1 High-energy acceleration

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Here, we should write the summary of Chapter 7 and in particular the parameters to be used by the other chapters, e.g. these exemplary and preliminary tables one obtains for a fixed survival rate of 90% per RCS.

Table 1.1: Summary table of the acceleration chain.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Number of bunches/species	-	-	1	1	1	1
Repetition rate	f_{rep}	[Hz]	5	5	5	5
Circumference	$2\pi R$	[m]	5990	5990	10700	26659
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	[1e12]	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Injection energy	E_{inj}	[GeV/u]	63	313.83	750	1500
Ejection energy	E_{ej}	[GeV/u]	313.830	750	1500	5000
Energy ratio	$E_{\text{ej}}/E_{\text{inj}}$	-	4.98	2.39	2.00	3.33
Planned Survival rate	$N_{\text{ej}}/N_{\text{inj}}$	-	0.9	0.9	0.9	0.9
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Number of turns	n_{turn}	-	17	55	66	72
Average Accel. Gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.755	7.930	11.364	48.611
Tot. straight section length	L_{str}	[m]	2334.7	2335.7	3975.7	4063.3
Vertical norm. Emittance	$\epsilon_{v,n}$	[mm]	25	25	25	25
Horiz. norm. Emittance	$\epsilon_{h,n}$	[mm]	25	25	25	25
Long. norm. emittance	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025
Total NC dipole length	L_{NC}	[m]	3655.3	2539.26	4366.29	18338.42
Total SC dipole length	L_{SC}	[m]	0	1115.02	2358.02	4257.27
Max. NC dipole field	B_{NC}	[T]	1.80	1.80	1.80	2.00
Max. SC dipole field	B_{SC}	[T]	-	10	10	16
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Max RF voltage	V_{RF}	[GV]	20.87	11.22	16.07	68.75
Number of cavities	-	-	696	374	536	2292

The first version of the tables prepared by Fabian Batsch *et al.* are presented at the 1st meeting on HEMAC discussions (https://indico.cern.ch/event/1132527/) on 22/02/2022. The up-to-date tables can be found in Tables 1.2, 1.3, 1.4, and 1.5 below. They should be used as concerns the 3 RCS for the 3TeV option. A parameter table exists also for a fourth RCS to go up to 5TeV by using the LHC tunnel or a dedicated tunnel. However, these parameters are still preliminary and will evolve in the near future. In agreement with the magnet people, the current magnetic field in the normal-conducting dipoles is 1.8T (instead of 1.5T as at the first begining) at extraction for all RCS. To fit the LHC tunnel, the maximum dipole field should go up to 2T. Going to 1.8T significantly changes the length of the NC sections and thus decreases the filling factor, i.e. it gives more space for RF cavities and so on. For the hybrid RCS2 and RCS3, the magnet field in the SC magnets is 10T. That was a compromise between a reduced filling factor and magnet price. Indeed, to protect the SC magnets from the decay products, the inner aperture of the SC magnets is larger and going to 16T implies a higher cost without a significant improvement of the machine performance. In the case of RCS4, the extraction high-energy and thus average magnetic field in the machine require a higher magnetic field in the SC magnets to fit with the LHC tunnel. That is why the current magnetic field is 16T in the SC dipoles of the RCS4. This requirement may evolve with the optimization of the high-energy chain. Indeed, the current optimization is based on MAP studies for the ring circumference and aims at a survival rate of 90% per ring. Including the cost optimization in the figures of merit may go to a slightly different table.

To mitigate the longitudinal emittance growth, the RF sections should be distributed along the RCS. The minimum number is 32 RF stations for RCS1 and 24 stations for the other to keep the longitudinal emittance growth below the 5% level (see Fig. 1.1). That is worth noting that the longitudinal dynamics used values of momentum compaction for an RCS lattice design based on FODO cells. The dynamics and thus the number of RF stations may also evolve with the optics design of the RCS.

Table 1.2: Tentative ramp parameters for the acceleration chain.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Injection energy	E_{inj}	[GeV/u]	63	313.83	750	1500
Ejection energy	E_{ej}	[GeV/u]	313.830	750	1500	5000
Energy ratio	$E_{\mathrm{ej}}/E_{\mathrm{inj}}$	-	4.98	2.39	2.00	3.33
Number of turns	n_{turn}	-	17	55	66	72
Ramp shape	-	-	Quasi-Linear			
Planned Survival rate	$N_{\mathrm{ej}}/N_{\mathrm{inj}}$	-	0.9	0.9	0.9	0.9
Total survival rate	N_{ej}/N_0	-	0.9	0.81	0.729	0.6561
Average Accel. Gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.755	7.930	11.364	48.611
Injection Lorentz factor	γ_{inj}	-	597	2971	7099	14198
Ejection Lorentz factor	γ_{ej}	-	2971	7099	14198	47323
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Repetition rate	-	[Hz]	5	5	5	5

Table 1.3: Tentative machine and lattice parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum dipole width and height do not include the required shielding and limitations coming from collective effects studies.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Radius	R	[m]	953.3	953.3	1703.0	4242.9
Circumference	$2\pi R$	[m]	5990	5990	10700	26659
Pack fraction	-	[%]	61	61	62.8	84.8
Bend radius	ρ_B	[m]	581.8	581.8	1070.2	3596.2
Tot. straight section length	L_{str}	[m]	2334.7	2335.7	3975.7	4063.3
Average Injection dipole field	B_{inj}	[T]	0.36	1.80	2.34	1.39
Average ejection dipole field	B_{ej}	[T]	1.8	4.30	4.68	4.64
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Repetition rate	-	[Hz]	5	5	5	5
Total NC dipole length	L_{NC}	[m]	3655.3	2539.26	4366.29	18338.42
Total SC length	L_{SC}	[m]	0	1115.02	2358.02	4257.27
Injection NC dipole field	$B_{NC,inj}$	[T]	0.36	-1.80	-1.80	-2.00
Ejection NC dipole field	$B_{NC,ej}$	[T]	1.80	1.80	1.80	2.00
SC dipole field	B_{SC}	[T]	-	10	10	16
Number of cells/arc	n_c	-	7	10	17	19
Cell length	L_c	[m]	21.4	19.6	20.6	45.9
Path length diff.	$\Delta\mathcal{C}$	[mm]	0	9.1	2.7	9.4
Orbit difference	$\Delta\bar{x}$	[mm]	0	12.2	5.9	13.2
Min. dipole width	w_d	[mm]	17.4	19.6	10.7	18.8
Min. dipole height	h_d	[mm]	14.8	6.4	4.2	4.4
Transition gamma	γ_{tr}	-	20.41	20.41	30.9	30.9

Table 1.4: Tentative beam parameters for the acceleration chain. The acceleration ramp is assumed to be linear.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	[1e12]	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Bunch length	4σ	[ns]	0.077	0.077	0.077	0.077
Number of bunches/species	-	-	1	1	1	1
Beam current per bunch	I	[mA]	20.38	19.50	9.88	4.00
Vertical norm. emittance	$\epsilon_{v,n}$	[mm]	25	25	25	25
Horizontal norm. emittance	$\epsilon_{t,n}$	[mm]	25	25	25	25
Long. norm. emittance $\sigma_E \times \sigma_z$	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025

Table 1.5: Tentative RF parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum required cavity gradient assumed that all the allocable space is filled with cavities by assuming an RF filling factor of the straight sections (to included the interconnections inside and between the cryomodules).

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Harmonic number	h	-	26000	26000	46370	115520
Revolution frequency	f_{rev}	[kHz]	50.08	50.08	28.04	11.25
Revolution period	T_{rev}	[ms]	20.0	20.0	35.7	88.9
Max. RF voltage	V_{RF}	[GV]	20.87	11.22	16.07	68.75
Max. RF power	P_{RF}	[kW]	850.6	437.4	317.6	550.3
Max. RF filling factor	-	-	0.4	0.4	0.45	0.45
Current RF filling factor	-	-	0.38	0.21	0.17	0.45
Minimum number RF stations	-	-	32	24	24	24
Number of cavities	-	-	696	374	536	2292
Assumed gradient in cavity	$\Delta E/L$	[MV/m]	30	30	30	45
Min. required gradient in cavity	$\Delta E/L$	[MV/m]	22.3	12.0	9.0	37.6
Stable phase	ϕ_S	[○]	135	135	135	135
Longitudinal emittance $\sigma_E \times \sigma_z$	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025
Injection bucket area	$A_{B,\mathrm{inj}}$	[eVs]	0.62	1.01	2.11	3.91
Ejection bucket area	$A_{B,\mathrm{ej}}$	[eVs]	1.37	1.56	2.99	7.15
Bucket area reduction factor	$A_B/A_{B,st}$	-	0.172	0.172	0.172	0.172
Injection synchrotron frequency	$f_{S,\mathrm{inj}}$	[kHz]	76.33	25.07	9.59	8.89
Ejection synchrotron frequency	$f_{S,\mathrm{ej}}$	[kHz]	34.20	16.22	6.78	4.87
Injection synchrotron tune	$Q_{s,\mathrm{inj}}$	-	1.52	0.50	0.34	0.79
Ejection synchrotron tune	$Q_{s,\mathrm{ej}}$	-	0.68	0.32	0.24	0.43
Momentum compaction factor	α_p	-	0.0024	0.0024	0.0010	0.0010

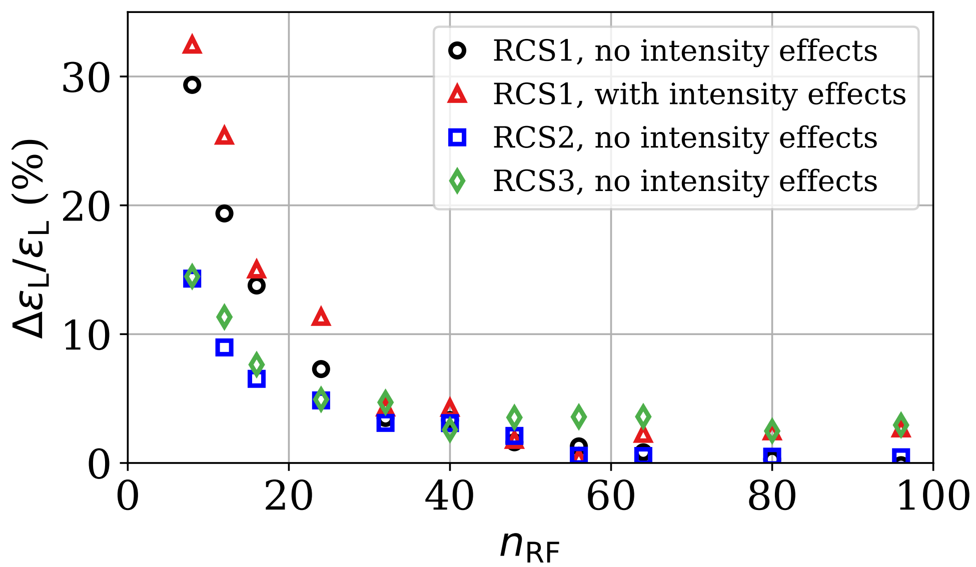

Fig. 1.1: Relative emittance growth at the end of the cycle with respect to the emittance at injection versus n_{RF} for RCS1 without (black circle) and with intensity effects (red triangle), and for RCS2 and RCS3 without intensity effects.

1.1 Longitudinal Emittance - Unit conversion

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When comparing literature, the longitudinal emittance of a particle bunch is usually given in different units. Also within the Muon Collider Collaboration, different units are used, with [mm], [eV-s] and [MeV-m] being the most common ones. To compare values in these different units, their conversion factors are determined as follows [1].

Between [MeV-m] and [eV-s]

The conversion factor between [MeV-m] and [eV-s] is c, the speed of light in vacuum. An emittance in units of [MeV-m] is converted to [eV-s] by dividing the value by c:

$$[\text{MeV-m}] \cdot \frac{1}{c} = [\text{eV-s}] \ . \tag{1.1}$$

As an example, the envisaged emittance of the muon collider of 7.5 MeV-m corresponds to 0.025 eV-s.

Between [mm] and [eV-s]

Papers from the MAP studies usually give longitudinal emittances in [mm]. The definition used there is [2]

$$\varepsilon_{\mathrm{L}} = \sigma_z \sigma_{\Delta p}/(m_\mu c) \ . \tag{1.2}$$

In practice, it is calculated as $\beta\gamma\sigma_z\sigma_{\Delta p/p}$. The normalization factor $m_\mu c$ is equal to $E_{\mu,0}/c$ where $E_{\mu,0} = 105.658\,\text{MeV}$ is the muon rest energy. The conversion factor therefore reads

$$[\text{mm}] \cdot 10^{-3} \cdot \frac{E_{\mu,0}}{c} = [\text{eV-s}] \ . \tag{1.3}$$

In [2], 24 mm are mentioned for muons at 244 MeV. This emittance corresponds to 0.11 eVs.

Between [mm] and [MeV-m]

The conversion factor between [mm] normalized to $[1/(m_\mu c)]$ and [MeV-m] is obtained by combining Eqs. 1.1 and 1.3:

$$[\text{mm}] \cdot 10^{-3} \cdot E_{\mu,0} = [\text{MeV-m}] \ . \tag{1.4}$$

1.2 The Survival Rate as a Function of Kinetic Energy

The survival rate of relativistic muons with an initial population of N_0 and a population of N after a certain time t is given as

$$\frac{N(t)}{N_0} = \exp\left(-\frac{t}{\gamma\tau_\mu}\right) \ , \tag{1.5}$$

where a constant γ is assumed. However, during acceleration, the kinetic energy of the muons and thus γ increases over time. For an acceleration time τ_{acc}, the survival rate is therefore calculated as

$$\frac{N(\tau_{acc})}{N_0} = \exp\left(-\frac{1}{\tau_\mu}\int_0^{\tau_{acc}} \frac{dt}{\gamma(t)}\right) \ . \tag{1.6}$$

Assuming a linear acceleration from injection energy E_{inj} with $\gamma_{inj} = \frac{E_{inj}}{m_\mu c^2} + 1$ to ejection energy E_{ej} and γ_{ej}, one can write the time-dependent Lorentz factor as

$$\gamma(t) = \gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj}) \,. \tag{1.7}$$

The integral in Eq. 1.6 becomes

$$\begin{aligned}\int_0^{\tau_{acc}} \frac{dt}{\gamma(t)} &= \int_0^{\tau_{acc}} \frac{1}{\gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj})} \\ &= \frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj})\right)\Bigg|_0^{\tau_{acc}} \\ &= \frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) .\end{aligned} \tag{1.8}$$

The survival rate is therefore

$$\frac{N(\tau_{acc})}{N_0} = \exp\left(-\frac{1}{\tau_\mu}\frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right)\right) = \left(\frac{\gamma_{ej}}{\gamma_{inj}}\right)^{-\frac{1}{\tau_\mu}\frac{\tau_{acc}}{\gamma_{ej}-\gamma_{inj}}} \,. \tag{1.9}$$

The term only depends on the energy ratio and energy difference of the muons. For a fixed survival rate $\frac{N_{ej}}{N_{inj}}$ and fixed injection and ejection energies, i.e., fixed γ_{inj} and γ_{ej}, the acceleration time can be written as

$$\tau_{acc} = -\tau_\mu(\gamma_{ej} - \gamma_{inj}) \ln\left(\frac{N_{ej}}{N_{inj}}\right) / \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) \,. \tag{1.10}$$

The number of turns in the machine for large particle energies, i.e. $\beta \approx 1$, is therefore:

$$\#\text{turns} = \frac{\tau_{acc}}{\tau_{rev}} = \frac{\tau_{acc} \cdot c}{2\pi R} \,. \tag{1.11}$$

1.3 Required Accelerating Gradient

We calculate the required accelerating gradient G_{acc} for a certain survival rate by rearranging Eq. 1.10:

$$\frac{\gamma_{ej} - \gamma_{inj}}{\tau_{acc}} = -\frac{1}{\tau_\mu} \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \,, \tag{1.12}$$

while using that for large kinetic energies ($\gamma \gg 1$) one can approximate $\frac{\gamma_{ej}}{\gamma_{inj}} \approx \frac{E_{ej}}{E_{inj}}$. Inserting the definition of the Lorentz factor gives

$$\frac{E_{ej} - E_{inj}}{m_\mu c^2 \tau_{acc}} = G_{acc}\frac{c}{m_\mu c^2} = -\frac{1}{\tau_\mu} \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \,, \tag{1.13}$$

or

$$G_{acc} = -\frac{1}{\tau_\mu} m_\mu c \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \,, \tag{1.14}$$

where the mass (and energy) is given in units of eV. For a certain required survival rate, the minimum required acceleration gradient therefore only depends on the energy ratio E_{ej}/E_{inj} of the accelerator.

References

[1] F. Batsch, Unit conversion for longitudinal emittances (https://cernbox.cern.ch/index.php/s/lvooRxgtE8pZQ4j).

[2] A. Bogacz, *Muon Acceleration - Linac and RLA*, https://indico.cern.ch/event/973753/contributions/4100198/attachments/2143992/3613246/Linac%20and%20RLA.pdf, 2020