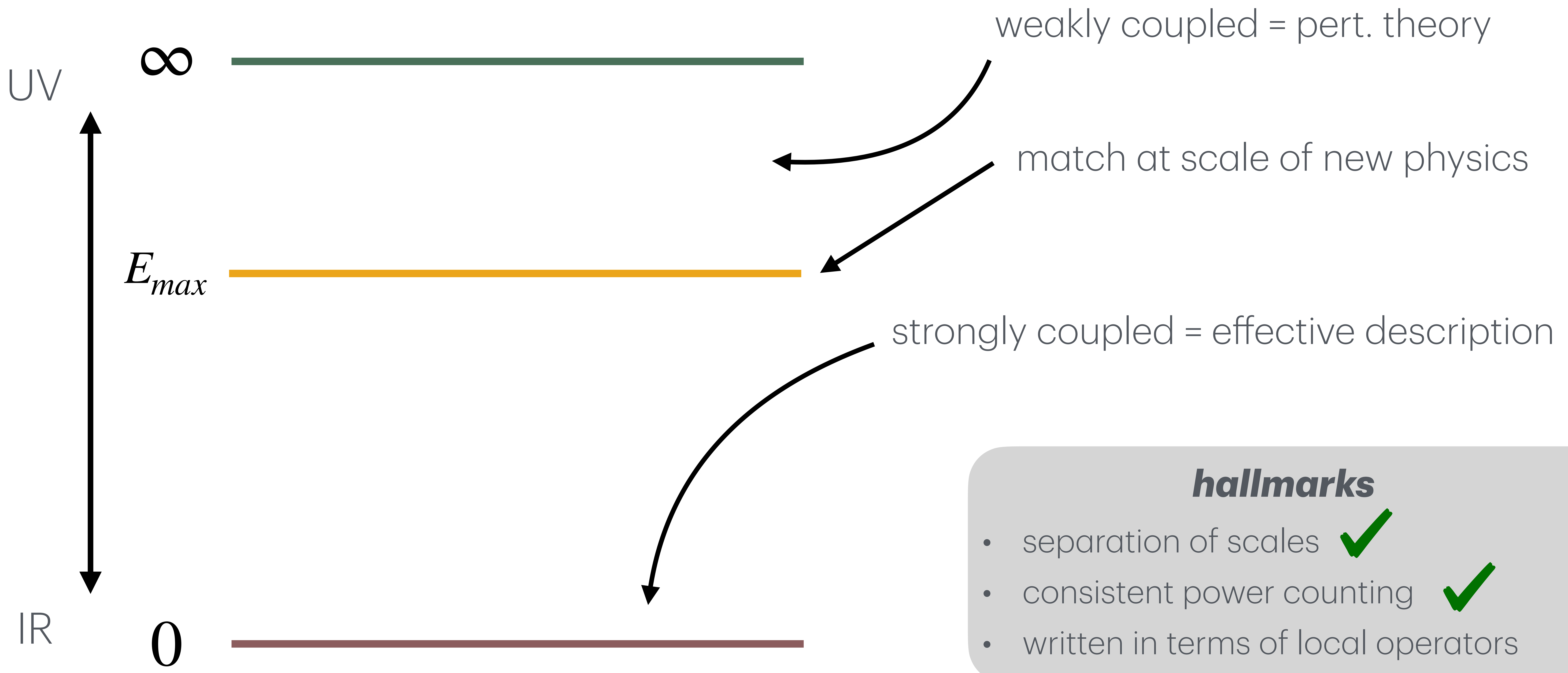

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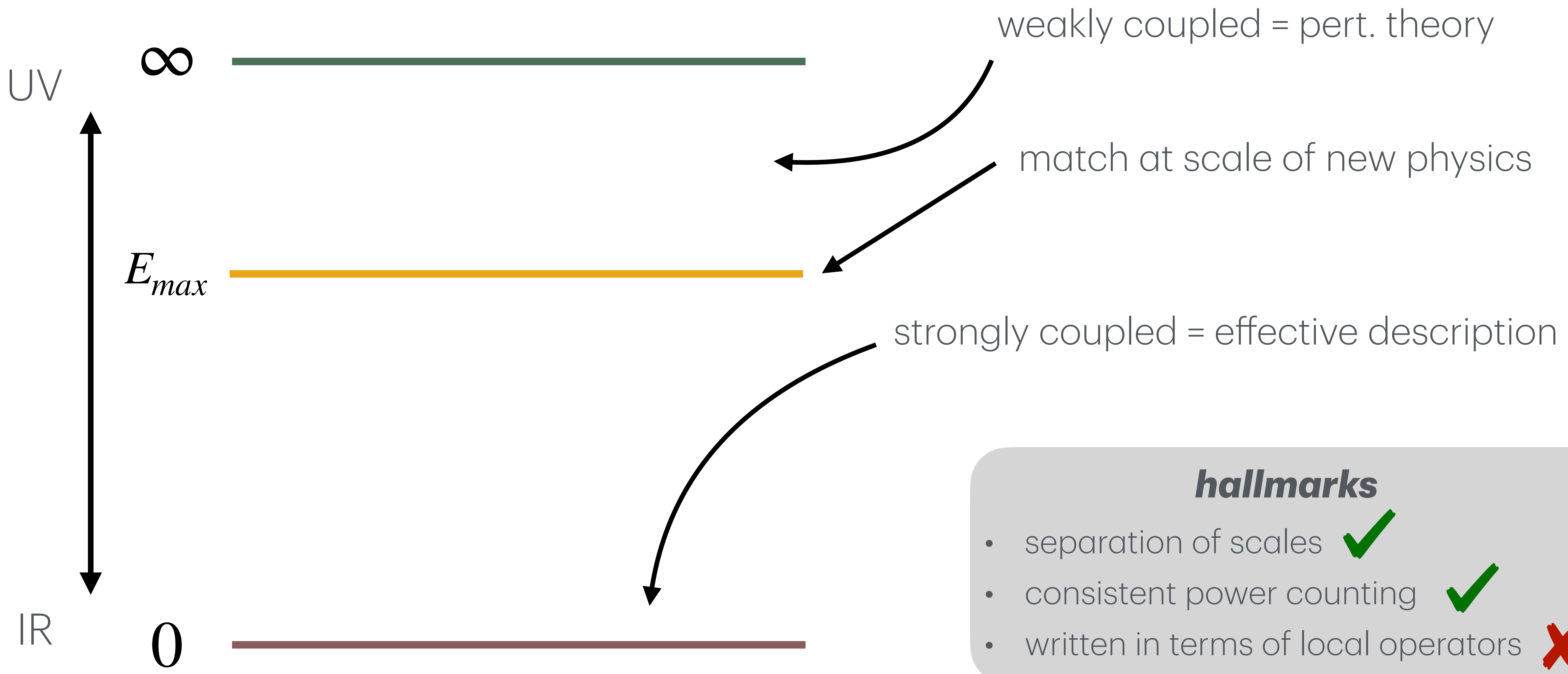

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Cohen, KF, Houtz, Luty, arXiv: 2110.08273

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full theory: $H | \Psi_i \rangle = \mathcal{E}_i | \Psi_i \rangle$ $\mathcal{E}_i = E_i + \mathcal{E}_{1i} + \mathcal{E}_{2i} + \cdots$

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effective theory: $H_{eff} = \underbrace{H_0 + H_1}_{H_{trunc}} + \underbrace{H_2 + \cdots}_{H_{corr}}$ $\qquad H_n = \mathcal{O}(V^n)$

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

What to match?

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eigenvectors? $\langle \Psi_f|i\rangle \Rightarrow \langle f|H_1|i\rangle = \langle f|V|i\rangle$ uniquely fixes H_{eff}

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

Example Theory 1: 2D $\lambda\phi^4$

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

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- strongly relevant interaction
- $\mathbb{Z}_2$ $(\phi \to -\phi)$ symmetry broken at strong coupling = phase transition
  - in universality class of Ising Model: good check

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

Power Counting for 2D $\lambda\phi^4$

$$[\lambda] = 2, \quad [\phi] = 0$$

$$H_n \simeq \frac{\lambda^n}{E_{max}^{2n-2}} \int dx \, (\textit{dimensionless})$$

$$H_2 \simeq \frac{\lambda^2}{E_{max}^2} \int dx \left(\phi^2 + \phi^4 + \mathcal{O}\left(\frac{E_f}{E_{max}}, \frac{R^{-1}}{E_{max}} \right) \right)$$

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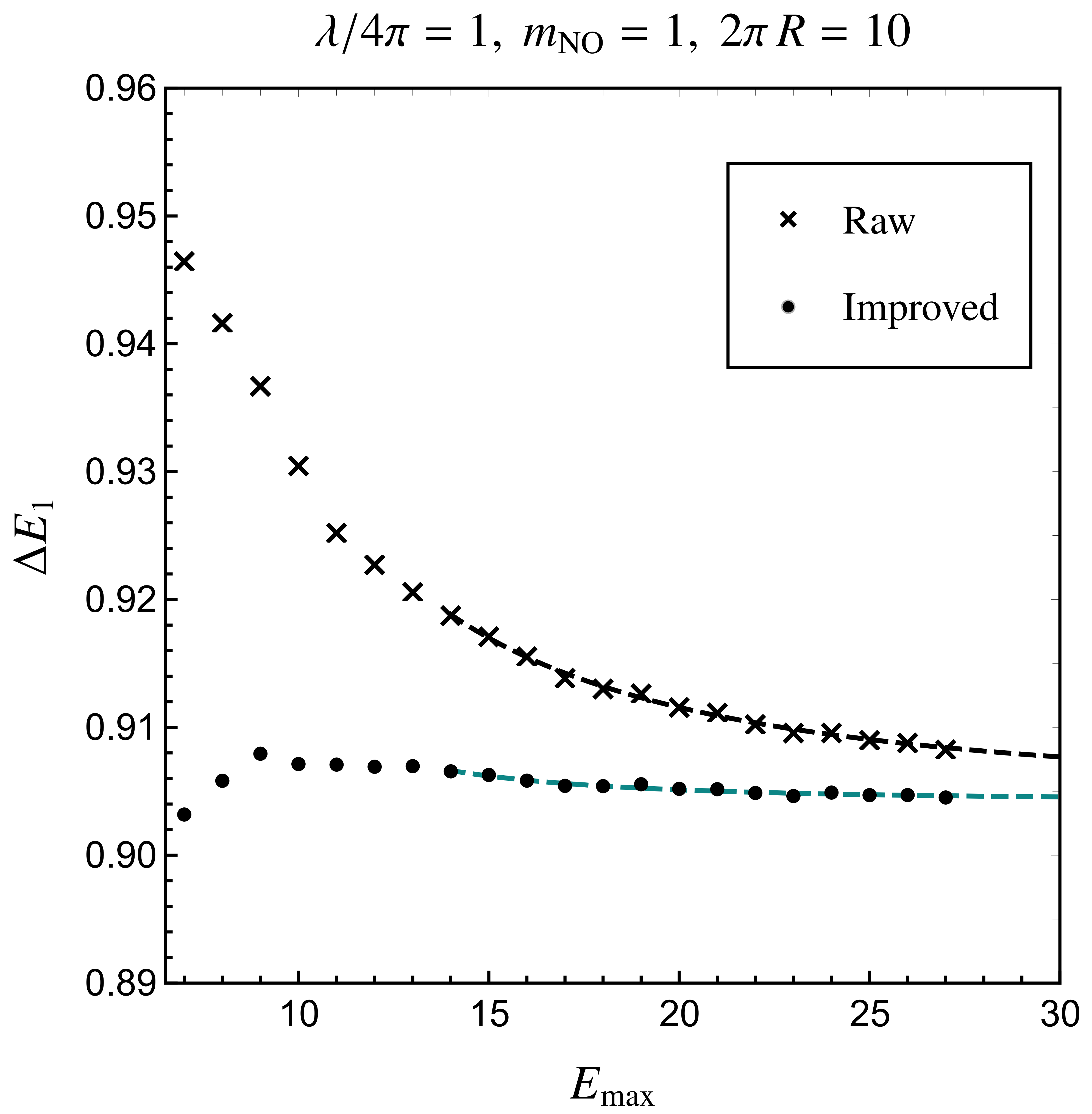
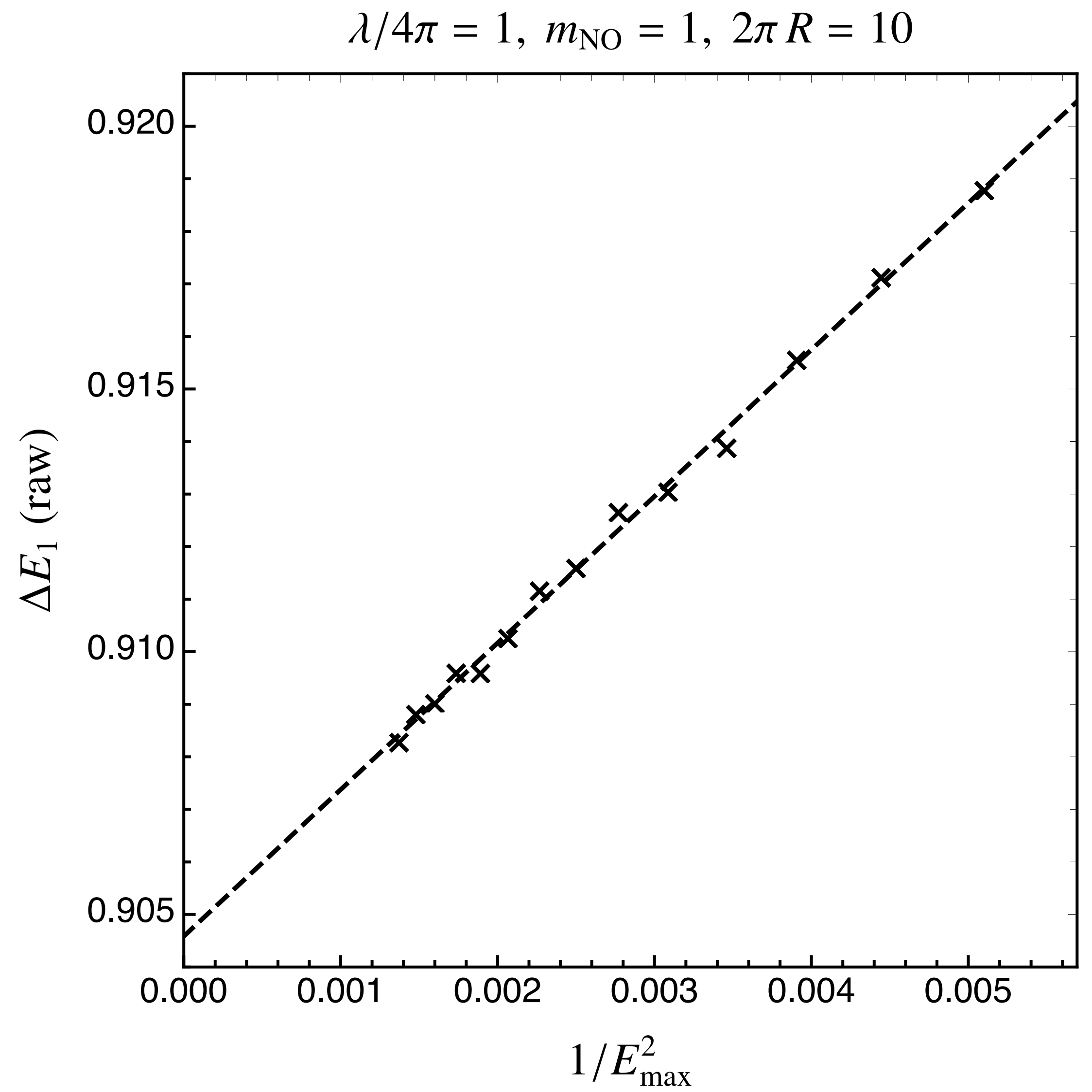
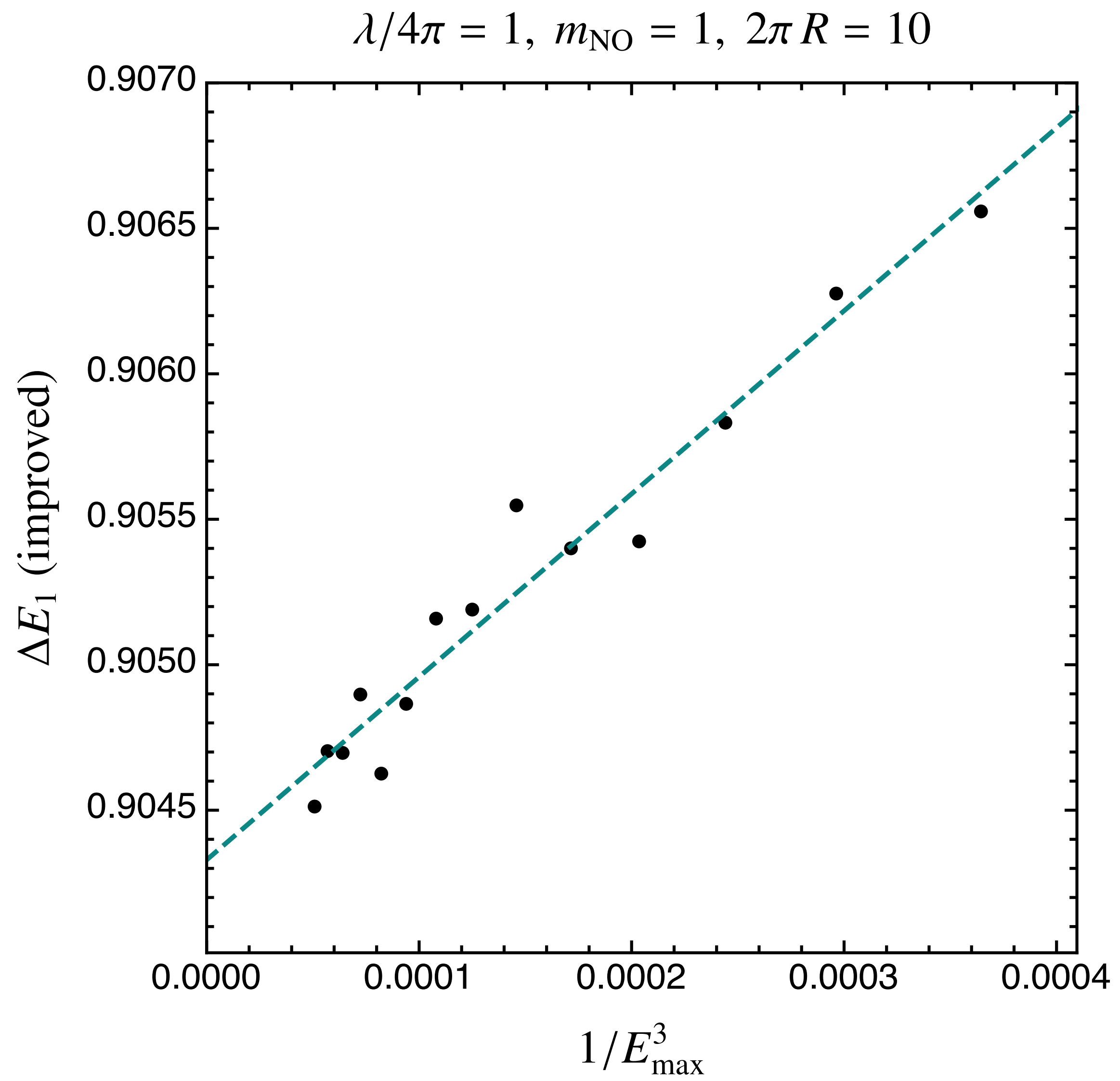
What we expect:

- error $\sim 1/E_{max}^2$ for raw truncation
- error $\sim 1/E_{max}^3$ after including corrections
- phase transition:
  - $\mathbb{Z}_2$ symmetry breaking at critical coupling
  - 2D Ising model

Scaling raw vs. improved







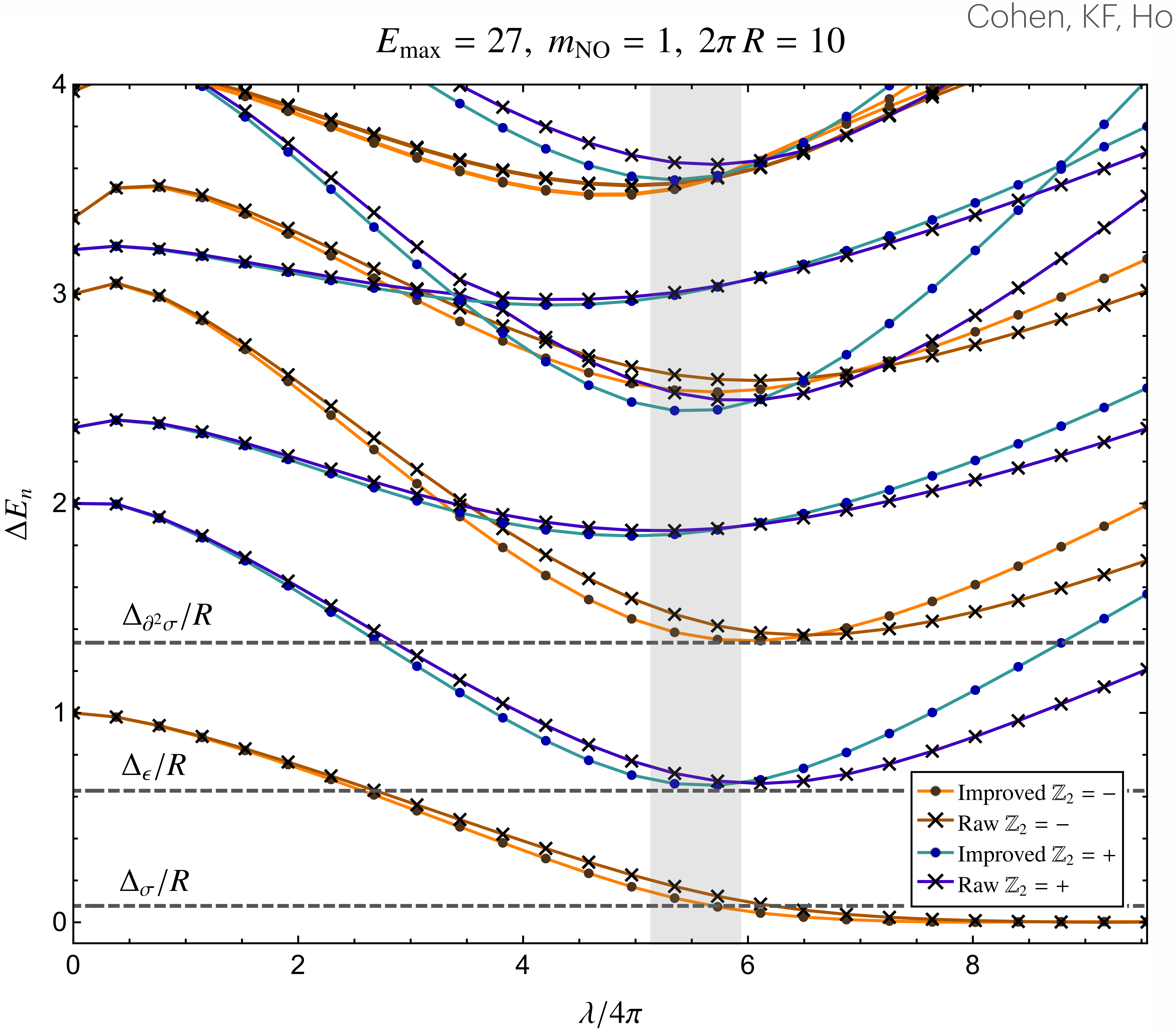
without EFT

$\sim 1/E_{max}^2$

with EFT

$\sim 1/E_{max}^3$

Cohen, KF, Houtz, Luty, arXiv: 2110.08273



What about 4D QCD?

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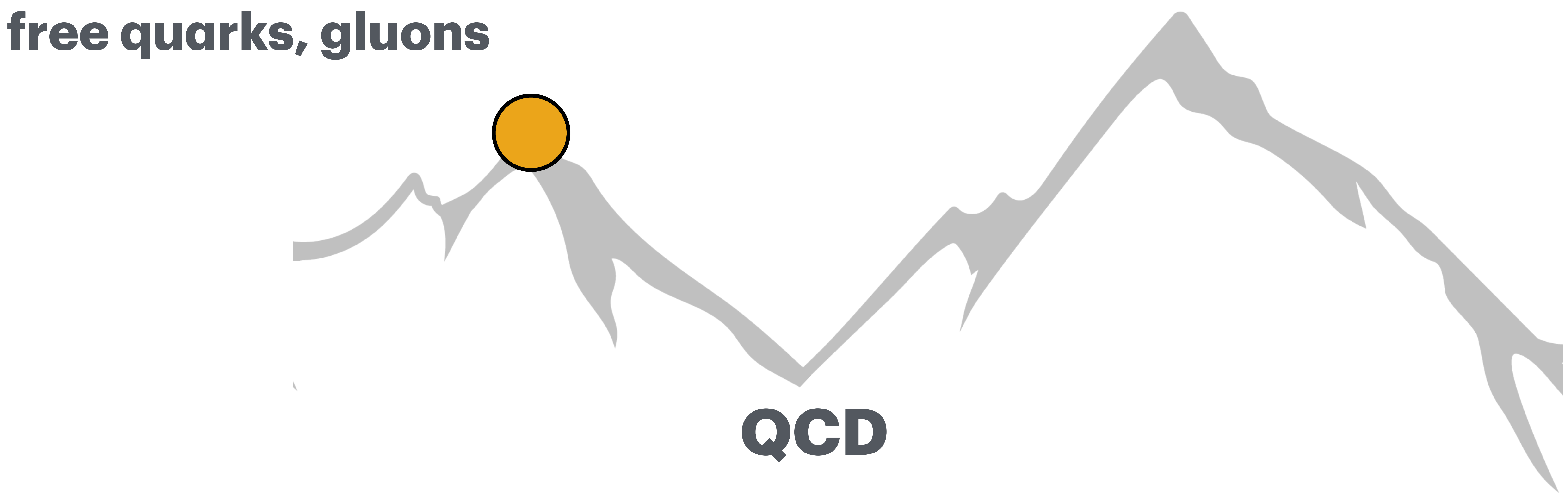

What about 4D QCD?

free quarks, gluons

$$+ gA^{\mu}J_{\mu} + g\partial A^{3} + g^{2}A^{4}$$

- marginal (convergence unknown)
- not gauge-invariant

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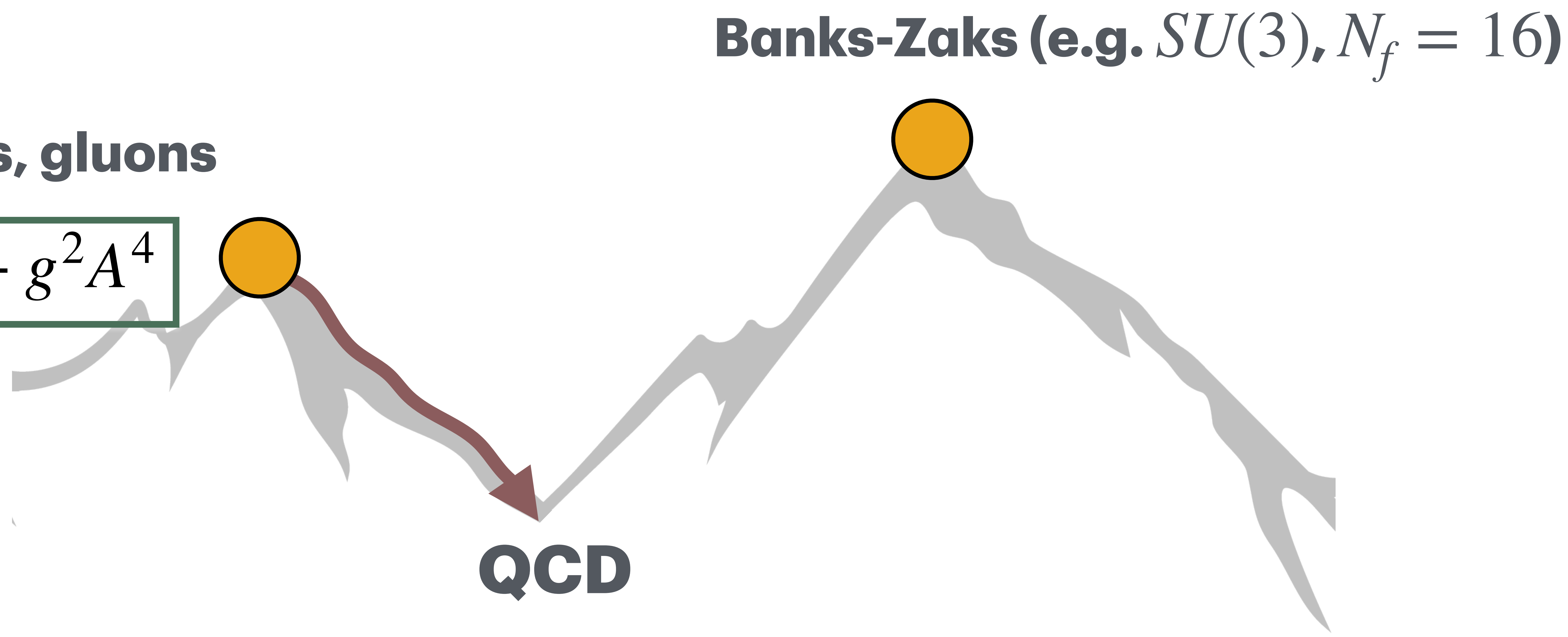

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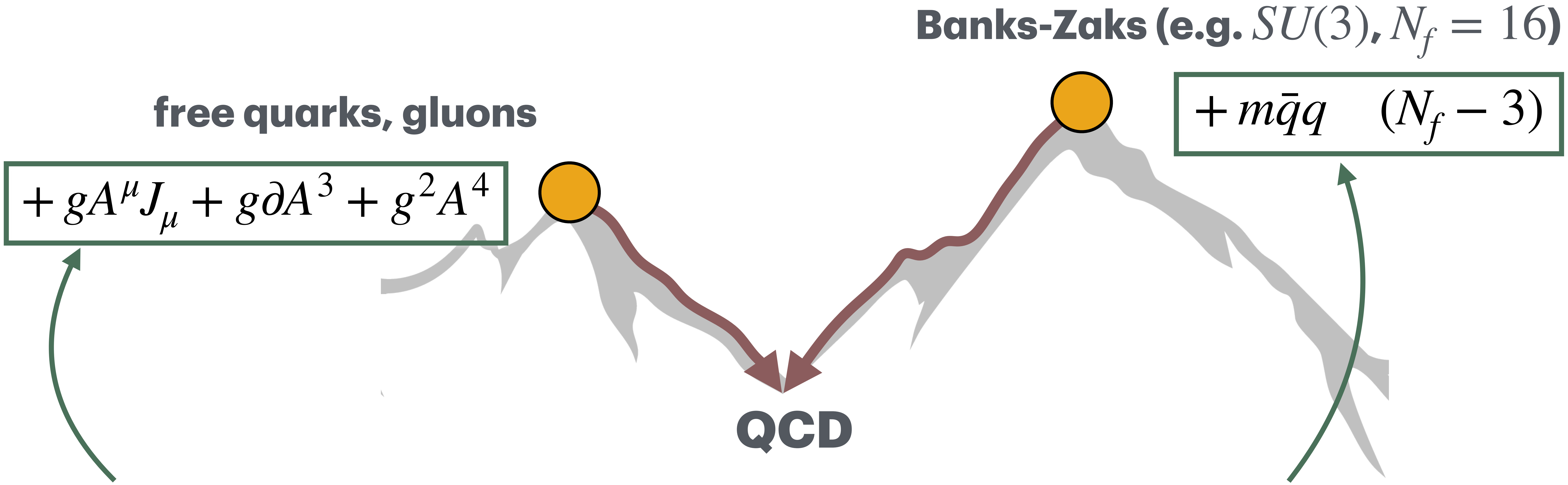


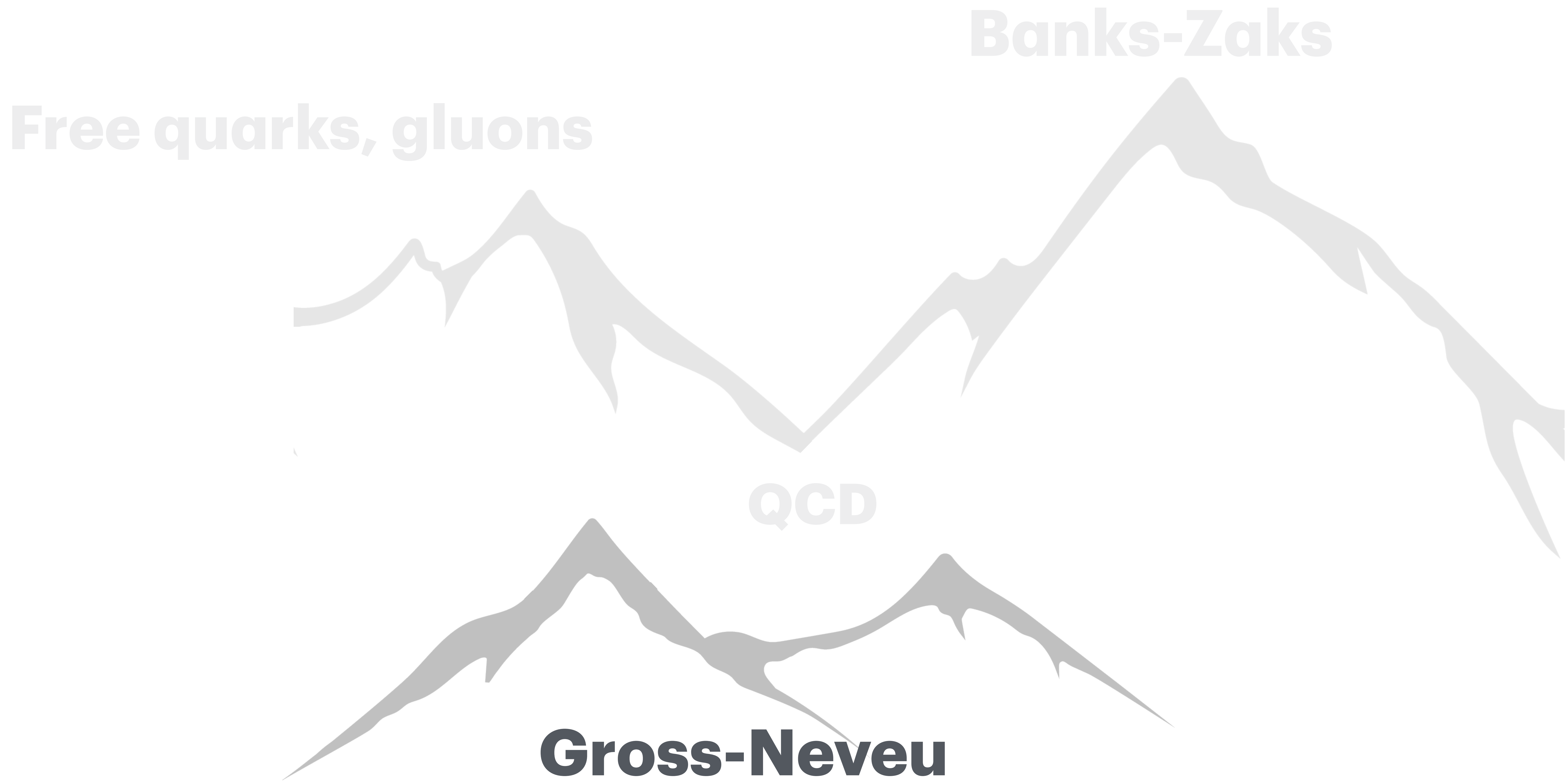
- marginal (convergence unknown)
- not gauge-invariant

- relevant (convergence under control)
- gauge-invariant

First step: 2D Gross-Neveu

in progress with Rachel Houtz, Francesco Riva and Dorian Wenzel

First step: 2D Gross-Neveu



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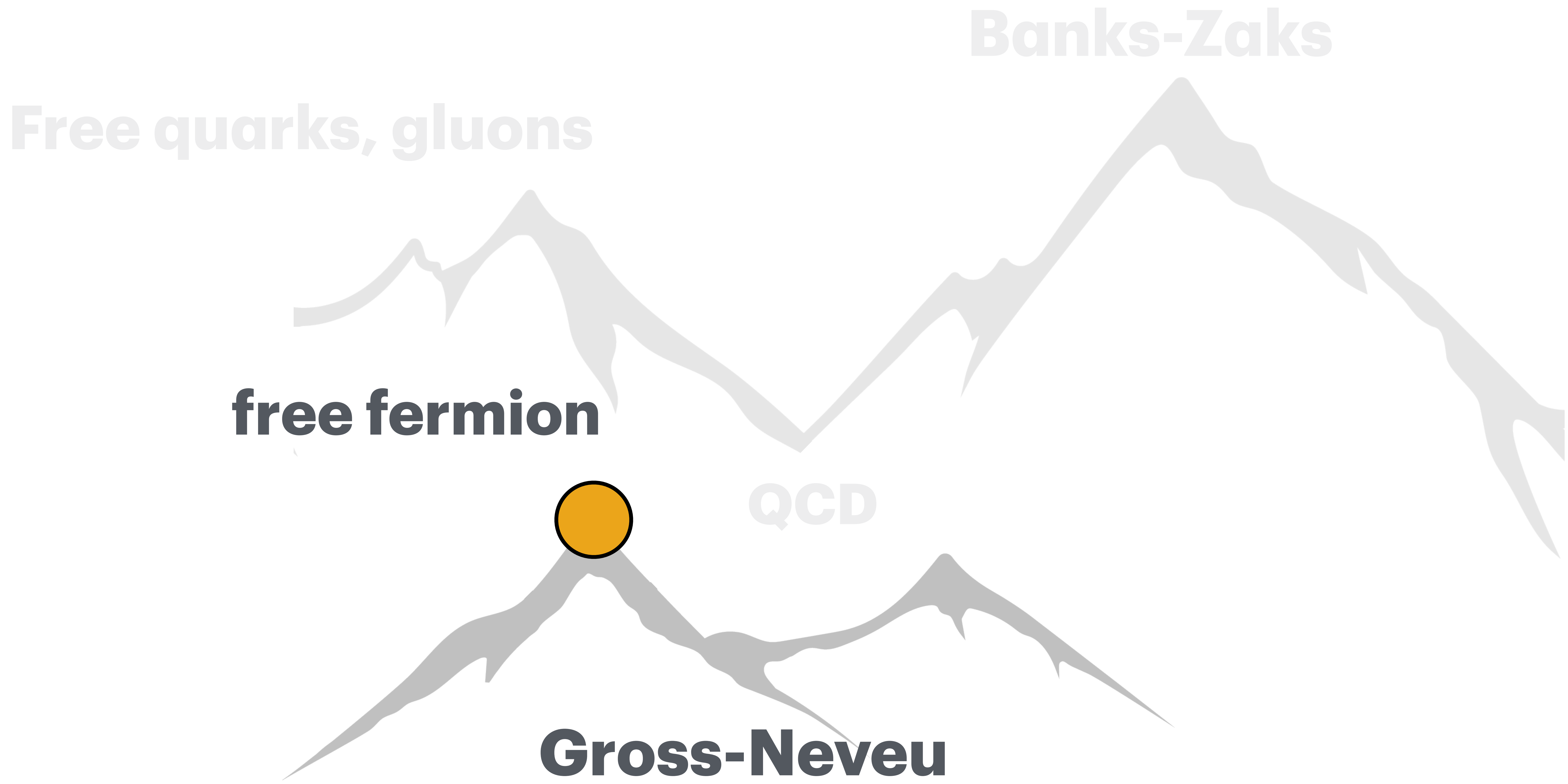

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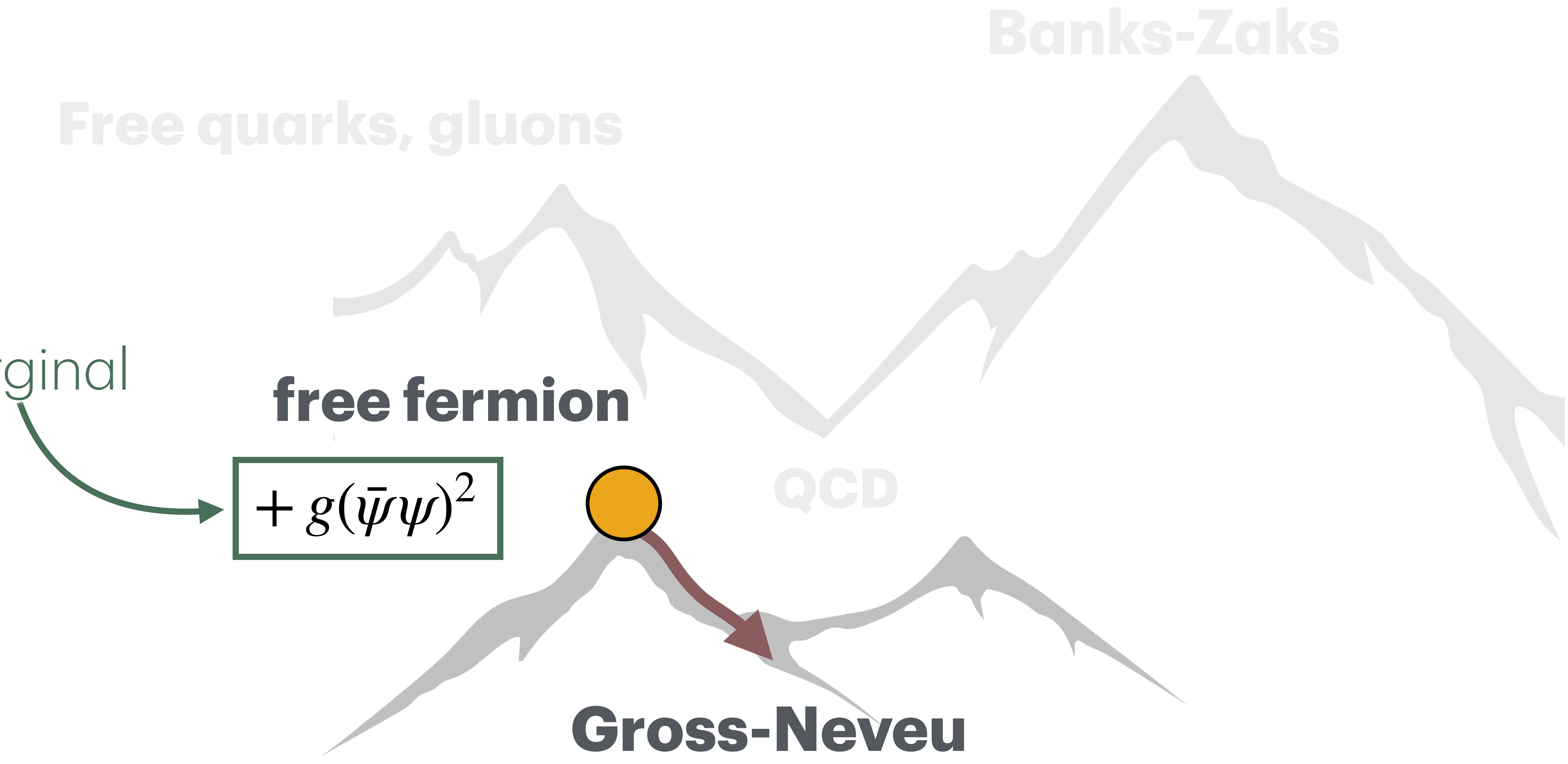

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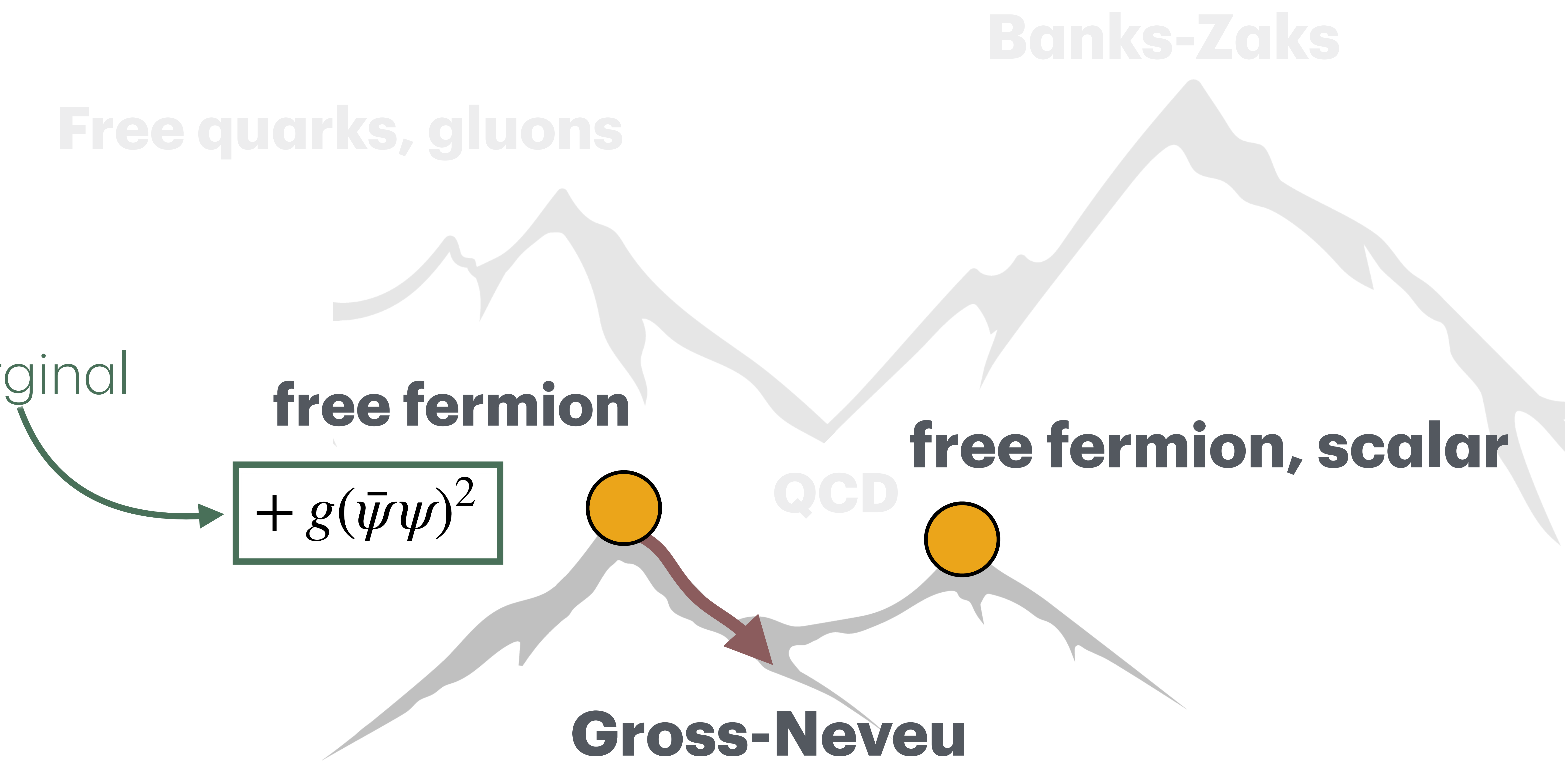

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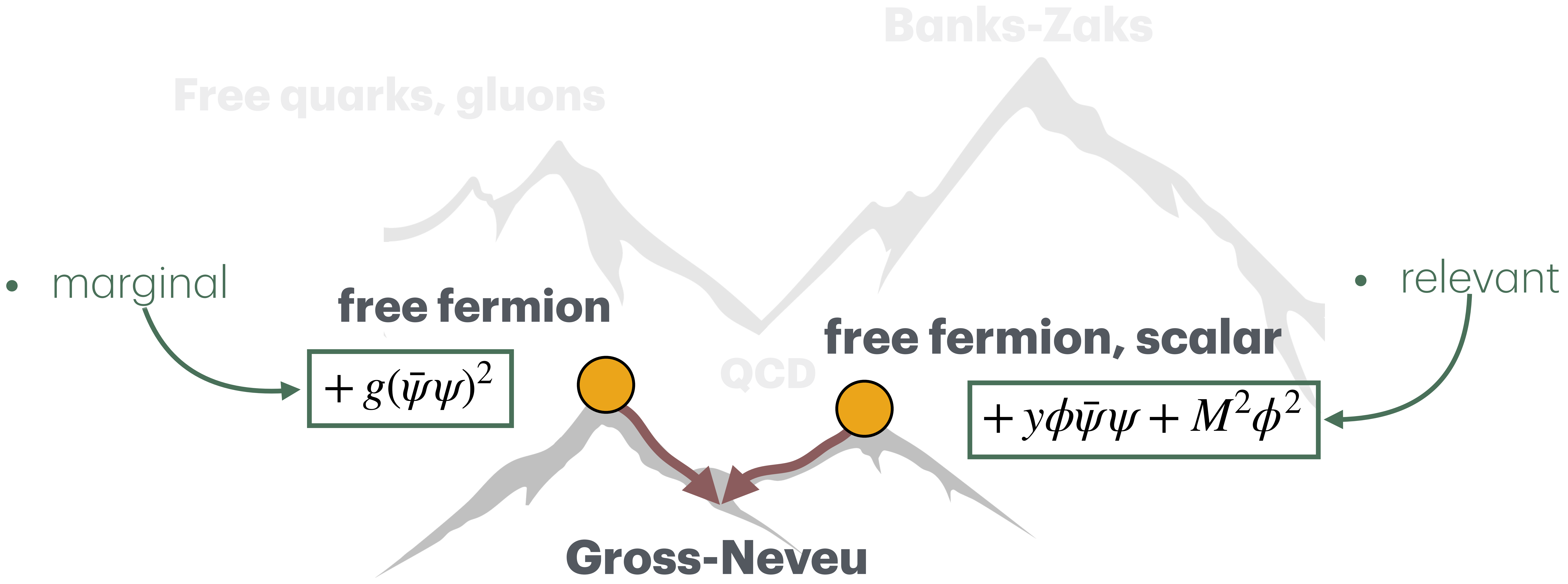

Simpler testing ground on way to QCD

in progress with Rachel Houtz, Francesco Riva and Dorian Wenzel

in progress with Dorian Wenzel

Example Theory 2: 2D Yukawa

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi + y\phi\bar{\psi}\psi$$

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in progress with Dorian Wenzel

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H_0 $\qquad$ V

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$$\phi(x,t) = \phi(x + 2\pi R, t)$$

$$\psi(x,t) = \psi(x + 2\pi R, t)$$

- now with **real** UV divergences at $\mathcal{O}(y^2)$
    - regulate with local counterterms
    - expect finite after including H_2 (DeLouche et al '23)
- charge conjugation symmetry

in progress with Dorian Wenzel

Power Counting for 2D Yukawa

$$[y] = 1, \quad [\phi] = 0, \quad [\psi] = 1/2$$

$$H_2 \simeq y^2 \int dx \left(\ln(E_{max}) + \phi^2 \ln(E_{max}) + \frac{1}{E_{max}} \bar{\psi}\psi + \frac{H_0}{E_{max}} \phi^2 + \cdots \right)$$

$$H_3 \simeq \frac{y^3}{E_{max}} \int dx \left(\frac{1}{E_{max}} \phi \bar{\psi}\psi + \cdots \right)$$

$$H_4 \simeq \frac{y^4}{E_{max}^2} \int dx \left(1 + \phi^2 + \frac{1}{E_{max}} \bar{\psi}\psi + \cdots \right)$$

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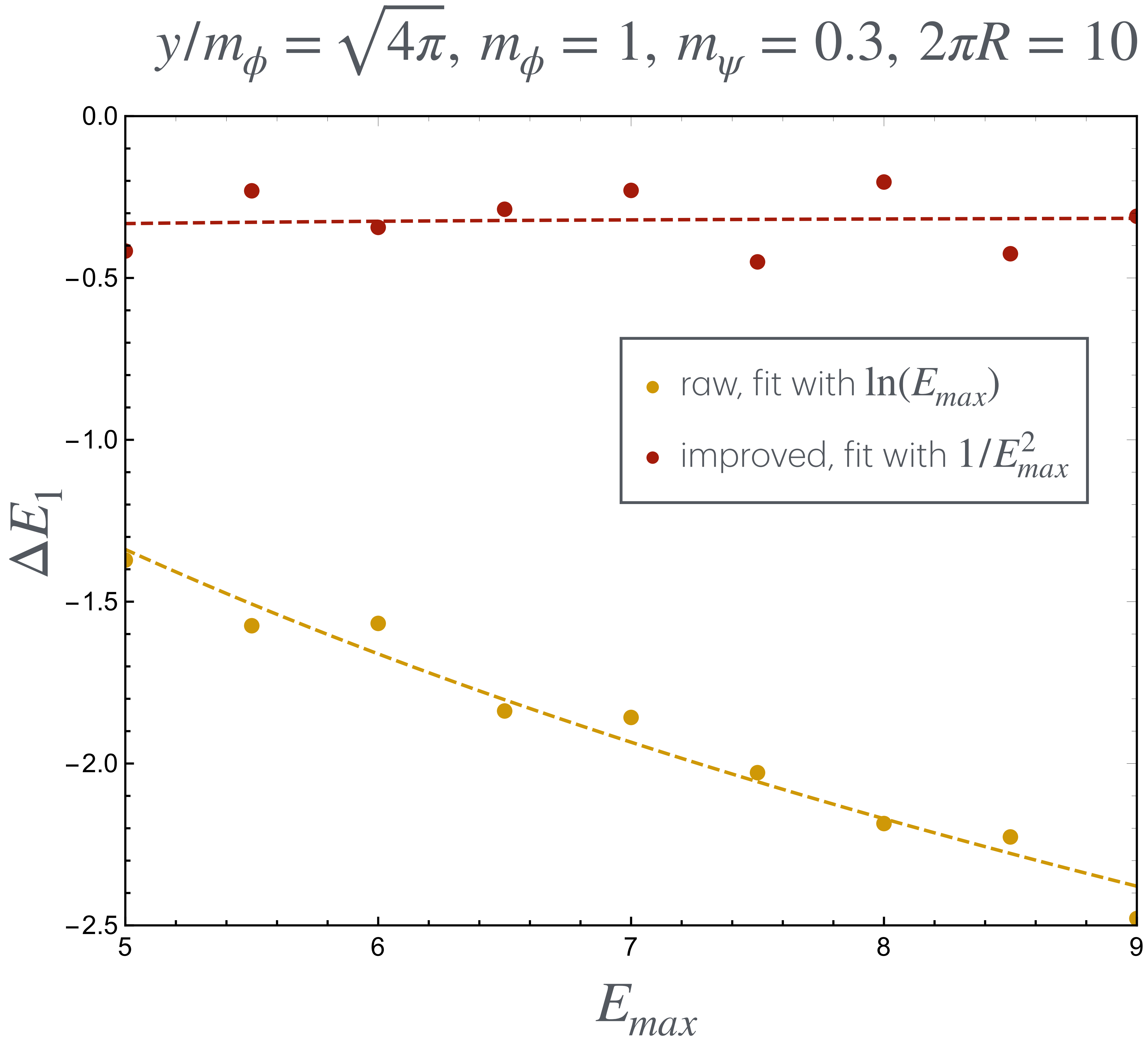
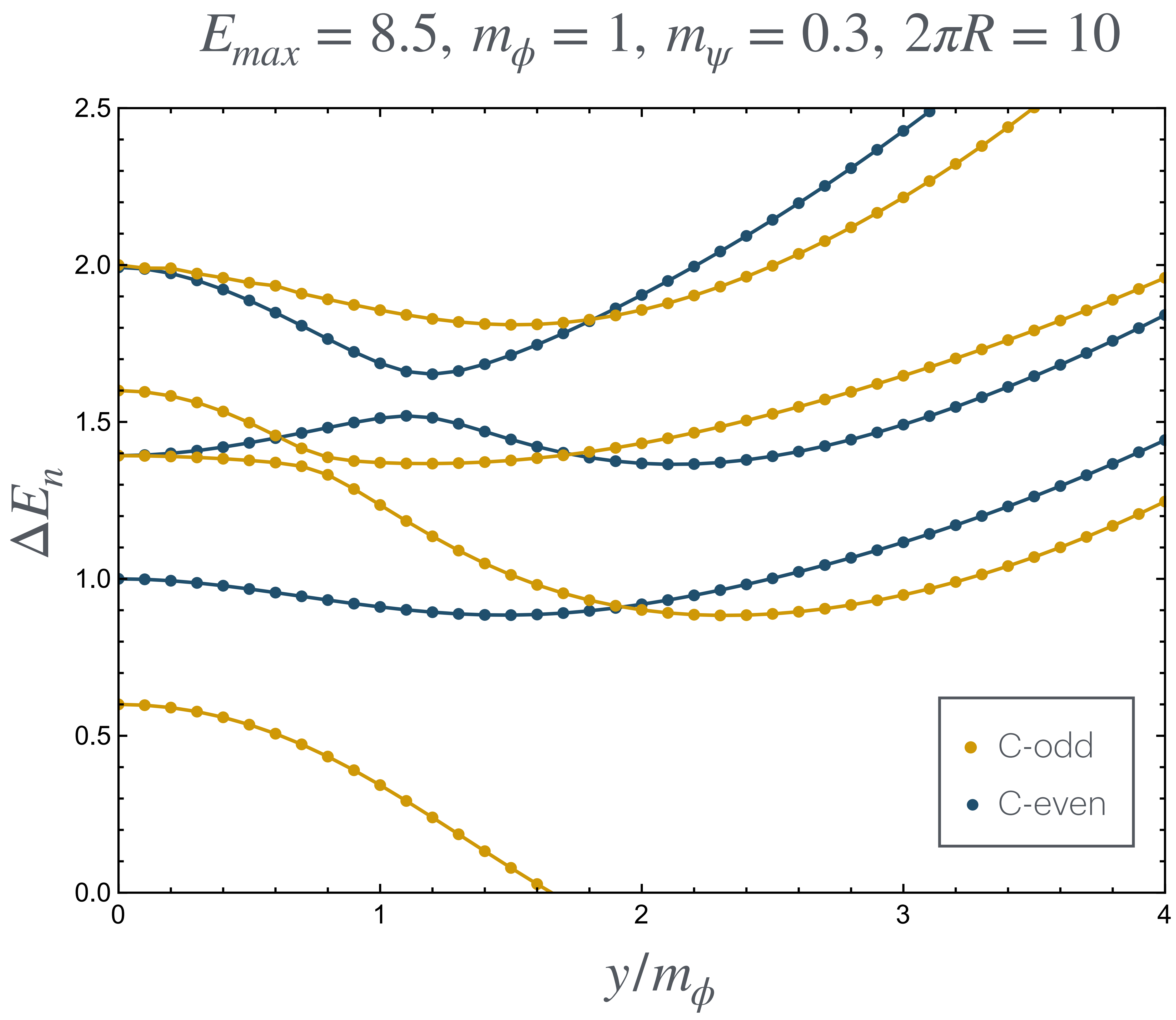
What we expect:

- diverge like $\ln(E_{max})$ with no correction
- error $\sim 1/E_{max}$ including local counterterms
- error $\sim 1/E_{max}^2$ after including corrections

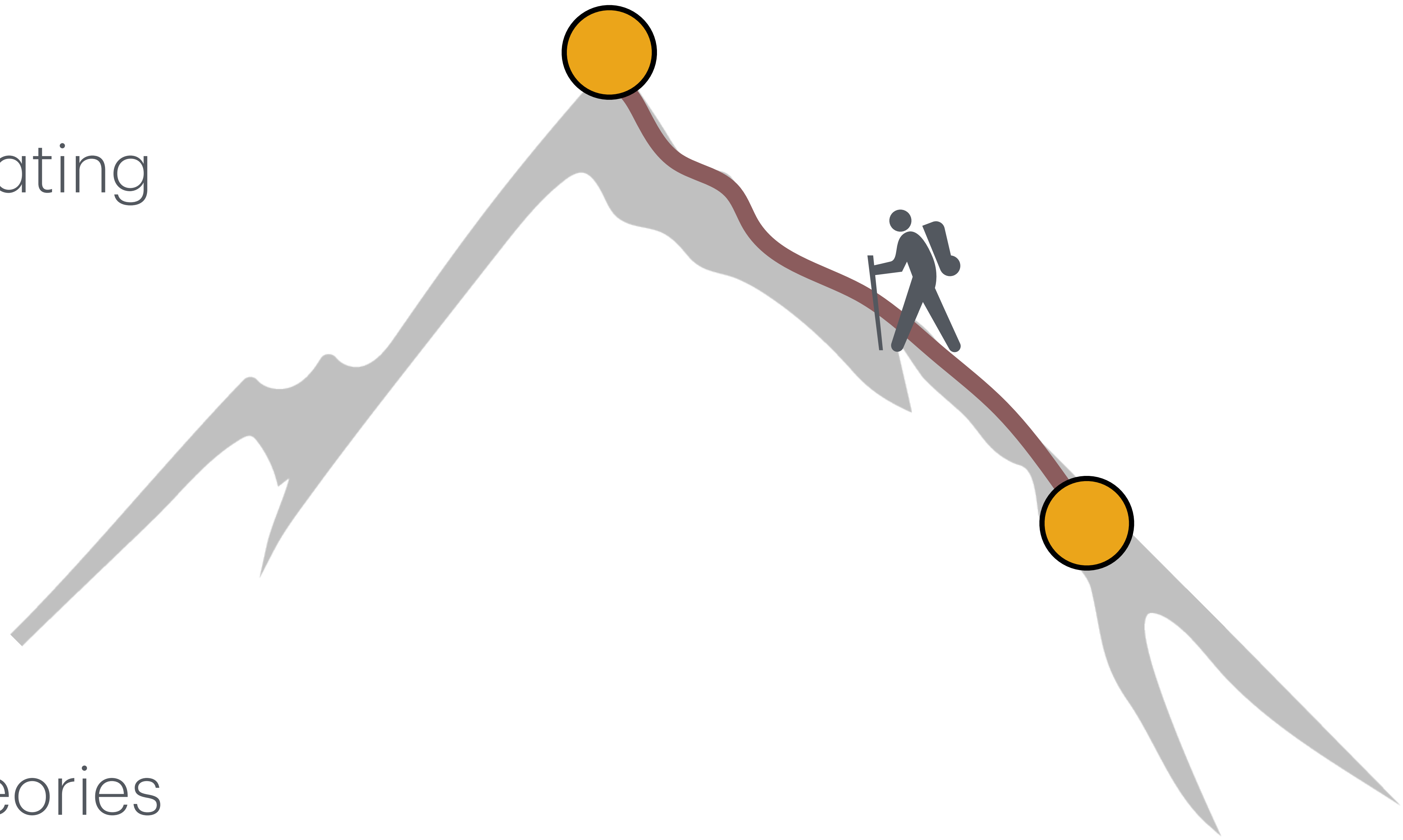
Preliminary plots





Summary

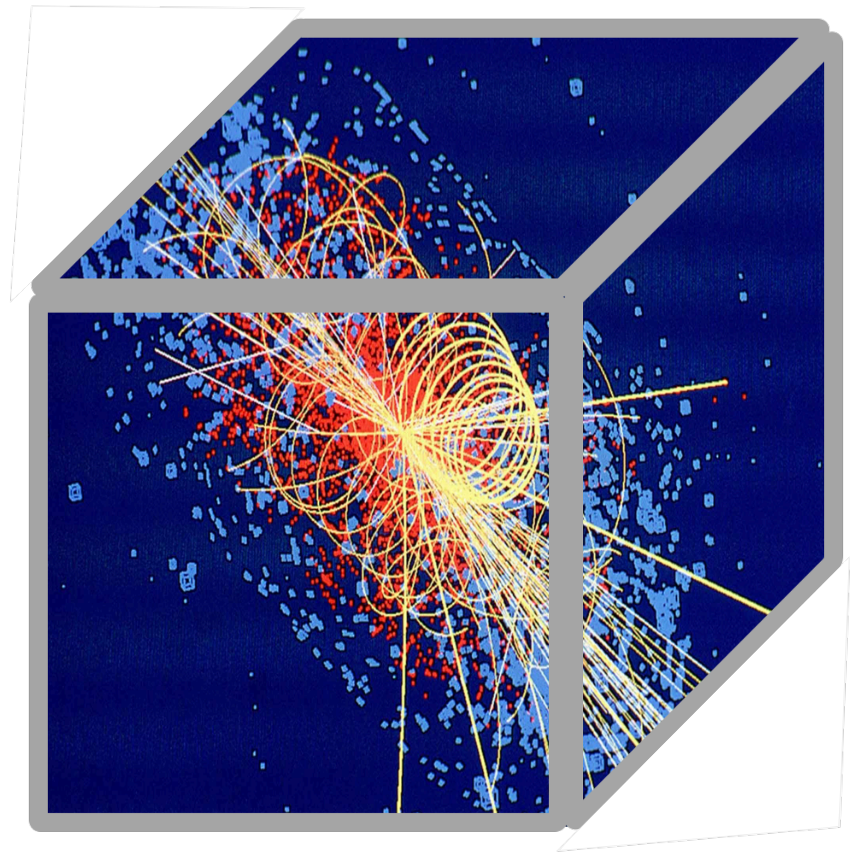
- Hamiltonian truncation = non-perturbative method for computing observables in strongly coupled QFTs
- effective field theory = powerful tool for compensating for ignorance
- effective field theory techniques **systematically improve** convergence in Hamiltonian truncation calculations
- **crucial** for higher dimensions, more complex theories



Future directions

- **improve this method** (next order, include more UV divergences)
    - work in progress with Rachel Houtz
- look at **new observables** (Wilson loops, entanglement entropy, energy correlators)
    - finite volume S-matrix, work in progress with Carl Beadle, Francesco Riva, Matthew Walters
- move to **higher dimensions**
- include **new fields** (gauge bosons)
- **curved spacetime** (cosmology, S-matrix)
- **connection** to other non-perturbative methods
- ... you tell me!

$$S_{fi} = \langle \Psi_f | \Psi_i \rangle \,?$$



Landscape of quantum field theory

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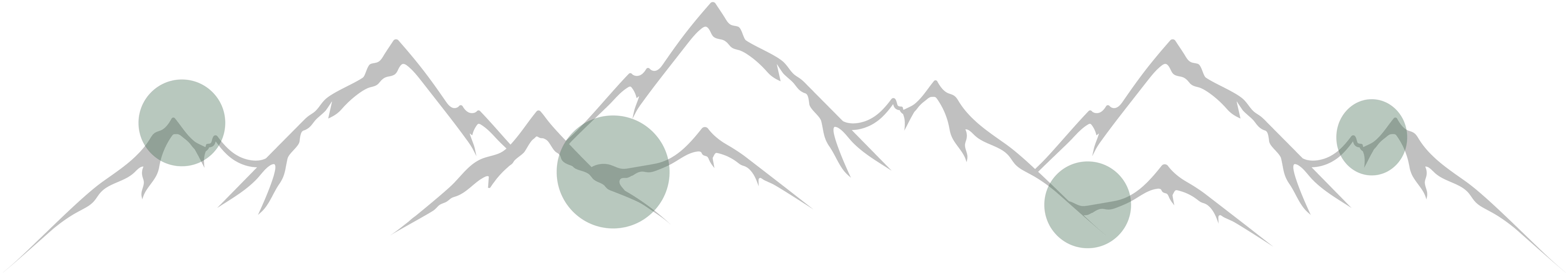

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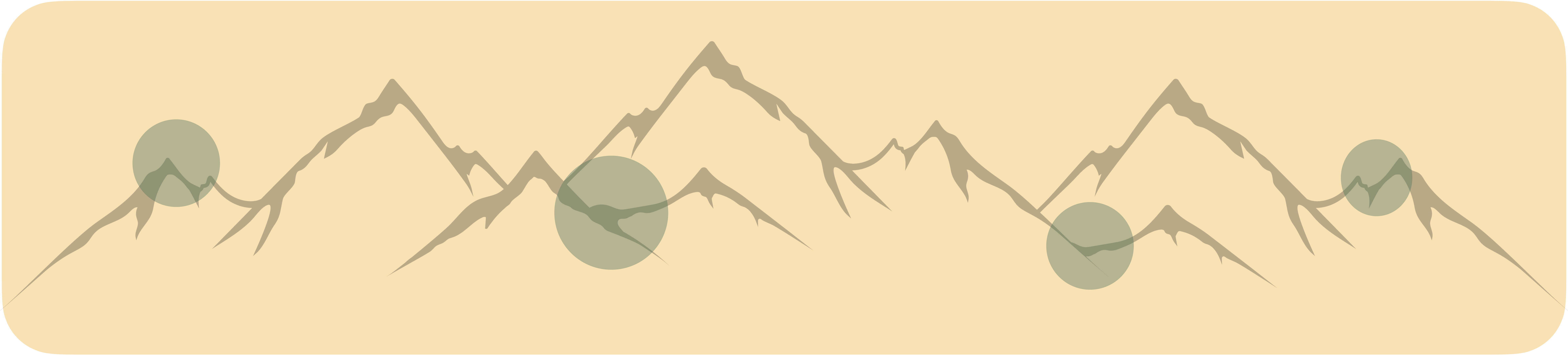
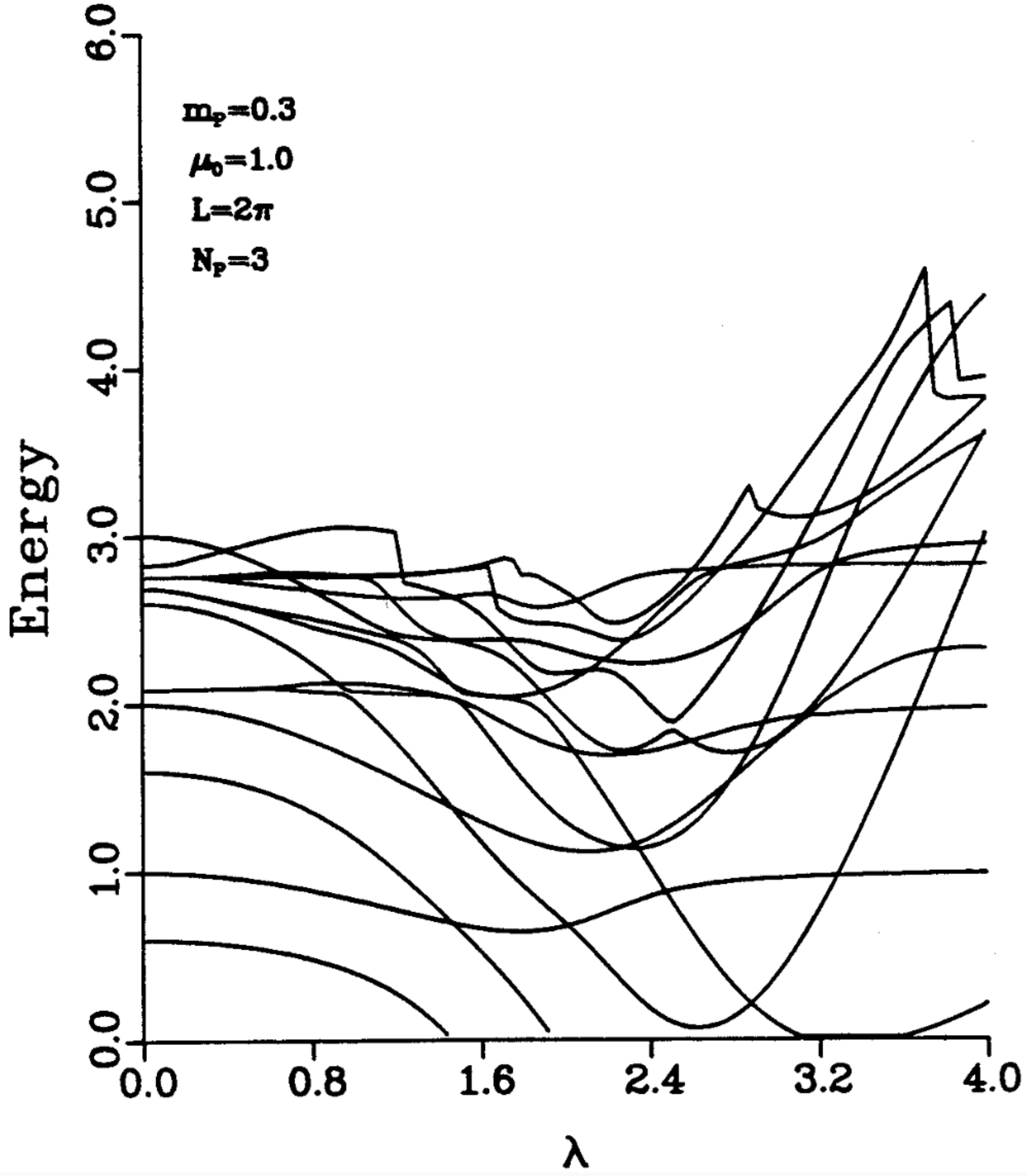
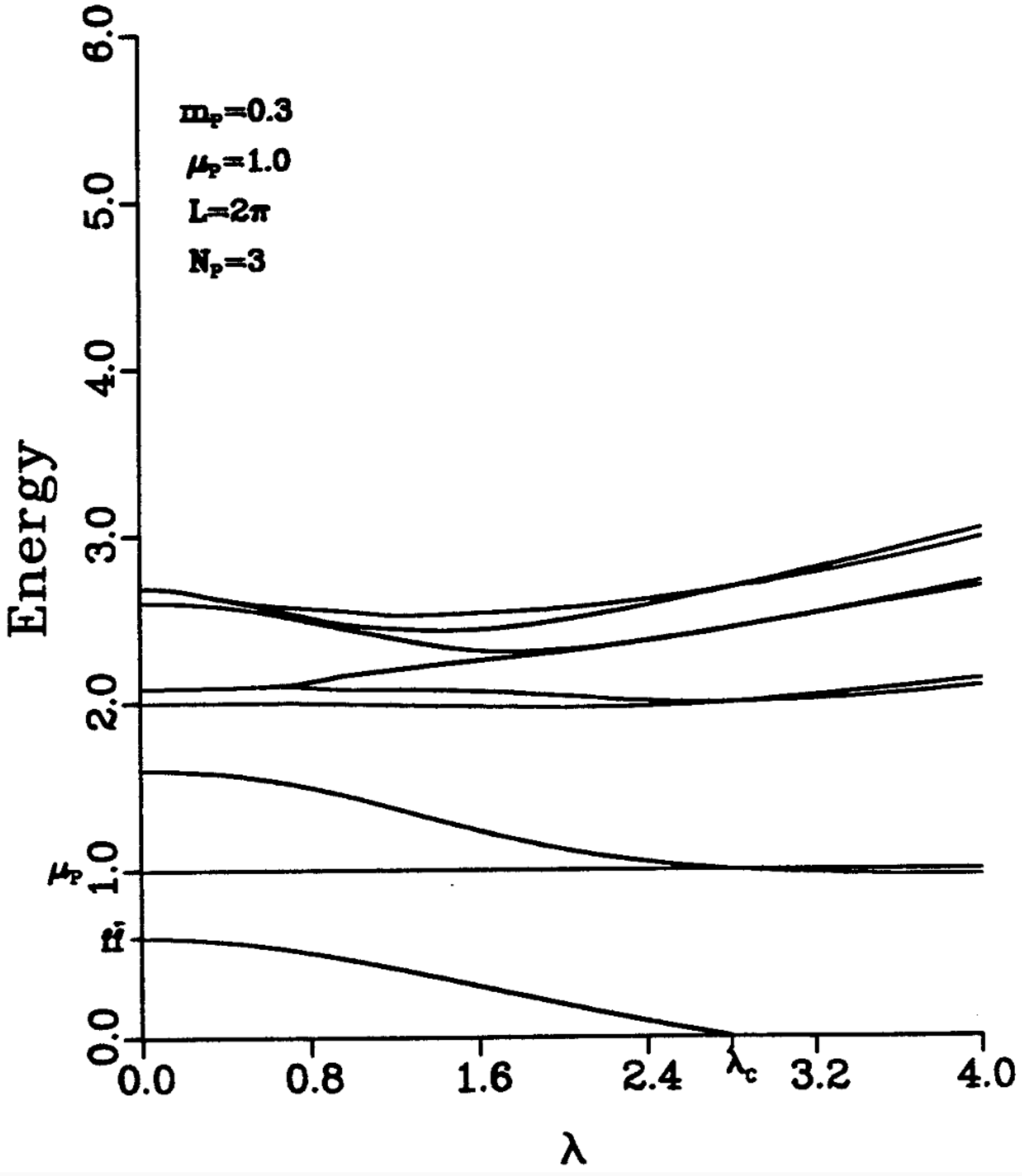


Thank You!

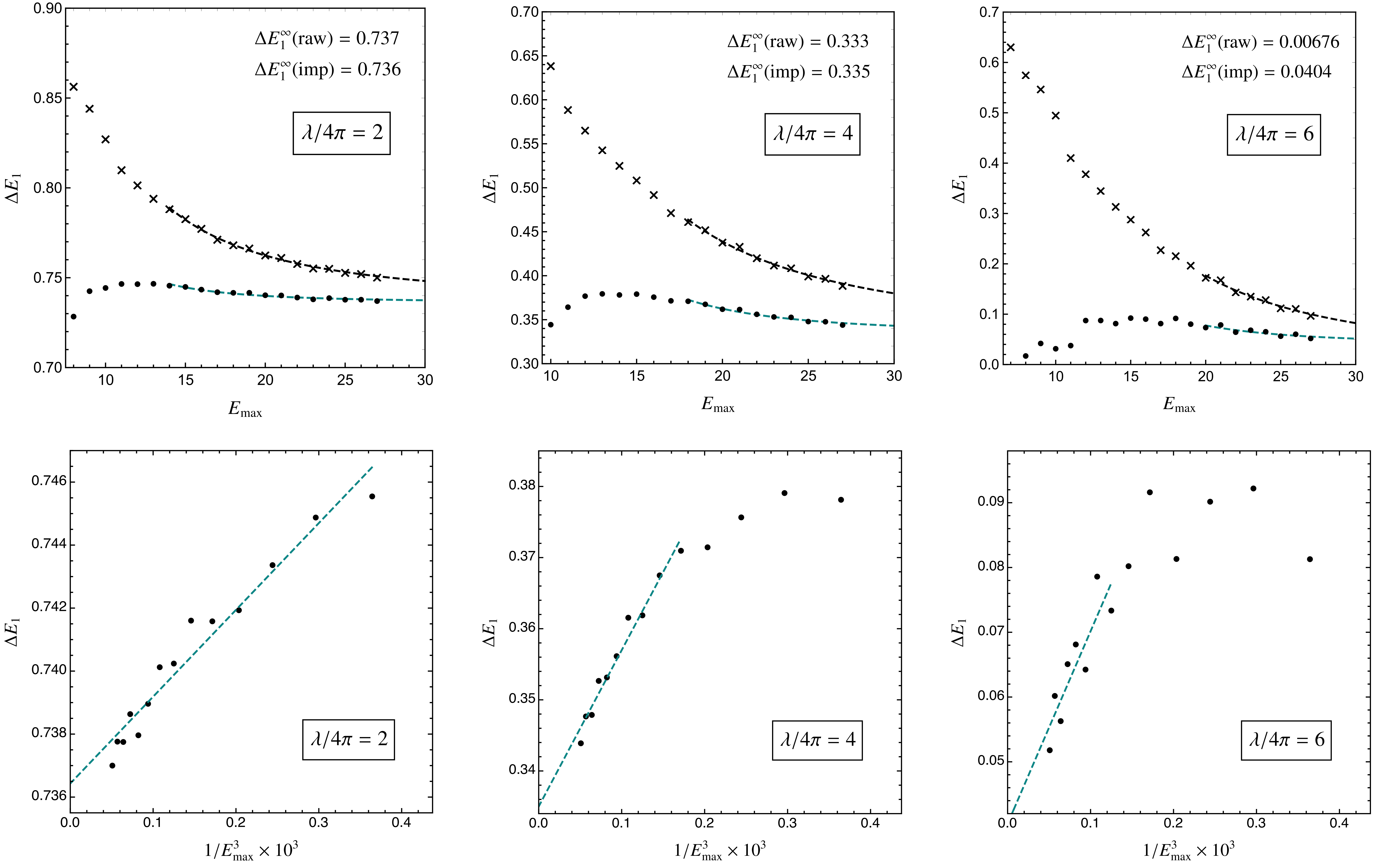
Old paper



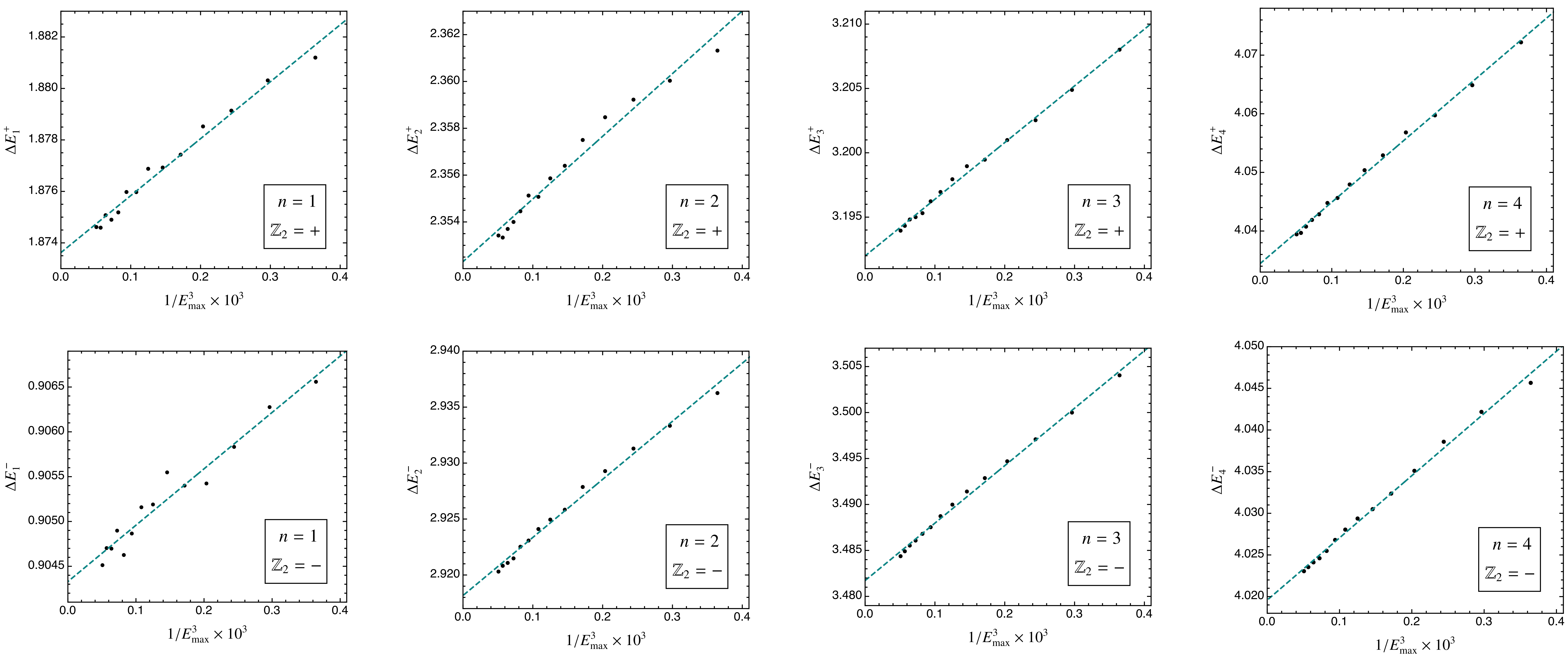


Brooks, Frautschi 1983

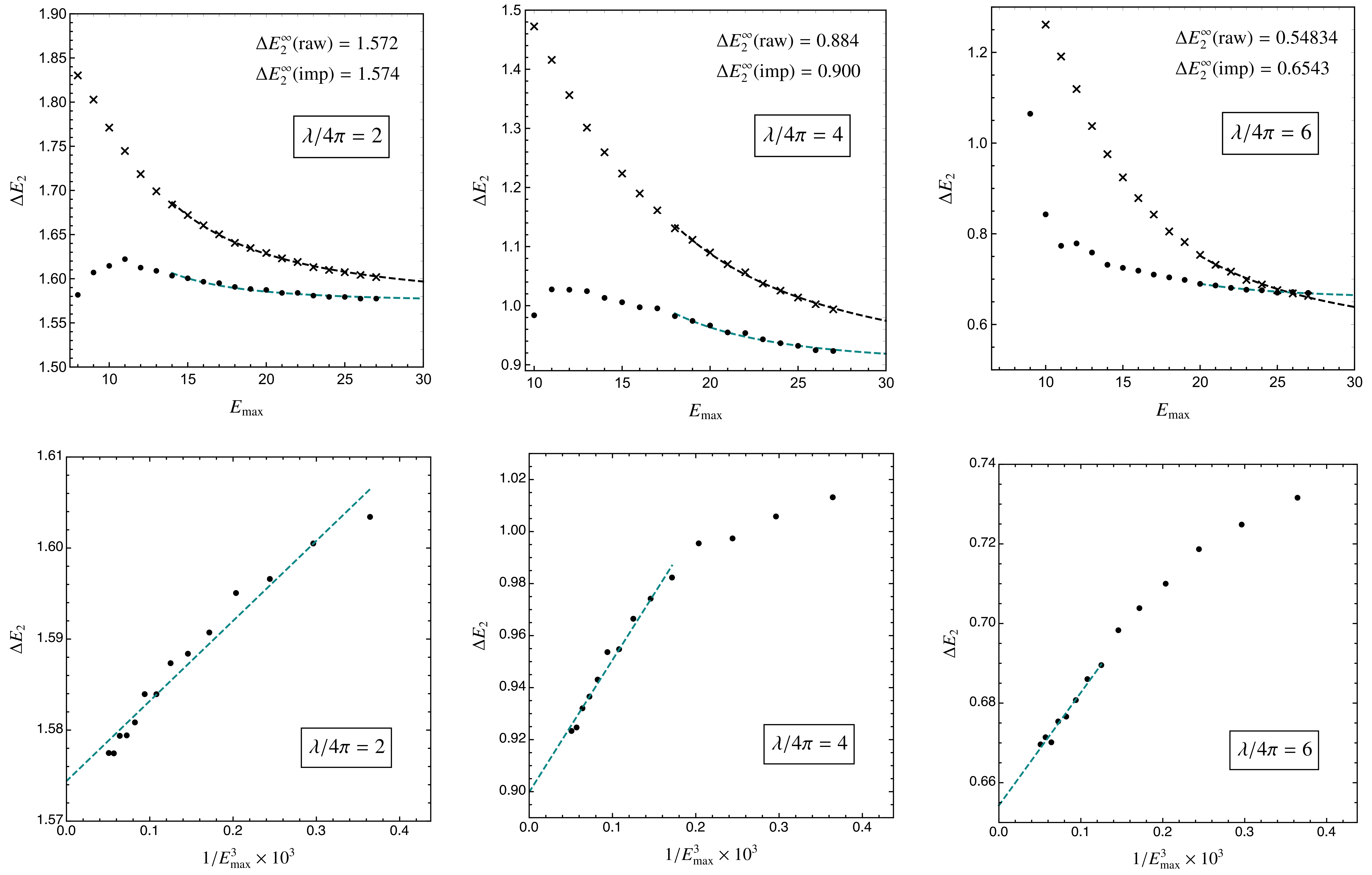
Higher coupling



Higher Excited States



Backup: Higher coupling ΔE_2



Cohen, KF, Houtz, Luty, arXiv: 2110.08273

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$$\langle f | H_2 | i \rangle = \sum_{\alpha \neq i}^{>} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | i \rangle}{E_f - E_\alpha}$$

$$\omega_i > 0$$

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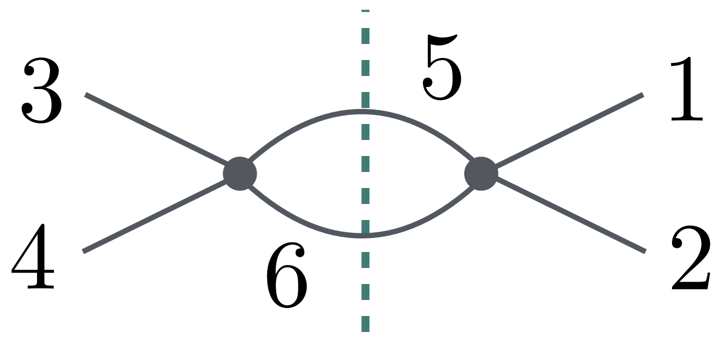

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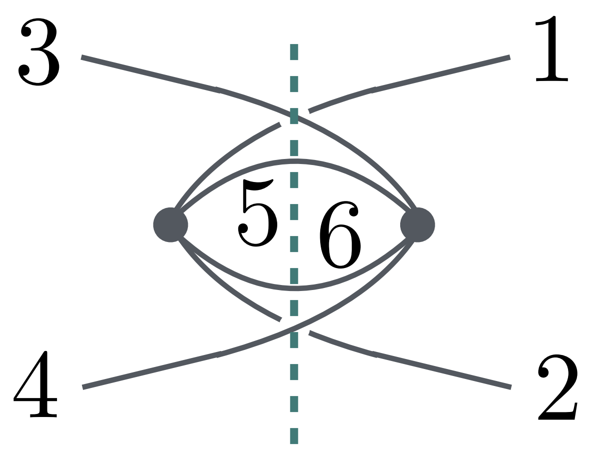

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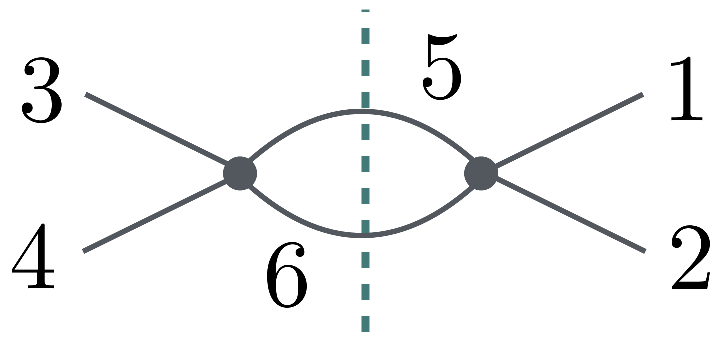

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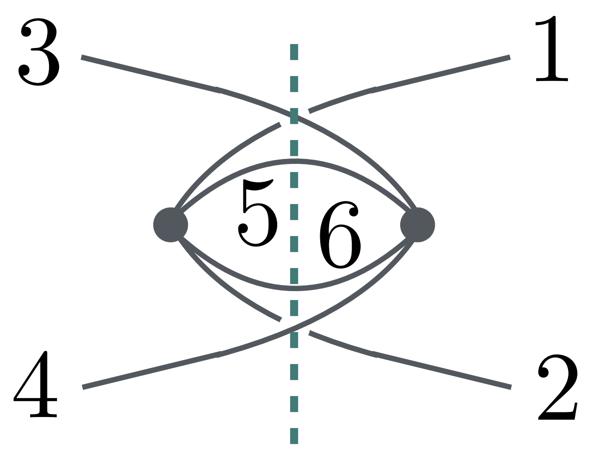

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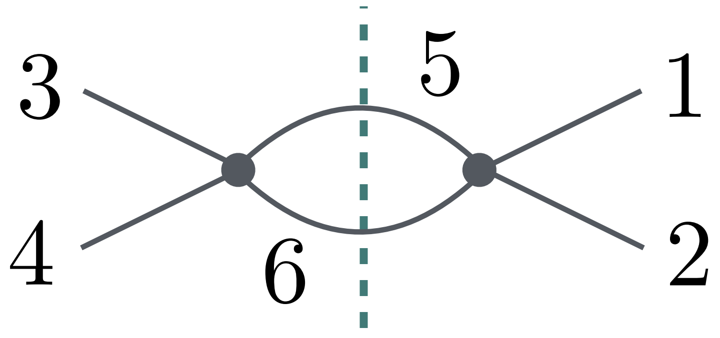

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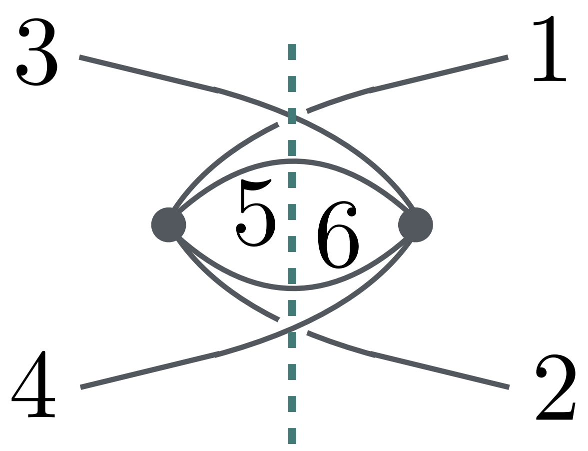

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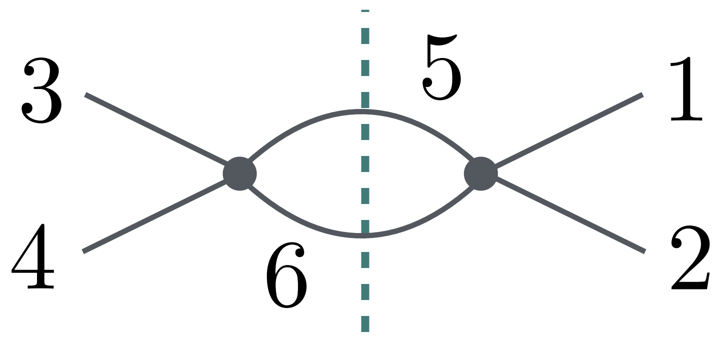

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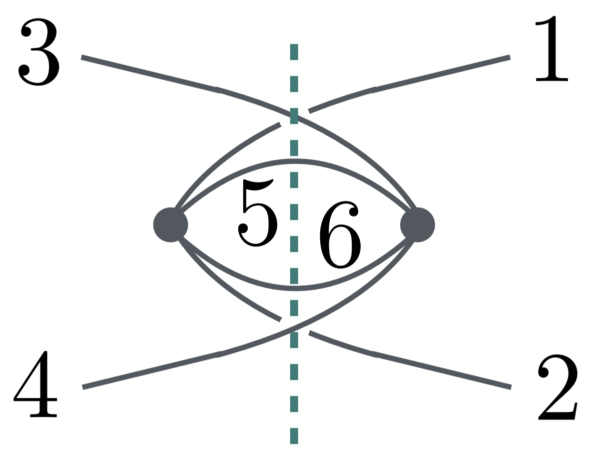

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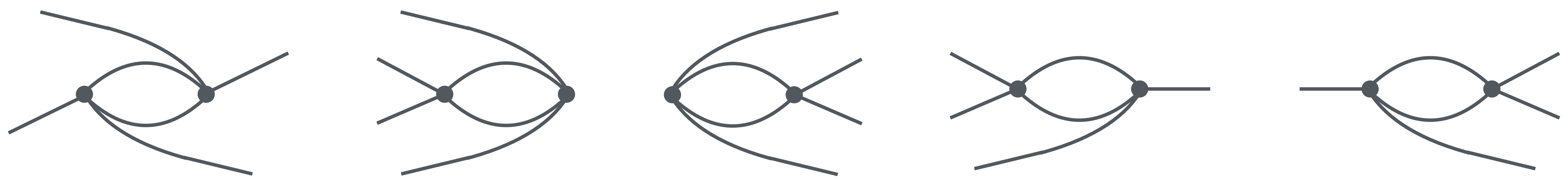

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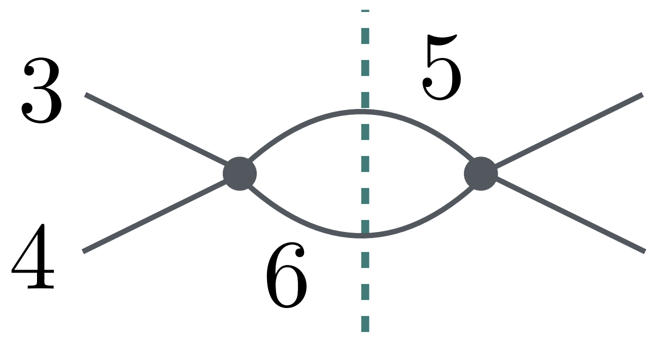

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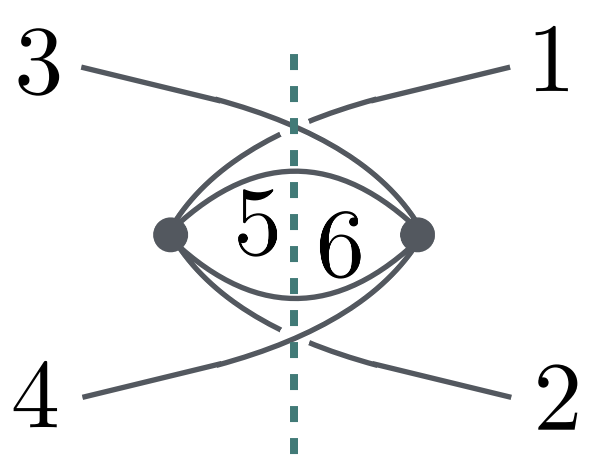
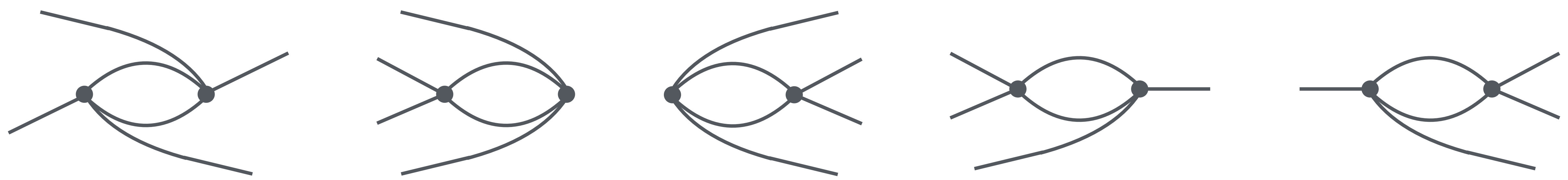
local approximation

$$\omega_{1,2,3,4} \lesssim E_{i,f} \ll E_{max}$$

$$E_\alpha = E_f - (-\omega_1 - \omega_2 - \omega_5 - \omega_6)$$



$$\Big|_{fund\,-\,eff} \sim \lambda^2 \sum_{5,6} \frac{\Theta(E_\alpha - E_{max})}{\omega_5\,\omega_6\,(-\omega_1 - \omega_2 - \omega_5 - \omega_6)}$$



$$\omega_i > 0$$

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

Example Theory 1: 2D $\lambda\phi^4$

$$\langle f|H_2|i\rangle = \sum_{\alpha\neq i}^{>} \frac{\langle f|V|\alpha\rangle\langle\alpha|V|i\rangle}{E_f - E_\alpha}$$

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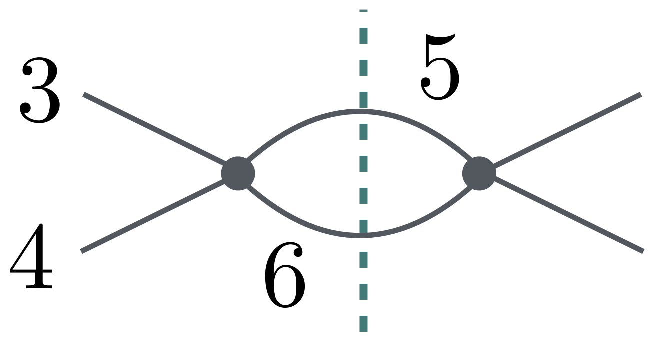

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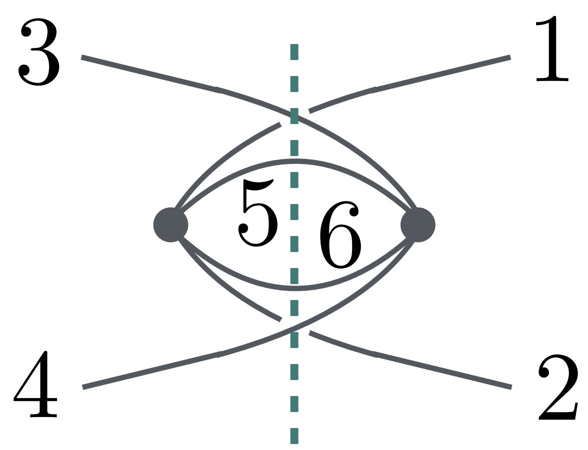

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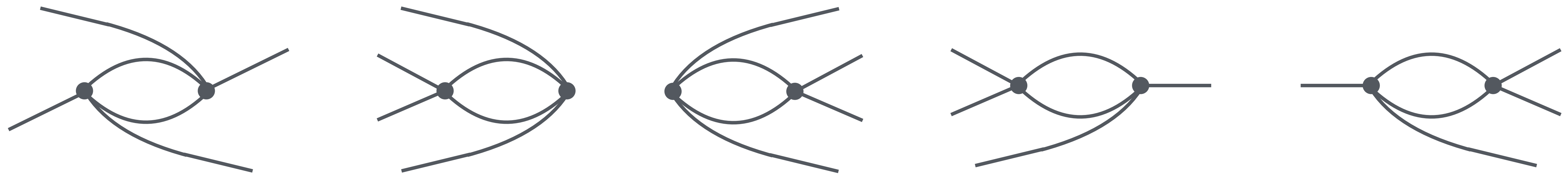

$$\sim \lambda^2 \sum_k \frac{\Theta(2\omega_k - E_{max})}{\omega_k^2\,(-2\omega_k)} \int dx\,\phi^4$$

$$\omega_i > 0$$

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

What to match? “S-matrix”?

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

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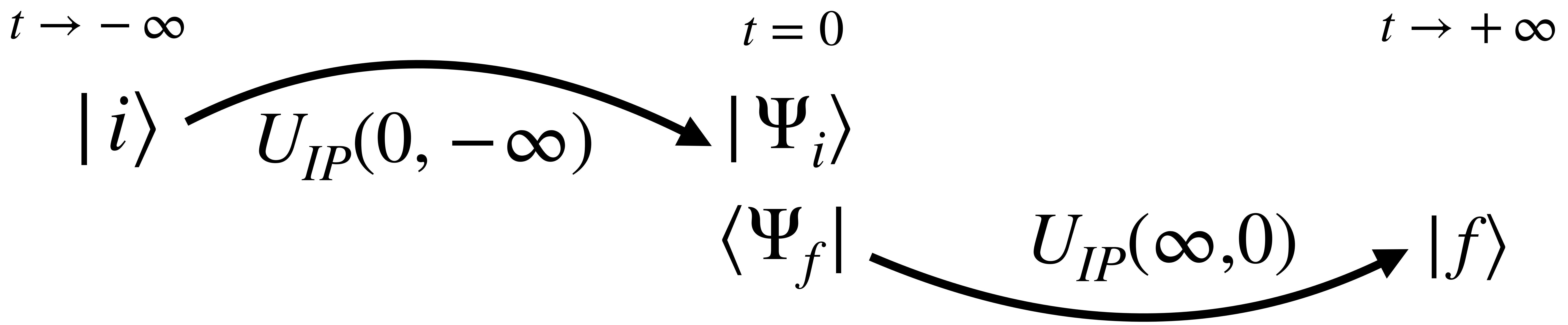

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