SYSTEMATIC IMPROVEMENT OF x-DEPENDENT PARTON DISTRIBUTION FUNCTION IN LATTICE QCD CALCULATION.

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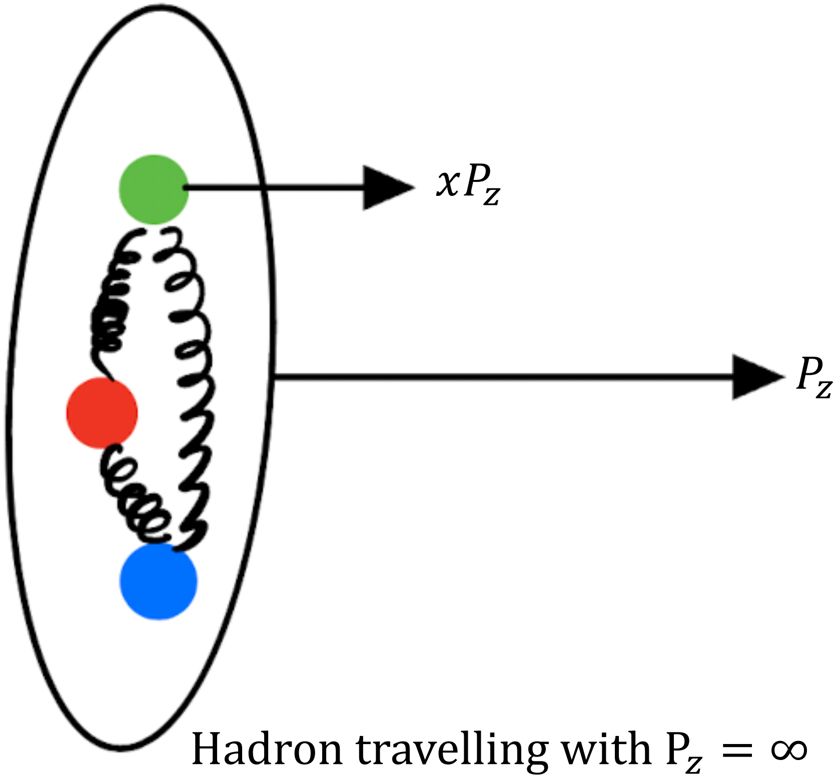
PARTON DISTRIBUTION FUNCTION

- Probability of finding a parton with momentum fraction in the interval $[x, x + dx]$ is $f(x)dx$ where $f(x)$ is the Parton Distribution Function (PDF).

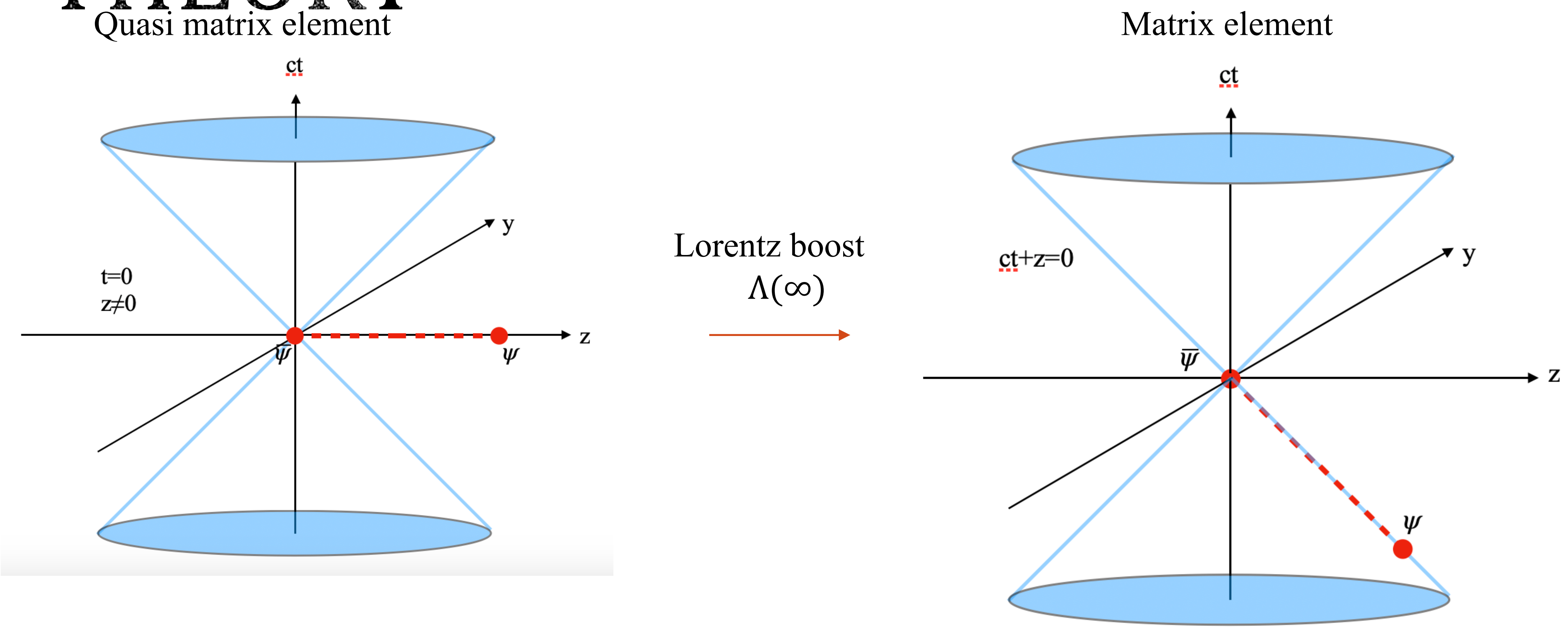
- Can be probed experimentally via deep inelastic scattering (DIS).

- The advent of large-momentum effective theory (LaMET)[1] in 2013 allowed PDFs to be computed on the Euclidean lattice.



[1] X. Ji. PRL, **110**, (2013), 262002

LARGE-MOMENTUM EFFECTIVE THEORY[1]



[1] X. Ji. PRL, **110**, (2013), 262002

FROM THE LATTICE

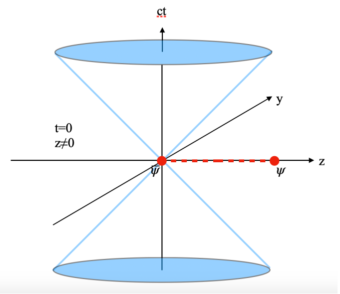
Fourier Transform:

$$\tilde{f}(x,P_z) = \int_{-\infty}^{\infty} \frac{P_z dz}{2\pi} e^{ixzP_z} h^R(z,P_z)$$

Renormalize in hybrid-ratio scheme[2]

$h^B(z,P_z)$ → $h^R(z,P_z)$ → $h^R(z\to\infty,P_z)$ → $\tilde{f}(x,P_z)$ → $f(x,\mu)$

Bare MEs from MILC collab[1].



Extrapolate to infinite distance[3]:

$$h^R(z\to\infty,P_z) \to \frac{Ae^{-mz}}{|zP_z|^d}$$

Fitting params.

Lightcone Matching:

$$f(x,\mu) = \mathcal{K}^{-1}(x,y,\mu,P_z) \otimes \tilde{f}(y,P_z)$$

[1]Bazavov et. al., PRD, **87**, 054505, 2013
[2]X. Ji et. al. NPB, **964**, 115311. 2021
[3]Gao et. al. PRL, **128**, 142003, 2022

LATTICE INFORMATION

- Lattice spacing, $a \approx 0.09$ fm.
- Box width, $L = 64a \approx 5.76$ fm.
- Physical pion mass, $m_\pi \approx 130\ MeV$.
- $N_f = 2 + 1 + 1$ flavors of highly improved staggered quarks.
- 501,760 measurements from 1960 lattice configurations.
- Boost momentum $P_z = 10 \times \frac{2\pi}{L} \approx 2.2\ GeV$.

- We will demonstrate the Renormalization Group Resummation (RGR)[1] and Leading Renormalon Resummation (LRR)[2] improvements on nucleon PDFs

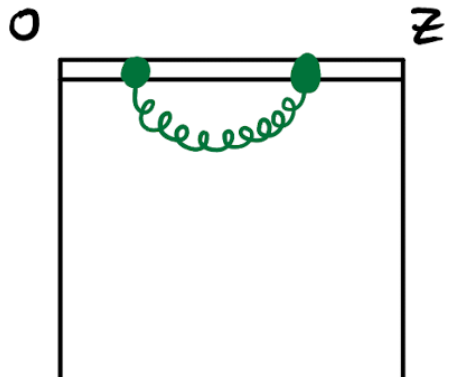
[1]Su, *JH* et. al. NPB, **991**, 116201, 2023

[2]Zhang, *JH* et. al. PLB, **844**, 138081

HYBRID-RATIO RENORMALIZATION

- Renormalize in the ratio scheme at short-distances ($z < z_s \approx 0.2\ fm$).
- Remove the linear divergence and renormalon ambiguity at large distances ($z \geq z_s$).

$$h^R(z, P_z) = \begin{cases} N \dfrac{h^B(z, P_z)}{h^B_\pi(z, P_z = 0)} & z < z_s \\ N e^{(\delta m + m_0)(z - z_s)} \dfrac{h^B(z, P_z)}{h^B_\pi(z_s, P_z = 0)} & z \geq z_s \end{cases}$$



- N is a constant s.t. $h^R(z = 0, P_z) = 1$.
- $h^B_\pi(z, P_z = 0)$: zero-momentum pion MEs from Lattice Parton Collab.[1]
- Linear divergence determined by fitting[2] $h^B(z, P_z)$ to $Ae^{-\delta m \times z}$.
- Renormalon ambiguity determined by demanding that $h^R(z, P_z)$ agrees with operator product expansion for $z \lesssim 0.2$ fm: Wilson coefficients[3].

[1]Huo et. al. NPB, **969**, 115443, 2021
[2]X. Ji et. al. NPB, **964**, 115311. 2021
[3]Zhang, *JH* et. al. PLB**, 844**, 138081

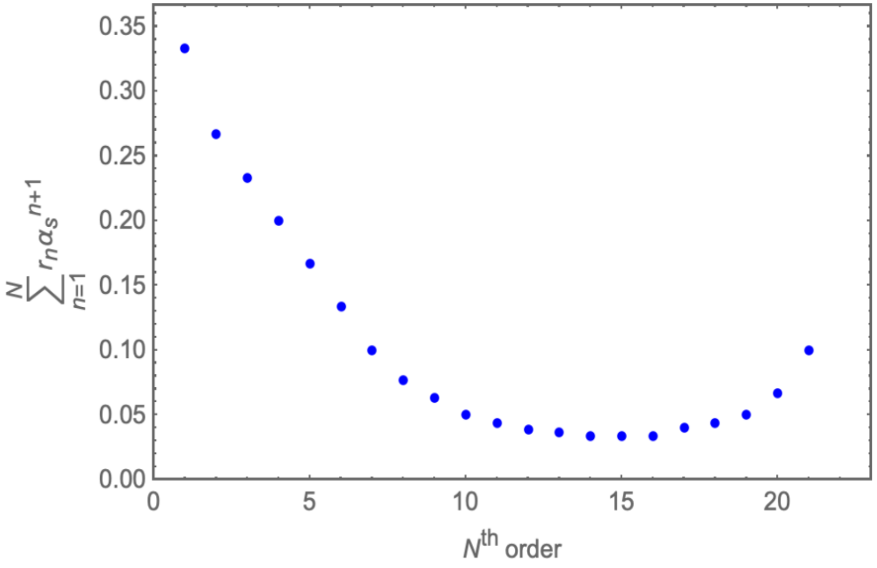
RENORMALIZATION GROUP RESUMATION

- Method of resumming large logarithms that appear in the renormalization and matching process.
- Improve Wilson coeffs.
- $C_0(z,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{5}{2}\right)$[1,2]. Unpolarized Wilson coeff. at NLO.
- Set initial scale to $\mu = c' \times \frac{2e^{-\gamma_E}}{z}$ before evolving with renormalization group.
- $c' = 1.0$ corresponds to central value. Upper and lower error bars derived from $c' = 0.75$ and $c' = 1.5$.
- The range $c' \in [0.75, 1.5]$ corresponds to a $\sim 15\%$ variation about $\alpha_s(\mu = 2.0\ GeV)$[3].

[1]Izubuchi et. al. PRD, **98**, 056004, 2018
[2]Y. Ji et. al. arXiv:2212.14415
[3]Zhang, *JH* et. al. PLB, **844**, 138081, 2023

LEADING REMORMALON RESUMMATION

- Perturbation series contain divergences.
- Divergence tends to emerge when we expand to order $n \sim 1/\alpha_s(\mu)$.
- When we apply RGR, this becomes important due to the variable energy scale, $\frac{2e^{-\gamma_E}}{z}$.
- Apply LRR to Wilson coeffs. to account for this divergence[1].



[1]Zhang, *JH* et. al. PLB, **844**, 138081

RGR (MATCHING)

- In the matching kernel, $\mathcal{K}(x, y, \mu, P_z)$, logarithms appear again.
- $\ln\left(\frac{\mu^2}{4x^2P_z^2}\right) \in \mathcal{K}(x, y, \mu, P_z)$[1,2].
- This time, set initial scale to $\mu = c' \times 2xP_z$ before evolving with DGLAP equation to desired scale: $\mu = 2.0\ GeV$[3].
- Same c' values as used in the renormalization process.
- DGLAP equation breaks down for $|x| \lesssim 0.2$ where α_s becomes non-perturbative.

[1]Chen et. al. PRL, **126**, 072002, 2021
[2]Li et. al. PRL, **126**, 072001, 2021
[3]Su, JH et. al. NPB, 991, 116201. 2023

LABELLING

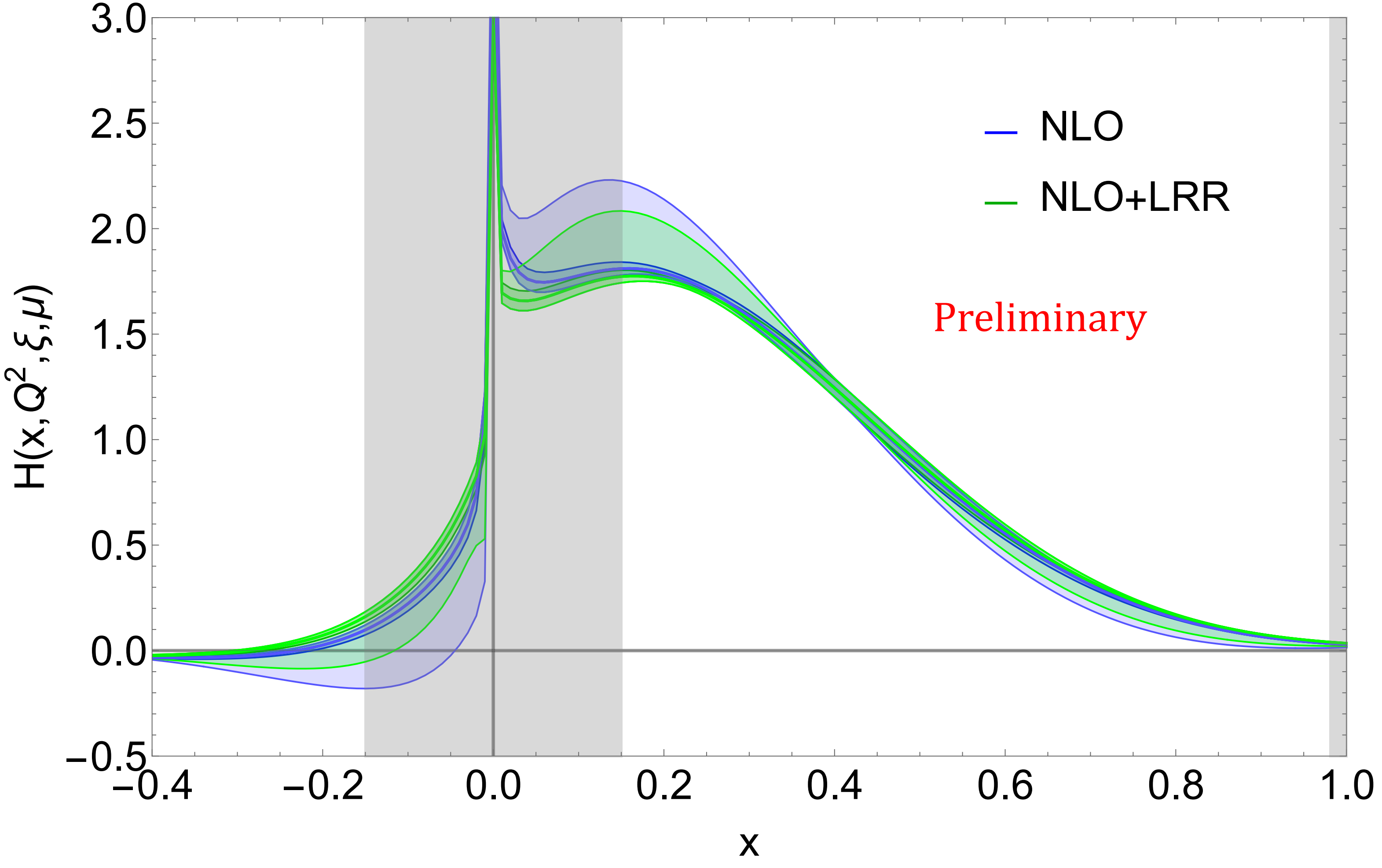
Renormalization & Matching	No additions	RGR only	LRR only	RGR and LRR
1st order	NLO	NLO×RGR	NLO+LRR	(NLO+LRR)×RGR
2nd order	NNLO	NNLO×RGR	NNLO+LRR	(NNLO+LRR)×RGR

Renormalize lightcone PDF in $\overline{MS}$ scheme at $\mu = 2.0$ GeV.
Unpolarized nucleon PDF:

$$f(x,\mu) = \begin{cases} f_u(x,\mu) - f_d(x,\mu), & x > 0 \\ f_{\bar{d}}(x,\mu) - f_{\bar{u}}(x,\mu), & x < 0 \end{cases}$$

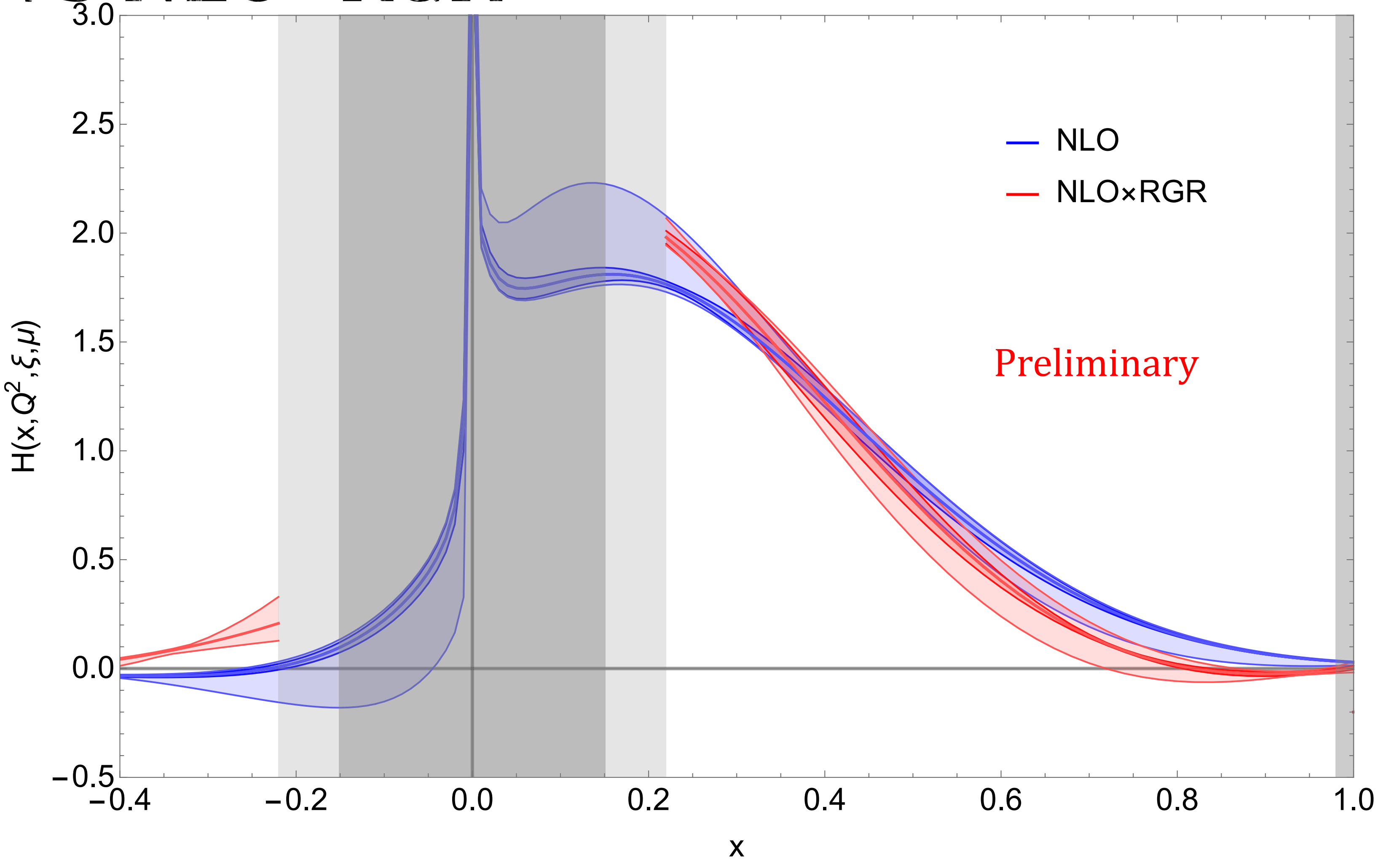
"quark region"
"antiquark region"

NLO VS NLO+LRR



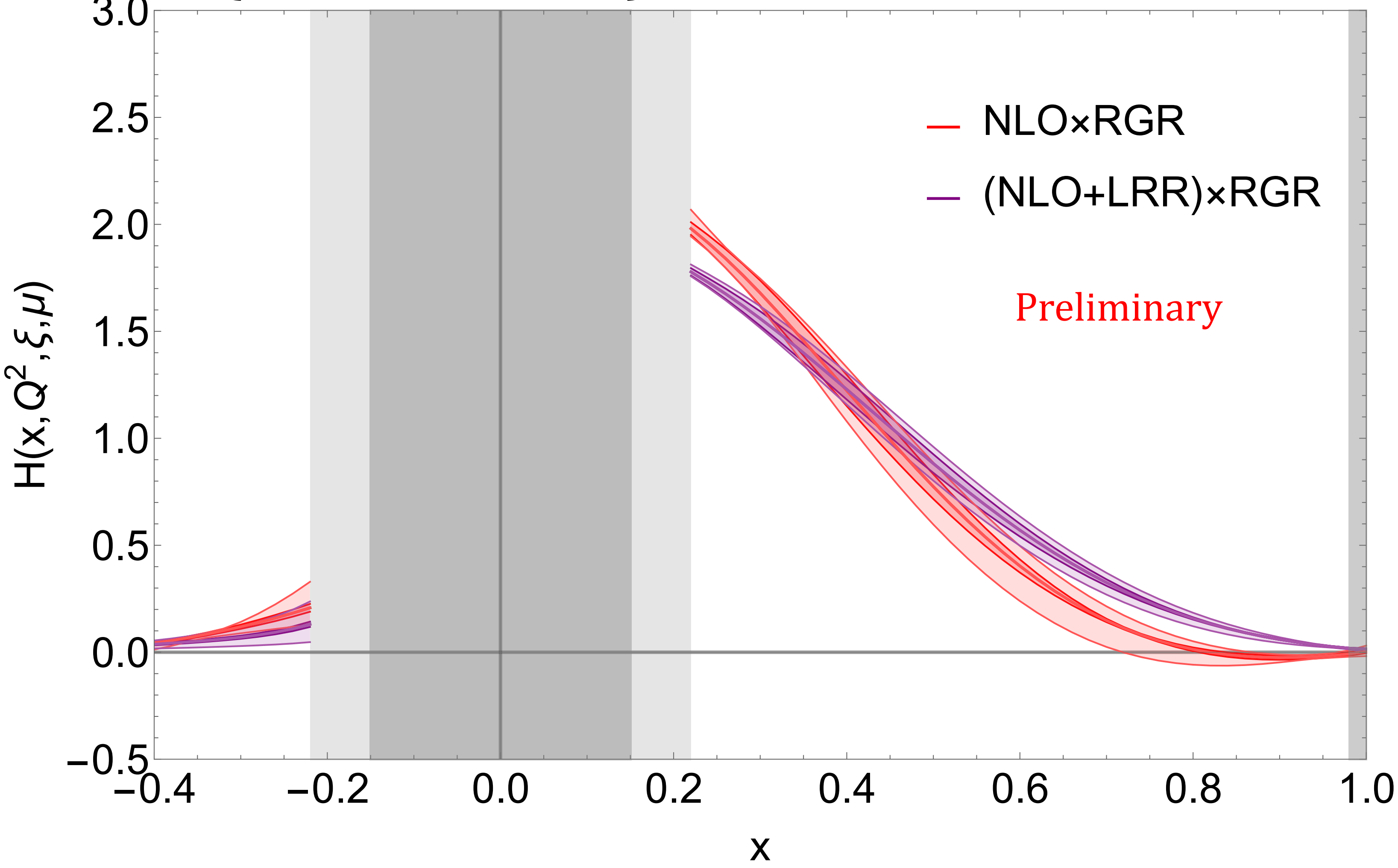
Not much change between the two schemes. Why bother, then, with the renormalon divergence?

NLO VS NLO×RGR



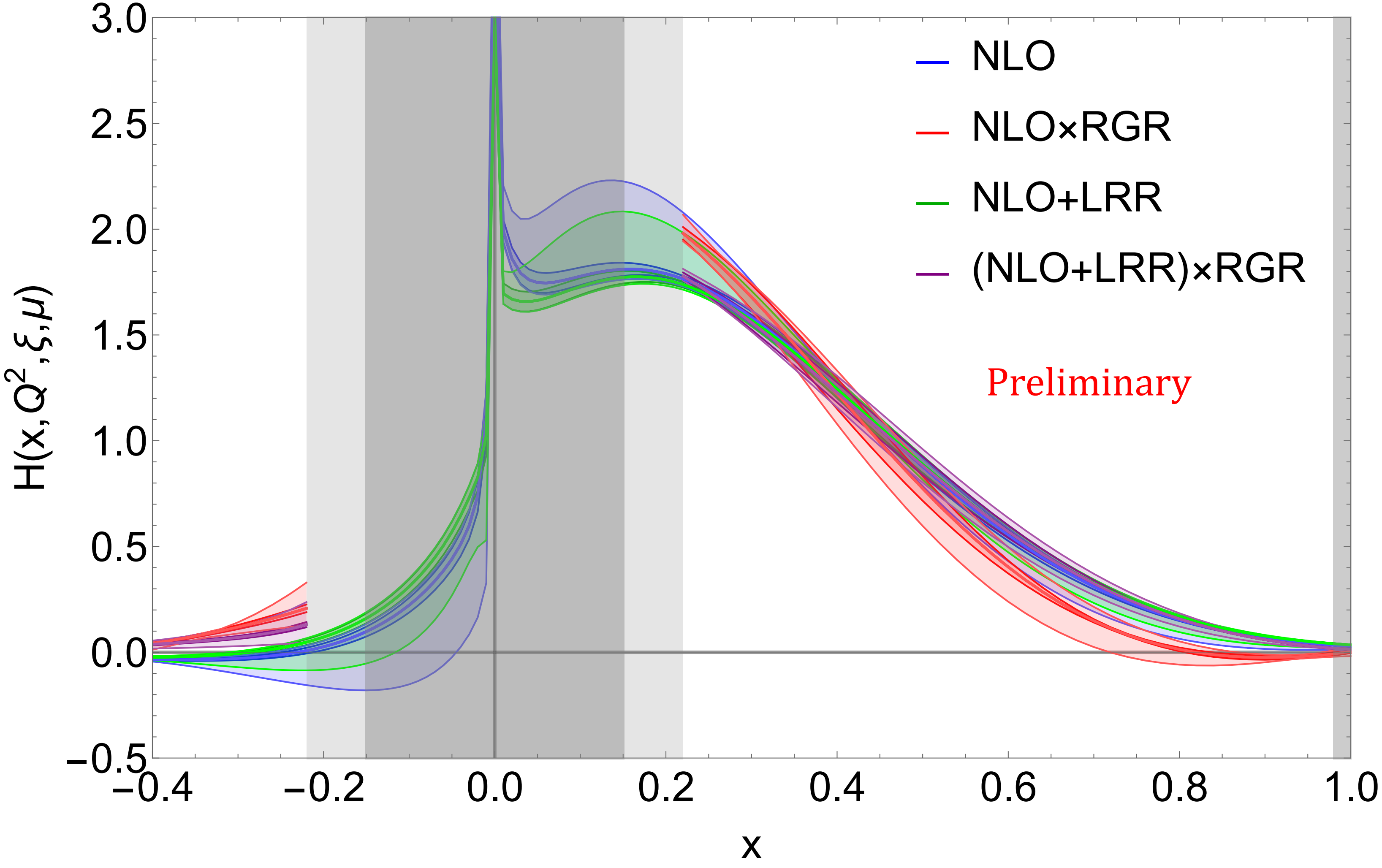
Significant difference between these two schemes. RGR is significant.

NLO×RGR VS (NLO+LRR)×RGR

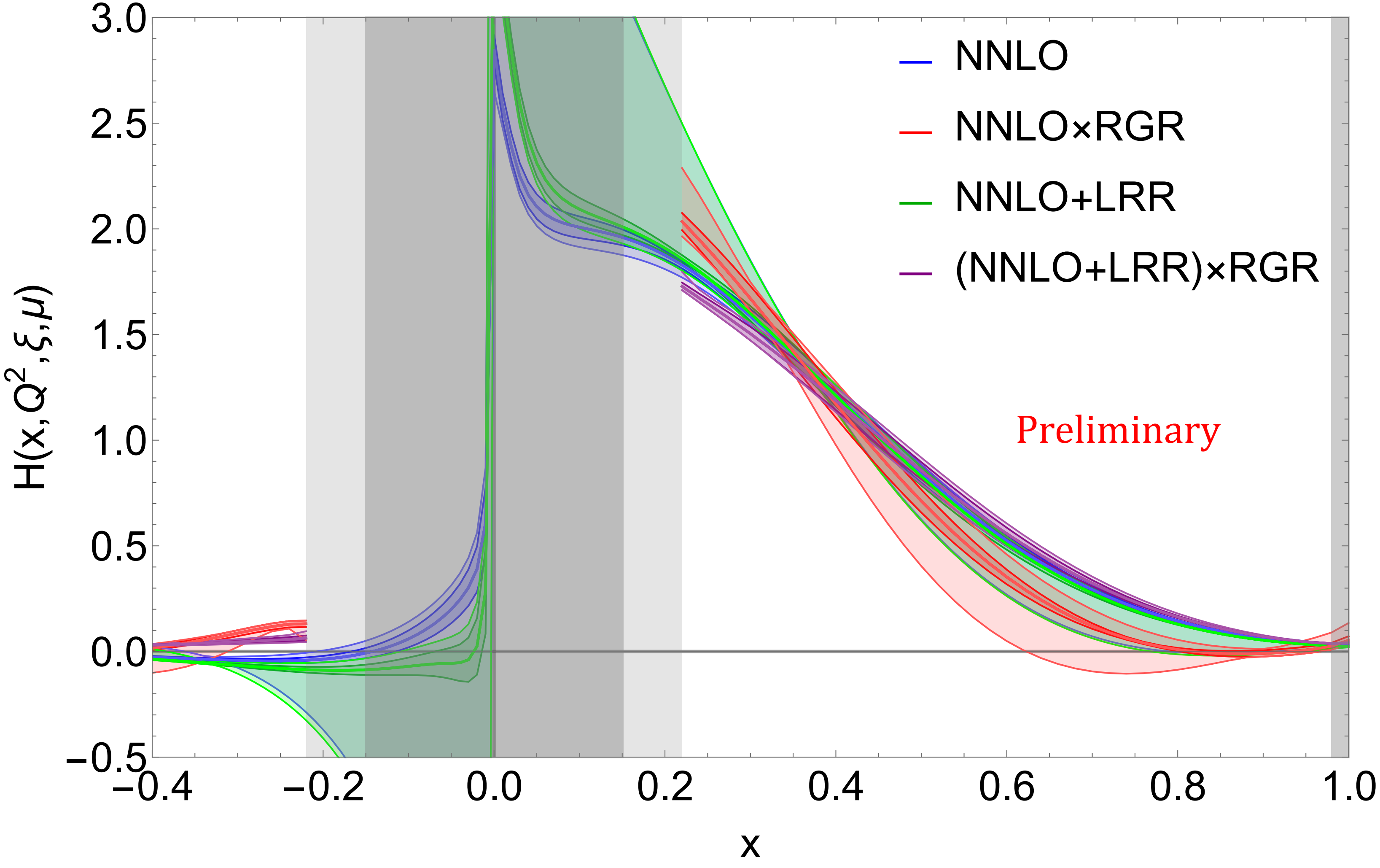


Significant difference between the two schemes. Reduced sys. and stat. errors.
Renormalon divergence is significant when RGR is used.

ALL NLO SCHEMES



ALL NNLO SCHEMES



Same behavior we see at NLO applies to a greater extent at NNLO

CONCLUSION

- Shown the first unpolarized nucleon isovector PDF matched to two loops with renormalization group resummation and leading renormalon resummation.
- Shown that the benefits afforded by RGR and LRR are much reduced systematics like in their original application to the pion PDF.
- Handling of systematic errors must keep pace with higher order renormalization and lightcone matching.