$SU(N)$ and $O(N)$ Representation Theory at non-integer N

Crossroads between Theory and Phenomenology
Jun 10-28, 2024

Xiaochuan Lu
University of California, San Diego

arXiv: 2312.10139
with Weiguang Cao and Tom Melia

This talk is not as ambitious as…

➢ **Continuous-Spin Particles, On Shell**

Brando Bellazzini (IPhT, Saclay), Stefano De Angelis (IPhT, Saclay), Marcello Romano (IPhT, Saclay) (Jun 24, 2024)

e-Print: 2406.17017 [hep-th]

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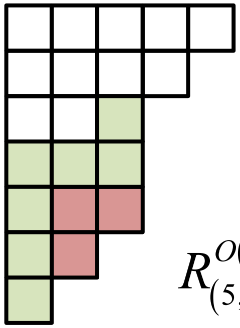
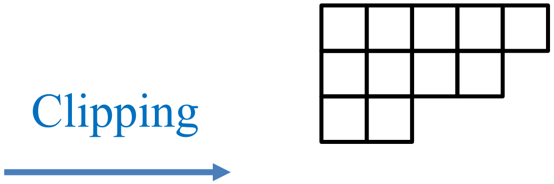
This talk is introducing a new Young diagram clipping algorithm for “Rep Specializations”





$$R^{O(7)}_{(5,4,3,3,3,2,1)} = -R^{O(7)}_{(5,4,3,3,1,1,1)} = -R^{O(7)}_{(5,4,2)}$$

Motivation

"Representation Theory" in this talk:

Decompositions of Tensors of Reps

$$R_1 \otimes R_2 = \bigoplus_\lambda m_{12}^\lambda R_\lambda$$

$SU(2)$

$$\mathbf{2} \times \mathbf{2} = \mathbf{3} + \mathbf{1}$$

$$\mathbf{2} \times \mathbf{2} \times \mathbf{2} = \mathbf{4} + 2(\mathbf{2})$$

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$$\mathbf{3} \times \mathbf{3} = \mathbf{6} + \overline{\mathbf{3}}$$

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{10} + 2(\mathbf{8}) + \mathbf{1}$$

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What about non-integer N, say $N = 3.98$?

Continuation: taking the large integer N limit

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holds for all integer $N \geq 3$

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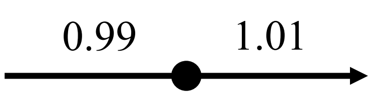

$N = 1$

low integer N are special

$$p_\mu q_\nu = \left[\frac{1}{2}\left(p_\mu q_\nu + p_\nu q_\mu\right) - \frac{1}{N}\eta_{\mu\nu}(p \cdot q)\right] + \frac{1}{N}\eta_{\mu\nu}(p \cdot q) + \frac{1}{2}\left(p_\mu q_\nu - p_\nu q_\mu\right)$$

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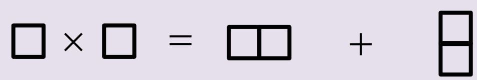

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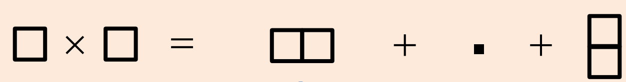

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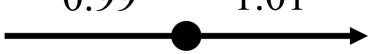

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"Specialization"

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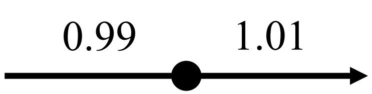

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"Specialization": encodes Evanescence

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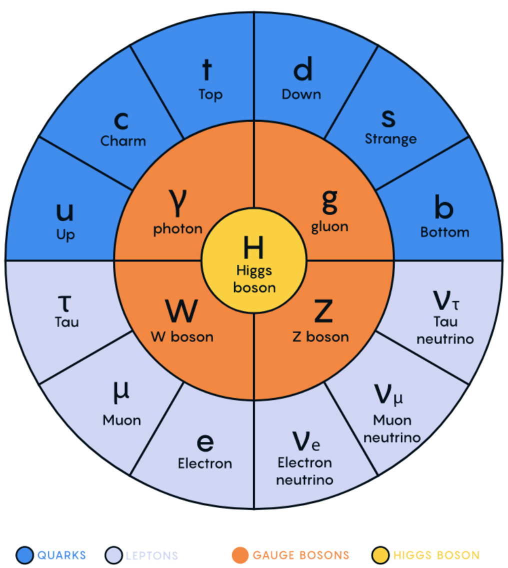
Standard Model Effective Field Theory (SMEFT)



Standard Model	Lorentz Group	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	mass dim
H	$(0,0)$	1	2	$+\frac{1}{2}$	1
q	$(\frac{1}{2},0)$	3	2	$+\frac{1}{6}$	3/2
u	$(0,\frac{1}{2})$	3	1	$+\frac{2}{3}$	3/2
d	$(0,\frac{1}{2})$	3	1	$-\frac{1}{3}$	3/2
l	$(\frac{1}{2},0)$	1	2	$-\frac{1}{2}$	3/2
e	$(0,\frac{1}{2})$	1	1	-1	3/2
$G^A_{\mu\nu}$	$(1,0)$	8	1	0	2
$W^a_{\mu\nu}$	$(1,0)$	1	3	0	2
$B_{\mu\nu}$	$(1,0)$	1	1	0	2

ϕ

$[\partial_\mu] = 1$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \underline{\sum_i c_i \mathcal{O}_i(\phi)}$$

mass dim $= 5, 6, 7, 8, \cdots$

Motivation

dim 6, $n_g = 1$ Grzadkowski, Iskrzynski, Misiak, and Rosiek, arXiv: 1008.4884

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi$		
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

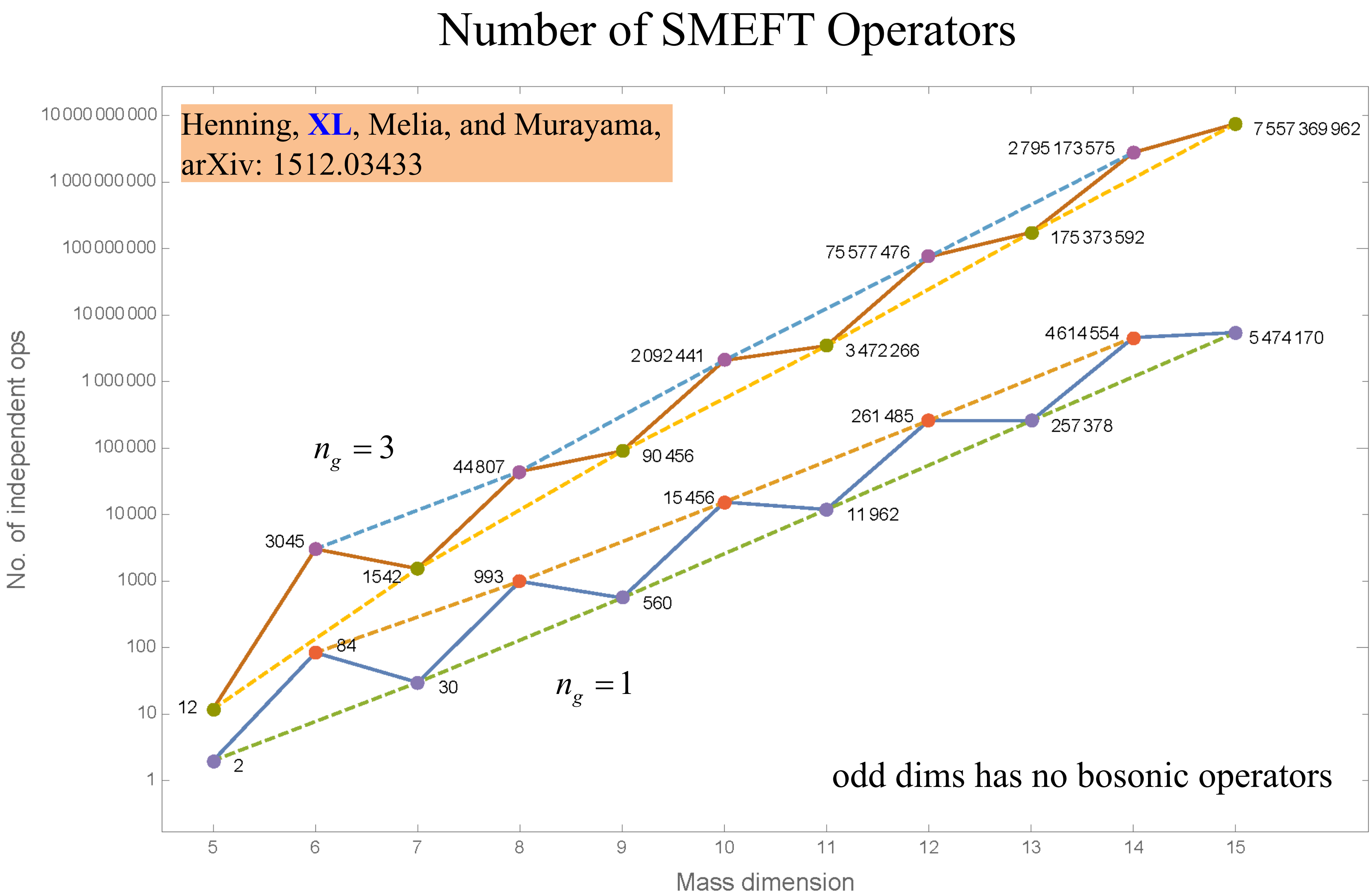
$X^2\varphi^2$		$\psi^2 X$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu}$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu}$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde W B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu}$

Table 2: Dimension-six operators

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Table 3: Four-fermion operators.



Number of SMEFT Operators
Henning, XL, Melia, and Murayama, arXiv: 1512.03433
No. of independent ops
Mass dimension
$n_g = 3$
$n_g = 1$
odd dims has no bosonic operators
12
2
3045
84
1542
30
44807
993
90456
560
2092441
15456
3472266
11962
75577476
261485
175373592
257378
2795173575
4614554
7557369962
5474170
10 000 000 000
1 000 000 000
100 000 000
10 000 000
1 000 000
100 000
10 000
1000
100
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1
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6
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12
13
14
15

Evanescent Operators can also be important

➢ EFT Matching Calculation

Evanescent operators in one-loop matching computations

Javier Fuentes-Martín (Granada U., Theor. Phys. Astrophys. and Mainz U., Inst. Phys. and U. Mainz, PRISMA), Matthias König (Munich, Tech. U.), Julie Pagès (UC, San Diego), Anders Eller Thomsen (Bern U.), Felix Wilsch (Zurich U.) (Nov 16, 2022)

Published in: *JHEP* 02 (2023) 031 • e-Print: 2211.09144 [hep-ph]

➢ EFT Running Calculation

Renormalization and non-renormalization of scalar EFTs at higher orders

Weiguang Cao (Tokyo U., IPMU and Tokyo U.), Franz Herzog (Edinburgh U.), Tom Melia (Tokyo U., IPMU), Jasper Roosmale Nepveu (Humboldt U., Berlin) (May 26, 2021)

Published in: *JHEP* 09 (2021) 014 • e-Print: 2105.12742 [hep-ph]

--- also discussed how Hilbert series may be used to systematically enumerate a class of evanescent operators in scalar EFTs

"Specializations" encode a class of evanescence

Outline

- Examples of Specializations; the Clipping Rules

  --- $SU(N)$: “West Coast” Clipping

  --- $O(N)$: “East Coast” Clipping

- (Not to be confused with) Racah-Speiser Algorithm

- Example Implication of Negative Specializations

  --- Degeneracies in SMEFT RGE

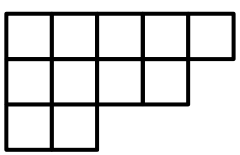
Clipping Rules for Specializations

Young Diagram Convention

- Label an irrep by its highest weight : $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_r)$,
  where r is the rank of the group

- $SU(N)$: $r = N - 1$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r \geq 0$ all integers

Each irrep λ corresponds to a Young diagram : λ_k boxes in the k-th row



$\lambda = (5, 4, 2)$

- $\lambda = (0, 0, \cdots, 0)$ the invariant irrep

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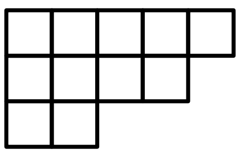
  where r is the rank of the group

- $SU(N)$: $r = N - 1$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r \geq 0$ all integers

Each irrep λ corresponds to a Young diagram : λ_k boxes in the k-th row



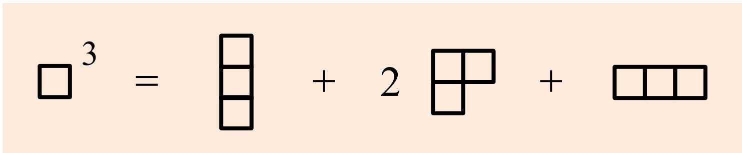
$\lambda = (5, 4, 2)$

- $\lambda = (0, 0, \cdots, 0)$ the invariant irrep

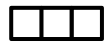
- $O(N)$: $r = \lfloor N/2 \rfloor$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r \geq 0$
  simultaneously integers (bosonic) or half odd integers (fermionic)

We focus on bosonic irreps only --- can be labeled by Young diagrams

Examples of $SU(N)$ Specializations



continued from integer $N \geq 4$

$N = 2$  $\mathbf{2}$  $\mathbf{4}$ $\mathbf{2} \times \mathbf{2} \times \mathbf{2} = 2(\mathbf{2}) + \mathbf{4}$

Examples of $SU(N)$ Specializations

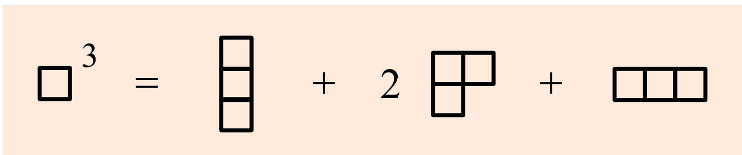
$$\square^{3} = \ydiagram{1,1,1} + 2\,\ydiagram{2,1} + \ydiagram{3}$$

continued from integer $N \geq 4$

$N = 2$ $\qquad \square_{\mathbf{2}} \qquad \ydiagram{3}_{\mathbf{4}} \qquad \mathbf{2} \times \mathbf{2} \times \mathbf{2} = 2(\mathbf{2}) + \mathbf{4}$

$N = 3$ $\qquad \cdot_{\mathbf{1}} \qquad \ydiagram{2,1}_{\mathbf{8}} \qquad \ydiagram{3}_{\mathbf{10}} \qquad \mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2(\mathbf{8}) + \mathbf{10}$

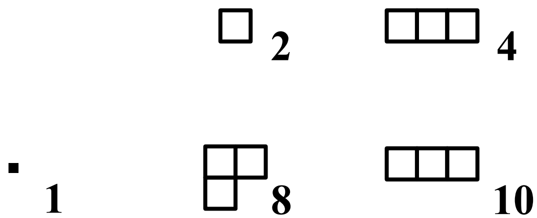
Examples of $SU(N)$ Specializations



continued from integer $N \geq 4$

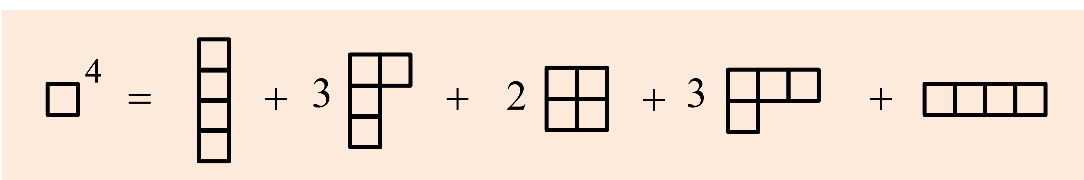
$N = 2$

$N = 3$



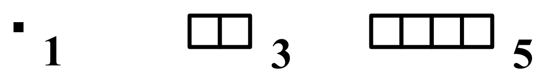
$\mathbf{2} \times \mathbf{2} \times \mathbf{2} = 2(\mathbf{2}) + \mathbf{4}$

$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2(\mathbf{8}) + \mathbf{10}$



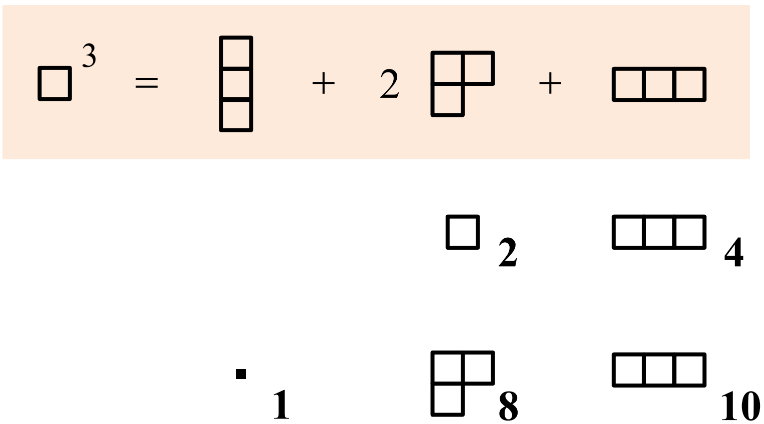
continued from integer $N \geq 5$

$N = 2$



$\mathbf{2}^4 = 2(\mathbf{1}) + 3(\mathbf{3}) + \mathbf{5}$

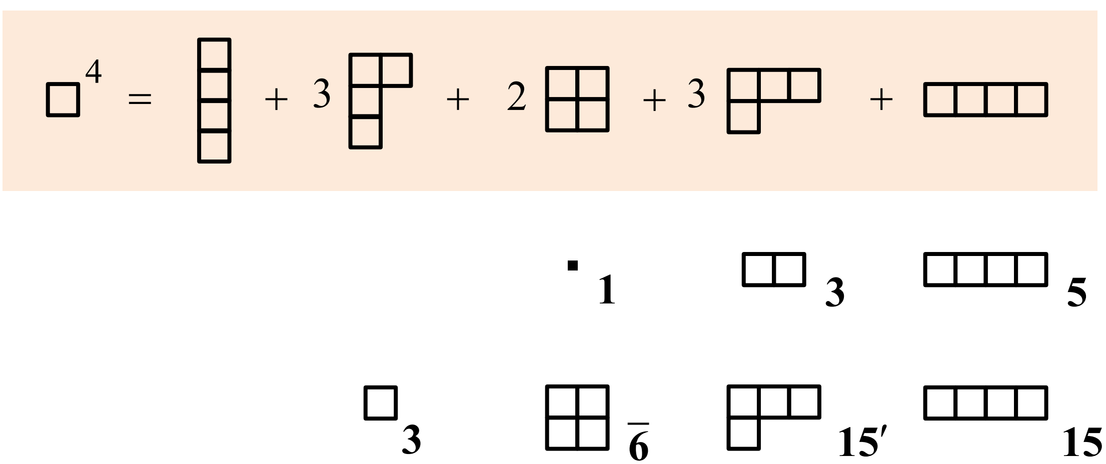
Examples of $SU(N)$ Specializations



continued from integer $N \geq 4$

$N=2$ $\qquad \mathbf{2}\times\mathbf{2}\times\mathbf{2} = 2(\mathbf{2}) + \mathbf{4}$

$N=3$ $\qquad \mathbf{3}\times\mathbf{3}\times\mathbf{3} = \mathbf{1} + 2(\mathbf{8}) + \mathbf{10}$



continued from integer $N \geq 5$

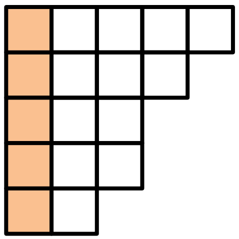
$N=2$ $\qquad \mathbf{2}^4 = 2(\mathbf{1}) + 3(\mathbf{3}) + \mathbf{5}$

$N=3$ $\qquad \mathbf{3}^4 = 3(\mathbf{3}) + 2(\overline{\mathbf{6}}) + 3(\mathbf{15}') + \mathbf{15}$

Clipping Rules for Specializations

$SU(N)$: West Coast Clipping

$\lambda = (5,4,3,3,2)$



West Coast has n_W boxes

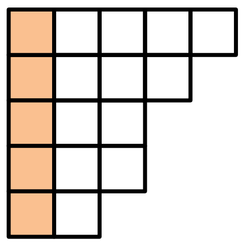
Clipping rule at a given integer N

$$\begin{cases} n_W < N: & \text{valid irrep, no clipping} \\ n_W > N: & 0\text{, no clipping} \\ n_W = N: & \text{clip off the west coast to get } \lambda_{\text{new}}\text{, and repeat} \end{cases}$$

Clipping Rules for Specializations

$SU(N)$: West Coast Clipping

$\lambda = (5,4,3,3,2)$



$R^{SU(6)}_{(5,4,3,3,2)}$

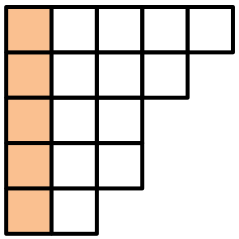
West Coast has n_W boxes

Clipping rule at a given integer N

$$\begin{cases} n_W < N: & \text{valid irrep, no clipping} \\ n_W > N: & 0\text{, no clipping} \\ n_W = N: & \text{clip off the west coast to get } \lambda_{\text{new}}\text{, and repeat} \end{cases}$$

$SU(N)$: West Coast Clipping

$\lambda = (5,4,3,3,2)$



$R^{SU(6)}_{(5,4,3,3,2)}$

$R^{SU(4)}_{(5,4,3,3,2)} = 0$

West Coast has n_W boxes

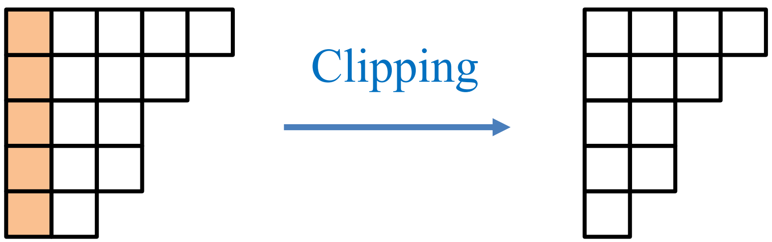
Clipping rule at a given integer N

$$\begin{cases} n_W < N: & \text{valid irrep, no clipping} \\ n_W > N: & \text{0, no clipping} \\ n_W = N: & \text{clip off the west coast to get } \lambda_{\text{new}}\text{, and repeat} \end{cases}$$

Clipping Rules for Specializations

$SU(N)$: West Coast Clipping

$\lambda = (5,4,3,3,2)$ $\qquad$ $\lambda_{\text{new}} = (4,3,2,2,1)$



$$R^{SU(6)}_{(5,4,3,3,2)}$$

$$R^{SU(4)}_{(5,4,3,3,2)} = 0$$

$$R^{SU(5)}_{(5,4,3,3,2)} = R^{SU(5)}_{(4,3,2,2,1)}$$

West Coast has n_W boxes

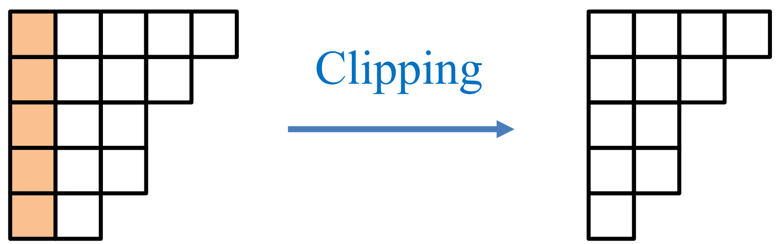
Clipping rule at a given integer N

$$\begin{cases} n_W < N: & \text{valid irrep, no clipping} \\ n_W > N: & \text{0, no clipping} \\ n_W = N: & \text{clip off the west coast to get } \lambda_{\text{new}}\text{, and repeat} \end{cases}$$

Clipping Rules for Specializations

$SU(N)$: West Coast Clipping

$\lambda = (5,4,3,3,2)$ $\qquad$ $\lambda_{\text{new}} = (4,3,2,2,1)$



$$R^{SU(6)}_{(5,4,3,3,2)}$$

$$R^{SU(4)}_{(5,4,3,3,2)} = 0$$

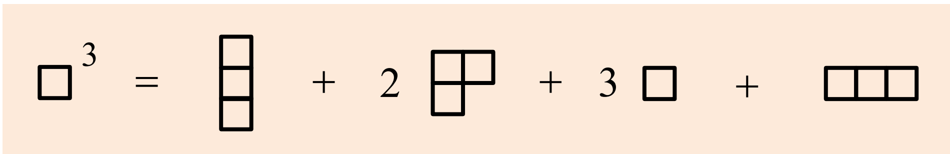
$$R^{SU(5)}_{(5,4,3,3,2)} = R^{SU(5)}_{(4,3,2,2,1)} = R^{SU(5)}_{(3,2,1,1)}$$

West Coast has n_W boxes

Clipping rule at a given integer N

$$\begin{cases} n_W < N: & \text{valid irrep, no clipping} \\ n_W > N: & 0\text{, no clipping} \\ n_W = N: & \text{clip off the west coast to get } \lambda_{\text{new}}\text{, and repeat} \end{cases}$$

Examples of $O(N)$ Specializations

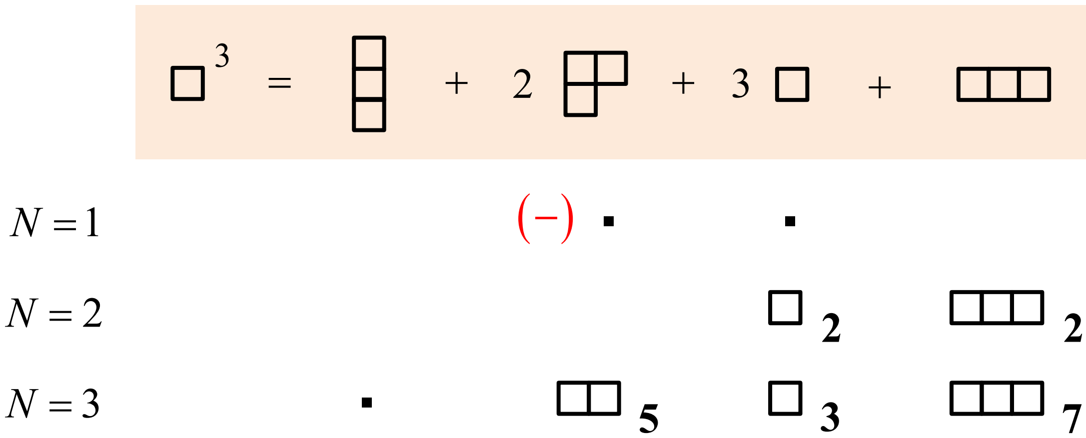


$N = 1$ $\qquad$ ${\color{red}(-)}\ \blacksquare \qquad \blacksquare$

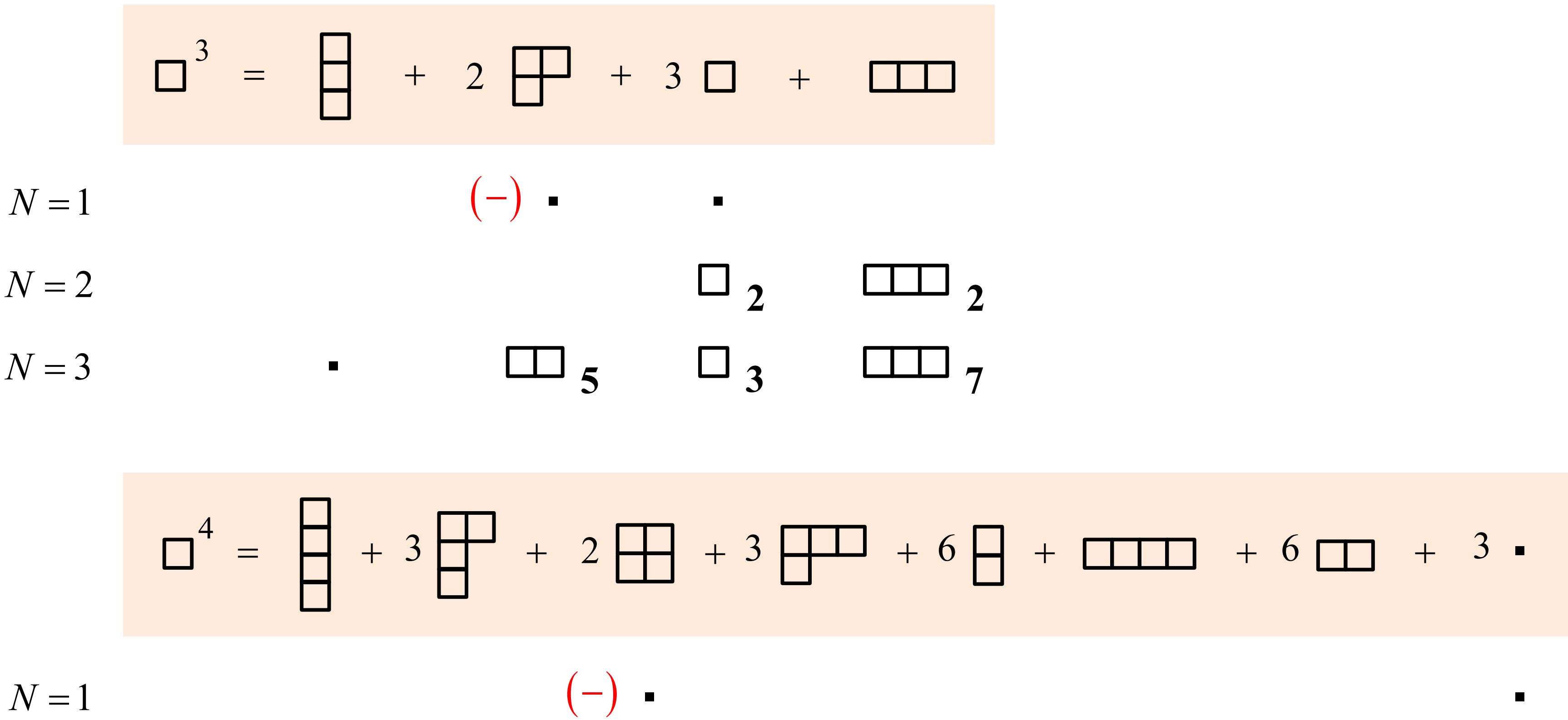
Examples of $O(N)$ Specializations

	$\ydiagram{1}^{\,3}$	$=$	$\ydiagram{1,1,1}$	$+$	$2\,\ydiagram{2,1}$	$+$	$3\,\ydiagram{1}$	$+$	$\ydiagram{3}$
$N=1$					$(-)\ \blacksquare$		$\blacksquare$		
$N=2$							$\ydiagram{1}_{\,\mathbf{2}}$		$\ydiagram{3}_{\,\mathbf{2}}$

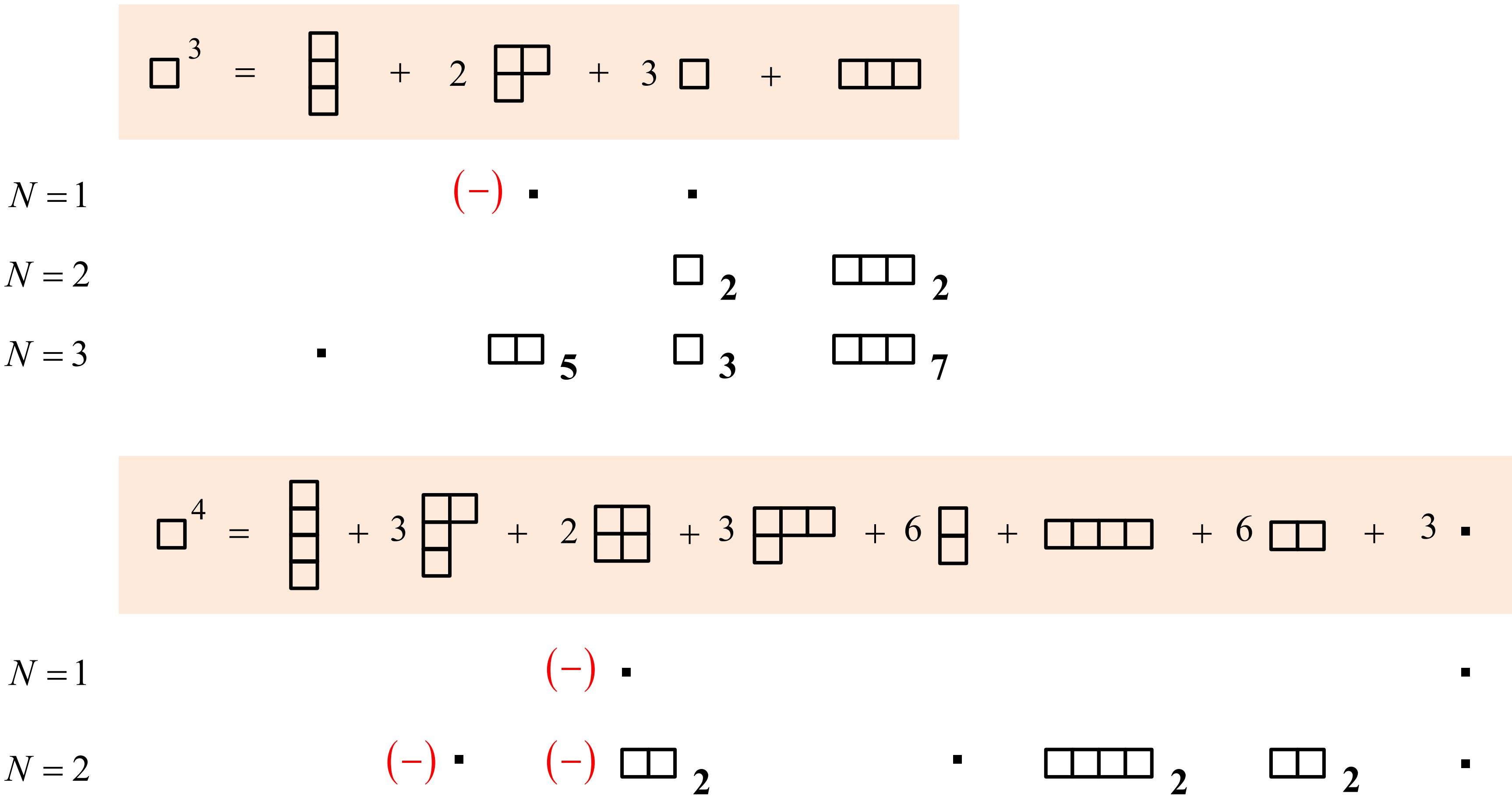
Examples of $O(N)$ Specializations



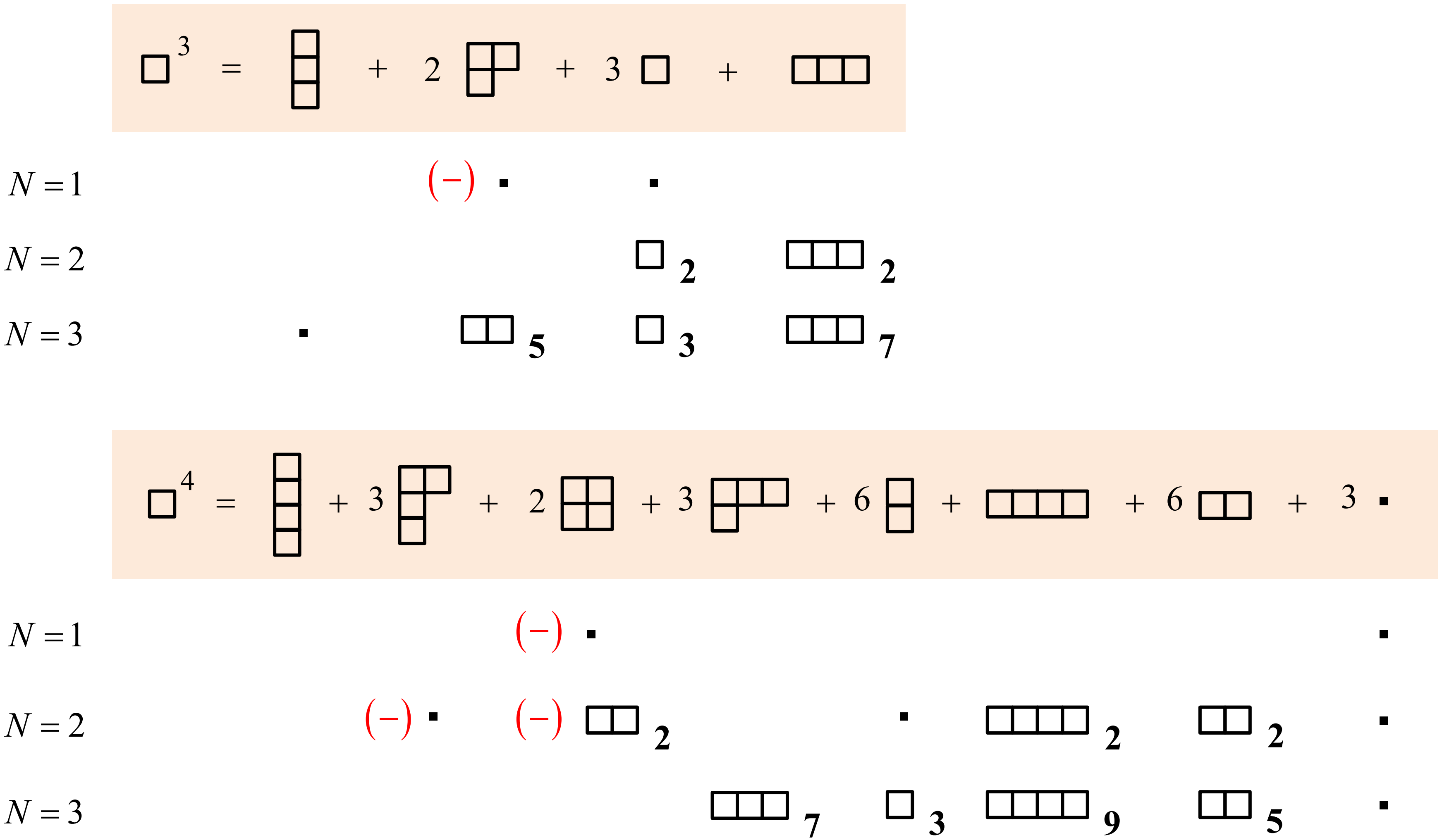
Examples of $O(N)$ Specializations



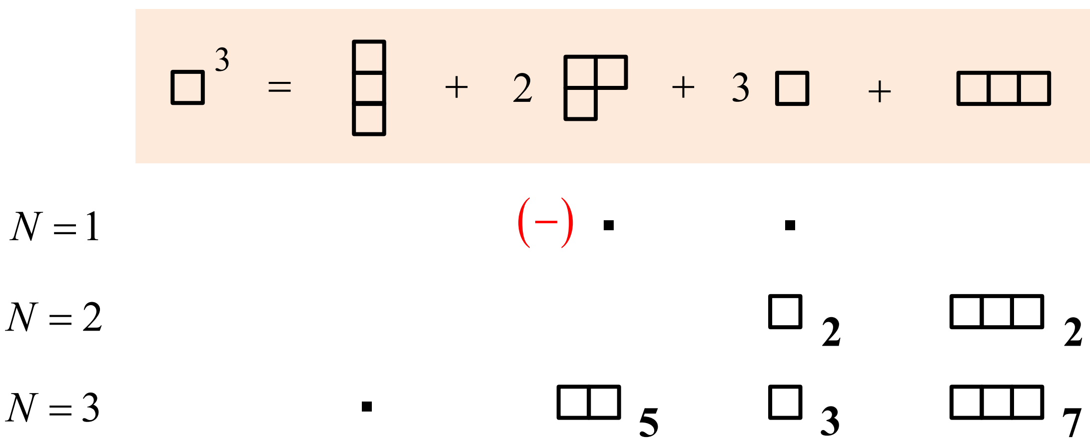
Examples of $O(N)$ Specializations



Examples of $O(N)$ Specializations



Examples of $O(N)$ Specializations

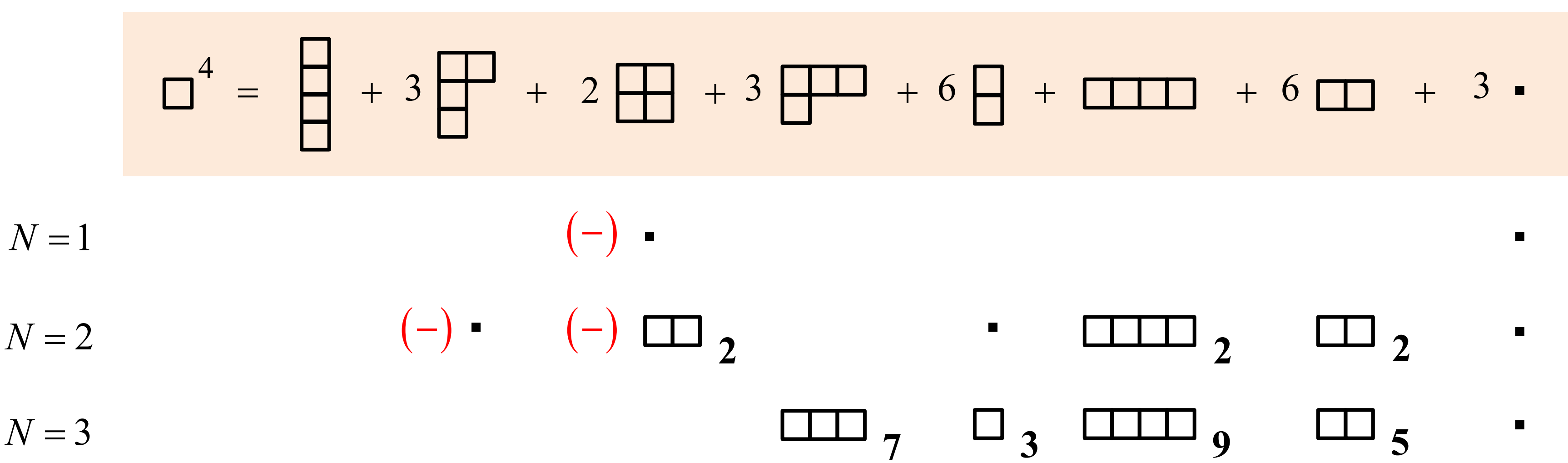


There are a lot of more complicated examples

$$R^{O(7)}_{(5,4,3,3,3,2,1)} = -R^{O(7)}_{(5,4,2)}$$

$$R^{O(7)}_{(5,4,3,3,1,1,1)} = +R^{O(7)}_{(5,4,2)}$$

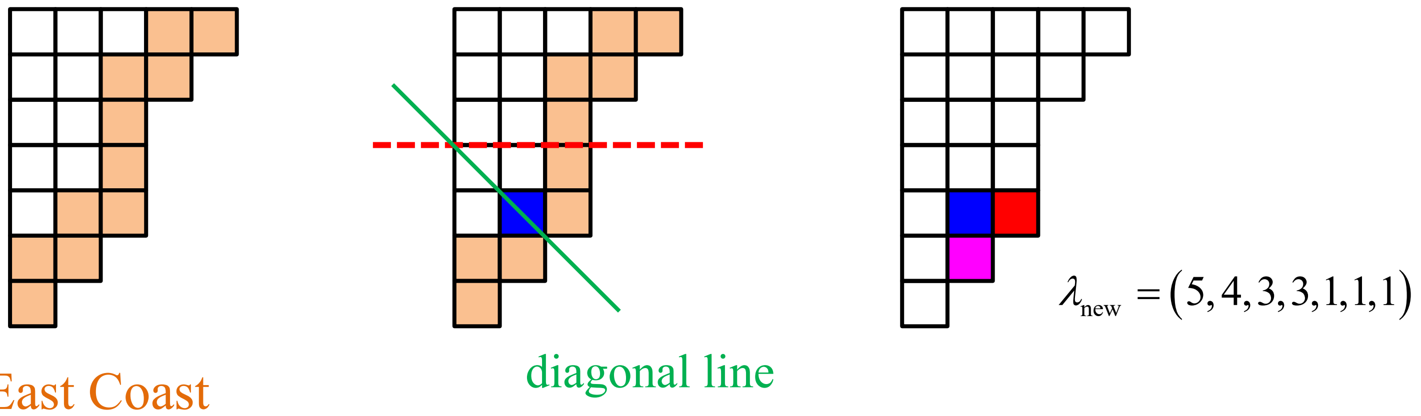
$$R^{O(8)}_{(5,4,3,3,3,3,1)} = -R^{O(8)}_{(5,4,3,2)}$$



Clipping Rules for Specializations

$O(N)$: East Coast Clipping

$\lambda = (5,4,3,3,3,2,1)$
at $N = 7$



Clipping Center : the box on the East Coast that is on the diagonal line

Clipping Patch Addition 1 : all (n_1) boxes that are strictly below the clipping center

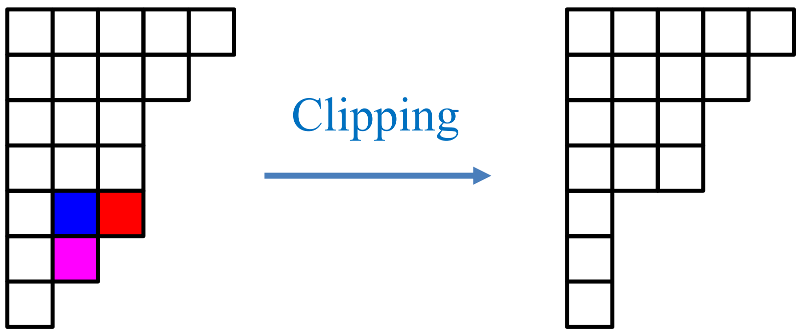
Clipping Patch Addition 2 : $n_1 + \frac{1}{2}\left[1+(-1)^N\right]$ boxes on the East Coast that are towards the east or north from the clipping center

Clipping Sign : $R_{\lambda}^{O(N)} = (-1)^{n_{\text{rows}}+N} R_{\lambda_{\text{new}}}^{O(N)}$

where n_{rows} is the number of rows that the clipping patch spans

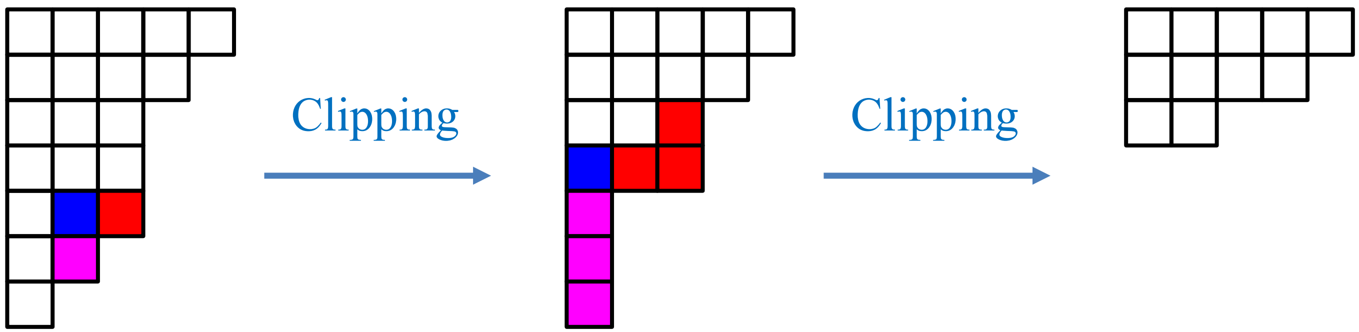
Clipping Rules for Specializations

$O(N)$: East Coast Clipping



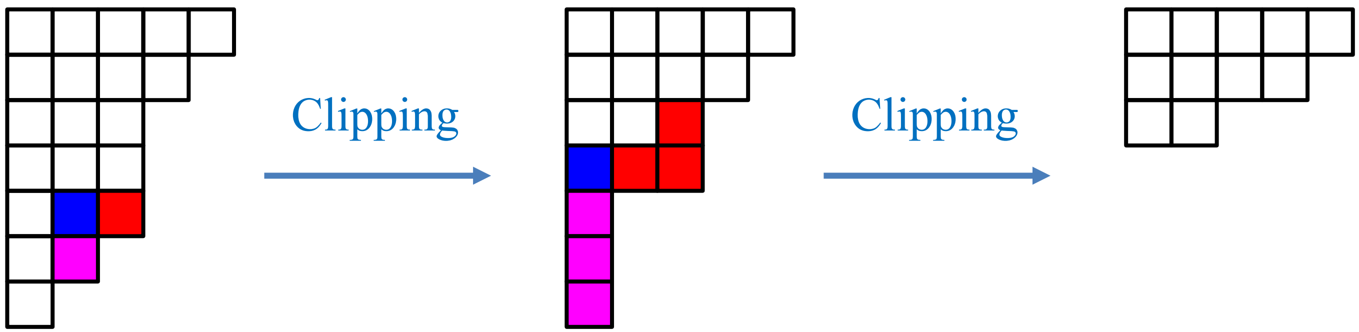
$$R^{O(7)}_{(5,4,3,3,3,2,1)} = -R^{O(7)}_{(5,4,3,3,1,1,1)}$$

$O(N)$: East Coast Clipping

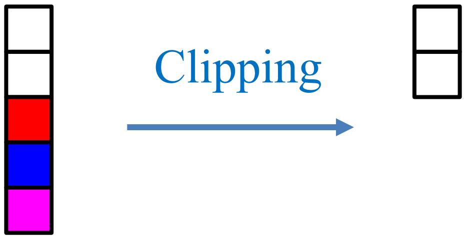


$$R^{O(7)}_{(5,4,3,3,3,2,1)} = -R^{O(7)}_{(5,4,3,3,1,1,1)} \qquad R^{O(7)}_{(5,4,3,3,1,1,1)} = +R^{O(7)}_{(5,4,2)}$$

$O(N)$: East Coast Clipping



$$R^{O(7)}_{(5,4,3,3,3,2,1)} = -R^{O(7)}_{(5,4,3,3,1,1,1)} \qquad R^{O(7)}_{(5,4,3,3,1,1,1)} = +R^{O(7)}_{(5,4,2)}$$



$$R^{O(7)}_{(1,1,1,1,1)} = R^{O(7)}_{(1,1)}$$

(not to be confused with) Racah-Speiser Algorithm

Racah-Speiser Algorithm

Anonymous Report 2 on 2024-5-10 *(Invited Report)*

Report

Generally I think the paper is mostly acceptable. However while the word degeneracy is no longer in the title it appears frequently in the introduction. I thought it was agreed that what washing described was not degeneracy in the usual sense but a necessary consistency condition. In my view the introductory paragraphs should be modified to reflect this

Secondly the results in (14a,b,c) and in Table II are a direct reflection of the Racah Speiser algorithm. For what it is worth a concise discussion of this is given in an appendix to hep-th/0209056. Similar results can be found in maths textbooks.

With some revision this paper would be acceptable.

Racah-Speiser Algorithm

$$\lambda_1 \otimes \lambda_2 = \bigoplus_{\lambda} m_{12}^{\lambda} \lambda$$

$\downarrow$ all weights in rep λ_2

$$\{\mu_i\} \rightarrow \{\tau_i = \mu_i + \lambda_1\}: \quad \text{A set of candidate weights}$$

(not to be confused with) Racah-Speiser Algorithm

Racah-Speiser Algorithm

$$\lambda_1 \otimes \lambda_2 = \bigoplus_{\lambda} m_{12}^{\lambda} \lambda$$

$\downarrow$ all weights in rep λ_2

$$\{\mu_i\} \to \{\tau_i = \mu_i + \lambda_1\}:$$ A set of candidate weights

$$\begin{cases} \tau_i \to (-1)^w \eta & \text{if } \rho + \tau_i = w(\rho + \eta) \text{ makes } \eta \text{ is dominant} \\ \tau_i \to 0 & \text{otherwise} \end{cases}$$

Racah-Speiser Algorithm

$$\lambda_1 \otimes \lambda_2 = \bigoplus_{\lambda} m_{12}^{\lambda}\, \lambda$$

$\downarrow$ all weights in rep λ_2

$$\begin{cases} \tau_i \to (-1)^w \eta & \text{if } \rho + \tau_i = w(\rho+\eta) \text{ makes } \eta \text{ is dominant} \\ \tau_i \to 0 & \text{otherwise} \end{cases}$$

$\{\mu_i\} \to \{\tau_i = \mu_i + \lambda_1\}$: A set of candidate weights

- An $SU(2)$ example

$$3 \otimes \tfrac{1}{2} = \tfrac{7}{2} \oplus \tfrac{5}{2}$$

$$\downarrow$$

$$\left\{\tfrac{1}{2}, -\tfrac{1}{2}\right\} \to \left\{\tfrac{7}{2}, \tfrac{5}{2}\right\}$$

Racah-Speiser Algorithm

$$\lambda_1 \otimes \lambda_2 = \bigoplus_{\lambda} m_{12}^{\lambda} \lambda$$

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$\{\mu_i\} \to \{\tau_i = \mu_i + \lambda_1\}$: A set of candidate weights

- An $SU(2)$ example

$$3 \otimes \tfrac{1}{2} = \tfrac{7}{2} \oplus \tfrac{5}{2}$$

$\downarrow$

$$\left\{\tfrac{1}{2}, -\tfrac{1}{2}\right\} \to \left\{\tfrac{7}{2}, \tfrac{5}{2}\right\}$$

$$\tfrac{1}{2} \otimes 3 = \tfrac{7}{2} \oplus \tfrac{5}{2}$$

$\downarrow$

$$\{3, 2, 1, 0, -1, -2, -3\} \to \left\{\tfrac{7}{2}, \tfrac{5}{2}, \tfrac{3}{2}, \tfrac{1}{2}, \underset{\uparrow}{\overset{0}{-\tfrac{1}{2}}}, \underset{\uparrow}{\overset{-\frac{1}{2}}{-\tfrac{3}{2}}}, \underset{\uparrow}{\overset{-\frac{3}{2}}{-\tfrac{5}{2}}}\right\}$$

Racah-Speiser Algorithm

$$\lambda_1 \otimes \lambda_2 = \bigoplus_{\lambda} m_{12}^{\lambda}\, \lambda$$

$\big\downarrow$ all weights in rep λ_2

$$\begin{cases} \tau_i \to (-1)^w \eta & \text{if } \rho + \tau_i = w(\rho+\eta) \text{ makes } \eta \text{ is dominant} \\ \tau_i \to 0 & \text{otherwise} \end{cases}$$

$$\{\mu_i\} \to \{\tau_i = \mu_i + \lambda_1\}: \quad \text{A set of candidate weights}$$

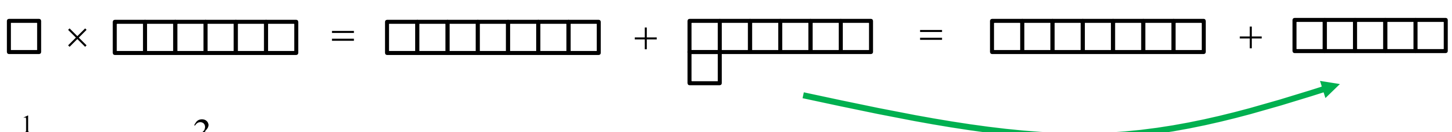
- An $SU(2)$ example

$$3 \otimes \tfrac{1}{2} = \tfrac{7}{2} \oplus \tfrac{5}{2} \qquad\qquad \tfrac{1}{2} \otimes 3 = \tfrac{7}{2} \oplus \tfrac{5}{2}$$

$$\big\downarrow \qquad\qquad\qquad \big\downarrow$$

$$\left\{\tfrac{1}{2}, -\tfrac{1}{2}\right\} \to \left\{\tfrac{7}{2}, \tfrac{5}{2}\right\} \qquad \{3, 2, 1, 0, -1, -2, -3\} \to \left\{\tfrac{7}{2}, \tfrac{5}{2}, \tfrac{3}{2}, \tfrac{1}{2}, \underset{\overset{\uparrow}{0}}{-\tfrac{1}{2}}, \underset{\overset{\uparrow}{-\frac{1}{2}}}{-\tfrac{3}{2}}, \underset{\overset{\uparrow}{-\frac{3}{2}}}{-\tfrac{5}{2}}\right\}$$

- Specialization is something different: High rank rep to low rank rep



$\tfrac{1}{2}$ 3

Example Implication of Negative Specializations

Degeneracies in dim-6 SMEFT RGE

$$Q_{qq}^{(1)} = \left(\bar{q}_p \gamma_\mu q_r\right)\left(\bar{q}_s \gamma^\mu q_t\right)$$

$$Q_{qq}^{(3)} = \left(\bar{q}_p \gamma_\mu \sigma^a q_r\right)\left(\bar{q}_s \gamma^\mu \sigma^a q_t\right)$$

$$Q_{ll} = \left(\bar{l}_p \gamma_\mu l_r\right)\left(\bar{l}_s \gamma^\mu l_t\right)$$

$$Q_{uu} = \left(\bar{u}_p \gamma_\mu u_r\right)\left(\bar{u}_s \gamma^\mu u_t\right)$$

$$Q_{dd} = \left(\bar{d}_p \gamma_\mu d_r\right)\left(\bar{d}_s \gamma^\mu d_t\right)$$

$$SU\left(N = n_g\right): \quad \mathrm{sym}^2\left(N \times \bar{N}\right) = s \times \bar{s} + a \times \bar{a} = \mathbf{1} + \mathbf{1} + \mathrm{adj} + \mathrm{adj} + \bar{s}s + \bar{a}a$$

$$s \equiv \Box\Box \quad , \quad a \equiv \begin{array}{c}\Box \\ \Box\end{array}$$

Alonso, Jenkins, Manohar, and Trott, arXiv: 1312.2014

Degeneracies in dim-6 SMEFT RGE

$$Q_{qq}^{(1)} = \left(\bar{q}_p \gamma_\mu q_r\right)\left(\bar{q}_s \gamma^\mu q_t\right)$$
$$Q_{qq}^{(3)} = \left(\bar{q}_p \gamma_\mu \sigma^a q_r\right)\left(\bar{q}_s \gamma^\mu \sigma^a q_t\right)$$
$$Q_{ll} = \left(\bar{l}_p \gamma_\mu l_r\right)\left(\bar{l}_s \gamma^\mu l_t\right)$$
$$Q_{uu} = \left(\bar{u}_p \gamma_\mu u_r\right)\left(\bar{u}_s \gamma^\mu u_t\right)$$
$$Q_{dd} = \left(\bar{d}_p \gamma_\mu d_r\right)\left(\bar{d}_s \gamma^\mu d_t\right)$$

$$\begin{cases} \dim(\mathbf{1}) = 1 \\ \dim(\mathrm{adj}) = N^2 - 1 \\ \dim(\bar{s}s) = \dfrac{N^2(N-1)(N+3)}{4} \\ \dim(\bar{a}a) = \dfrac{N^2(N+1)(N-3)}{4} \end{cases}$$

$$\dim(Q) = \frac{1}{2} N^2 \left(N^2 + 1\right)$$

$$SU\left(N = n_g\right): \quad \mathrm{sym}^2\left(N \times \bar{N}\right) = s \times \bar{s} + a \times \bar{a} = \mathbf{1} + \mathbf{1} + \mathrm{adj} + \mathrm{adj} + \bar{s}s + \bar{a}a$$

Alonso, Jenkins, Manohar, and Trott, arXiv: 1312.2014

$$s \equiv \square\square \quad , \quad a \equiv \begin{array}{c}\square\\[-4pt]\square\end{array}$$

Degeneracies in dim-6 SMEFT RGE

$$Q_{qq}^{(1)} = \left(\bar{q}_p \gamma_\mu q_r\right)\left(\bar{q}_s \gamma^\mu q_t\right)$$
$$Q_{qq}^{(3)} = \left(\bar{q}_p \gamma_\mu \sigma^a q_r\right)\left(\bar{q}_s \gamma^\mu \sigma^a q_t\right)$$
$$Q_{ll} = \left(\bar{l}_p \gamma_\mu l_r\right)\left(\bar{l}_s \gamma^\mu l_t\right)$$
$$Q_{uu} = \left(\bar{u}_p \gamma_\mu u_r\right)\left(\bar{u}_s \gamma^\mu u_t\right)$$
$$Q_{dd} = \left(\bar{d}_p \gamma_\mu d_r\right)\left(\bar{d}_s \gamma^\mu d_t\right)$$

$$\begin{cases} \dim(\mathbf{1}) = 1 \\ \dim(\mathrm{adj}) = N^2 - 1 \\ \dim(\bar{s}s) = \dfrac{N^2(N-1)(N+3)}{4} \\ \dim(\bar{a}a) = \dfrac{N^2(N+1)\color{red}{(N-3)}}{4} \end{cases}$$

$$\dim(Q) = \frac{1}{2} N^2 \left(N^2 + 1\right)$$

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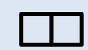
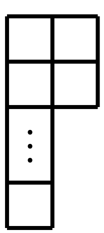
Alonso, Jenkins, Manohar, and Trott, arXiv: 1312.2014

$s \equiv$  , $a \equiv$ 

$\bar{a}a$



$N-2$

$$a \times \bar{a} = \mathbf{1} + \mathrm{adj} + \bar{a}a$$

continued from integer $N \geq 4$

Example Implication of Negative Specializations

Degeneracies in dim-6 SMEFT RGE

$$Q_{qq}^{(1)} = \left(\bar{q}_p \gamma_\mu q_r\right)\left(\bar{q}_s \gamma^\mu q_t\right)$$

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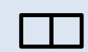
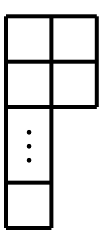
$s \equiv$  , $a \equiv$ 

$\bar{a}a$



$N-2$

$$a \times \bar{a} = \mathbf{1} + \mathrm{adj} + \bar{a}a$$

continued from integer $N \geq 4$

$N = 1$ $\quad (-)\,\mathbf{1}$

Degeneracies in dim-6 SMEFT RGE

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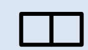
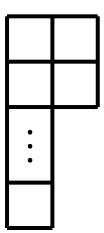
$s \equiv$  , $a \equiv$ 

$\bar{a}a$



$N-2$

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continued from integer $N \geq 4$

$N = 1$ $\quad (-)\,\mathbf{1}$

$N = 2$ $\quad (-)\,\mathrm{adj}$

Degeneracies in dim-6 SMEFT RGE

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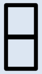
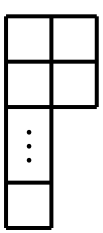
$s\equiv$  , $a\equiv$ 

$\bar{a}a$



$N-2$

$$a\times\bar{a}=\mathbf{1}+\mathrm{adj}+\bar{a}a$$

continued from integer $N\geq 4$

$N=1$ $\quad(-)\mathbf{1}$

$N=2$ $\quad(-)\ \mathrm{adj}$

$N=3$ $\quad 0$

Example Implication of Negative Specializations

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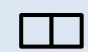
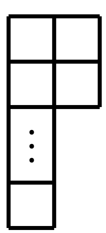
$s \equiv$  , $a \equiv$ 

$\bar{a}a$



$N-2$

$$a \times \bar{a} = \mathbf{1} + \text{adj} + \bar{a}a$$

continued from integer $N \geq 4$

$N = 1$ $\quad (-)\,\mathbf{1}$ $\quad \gamma_{\bar{a}a} = \gamma_{\mathbf{1}}$

$N = 2$ $\quad (-)$ adj

$N = 3$ $\quad 0$

Example Implication of Negative Specializations

Degeneracies in dim-6 SMEFT RGE

$$Q_{qq}^{(1)}=\left(\bar{q}_p\gamma_\mu q_r\right)\left(\bar{q}_s\gamma^\mu q_t\right)$$
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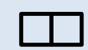
$s\equiv$  , $a\equiv$ 

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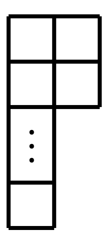

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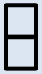
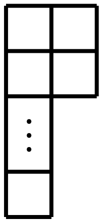
$s \equiv$  , $a \equiv$ 

$\bar{a}a$



$N-2$

$$a \times \bar{a} = \mathbf{1} + \text{adj} + \bar{a}a$$

continued from integer $N \geq 4$

$N = 1$	$\color{red}{(-)}\,\mathbf{1}$
$N = 2$	$\color{red}{(-)}\,\text{adj}$
$N = 3$	0

$$\boxed{\begin{aligned} \gamma_{\bar{a}a} &= \gamma_{\mathbf{1}} \\ \gamma_{\bar{a}a} &= \gamma_{\text{adj}} \end{aligned}}$$

Degeneracies in anomalous dimensions

Summary

- A natural definition (continuation) of the representation theory of $SU(N)$ and $O(N)$ at non-integer N is to take the large integer N limit
- In this point of view, low integer N are special
- The "specializations" encode (a class of) evanescence
- For $O(N)$ specializations, we have discovered an efficient Young diagram clipping rule (which is not to be confused with Racah-Speiser algorithm)
- Negative specializations indicate degeneracies