Gravitational wave signals inflated string bounded domain walls

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Work in collaboration with Yunjia Bao and Keisuke Harigaya to appear

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Why topological defect?

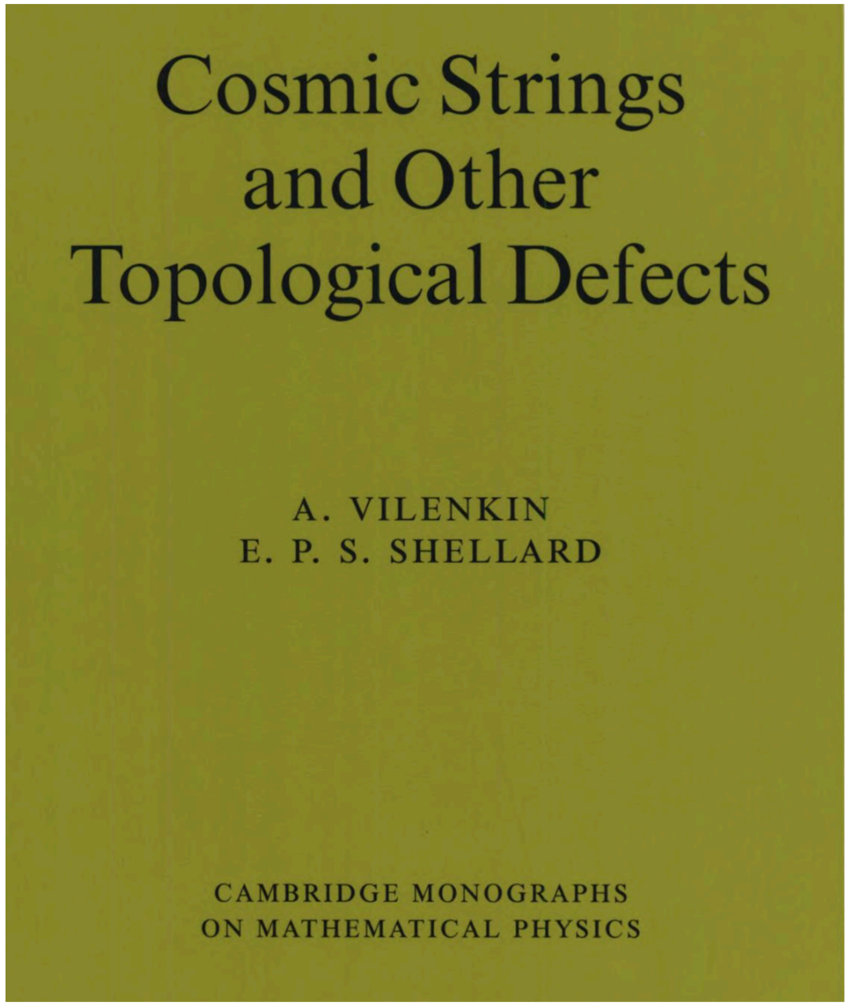
- Old subject.

  A. Vilenkin et al. + many others, 40+ years ago.

- Beautiful topic

  Can reveal fascinating info about BSM physics.

  Cosmo implications well studied.



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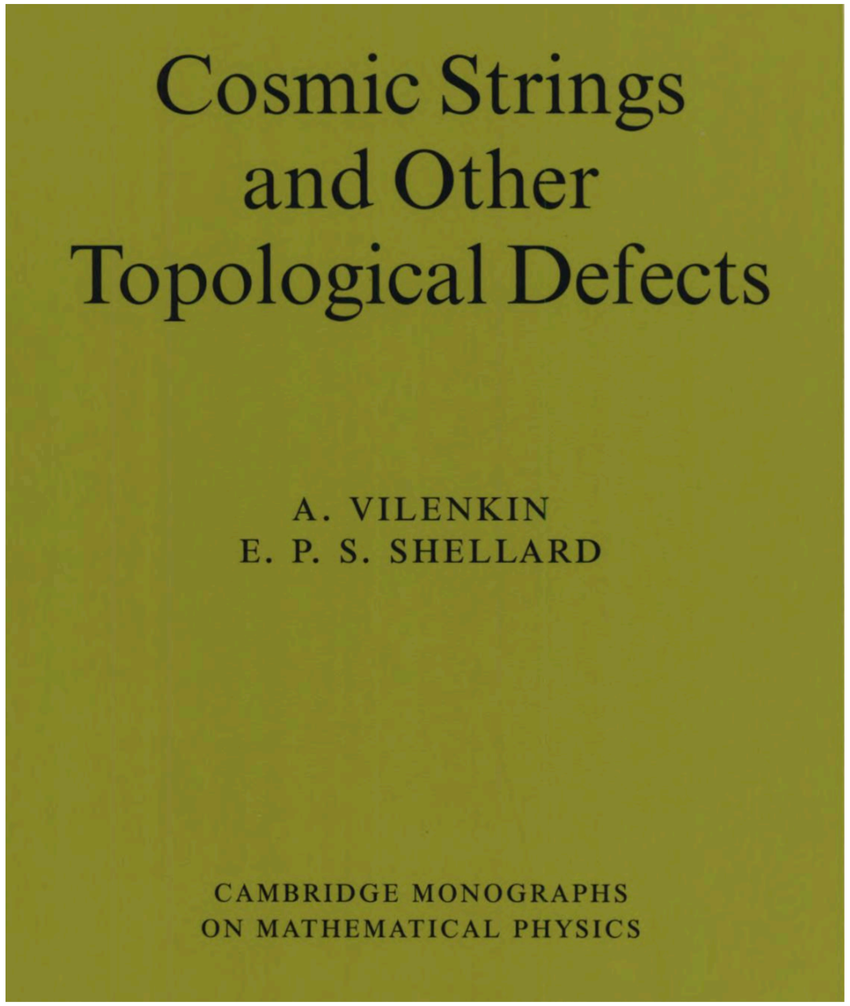
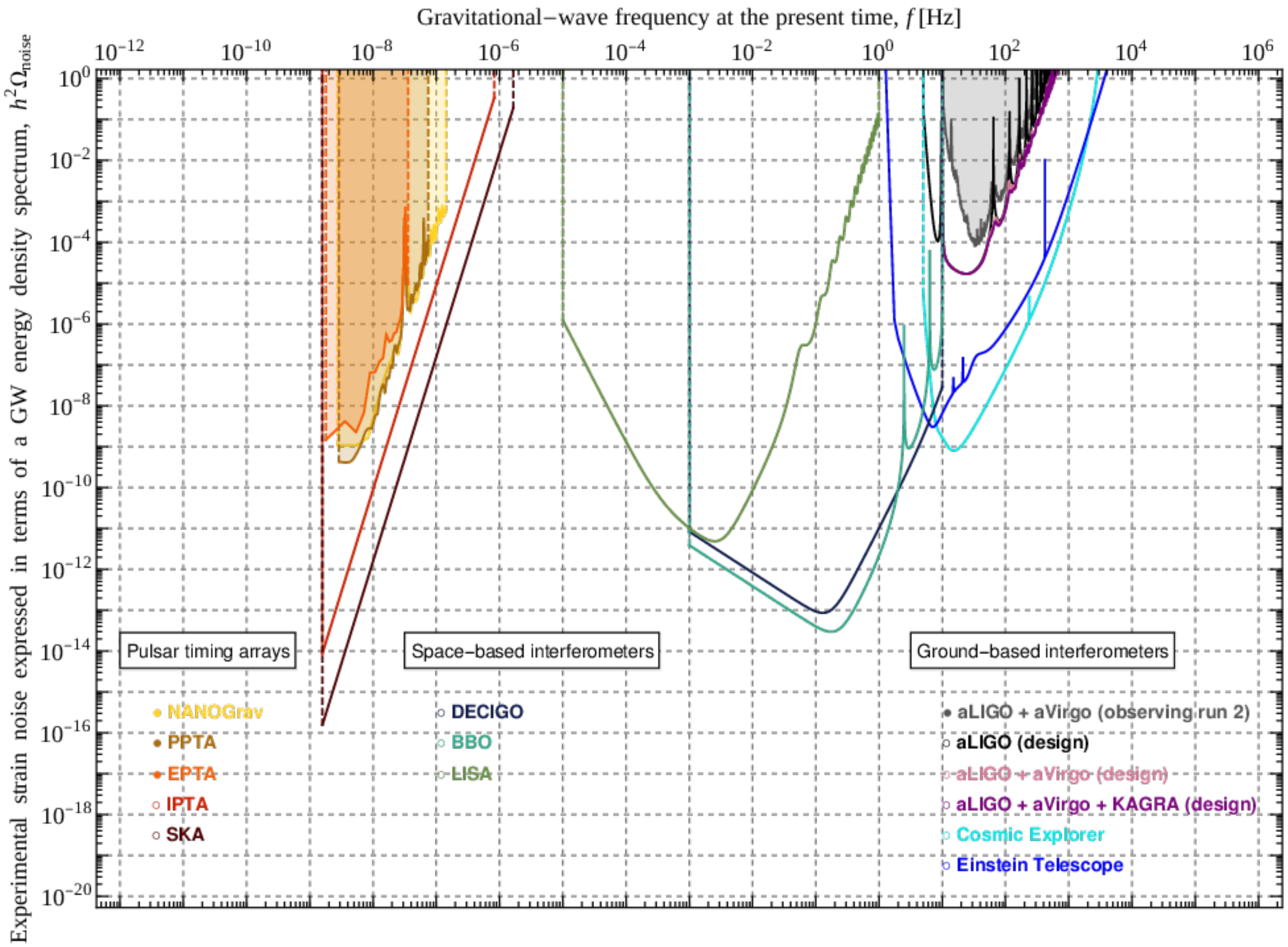


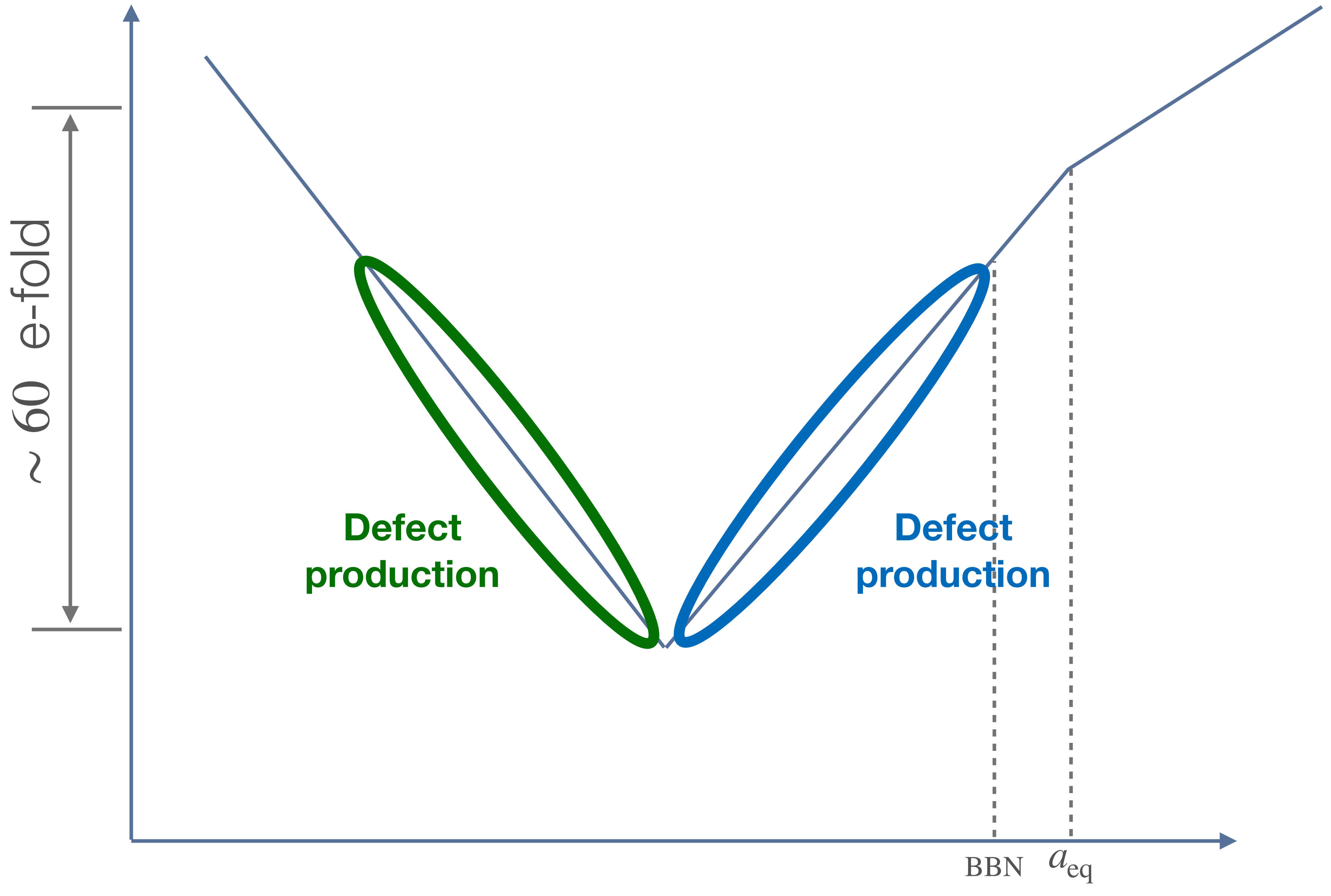
So, why the recent interest?

Gravitational wave signal

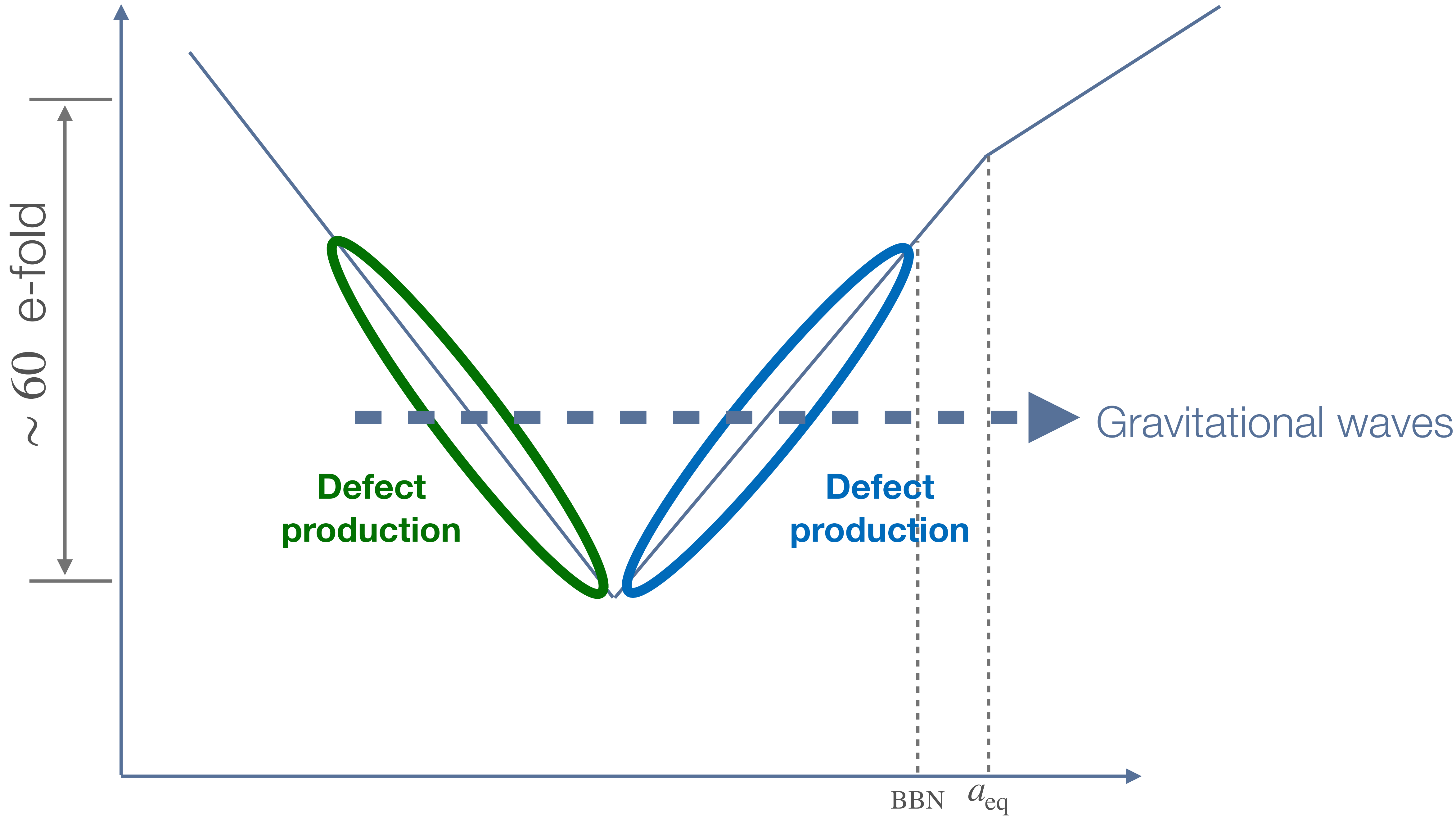


K. Schmitz, 2002.04615

Gravitational wave signal



Gravitational wave signal



Defect networks

Two stages of of symmetry breaking

$$G \to H \to K$$

Example 1: monopole-string network

$\pi_2(G/H) \neq 1 \to$ monopoles, $\pi_1(H/K) \neq 1 \to$ strings

If $\pi_1(G/K) = 1 \to$ string monopole network unstable

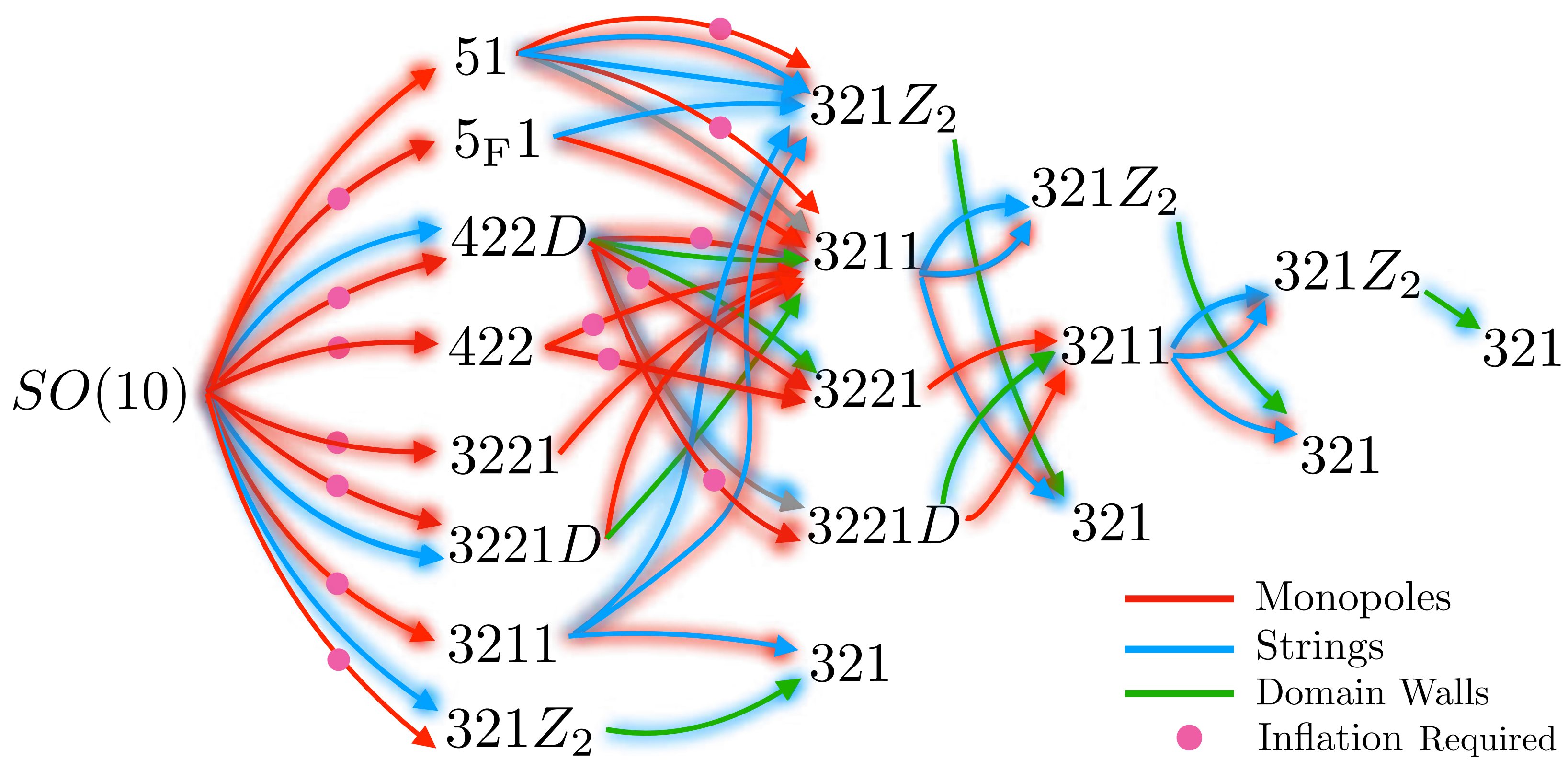
Example 2: string wall network

$\pi_1(G/H) \neq 1 \to$ strings, $\pi_0(H/K) \neq 1 \to$ walls

If $\pi_0(G/K) = 1 \to$ string wall network unstable

Defect networks

Maybe not crazy.



Dunsky, Ghoshal, Murayama, Sakakihara, White, "gastronomy" 2111.08750

Toy model

Two scalar: ϕ_1 and ϕ_2

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 + \lambda_1 \left(|\phi_1|^2 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(|\phi_2|^2 - \frac{v_2^2}{2} \right)^2 + \mu_m \left(\phi_2^* {\phi_1}^2 + \mathrm{h.c.} \right)$$

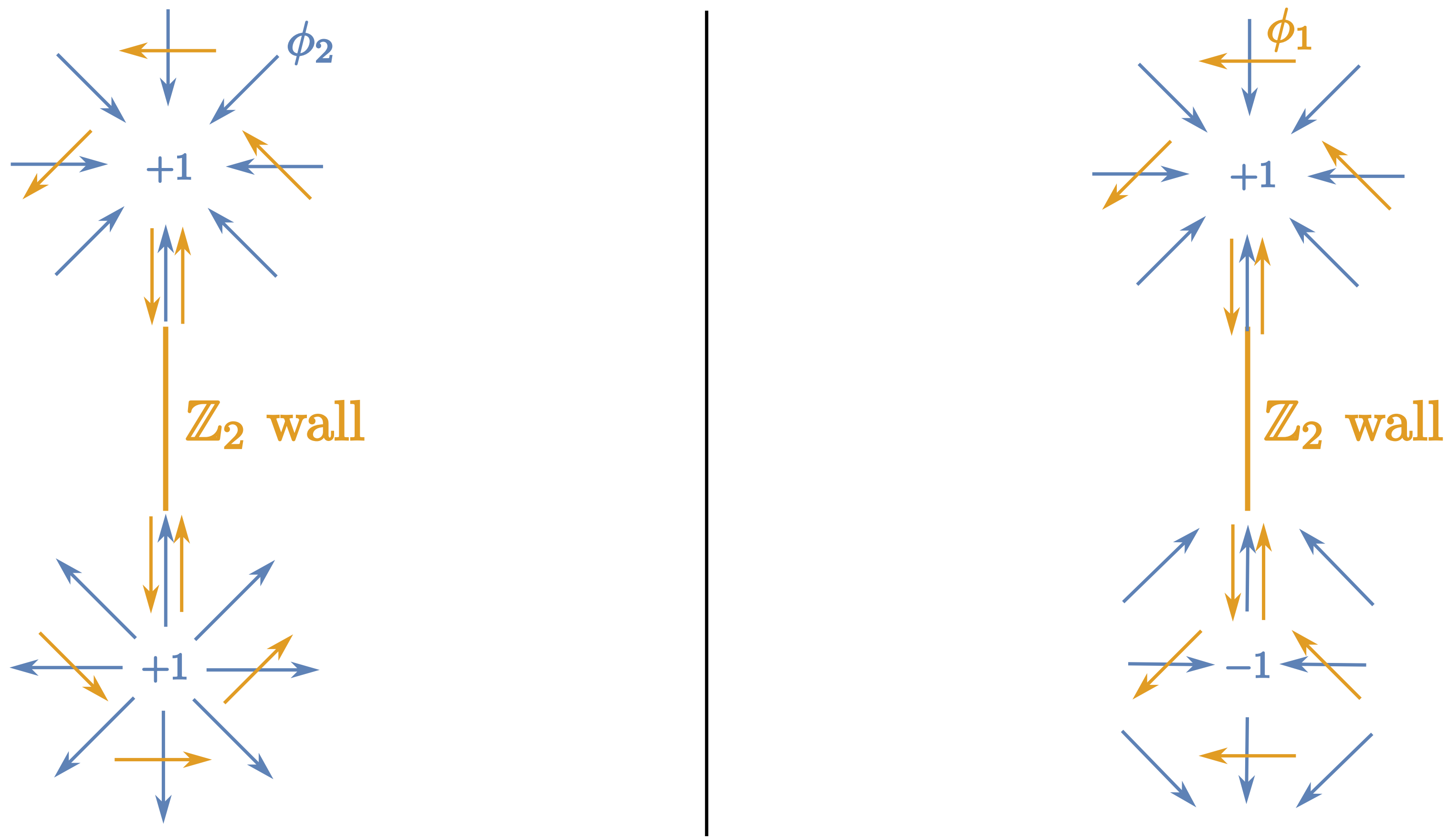
$$\langle \phi_1 \rangle = v_1, \ \ \langle \phi_2 \rangle = v_2$$

Assume $v_2 \gg v_1$, two stages $\mathrm{U}(1) \to \mathbb{Z}_2 \to \emptyset$

Formation of string-domain wall network.

String+wall network

Two possible configurations

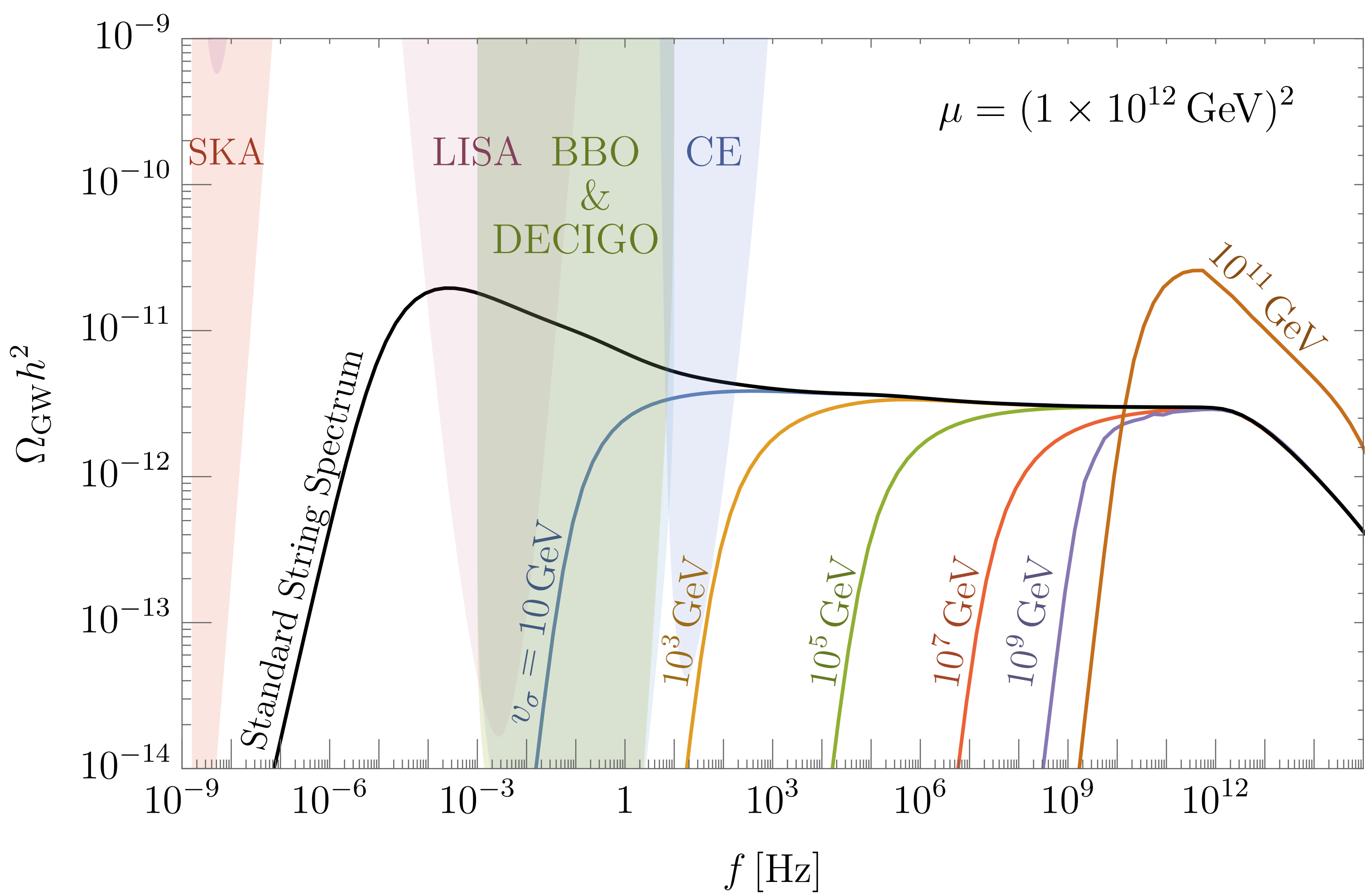


$$\mathcal{L} \supset \mu_m(\phi_1^2\phi_2^* + \text{h.c.}) \to \frac{\mu_m}{\sqrt{2}} v_1^2 v_2 \cos(2\theta_1 - \theta_2)$$

Production of the network

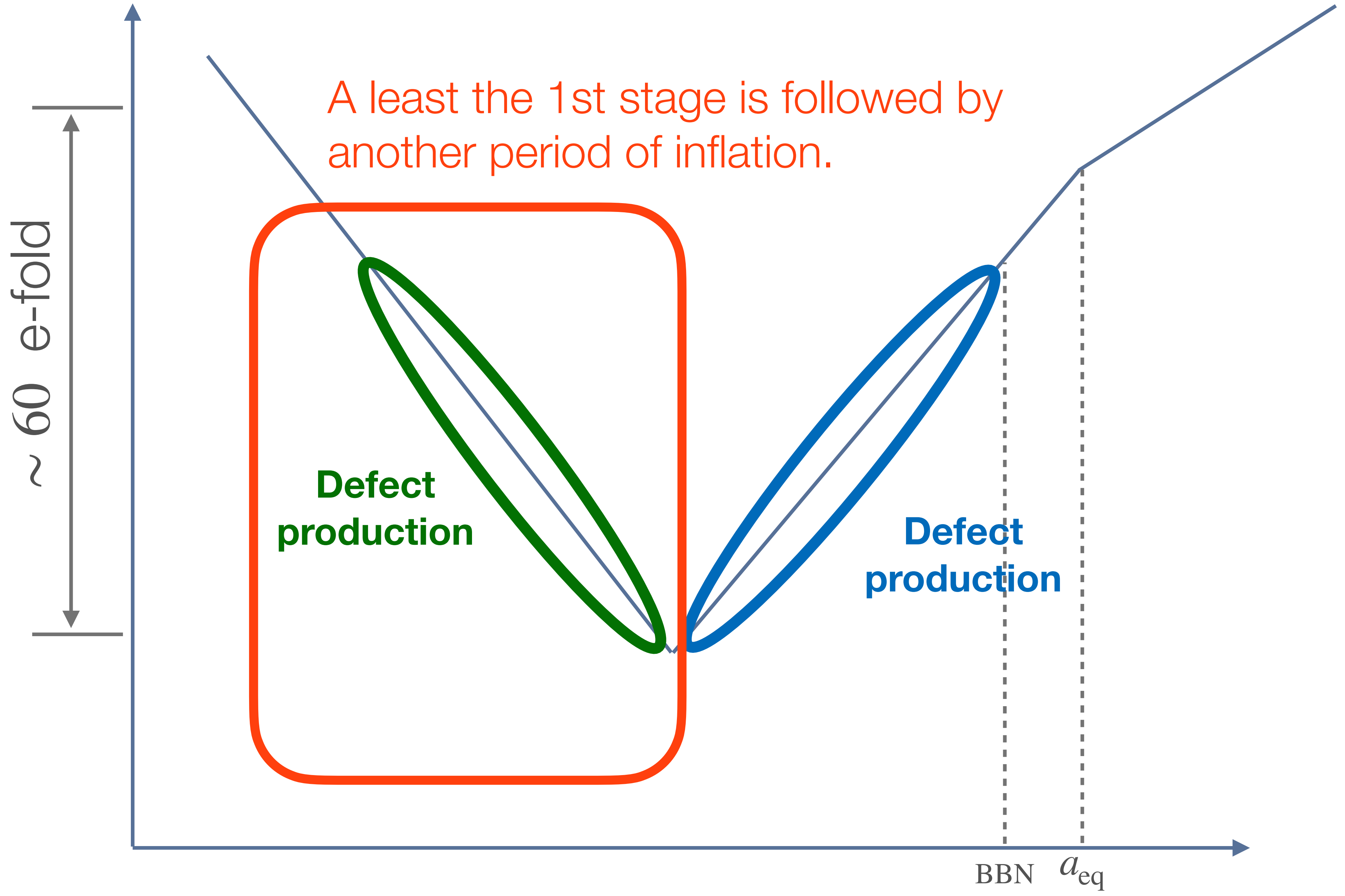
- Production after reheating (post inflationary).

Dunsky, Ghoshal, Murayama, Sakakihara, White, “gastronomy”, 2111.08750



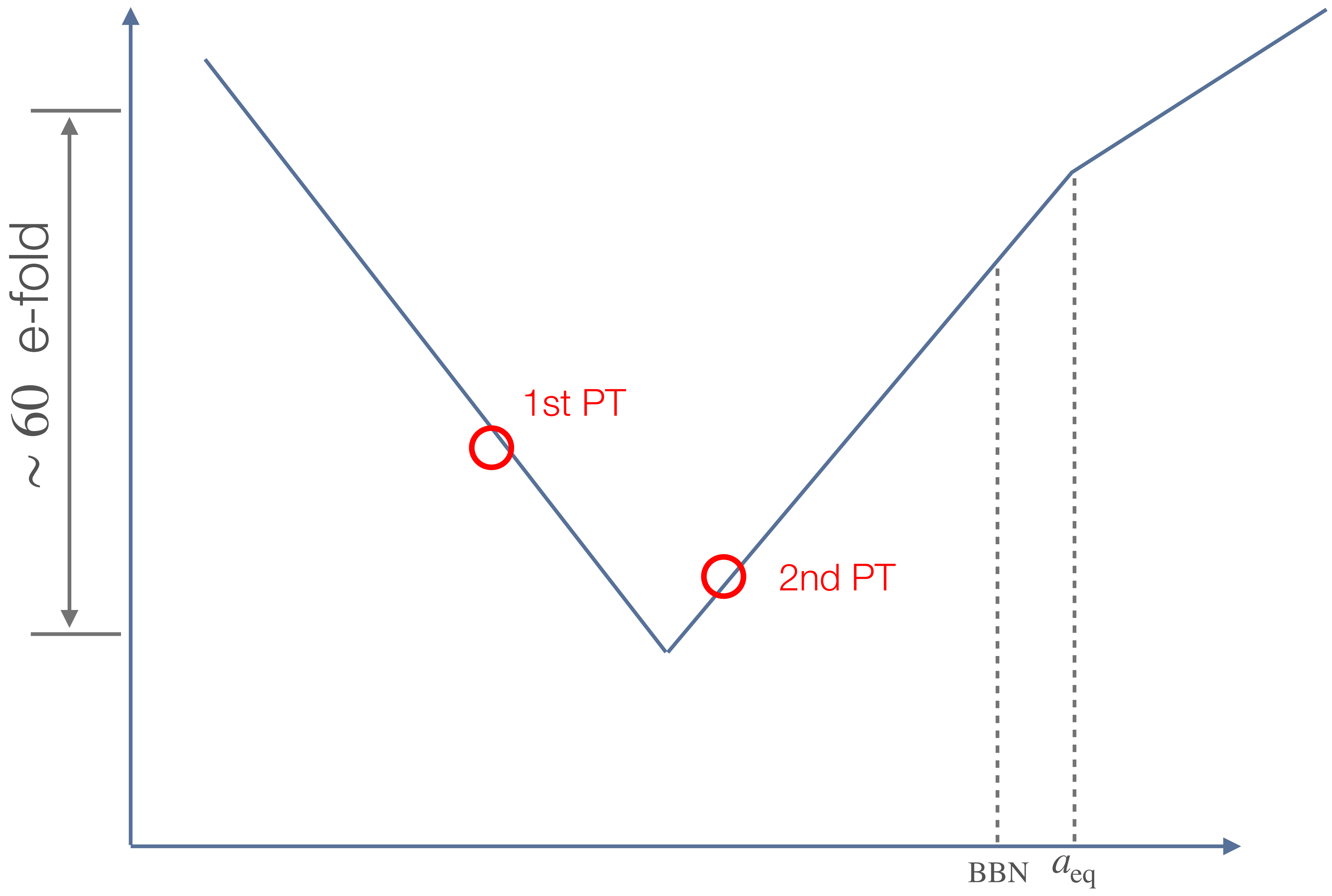
Network collapse cuts off the GW spectrum.

Our work



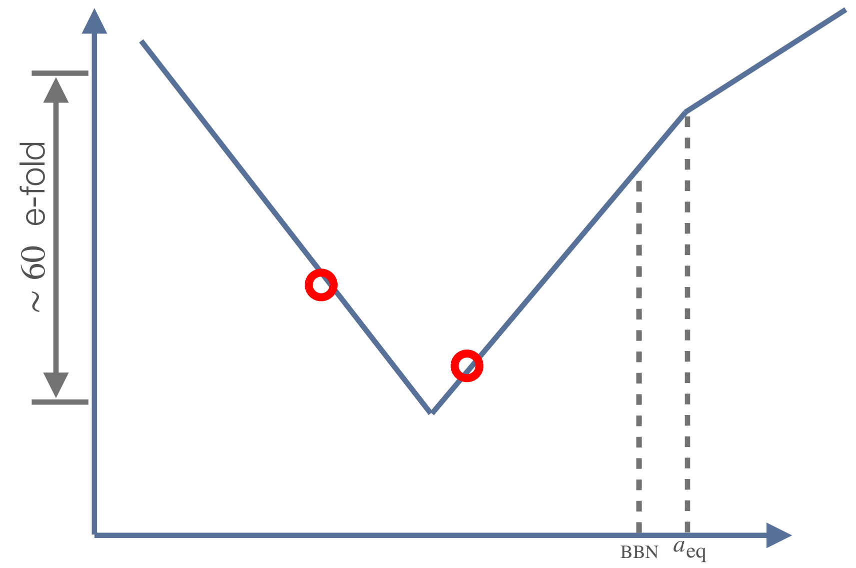
A least the 1st stage is followed by
another period of inflation.
~ 60 e-fold
Defect
production
Defect
production
BBN
a_{eq}

Our work



Why is this plausible?

Our work



Why is this plausible?

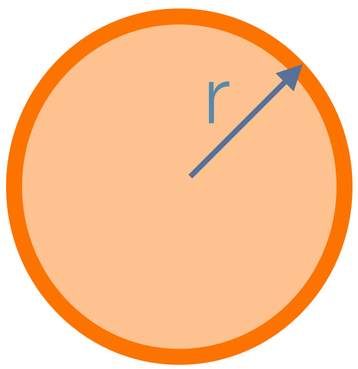
If the scales of the two stages are well separated, easy to fit in another stage of inflation in between.

Rolling inflaton can trigger the 1st phase transition.

Natural to have the second period to be thermal inflation.

To have sizable GW signal, second stage of (thermal) inflation expected! (More on this later)

GW from disk bounded by string



Oscillation and radiation

$$r(t) \approx \bar{r}(t) \cos\left(\frac{t}{\bar{r}(t)}\right)$$

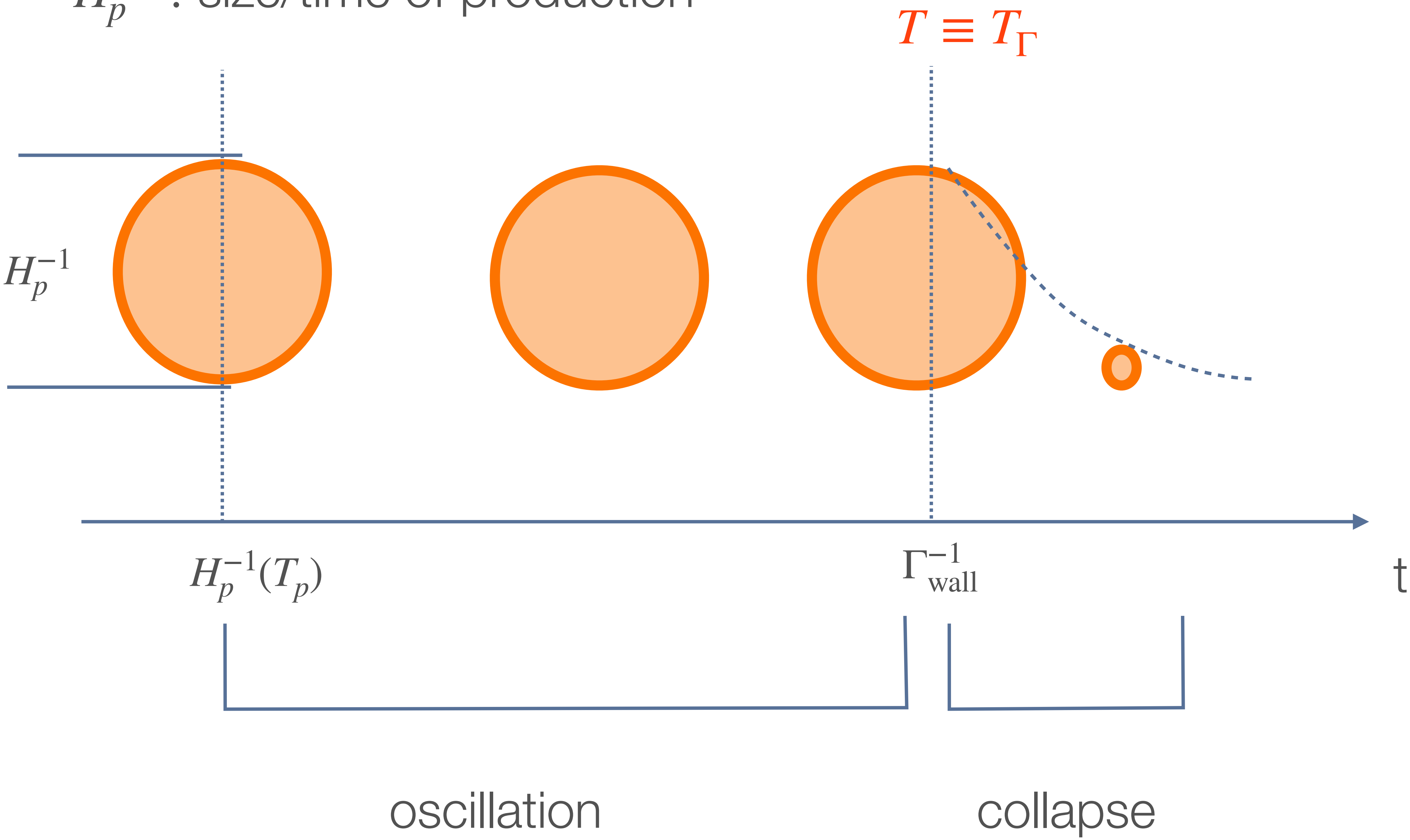
Radiation power and rate:

$$P = \frac{\mathrm{d}E}{\mathrm{d}t} \approx \frac{\left\langle \dddot{Q}^2 \right\rangle}{8\pi M_{\mathrm{Pl}}^2} \approx \frac{1}{8\pi M_{\mathrm{Pl}}^2} \left| \frac{\mathrm{d}^3}{\mathrm{d}t^3} \left(\sigma \pi \bar{r}^4 \cos^4 \left(\frac{t}{\bar{r}} \right) \right) \right|^2 \approx \frac{\pi \sigma^2 \bar{r}^2}{M_{\mathrm{Pl}}^2}$$

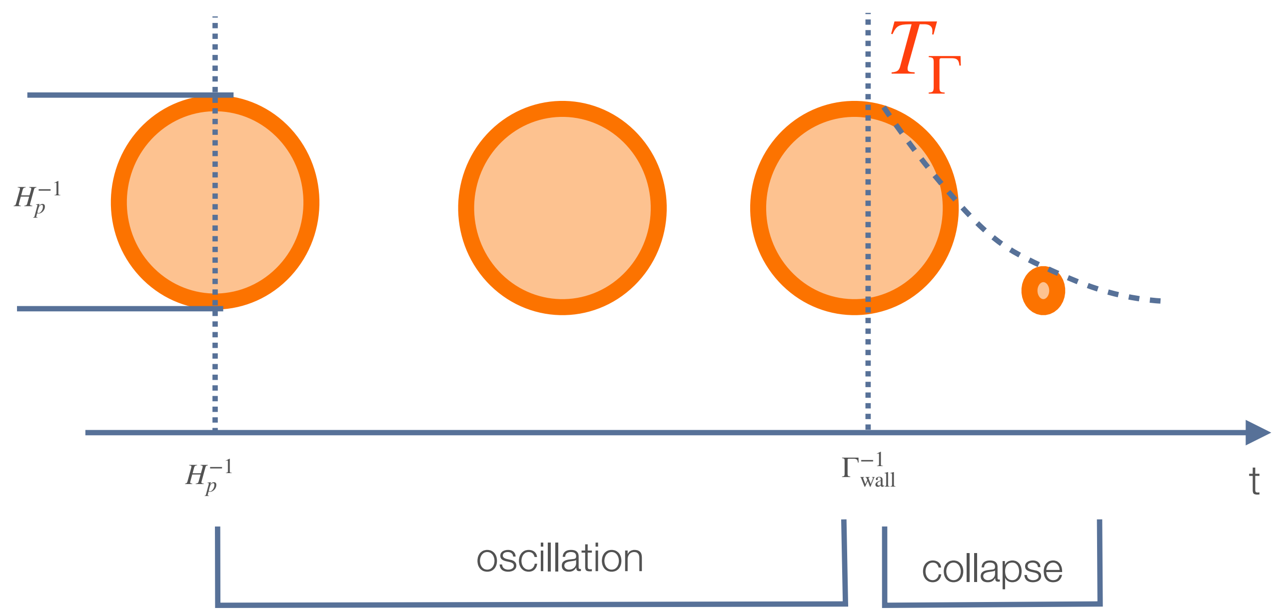
$$P = \frac{\mathrm{d}(\sigma \bar{r}^2)}{\mathrm{d}t} \approx -\Gamma_{\mathrm{wall}} \sigma \bar{r}^2 \qquad \Gamma_{\mathrm{wall}} \approx \frac{\sigma}{M_{\mathrm{Pl}}^2}, \ \sigma = \text{wall tension}$$

GW from disk

H_p^{-1} : size/time of production



GW from disk



$$\Omega_{\rm GW} \approx \frac{1}{T_\Gamma^4} \sigma H_p \left(\frac{T_\Gamma}{T_p} \right)^3$$

Wall energy density

Red-shift like a^{-3} after re-entry

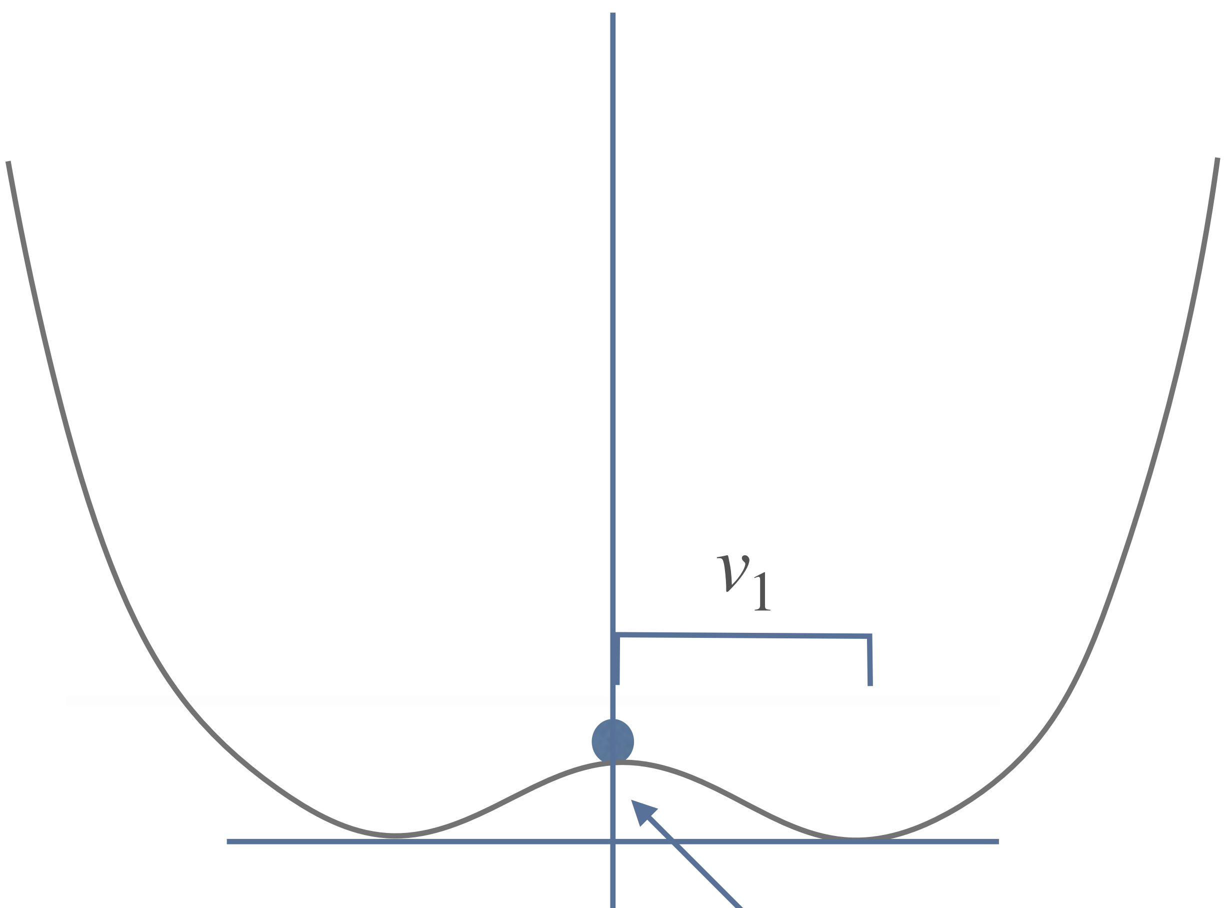
T_p: temperature at wall production

using $H_\Gamma \approx \frac{T_\Gamma^2}{M_{\rm Pl}^2} \approx \Gamma_{\rm wall} \to \Omega_{\rm GW} \approx \frac{\sigma^{1/2}}{M_{\rm Pl}^{1/2} T_p}$

Post inflationary production

Post-inflationary production

$$V = -\, m_1^2 \phi_1^2 + \ldots$$



$$V(\phi_1 = 0) \sim m_1^2 v_1^2$$

$$\Omega_{\rm GW} \approx \frac{\sigma^{1/2}}{M_{\rm Pl}^{1/2} T_p} \qquad \sigma \approx m_1 v_1^2,$$

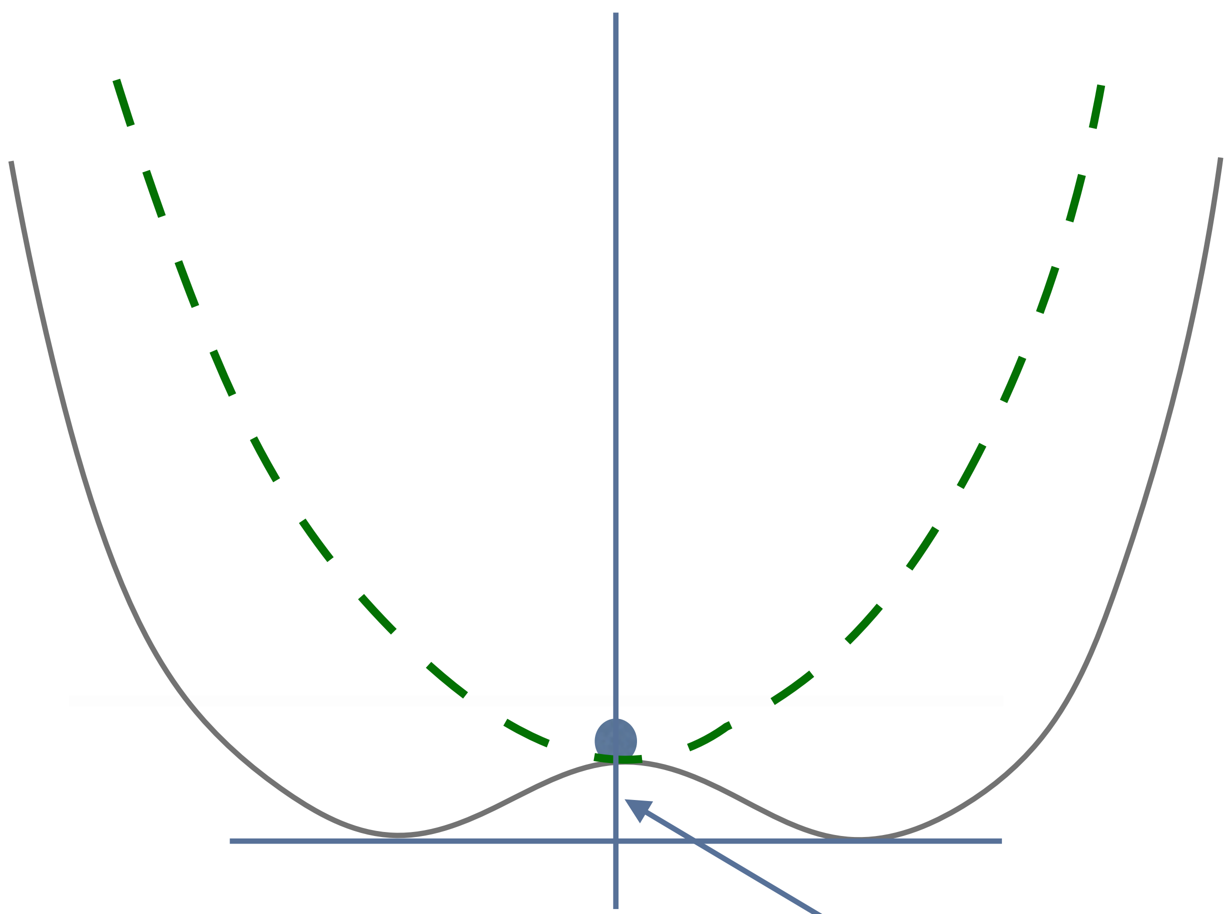
$$\approx \left(\frac{v_1}{M_{\rm Pl}}\right)^{1/2} \left(\frac{V(\phi_1 = 0)}{T_p^4}\right)^{1/4}$$

Large $\Omega_{\rm GW} \to V(\phi_1 = 0) \gg T_p$

Vacuum domination, inflation!

Post-inflationary production

In the context of thermal phase transition:



$V(\phi_1 = 0) \sim m_1^2 v_1^2$

$$V = -\, m_1^2 \phi_1^2 + \ldots$$

$$+ y^2 T^2 \phi_1^2$$

Phase transition happens at

$$T_p \approx \frac{m_1}{y}$$

Post-inflationary production

Using $T_p \approx \frac{m_1}{y}$

$$\Omega_{\rm GW} \approx \frac{\sigma^{1/2}}{M_{\rm Pl}^{1/2} T_p} \approx \left(\frac{v_1}{M_{\rm Pl}}\right)^{1/2} \cdot \kappa, \qquad \kappa \equiv \frac{y v_1^{1/2}}{m_1^{1/2}}$$

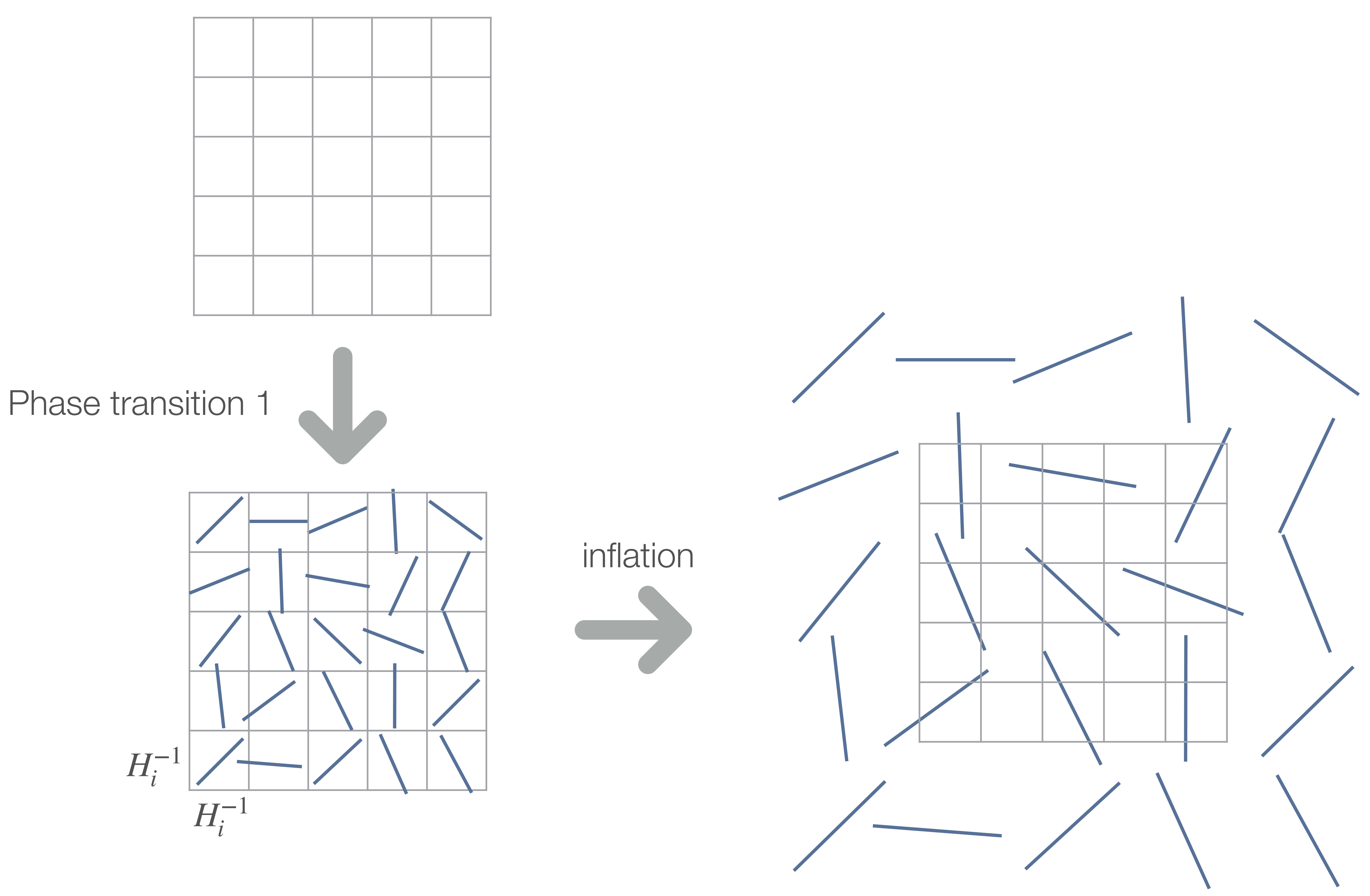
Sizable gravitational wave ⇒ large κ

However, $\frac{V(\phi_1 = 0)}{T_p^4} \approx \frac{y^4 v_1^2}{m_1^2} = \kappa^4$

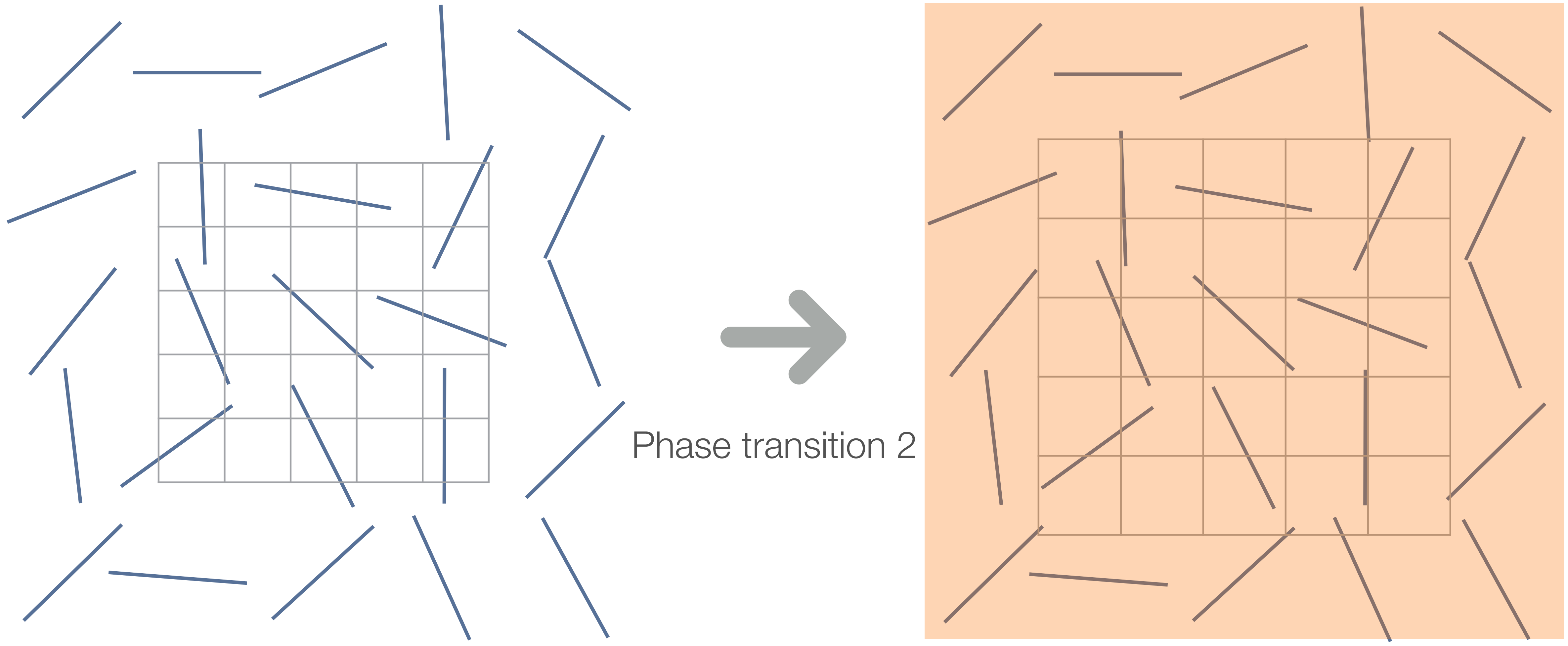
Hence, before second phase transition and during $(m_1 v_1)^{1/2}\kappa > T > m_1/y$,
ϕ_1 trapped at origin and vacuums energy dominates ⇒ thermal inflation

Back to our work

Network production and evolution.



Network production and evolution.



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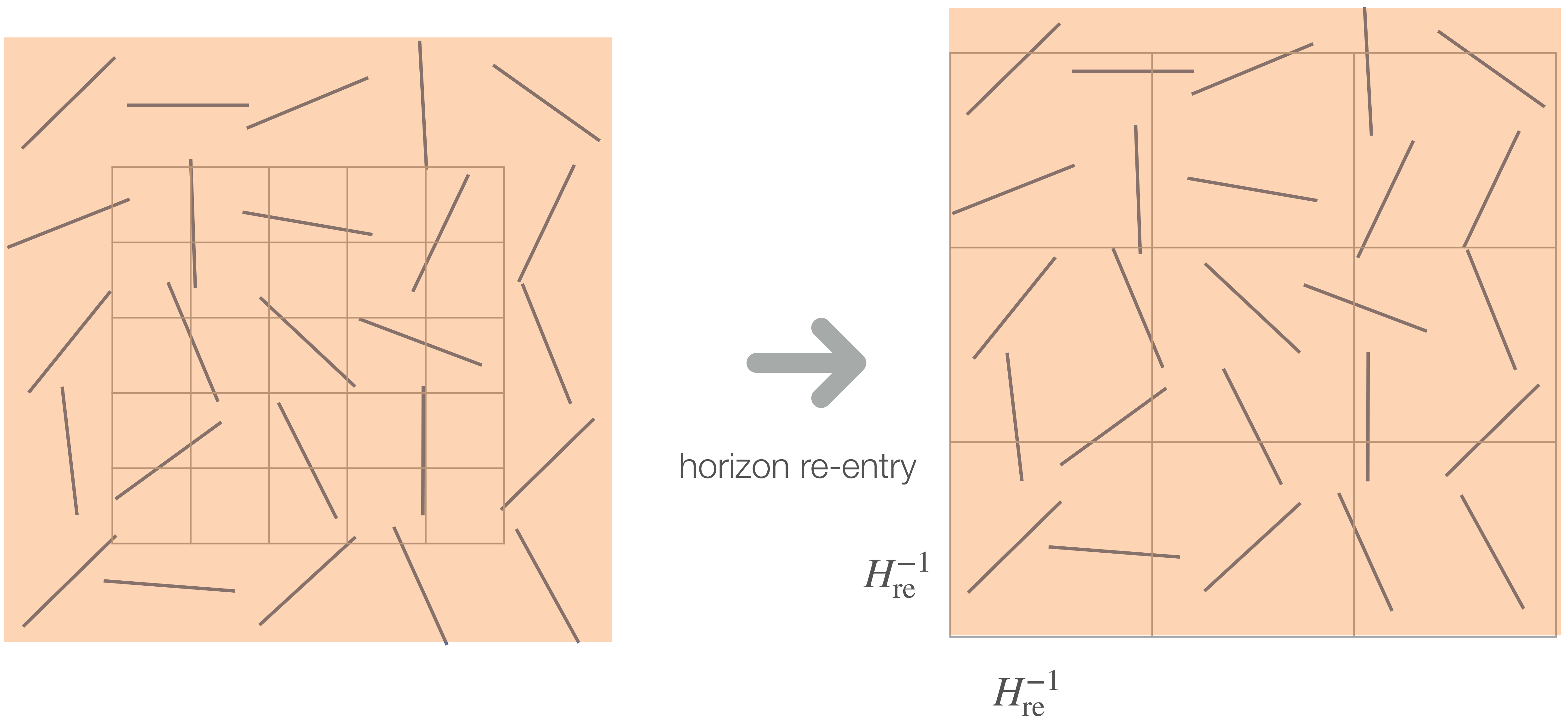
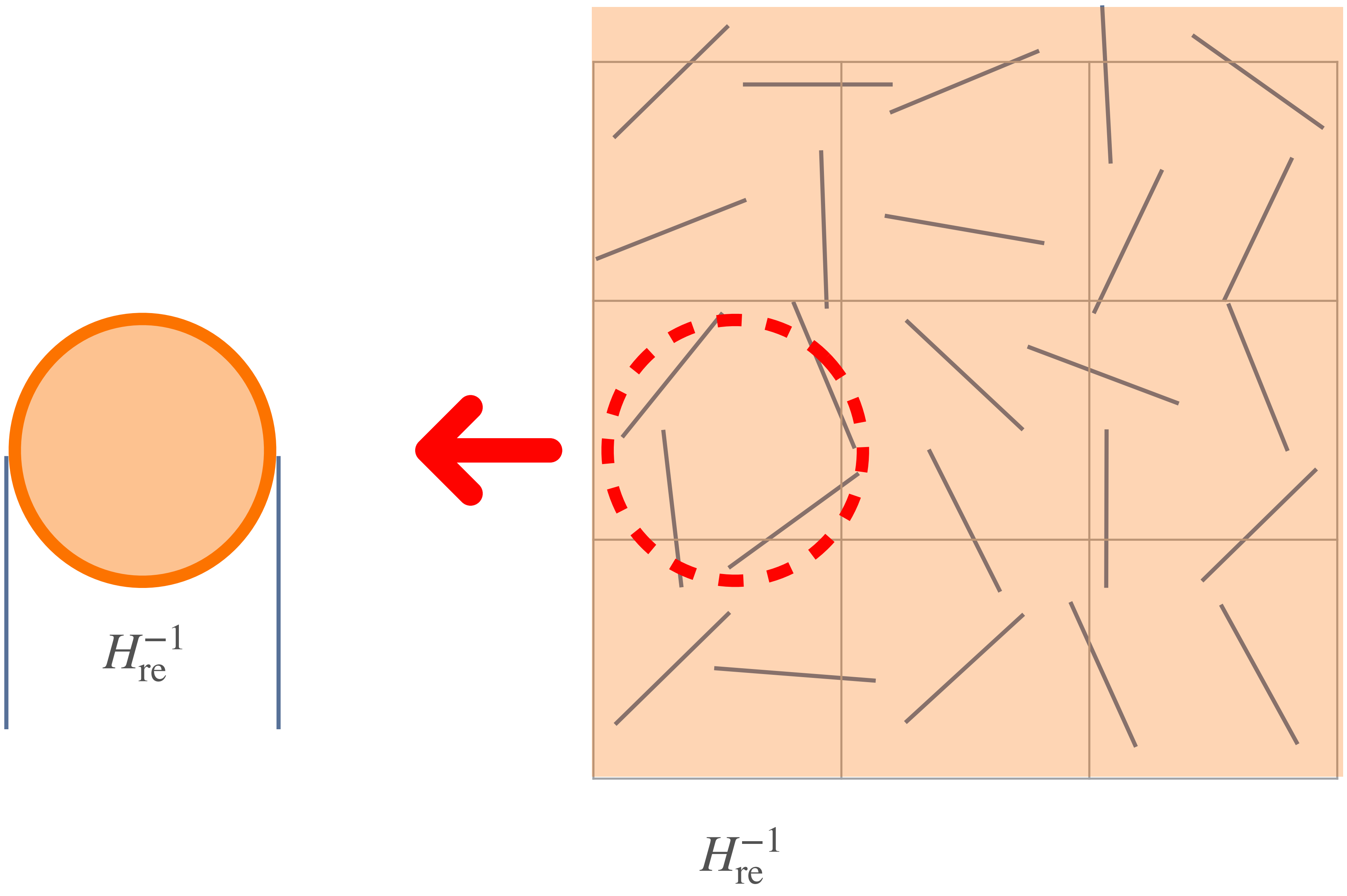
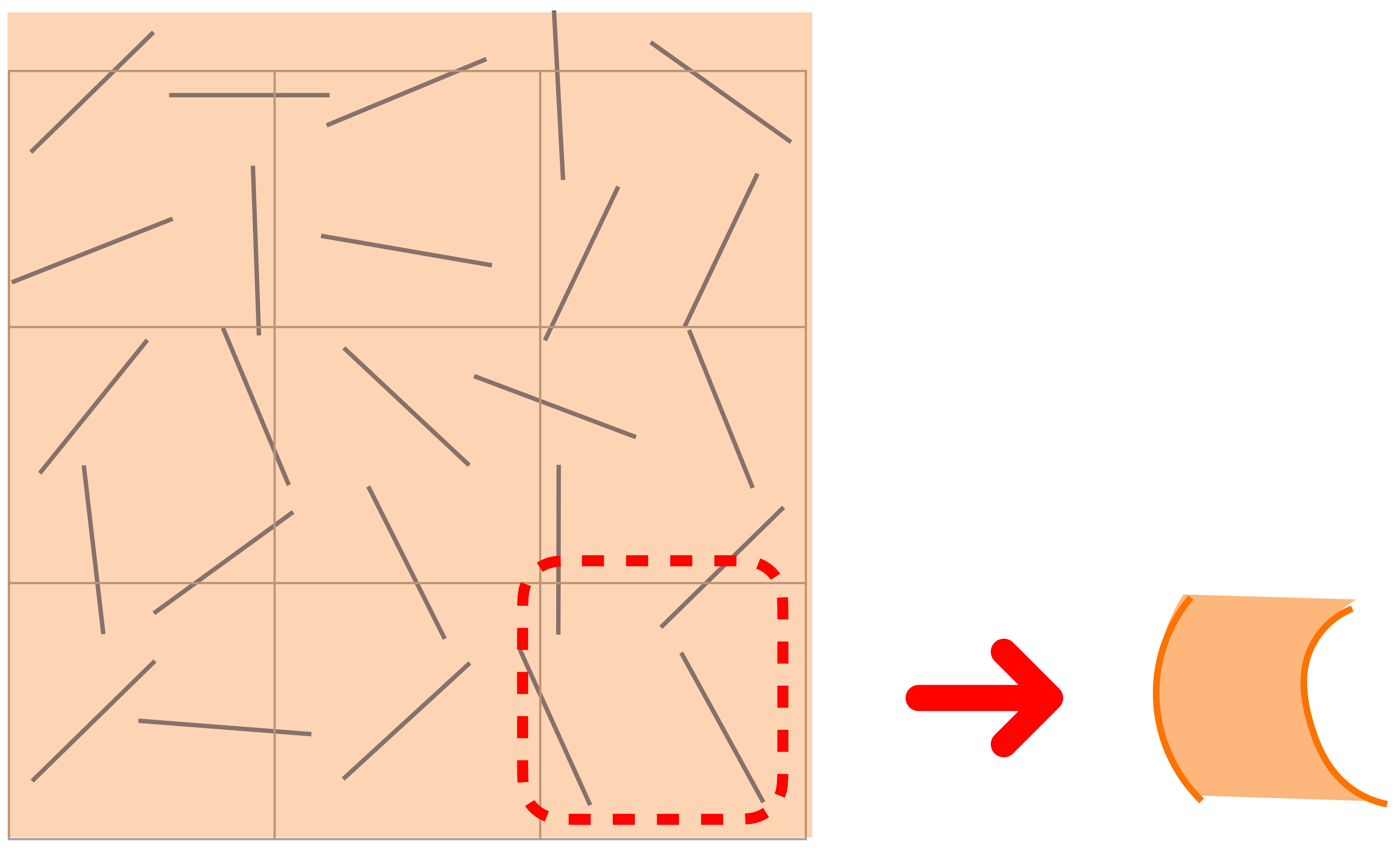
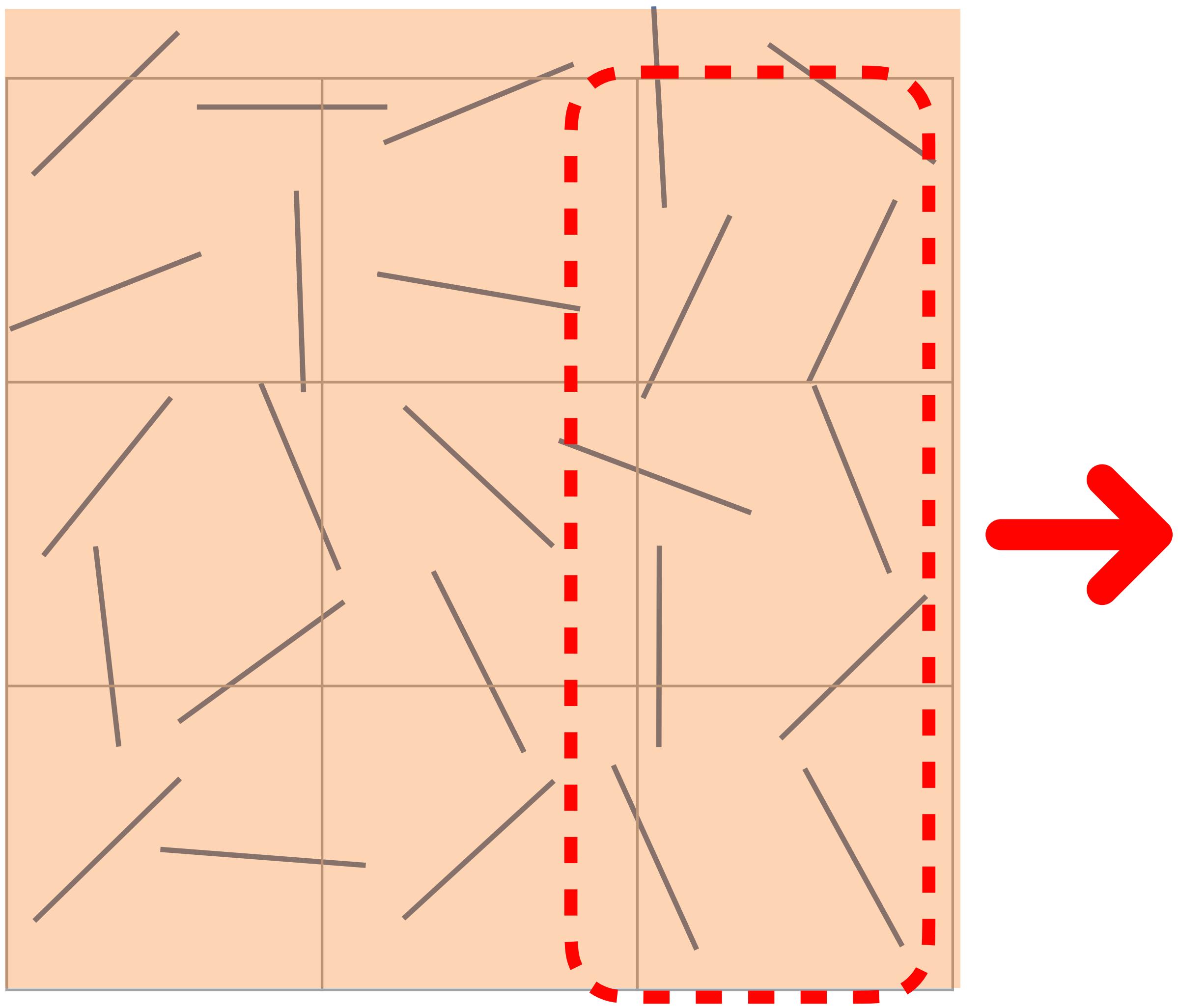


Disk

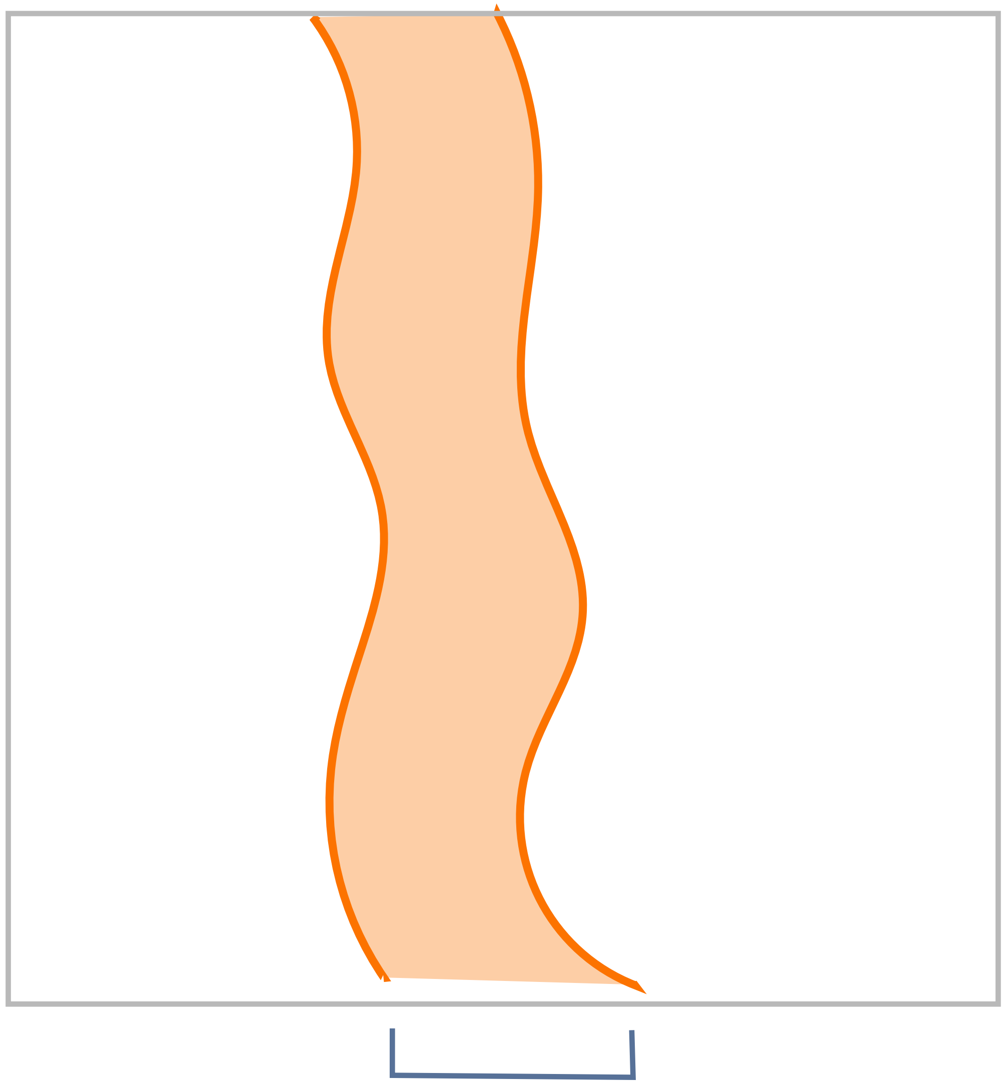




Belt

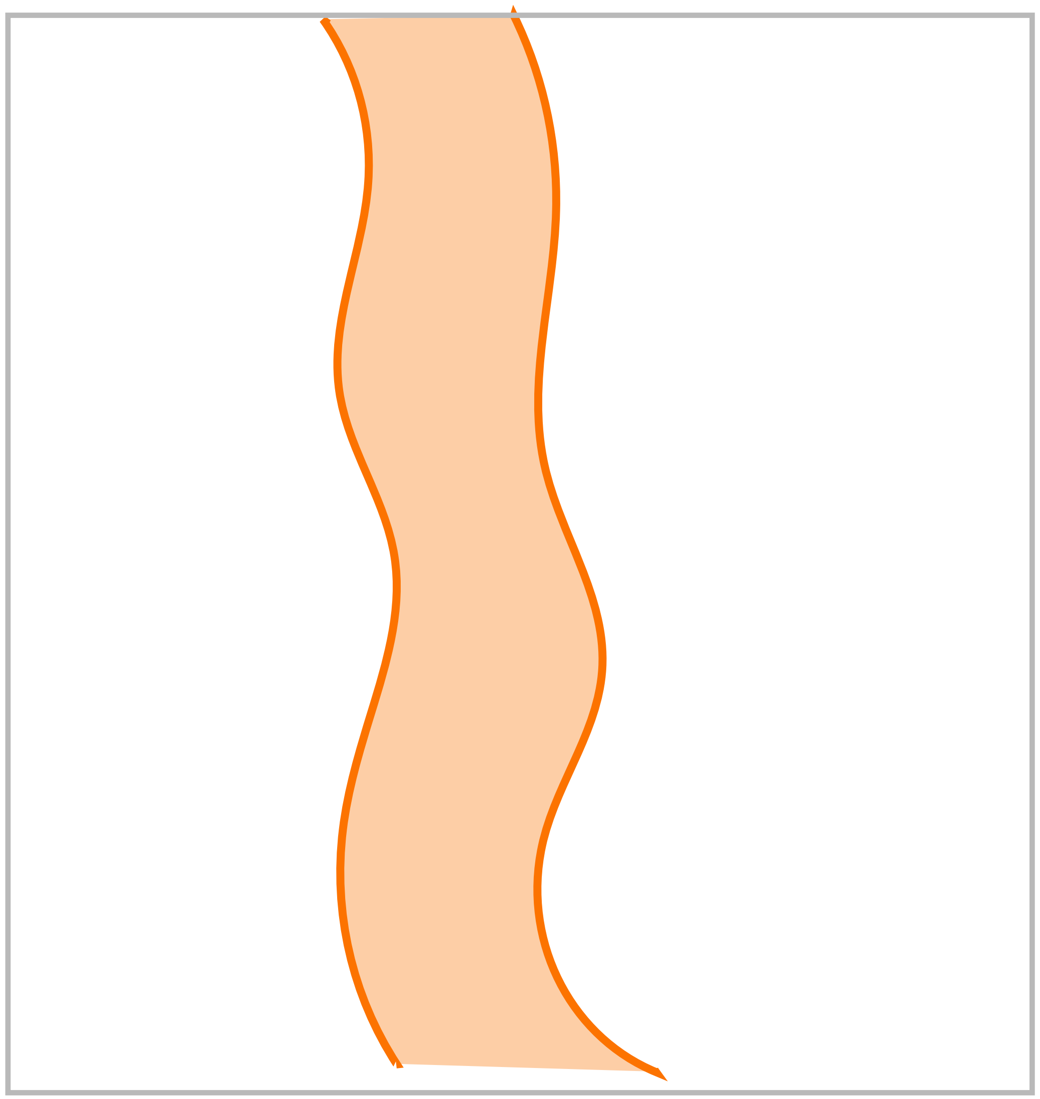


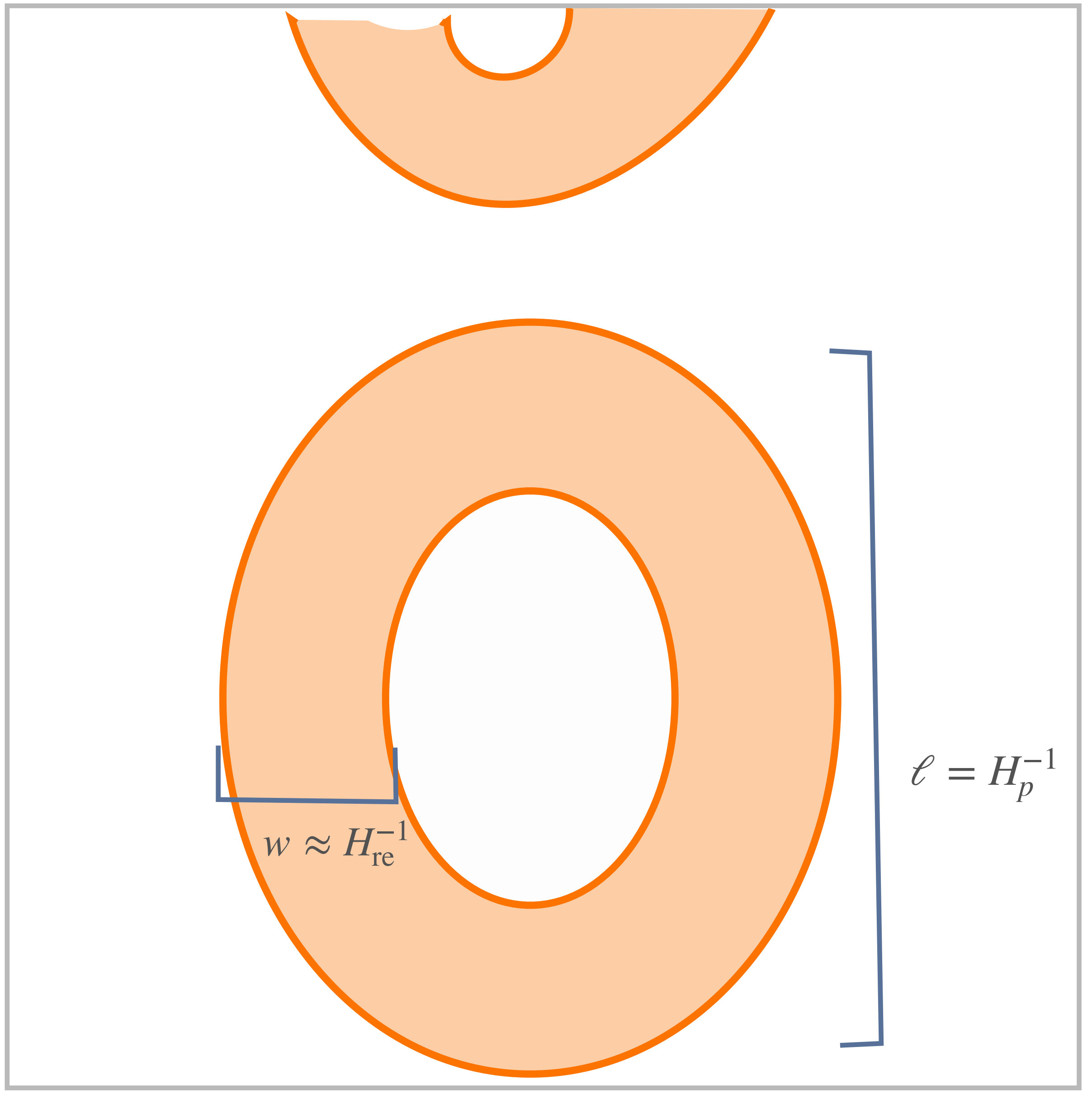
scaling-like



$w \approx H_{\mathrm{re}}^{-1}$

Ring

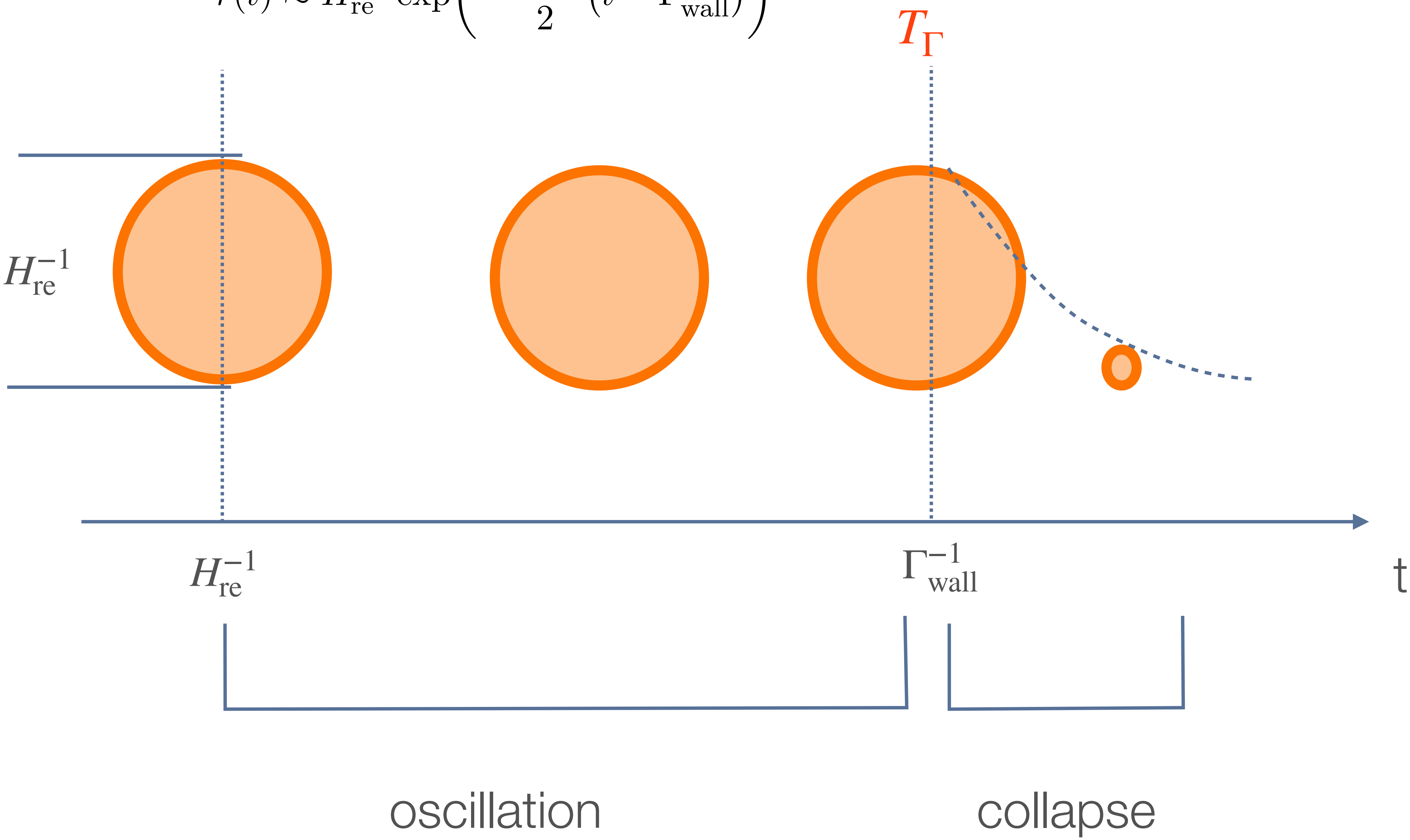




GW from disk

$H_p = H_{\rm re}$

$$\bar{r}(t) \approx H_{\rm re}^{-1} \exp\left(-\frac{\Gamma_{\rm wall}}{2}\left(t - \Gamma_{\rm wall}^{-1}\right)\right)$$



GW: string vs wall

Wall mode: $\Gamma_{\rm wall} \approx \dfrac{\sigma}{M_{\rm Pl}^2}$

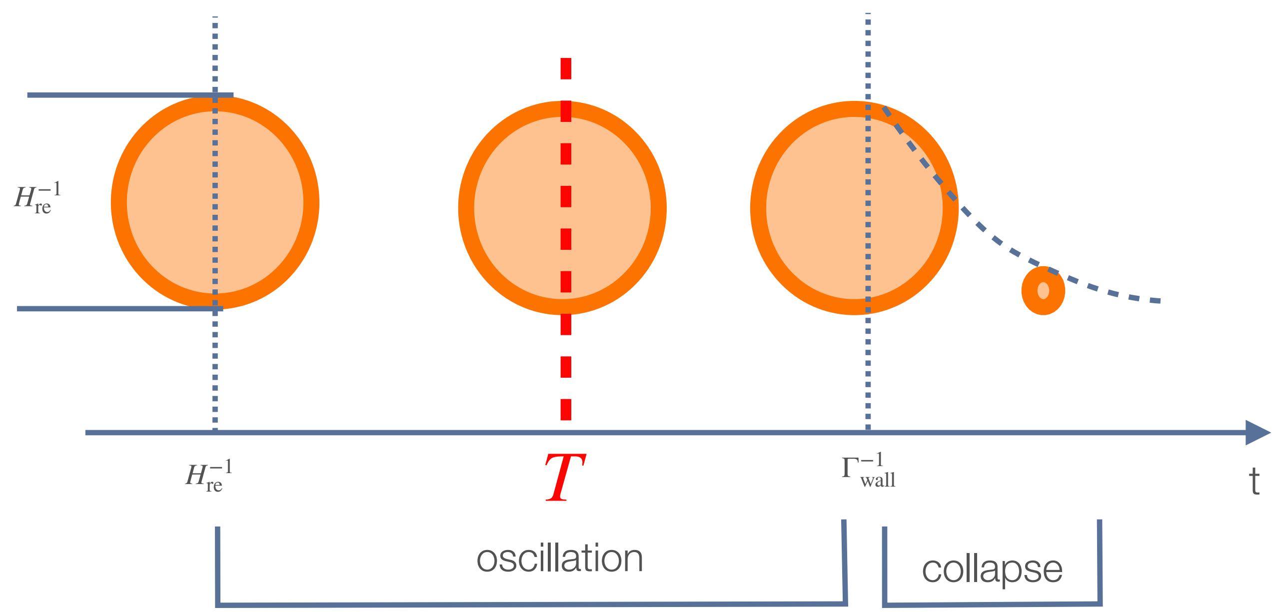
String mode: $\Gamma_{\rm str} \approx \dfrac{\mu}{M_{\rm Pl}^2}\dfrac{1}{\ell}$ $\quad \ell$ string length

For disk: $\ell \approx H_{\rm re}^{-1}$

Wall domination: $\sigma\ell > \mu \to \Gamma_{\rm GW} > \Gamma_{\rm str}$

$P_{\rm GW}({\rm wall}) > P_{\rm GW}({\rm string})$

GW from disk: oscillation



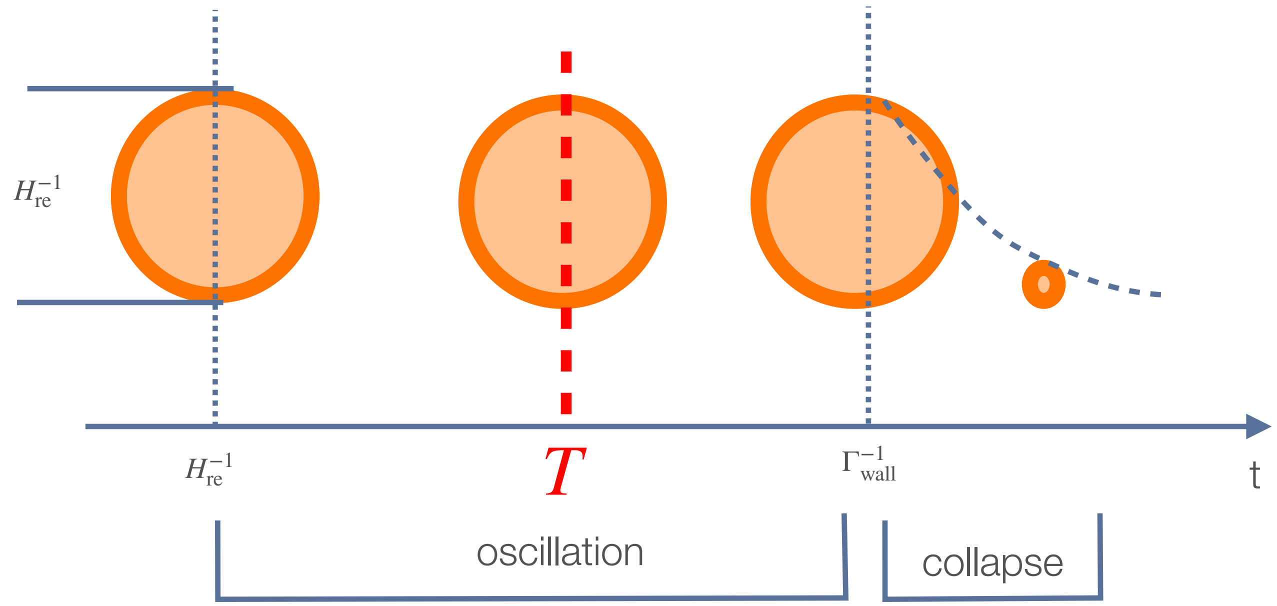
Oscillation stage: $t < \Gamma_{\rm wall}^{-1}$

$$\left.\frac{\partial \Omega_{\rm GW,\ osc.}}{\partial \ln k}\right|_{t=\Gamma_{\rm wall}^{-1}} \approx \frac{1}{T^4}(\sigma H_{\rm re})\frac{\Gamma_{\rm wall}}{H}\left(\frac{T}{T_{\rm re}}\right)^3$$

Wall energy density

Red-shite like a^{-3} after re-entry

GW from disk: oscillation

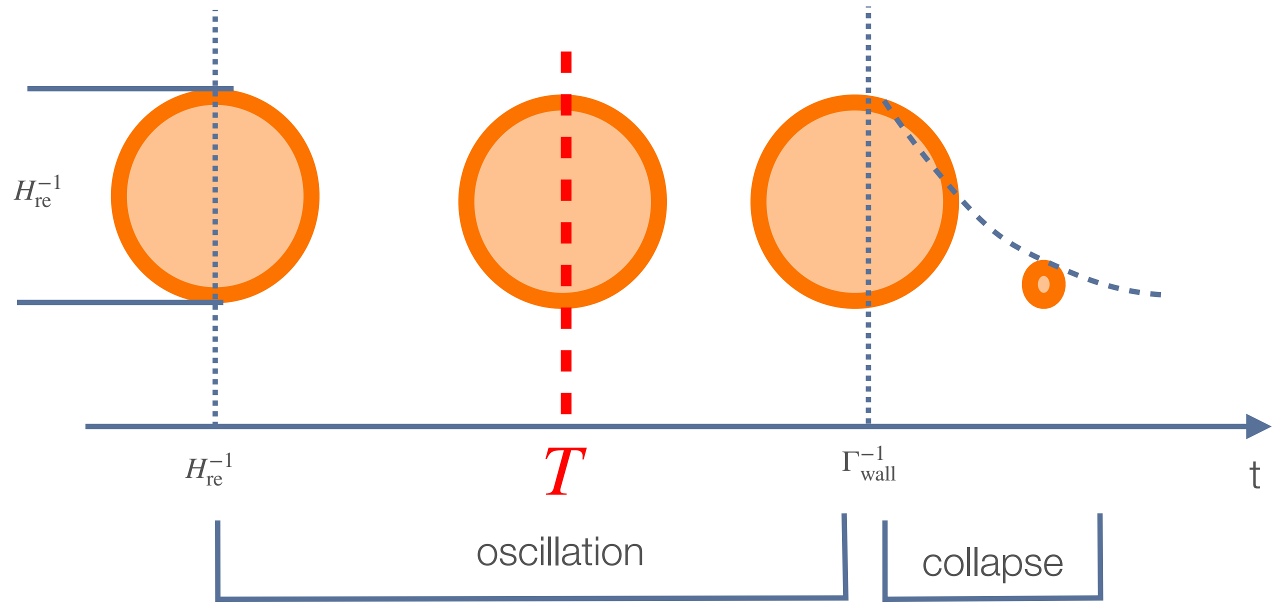


Oscillation stage: $t < \Gamma_{wall}^{-1}$

$$\left.\frac{\partial \Omega_{GW,\ osc.}}{\partial \ln k}\right|_{t=\Gamma_{wall}^{-1}} \approx \frac{1}{T^4}(\sigma H_{re}) \boxed{\frac{\Gamma_{wall}}{H}} \left(\frac{T}{T_{re}}\right)^3$$

$$\approx \Gamma_{wall} \cdot t$$

GW from disk: oscillation



Oscillation stage: $t < \Gamma_{\rm wall}^{-1}$

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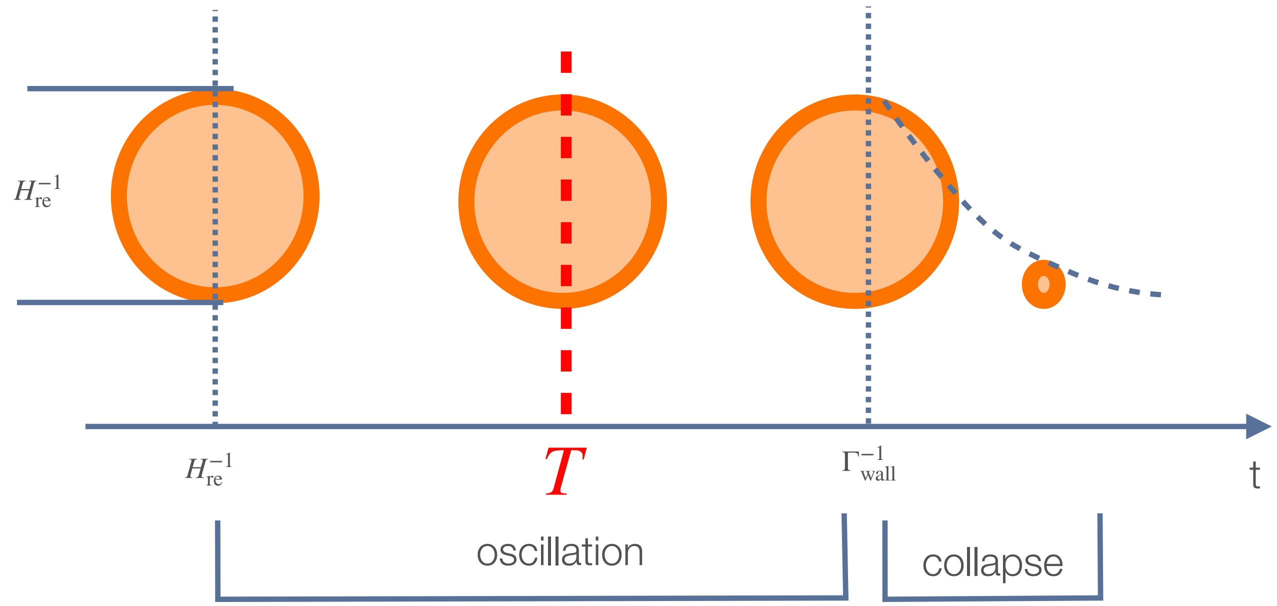
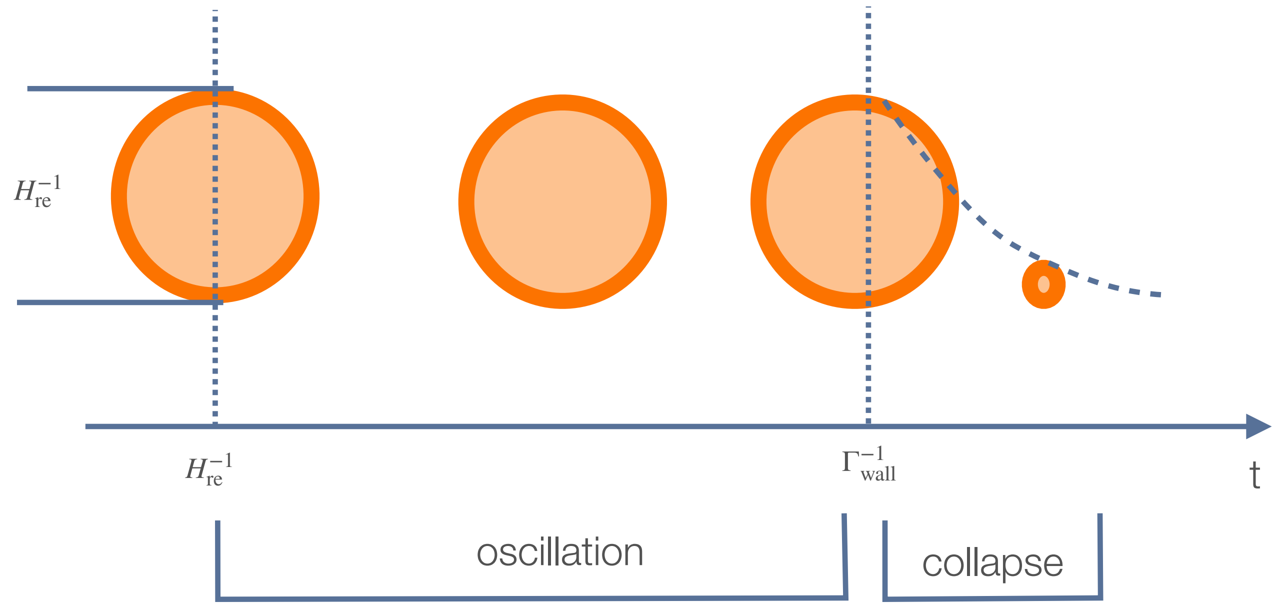
radiation frequency, red-shifted : $k \cdot T = \text{constant}$

GW from disk: oscillation



$$\left.\frac{\partial \Omega_{\rm GW,\ osc.}}{\partial \ln k}\right|_{t=\Gamma_{\rm wall}^{-1}} \approx \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2} \Gamma_{\rm wall}^{3/2}} \left(\frac{k}{H_{\rm re}}\right)^3$$

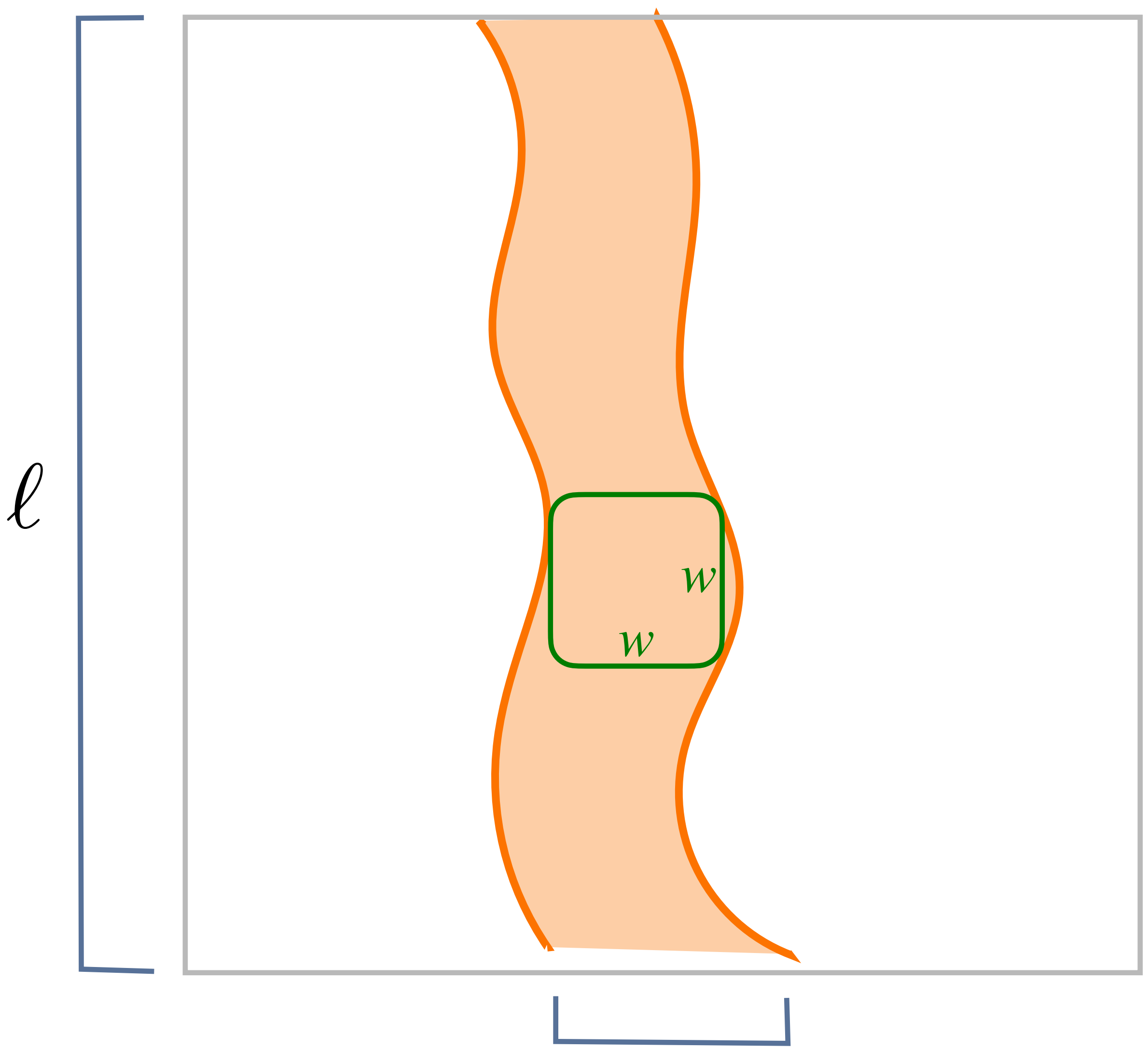
GW from disk: collapse



Collapse stage: $\bar{r} \approx k^{-1}$

$$\left.\frac{\partial \Omega_{\rm GW,\ col.}}{\partial \ln k}\right|_{t=\Gamma^{-1}} \approx \frac{1}{T_\Gamma^4}\sigma H_{\rm re}\left(\frac{T_\Gamma}{T_{\rm re}}\right)^3 \frac{\Gamma_{\rm GW}}{\Gamma}(\bar{r}H_{\rm re})^2 \approx \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma^{3/2}}\left(\frac{H_{\rm re}}{k}\right)^2$$

GW from belts



$w \approx H_{\rm re}^{-1}$

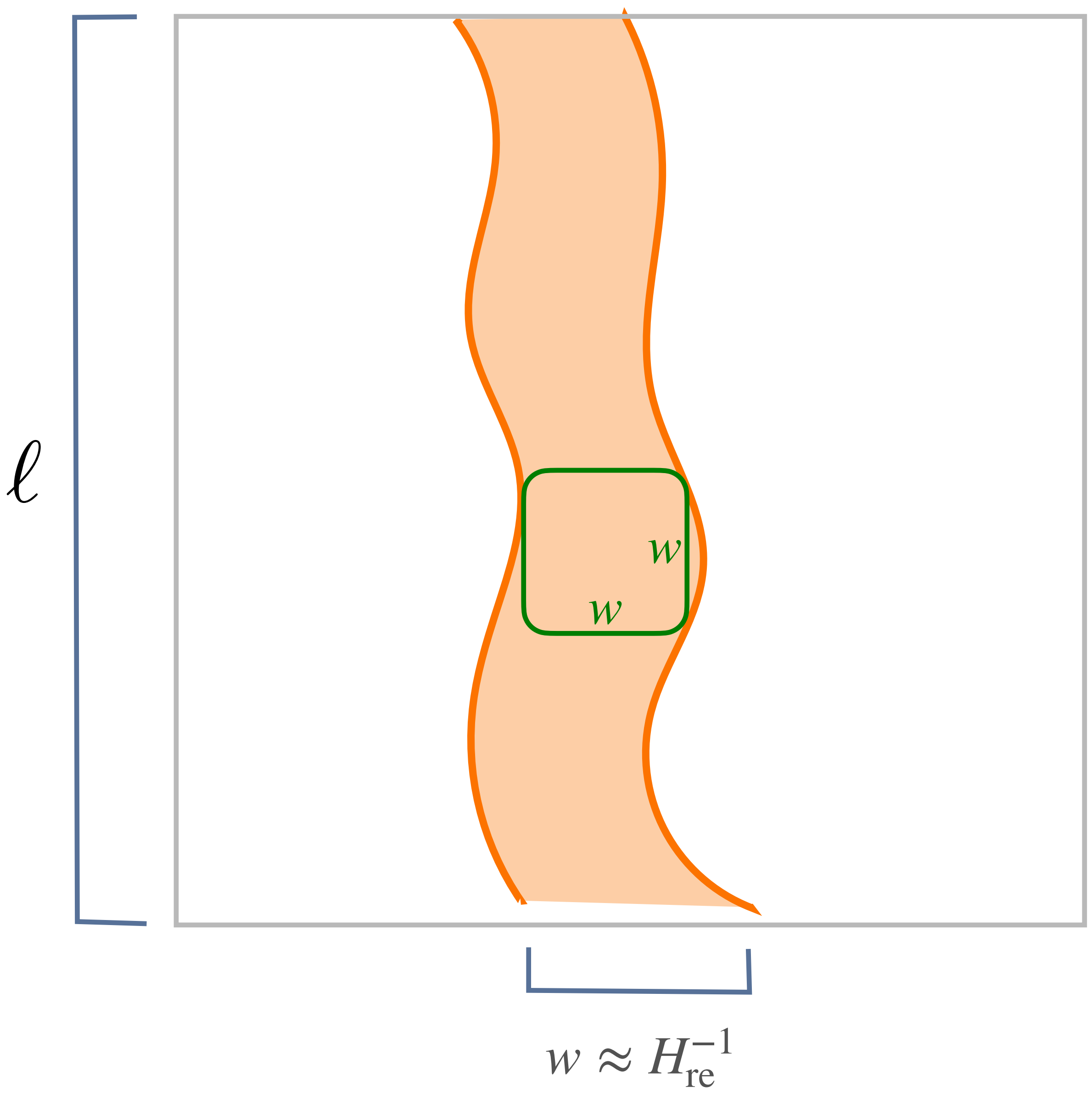
$$\rho_{\rm belts} \approx \sigma \ell w H^3 \approx \frac{\sigma H^2}{H_{\rm re}}$$

$$P_{\rm GW}^{w\times w} \approx \frac{1}{M_{\rm Pl}^2}\left(\sigma w^4 \frac{1}{w^3}\right)^2$$

$$P_{\rm GW} \approx \frac{\left\langle \dddot{Q}^2 \right\rangle}{M_{\rm Pl}^2} \approx \frac{1}{M_{\rm Pl}^2}\left(\sum_{w\times w \text{ patches}} \frac{\sigma w^4}{w^3}\right)^2 \approx \frac{\sigma^2 w^2}{M_{\rm Pl}^2}\frac{\ell}{w}$$

$$\Gamma_{\rm wall} \approx \frac{\sigma}{M_{\rm Pl}^2}$$

GW from belts

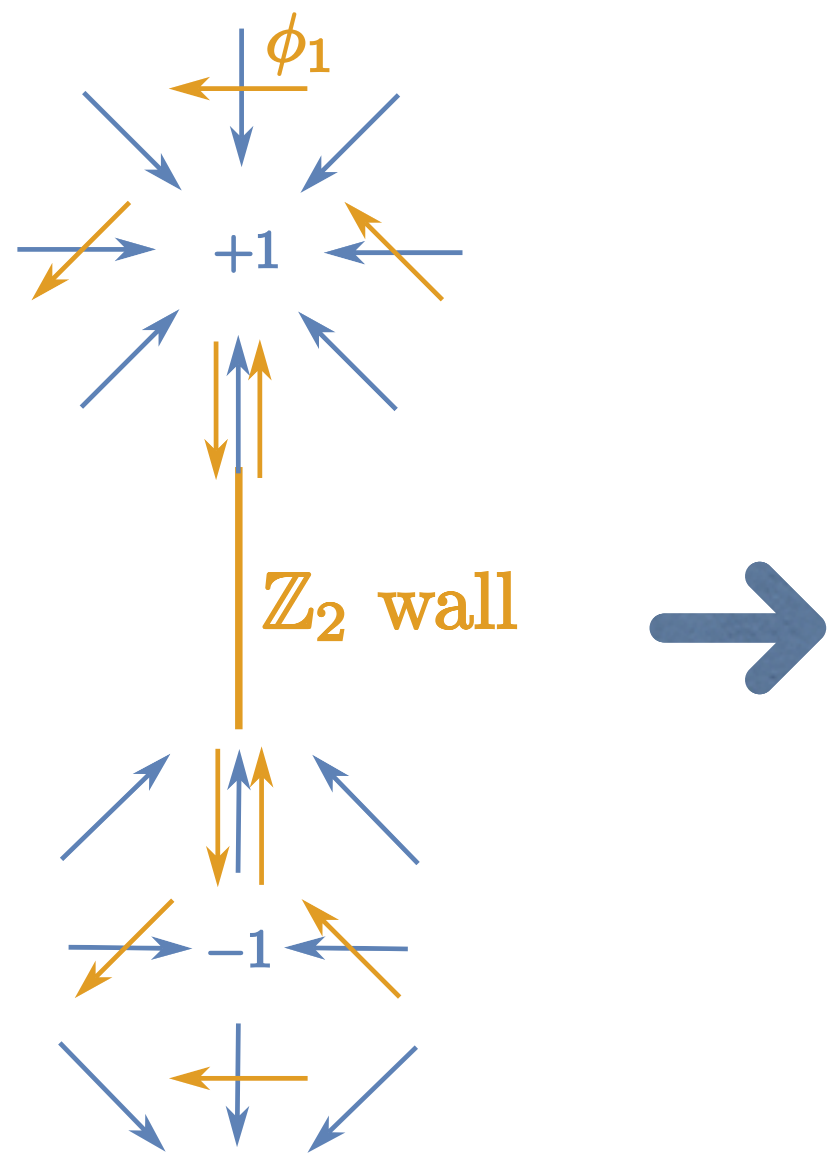


$$\Gamma_{\rm wall} \approx \frac{\sigma}{M_{\rm Pl}^2}$$

$$\Gamma_{\rm str} \approx \frac{\mu}{M_{\rm Pl}^2}\frac{1}{\ell}$$

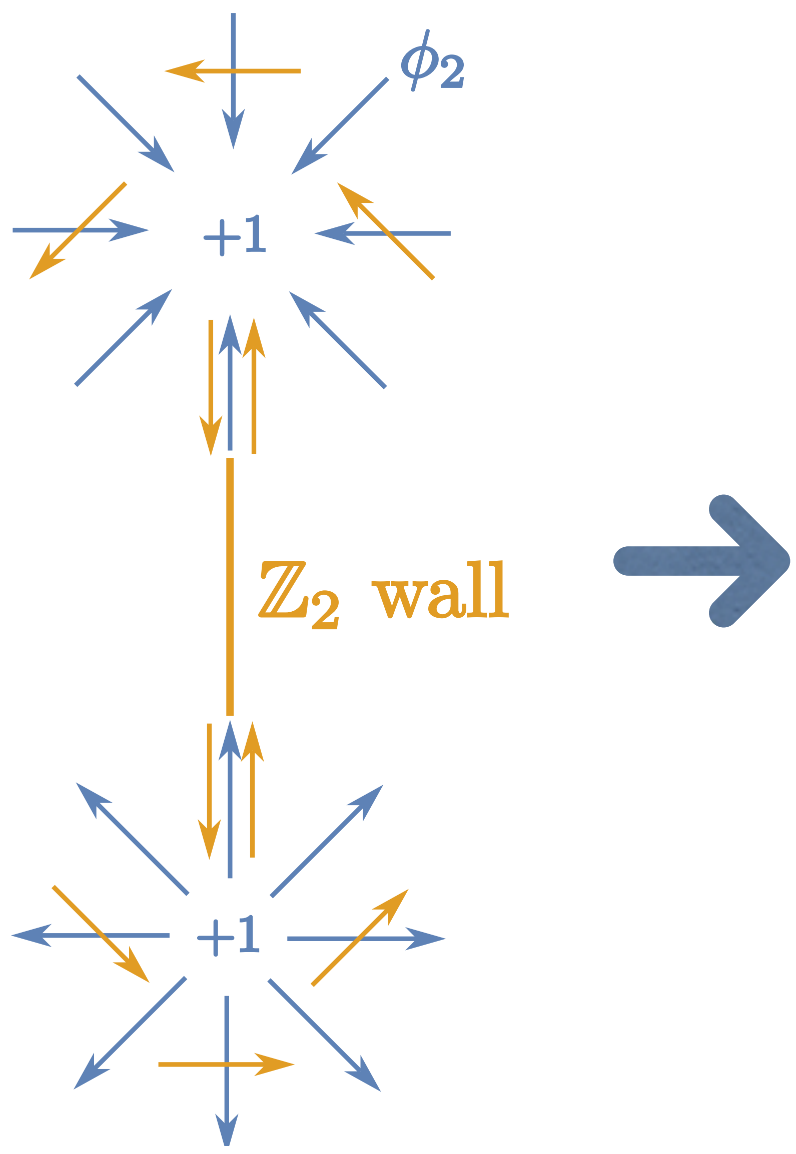
$$\Gamma_{\rm wall} > \Gamma_{\rm str} \text{ for } \ell > H_{\rm re}^{-1}$$

End point of strip collapse



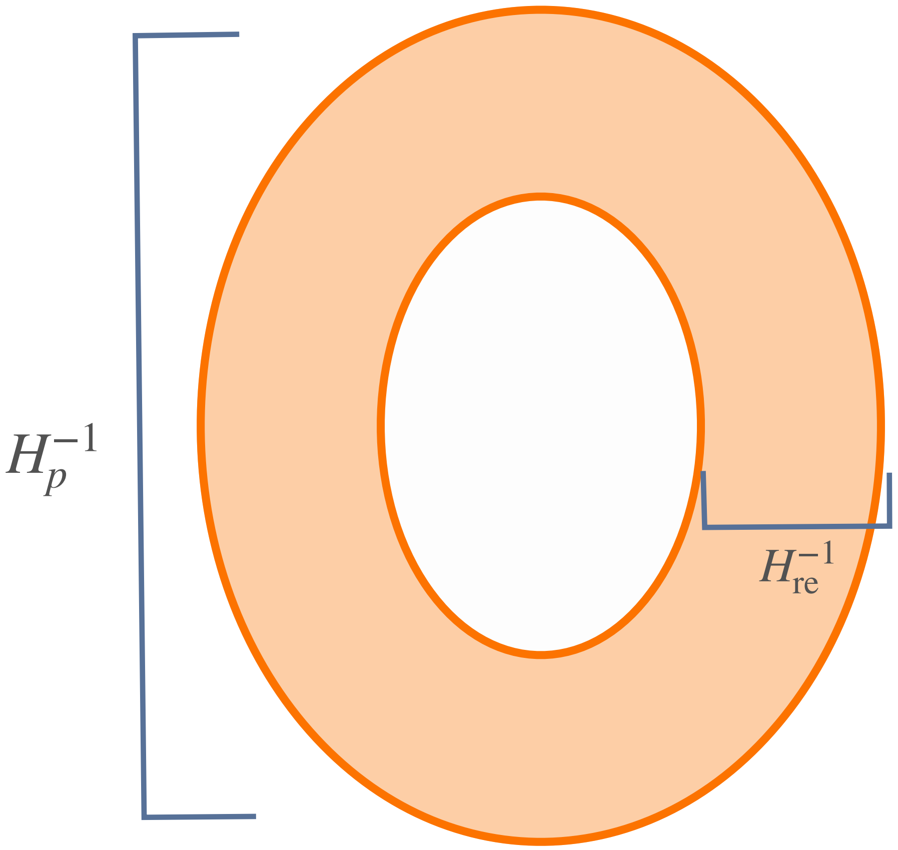
String annihilation

End point of strip collapse



String bundle

GW from rings



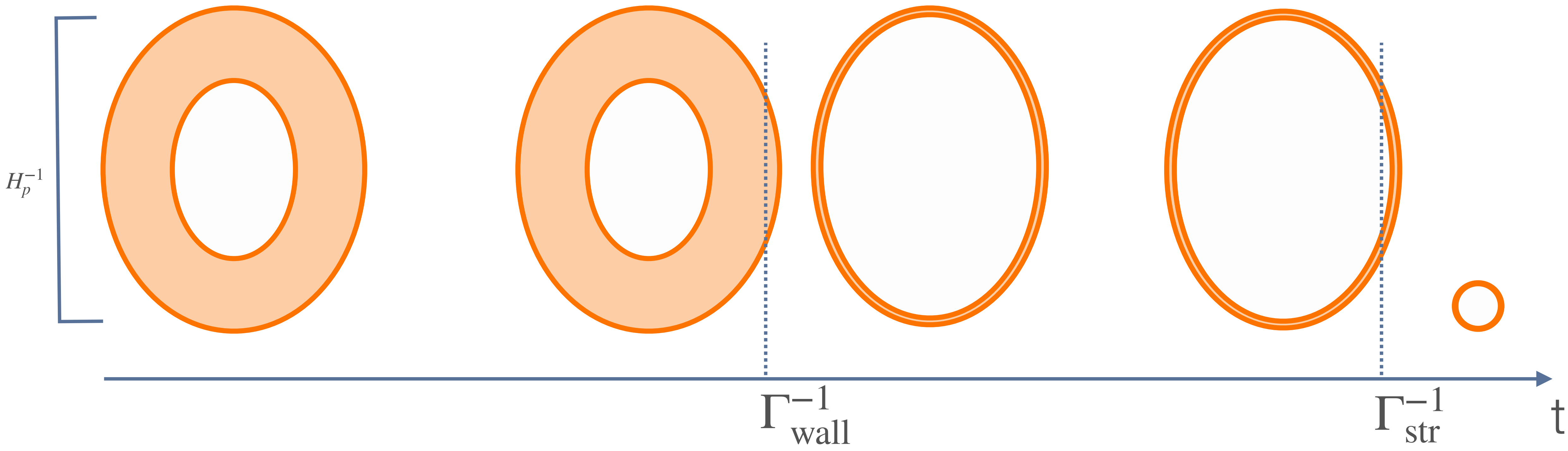
H_p^{-1} : Size of the ring at production

GW from

Wall mode: frequency = $H_{\rm re}$

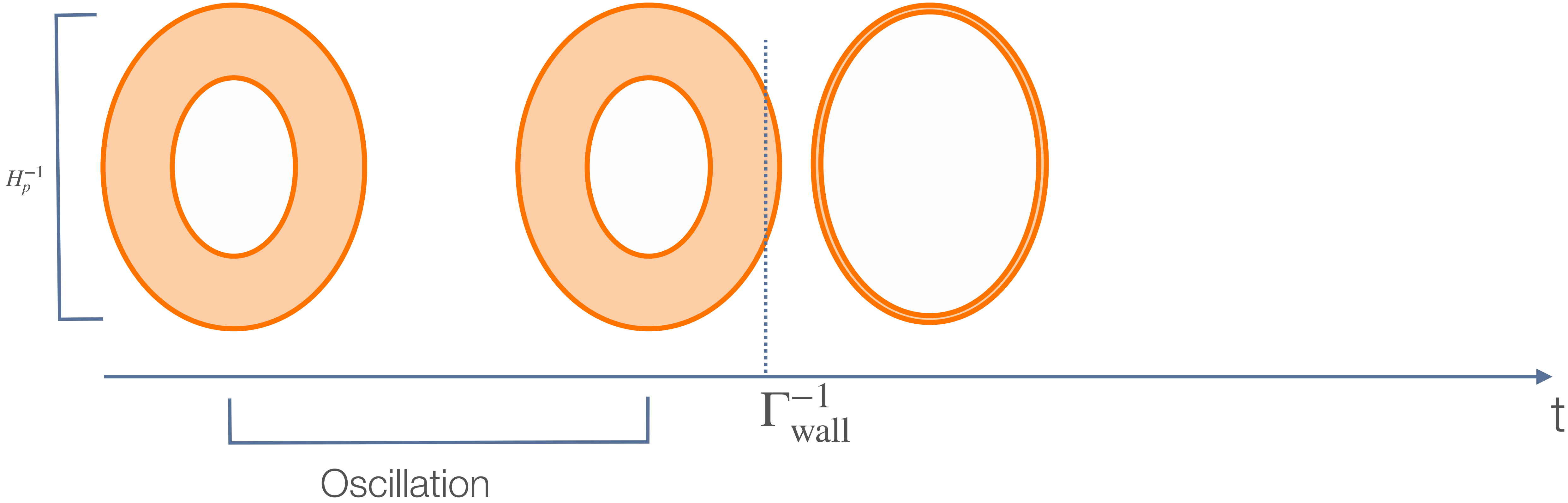
String mode: frequency = H_p

GW from rings



$\Gamma_{\rm wall} > \Gamma_{\rm str}$

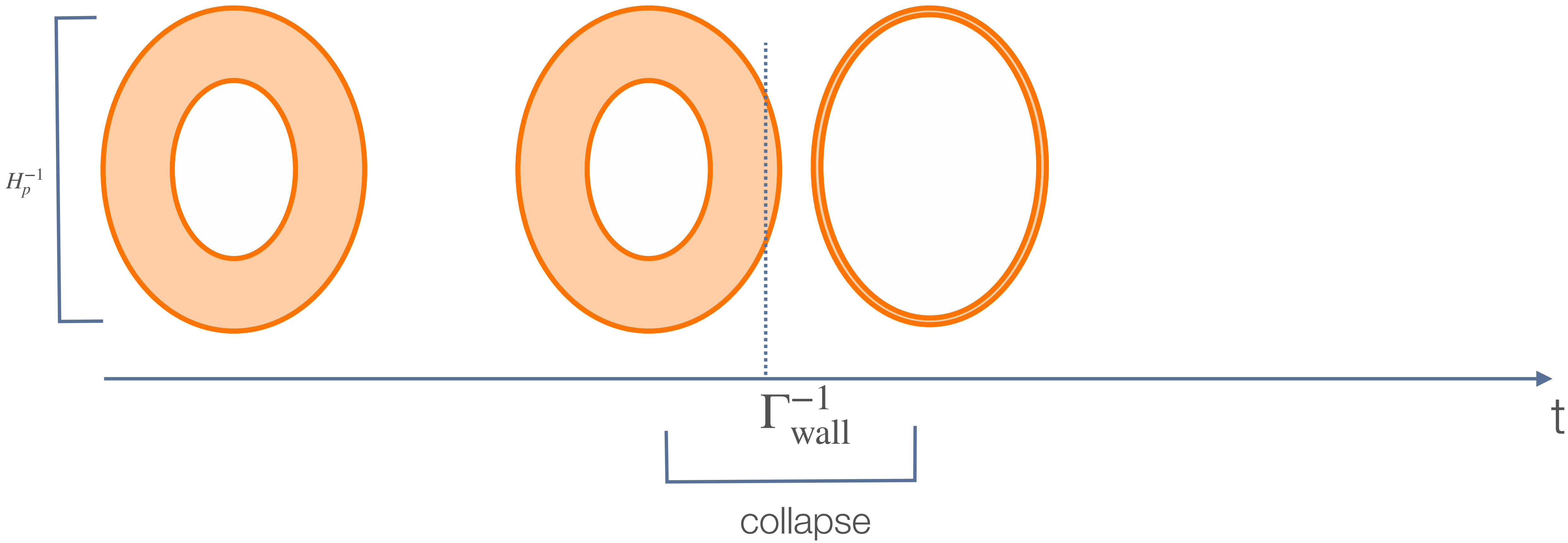
GW from rings: wall mode



$$\left.\frac{\partial^2 \Omega_{\rm GW,\ ring,\ wall\ osc.}}{\partial \ln k \partial \ln H_p}\right|_{t=\Gamma^{-1}} \approx \frac{1}{T^4}\left(\frac{\sigma}{H_{\rm re}} H_p^2\right)\left(\frac{T}{T_p}\right)^3 \frac{\Gamma_{\rm GW}}{H}$$

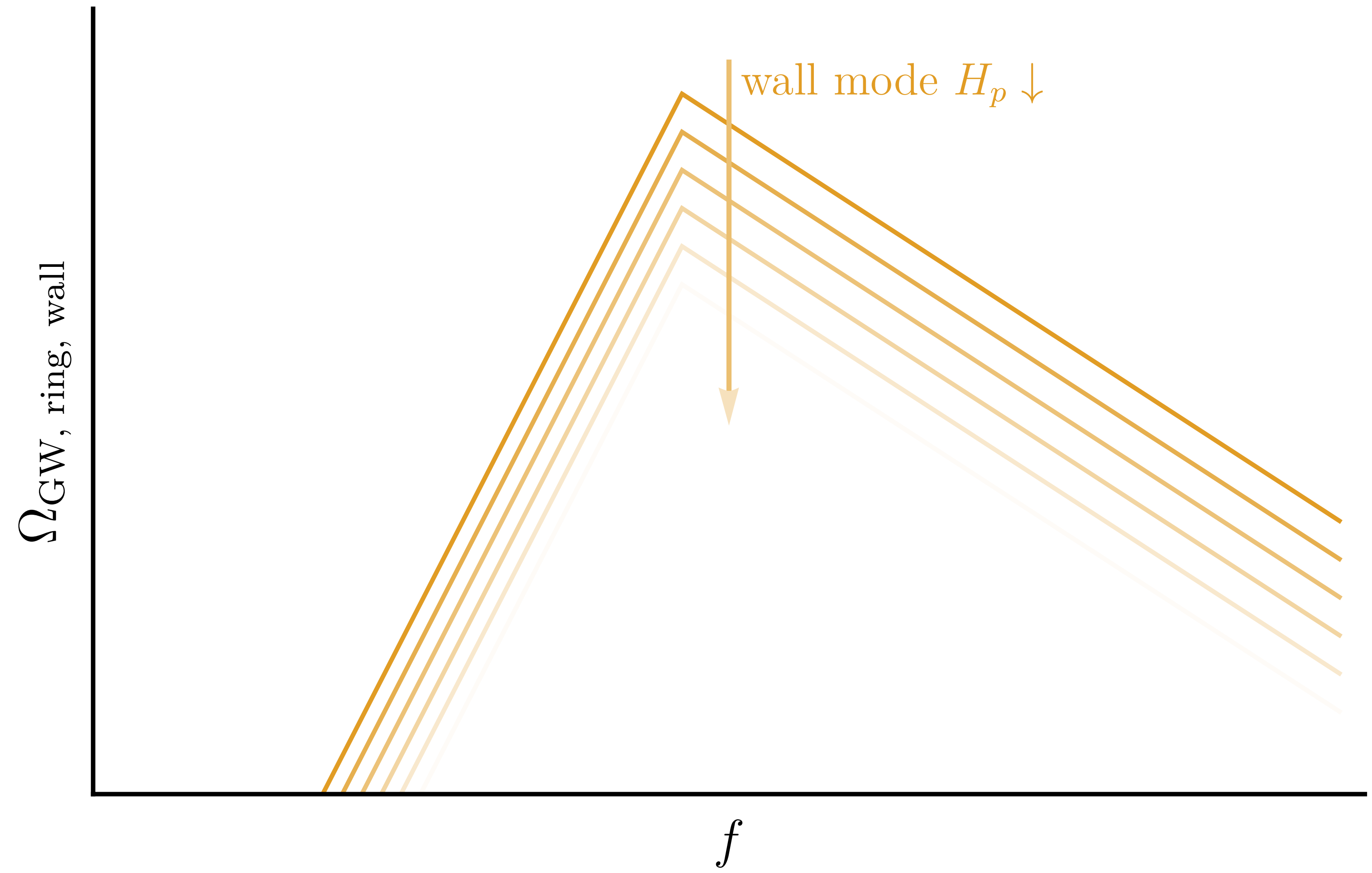
$$\approx \left(\frac{H_p}{H_{\rm re}}\right)^{1/2} \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2} \Gamma^{3/2}} \left(\frac{k}{H_{\rm re}}\right)^3, \quad k \lesssim H_{\rm re}$$

GW from rings: wall mode



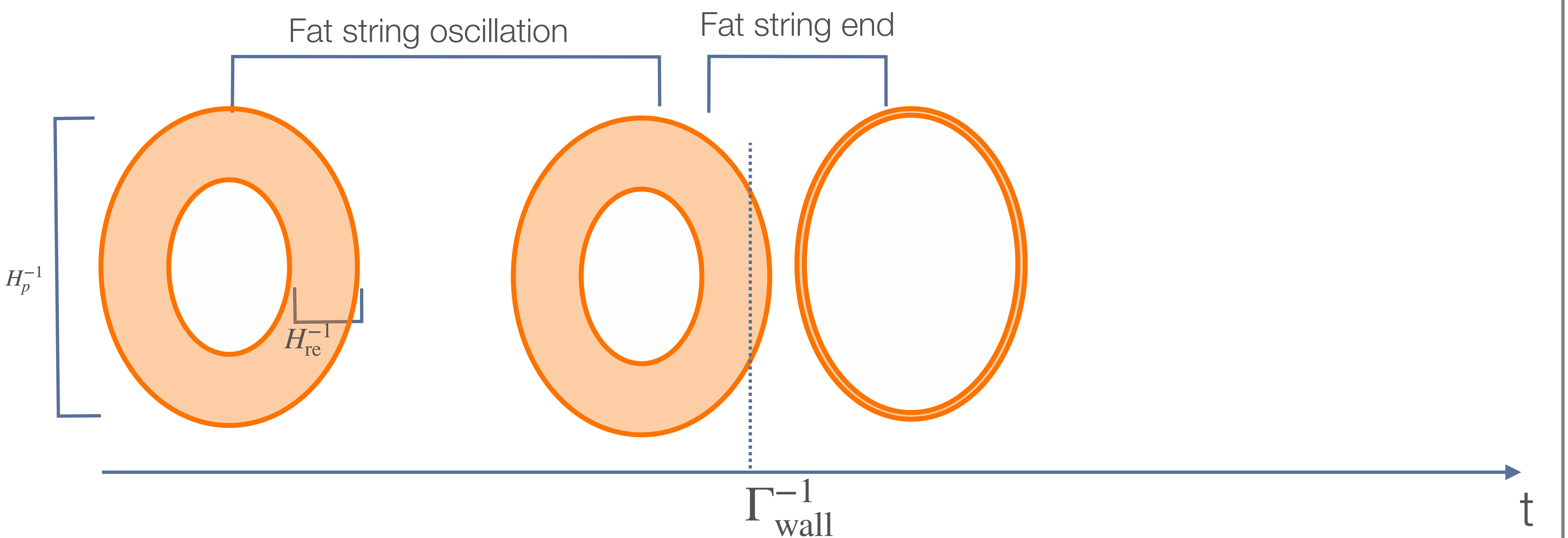
$$\left.\frac{\partial^2\Omega_{\text{GW, ring, wall col.}}}{\partial\ln k\,\partial\ln H_p}\right|_{t=\Gamma^{-1}} \approx \left(\frac{H_p}{H_{\rm re}}\right)^{1/2}\frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma^{3/2}}\left(\frac{H_{\rm re}}{k}\right), \quad k \gtrsim H_{\rm re}.$$

GW from rings: wall mode



Adding up all possible H_p

GW from rings: fat string mode

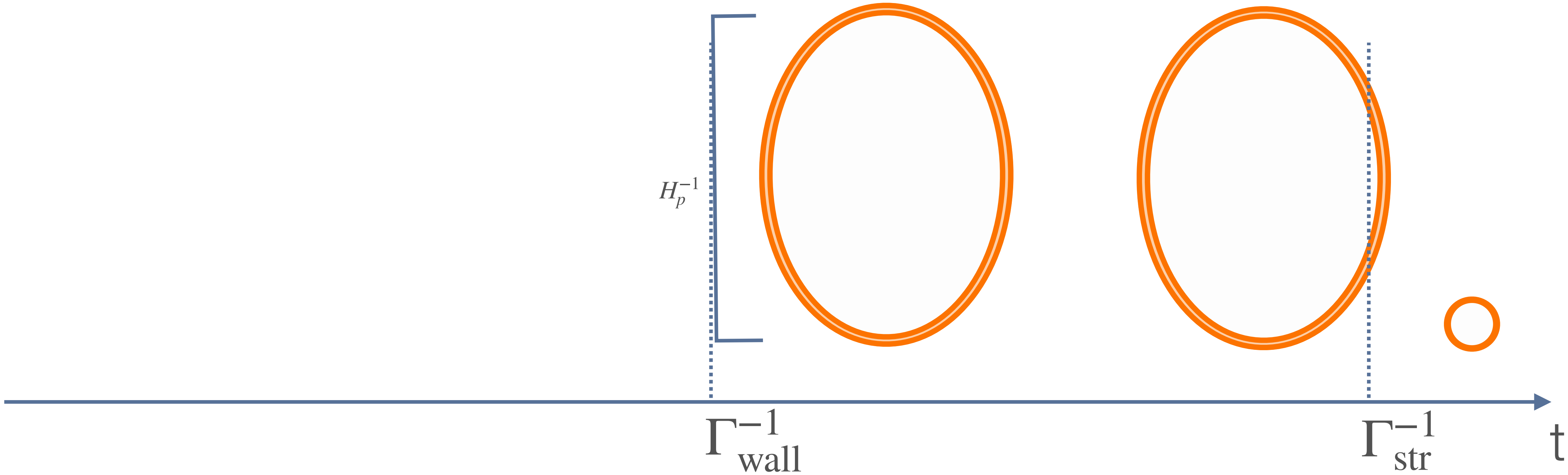


Effective string tension: $\mu_{\rm eff} \approx \dfrac{\sigma}{H_{\rm re}}$

$$\left.\frac{\partial \Omega_{\rm GW,\ ring,\ fat\ str}}{\partial \ln k}\right|_{t=\Gamma^{-1}} \approx \frac{1}{T^4}\left(\frac{\sigma H_p^2}{H_{\rm re}}\right)\left(\frac{T}{T_p}\right)^3 \frac{\Gamma^{\rm str}_{\rm GW,\ ring}}{H}$$

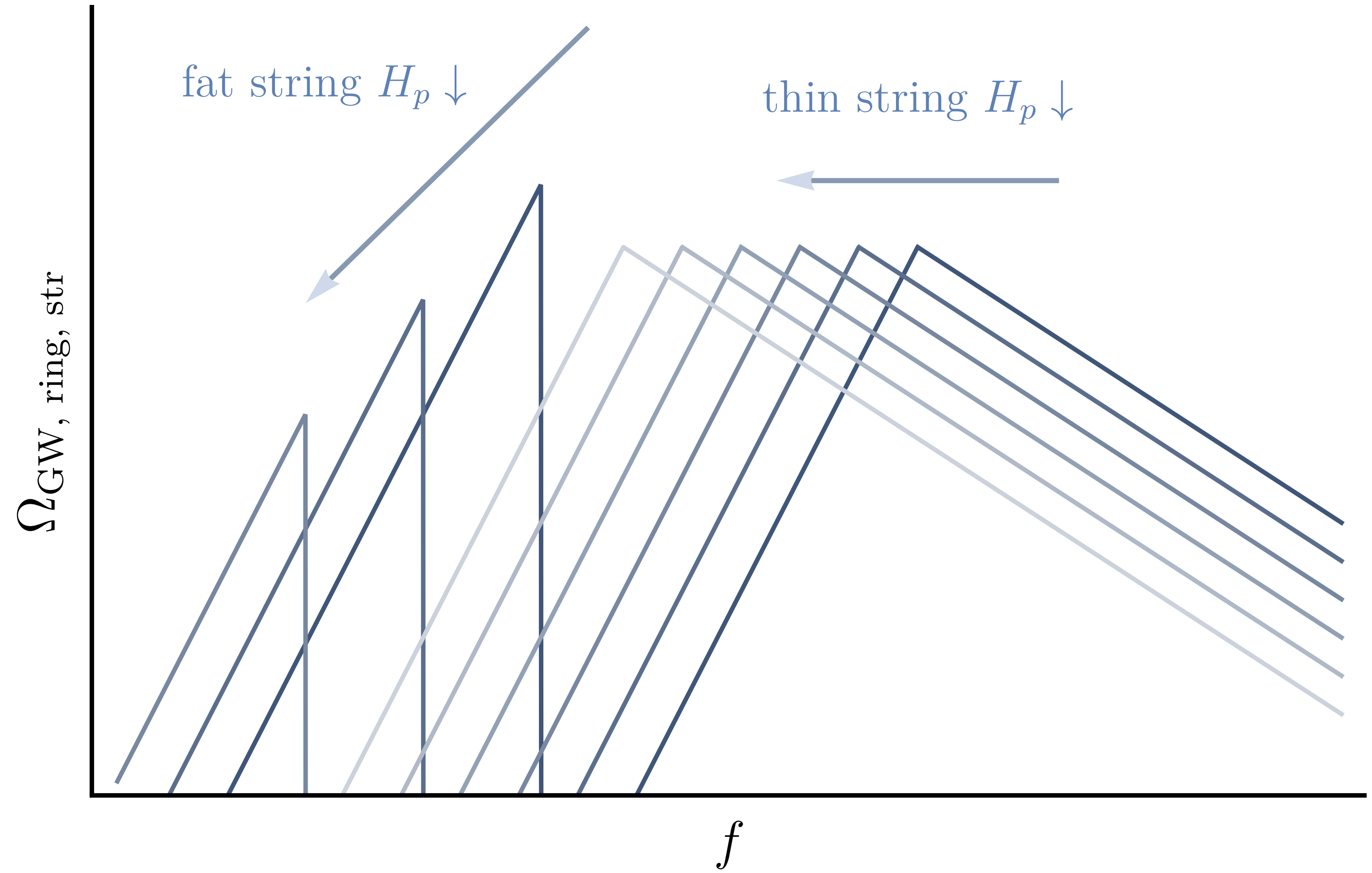
$$\approx \left(\frac{H_p}{H_{\rm re}}\right)^{3/2} \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma^{3/2}}\left(\frac{k}{H_p}\right)^3, \quad k \lesssim H_p.$$

GW from rings, thin string

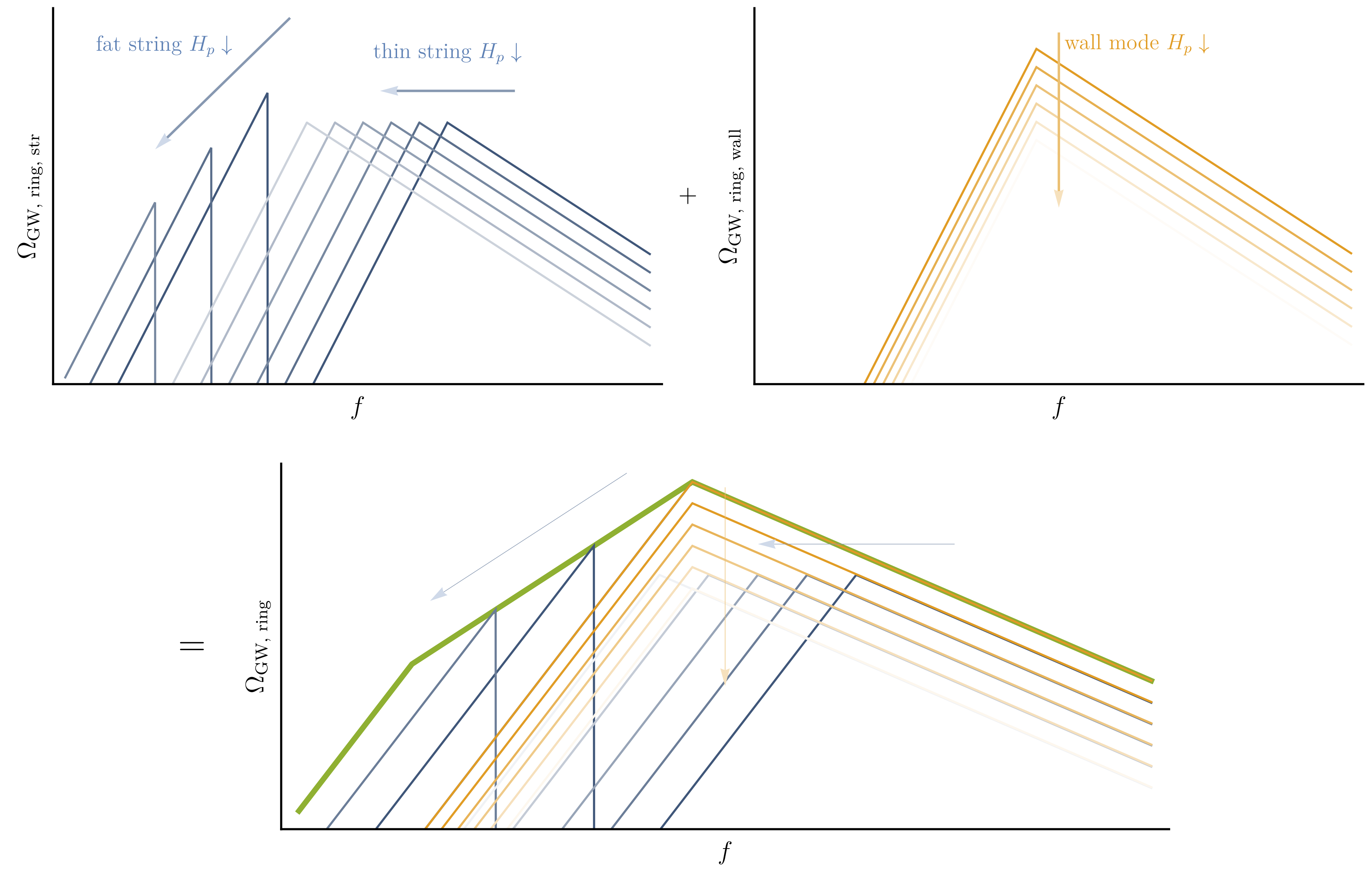


Similar to the usual cosmic string

GW from rings: strings



GW from rings:



GW (total)

$$\left.\frac{\partial \Omega_{\rm GW}}{\partial \ln k}\right|_{t=\Gamma_{\rm wall}^{-1}} \approx \frac{2\pi\sigma^2}{3M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma_{\rm wall}^{3/2}} \begin{cases} \left(\frac{k}{\Gamma_{\rm wall}}\right)^3 \left(\frac{\Gamma_{\rm wall}}{H_{\rm re}}\right)^{3/2}, & k \lesssim \Gamma_{\rm wall}, \\ \left(\frac{k}{H_{\rm re}}\right)^{3/2}, & \Gamma_{\rm wall} \lesssim k \lesssim H_{\rm re} \\ \left(\frac{H_{\rm re}}{k}\right), & k \gtrsim H_{\rm re}. \end{cases}$$

In comparison with post inflation production

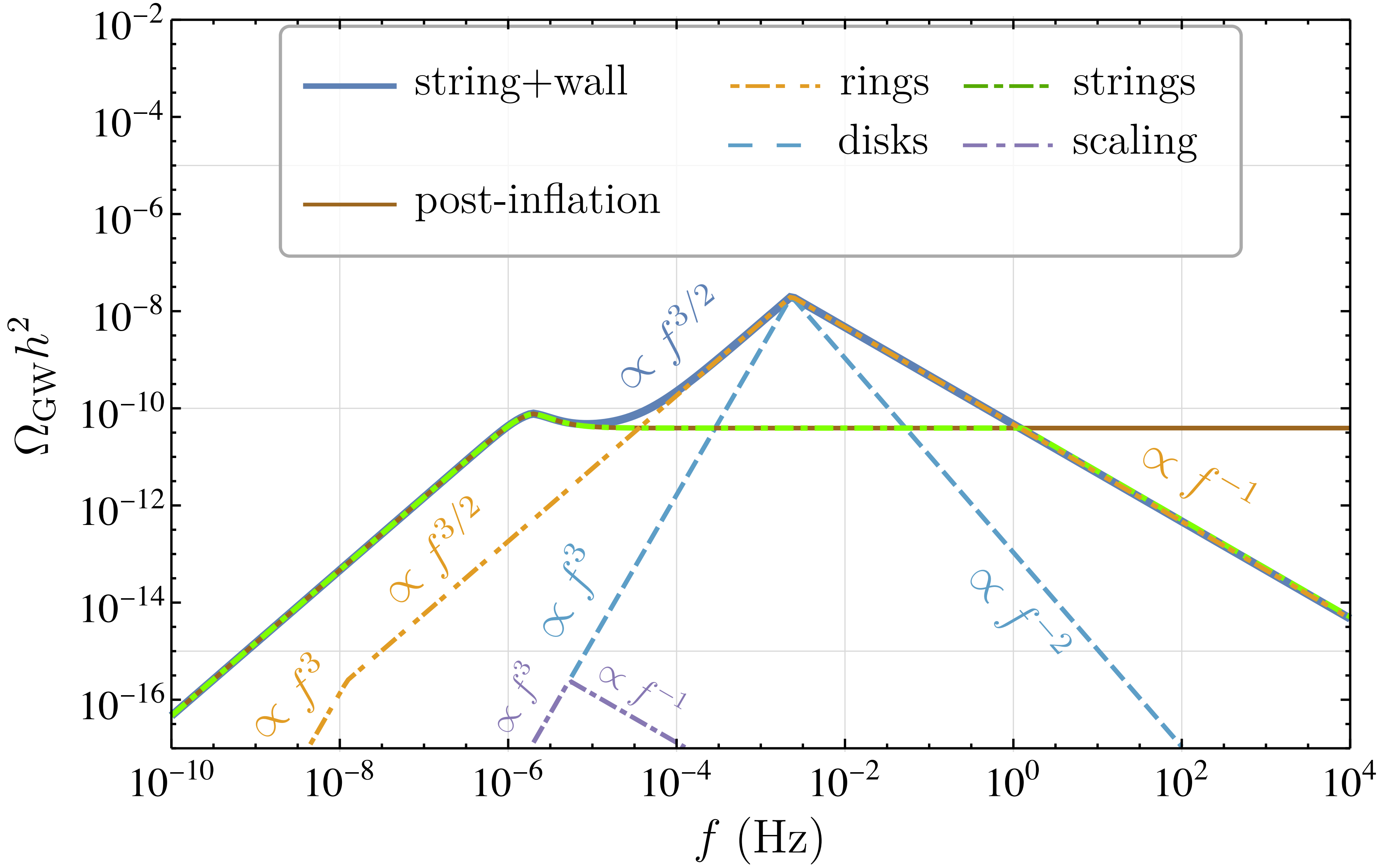
$H_{\rm re}$ is an independent parameter, not tied to string/wall parameters

Small $H_{\rm re}$ enhances GW signal

$$\left.\Omega_{\rm GW} h^2\right|_{T_0, f_{\rm peak}} \approx 2 \times 10^{-8} \left(\frac{\sigma^{1/3}}{10^6\,{\rm GeV}}\right)^6 \left(\frac{5.3 \times 10^{-19}\,{\rm GeV}}{\Gamma_{\rm wall}}\right)^{3/2} \left(\frac{10^{-14}\,{\rm GeV}}{H_{\rm re}}\right)^{1/2}$$

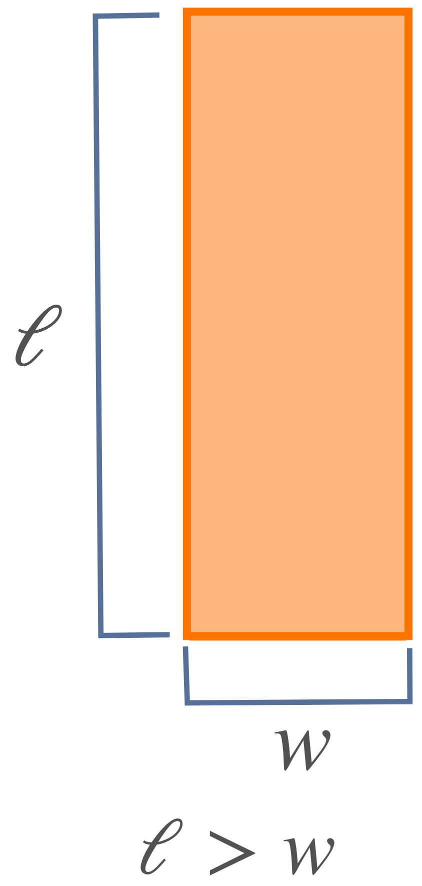
$$\left.f_{\rm peak}\right|_{T_0} \approx 2\times 10^{-3}\,{\rm Hz} \left(\frac{106.75}{g_{*,s}(T_{\Gamma_{\rm wall}})}\right)^{1/3} \left(\frac{g_{*,\rho}(T_{\Gamma_{\rm wall}})}{106.75}\right)^{1/4} \left(\frac{H_{\rm re}}{10^{-13}\,{\rm GeV}}\right) \left(\frac{5.3 \times 10^{-19}\,{\rm GeV}}{\Gamma_{\rm wall}}\right)^{1/2}$$

GW (total)



$\mu^{1/2} = 10^{13}\,\text{GeV} \quad \sigma^{1/3} = 10^6\,\text{GeV} \quad \Gamma_{\rm GW} \approx 5.3 \times 10^{-19}\,\text{GeV} \quad H_{\rm re} = 10^{-14}\,\text{GeV}$

Global string bounding wall



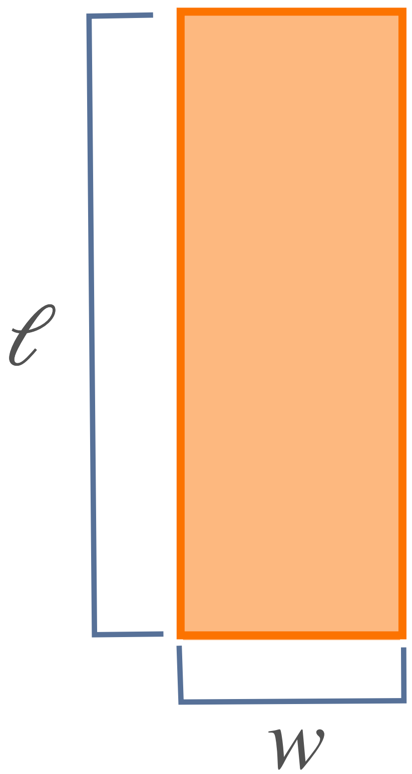
Axion radiation dominantly from the string.

Radiation will damp wall oscillation, with $k \approx \frac{1}{w}$.

$$P_{\rm NGB} \approx \gamma_a \mu k \ell \approx \gamma_a \mu \frac{\ell}{w}, \quad \mu \sim v_2^2 \quad \gamma_a \sim 60$$

$$\Gamma_{\rm NGB} \approx \frac{\gamma_a \mu}{\sigma w^2} \approx \gamma_a \mu k^2$$

Global string bounding wall



$\ell > w$

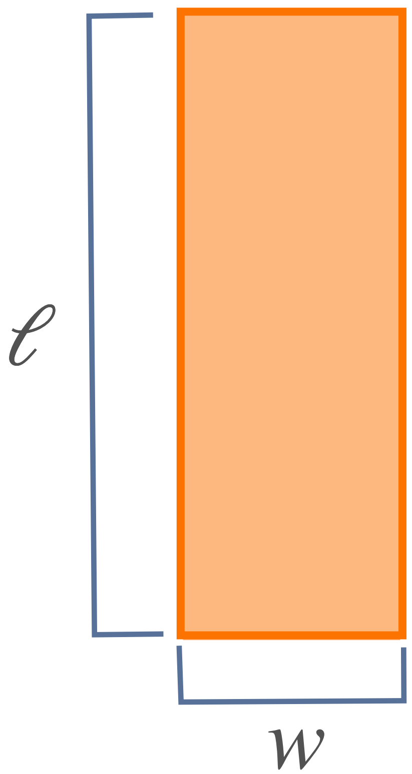
$$\frac{d(\sigma w \ell)}{dt} = -\left(\Gamma_{\text{wall}} + \Gamma_{\text{NGB}}\right)\sigma w \ell$$

$$\Gamma_{\text{wall}} = \frac{\sigma}{M_{\text{Pl}}^2} \qquad \Gamma_{\text{NGB}} \approx \frac{\gamma_a \mu}{\sigma w^2} \approx \gamma_a \mu k^2$$

Threshold where $\Gamma_{\text{wall}} = \Gamma_{\text{NGB}}$

$$k_{\text{NGB}} \approx \frac{\sigma}{M_{\text{Pl}} v_2} \frac{1}{\gamma_a^{1/2}}$$

Global string bounding wall



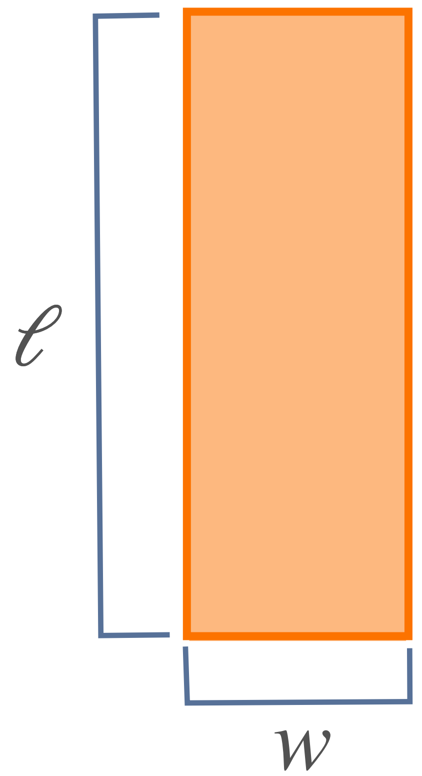
$$\ell > w$$

$$k_{\rm NGB} \approx \frac{\sigma}{M_{\rm Pl} v_2} \frac{1}{\gamma_a^{1/2}}$$

$$\frac{1}{w} \approx k < k_{\rm NGB}$$

GW radiation dominates

Global string bounding wall



$$\ell > w$$

$$k_{\rm NGB} \approx \frac{\sigma}{M_{\rm Pl} v_2} \frac{1}{\gamma_a^{1/2}}$$

$$\frac{1}{w} \approx k < k_{\rm NGB}$$

GW radiation dominates

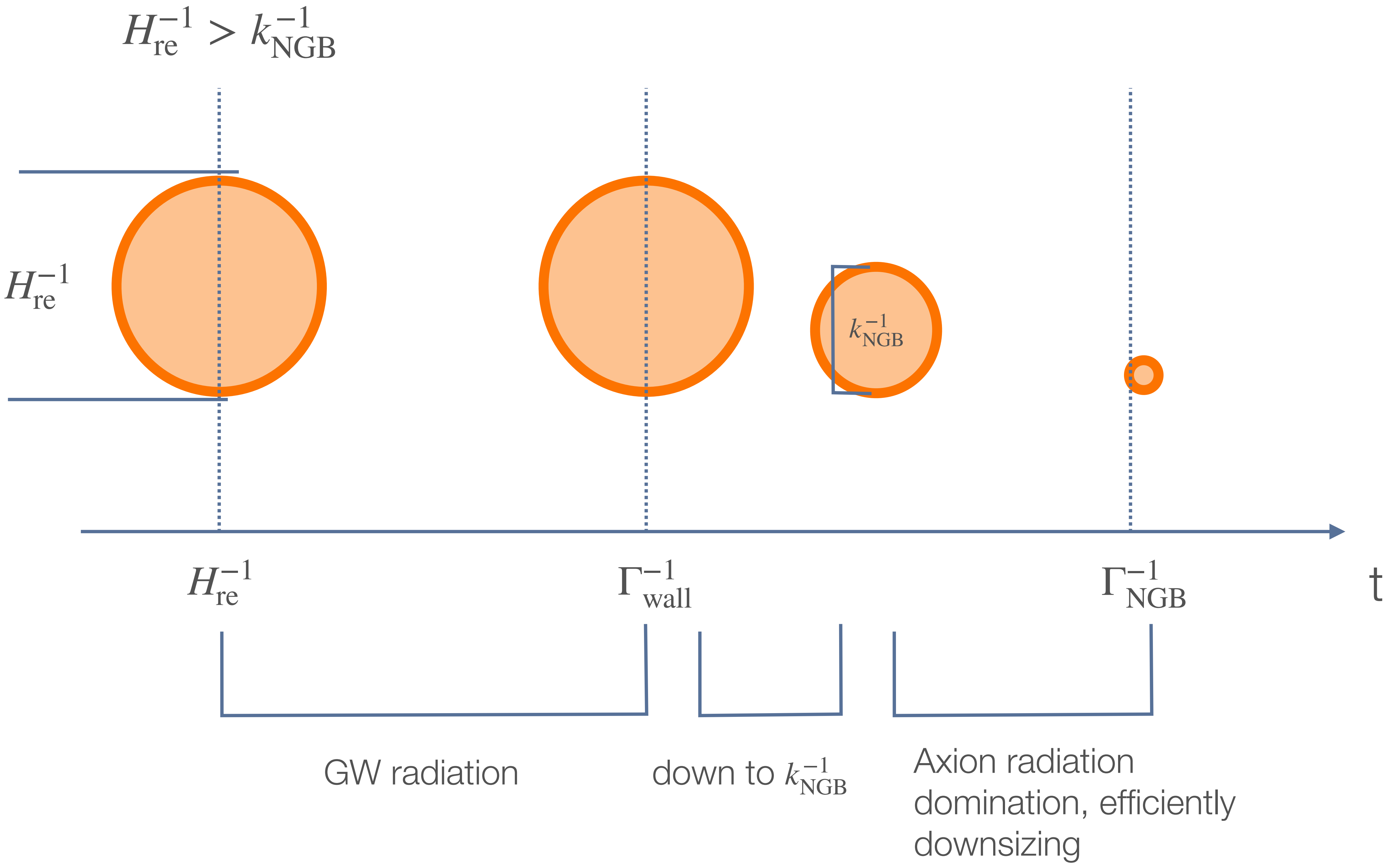
$$\frac{1}{w} \approx k > k_{\rm NGB}$$

Goldstone radiation dominates, efficiently reducing w.

$$\Omega_{\rm GW} \propto \frac{\Gamma_{\rm wall}}{\Gamma_{\rm NGB}(k)} \qquad \Gamma_{\rm NGB}(k) \approx \Gamma_{\rm wall} \left(\frac{k}{k_{\rm NGB}} \right)^2$$

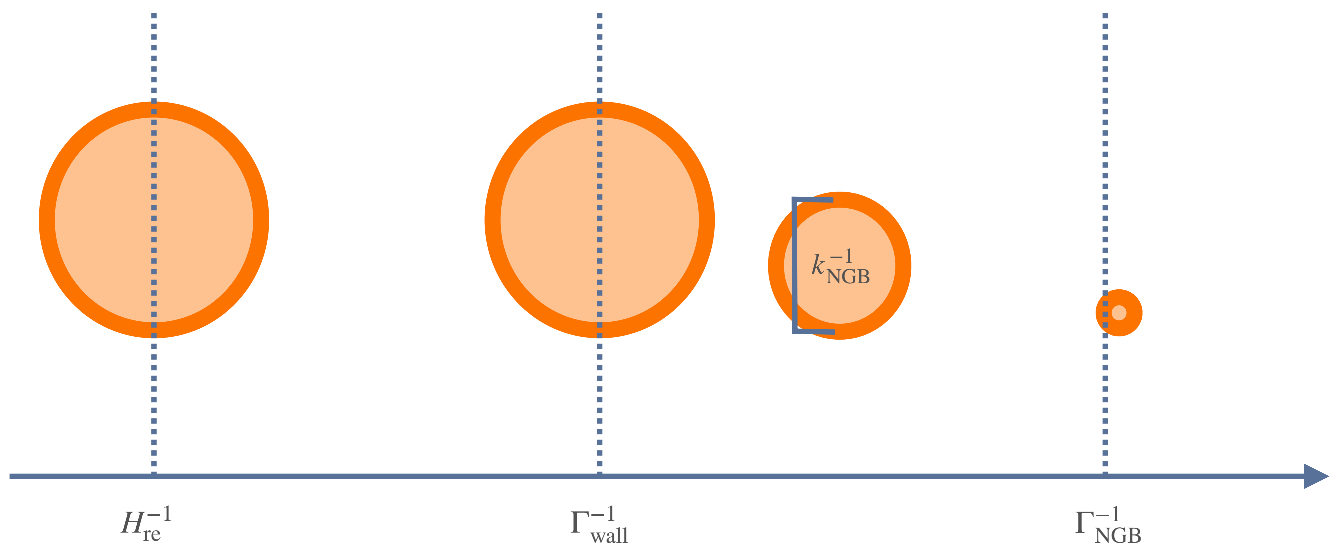
Additional factor of k^{-2} during axion radiation domination

Global string bounded disk



Global string bounded disk

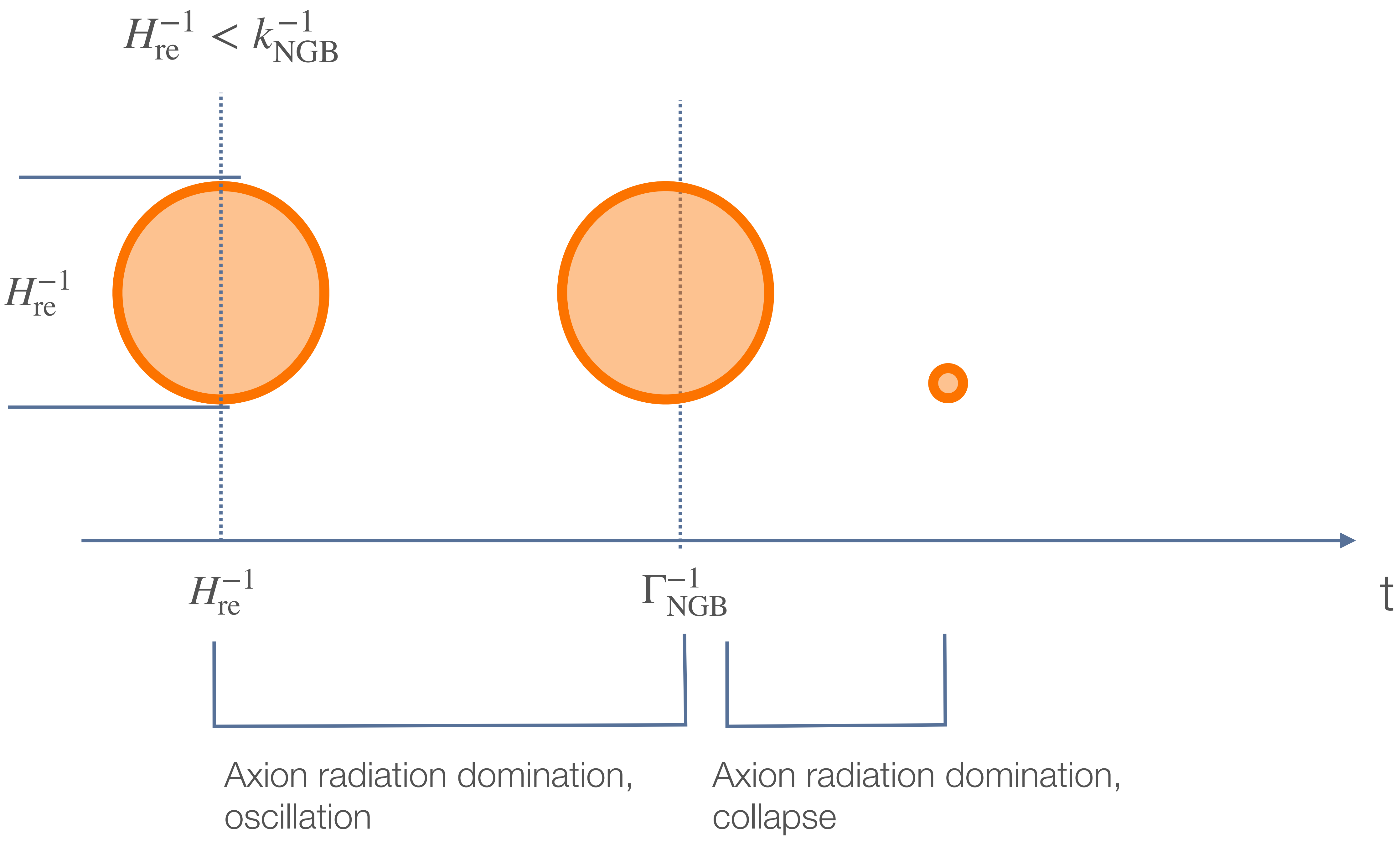
$H_{\rm re}^{-1} > k_{\rm NGB}^{-1}$



$$\left.\frac{\partial \Omega_{\rm GW,\,col.}}{\partial \ln k}\right|_{t=\Gamma_{\rm wall}^{-1}} \approx \frac{1}{T_{\Gamma_{\rm wall}}^4}(\sigma H_{\rm re})\left(\frac{T_{\Gamma_{\rm wall}}}{T_{\rm re}}\right)^3 \frac{\Gamma_{\rm wall}}{\Gamma_{\rm tot}}(r H_{\rm re})^2$$

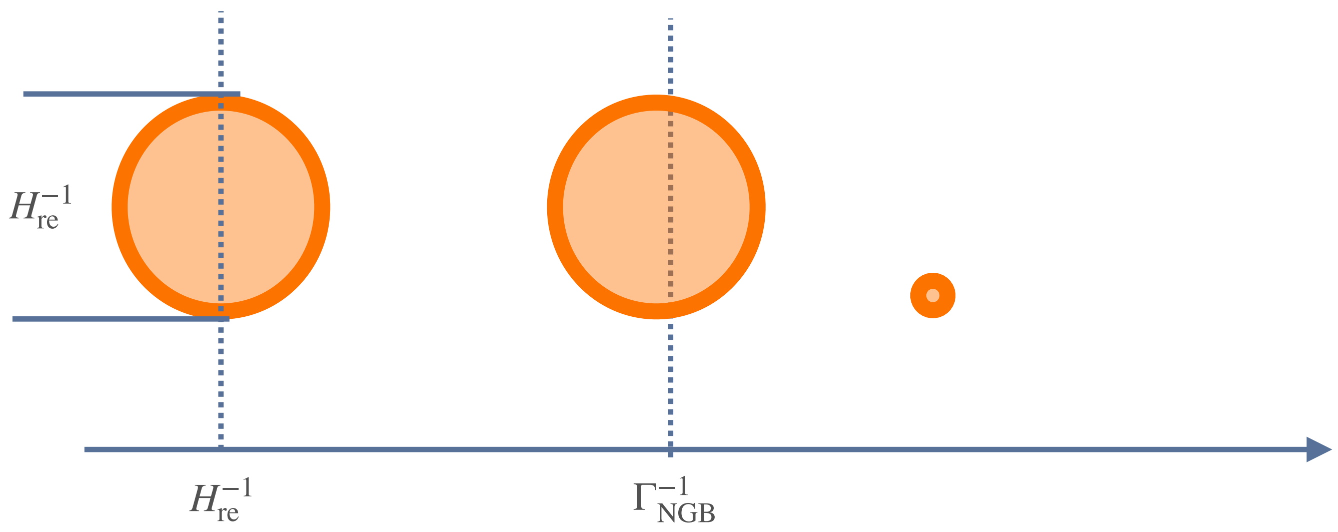
$$\approx \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma_{\rm wall}^{3/2}} \begin{cases} \left(\dfrac{H_{\rm re}}{k}\right)^2, & H_{\rm re} \lesssim k \lesssim k_{\rm NGB} \\ \left(\dfrac{H_{\rm re}}{k}\right)^2 \left(\dfrac{k_{\rm NGB}}{k}\right)^2, & k \gtrsim k_{\rm NGB}, \end{cases}$$

Global string bounded disk



Global string bounded disk

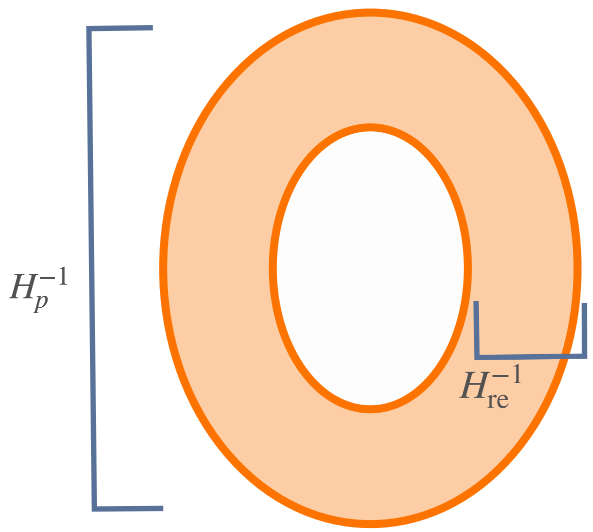
$H_{\rm re}^{-1} < k_{\rm NGB}^{-1}$



$$\left.\frac{\partial \Omega_{\rm GW,\,osc.}}{\partial \ln k}\right|_{t={\Gamma^{\rm wall}_{\rm NGB}}^{-1}} \approx \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2} (\Gamma^{\rm wall}_{\rm NGB})^{3/2}} \left(\frac{k}{H_{\rm re}}\right)^3$$

$$\left.\frac{\partial \Omega_{\rm GW,\,col.}}{\partial \ln k}\right|_{t={\Gamma^{\rm wall}_{\rm NGB}}^{-1}} \approx \frac{\sigma^2}{M_{\rm Pl}^4 H_{\rm re}^{1/2} (\Gamma^{\rm wall}_{\rm NGB})^{3/2}} \left(\frac{H_{\rm re}}{k}\right)^4$$

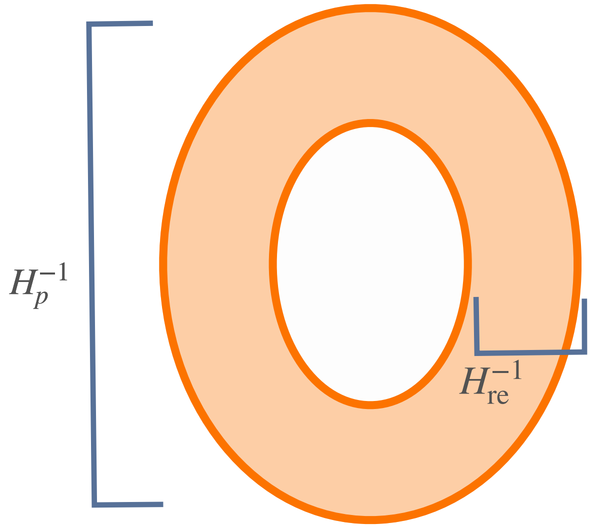
Global string bounded ring



$H_{\rm re}^{-1} > k_{\rm NGB}^{-1}$ GW radiation first, then axion radiation dominates

$$\left.\frac{\partial\Omega_{\rm GW}}{\partial\ln k}\right|_{t=\Gamma_{\rm wall}^{-1}} \approx \frac{2\pi\sigma^2}{3M_{\rm Pl}^4 H_{\rm re}^{1/2}\Gamma_{\rm wall}^{3/2}} \begin{cases} \left(\frac{k}{\Gamma_{\rm wall}}\right)^3\left(\frac{\Gamma_{\rm wall}}{H_{\rm re}}\right)^{3/2}, & k \lesssim \Gamma_{\rm wall}, \\ \left(\frac{k}{H_{\rm re}}\right)^{3/2}, & \Gamma_{\rm wall} \lesssim k \lesssim H_{\rm re}, \\ \frac{H_{\rm re}}{k}, & H_{\rm re} \lesssim k \lesssim k_{\rm NGB}. \\ \frac{H_{\rm re}}{k}\left(\frac{k_{\rm NGB}}{k}\right)^2, & k \gtrsim k_{\rm NGB}. \end{cases}$$

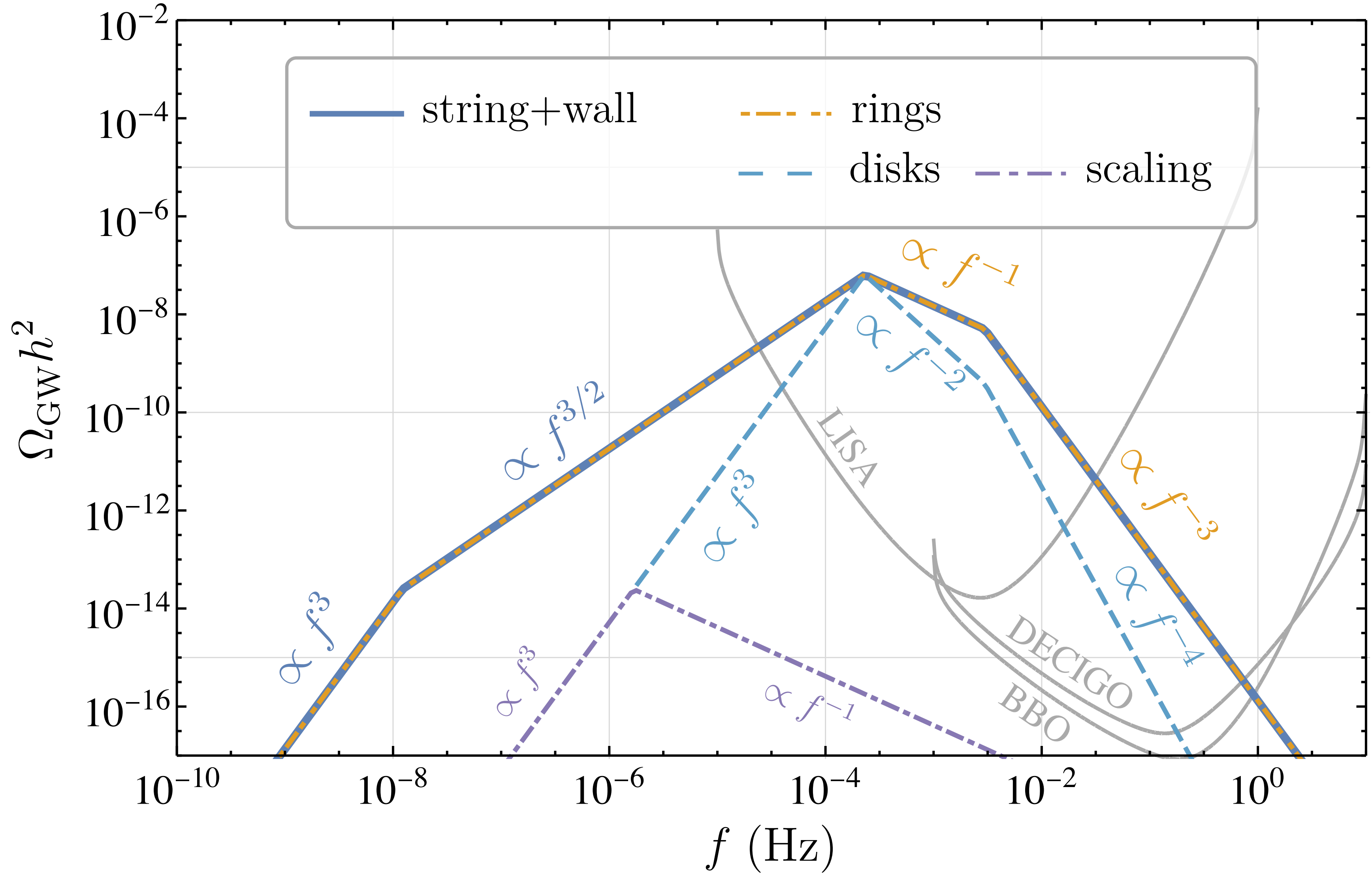
Global string bounded ring



$H_{\rm re}^{-1} < k_{\rm NGB}^{-1}$ Axion radiation dominated

$$\left.\frac{\partial \Omega_{\rm GW}}{\partial \ln k}\right|_{t={\Gamma_{\rm NGB}^{\rm wall}}^{-1}} = \frac{2\pi\sigma^2}{3M_{\rm Pl}^4 H_{\rm re}^{1/2} {\Gamma_{\rm NGB}^{\rm wall}}^{3/2}} \begin{cases} \left(\frac{k}{\Gamma_{\rm NGB}^{\rm wall}}\right)^3 \left(\frac{\Gamma_{\rm NGB}^{\rm wall}}{k_{\rm NGB}}\right)^{3/2}, & k \lesssim \Gamma_{\rm NGB}^{\rm wall}, \\ \left(\frac{k}{k_{\rm NGB}}\right)^{3/2}, & \Gamma_{\rm NGB}^{\rm wall} \lesssim k \lesssim k_{\rm NGB}, \\ \left(\frac{k_{\rm NGB}}{k}\right)^3, & k \gtrsim k_{\rm NGB}. \end{cases}$$

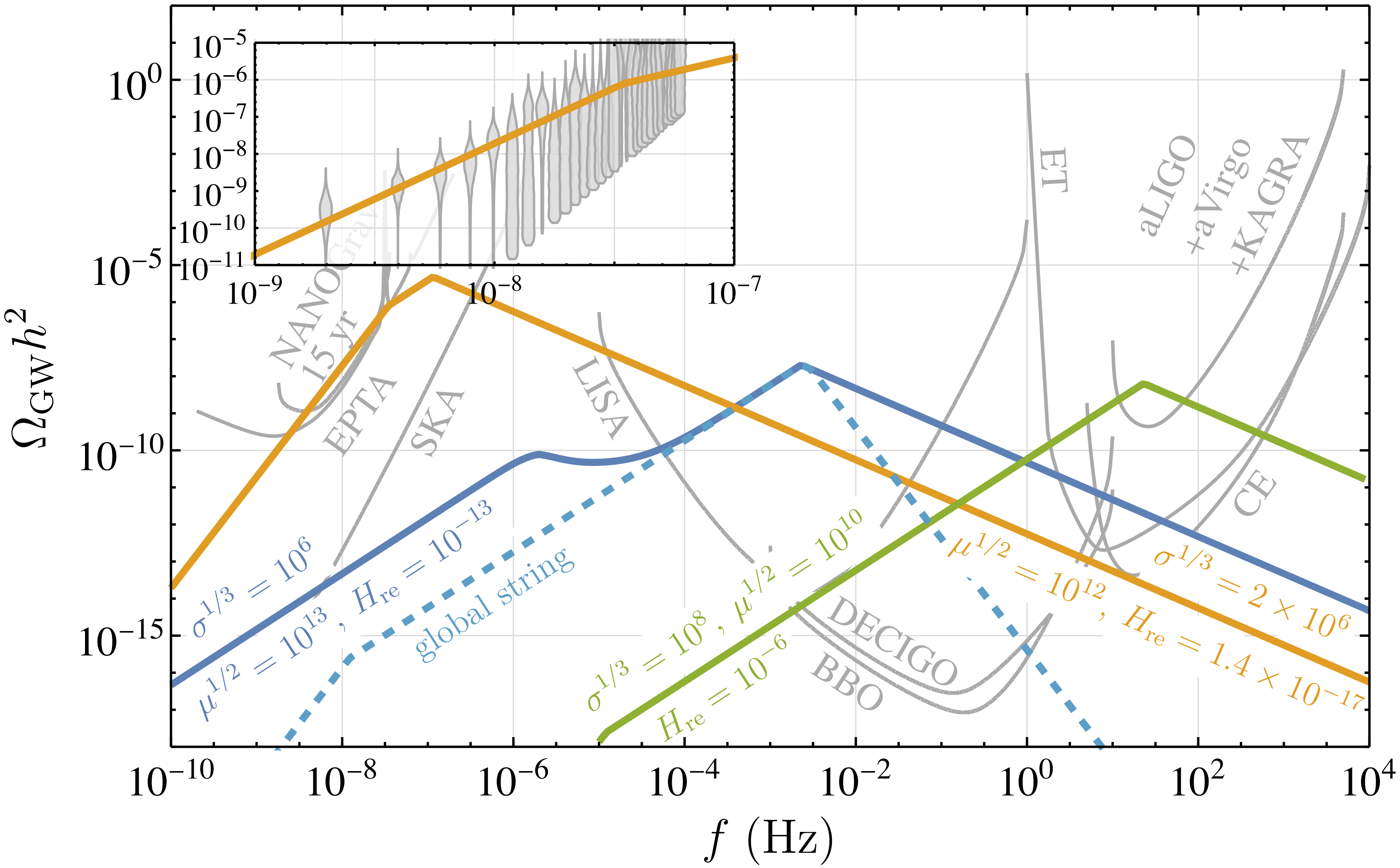
Global string bounded ring



$\mu^{1/2} = 10^{13}\,\text{GeV}$ $\sigma^{1/3} = 10^{6}\,\text{GeV}$ $\Gamma_{\rm GW} \approx 5.3 \times 10^{-19}\,\text{GeV}$

$H_{\rm re} = 10^{-14}\,\text{GeV}$ $k_{\rm NGB} \approx 10^{-13}\,\text{GeV}$

Some benchmarks



$$\Omega_{\rm GW}h^2\Big|_{T_0,f_{\rm peak}} \approx 2\times10^{-8}\left(\frac{\sigma^{1/3}}{10^6\,{\rm GeV}}\right)^6\left(\frac{5.3\times10^{-19}\,{\rm GeV}}{\Gamma_{\rm wall}}\right)^{3/2}\left(\frac{10^{-14}\,{\rm GeV}}{H_{\rm re}}\right)^{1/2}$$

$$f_{\rm peak}\Big|_{T_0} \approx 2\times10^{-3}\,{\rm Hz}\left(\frac{106.75}{g_{*,s}(T_{\Gamma_{\rm wall}})}\right)^{1/3}\left(\frac{g_{*,\rho}(T_{\Gamma_{\rm wall}})}{106.75}\right)^{1/4}\left(\frac{H_{\rm re}}{10^{-13}\,{\rm GeV}}\right)\left(\frac{5.3\times10^{-19}\,{\rm GeV}}{\Gamma_{\rm wall}}\right)^{1/2}$$

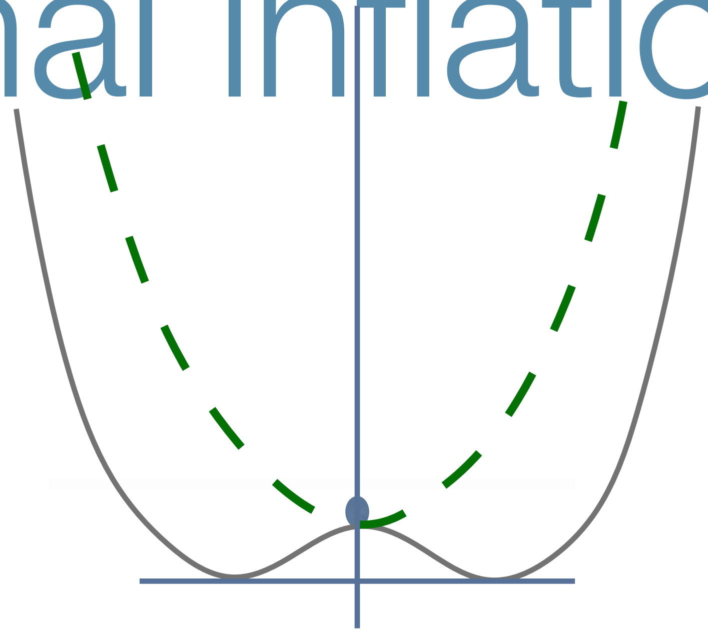
Comparisons

source	spectral shape
gauge str. + inf. + wall	$f^3 \to f^{3/2} \to f^{-1}$
global str. ($w_i \gtrsim k_{\rm NGB}$) + inf. + wall	$f^3 \to f^{3/2} \to f^{-1} \to f^{-3}$
global str. ($w_i \lesssim k_{\rm NGB}$) + inf. + wall	$f^3 \to f^{3/2} \to f^{-3}$
primordial metric perturbation	$f^{n_T} \to f^{n_T-2}$
secondary GW (log-normal P_ζ)	$f^3 \ln^2 f \to$ cutoff
secondary GW (Dirac delta P_ζ)	$f^2 \ln^2 f \to$ cutoff
secondary GW ($k^{n_{\rm IR}} \to k^{-n_{\rm UV}}$)	$f^3 \ln^2 f \to f^{-2n_{\rm UV}}$
phase transition, turbulence, analytical	$f^3 \to f^{-7/2}$
phase transition, turbulence, numerical	$f^1 \to f^{-8/3}$
phase transition, sound wave	$f^9 \to f^{-3}$
domain wall	$f^3 \to f^{-1}$
cosmic gauge string	$f^{3/2} \to f^0 \to f^{-1}$
gauge string in kination domination	$f^1 \to f^{-2}$ bump
supermassive black hole binary	$f^{2/3}$

A model of thermal inflation

$$V(\phi_1) = -m_1^2|\phi_1|^2 + V_{\rm up}(\phi_1),$$

$$V \supset y^2 T^2 |\phi_1|^2$$



Radiation became subdominant at $T_i \approx \sqrt{m_1 v_1}$

Inflation ends, phase transition complete at $T_f \lesssim m_1/y$

Inflation lasts $N_{\rm inf} \approx \ln\left(\frac{T_i}{T_f}\right) \approx \frac{1}{2}\ln\left(\frac{y^2 v_1}{m_1}\right)$

Efficient reheating: $H_{\rm re} \approx e^{-2N_{\rm inf}} H_i \approx \frac{1}{y^2}\frac{m_1^2}{M_{\rm Pl}}$

More dignified

$$W = X\left(\phi_2\phi_{-2} - v_2^2\right) + \lambda\phi_1^2\phi_{-2} + y\phi_1\bar{\psi}\psi \qquad F\text{-flat direction } \phi_2\phi_{-2} = v_2^2$$

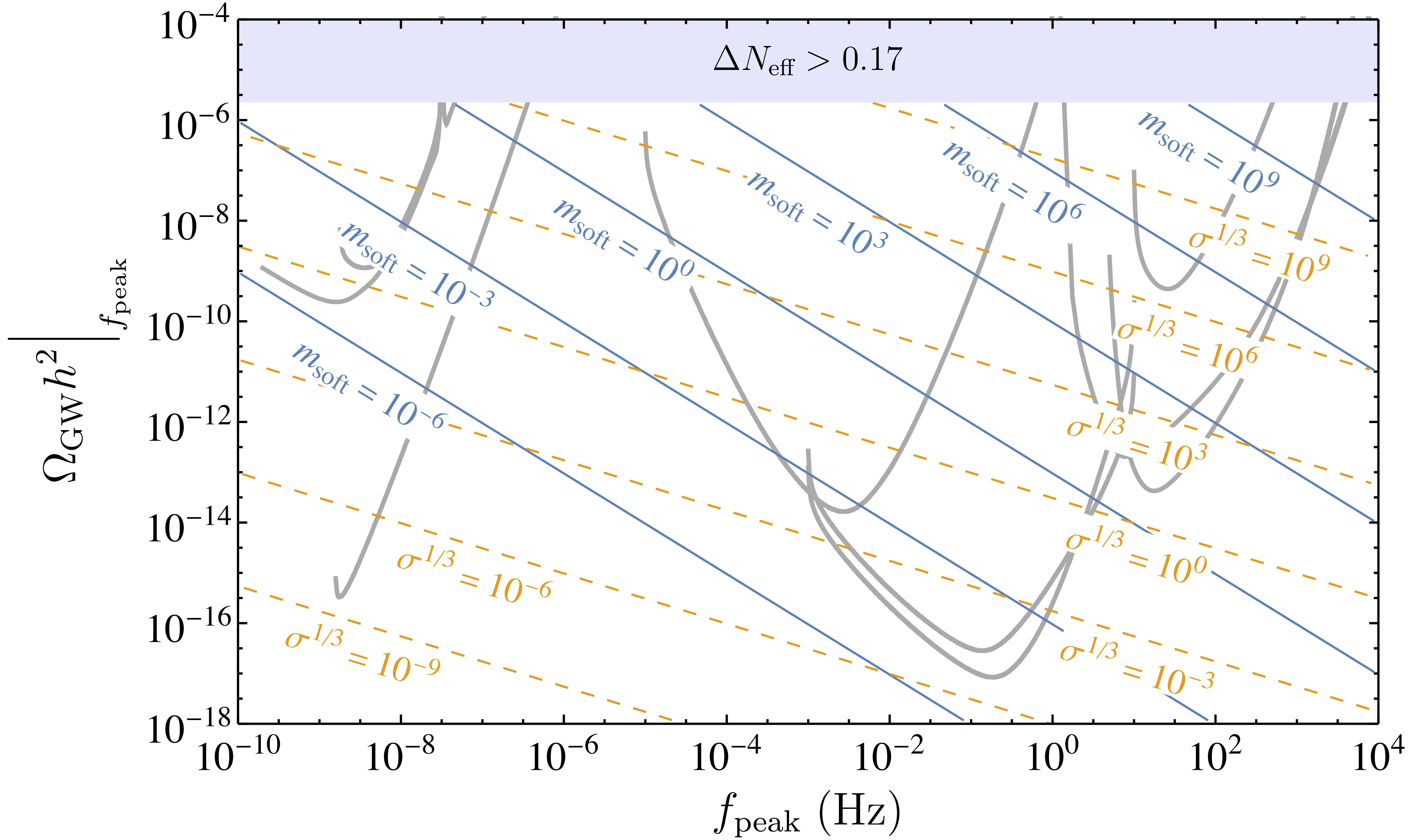
Soft breaking stabilizes: $\phi_2 \approx \phi_{-2} \approx v_2$

$$V_{\cancel{\mathrm{SUSY}},|\phi_1|^2} \approx -m_{\mathrm{soft}}^2|\phi_1|^2 + \frac{y^2 m_{\mathrm{soft}}^2}{(4\pi)^2}|\phi_1|^2 \ln\left(\frac{|\phi_1|^2}{\Lambda^2}\right)$$

$$V_{\mathrm{thermal},|\phi_1|^2} \approx y^2T^2|\phi_1|^2 \qquad H_{\mathrm{re}} \approx \frac{m_{\mathrm{soft}}^2}{M_{\mathrm{Pl}}}$$

$$V_{\cancel{\mathrm{SUSY}},\mathrm{tri.}} \approx -m_{\mathrm{soft}}\lambda\left(\phi_1^2\phi_{-2} + \mathrm{h.c.}\right) \approx -m_{\mathrm{soft}}\lambda v_1^2 v_2 \cos\left(\frac{2a_1}{v_1}\right)$$

$$m_a \approx \sqrt{\lambda m_{\mathrm{soft}} v_2} \qquad \sigma \approx m_a v_1^2 \approx \sqrt{\lambda m_{\mathrm{soft}} v_2}\, v_1^2$$



$$f_{\rm peak}\Big|_{T_0} \propto m_{\rm soft}^2 \sigma^{-1/2} \qquad \Omega_{\rm GW} h^2\Big|_{T_0, f_{\rm peak}} \propto \sigma^{1/2} m_{\rm soft}^{-1}$$

Can in principle measure $m_{\rm soft}$

Conclusions

- More observations of stochastic gravitational wave in the coming decades.
- Topological defects are prominent sources.
- Topological defect networks give promising signals.

  Generic to expect a period of inflation between two stages of phase transitions.

  Sizable gravitational wave signal. Shapes can be very informative.

Why topological defect

- Old subject.

A. Vilenkin et al. + many others, 40+ years ago.

Why topological defect

- Old subject.

  A. Vilenkin et al. + many others, 40+ years ago.

- Perhaps, old = new trend?

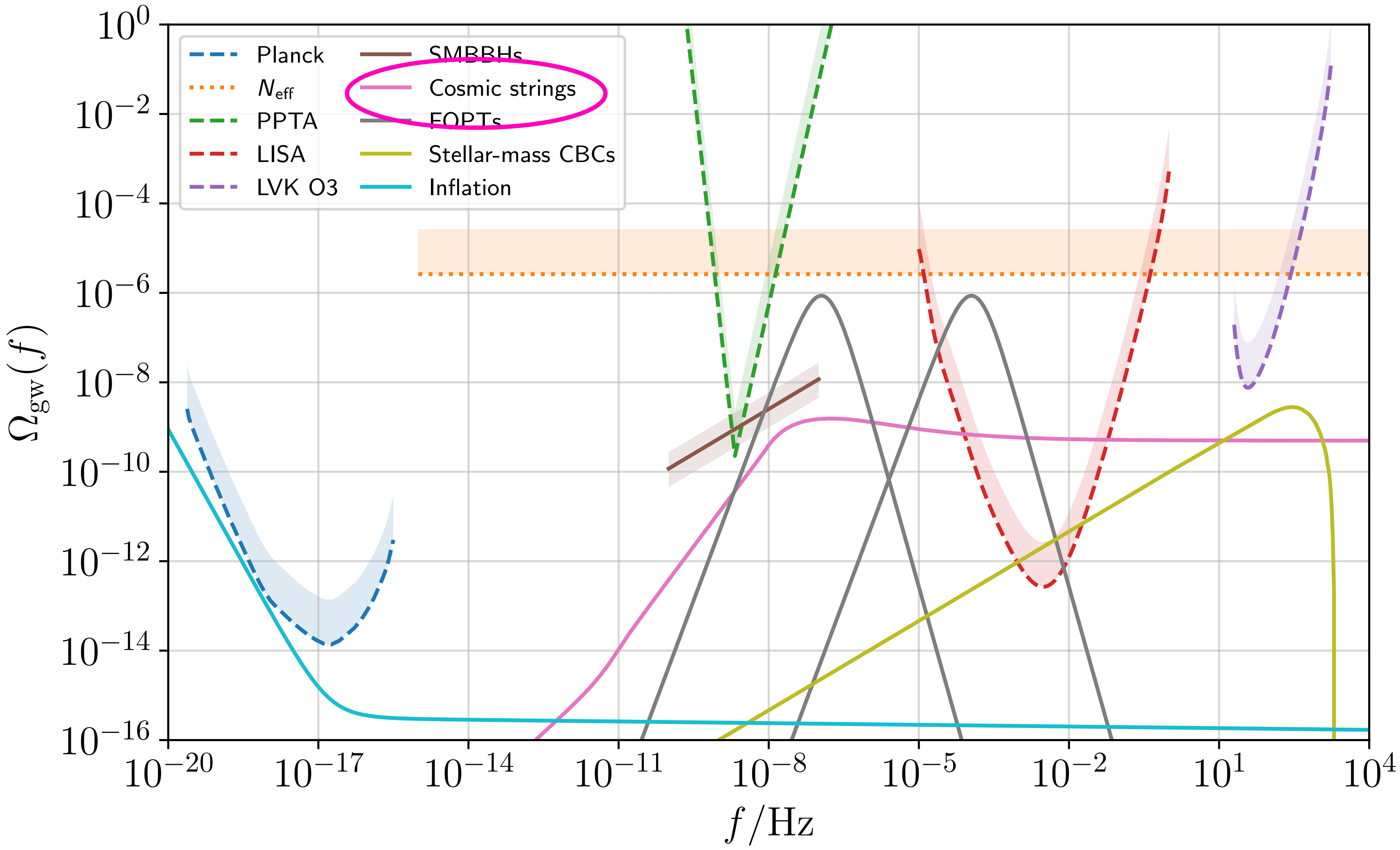
  Dark photon (mili-charge), B. Holdom, 1985

  Axion, Peccei and Quinn 1977, F. Wilczek 1978, S. Weinberg 1978. Invisible, 1979-1981

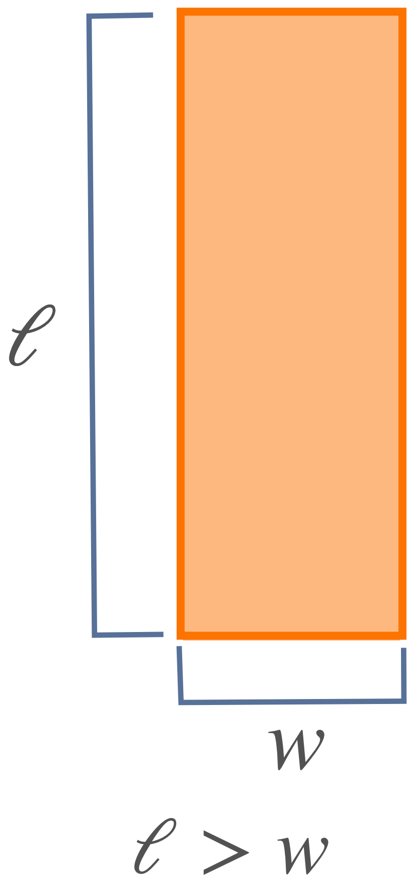
GW from Strings and walls

Well studied.

Review by Renzini et al, 2202.00178



Global string bounded



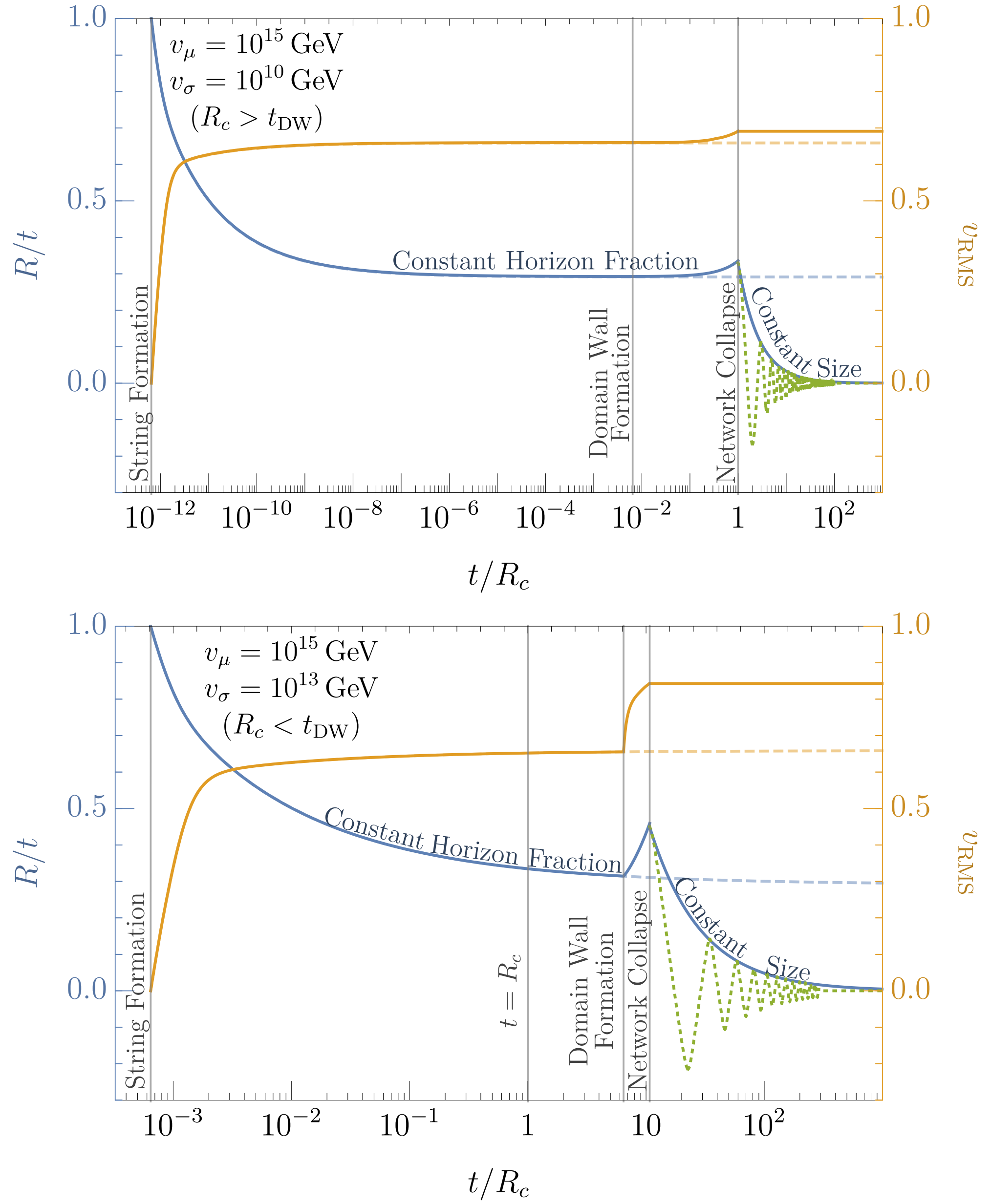
wall	P	Γ
GW	$\dfrac{\sigma^2}{M_{\rm Pl}^2} w\ell,$	$\dfrac{\sigma}{M_{\rm Pl}^2},$
NGB	$\dfrac{\mu}{\pi}\left(\dfrac{\ell}{w}\right),$	$\dfrac{\mu}{\pi\sigma} H_{\rm re}^2,$

string	P	Γ
GW	$\dfrac{\sigma^2}{M_{\rm Pl}^2} w^2,$	$\dfrac{\sigma}{M_{\rm Pl}^2}\dfrac{H_p}{H_{\rm re}},$
NGB	$\dfrac{\mu}{\pi},$	$\dfrac{\mu}{\pi\sigma} H_{\rm re} H_p.$

$$H_{\rm re} \lesssim \sigma/\mu \quad \Rightarrow \quad H_{\rm re}^{-1} < \frac{\sigma}{\mu H_{\rm re}} H_{\rm re}^{-1} \approx {\Gamma_{\rm NGB}^{\rm wall}}^{-1}$$

Axion induced decay happens after horizon re-entry

Wall does not dominate from the beginning



Dunsky, Ghoshal, Murayama, Sakakihara, White, "gastronomy" 2111.08750

$$\Omega_{\rm GW} \sim \frac{m_1^{1/2} v_1}{M_{\rm Pl}^{1/2} T_p}$$

T_p: temperature at wall production

Assuming thermal phase transition

$$T_p \sim \frac{m_1}{y}, \quad \Omega_{\rm GW} \sim \left(\frac{v_1}{M_{\rm Pl}}\right)^{1/2} \frac{y v_1^{1/2}}{m_1^{1/2}}$$

If $\quad \Omega_{\rm GW} \sim 1 \to \dfrac{y v_1^{1/2}}{m_1^{1/2}} \gg 1 \qquad \to \dfrac{T_p^4}{m_1^2 v_1^2} = \left(\dfrac{y v_1^{1/2}}{m_1^{1/2}}\right)^{-4} \ll 1$

Assuming phase transition just happens at T_p

If no inflation, $T_p^4 > m_1^2 v_1^2 \to T_p > (m_1 v_1)^{1/2}$

$$\to \Omega_{\rm GW} < \left(\frac{v_1}{M_{\rm Pl}}\right)^{1/2}$$

Why topological defect

- However, do more harm than good?

    Can’t (easily) be dark matter.

    Lead to disaster: wall, monopole domination

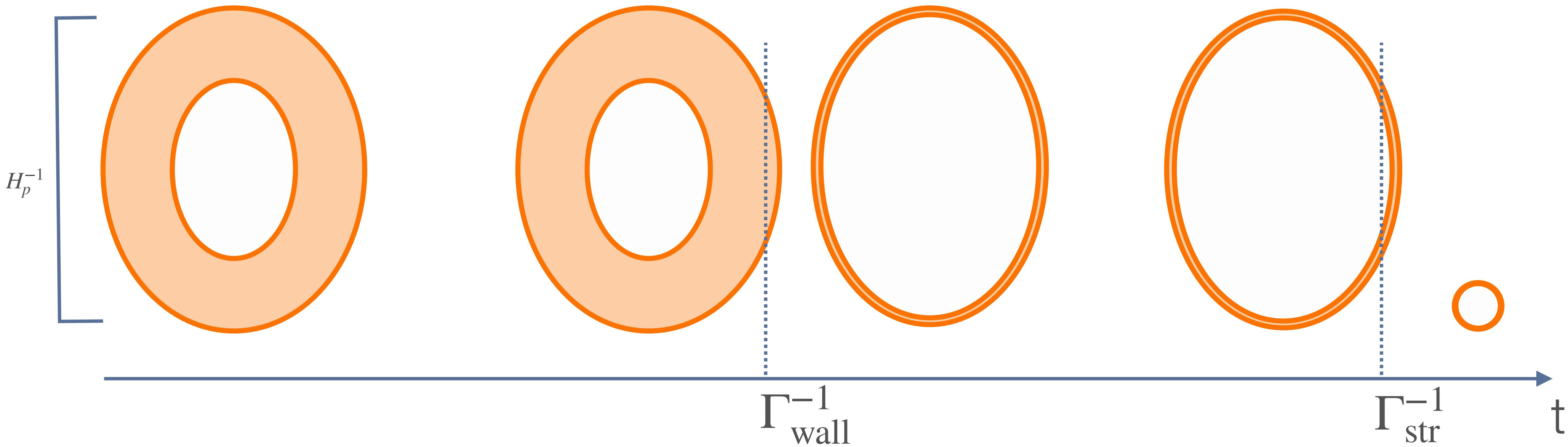
- Can get rid of them

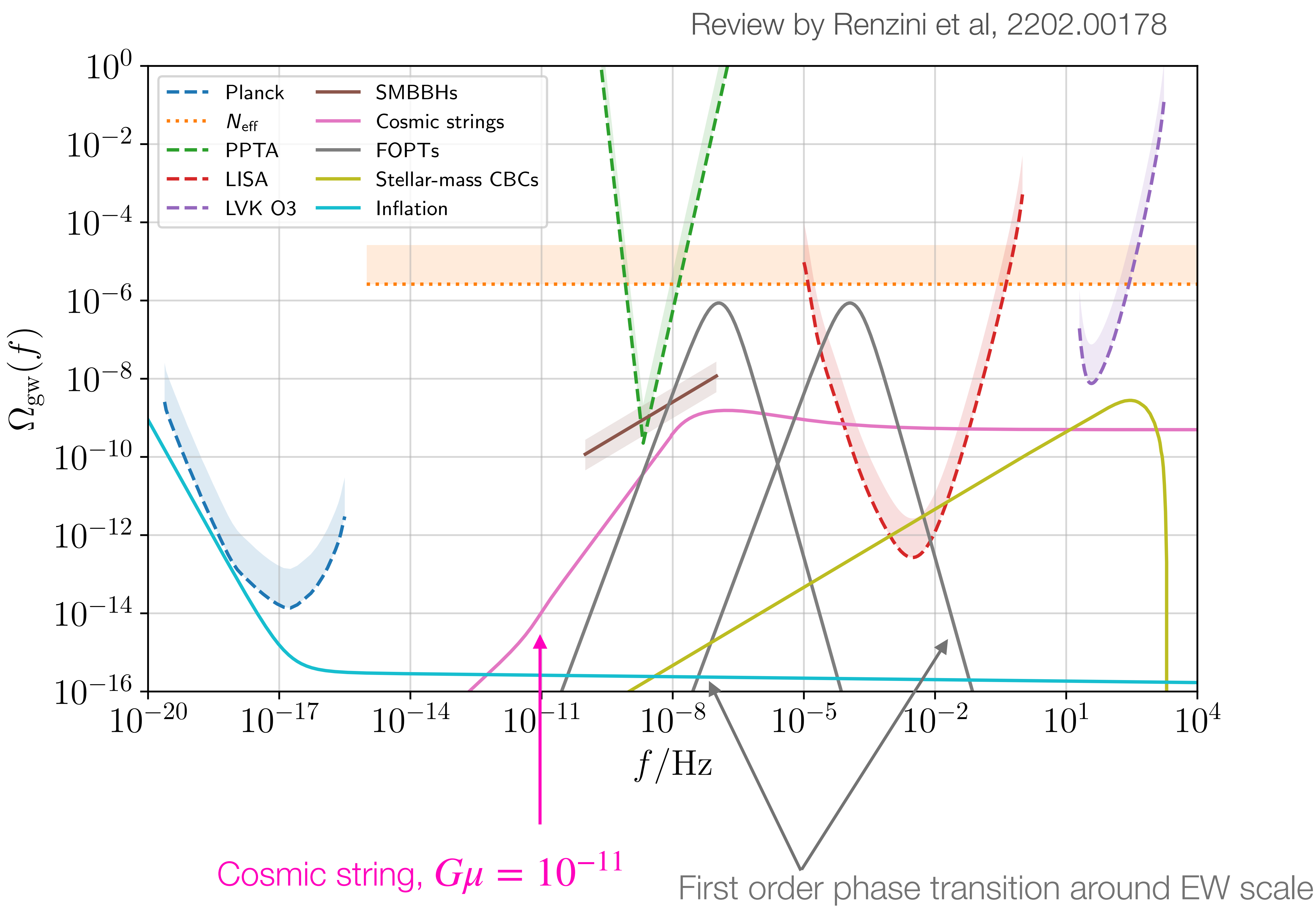
    Inflation + other mechanisms.

    bias induced wall collapse, confine monopoles, …

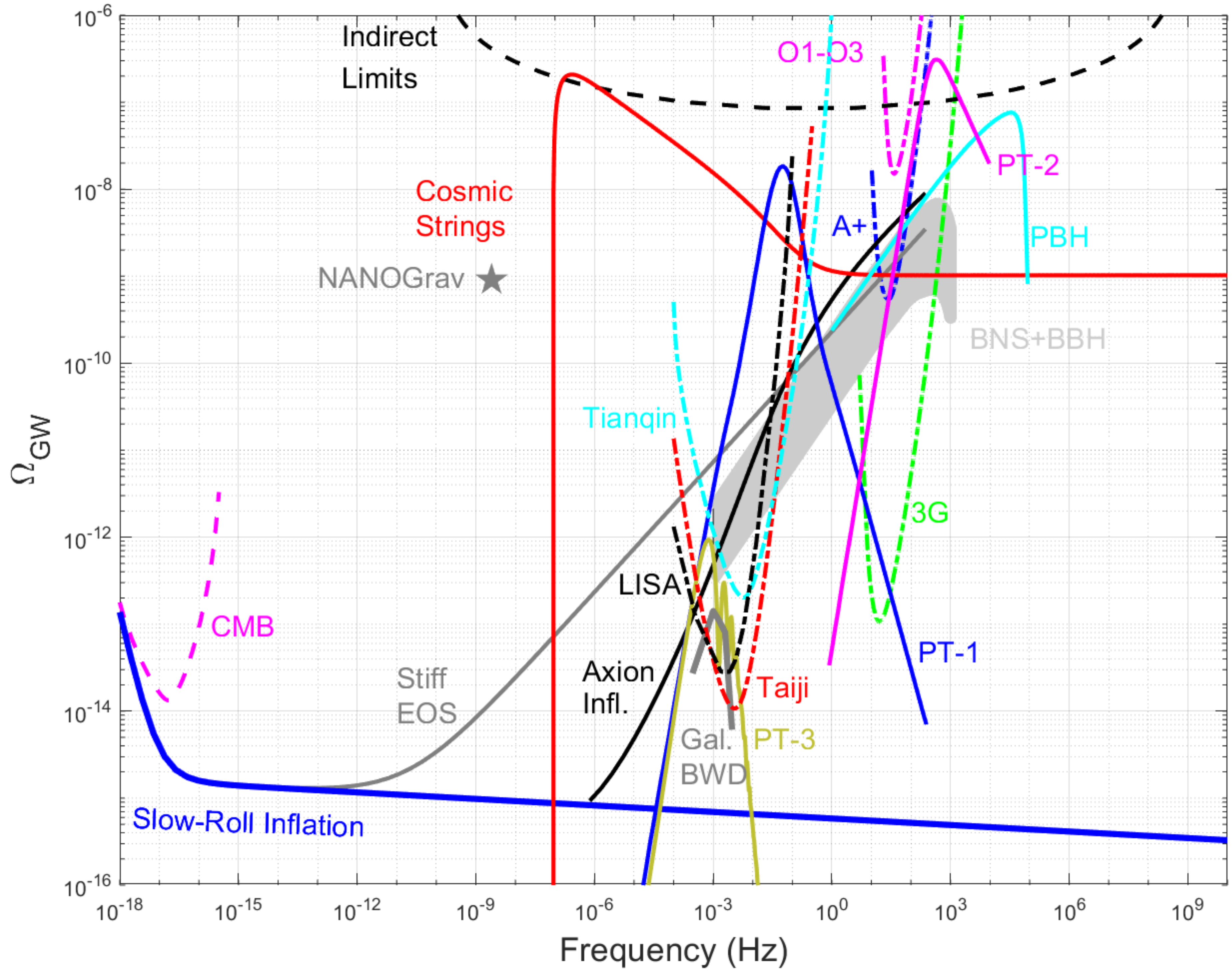
    Any signals?

GW from rings, thin string





Review by Renzini et al, 2202.00178
Planck
N_{eff}
PPTA
LISA
LVK O3
SMBBHs
Cosmic strings
FOPTs
Stellar-mass CBCs
Inflation
$\Omega_{\text{gw}}(f)$
f/Hz
Cosmic string, $G\mu = 10^{-11}$
First order phase transition around EW scale



Indirect
Limits
O1-O3
PT-2
Cosmic
Strings
A+
PBH
NANOGrav
BNS+BBH
Tianqin
3G
LISA
CMB
PT-1
Stiff
EOS
Axion
Infl.
Taiji
Gal.
BWD
PT-3
Slow-Roll Inflation
Ω_{GW}
Frequency (Hz)
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-15}
10^{-12}
10^{-9}
10^{-6}
10^{-3}
10^{0}
10^{3}
10^{6}
10^{9}