Baseline RCS and RF parameters for IMCC parameter document

F. Batsch, I. Karpov, H. Damerau, R. Calaga, L. Thiele

HEMAC meeting #20
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IMCC parameter paper

- **Comparing Fabian's table and Antoine's - Summary table**
- **Sorting of parameters to be done**

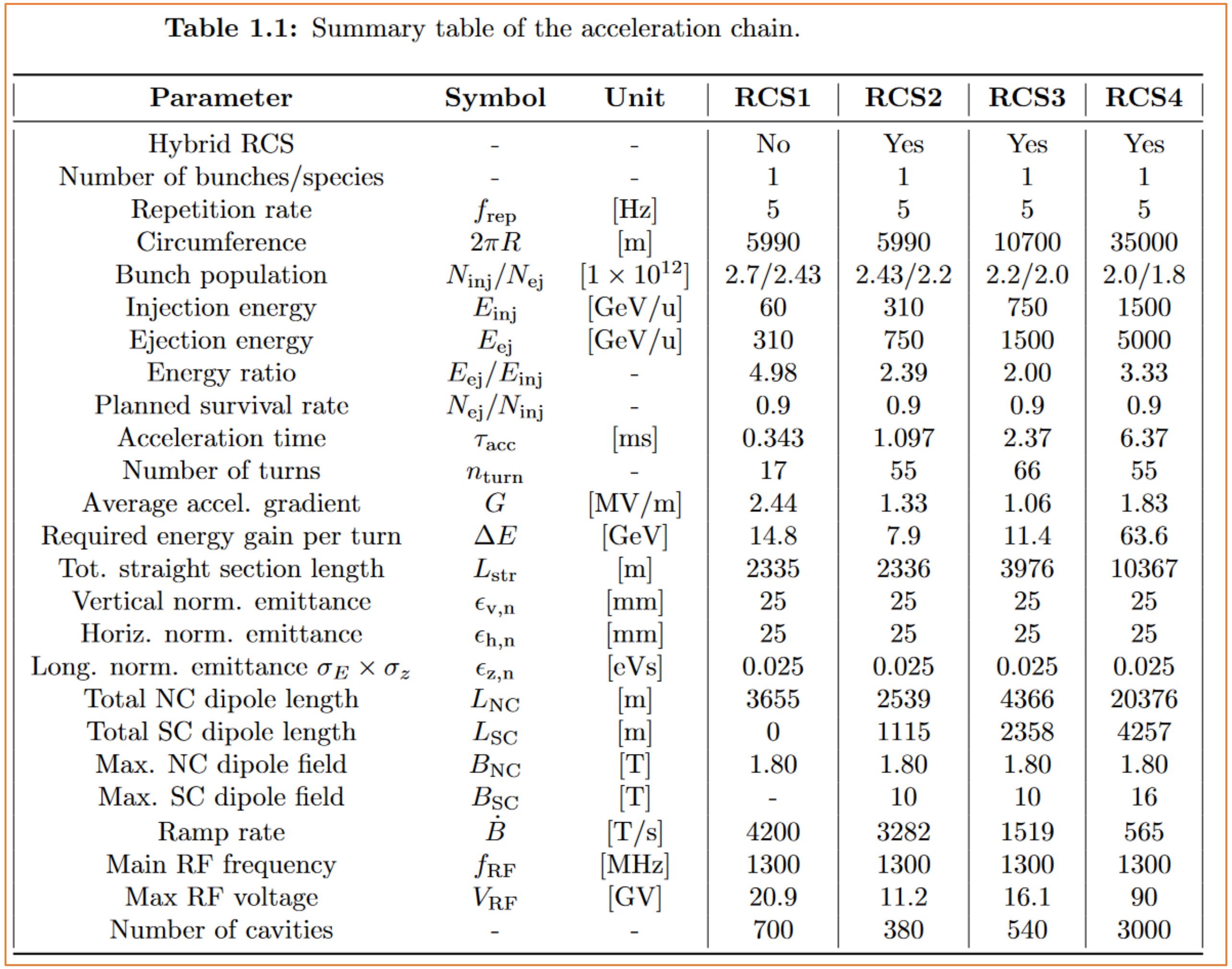

Table 1.1: Summary table of the acceleration chain.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Number of bunches/species	-	-	1	1	1	1
Repetition rate	f_{rep}	[Hz]	5	5	5	5
Circumference	$2\pi R$	[m]	5990	5990	10700	35000
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	$[1 \times 10^{12}]$	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Injection energy	E_{inj}	[GeV/u]	60	310	750	1500
Ejection energy	E_{ej}	[GeV/u]	310	750	1500	5000
Energy ratio	$E_{\text{ej}}/E_{\text{inj}}$	-	4.98	2.39	2.00	3.33
Planned survival rate	$N_{\text{ej}}/N_{\text{inj}}$	-	0.9	0.9	0.9	0.9
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Number of turns	n_{turn}	-	17	55	66	55
Average accel. gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.8	7.9	11.4	63.6
Tot. straight section length	L_{str}	[m]	2335	2336	3976	10367
Vertical norm. emittance	$\epsilon_{\text{v,n}}$	[mm]	25	25	25	25
Horiz. norm. emittance	$\epsilon_{\text{h,n}}$	[mm]	25	25	25	25
Long. norm. emittance $\sigma_E \times \sigma_z$	$\epsilon_{\text{z,n}}$	[eVs]	0.025	0.025	0.025	0.025
Total NC dipole length	L_{NC}	[m]	3655	2539	4366	20376
Total SC dipole length	L_{SC}	[m]	0	1115	2358	4257
Max. NC dipole field	B_{NC}	[T]	1.80	1.80	1.80	1.80
Max. SC dipole field	B_{SC}	[T]	-	10	10	16
Ramp rate	$\dot{B}$	[T/s]	4200	3282	1519	565
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Max RF voltage	V_{RF}	[GV]	20.9	11.2	16.1	90
Number of cavities	-	-	700	380	540	3000

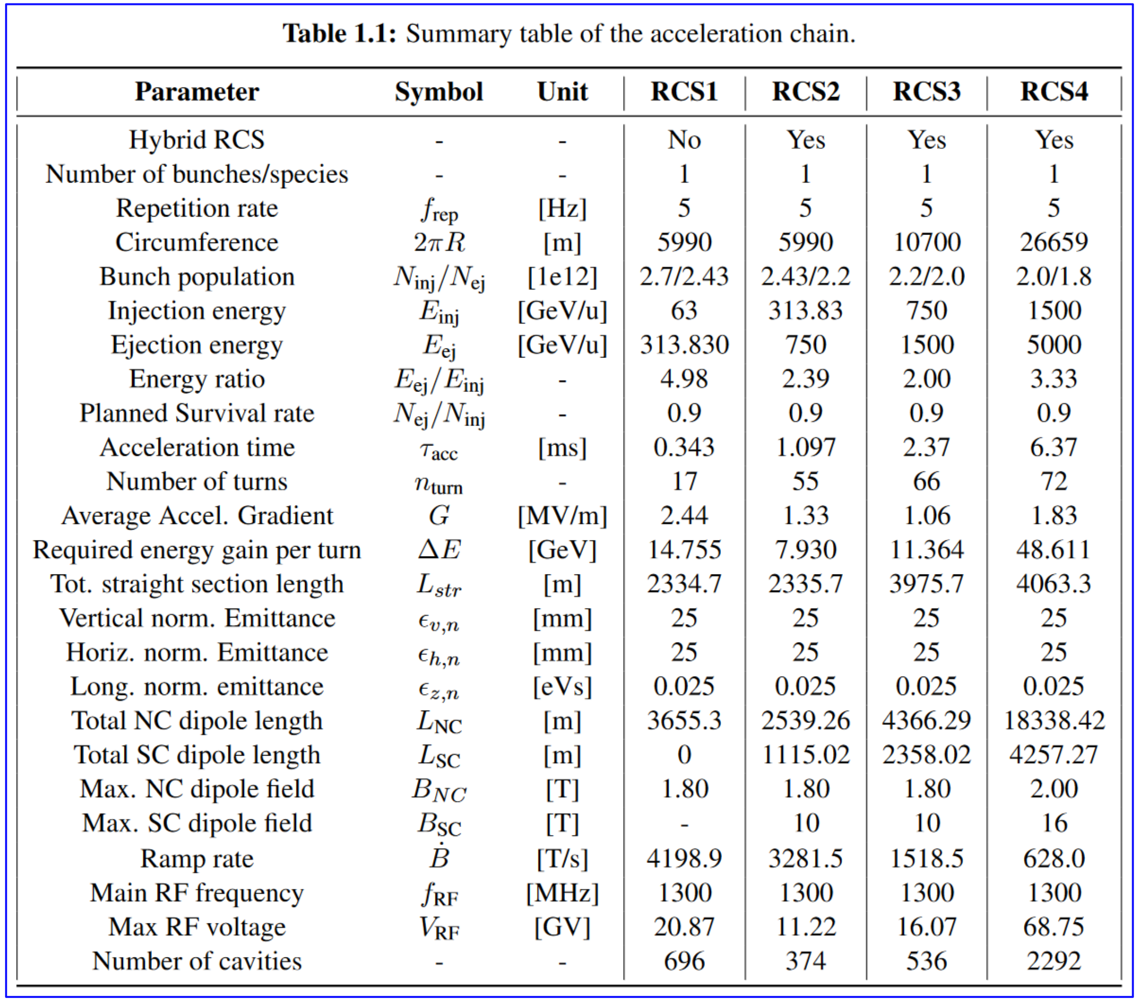

Table 1.1: Summary table of the acceleration chain.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Number of bunches/species	-	-	1	1	1	1
Repetition rate	f_{rep}	[Hz]	5	5	5	5
Circumference	$2\pi R$	[m]	5990	5990	10700	26659
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	[1e12]	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Injection energy	E_{inj}	[GeV/u]	63	313.83	750	1500
Ejection energy	E_{ej}	[GeV/u]	313.830	750	1500	5000
Energy ratio	$E_{\text{ej}}/E_{\text{inj}}$	-	4.98	2.39	2.00	3.33
Planned Survival rate	$N_{\text{ej}}/N_{\text{inj}}$	-	0.9	0.9	0.9	0.9
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Number of turns	n_{turn}	-	17	55	66	72
Average Accel. Gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.755	7.930	11.364	48.611
Tot. straight section length	L_{str}	[m]	2334.7	2335.7	3975.7	4063.3
Vertical norm. Emittance	$\epsilon_{v,n}$	[mm]	25	25	25	25
Horiz. norm. Emittance	$\epsilon_{h,n}$	[mm]	25	25	25	25
Long. norm. emittance	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025
Total NC dipole length	L_{NC}	[m]	3655.3	2539.26	4366.29	18338.42
Total SC dipole length	L_{SC}	[m]	0	1115.02	2358.02	4257.27
Max. NC dipole field	B_{NC}	[T]	1.80	1.80	1.80	2.00
Max. SC dipole field	B_{SC}	[T]	-	10	10	16
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Max RF voltage	V_{RF}	[GV]	20.87	11.22	16.07	68.75
Number of cavities	-	-	696	374	536	2292

IMCC parameter paper

- **Comparing Fabian's table and Antoine's - accelerator chain**
- **Sorting of parameters to be done**

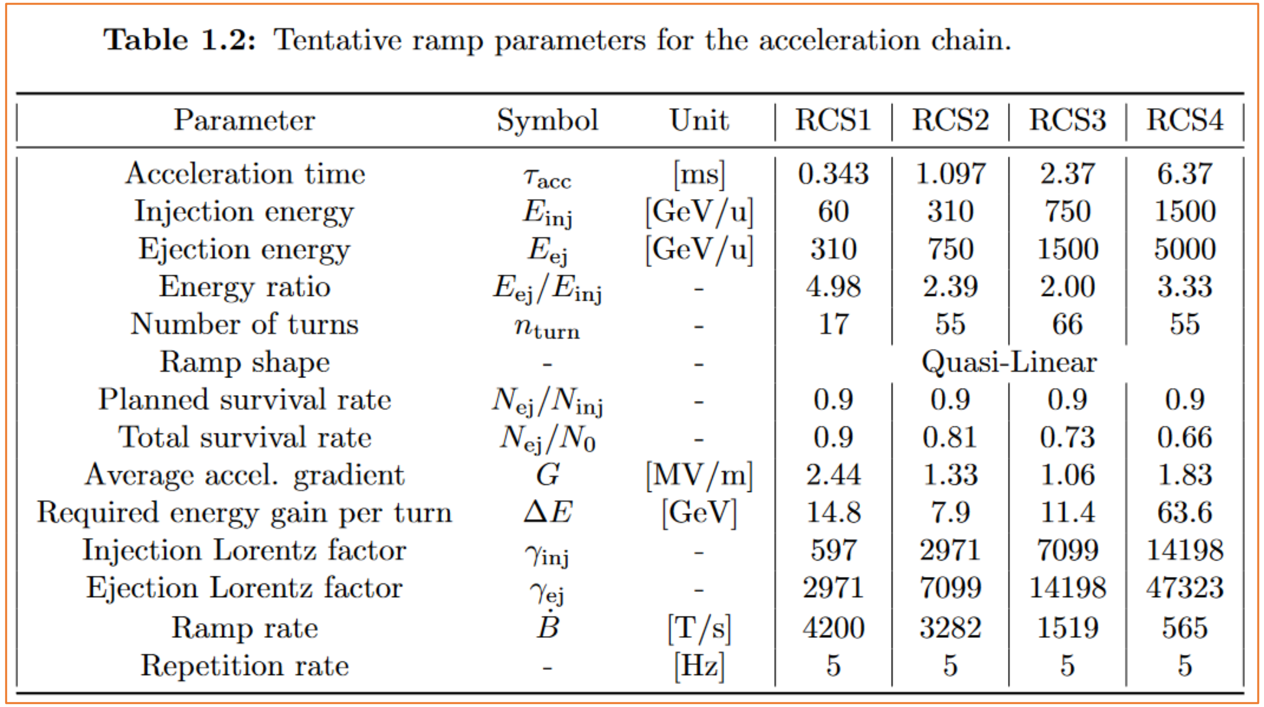

Table 1.2: Tentative ramp parameters for the acceleration chain.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Injection energy	E_{inj}	[GeV/u]	60	310	750	1500
Ejection energy	E_{ej}	[GeV/u]	310	750	1500	5000
Energy ratio	$E_{\text{ej}}/E_{\text{inj}}$	-	4.98	2.39	2.00	3.33
Number of turns	n_{turn}	-	17	55	66	55
Ramp shape	-	-	Quasi-Linear			
Planned survival rate	$N_{\text{ej}}/N_{\text{inj}}$	-	0.9	0.9	0.9	0.9
Total survival rate	N_{ej}/N_0	-	0.9	0.81	0.73	0.66
Average accel. gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.8	7.9	11.4	63.6
Injection Lorentz factor	γ_{inj}	-	597	2971	7099	14198
Ejection Lorentz factor	γ_{ej}	-	2971	7099	14198	47323
Ramp rate	$\dot{B}$	[T/s]	4200	3282	1519	565
Repetition rate	-	[Hz]	5	5	5	5

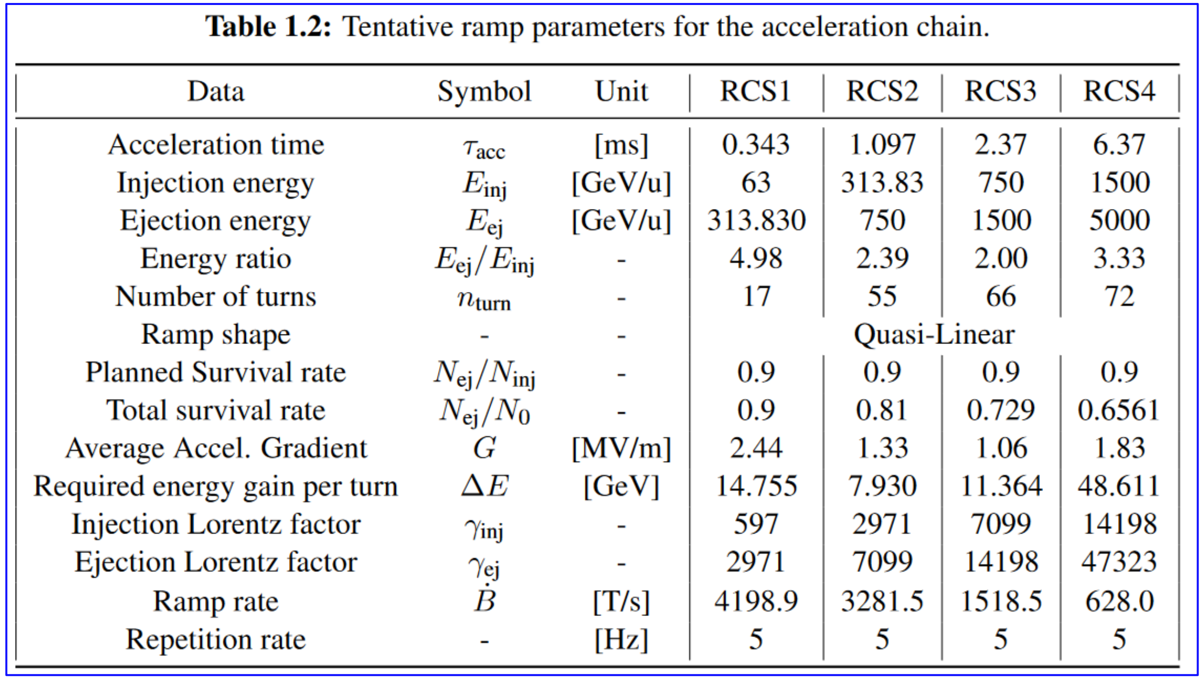

Table 1.2: Tentative ramp parameters for the acceleration chain.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Acceleration time	τ_{acc}	[ms]	0.343	1.097	2.37	6.37
Injection energy	E_{inj}	[GeV/u]	63	313.83	750	1500
Ejection energy	E_{ej}	[GeV/u]	313.830	750	1500	5000
Energy ratio	$E_{\text{ej}}/E_{\text{inj}}$	-	4.98	2.39	2.00	3.33
Number of turns	n_{turn}	-	17	55	66	72
Ramp shape	-	-	Quasi-Linear			
Planned Survival rate	$N_{\text{ej}}/N_{\text{inj}}$	-	0.9	0.9	0.9	0.9
Total survival rate	N_{ej}/N_0	-	0.9	0.81	0.729	0.6561
Average Accel. Gradient	G	[MV/m]	2.44	1.33	1.06	1.83
Required energy gain per turn	ΔE	[GeV]	14.755	7.930	11.364	48.611
Injection Lorentz factor	γ_{inj}	-	597	2971	7099	14198
Ejection Lorentz factor	γ_{ej}	-	2971	7099	14198	47323
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Repetition rate	-	[Hz]	5	5	5	5

IMCC parameter paper

- **Comparing Fabian's table and Antoine's - accelerator chain**
- **Sorting of parameters to be done, not all can be commented on**

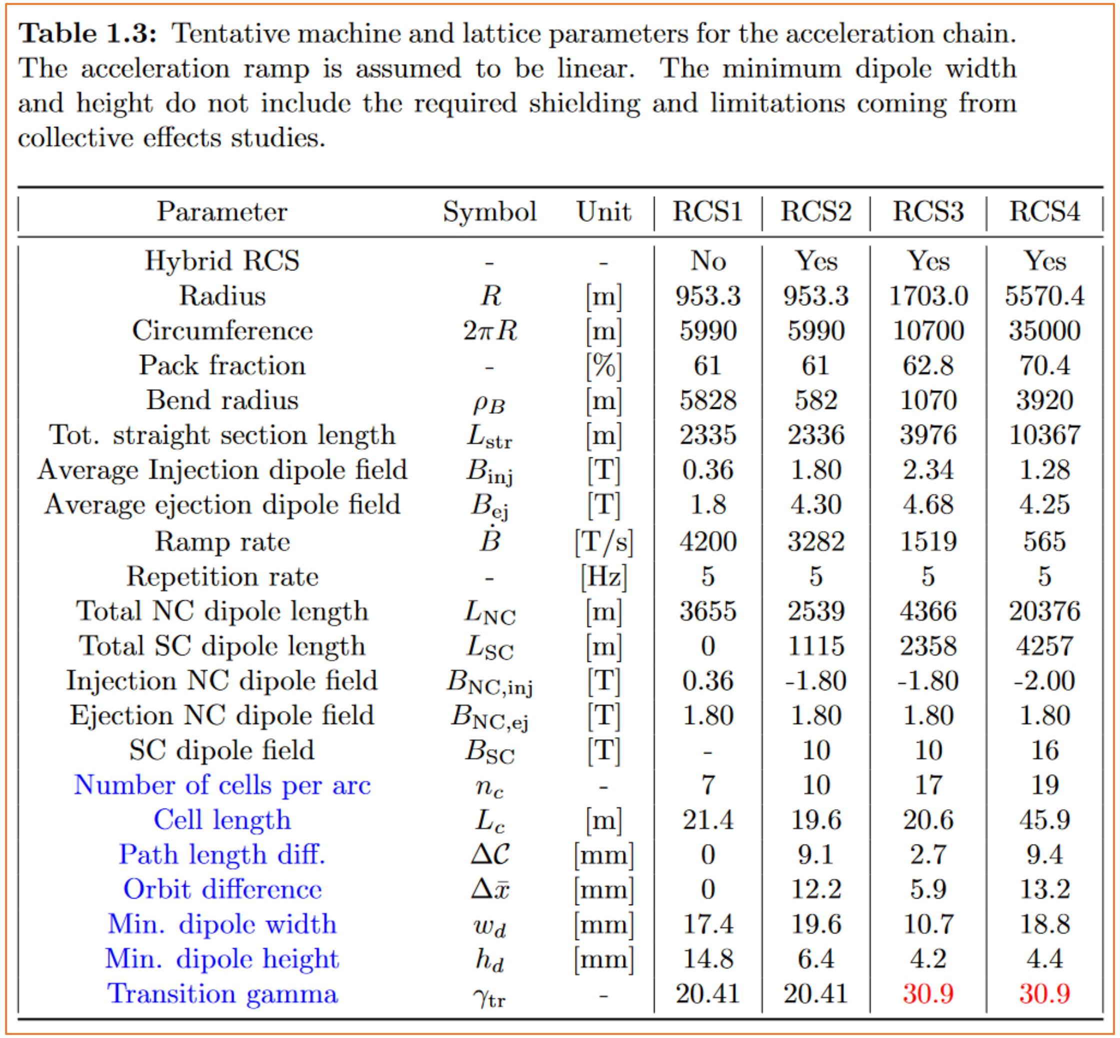

Table 1.3: Tentative machine and lattice parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum dipole width and height do not include the required shielding and limitations coming from collective effects studies.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Radius	R	[m]	953.3	953.3	1703.0	5570.4
Circumference	$2\pi R$	[m]	5990	5990	10700	35000
Pack fraction	-	[%]	61	61	62.8	70.4
Bend radius	ρ_B	[m]	5828	582	1070	3920
Tot. straight section length	L_{str}	[m]	2335	2336	3976	10367
Average Injection dipole field	B_{inj}	[T]	0.36	1.80	2.34	1.28
Average ejection dipole field	B_{ej}	[T]	1.8	4.30	4.68	4.25
Ramp rate	$\dot{B}$	[T/s]	4200	3282	1519	565
Repetition rate	-	[Hz]	5	5	5	5
Total NC dipole length	L_{NC}	[m]	3655	2539	4366	20376
Total SC dipole length	L_{SC}	[m]	0	1115	2358	4257
Injection NC dipole field	$B_{\mathrm{NC,inj}}$	[T]	0.36	-1.80	-1.80	-2.00
Ejection NC dipole field	$B_{\mathrm{NC,ej}}$	[T]	1.80	1.80	1.80	1.80
SC dipole field	B_{SC}	[T]	-	10	10	16
Number of cells per arc	n_c	-	7	10	17	19
Cell length	L_c	[m]	21.4	19.6	20.6	45.9
Path length diff.	$\Delta\mathcal{C}$	[mm]	0	9.1	2.7	9.4
Orbit difference	$\Delta\bar{x}$	[mm]	0	12.2	5.9	13.2
Min. dipole width	w_d	[mm]	17.4	19.6	10.7	18.8
Min. dipole height	h_d	[mm]	14.8	6.4	4.2	4.4
Transition gamma	γ_{tr}	-	20.41	20.41	30.9	30.9

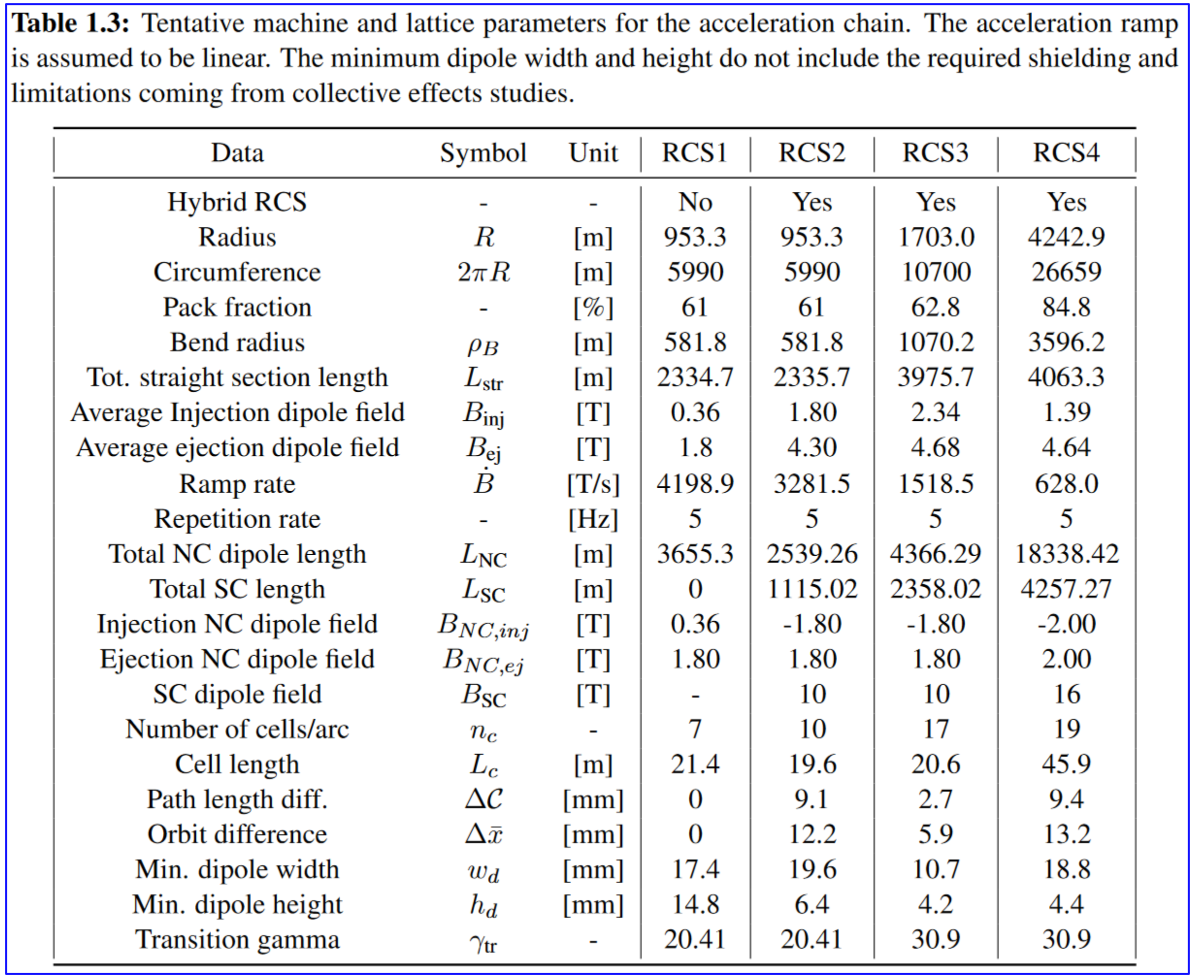

Table 1.3: Tentative machine and lattice parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum dipole width and height do not include the required shielding and limitations coming from collective effects studies.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Hybrid RCS	-	-	No	Yes	Yes	Yes
Radius	R	[m]	953.3	953.3	1703.0	4242.9
Circumference	$2\pi R$	[m]	5990	5990	10700	26659
Pack fraction	-	[%]	61	61	62.8	84.8
Bend radius	ρ_B	[m]	581.8	581.8	1070.2	3596.2
Tot. straight section length	L_{str}	[m]	2334.7	2335.7	3975.7	4063.3
Average Injection dipole field	B_{inj}	[T]	0.36	1.80	2.34	1.39
Average ejection dipole field	B_{ej}	[T]	1.8	4.30	4.68	4.64
Ramp rate	$\dot{B}$	[T/s]	4198.9	3281.5	1518.5	628.0
Repetition rate	-	[Hz]	5	5	5	5
Total NC dipole length	L_{NC}	[m]	3655.3	2539.26	4366.29	18338.42
Total SC length	L_{SC}	[m]	0	1115.02	2358.02	4257.27
Injection NC dipole field	$B_{NC,inj}$	[T]	0.36	-1.80	-1.80	-2.00
Ejection NC dipole field	$B_{NC,ej}$	[T]	1.80	1.80	1.80	2.00
SC dipole field	B_{SC}	[T]	-	10	10	16
Number of cells/arc	n_c	-	7	10	17	19
Cell length	L_c	[m]	21.4	19.6	20.6	45.9
Path length diff.	$\Delta\mathcal{C}$	[mm]	0	9.1	2.7	9.4
Orbit difference	$\Delta\bar{x}$	[mm]	0	12.2	5.9	13.2
Min. dipole width	w_d	[mm]	17.4	19.6	10.7	18.8
Min. dipole height	h_d	[mm]	14.8	6.4	4.2	4.4
Transition gamma	γ_{tr}	-	20.41	20.41	30.9	30.9

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- **Comparing Fabian's table and Antoine's - beam parameter**
- **Sorting of parameters to be done, some units to be discussed**

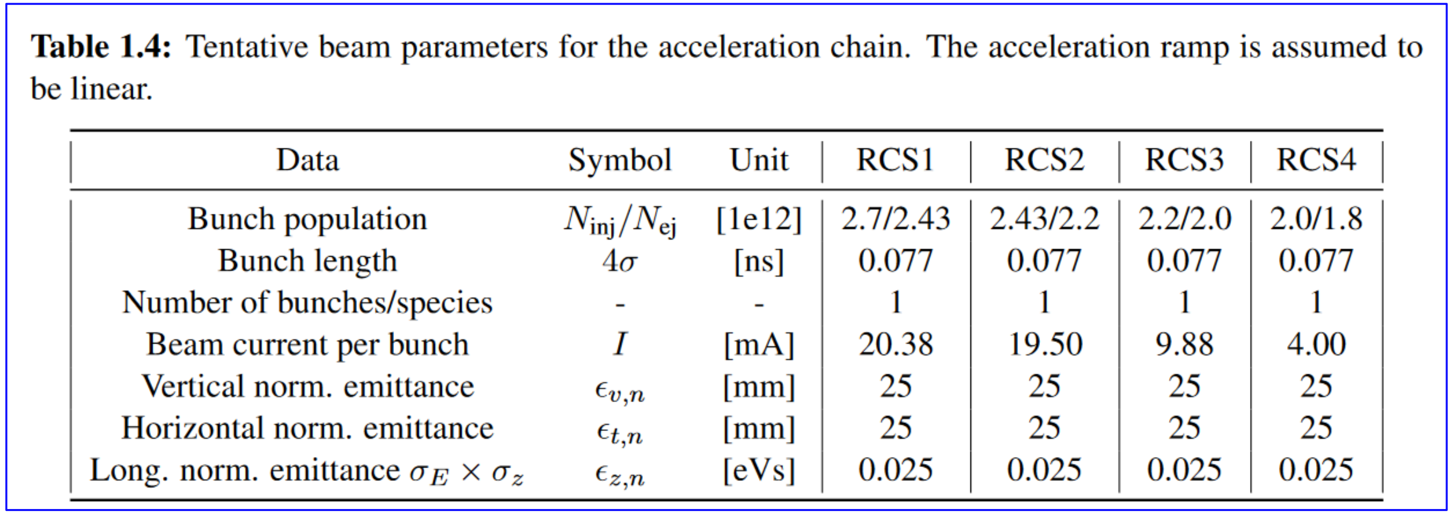

Table 1.4: Tentative beam parameters for the acceleration chain. The acceleration ramp is assumed to be linear.

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	[1e12]	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Bunch length	4σ	[ns]	0.077	0.077	0.077	0.077
Number of bunches/species	-	-	1	1	1	1
Beam current per bunch	I	[mA]	20.38	19.50	9.88	4.00
Vertical norm. emittance	$\epsilon_{v,n}$	[mm]	25	25	25	25
Horizontal norm. emittance	$\epsilon_{t,n}$	[mm]	25	25	25	25
Long. norm. emittance $\sigma_E \times \sigma_z$	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025

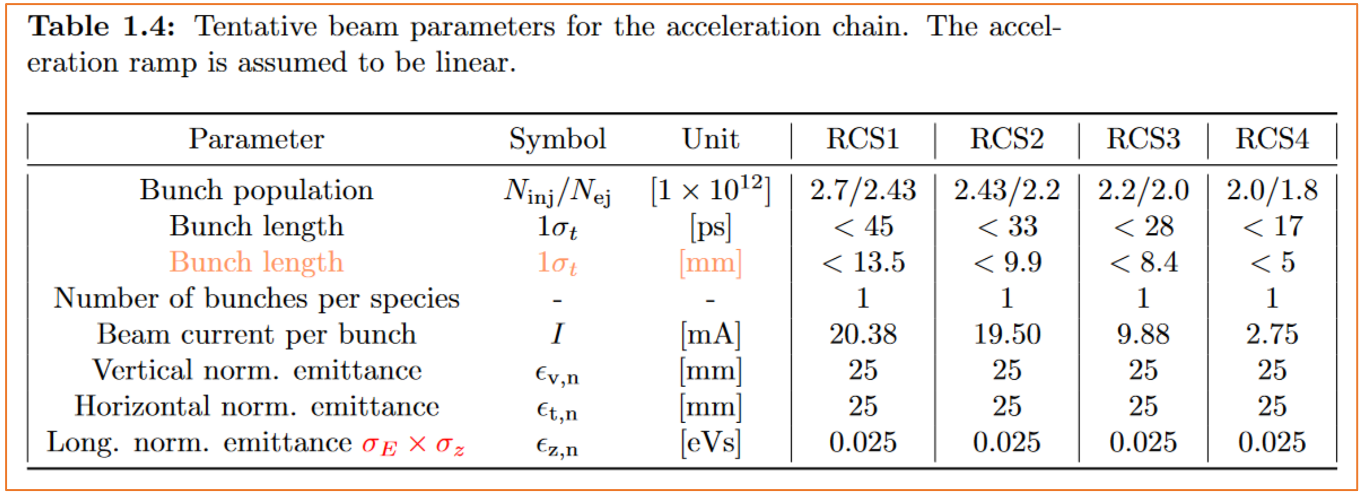

Table 1.4: Tentative beam parameters for the acceleration chain. The acceleration ramp is assumed to be linear.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Bunch population	$N_{\text{inj}}/N_{\text{ej}}$	$[1 \times 10^{12}]$	2.7/2.43	2.43/2.2	2.2/2.0	2.0/1.8
Bunch length	$1\sigma_t$	[ps]	< 45	< 33	< 28	< 17
Bunch length	$1\sigma_t$	[mm]	< 13.5	< 9.9	< 8.4	< 5
Number of bunches per species	-	-	1	1	1	1
Beam current per bunch	I	[mA]	20.38	19.50	9.88	2.75
Vertical norm. emittance	$\epsilon_{\text{v,n}}$	[mm]	25	25	25	25
Horizontal norm. emittance	$\epsilon_{\text{t,n}}$	[mm]	25	25	25	25
Long. norm. emittance $\sigma_E \times \sigma_z$	$\epsilon_{\text{z,n}}$	[eVs]	0.025	0.025	0.025	0.025

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- **Comparing Fabian's table and Antoine's - RF parameter**
- **Sorting of parameters to be done, not all can be commented on**

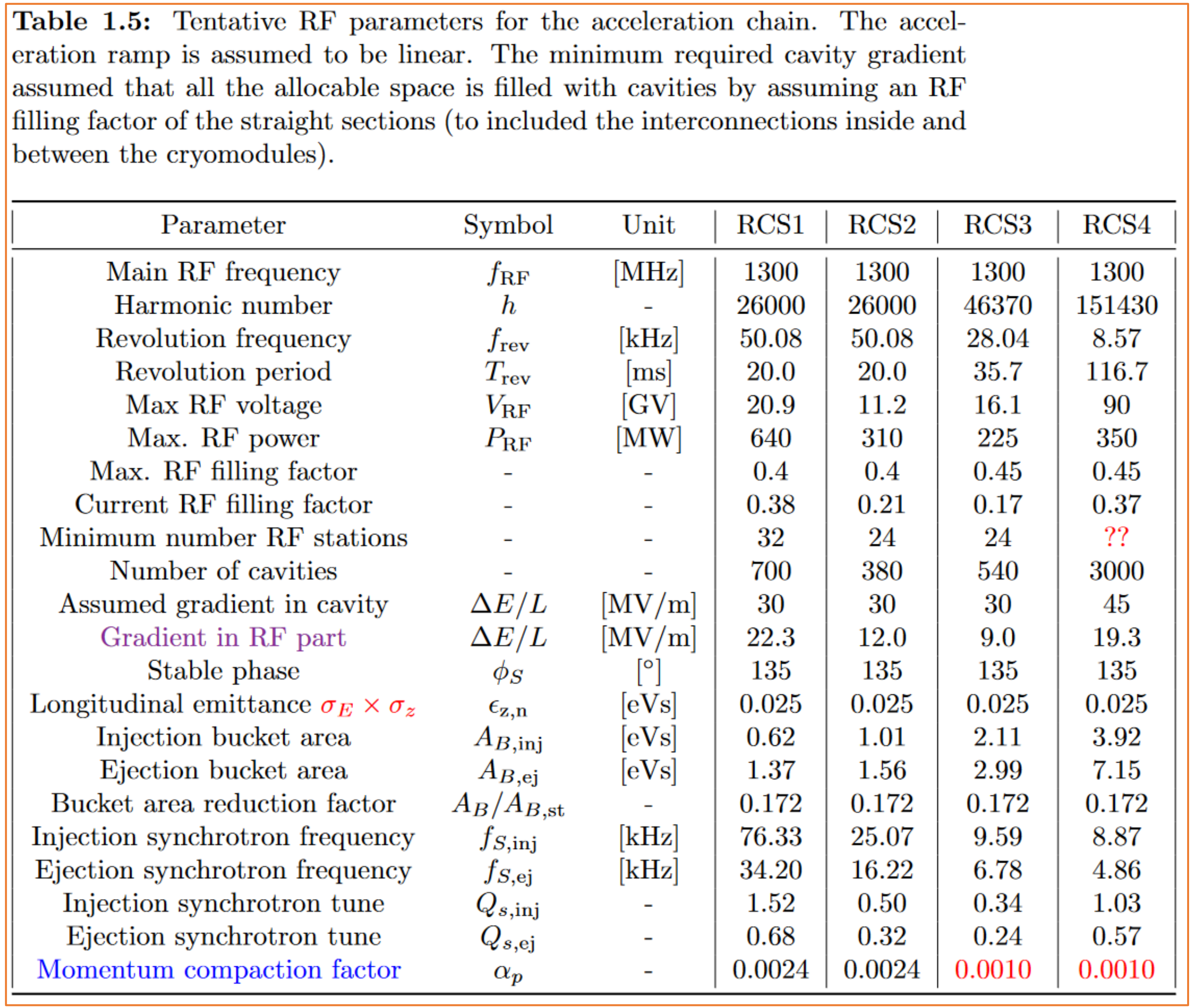

Table 1.5: Tentative RF parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum required cavity gradient assumed that all the allocable space is filled with cavities by assuming an RF filling factor of the straight sections (to included the interconnections inside and between the cryomodules).

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Harmonic number	h	-	26000	26000	46370	151430
Revolution frequency	f_{rev}	[kHz]	50.08	50.08	28.04	8.57
Revolution period	T_{rev}	[ms]	20.0	20.0	35.7	116.7
Max RF voltage	V_{RF}	[GV]	20.9	11.2	16.1	90
Max. RF power	P_{RF}	[MW]	640	310	225	350
Max. RF filling factor	-	-	0.4	0.4	0.45	0.45
Current RF filling factor	-	-	0.38	0.21	0.17	0.37
Minimum number RF stations	-	-	32	24	24	??
Number of cavities	-	-	700	380	540	3000
Assumed gradient in cavity	$\Delta E/L$	[MV/m]	30	30	30	45
Gradient in RF part	$\Delta E/L$	[MV/m]	22.3	12.0	9.0	19.3
Stable phase	ϕ_S	[°]	135	135	135	135
Longitudinal emittance $\sigma_E \times \sigma_z$	$\epsilon_{\mathrm{z,n}}$	[eVs]	0.025	0.025	0.025	0.025
Injection bucket area	$A_{B,\mathrm{inj}}$	[eVs]	0.62	1.01	2.11	3.92
Ejection bucket area	$A_{B,\mathrm{ej}}$	[eVs]	1.37	1.56	2.99	7.15
Bucket area reduction factor	$A_B/A_{B,\mathrm{st}}$	-	0.172	0.172	0.172	0.172
Injection synchrotron frequency	$f_{S,\mathrm{inj}}$	[kHz]	76.33	25.07	9.59	8.87
Ejection synchrotron frequency	$f_{S,\mathrm{ej}}$	[kHz]	34.20	16.22	6.78	4.86
Injection synchrotron tune	$Q_{s,\mathrm{inj}}$	-	1.52	0.50	0.34	1.03
Ejection synchrotron tune	$Q_{s,\mathrm{ej}}$	-	0.68	0.32	0.24	0.57
Momentum compaction factor	α_p	-	0.0024	0.0024	0.0010	0.0010

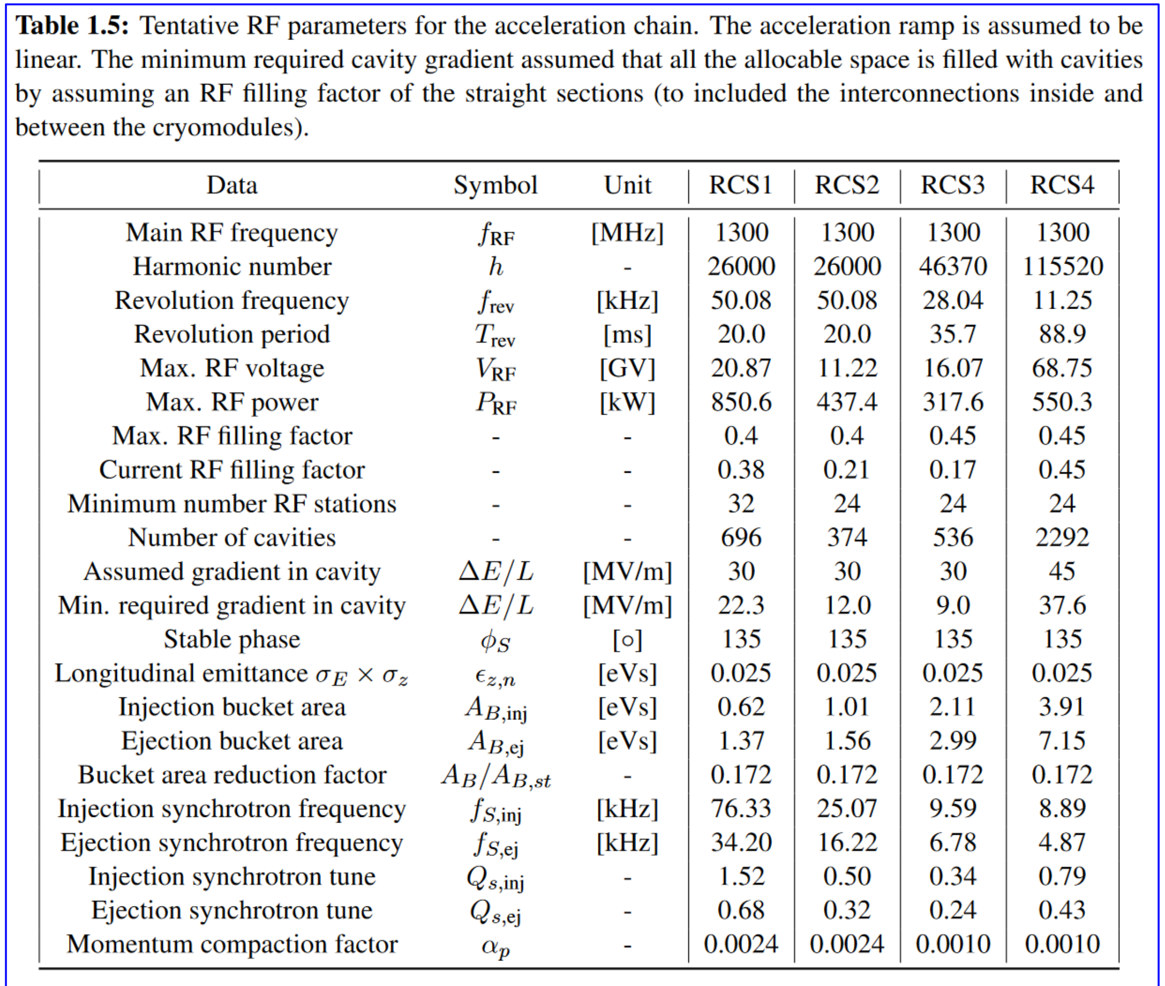

Table 1.5: Tentative RF parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum required cavity gradient assumed that all the allocable space is filled with cavities by assuming an RF filling factor of the straight sections (to included the interconnections inside and between the cryomodules).

Data	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Harmonic number	h	-	26000	26000	46370	115520
Revolution frequency	f_{rev}	[kHz]	50.08	50.08	28.04	11.25
Revolution period	T_{rev}	[ms]	20.0	20.0	35.7	88.9
Max. RF voltage	V_{RF}	[GV]	20.87	11.22	16.07	68.75
Max. RF power	P_{RF}	[kW]	850.6	437.4	317.6	550.3
Max. RF filling factor	-	-	0.4	0.4	0.45	0.45
Current RF filling factor	-	-	0.38	0.21	0.17	0.45
Minimum number RF stations	-	-	32	24	24	24
Number of cavities	-	-	696	374	536	2292
Assumed gradient in cavity	$\Delta E/L$	[MV/m]	30	30	30	45
Min. required gradient in cavity	$\Delta E/L$	[MV/m]	22.3	12.0	9.0	37.6
Stable phase	ϕ_S	[$\circ$]	135	135	135	135
Longitudinal emittance $\sigma_E \times \sigma_z$	$\epsilon_{z,n}$	[eVs]	0.025	0.025	0.025	0.025
Injection bucket area	$A_{B,\mathrm{inj}}$	[eVs]	0.62	1.01	2.11	3.91
Ejection bucket area	$A_{B,\mathrm{ej}}$	[eVs]	1.37	1.56	2.99	7.15
Bucket area reduction factor	$A_B/A_{B,st}$	-	0.172	0.172	0.172	0.172
Injection synchrotron frequency	$f_{S,\mathrm{inj}}$	[kHz]	76.33	25.07	9.59	8.89
Ejection synchrotron frequency	$f_{S,\mathrm{ej}}$	[kHz]	34.20	16.22	6.78	4.87
Injection synchrotron tune	$Q_{s,\mathrm{inj}}$	-	1.52	0.50	0.34	0.79
Ejection synchrotron tune	$Q_{s,\mathrm{ej}}$	-	0.68	0.32	0.24	0.43
Momentum compaction factor	α_p	-	0.0024	0.0024	0.0010	0.0010

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- **RF parameter table**
- **This parameters could move from Chapter 7 to Chapter 12**
- **Tables by Leonard:**

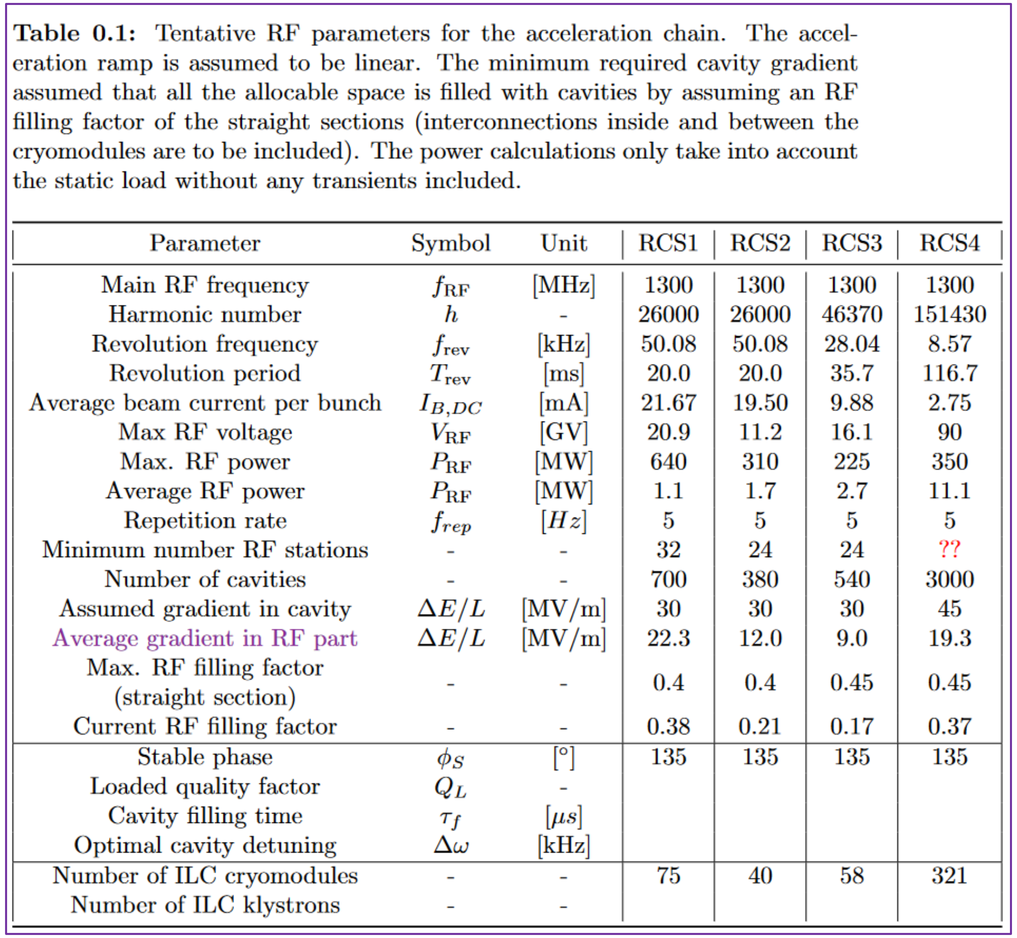


Table 0.1: Tentative RF parameters for the acceleration chain. The acceleration ramp is assumed to be linear. The minimum required cavity gradient assumed that all the allocable space is filled with cavities by assuming an RF filling factor of the straight sections (interconnections inside and between the cryomodules are to be included). The power calculations only take into account the static load without any transients included.

Parameter	Symbol	Unit	RCS1	RCS2	RCS3	RCS4
Main RF frequency	f_{RF}	[MHz]	1300	1300	1300	1300
Harmonic number	h	-	26000	26000	46370	151430
Revolution frequency	f_{rev}	[kHz]	50.08	50.08	28.04	8.57
Revolution period	T_{rev}	[ms]	20.0	20.0	35.7	116.7
Average beam current per bunch	$I_{B,DC}$	[mA]	21.67	19.50	9.88	2.75
Max RF voltage	V_{RF}	[GV]	20.9	11.2	16.1	90
Max. RF power	P_{RF}	[MW]	640	310	225	350
Average RF power	P_{RF}	[MW]	1.1	1.7	2.7	11.1
Repetition rate	f_{rep}	$[Hz]$	5	5	5	5
Minimum number RF stations	-	-	32	24	24	??
Number of cavities	-	-	700	380	540	3000
Assumed gradient in cavity	$\Delta E/L$	[MV/m]	30	30	30	45
Average gradient in RF part	$\Delta E/L$	[MV/m]	22.3	12.0	9.0	19.3
Max. RF filling factor (straight section)	-	-	0.4	0.4	0.45	0.45
Current RF filling factor	-	-	0.38	0.21	0.17	0.37
Stable phase	ϕ_S	[°]	135	135	135	135
Loaded quality factor	Q_L	-				
Cavity filling time	τ_f	[μs]				
Optimal cavity detuning	$\Delta\omega$	[kHz]				
Number of ILC cryomodules	-	-	75	40	58	321
Number of ILC klystrons	-	-				

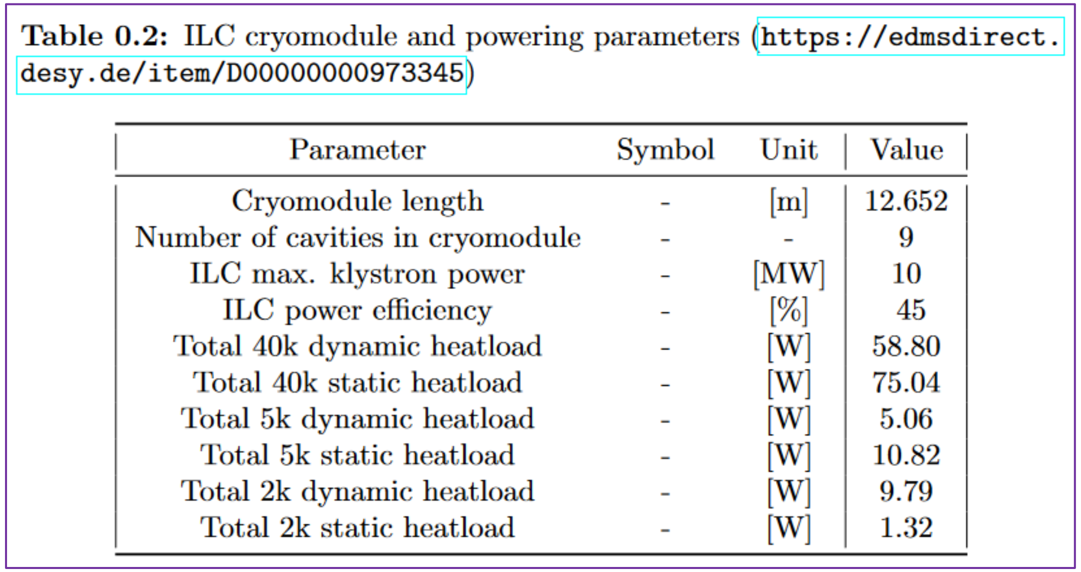


Table 0.2: ILC cryomodule and powering parameters (`https://edmsdirect.desy.de/item/D00000000973345`)

Parameter	Symbol	Unit	Value
Cryomodule length	-	[m]	12.652
Number of cavities in cryomodule	-	-	9
ILC max. klystron power	-	[MW]	10
ILC power efficiency	-	[%]	45
Total 40k dynamic heatload	-	[W]	58.80
Total 40k static heatload	-	[W]	75.04
Total 5k dynamic heatload	-	[W]	5.06
Total 5k static heatload	-	[W]	10.82
Total 2k dynamic heatload	-	[W]	9.79
Total 2k static heatload	-	[W]	1.32

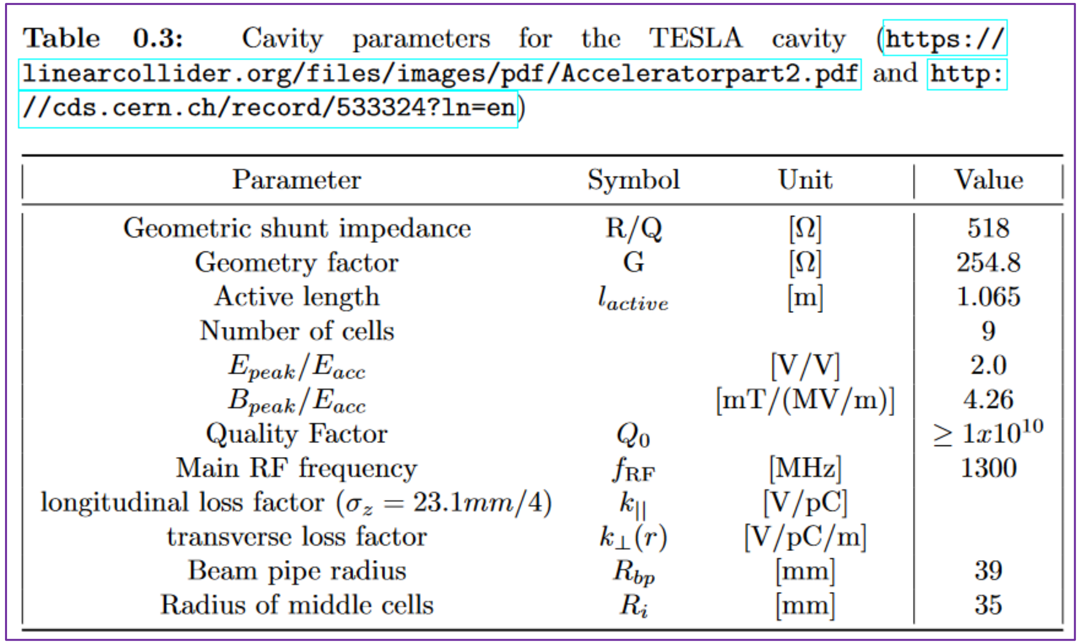


Table 0.3: Cavity parameters for the TESLA cavity (`https://linearcollider.org/files/images/pdf/Acceleratorpart2.pdf` and `http://cds.cern.ch/record/533324?ln=en`)

Parameter	Symbol	Unit	Value
Geometric shunt impedance	R/Q	[Ω]	518
Geometry factor	G	[Ω]	254.8
Active length	l_{active}	[m]	1.065
Number of cells			9
E_{peak}/E_{acc}		[V/V]	2.0
B_{peak}/E_{acc}		[mT/(MV/m)]	4.26
Quality Factor	Q_0		$\geq 1x10^{10}$
Main RF frequency	f_{RF}	[MHz]	1300
longitudinal loss factor ($\sigma_z = 23.1mm/4$)	$k_{\|\|}$	[V/pC]	
transverse loss factor	$k_{\perp}(r)$	[V/pC/m]	
Beam pipe radius	R_{bp}	[mm]	39
Radius of middle cells	R_i	[mm]	35

IMCC parameter paper

- **Other text elements to be discussed**

WHAT TO DO WITH THE FOLLOWING PARTS?

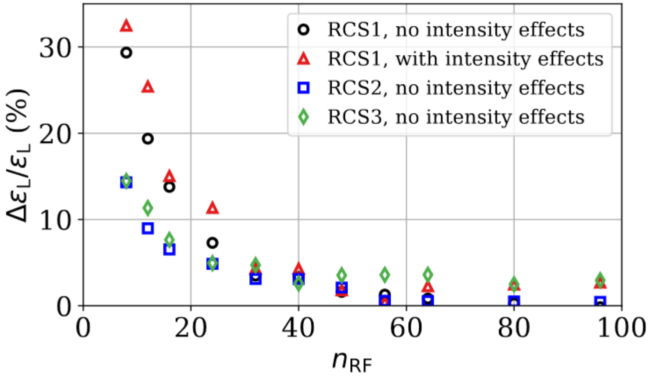
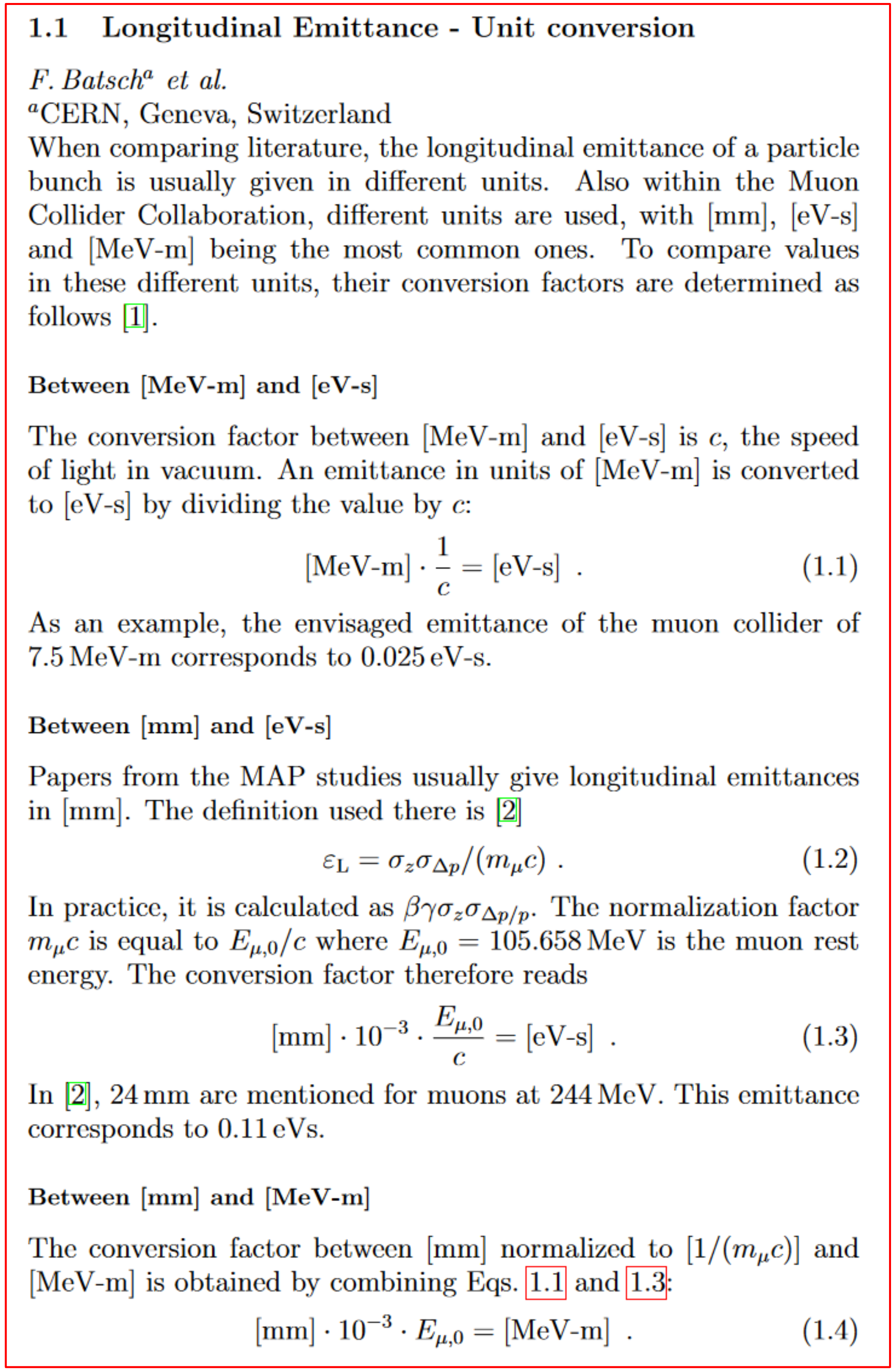


Fig. 1.1: Relative emittance growth at the end of the cycle with respect to the emittance at injection versus n_{RF} for RCS1 without (black circle) and with intensity effects (red triangle), and for RCS2 and RCS3 without intensity effects.



1.1 Longitudinal Emittance - Unit conversion

F. Batsch[a] *et al.*

[a]CERN, Geneva, Switzerland

When comparing literature, the longitudinal emittance of a particle bunch is usually given in different units. Also within the Muon Collider Collaboration, different units are used, with [mm], [eV-s] and [MeV-m] being the most common ones. To compare values in these different units, their conversion factors are determined as follows [1].

Between [MeV-m] and [eV-s]

The conversion factor between [MeV-m] and [eV-s] is c, the speed of light in vacuum. An emittance in units of [MeV-m] is converted to [eV-s] by dividing the value by c:

$$[\text{MeV-m}] \cdot \frac{1}{c} = [\text{eV-s}] \quad . \tag{1.1}$$

As an example, the envisaged emittance of the muon collider of 7.5 MeV-m corresponds to 0.025 eV-s.

Between [mm] and [eV-s]

Papers from the MAP studies usually give longitudinal emittances in [mm]. The definition used there is [2]

$$\varepsilon_L = \sigma_z \sigma_{\Delta p}/(m_\mu c) \ . \tag{1.2}$$

In practice, it is calculated as $\beta\gamma\sigma_z\sigma_{\Delta p/p}$. The normalization factor $m_\mu c$ is equal to $E_{\mu,0}/c$ where $E_{\mu,0} = 105.658$ MeV is the muon rest energy. The conversion factor therefore reads

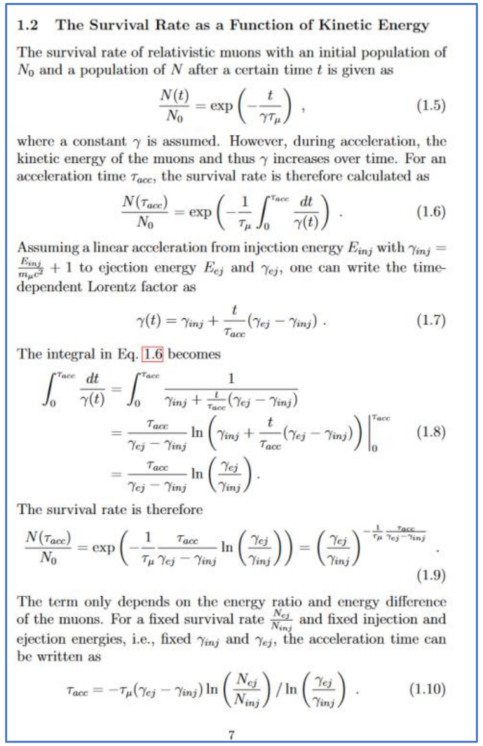
$$[\text{mm}] \cdot 10^{-3} \cdot \frac{E_{\mu,0}}{c} = [\text{eV-s}] \quad . \tag{1.3}$$

In [2], 24 mm are mentioned for muons at 244 MeV. This emittance corresponds to 0.11 eVs.

Between [mm] and [MeV-m]

The conversion factor between [mm] normalized to $[1/(m_\mu c)]$ and [MeV-m] is obtained by combining Eqs. 1.1 and 1.3:

$$[\text{mm}] \cdot 10^{-3} \cdot E_{\mu,0} = [\text{MeV-m}] \quad . \tag{1.4}$$



1.2 The Survival Rate as a Function of Kinetic Energy

The survival rate of relativistic muons with an initial population of N_0 and a population of N after a certain time t is given as

$$\frac{N(t)}{N_0} = \exp\left(-\frac{t}{\gamma\tau_\mu}\right) \ , \tag{1.5}$$

where a constant γ is assumed. However, during acceleration, the kinetic energy of the muons and thus γ increases over time. For an acceleration time τ_{acc}, the survival rate is therefore calculated as

$$\frac{N(\tau_{acc})}{N_0} = \exp\left(-\frac{1}{\tau_\mu}\int_0^{\tau_{acc}} \frac{dt}{\gamma(t)}\right) \ . \tag{1.6}$$

Assuming a linear acceleration from injection energy E_{inj} with $\gamma_{inj} = \frac{E_{inj}}{m_\mu c^2} + 1$ to ejection energy E_{ej} and γ_{ej}, one can write the time-dependent Lorentz factor as

$$\gamma(t) = \gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj}) \ . \tag{1.7}$$

The integral in Eq. 1.6 becomes

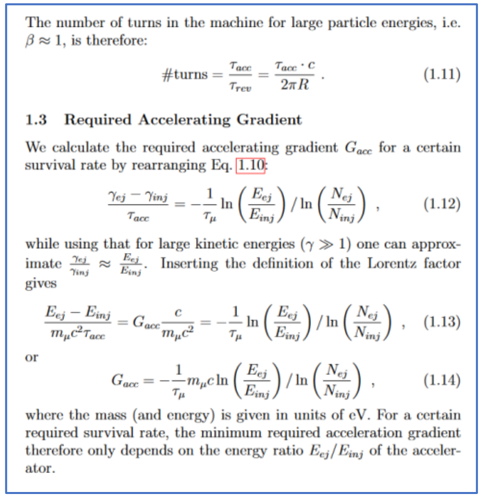
$$\begin{aligned} \int_0^{\tau_{acc}} \frac{dt}{\gamma(t)} &= \int_0^{\tau_{acc}} \frac{1}{\gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj})} \\ &= \frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\gamma_{inj} + \frac{t}{\tau_{acc}}(\gamma_{ej} - \gamma_{inj})\right)\Big|_0^{\tau_{acc}} \qquad (1.8) \\ &= \frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) . \end{aligned}$$

The survival rate is therefore

$$\frac{N(\tau_{acc})}{N_0} = \exp\left(-\frac{1}{\tau_\mu}\frac{\tau_{acc}}{\gamma_{ej} - \gamma_{inj}} \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right)\right) = \left(\frac{\gamma_{ej}}{\gamma_{inj}}\right)^{-\frac{1}{\tau_\mu}\frac{\tau_{acc}}{\gamma_{ej}-\gamma_{inj}}} \ . \tag{1.9}$$

The term only depends on the energy ratio and energy difference of the muons. For a fixed survival rate $\frac{N_{ej}}{N_{inj}}$ and fixed injection and ejection energies, i.e., fixed γ_{inj} and γ_{ej}, the acceleration time can be written as

$$\tau_{acc} = -\tau_\mu(\gamma_{ej} - \gamma_{inj}) \ln\left(\frac{N_{ej}}{N_{inj}}\right) / \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) \ . \tag{1.10}$$



The number of turns in the machine for large particle energies, i.e. $\beta \approx 1$, is therefore:

$$\#\text{turns} = \frac{\tau_{acc}}{\tau_{rev}} = \frac{\tau_{acc} \cdot c}{2\pi R} \ . \tag{1.11}$$

1.3 Required Accelerating Gradient

We calculate the required accelerating gradient G_{acc} for a certain survival rate by rearranging Eq. 1.10:

$$\frac{\gamma_{ej} - \gamma_{inj}}{\tau_{acc}} = -\frac{1}{\tau_\mu} \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \ , \tag{1.12}$$

while using that for large kinetic energies ($\gamma \gg 1$) one can approximate $\frac{\gamma_{ej}}{\gamma_{inj}} \approx \frac{E_{ej}}{E_{inj}}$. Inserting the definition of the Lorentz factor gives

$$\frac{E_{ej} - E_{inj}}{m_\mu c^2 \tau_{acc}} = G_{acc}\frac{c}{m_\mu c^2} = -\frac{1}{\tau_\mu} \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \ , \tag{1.13}$$

or

$$G_{acc} = -\frac{1}{\tau_\mu} m_\mu c \ln\left(\frac{E_{ej}}{E_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) \ , \tag{1.14}$$

where the mass (and energy) is given in units of eV. For a certain required survival rate, the minimum required acceleration gradient therefore only depends on the energy ratio E_{ej}/E_{inj} of the accelerator.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