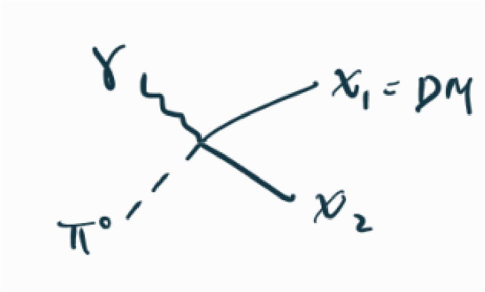
Topological Portal
to the
Dark Sector

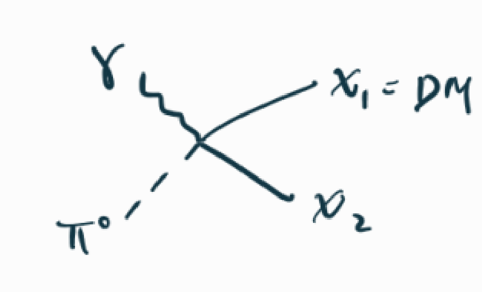
Joe Davighi, CERN

2401.09528 with A. Greljo, N. Selimović

Work in progress with N. Lohitsiri

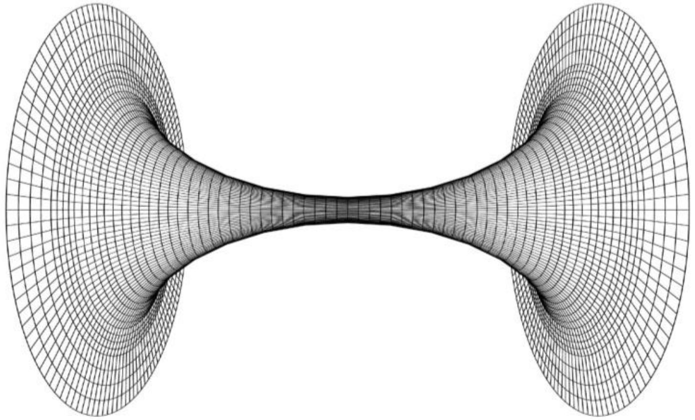


$$\frac{1}{f_\pi f_D^2}\epsilon^{\mu\nu\rho\sigma}\left(e\pi^0 F_{\mu\nu}+\frac{1}{f_\pi^2}f_{abc}\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\right)\partial_\rho\chi_1\partial_\sigma\chi_2$$



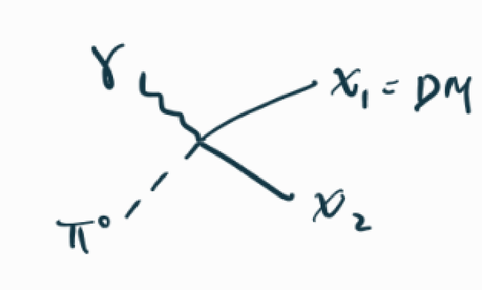
$$\frac{1}{f_\pi f_D^2}\epsilon^{\mu\nu\rho\sigma}\left(e\pi^0 F_{\mu\nu} + \frac{1}{f_\pi^2} f_{abc}\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\right)\partial_\rho\chi_1\partial_\sigma\chi_2$$

Visible Sector



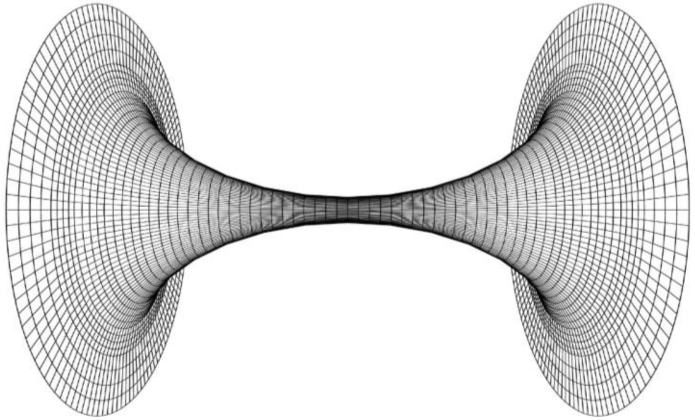
Dark Sector

(Almost) **unique** interaction between low-energy DOFs of SM (pions, photons, …) and dark particles that is **topological**



$$\frac{1}{f_\pi f_D^2}\epsilon^{\mu\nu\rho\sigma}\left(e\pi^0 F_{\mu\nu} + \frac{1}{f_\pi^2} f_{abc}\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\right)\partial_\rho\chi_1\partial_\sigma\chi_2$$

Origin of
Dark Matter



Topological Interactions;
Generalised Symmetries

PHENOMENOLOGY

FORMAL THEORY

Outline

Part 0

- Intro to topological terms in QFT

Part I: EFT

2401.09528

- Identify a novel *mixed* topological term in sigma model of QCD pions & dark pions
- Elegant realization of light thermal inelastic dark matter
- Collider signatures in Belle II

Part II: UV

work in progress

- 2-group generalised symmetry and a no go theorem
- "Symmetry matching" via a weakly coupled UV completion: QCD + Linear Sigma Model

Part 0: Intro

Topological Terms in QFT

Let's consider scalar field theory in d dimensions. Fields = maps $\phi\ (x)\colon \Sigma_{\mathrm{d}} \to X$

"Definition":

> A term in the exponentiated action $\mathrm{e}^{iS[\phi(x)]}$ is **topological** if it can be written **without a metric** on spacetime OR target space

Topological Terms in QFT

Let's consider scalar field theory in d dimensions. Fields = maps $\phi\,(x)\colon \Sigma_{\mathrm{d}} \to X$

"<u>Definition</u>":

A term in the exponentiated action $\mathrm{e}^{iS[\phi(x)]}$ is **topological** if it can be written **without a metric** on spacetime OR target space

One source of topological terms:

$$S_{\mathrm{top}} \sim \int_{\Sigma_{\mathrm{d}}} (\alpha = \text{differential } d \text{ form}), \qquad \text{e.g. } \alpha = \phi^*(d \text{ form on } X)$$

Recall: A differential k-form on manifold X is a totally antisymmetric tensor of type $(0, k)$
Can expand in coordinate basis 1-forms dx^{μ}, as $\alpha = \frac{1}{k!}\alpha_{[\mu_1 \dots \mu_d]} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_d}$

Topological Terms in QFT

Let's consider scalar field theory in d dimensions. Fields = maps $\phi\ (x)\colon \Sigma_{\mathrm{d}} \to X$

"<u>Definition</u>":

> A term in the exponentiated action $\mathrm{e}^{iS[\phi(x)]}$ is **topological** if it can be written **without a metric** on spacetime OR target space

One source of topological terms:

$$S_{\mathrm{top}} \sim \int_{\Sigma_{\mathrm{d}}} (\alpha = \text{differential } d \text{ form}), \qquad \text{e.g. } \alpha = \phi^{*}(d \text{ form on } X)$$

Going beyond just scalars, **gauge fields** A and their field strengths $F = F_{\mu\nu} dx^{\mu} \wedge dx^{\nu}$ provide other differential forms that we use to build topological terms

Going beyond differential forms, there can be more subtle "**torsion** terms" e.g. a discrete theta angle in e.g. SO gauge theory in 4d.

(we will stick with differential forms)

Type I: theta-like terms

In *d*-dimensional theories, integrate a **closed *d*-form** ($d\alpha = 0$)

Type I: theta-like terms

In *d*-dimensional theories, integrate a **closed *d*-form** ($d\alpha = 0$)

Example 1. QM on a circle $q(t): S^1 \to S^1$, coupled to a solenoid (or any gauge field A with $F = 0$):

$S_{\mathrm{AB}} = \theta \int \frac{dq}{2\pi} = \theta W$, winding number $W \in \mathbb{Z}$ — Aharonov-Bohm phase

Shifts the discrete spectrum $E_k = \frac{k^2}{2m} \to \frac{1}{2m}\left(k - \frac{\theta}{2\pi}\right)^2$

$\frac{dq}{2\pi}$ is a closed 1-form

Type I: theta-like terms

In d-dimensional theories, integrate a **closed d-form** ($d\alpha = 0$)

Example 1. QM on a circle $q(t): S^1 \to S^1$, coupled to a solenoid (or any gauge field A with $F = 0$):

$S_{\text{AB}} = \theta \int \frac{dq}{2\pi} = \theta W$, winding number $W \in \mathbb{Z}$ — Aharonov-Bohm phase

Shifts the discrete spectrum $E_k = \frac{k^2}{2m} \to \frac{1}{2m}\left(k - \frac{\theta}{2\pi}\right)^2$

Example 2. 4d gauge theory:

$S_\theta = \theta \int \frac{\text{Tr}\,(F \wedge F)}{8\pi^2} = \theta N_{\text{inst}}$ — Instanton effects in IR e.g. $d_n \propto \theta$

$\frac{\text{Tr}\,(F \wedge F)}{8\pi^2}$ is a closed 4-form,
where $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$

Type I: theta-like terms

In d-dimensional theories, integrate a **closed d-form** $(d\alpha = 0)$

Example 1. QM on a circle $q(t): S^1 \to S^1$, coupled to a solenoid (or any gauge field A with $F = 0$):

$S_{\text{AB}} = \theta \int \frac{dq}{2\pi} = \theta W$, winding number $W \in \mathbb{Z}$ — Aharonov-Bohm phase

Shifts the discrete spectrum $E_k = \frac{k^2}{2m} \to \frac{1}{2m}\left(k - \frac{\theta}{2\pi}\right)^2$

Example 2. 4d gauge theory:

$S_\theta = \theta \int \frac{\text{Tr}\,(F \wedge F)}{8\pi^2} = \theta N_{\text{inst}}$ — Instanton effects in IR e.g. $d_n \propto \theta$

- Theta terms have no effects perturbatively because they are locally total derivatives (Poincaré lemma)
- Affect IR structure; not scattering amplitudes
- The action is a true topological invariant of the target space

Type II: WZW-like terms

In *d*-dimensions, integrate a *d*-form α that is NOT closed (and maybe only locally-defined $\{\alpha_i, \ldots\}$)

Witten's formulation: $\int_{\Sigma_d} \alpha_i = \int_{Y_{d+1}} \omega = d\alpha_i$, assuming $\partial Y_{d+1} = \Sigma_d$. NOT a total derivative locally

Witten Nucl. Phys. B 1983

Type II: WZW-like terms

In *d*-dimensions, integrate a *d*-form α that is NOT closed (and maybe only locally-defined $\{\alpha_i, \dots\}$)

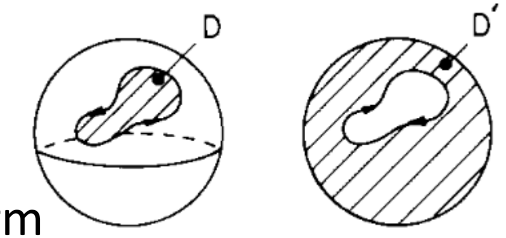
Witten's formulation: $\int_{\Sigma_d} \alpha_i = \int_{Y_{d+1}} \omega = d\alpha_i$, assuming $\partial Y_{d+1} = \Sigma_d$. NOT a total derivative locally

Example 3. QM on sphere $\gamma: S^1 \to S^2 = SU(2)/SO(2)$ + magnetic monopole $F \neq 0$:

$$S_{\text{Dirac}} = n \int_{D \subset S^2} \frac{\sin\theta \, d\theta d\phi}{4\pi}, \qquad \partial D = \text{Im}\,(\gamma).$$

Consistency $\Longrightarrow n \in \mathbb{Z}$

We integrate $\omega_2 = n \frac{\sin\theta d\theta d\phi}{4\pi}$, a closed, $SU(2)$-invariant, integral 2-form



On a local patch, can write a Lagrangian $L \sim \frac{n}{4\pi}(1 - \cos\theta)\dot{\phi}$

⚠ Lagrangian not globally-defined, and not $SU(2)$ invariant!

Dirac Proc. Royal Soc. A, 1931
Wu, Yang Phys. Rev. D, 1976
Witten Nucl. Phys. B, 1983

Type II: WZW-like terms

In *d*-dimensions, integrate a *d*-form α that is NOT closed (and maybe only locally-defined $\{\alpha_i, \dots\}$)

Witten's formulation: $\int_{\Sigma_d} \alpha_i = \int_{Y_{d+1}} \omega = d\alpha_i$, assuming $\partial Y_{d+1} = \Sigma_d$. NOT a total derivative locally

- WZW-like terms are obtained from closed ***d+1*-forms** for *d*-dimensional theories
- Have $\mathbb{Z}$ or $\mathbb{R}$ valued coefficients
- Affect classical equations of motion (action is not actually a topological invariant)
- In QFT they are seen in **perturbation theory** (give new Feynman diagrams; affect scattering)

We will be interested in this WZW type of topological term to get a new **local interaction with dark matter**

Type II: WZW-like terms

Example 4. WZW term in low-energy limit of 4d QCD:

EFT of pion fields $g(x) = e^{\frac{2i}{f_\pi} t_a \pi_a(x)} : \Sigma_4 \to X = \frac{SU(3)_L \times SU(3)_R}{SU(3)_{L+R}} \cong SU(3)$ [χ symmetry breaking]

$S_{\mathrm{WZW}} = n \int_{B \subset SU(3)} \omega_5$, $\partial B = \mathrm{Im}\,(\Sigma)$. Consistency $\Longrightarrow n \in \mathbb{Z}$ Witten Nucl. Phys. B 1983

Integrand $\omega_5 = \frac{-in}{480\pi^3} \mathrm{Tr}\,(g^{-1}dg)^5$ is a closed, $SU(3)_L \times SU(3)_R$-invariant, integral 5-form

Expanding locally $g = 1 + \frac{2i}{f_\pi} t_a \pi_a(x) + \cdots$ can write a local Lagrangian

$$L \sim \epsilon^{\mu\nu\rho\sigma} \pi_0 \partial_\mu \pi^+ \partial_\nu \pi^- \partial_\rho K^+ \partial_\sigma K^- + \cdots$$

which again is not $\mathrm{G} = SU(3)_L \times SU(3)_R$ invariant, as for the Dirac monopole
This term gives many new vertices in perturbation theory

Physics of the WZW term in 4d QCD

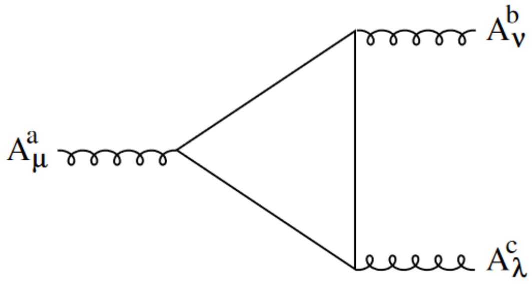
The rest of the chiral Lagrangian (à la CCWZ) respects certain symmetries not possessed by QCD:

Callan, Coleman, Wess, Zumino [Phys. Rev. 1969]

- Discrete $P_0 : x_i \to -x_i$ and pion number mod 2 – but we observe $\phi \to K^+K^-$ and $\phi \to \pi^0\pi^+\pi^-$
  - WZW violates each, but preserves $P_0(-1)^{n_\pi}$

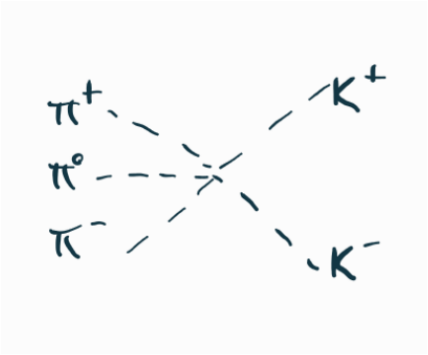
Wess, Zumino [Phys. Lett. 1971]

- Chiral $U_L \neq U_R$ transformations (on quark triplets in UV) with background gauge field (e.g. QED)
  - WZW **matches this chiral anomaly** in IR, for fixed coefficient $n = N_c = 3$



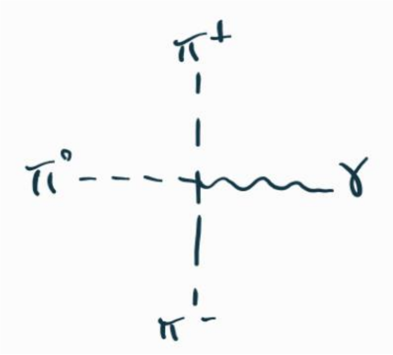
Physics of the WZW term in 4d QCD

Leading order phenomenological effects after *gauging QED* (complicated…)



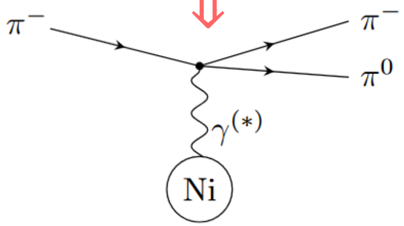
$$\sim \frac{1}{f_\pi^5}\pi_0 d\pi^+ d\pi^- dK^+ dK^-$$

Unobserved



$$\sim \frac{1}{f_\pi^3}\pi_0 d\pi^+ d\pi^- F$$

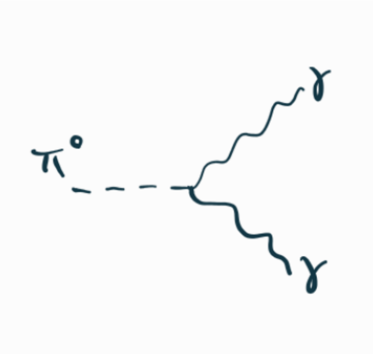
$$\Downarrow$$



$$F_{3\pi}^{\text{theory}} = \frac{eN_c}{12\pi^2 F_\pi^3} = (9.78 \pm 0.04)\ \text{GeV}^{-3}$$

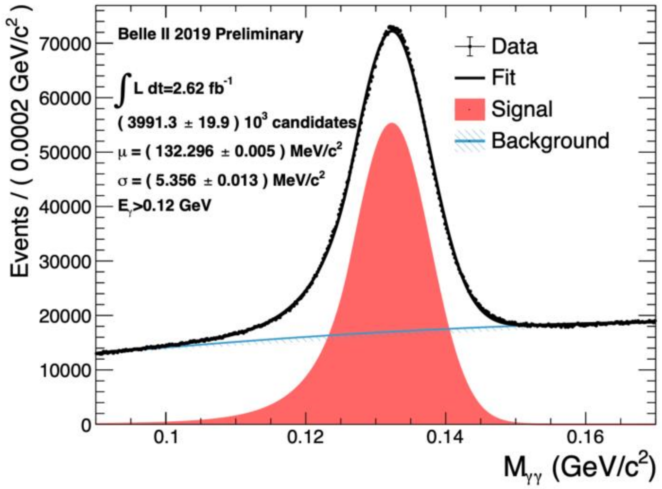
$$F_{3\pi}^{\text{preliminary}} = (10.3 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}})\ \text{GeV}^{-3}$$

COMPASS experiment @ CERN 2310.09138



$$\sim \frac{1}{f_\pi}\pi_0 FF$$

$$\Downarrow$$



Part I: EFT

Observation: QCD also has an invariant 3-form $\omega_3 \sim \frac{1}{24\pi^2}\mathrm{Tr}\,(g^{-1}dg)^3 \sim f_{abc}d\pi^a d\pi^b d\pi^c$

ω_3 and ω_5 are the *only* *bi*-invariant forms on $SU(3)$ - closed or otherwise!

Observation: QCD also has an invariant 3-form $\omega_3 \sim \frac{1}{24\pi^2}\mathrm{Tr}\,(g^{-1}dg)^3 \sim f_{abc}d\pi^a d\pi^b d\pi^c$

ω_3 and ω_5 are the *only bi*-invariant forms on $SU(3)$ - closed or otherwise!

ω_3 does not appear in building the EFT action in pure QCD, just because of the spacetime dimension

(It would match an anomaly in 2d QCD)

However:

- Integrating ω_3 over spatial infinity measures winding number of pion configuration
- Get a non-zero integer for solitons, such as Skyrmions   Skyrme, Proc.Roy.Soc.Lond.A, 1961
- $\star\,\omega_3 = j_B$ is identified as **topological baryon number current** in low-energy QCD

Balachandran, Nair, Rajeev, Stern, PRL, 1982
Witten Nucl. Phys. B, 1983

Observation: QCD also has an invariant 3-form $\omega_3 \sim \frac{1}{24\pi^2}\mathrm{Tr}\,(g^{-1}dg)^3 \sim f_{abc}d\pi^a d\pi^b d\pi^c$

ω_3 and ω_5 are the *only bi*-invariant forms on $SU(3)$ - closed or otherwise!

ω_3 does not appear in building the EFT action in pure QCD, just because of the spacetime dimension

(It would match an anomaly in 2d QCD)

However:

- Integrating ω_3 over spatial infinity measures winding number of pion configuration
- Get a non-zero integer for solitons, such as Skyrmions — Skyrme, Proc.Roy.Soc.Lond.A, 1961
- $\star\,\omega_3 = j_B$ is identified as **topological baryon number current** in low-energy QCD

Balachandran, Nair, Rajeev, Stern, PRL, 1982
Witten Nucl. Phys. B, 1983

Challenge: by extending QCD by a dark sector $X = SU(3) \to SU(3) \times (G/H)_D$, can we use ω_3 to write a second topological WZW-like term involving QCD pions and dark pions?

EFT Setup: QCD x Dark Sector

We study an EFT of scalar fields that we suppose is valid near the GeV scale

- 3-flavour QCD. Chiral symmetry breaking $\langle \overline{\psi}_i \psi_j \rangle \sim \Lambda^3_{\mathrm{QCD}} \delta_{ij} \implies$ 8 QCD pions $\left\{\pi^0, \pi^\pm, K^0, \overline{K}^0, K^\pm, \eta\right\}$
- Dark sector global symmetry breaking $G_D \to H_D$: DM = dark pions χ_i on coset G_D/H_D

EFT is a non-linear sigma model on

$$X = \frac{SU(3)_L \times SU(3)_R \times G_D}{SU(3)_{L+R} \times H_D}$$

Classification of WZW terms

Toplogical terms in sigma models can be classified algebraically, e.g. using

- Cohomology [d'Hoker, Weinberg, hep-ph/9409402]; good enough in simple situations
- Invariant (differential) cohomology [JD, Gripaios, 1803.07585, JD, Gripaios, Randal-Williams, 2011.05768];
  - Invariance refinement ($\iota_X \omega$ is exact) is needed when there are non-trivial d-cycles in X, e.g. QM on T^2;
  - *Differential* refinement further captures WZW terms that are de Rham exact, e.g. QM on $\mathbb{R}^2$ (Landau levels)
- Bordism [Freed hep-th/0607134; Lee, Ohmori, Tachikawa, 2009.00033]; needed for correct quantization of coefficients

Classification of WZW terms

Toplogical terms in sigma models can be classified algebraically, e.g. using

- Cohomology [d'Hoker, Weinberg, hep-ph/9409402]; good enough in simple situations
- Invariant (differential) cohomology [JD, Gripaios, 1803.07585, JD, Gripaios, Randal-Williams, 2011.05768];
  - Invariance refinement ($\iota_X \omega$ is exact) is needed when there are non-trivial d-cycles in X, e.g. QM on T^2;
  - *Differential* refinement further captures WZW terms that are de Rham exact, e.g. QM on $\mathbb{R}^2$ (Landau levels)
- Bordism [Freed hep-th/0607134; Lee, Ohmori, Tachikawa, 2009.00033]; needed for correct quantization of coefficients

__Upshot__: want to integrate a closed, $SU(3)_L \times SU(3)_R \times G_D$-invariant*, integral 5-form ω_5' on $X \cong \frac{SU(3)_L \times SU(3)_R \times G_D}{SU(3)_{L+R} \times H_D}$

1. Invariance + product structure $\Longrightarrow$ separately QCD-invariant and G_D-invariant $\Longrightarrow \omega_5' = \omega_3^{\text{QCD}} \wedge \omega_2^D$
2. Closure: $d\omega_5' = 0 \;\Longrightarrow d\omega_2^D = 0$ closed
3. Integrality: cycles also "factorise", so can take minimal normalisation of ω_3^{QCD} and ω_2^D separately

So we want a dark coset G_D/H_D that features a **G_D-invariant closed 2-form**

*Only valid assuming G_D is semi-simple

Hunting for a topological portal

We want a dark coset G_D/H_D that features a $\boldsymbol{G_D}$**-invariant closed 2-form**

Consider cosets that we expect to arise from chiral symmetry breaking in a dark QCD-like sector:

$$\frac{G}{H} = \left\{ \frac{SU(N)_L \times SU(N)_R}{SU(N)_{L+R}}, \frac{SU(N)}{SO(N)}, \frac{SU(2N)}{Sp(2N)} \right\} \text{ if dark quarks are in } \{\mathbb{C}, \mathbb{R}, i\mathbb{R}\} \text{ representation}$$

These are all not just homogeneous spaces, but *symmetric spaces:*

Theorem: **any** $\boldsymbol{G}$**-invariant form** on symmetric space G/H **is closed**

Corollary: there are no exact forms in the cohomology of invariant forms on symmetric spaces

Therefore, G-invariant forms on these G/H are in **1-to-1 with cohomology classes**

See e.g. Schwarz, Springer New York, 1991

Hunting for a topological portal

We can look up these de Rham cohomology groups in an algebraic topology book:

p	1	Portal 2	3	4	SIMP 5
$H^p(SU(2))$	0	0	$\mathbb{R}$	$-$	$-$
$H^p(SU(n)),\ n \geq 3$	0	0	$\mathbb{R}$	0	$\mathbb{R}$
$H^p(SU(2)/SO(2))$	0	$\mathbb{R}$	$-$	$-$	$-$
$H^p(SU(3)/SO(3))$	0	0	0	0	$\mathbb{R}$
$H^p(SU(4)/SO(4))$	0	0	0	$\mathbb{R}$	$\mathbb{R}$
$H^p(SU(n)/SO(n)),\ n \geq 5$	0	0	0	0	$\mathbb{R}$
$H^p(SU(2n)/Sp(2n)),\ n \geq 2$	0	0	0	0	$\mathbb{R}$

Cartan, Séminaire Henri Cartan, 1959

SIMP
= Strongly Interacting Massive Particle
WZW ⇒ 3 → 2 process within dark sector

Hochberg, Kuflik, Volansky, Wacker, 1402.5143
Hochberg, Kuflik, Murayama, Volansky, Wacker, 1411.3727

Only $SU(2)/SO(2) = S^2$ has a non-zero invariant 2-form - by fluke of its low-dimension

So QCD x Dark Sector admits a topological portal for a **unique choice** of dark coset in this class

Aside: is there a weak scale version?

- Question: If Higgs is composite (a pNGB on some G/H), can we use similar tricks to find an analogous topological portal around the TeV scale?

- Answer: Not really! Custodially-invariant differential form requires all 4 Higgs components:

$$\omega_5 \sim d^4 H \wedge \omega_1^{\text{Dark}}$$

c.f. $\omega_5' = \omega_3^{\text{QCD}} \wedge \omega_2^{\text{Dark}}$ for QCD version

- Only one direction left! Take $\omega_1^{\text{Dark}} = d\eta$.
  - No stabilizing $\mathbb{Z}_2$ symmetry
  - Even worse: when I gauge $SU(2)_L \times U(1)_Y \subset \text{Cust}$, I get $L \supset \eta F F$, so $\eta \to \gamma\gamma$ like the pion… it's certainly not dark matter!

- This illustrates the "finiteness" of topological effects (once you fix your spacetime dimension, and you have global symmetries)

EFT for the topological portal operator

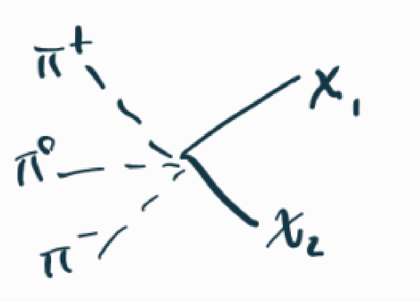
$\omega_2^D = \mathrm{Vol}_{S^2}$ is just the volume form on S^2 c.f. Dirac monopole; $\omega_2^D = \frac{1}{4\pi f_D^2} \cos\chi_1 d\chi_1 d\chi_2$

5-form for the mixed WZW term is ($N \in \mathbb{Z}$)

$$\omega_5' = \frac{1}{24\pi^2} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \omega_2^D = \frac{N}{96\pi^3 f_\pi^3 f_D^2} f^{abc} \epsilon^{ij} d\pi_a d\pi_b d\pi_c d\chi_i d\chi_j + \mathcal{O}(\pi^4\chi^2, \pi^3\chi^3)$$

Corresponding 4d Lagrangian on a local patch is

$$L = \frac{N}{48\pi^2 f_\pi^3 f_D^2} i\epsilon^{\mu\nu\rho\sigma} f^{abc} \epsilon^{ij} \pi_a \partial_\mu \pi_b \partial_\nu \pi_c \partial_\rho \chi_i \partial_\sigma \chi_j$$



N.b. contrast to SIMP mechanism, which features a 5-dark-pion interaction $d\chi_i \wedge d\chi_j \wedge d\chi_k \wedge d\chi_l \wedge d\chi_m$

Hochberg, Kuflik, Volansky, Wacker, 1402.5143
Hochberg, Kuflik, Murayama, Volansky, Wacker, 1411.3727

Gauging the topological portal

$L = \frac{N}{48\pi^2 f_\pi^3 f_D^2} i\epsilon^{\mu\nu\rho\sigma} f^{abc} \epsilon^{ij} \pi_a \partial_\mu \pi_b \partial_\nu \pi_c \partial_\rho \chi_i \partial_\sigma \chi_j$ is dimension-9.

As for the ordinary QCD WZW term, leading effect comes after gauging QED, roughly $\frac{1}{f_\pi^2}\partial_\mu \pi^+ \partial_\nu \pi^- \to eF_{\mu\nu}$

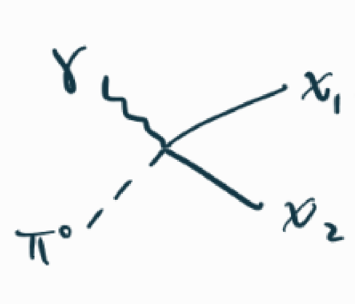
QED is a subgroup of the unbroken $SU(3)_{L+R}$ generated by $Q = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}$

Effectively we gauge a 2d WZW term: replace $\omega_3^{\mathrm{QCD}} \to \omega_3^{\mathrm{QCD}} - \frac{e}{4\pi} F \wedge \mathrm{Tr}\,(Qg^{-1}dg)$

E.g. Yonekura, 2009.04692

$$\Delta L = \frac{N}{16\pi^2 f_\pi f_D^2} \epsilon^{\mu\nu\rho\sigma} \left(\pi_0 + \frac{\eta}{\sqrt{3}}\right) F_{\mu\nu}\, \partial_\rho \chi_1\, \partial_\sigma \chi_2$$

(dimension-7)



Relic Abundance from Topological Portal

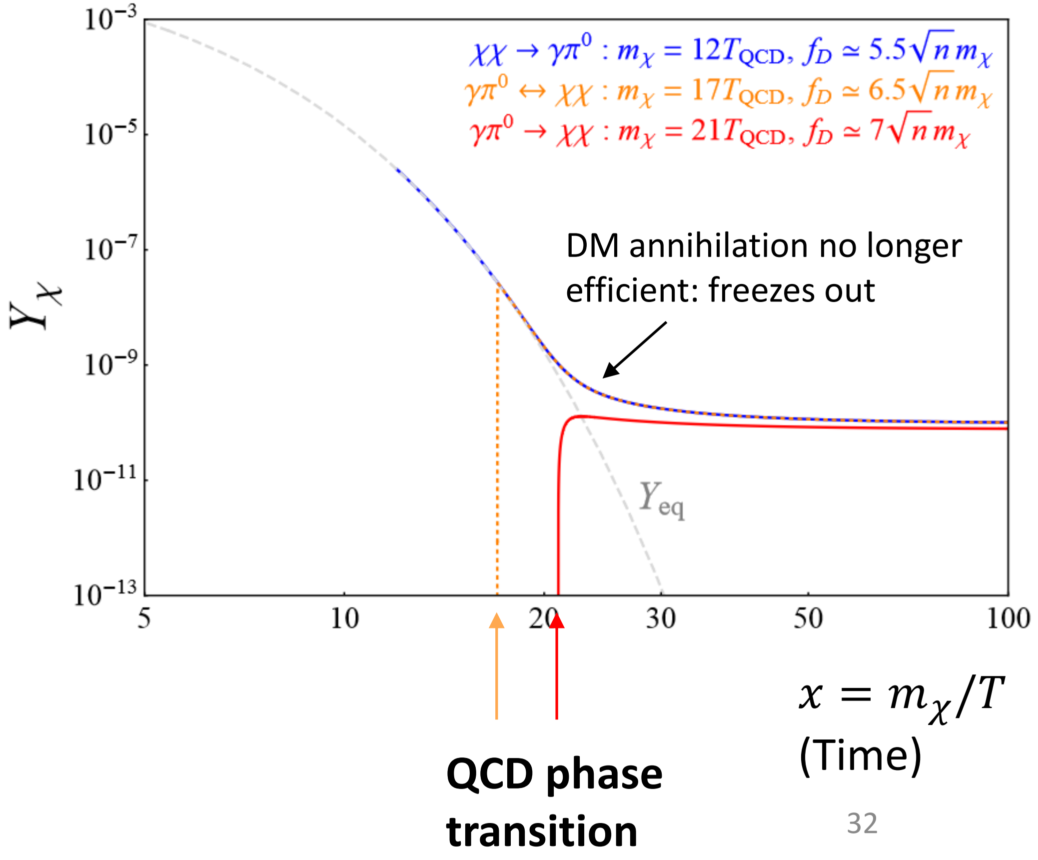
DM freezes in/out by $\pi^0\gamma \leftrightarrow \chi_1\chi_2$ co-annihilations, $M \sim \frac{ne}{f_\pi f_D^2}\epsilon^{\mu\nu\rho\sigma}p_\pi^\mu \varepsilon^\nu p_1^\rho p_2^\sigma$

Finite $\Delta m_\chi \coloneqq m_{\chi_2} - m_{\chi_1} > 0 \Longrightarrow \chi_2 \to \chi_1$ shortly after freeze-out, leaving χ_1 as relic DM

[Masses (and splitting) from small explicit breaking]

- Our EFT applies when freeze-out occurs after QCD phase transition $T_{\rm QCD} \approx 160$ MeV
- Once portal operator turns on, DM rapidly thermalises if not already in the bath (orange / red)
- To freeze-out at large enough abundance, topological portal *cannot turn on too late* (in units of m_χ)
- Numerically, need $\frac{m_\chi}{T_{\rm QCD}} \lesssim 23 \Longrightarrow m_\chi \lesssim 3.7$ GeV
- Adjust coupling in range $f_D \sim \mathcal{O}(5-7)\sqrt{n}\, m_\chi$ to fit RA

JD, Greljo, Selimović 2401.09528



Dark Matter Phenomenology

Need χ_2 decay before BBN $\Rightarrow \Delta m_\chi > m_{\pi^0}$; otherwise χ_2 decays through suppressed $\chi_2 \to \chi_1 \gamma\gamma\gamma$ $(\tau > 1\text{s})$

- Thermal relic **DM is only χ_1**
- Finite Δm_χ Boltzmann suppresses co-annihilation

Kawasaki, Kohri, Moroi, Takaesu, 1709.01211
D'Agnolo, Mondino, Ruderman, Wang, 1803.02901

Indirect Detection

Because the interaction is **topological** i.e. a differential form, it is **perfectly antisymmetric** $(d\chi_1 \wedge d\chi_2)$

- Absence of "diagonal" interactions $\chi_i \chi_i \to \text{SM}$
- No late time DM annihilation – model for **completely inelastic** light thermal DM

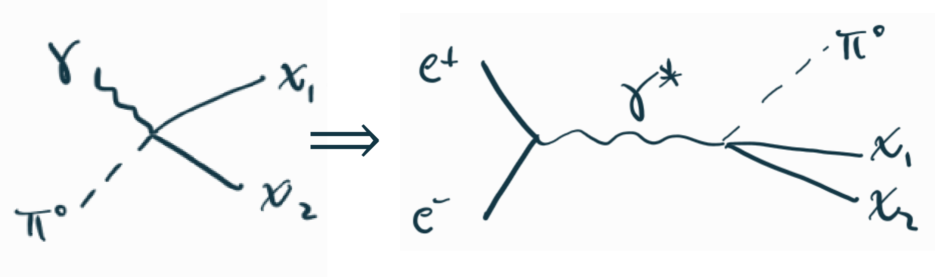
Tucker-Smith, Weiner, hep-ph/0101138
ADD OTHER REFS

Direct Detection

Insufficient energy to up-scatter dark matter in lab $\chi_1 \to \chi_2$ via portal

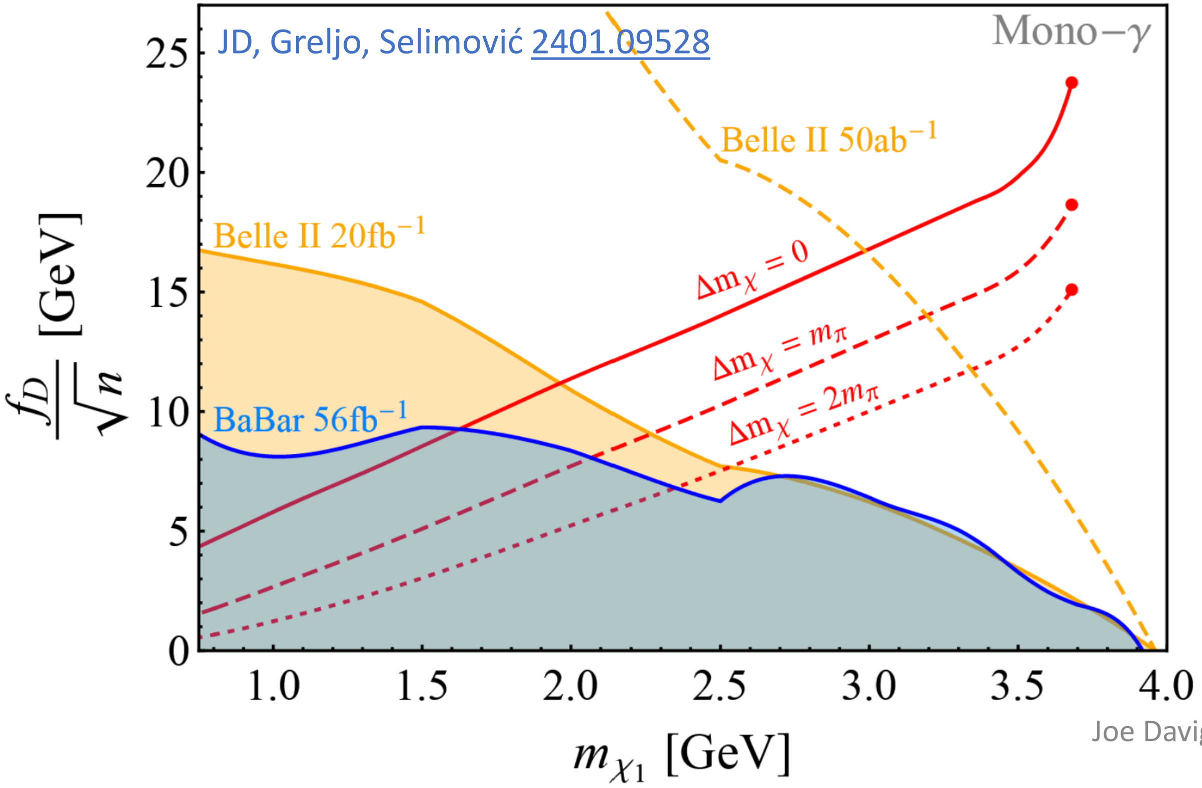
[note derivative suppression by $(\partial/f)^3$ in top portal is huge for DM velocity $v \ll c$].

Prospects in e^+e^- colliders



Δm_χ	$\lesssim 1.7 m_{\pi^0}$	$\gtrsim 1.7 m_{\pi^0}$
Signature	$\pi^0 + \not{E}_T$	$\pi^0 + \not{E}_T + \mathrm{DV}(\pi^0 \gamma \not{E}_T)$

Final state π^0 reconstructed as photon if boosted to few GeV, so can recast $\gamma + \mathrm{Inv}$ searches for signature 1



Our monophoton recast demonstrates **tremendous prospects at Belle II**

There is no data relevant to the DV region (currently veto-ed in these mono-photon searches)

Bonus: if observe a signal, definite prediction in **channel $e^+e^- \to \eta\chi_1\chi_2$ provides smoking gun!**

Part II: UV

Work in progress with Nakarin Lohitsiri

UV?

↓

$$\int_{Y_5} \frac{1}{24\pi^2} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \mathrm{Vol}_{S^2}$$

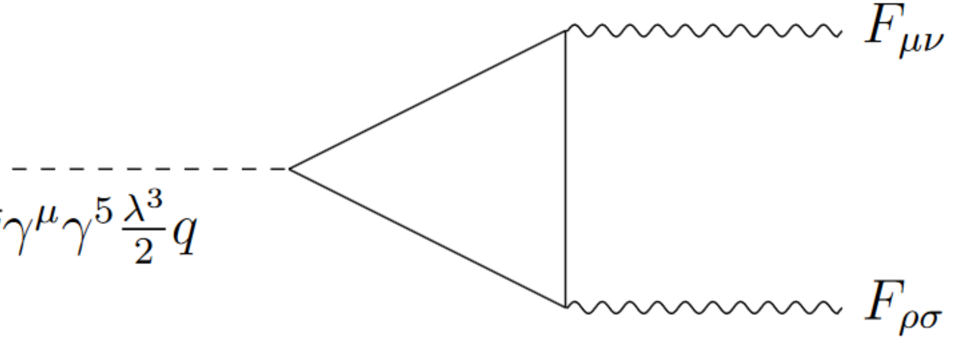
Where do topological terms come from?

QCD-informed answer: anomalies!

Where do topological terms come from?

QCD-informed answer: anomalies!

UV



$$j_A^{\mu 3} = \overline{q}\gamma^\mu\gamma^5\frac{\lambda^3}{2}q$$

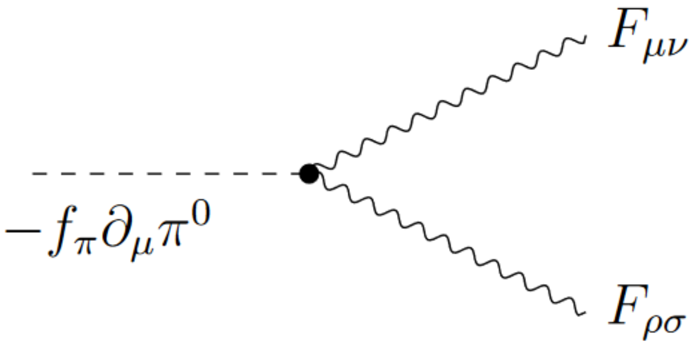
Compute $\partial_\mu j_A^{\mu 3} \sim \frac{n_c e^2}{16\pi^2} FF\,\mathrm{Tr}\left(\frac{\lambda^3}{2}Q^2\right)$

Find the answer is the index of the Dirac operator – an integer!

… ⇒ anomaly not renormalised

↓

IR



$$j_A^{\mu 3} = -f_\pi \partial_\mu \pi^0$$

$$L_{\mathrm{WZW}}(A) \sim \frac{n_c e^2}{96\pi^2}\frac{\pi^0}{f_\pi}FF + \cdots$$

[WZW also quantized for consistency!]

A Puzzle!

- For QCD, the $\omega_5 = \frac{-in}{480\pi^3} \mathrm{Tr}\,(g^{-1}dg)^5$ term matches all ‘t Hooft anomalies associated with gauging any subgroup of $SU(3)_L$ and $SU(3)_R$: essentially from $\mathrm{Tr}\,F_L^3$ and $\mathrm{Tr}\,F_R^3$ anomalies

- The mixed WZW term $\omega_3^{\mathrm{QCD}} \wedge \omega_2^{\mathrm{Dark}}$ looks set to match a *mixed anomaly* between $SU(3)_{L/R}$ and $SU(2)_D$ - but ***there is no such anomaly***! Regardless of chiral fermion content

$$\mathrm{Tr}\,F_{SU(3)}F_{SU(3)}f_{SU(2)} = 0$$

So, the mixed WZW term does not match anomalies. Peculiar!

A Clue…

- Global 0-form symmetries are $SU(3)_L \times SU(3)_R \times SU(2)_D$
- Anomalies in this 0-form symmetry are not telling us much
- But upon closer inspection, **there are more symmetries**!

A Clue…

- Global 0-form symmetries are $SU(3)_L \times SU(3)_R \times SU(2)_D$
- Anomalies in this 0-form symmetry are not telling us much
- But upon closer inspection, **there are more symmetries**!

- There is a **1-form symmetry** corresponding to a conserved 2-form current ($\partial_\mu j^{[\mu\nu]} = 0$)

$$j^{(2)}_{\mathrm{wind}} = \star\, \mathrm{Vol}_{S^2},\;\; d \star j^{(2)}_{\mathrm{wind}} = d\mathrm{Vol}_{S^2} = 0$$

- **Specific to this choice of dark coset** which features an invariant closed 2-form – recall this was the condition for the QCD x dark sector to feature the mixed WZW term

Generalised Symmetries
[Gaiotto, Kapustin, Seiberg, Willet, 1412.5148]

Ordinary "0-form" symmetries can be derived from varying action.
- Noether procedure $\Longrightarrow$ conserved current $\partial^\mu j_\mu = 0$.
- Defines a 1-form $j = j_\mu dx^\mu$

Higher "p-form" symmetries are associated with conserved **p+1 forms**, $d \star j^{(p+1)} = 0$
- Not deduced from variation of the action!
- Do not act on local operators, but on extended gauge-invariant operators e.g. "Wilson lines"

See Seth's talk last week

But that's not all!

The 0-form and 1-form symmetries are ***inter-linked*** by the mixed WZW term

$$S_{\text{mix}} = \int_{Y_5} \frac{1}{24\pi^2} \text{Tr}\,(g^{-1}dg)^3 \wedge \text{Vol}_{S^2}$$

$\uparrow$ $j^{(2)}_{\text{wind}}$

The correct global symmetry structure appears to be a "2 group"

2-group global symmetry

One way to see this is to introduce background fields (~ ‘t Hooft anomaly matching):

- 0-form symmetries $SU(3)_L \times SU(3)_R \times SU(2)_D$, background gauge fields $A_L^{(1)}$, $A_R^{(1)}$, $A_D^{(1)}$
- 1-form symmetry $U(1)^{[1]}_{\text{wind}}$, background gauge field $B^{(2)}$: minimal coupling $S = \int_{\Sigma_4} \star\, j^{(2)}_{\text{wind}} \wedge B^{(2)}$

2-group global symmetry

One way to see this is to introduce background fields (~ ‘t Hooft anomaly matching):

- 0-form symmetries $SU(3)_L \times SU(3)_R \times SU(2)_D$, background gauge fields $A_L^{(1)}$, $A_R^{(1)}$, $A_D^{(1)}$
- 1-form symmetry $U(1)^{[1]}_{\text{wind}}$, background gauge field $B^{(2)}$: minimal coupling $S = \int_{\Sigma_4} \star\, j^{(2)}_{\text{wind}} \wedge B^{(2)}$

The mixed WZW modifies the anomalous variation of the QCD 0-form flavour currents. E.g. under a background $SU(3)_L$ gauge transformation $A_L^{(1)} \to g_L A_L^{(1)} g_L^\dagger + g_L d g_L^\dagger$, pick up a new piece

$$\delta_L \int_{Y_5} n\, \omega_3^{\text{QCD}}(A_L, A_R) \wedge \omega_2^D \sim \int_{\Sigma_4} n \, \text{Tr}\left(g_L d g_L^\dagger\, F_L\right) \wedge j^{(2)}_{\text{wind}}$$

n is the integer-quantized coefficient of the topological portal term

- This signals the 0-form flavour symmetry is *mixed with the 1-form symmetry* in what’s known as a 2-group global symmetry structure.* [The Postnikov class equals n, the coefficient of the WZW]

Sharpe, 1508.04770
Cordova, Dumitrescu, Intriligator, 1802.04790
Benini, Cordova, Hsin, 1803.09336
Hsin, Lam, 2007.05915
Lee, Ohmori, Tachikawa, 2108.05369

*If one modifies the gauge transformation law for $B^{(2)}$ to be g_L-dependent, encoding a “2-connection”, the anomalous variation returns to *zero*

How is this useful? “Symmetry Matching”!

(Recall a comment of Sungwoo last week)

The 2-group structure is like a ‘t Hooft anomaly: it is integer-quantized ⇒ **preserved along RG flow**

There is a **2-group current algebra** analogous to the familiar anomalous current algebra:

$$\left\langle \partial^\mu j_\mu^{L,A}(x) j_\nu^{L,B}(y) - \delta(x-y) f^{ABC} j_\nu^C(y) \right\rangle \sim \left\langle n\, \delta^{AB} \partial^\rho \delta(x-y) j_{\rho\nu}^{\text{wind}}(y) \right\rangle$$

Quantized coefficient n

Cordova, Dumitrescu, Intriligator 1802.04790

The mixed WZW term encodes non-zero 2-group current algebra **in the IR**

⇒ Same current algebra must be manifest **in the UV completion**

(or else the whole structure is emergent, including the 0-form flavour symmetry)

The *“2-group emergence theorem”*
Cordova, Dumitrescu, Intriligator 1802.04790

As used e.g. in Cordova, Koren 2212.13193 to study GUT embeddings of SM

No-go!

A non-abelian dark gauge sector with semi-simple gauge group does not have a (continuous) 1-form symmetry ⇒ 2-group would not close in such a UV theory! Incompatible with exact flavour symmetry

Can formulate a **no-go theorem**:

Dark QCD-like UV theory has no 1-form ergo no 2-group symmetry, so it cannot give rise to an IR theory with the mixed WZW term (the topological portal) while preserving QCD flavour symmetry

Generalised symmetry immediately **precludes naïve QCD-like UV completions** that otherwise give the right coset, e.g. $SU \times SO$* gauge theory or SU gauge theory with fundamental + adjoint quarks

… Instead, we are guided to UV theories that (at least) **have a 1-form symmetry**

* $SO(n_D)$ gauge theory has only a $\mathbb{Z}_2$-valued 1-form symmetry

See e.g. Lee, Ohmori, Tachikawa, 2108.05369

Try an Abelian gauge field

Strategy: lift $j^{(2)}_{\rm wind} = \star\,\mathrm{Vol}_{S^2}$ (IR) to $j^{(2)}_{\rm wind} = \frac{h}{2\pi}$ in UV, where $h = da$ is an **abelian** field strength

i.e. want dark pion winding number (IR) = abelian monopole charge (UV)

Then the mixed WZW term would look like (below $\Lambda_{\rm QCD}$, assuming now that $\mathrm{f_D} < \Lambda_{\rm QCD}$)

$$S_{\rm mix} = \int_{Y_5} \frac{1}{24\pi^2} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \frac{da}{2\pi} = \int_{\Sigma_4} \frac{1}{24\pi^2} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \frac{a}{2\pi} = \int_{\Sigma_4} \star j \wedge \frac{a}{2\pi}$$

Recall $\star\,\omega_3 = j_B$ is identified as **topological baryon number current** in low-energy QCD!

We know that we can robustly identify this current past the QCD phase transition to the UV ($E > \Lambda_{\rm QCD}$):

$$j_q = n_c j_B, \qquad j_q^\mu = \sum_q \overline{\Psi}_q \gamma^\mu \Psi_q$$

Suggests $S_{\rm mix}$ can be realised just as **coupling of the abelian gauge field a to baryon number** in the UV!

A UV model: QCD + Linear Sigma Model

Weak coupling UV completion of the dark part:

- x2 $\mathbb{C}$ scalars ϕ_i with charge N_ϕ under $U(1)_X$; n_f (3) quarks coupled with unit charge
- $U(2)$-invariant scalar potential $V(\phi) = -\mu^2 \phi_i^\dagger \phi_i + \lambda\left(\phi_i^\dagger \phi_i\right)^2$
- Faithful global symmetry in UV is $\left[U(1)_q \times SU(n_f)_L \times SU(n_f)_R \times SU(2)_\phi\right]/\Gamma \times U(1)_m^{[1]}$

Symmetry breaking: $\langle \phi_i^\dagger \phi_i \rangle \neq 0$

- Higgses $U(1)_X \Rightarrow$ **heavy Z'** gauge boson
- Breaks $SU(2)_\phi \to U(1)_\phi \Rightarrow$ $\boldsymbol{\chi_{1,2}}$ **pNGBs** [combination of $U(1) \subset SU(2)_\phi$ and gauged $U(1)_X$ unbroken]
- Integrate out Z' out: EOM is

$$a = -\frac{1}{2} j_q + \chi_1 d\chi_2, \qquad \int_{S^2 \subset \Sigma_4} da = \int_{S^2 \subset \Sigma_4} \mathrm{Vol}_{S^2} \text{ exactly as we wanted!}$$

Interaction | 0-form | 1-form | 2-group

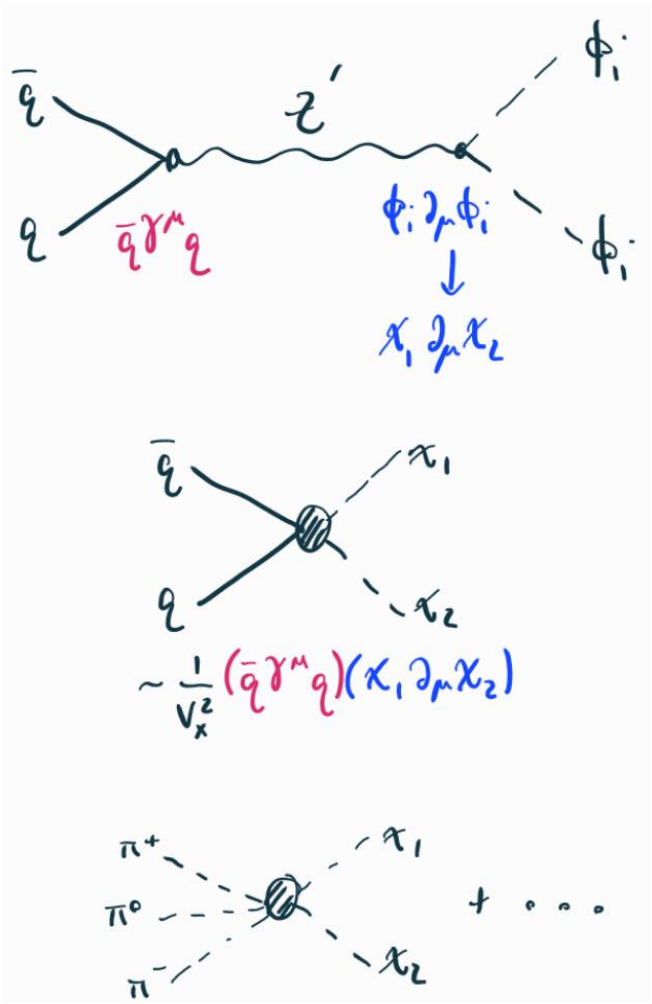
UV

$\langle \phi \rangle \neq 0$

Visible

$\langle \overline{q_L} q_R \rangle \neq 0$

IR



0-form

$j_L^{\mu a} = \bar{q}\gamma^\mu P_L \frac{\lambda^a}{2} q$

$j_q^\mu = \bar{q}\gamma^\mu q$

$j_L^{\mu a} = -\frac{f_\pi}{2}\partial_\mu \pi^a$

$j_B = \star \frac{\mathrm{Tr}\left(g^{-1}dg\right)^3}{24\pi^2}$

1-form

$j_m^{(2)} = \frac{da}{2\pi}$

Works only for dark coset that can match the monopoles!*

$j_{\mathrm{wind}}^{(2)} = \mathrm{Vol}_{S^2}$

Coupling to baryon current in UV is special! Identified with *topologically conserved* IR current

2-group

$F_L F_L da$ "anomaly"

UV 2-group manifest as "operator-valued mixed anomaly"

$\int_{Y_5} \frac{\mathrm{Tr}\left(g^{-1}dg\right)^3}{24\pi^2} \wedge \mathrm{Vol}_{S^2}$

(top term in action)

*It's not just that $H^2(X) \neq 0$; doesn't work for "pair of axions" on $X = S^1 \times S^1$; matches low-energy WZW classification [JD, Gripaios, 1803.07585]

What's Next?

1. **TH**: Complete story of generalized symmetry structure, and implications for UV completion; explore related examples of 2-group symmetry matching by WZW

   With N. Lohitsiri

2. **PH**: Can we actually use this to UV complete the dark matter topological portal?
   - Coupling through j_B makes clear there is important $\pi^+\pi^- \to \rho \to \chi_1\chi_2$ channel, at least
   - Revisit story for relic abundance with more complete picture
   - Is there viable parameter space given all current data?
   - Elucidate correlated collider phenomenology e.g. look for (hide?) the Z' (light!) in LHC

   With A. Greljo and N. Selimović

3. **EXP**: Proper Experimental Study for clear Belle II signatures, including displaced vertex region

   With A. Greljo and N. Selimović
   + Belle II person (hopefully)

Thank you!

Aside 2: other low-scale options?

Can use gauge fields directly to write down two more possible couplings:

1. $\omega_5' = \omega_3^{\mathrm{QCD}} \wedge F^{\mathrm{Dark}}$

- Upon gauging QED, gives $\pi^0 F F^{\mathrm{Dark}}$: require $m_{\gamma_D} > m_{\pi^0}$ to not spoil pion decay, but then $\gamma_D \to \pi^0 \gamma$
- Dark photon could nonetheless be a mediator

2. $\omega_5 \sim F \wedge \omega_3^{\mathrm{Dark}}$

- Modification to SIMP picture?