Pion Gluon Momentum Fraction and PDF Updates from Lattice QCD

11.09.2023

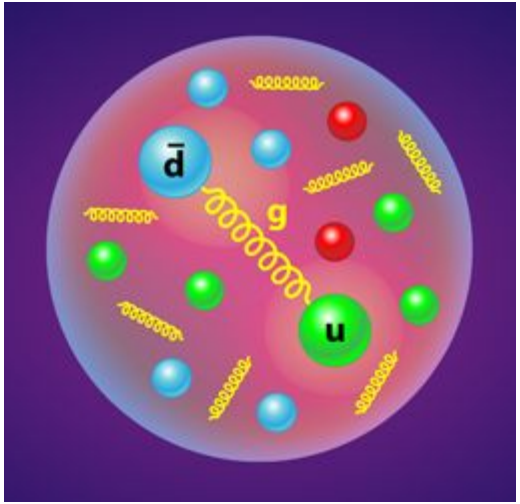
Bill Good

In collaboration with

Kinza Hasan, Allison Chevis, Huey-Wen Lin

Results from arXiv:2310.12034v1 [hep-lat]

Introduction to the Pion

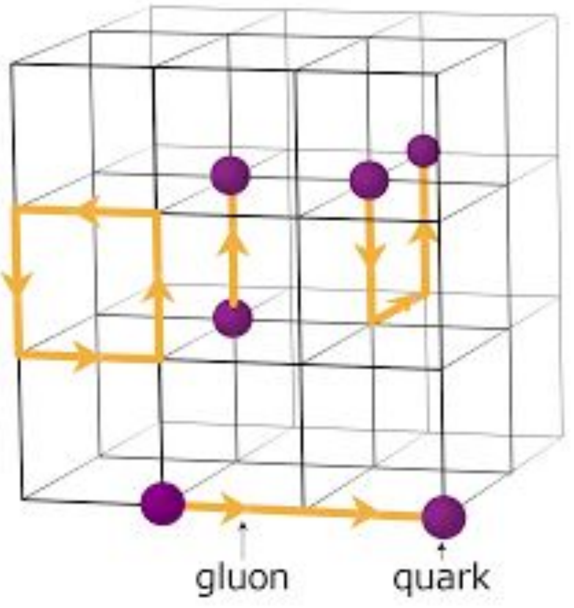


APS/Alan Stonebraker

- Understanding pion structure is important to furthering our knowledge of QCD
  - Pseudo-goldstone boson associated with chiral symmetry breaking, important to low energy interactions
  - Partonic structure comes out in high energy experiment
- It is difficult to study directly due to its short lifetime
- Future EIC, EicC, COMPASS++, and AMBER experiments will help reveal more information about the pion
- In the meantime we have global analyses of existing data and Lattice QCD
- We are interested in the gluon momentum fraction of the pion:

$$\langle x \rangle_g = \int_0^1 x g(x) dx$$

Lattice QCD



- LQCD is just QCD in discrete Euclidean space-time
    - Quarks exist on vertices and gluons act as links between the vertices
- We control pion mass, lattice spacing, and volume
    - These are the main sources of systematic uncertainties
    - We vary these parameters to extrapolate to the physical-continuum limit values of quantities
- Can measure operators on the lattice that relate to hadron structure quantities and functions such as PDFs, GPDs, TMDs, etc.
    - Measurements are done on top of a “QCD vacuum configuration”
    - Previous study used the pseudo-PDF method to calculate the pion gluon PDF divided out by the momentum fraction $xg(x)/\langle x \rangle_g$

A. V. Radyushkin, PRD 96:034025, 2017.
Z. Fan, et al. PLB 823:136778, 2021.

Lattice Details

Follana et al. PRD 75:054502, 2007.
A. Bazavov, et al. [MILC], PRD 82:074501, 2010.
A. Bazavov, et al. [MILC], PRD 87:054505, 2013.
A. Hasenfratz, et al., PRD 64:034504, 2001.

- Calculation carried out with $N_f = 2 + 1 + 1$ highly improved staggered quarks (HISQ) generated by MILC collaboration
- Wilson-clover fermions used in valence sector
- Lattice spacing $a \approx 0.09, 0.12, 0.15$ fm
- Valence quarks tuned to reproduce light and strange pion masses $M_\pi \approx 220, 310$ MeV and 690 MeV
- $O(10^5 - 10^6)$ 2pt correlator measurements over $O(10^3)$ configurations
- Gaussian momentum smearing on quark fields
- 5 steps of hypercubic smearing on the gluon fields

Calculating the Momentum Fraction

Calculate 2pt and 3pt correlators at various P_z

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↓

Fit $\langle 0|O_g|0\rangle = Z^{-1}\langle x\rangle_g$ for multiple P_z

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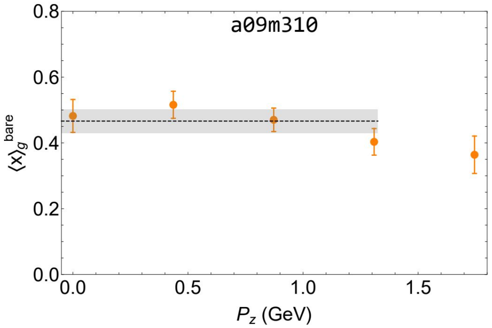
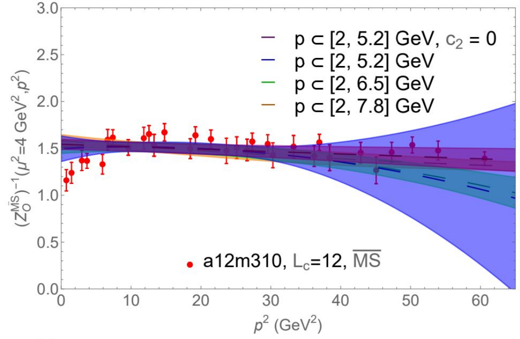
Constant fit over momentum



Calculating the Momentum Fraction

Z. Fan, et al. PRD 107:034505, 2023.



Calculate 2pt and 3pt correlators at various P_z

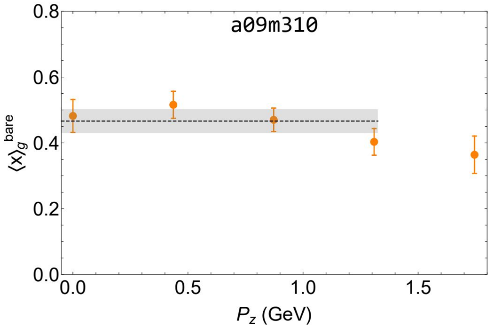
(Previous work)

Obtain renormalization coefficient $Z^{-1}(\mu)$

Fit $\langle 0|O_g|0\rangle = Z^{-1}\langle x\rangle_g$ for multiple P_z

Constant fit over momentum

Obtain each $\langle x\rangle_g(a, M_\pi, \mu)$



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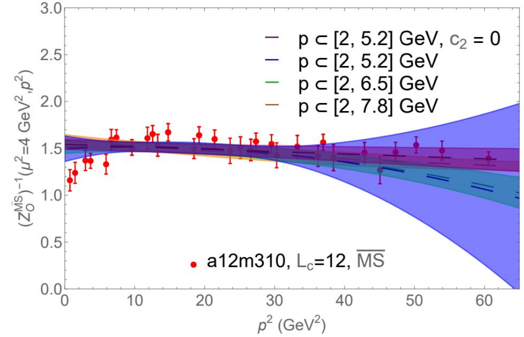
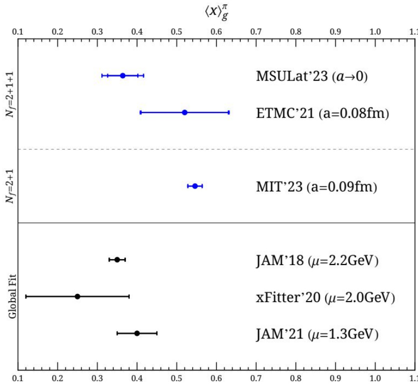
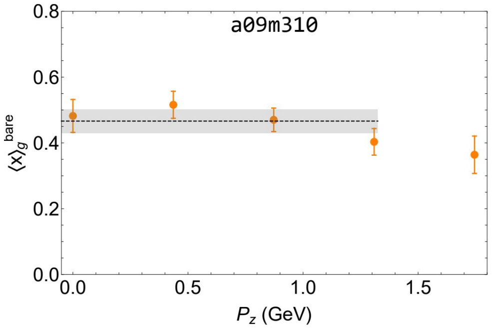
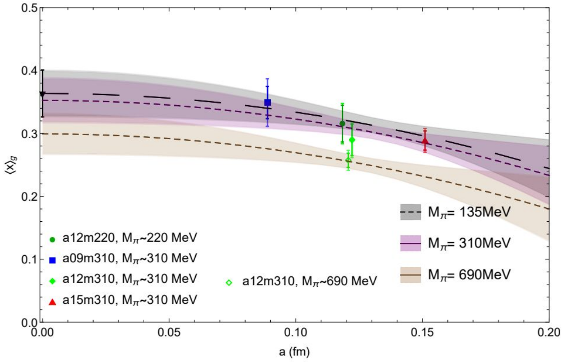


Obtain each $\langle x\rangle_g(a, M_\pi, \mu)$

Extrapolate to get $\langle x\rangle_g^{\text{cont.}} = \langle x\rangle_g(\mu)$



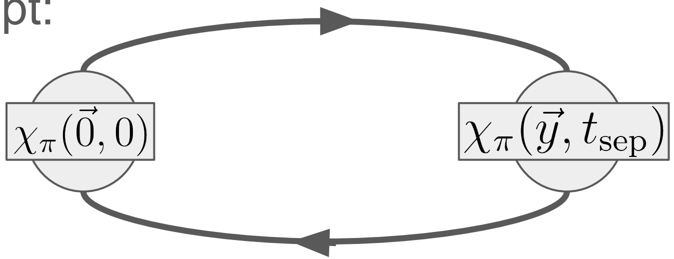




Correlators

$$\chi_\pi(\vec{y},t) = \overline{q}_1\gamma_5 q_2$$

2pt:



$$C_\pi^{2\text{pt}}(P_z;t) = \int d^3y\, e^{-iy_zP_z} \left\langle \chi_\pi(\vec{y},t) | \chi_\pi(\vec{0},0) \right\rangle$$

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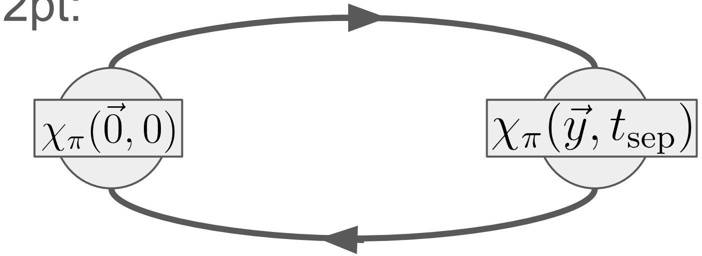
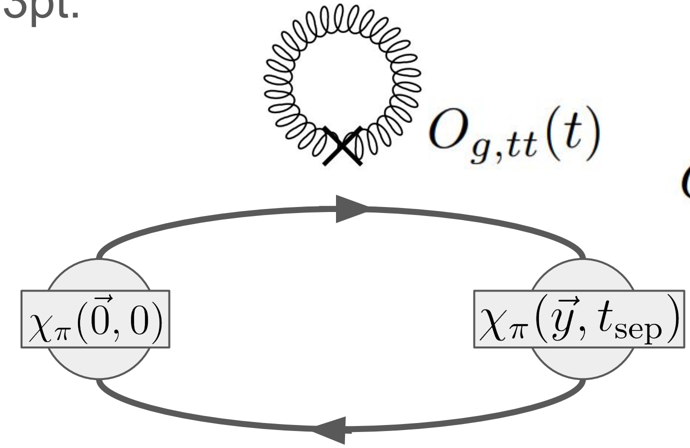


$$C_\pi^{\mathrm{2pt}}(P_z;t) = \int d^3y\, e^{-iy_zP_z} \left\langle \chi_\pi(\vec{y},t) | \chi_\pi(\vec{0},0) \right\rangle$$

3pt:



$$C_\pi^{\mathrm{3pt}}(P_z;t_{\mathrm{sep}},t) = \int d^3y\, e^{-iy_zp_z} \left\langle \chi_\pi(\vec{y},t_{\mathrm{sep}}) | O_{g,tt}(t) | \chi_\pi(\vec{0},0) \right\rangle$$

Operator

$$O_g \equiv \sum_{i=x,y,z,t} F^{\mu i} F^{\nu i} - \frac{1}{4} \sum_{i,j=x,y,z,t} F^{ij} F^{ij}$$

evaluated for $\mu = \nu = t$

$$\langle 0 | O_g | 0 \rangle = \frac{3E_0^2 + P_z^2}{4E_0^2} \langle x \rangle_g^{\mathrm{bare}}$$

$$\langle x \rangle_g = Z(\mu) \langle x \rangle_g^{\mathrm{bare}}$$

2pt and 3pt Correlator Form

- 2pt correlator expands as:

$$C_\pi^{\mathrm{2pt}}(P_z, t_{\mathrm{sep}}) = |A_{\pi,0}|^2 e^{-E_{\pi,0} t_{\mathrm{sep}}} + |A_{\pi,1}|^2 e^{-E_{\pi,1} t_{\mathrm{sep}}} + \dots$$

- 3pt correlator expands as:

$$C_\pi^{\mathrm{3pt}}(z, P_z; t_{\mathrm{sep}}, t) = \\ |A_{\pi,0}|^2 \langle 0|O_g|0\rangle e^{-E_{\pi,0} t_{\mathrm{sep}}} + |A_{\pi,0}||A_{\pi,1}| \langle 0|O_g|1\rangle e^{-E_{\pi,0}(t_{\mathrm{sep}}-t)} e^{-E_{\pi,1} t} + \\ |A_{\pi,0}||A_{\pi,1}| \langle 1|O_g|0\rangle e^{-E_{\pi,1}(t_{\mathrm{sep}}-t)} e^{-E_{\pi,0} t} + |A_{\pi,1}|^2 \langle 1|O_g|1\rangle e^{-E_{\pi,1} t_{\mathrm{sep}}} + \dots$$

- Looking at the ratio, we see: $R_\pi(z, P_z, t_{\mathrm{sep}}, t) = \dfrac{C_\pi^{\mathrm{3pt}}(z, P_z, t, t_{\mathrm{sep}})}{C_\pi^{\mathrm{2pt}}(P_z, t_{sep})} \xrightarrow{t_{sep}\to\infty} \langle 0|O_g|0\rangle$

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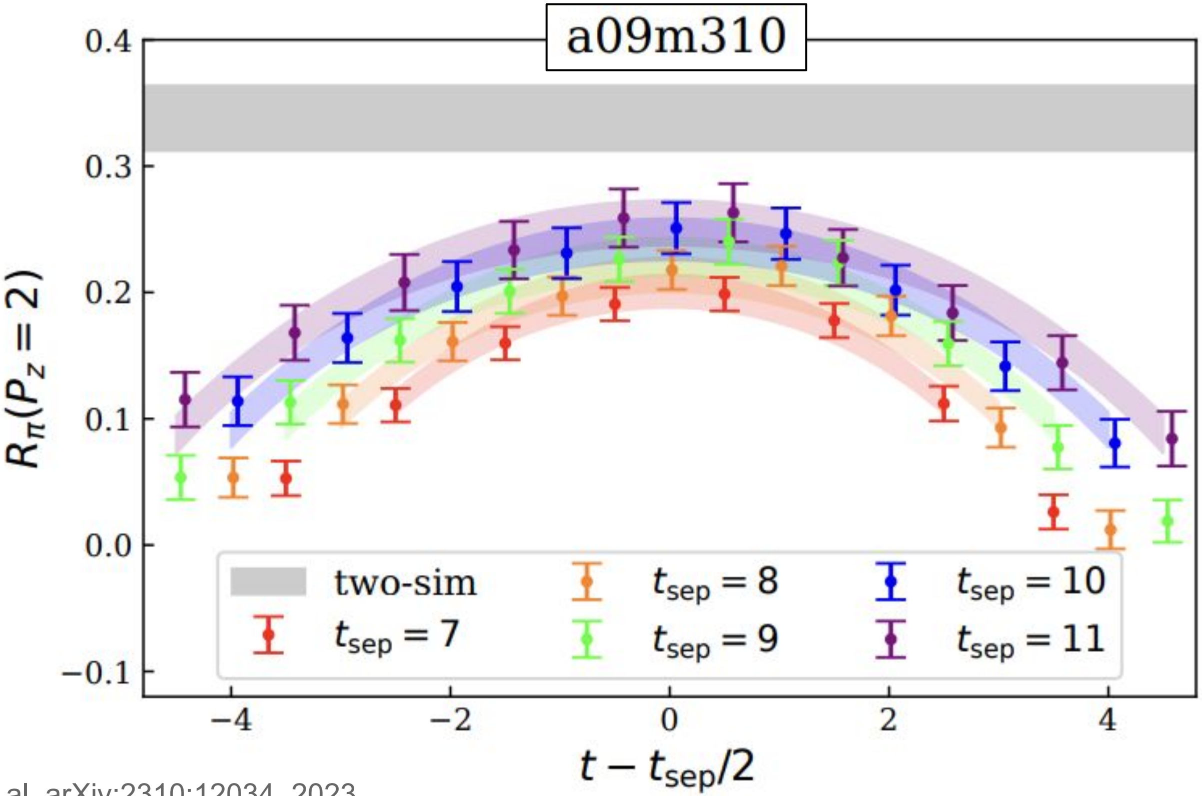
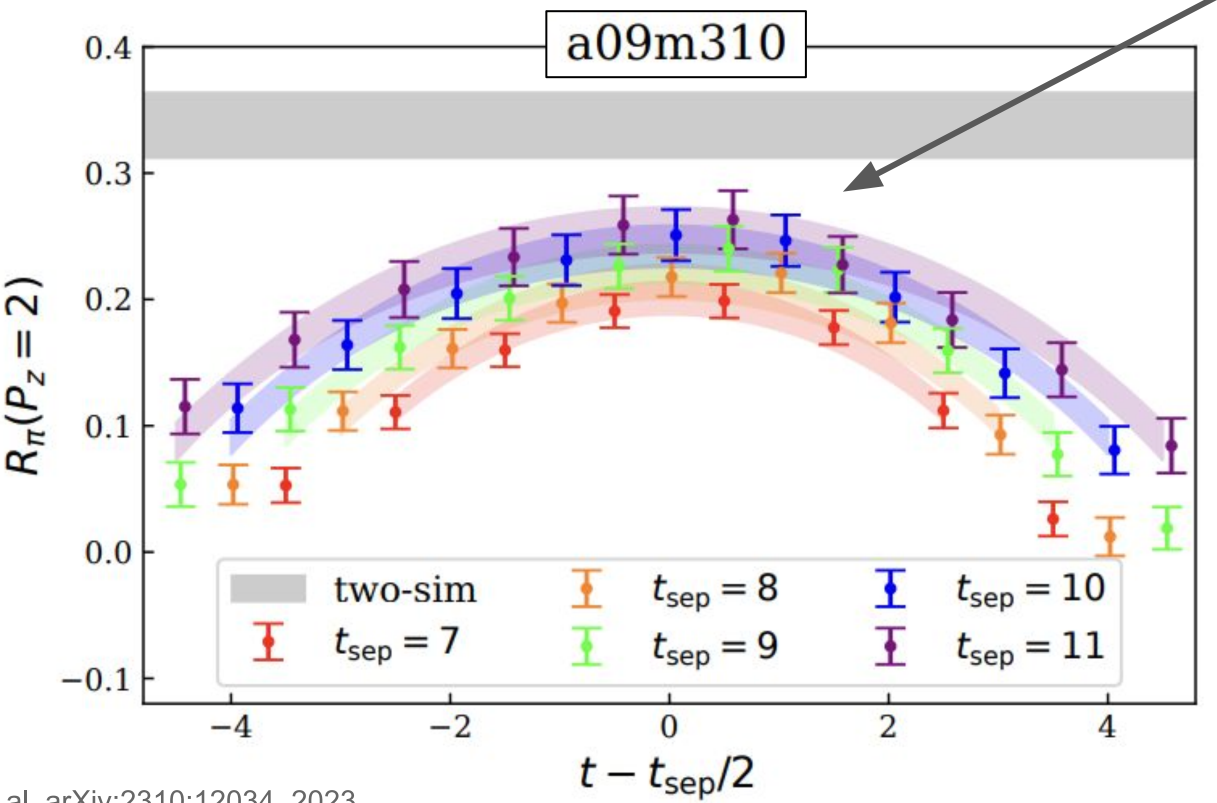
Ratio Plot and Simultaneous Fit



W. G., et al. arXiv:2310:12034, 2023.

Ratio Plot and Simultaneous Fit

Approaches ground fitted ground state matrix element as expected



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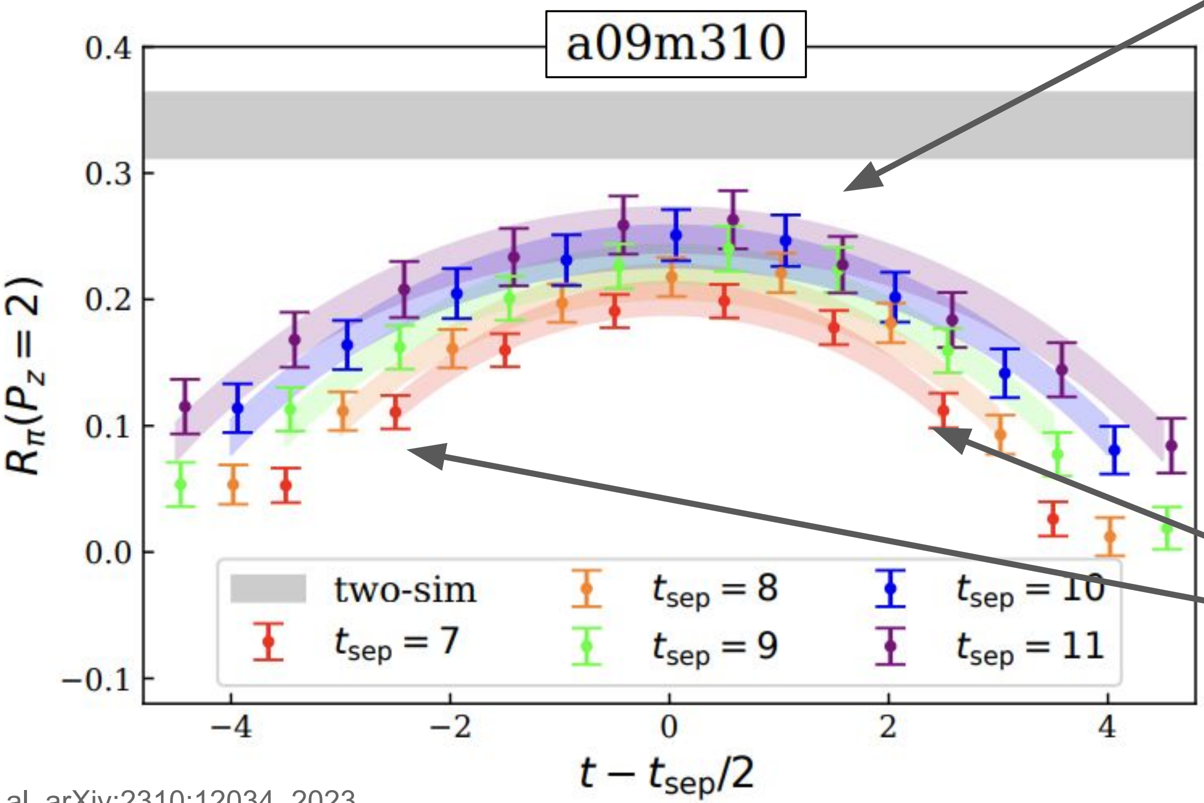


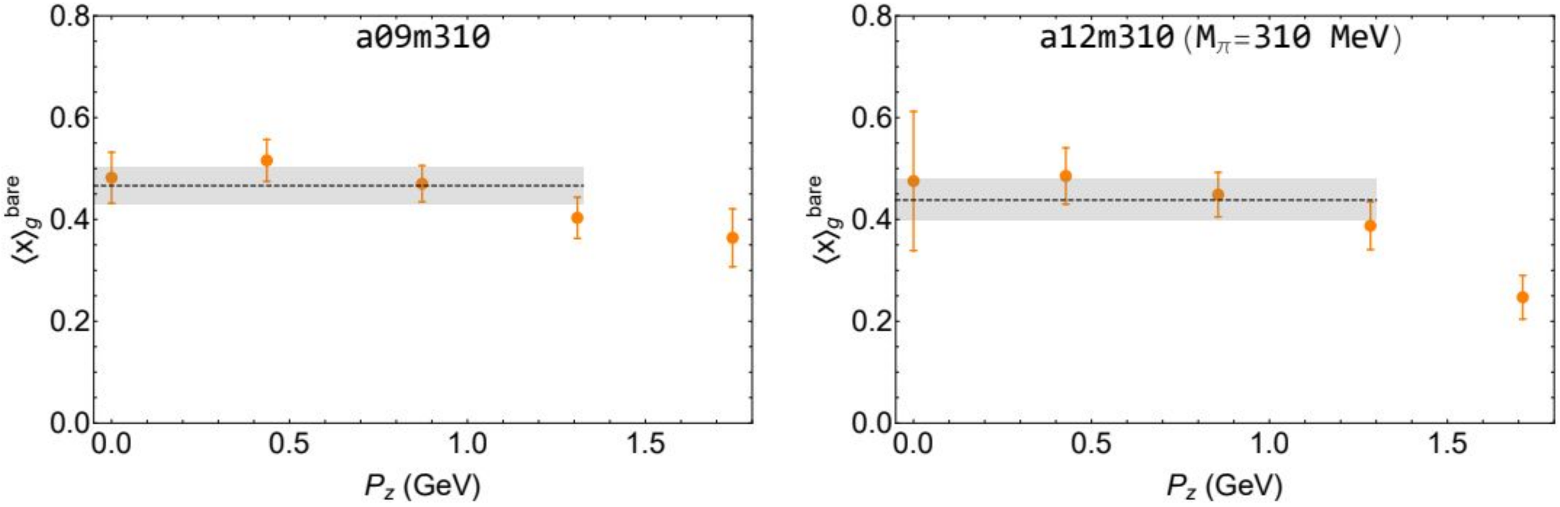
Approaches ground fitted ground state matrix element as expected

Relatively symmetric

W. G., et al. arXiv:2310:12034, 2023.

Momentum Averaged $\langle x \rangle_g^{\text{bare}}$

Take a weighted average (equivalent to a constant fit) of the bare momentum fractions for each ensemble. A couple examples:



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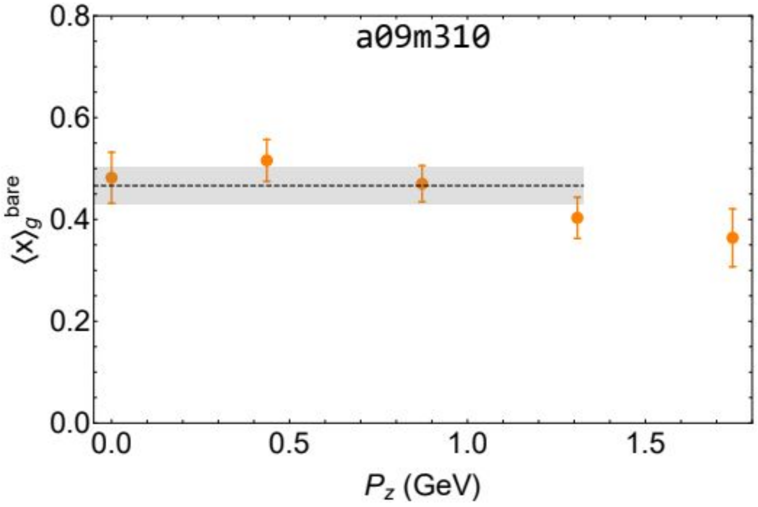
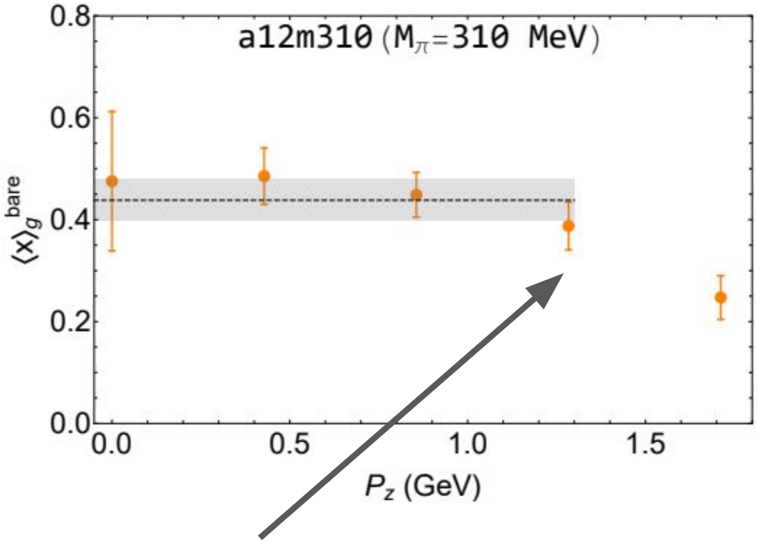




We see some momentum discretization effects, so we choose range of momentum that obtains best χ^2/dof

W. G., et al. arXiv:2310:12034, 2023.

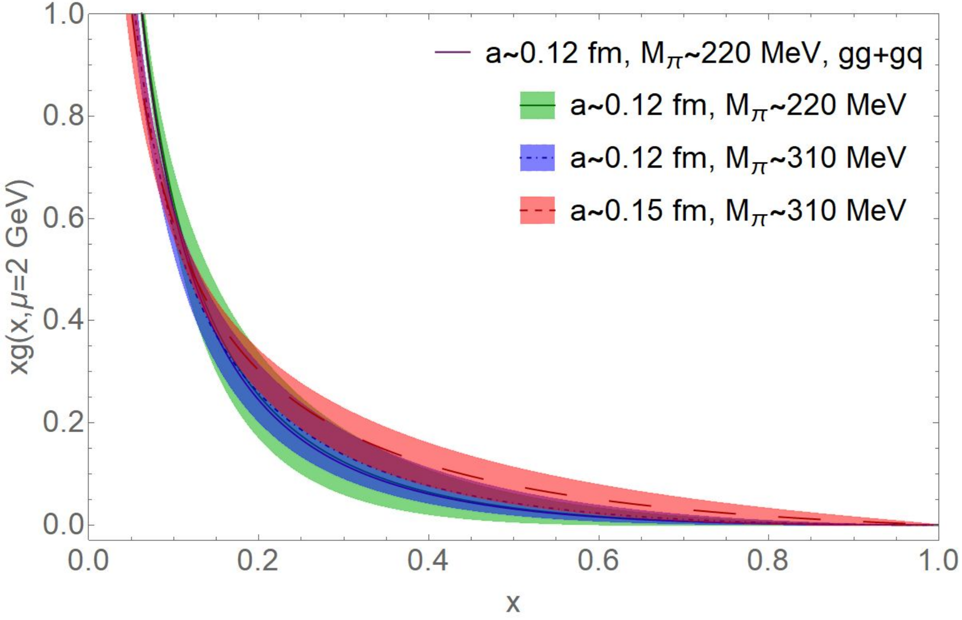
Renormalization

- The renormalization coefficients $(Z^{\overline{\mathrm{MS}}})^{-1}(\mu)$ were obtained from previous work by our group at $\mu = 2$ GeV Z. Fan, et al. PRD 107:034505, 2023.
- Non-perturbatively renormalize the gluon operator in the regularization-independent momentum subtraction (RI/MOM) scheme on the lattice
- Convert to the modified minimal-subtraction $\overline{\mathrm{MS}}$ scheme

Pion PDF Updates

Multiplied our PDFs $xg(x)/\langle x\rangle_g$ through by $\langle x\rangle_g$

Z. Fan, et al. PLB 823:136778, 2021.



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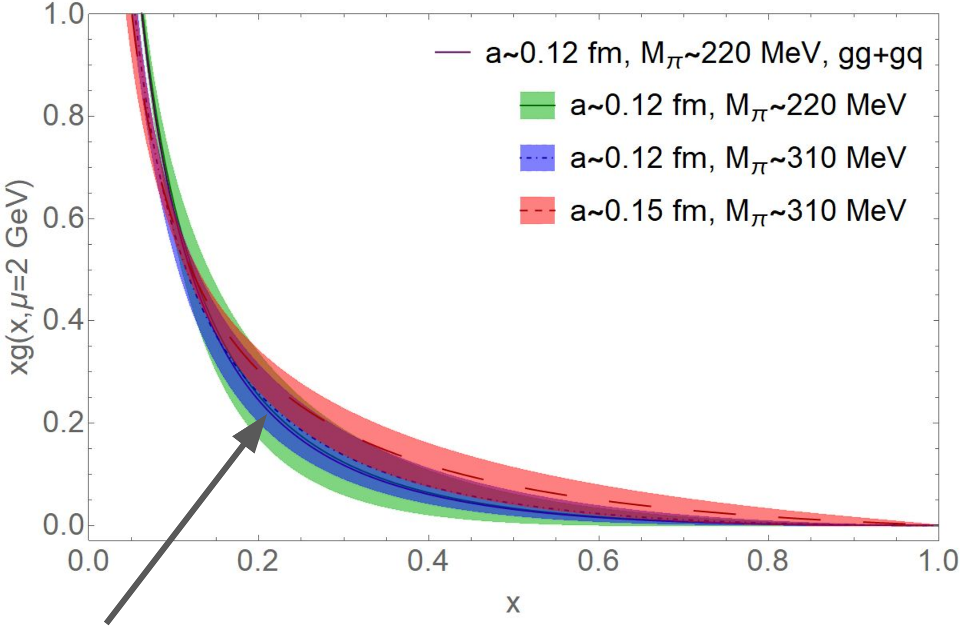

All of our lattice results in good agreement

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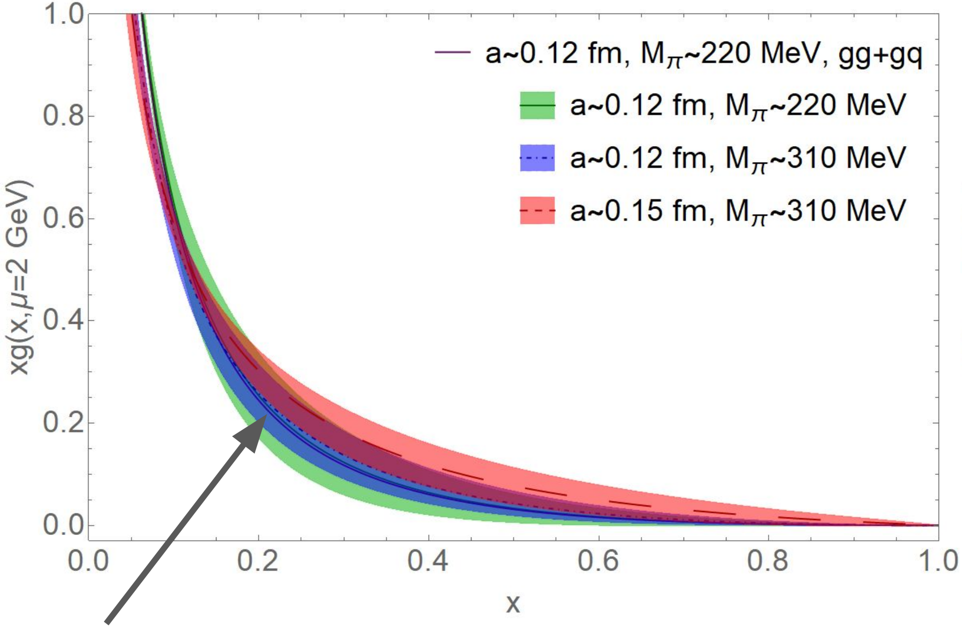

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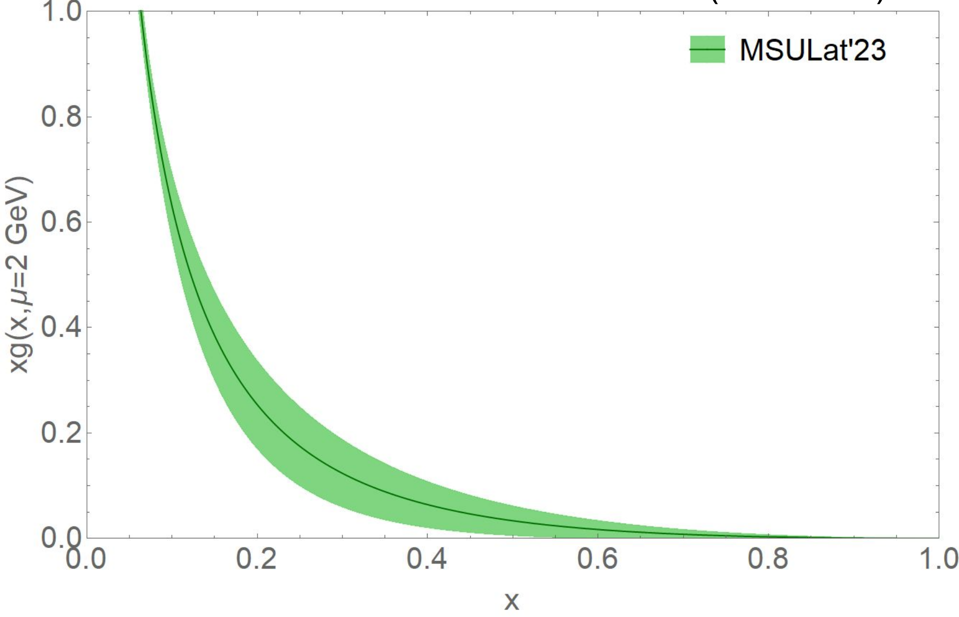

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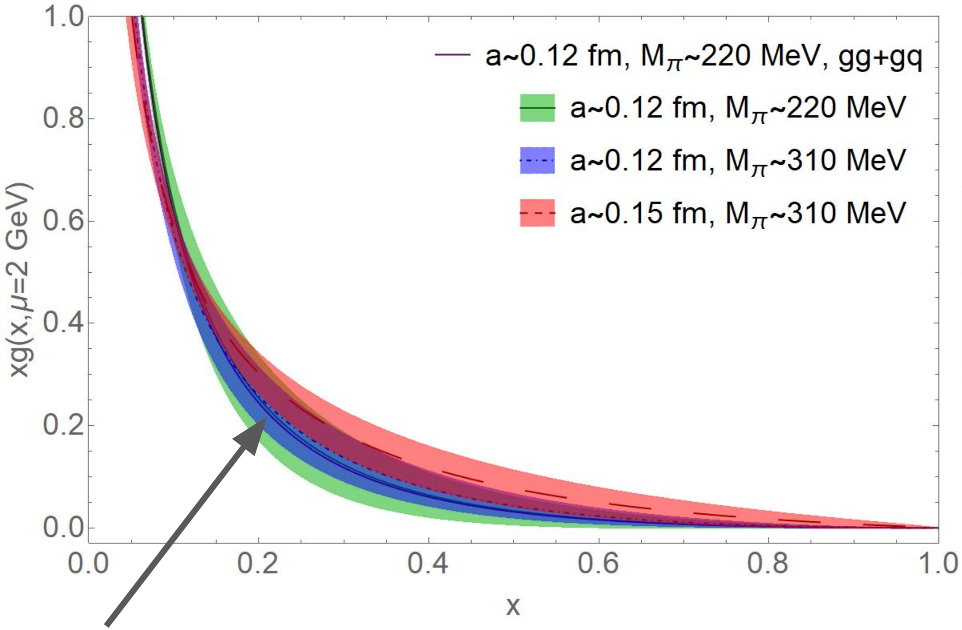

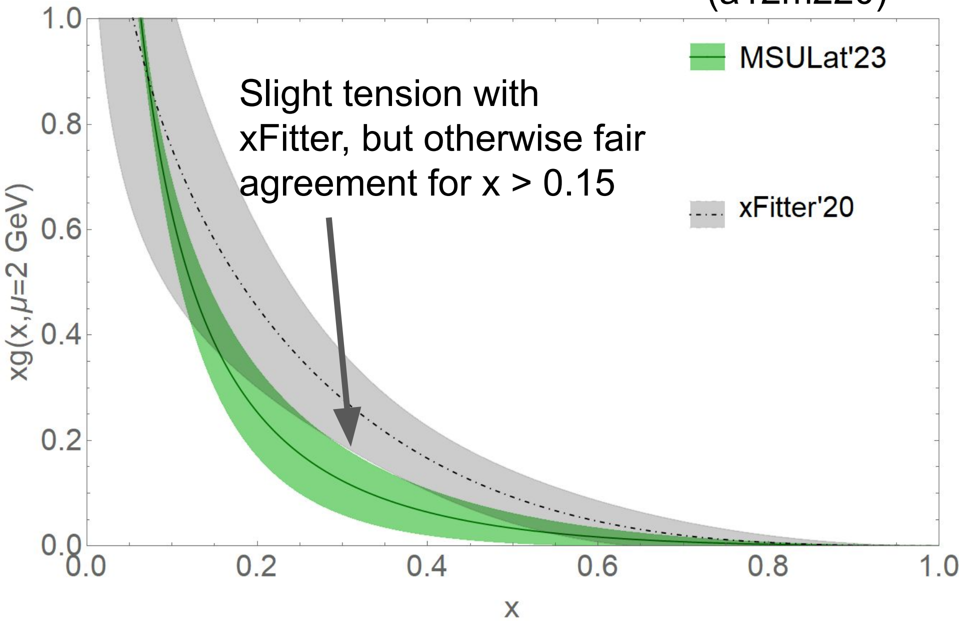

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xFitter: I. Novikov, et al. PRD 102:014040, 2020

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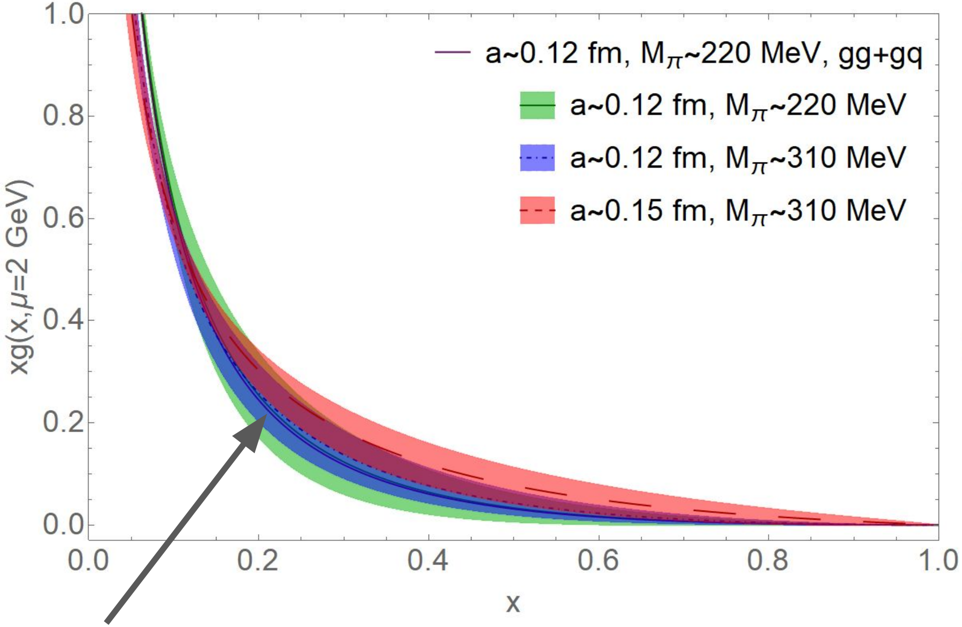

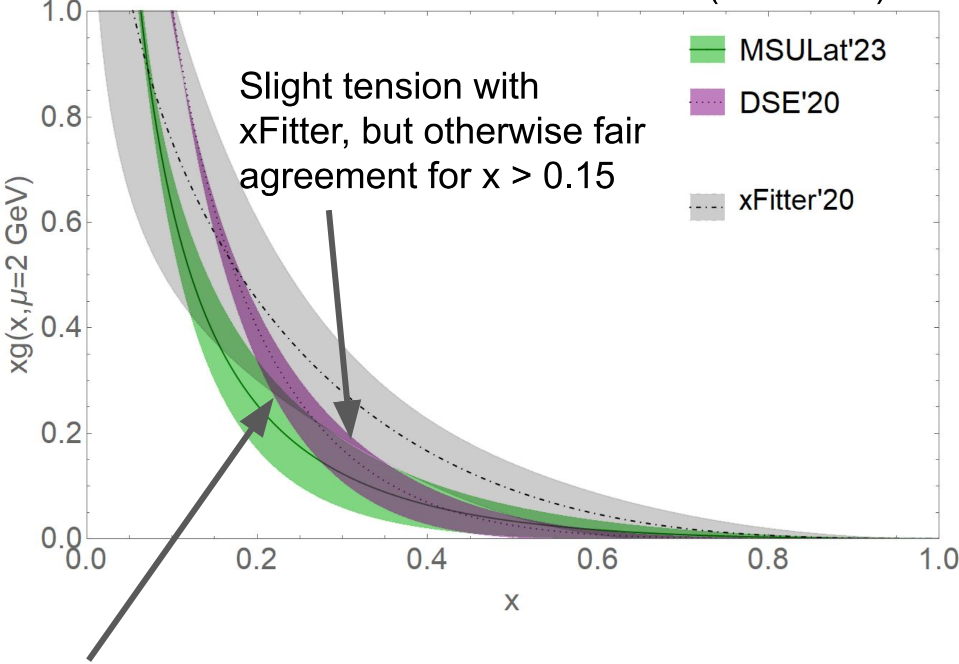

All of our lattice results in good agreement

DSE gives good agreement for x>0.2

DSE: Z.-F. Cui, et al. EPJC 80:1064 2020

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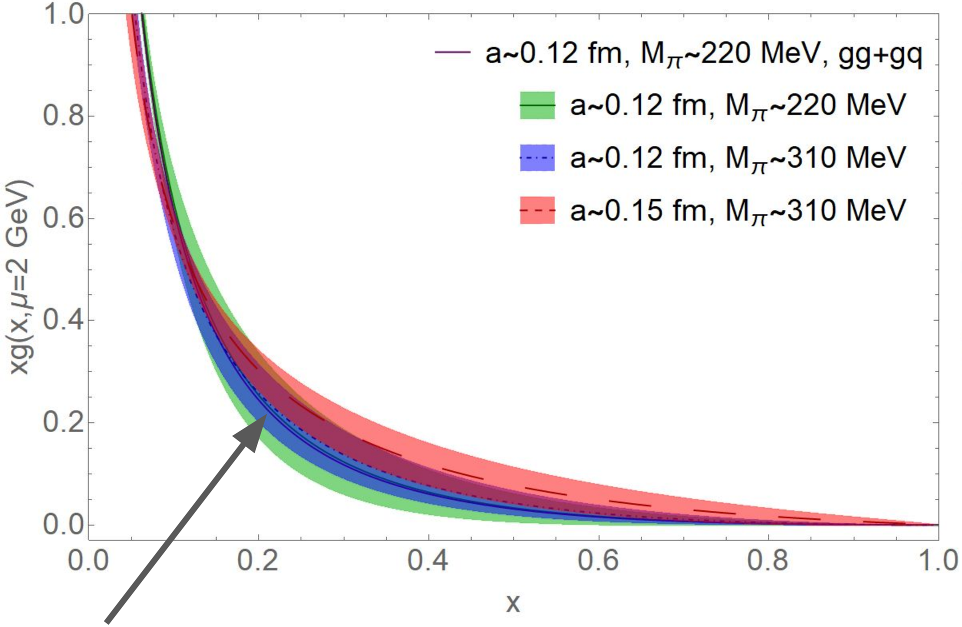

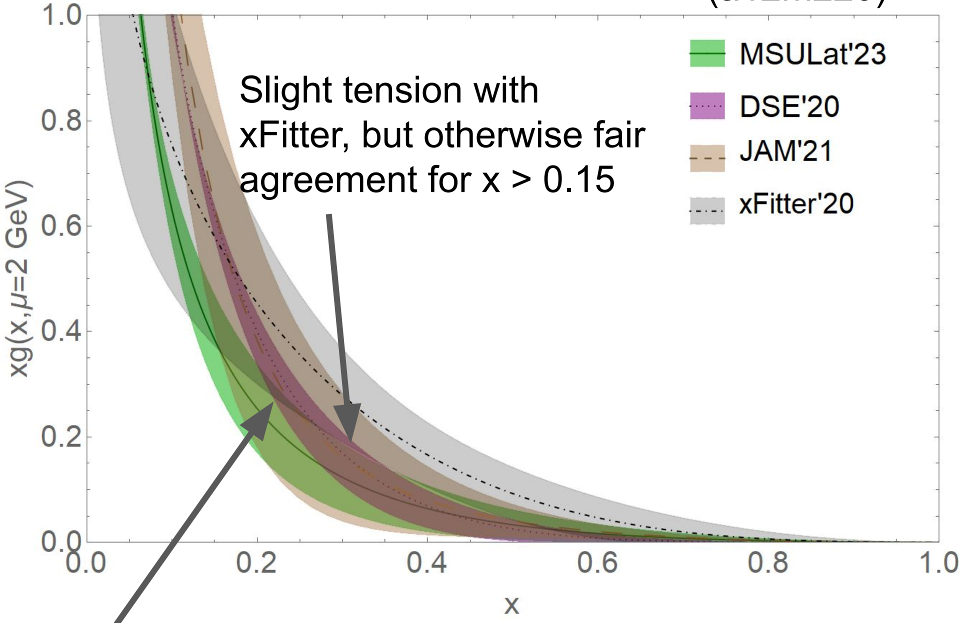

Slight tension with xFitter, but otherwise fair agreement for x > 0.15

All of our lattice results in good agreement

DSE and JAM give good agreement for x>0.2

DSE: Z.-F. Cui, et al. EPJC 80:1064 2020
JAM: P. C. Berry, et al. PRL 127:232001 2021
xFitter: I. Novikov, et al. PRD 102:014040, 2020

W. G., et al. arXiv:2310:12034, 2023.

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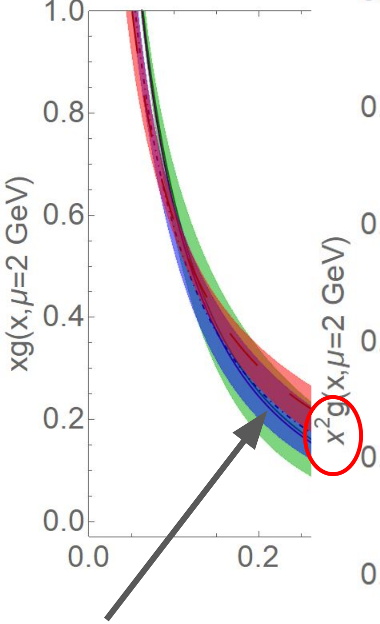

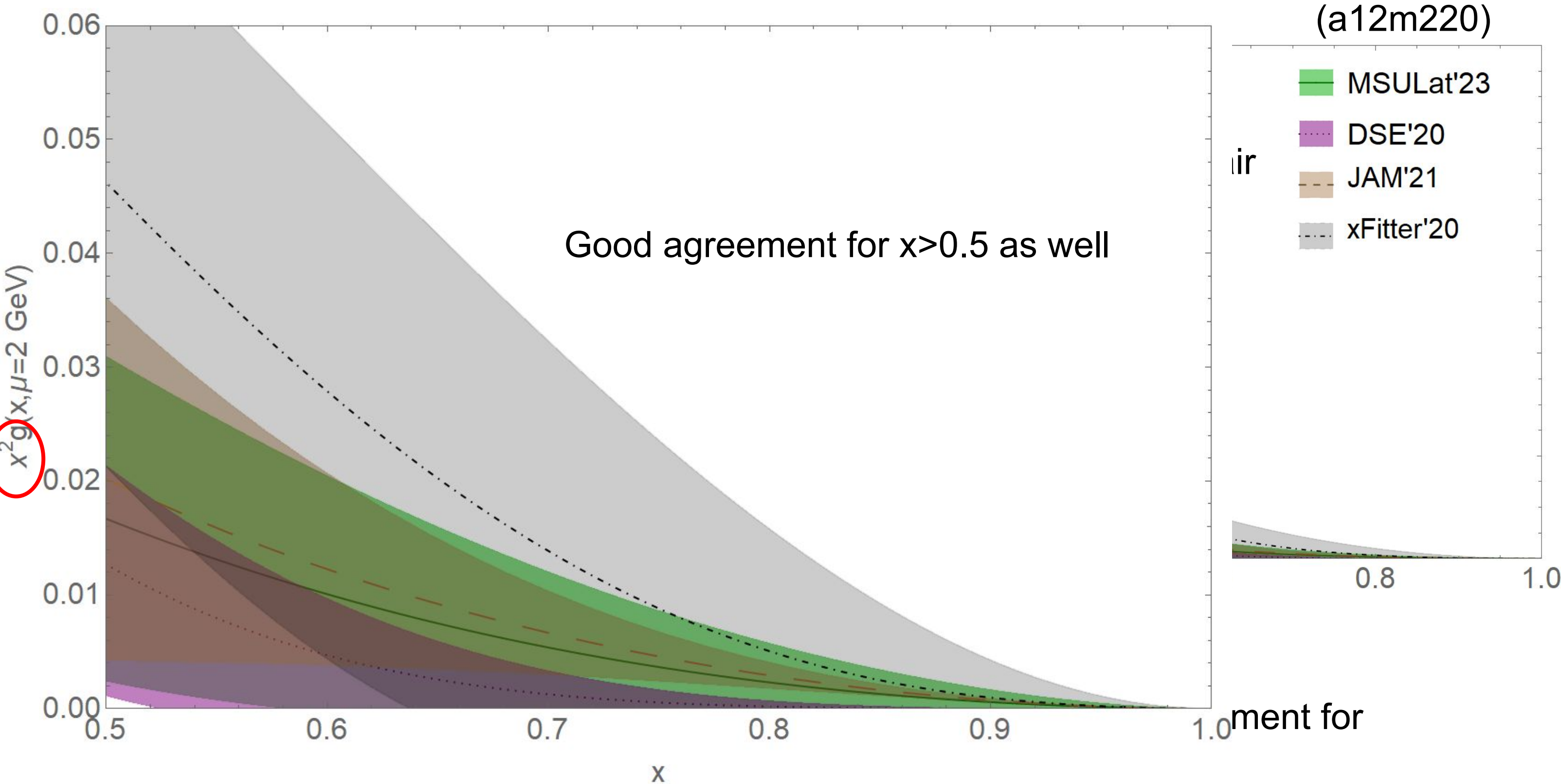

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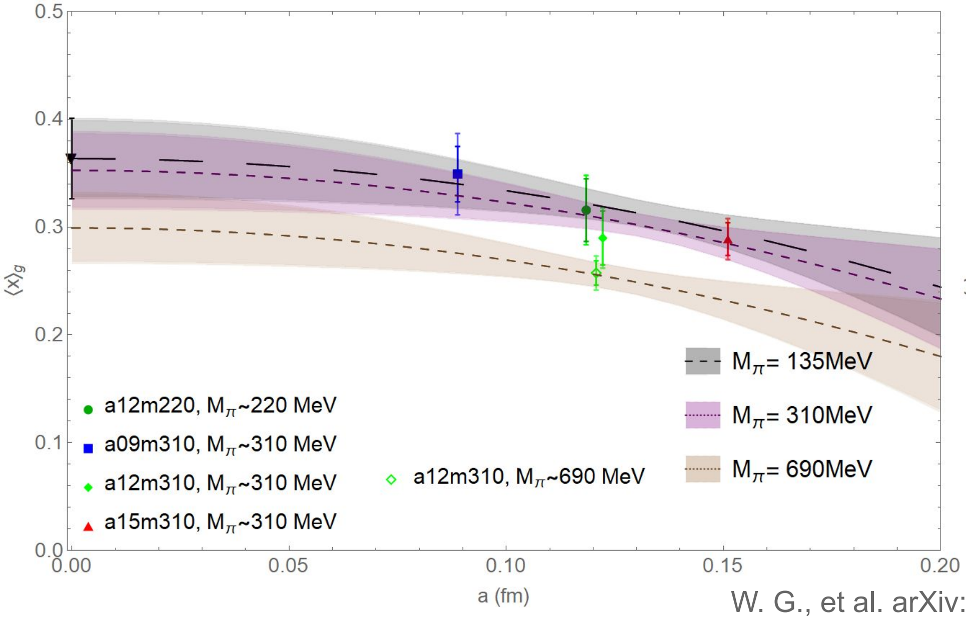
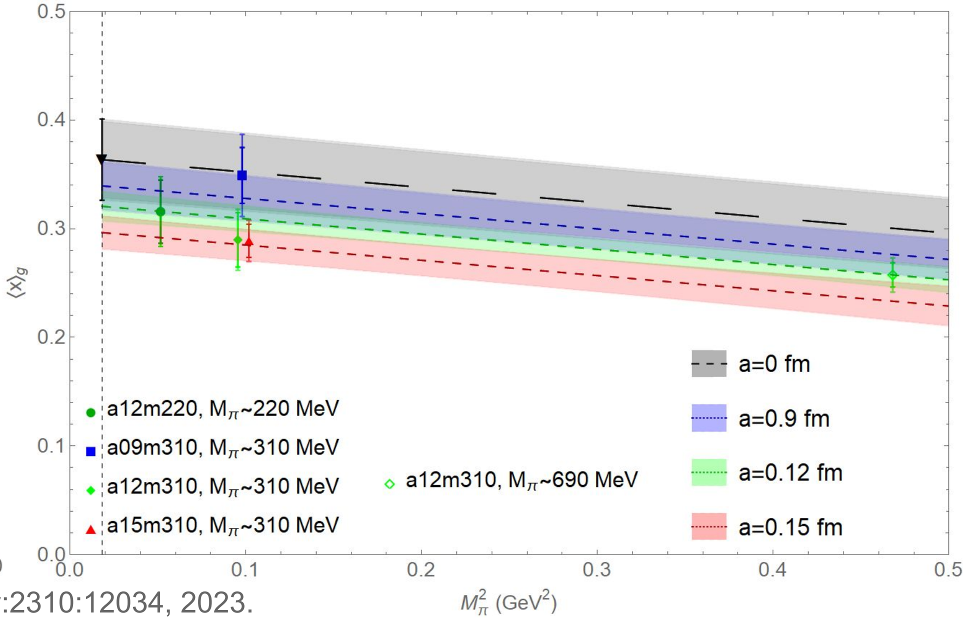
Continuum Extrapolation

$$\langle x \rangle_g(M_\pi, a) = \langle x \rangle_g^{\text{cont}} + k_M(M_\pi^2 - (M_\pi^{\text{phys}})^2) + k_a a^2$$

Use an extrapolation fit form linear in M_π^2 and a^2 we plot fit bands vs M_π^2 and a





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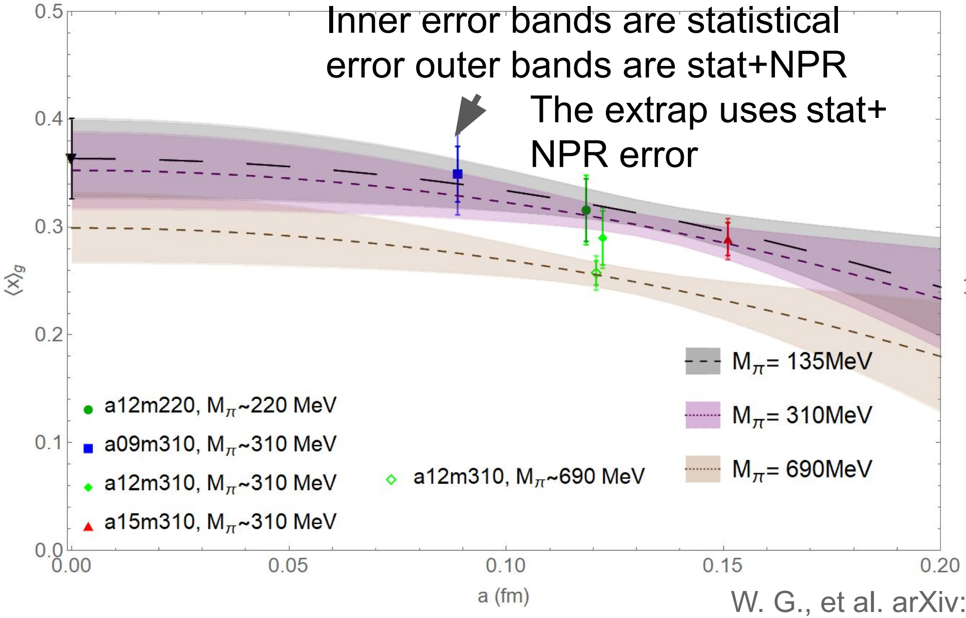

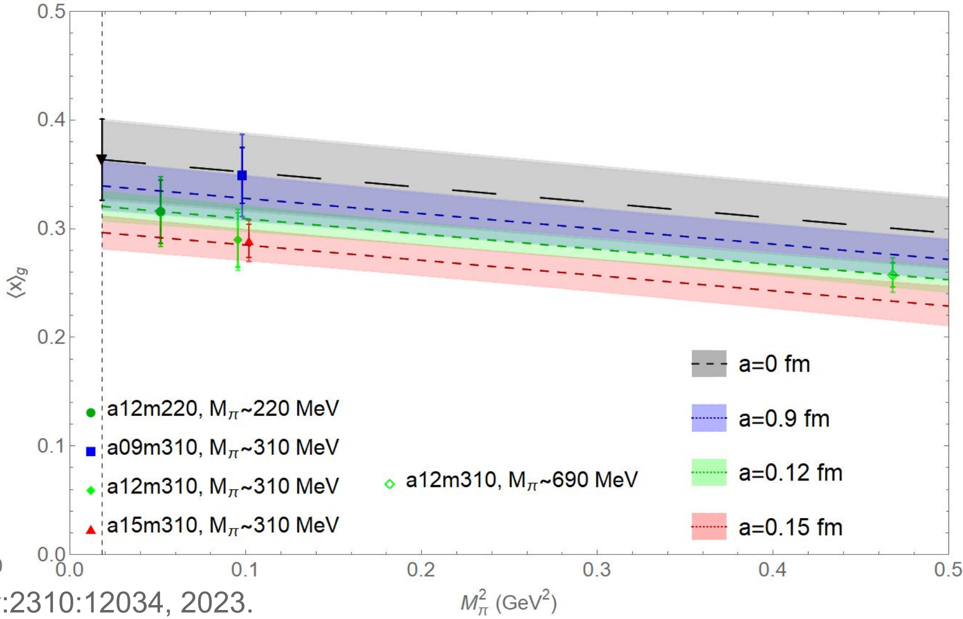

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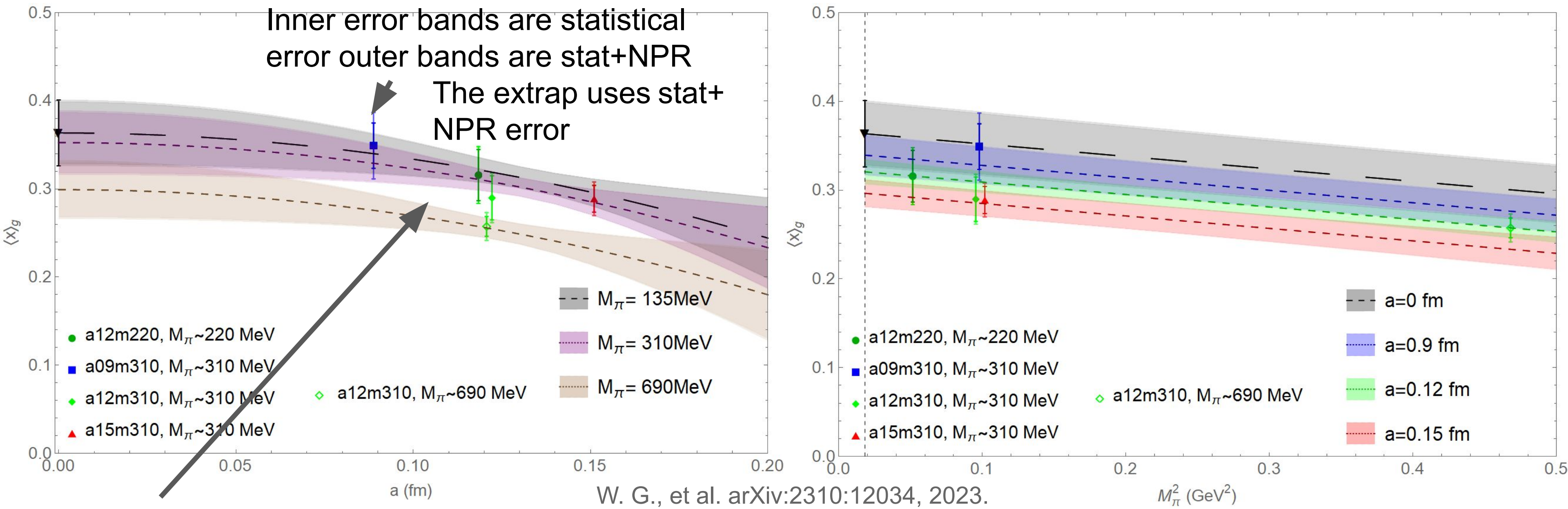

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Good agreement between data and fit bands

Lighter pion mass agrees well with physical pion mass

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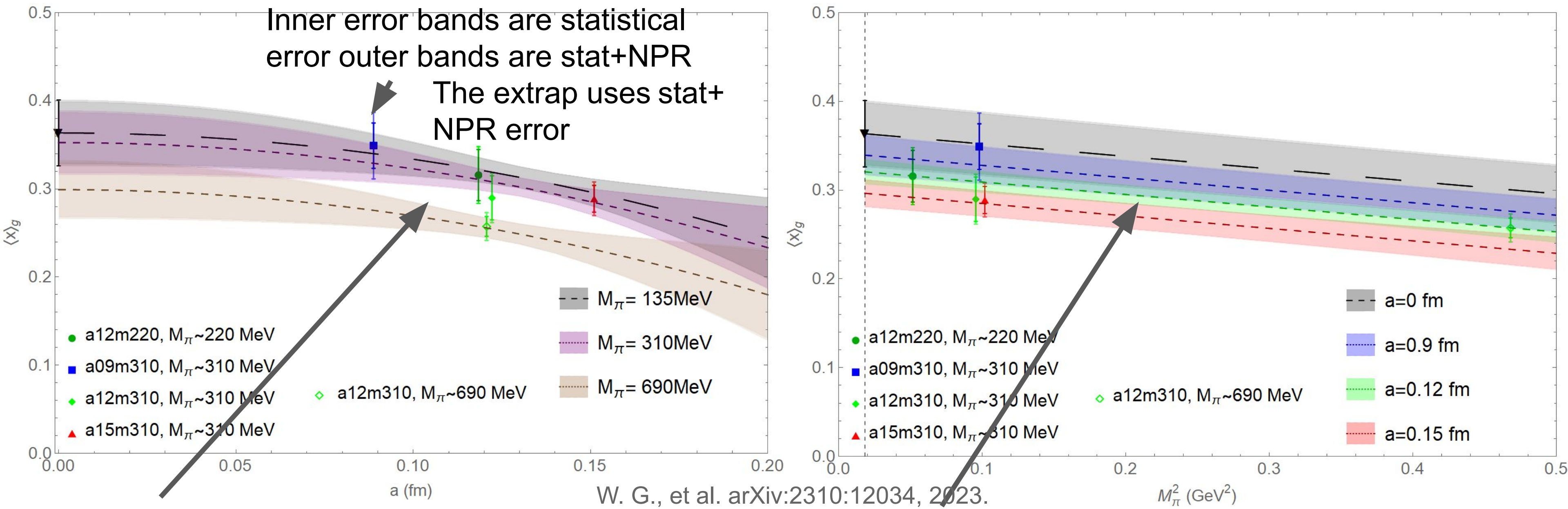

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Good agreement as well
Larger lattice spacing tends towards smaller momentum fraction

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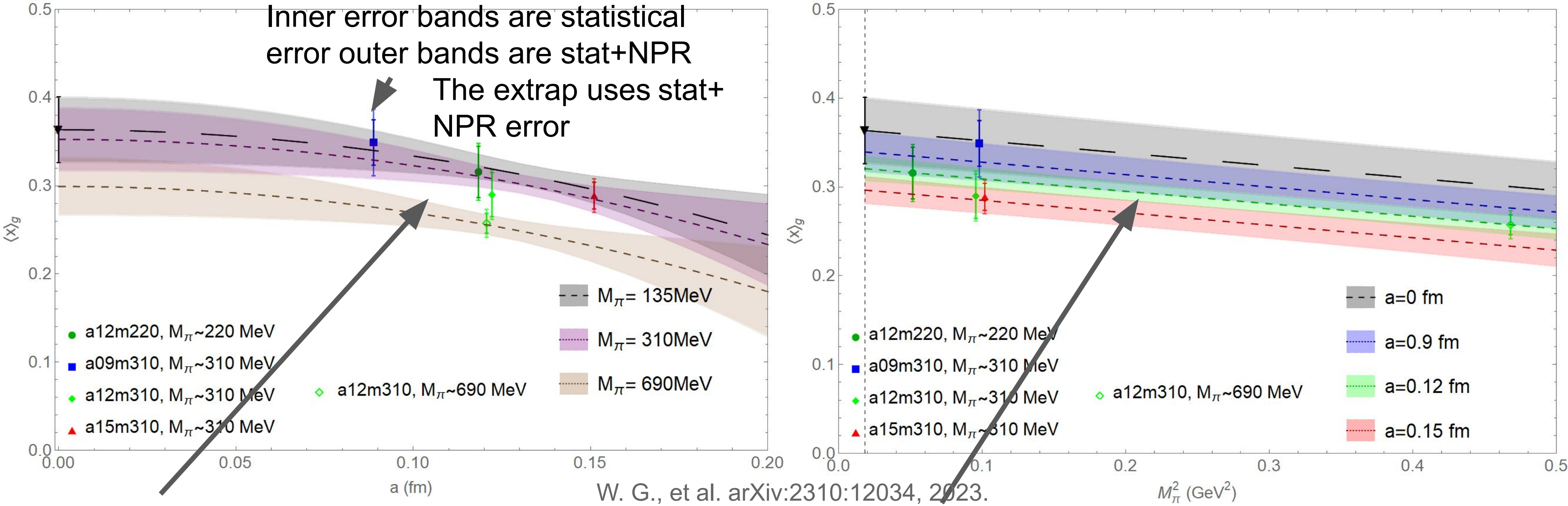

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Larger lattice spacing tends towards smaller momentum fraction

Overall, more M_π and a dependence than nucleon

Z. Fan, et al. PRD 107:034505, 2023.

Mixing

- Bare gluon operator can mix with singlet quark operators through the renormalized operator

$$O_g = Z_{gg} O_g^{\mathrm{bare}} + Z_{gq} \sum_{i=u,d,s} O_{q,i}^{\mathrm{bare}}$$

- We do not calculate this mixing, and instead estimate a 10% systematic uncertainty on the final result based on the range of mixing seen in other lattice results (1%-20%)

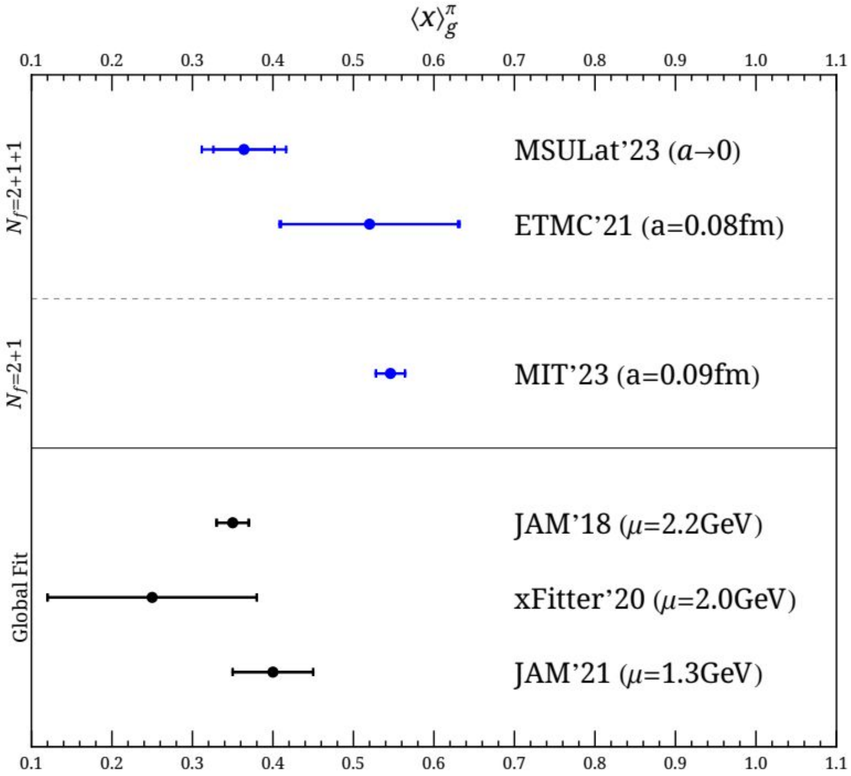
C. Alexandrou, et al. PRD 96:054503, 2017.
C. Alexandrou, et al. PRD 101:094513, 2020.
D.C. Hackett, et al. arXiv:2307:11707, 2023.

Final Result

- We quote:

$$\langle x \rangle_g(\mu = 2\text{GeV}) = 0.364(38)_{\text{stat+NPR}}(36)_{\text{mixing}}$$

- Our result is lower than other most recent lattice calculations
  - Single lattice spacings
  - Different actions
- Reasonable agreement with global fit numbers
- Further information about systematics and greater statistics may reveal more information



MSULat'23: W. G., et al. arXiv:2310:12034, 2023.
ETMC'21: C. Alexandrou, et al. PRL 127:252001, 2021.
MIT'23: D.C. Hackett, et al. arXiv:2307:11707, 2023.
JAM'18: P. C. Berry, et al. PRL 121:152001 2018
xFitter'20: I. Novikov, et al. PRD 102:014040, 2020
JAM'21: P. C. Berry, et al. PRL 127:232001 2021

Conclusion

- Showed how we extract matrix elements related to $\langle x \rangle_g$
- Took a momentum average of $\langle x \rangle_g$
- Showed PDFs updated with the momentum fraction
- Did a continuum extrapolation on to get our final result
    - Estimated 10% mixing as a systematic
- Our results show tension with other lattice results, but agree with the global analysis results with non-zero lattice spacing and different actions
- Pion data is extremely limited, so future experiments will lead to updates in the global analysis work

Backup Slides

Ensemble Information Table

ensemble	a09m310	a12m220	a12m310 (310 MeV)	a12m310 (690 MeV)	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1207(11)	0.1510(20)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
M_π^{val} (MeV)	313.1(13)	226.6(3)	309.0(11)	687.3(6)	319.1(31)
P_z (GeV)	$[0, 1.75]$	$[0, 1.64]$	$[0, 1.71]$	$[0, 1.71]$	$[0, 1.54]$
N_{cfg}	1009	957	1013	1013	900
N_{meas}	$\{387, 456\}$	1,466,944	324,160	324,160	259,200
t_{sep}	$[7, 11]$	$[5, 9]$	$[5, 9]$	$[4, 8]$	$[4, 8]$

Results for Each Ensemble

ensemble	M_π^{val} (MeV)	$\langle x \rangle_g^{\text{bare}}$	$\left(Z_{O_g}^{\overline{\text{MS}}}\right)^{-1}$	$\langle x \rangle_g^{\overline{\text{MS}}}$
a12m220	226.6(3)	0.477(45)	1.512(65)	$0.316(29)_{\text{stat}}(14)_{\text{NPR}}$
a09m310	313.1(13)	0.466(36)	1.336(106)	$0.349(26)_{\text{stat}}(28)_{\text{NPR}}$
a12m310	309.0(11)	0.438(40)	1.512(65)	$0.290(25)_{\text{stat}}(13)_{\text{NPR}}$
	684.1(6)	0.389(18)	1.512(65)	$0.257(11)_{\text{stat}}(11)_{\text{NPR}}$
a15m310	319.1(31)	0.302(16)	1.047(41)	$0.289(15)_{\text{stat}}(11)_{\text{NPR}}$