One-loop power spectrum in ultra slow-roll inflation and implications for primordial black hole dark matter

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based on
arXiv:2404.07196, with G. Ballesteros

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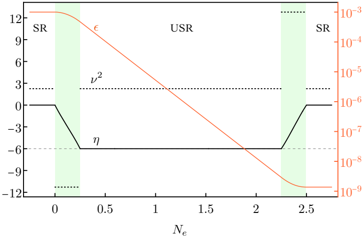
Motivation

- PBH as a DM candidate $\Rightarrow \mathcal{P}_\zeta \sim 10^{-2} \gg \mathcal{P}_\zeta^{\rm CMB}$
- Enhancement of $\mathcal{P}_\zeta \Rightarrow$ USR inflationary model
- If ζ is large, is perturbation theory still valid?
- Kristiano & Yokoyama arXiv:2211.03395: loop correction larger than the tree-level $\Rightarrow$ ruling out PBHs
- In our work, we include all the interactions and we renormalize $\mathcal{P}_\zeta$ with counterterms

The validity of PT depends on the duration of the transition between SR and USR

Model of inflation



- Background evolution

SR - USR (ΔN) - SR + smooth transitions (δN)

- Fluctuations

  Scalar dof in the scalar field: $\delta\phi$-gauge

- In our model, at leading order in $\epsilon = -\dot{H}/H^2$ ($\ll 1$), the interactions are $\mathcal{L} \supset \sqrt{-g}\, V(\phi) \sim a^4\, V_3\, \delta\phi^3 + a^4\, V_4\, \delta\phi^4$

$$V_2 = -H^2(\nu^2 - 9/4)\,, \quad V_n \sim \frac{\mathrm{d}^{(n-2)}\nu^2}{\mathrm{d}\tau^{(n-2)}}\,, \quad \nu^2 \equiv \frac{9}{4} + \frac{1}{2}\left(3\,\eta + \frac{\eta^2}{2} + \frac{\eta'}{aH}\right)$$

- We impose that $\nu^2 = \text{const.} \Rightarrow V_{3,4} \sim \Delta\nu^2\delta(\tau - \tau_*)$
- Instantaneous transitions: $\nu^2 \xrightarrow{\delta N \to 0} \pm 3/\delta N \Rightarrow$ interactions diverge

In-in formalism

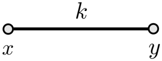
- Our goal is to calculate $\mathcal{P}_\zeta$ at one-loop

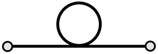
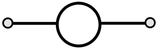
$$\mathcal{P}_\zeta(\tau, k) = \int \frac{\mathrm{d}^3 \mathrm{r}}{(2\pi)^3} e^{-ik\mathrm{r}} \frac{4\pi k^3}{2\epsilon(\tau) M_P^2} \langle \delta\phi(\mathrm{x}+\mathrm{r})\delta\phi(\mathrm{x})\rangle$$

- In-in formalism:

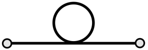
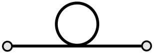
$$\langle \delta\phi(\mathrm{x})\delta\phi(\mathrm{y})\rangle = \langle 0| \left(F(t, -\infty^-)\right)^\dagger \delta\phi(\mathrm{x})\delta\phi(\mathrm{y}) F(t, -\infty^-) |0\rangle$$

$$F(t, -\infty^-) = T \exp\left(-i \int_{-\infty(1-i\omega)}^{t} \mathrm{d}t' H_I(t') \right)$$

$\mathcal{O}(H_I^0)$: 

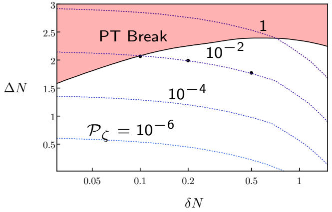
$\mathcal{O}(H_I^1)$: 
$\mathcal{O}(H_I^2)$: 

- The one-loop diverges in the UV $\Rightarrow$ we regularise with a cutoff Λ_{UV}
- Counterterms to absorb the divergences: $H_{ct} \sim \delta\phi^2$

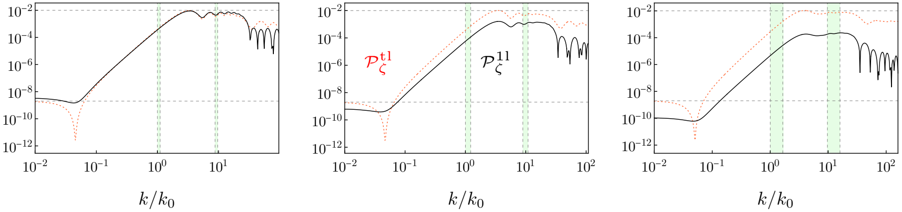
$\mathcal{O}(H_I^1)$:  $\Rightarrow$ $\mathcal{O}(H_I^1)$:  

Results: Analysis of the validity of perturbation theory

- Is perturbation theory valid? $\Rightarrow$ We need to compare $\mathcal{P}_\zeta^{1l}$ with $\mathcal{P}_\zeta^{tl}$



- The validity of PT depends on the values chosen for $\{\Delta N\,, \delta N\}$
- If $\delta N > 0.1 \Rightarrow$ PT is valid!



- The one-loop contribution grows as δN decreases ($\nu^2 \sim 1/\delta N$)

Dependence on δN in the $\delta N \to 0$ limit

- In previous work, the one-loop is finite in the limit $\delta N \to 0$
- Dominant interaction in the ζ-gauge

$$S \supset \int \mathrm{d}^4 x\, M_P^2\, a^2\, \epsilon\, \frac{\eta'}{2}\, \zeta'\, \zeta^2$$

- This interaction induces a cubic and a quartic interaction Hamiltonian

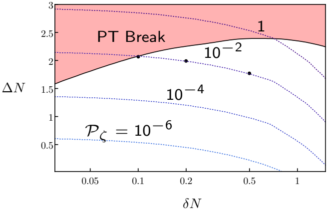
$$H_I(\tau) = \int \mathrm{d}^3 x\, M_P^2 a^2 \epsilon \left(-\frac{\eta'}{2} \zeta' \zeta^2 + \frac{(\eta')^2}{16} \zeta^4 \right)$$

The term $H \sim (\eta')^2\, \zeta^4$ is responsible for the divergence of $\mathcal{P}_\zeta$ when $\delta N \to 0$

Including the induced quartic term, the result coincides with that calculated in the $\delta\phi$-gauge

Summary

- We analyse a SR - USR - SR model with smooth transitions δN
- We introduce all the relevant interactions and the counterterms that absorb the dependence of $\mathcal{P}_\zeta$ on the regulator



- We obtain that for a large parameter space $\{\Delta N\,, \delta N\}$, perturbation theory is valid

- The one-loop power spectrum grows if δN decreases, being divergent in the limit $\delta N \to 0$