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SCALE-INVARIANT INFLATION

Based on arXiv:2403.04316, C. Cecchini, M. De Angelis, W. Giarè, M. Rinaldi, & S. Vagnozzi

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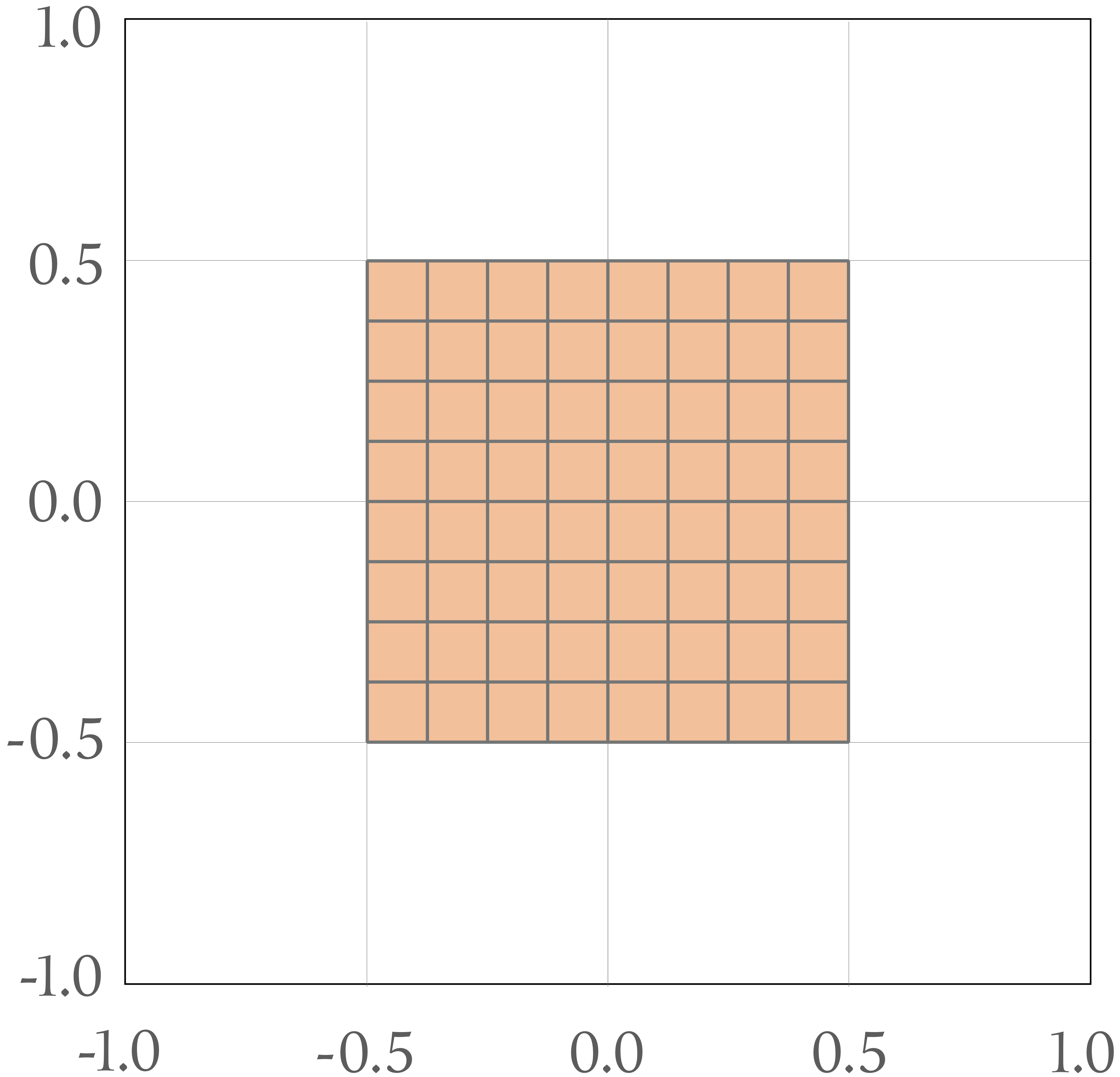
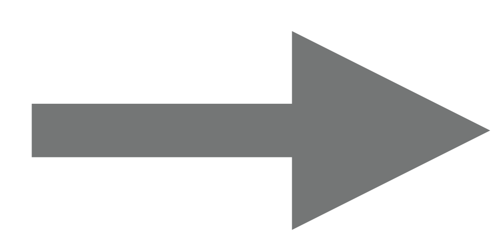
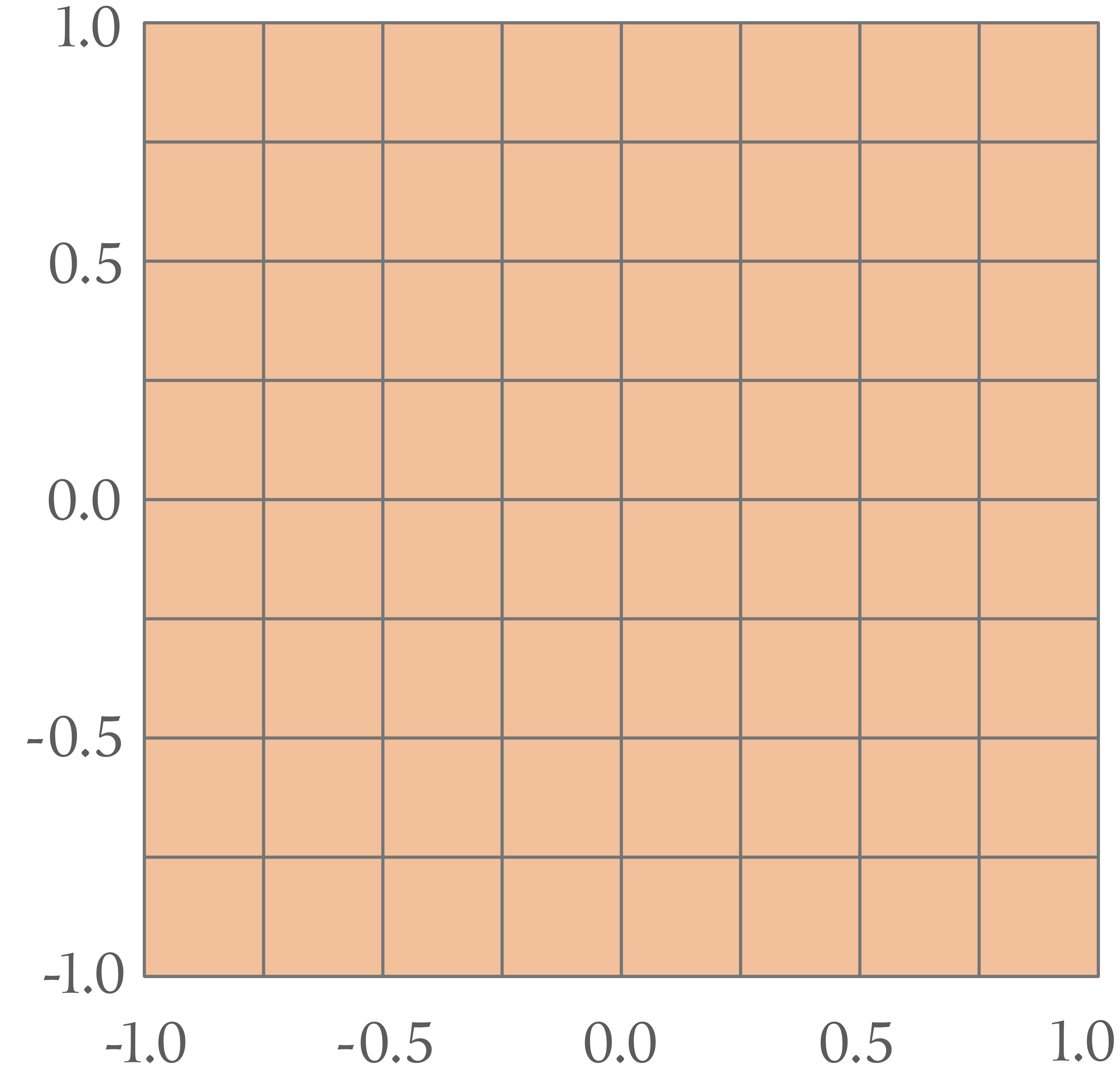
FUNDAMENTAL SCALE INVARIANCE

SCALE TRANSFORMATION IN 2 DIMENSIONS







WHY SCALE INVARIANCE?

RENORMALIZABILITY

Naturally renormalizable, but also avoids fixing the relevant parameters of the theory

- Fixed points are exact scaling solutions
- Higher predictive power

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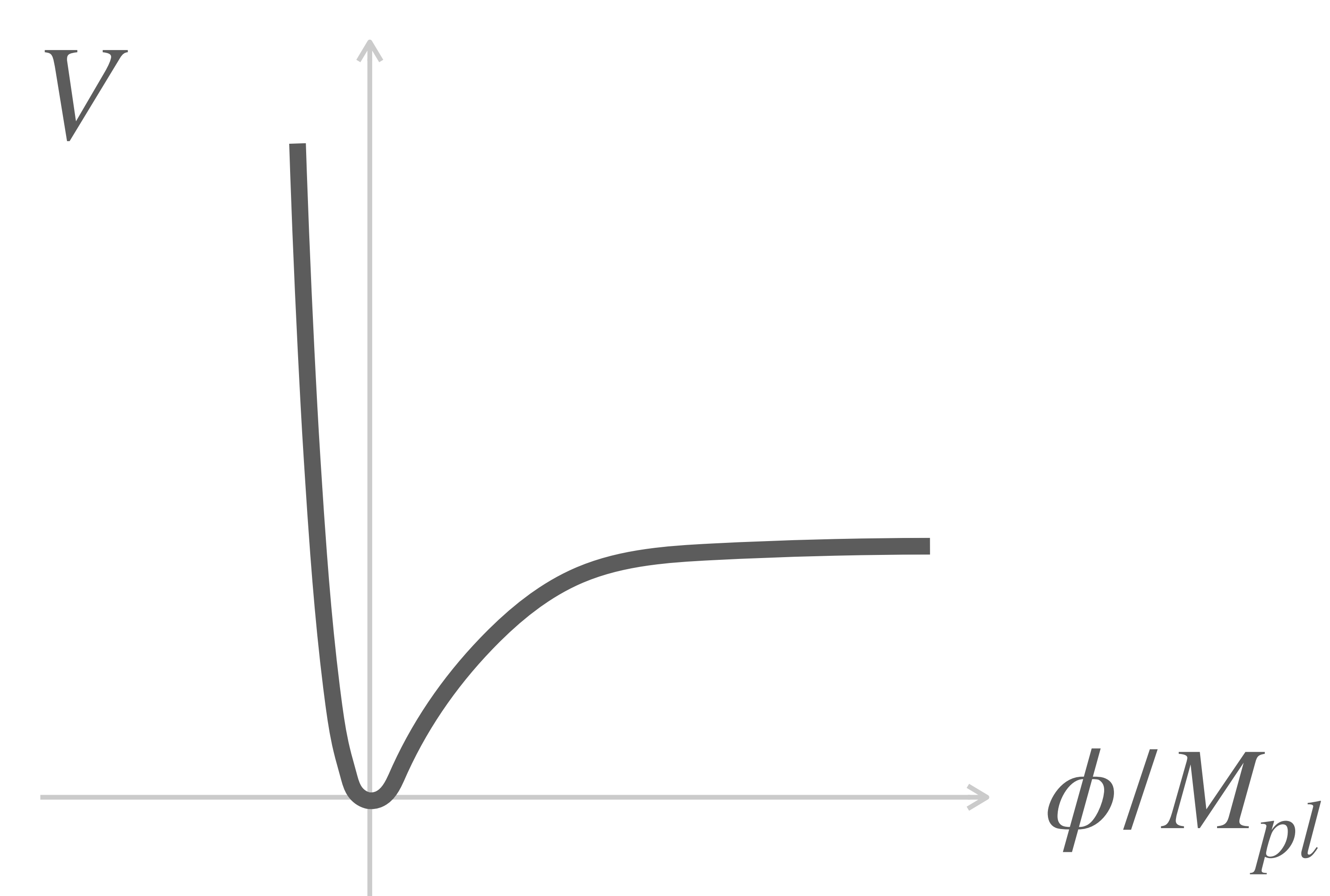
INFLATION

Flat inflationary potentials without fine-tuning



SCALE-INVARIANT QUADRATIC GRAVITY

THE MODEL

M. Rinaldi and L. Vanzo PR D 94 (2016)

- $\mathscr{L}_{EH} \longrightarrow f(R,\phi)$: scalar-tensor theory of modified gravity

$$\mathscr{L}_J = \sqrt{-g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right], \qquad \alpha, \lambda, \xi > 0$$

Higher order term in R

Scalar field

- Two additional scalar degrees of freedom

SCALE TRANSFORMATION

- $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$
- $\bar{\phi}(x) = \ell\phi(\ell x)$

$\longrightarrow \bar{\mathscr{L}} = \mathscr{L}$

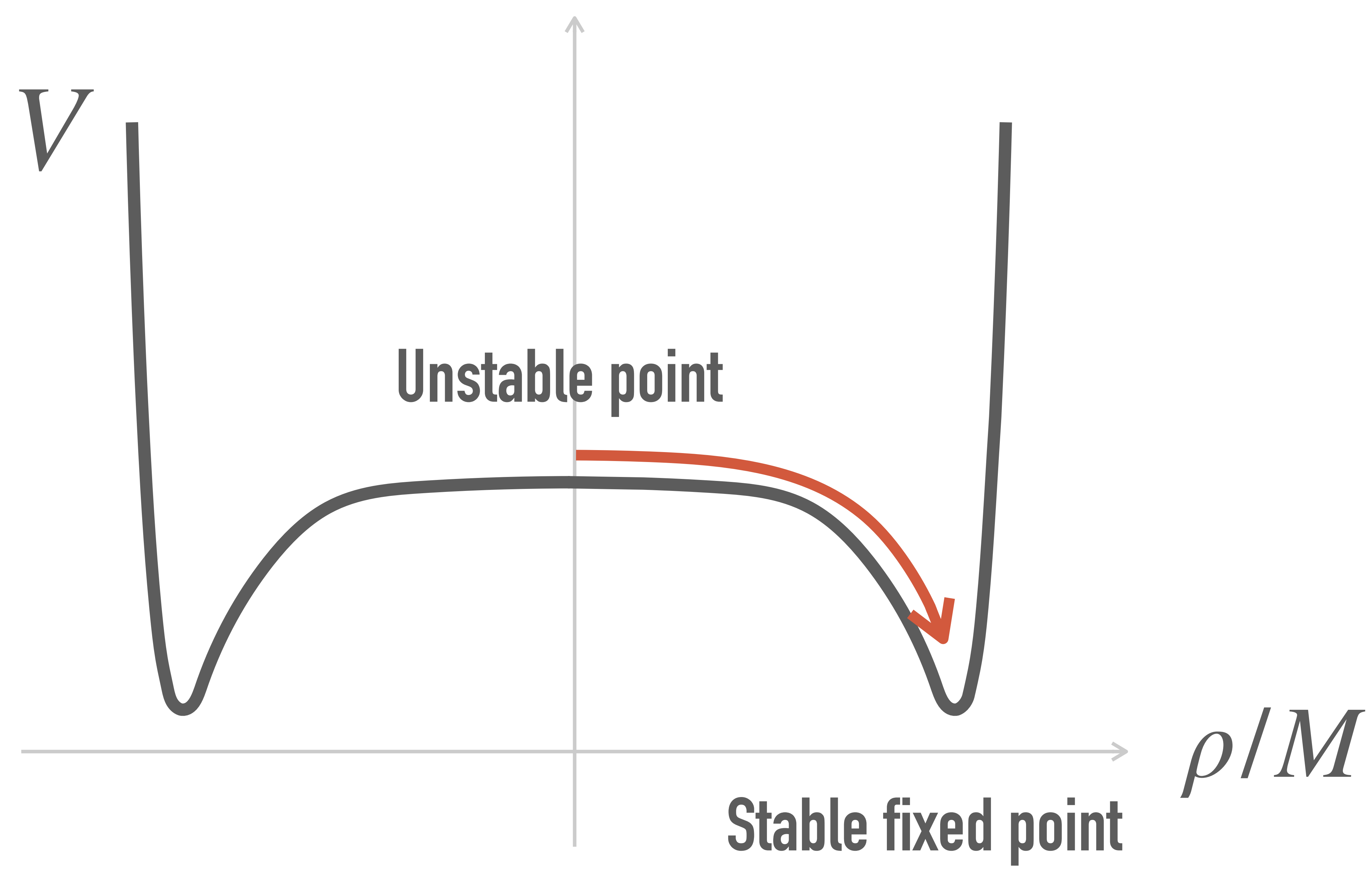
SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME: SINGLE-FIELD POTENTIAL

- Noether's current conservation
- Naturally flat plateau: no fine-tuning

χ GOLDSTONE BOSON

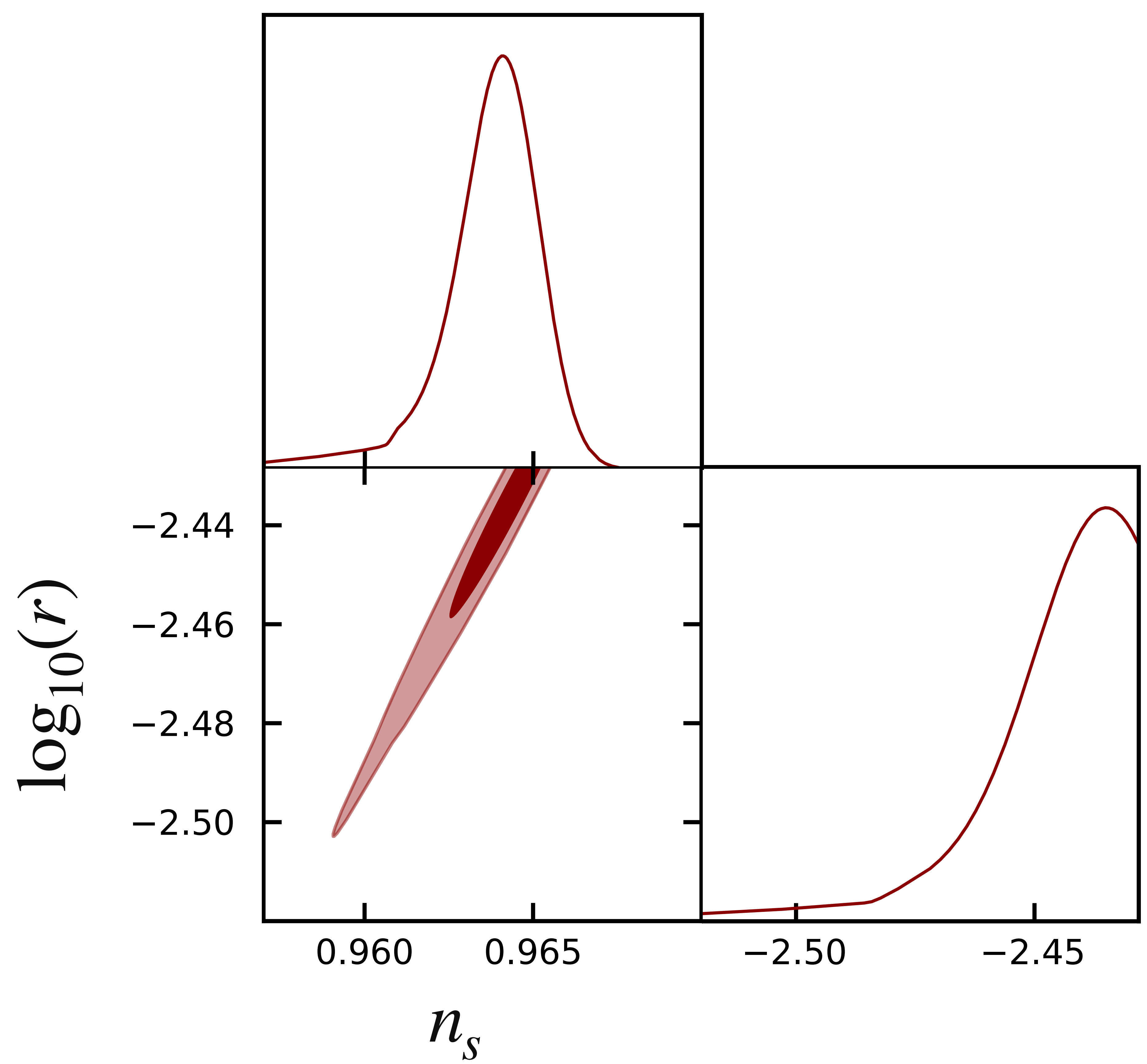
ρ INFLATON



SOME RESULTS

ANALYTICAL AND NUMERICAL ANALYSIS

➤ Lower bound on the tensor-to-scalar ratio: $r > 0.003$;



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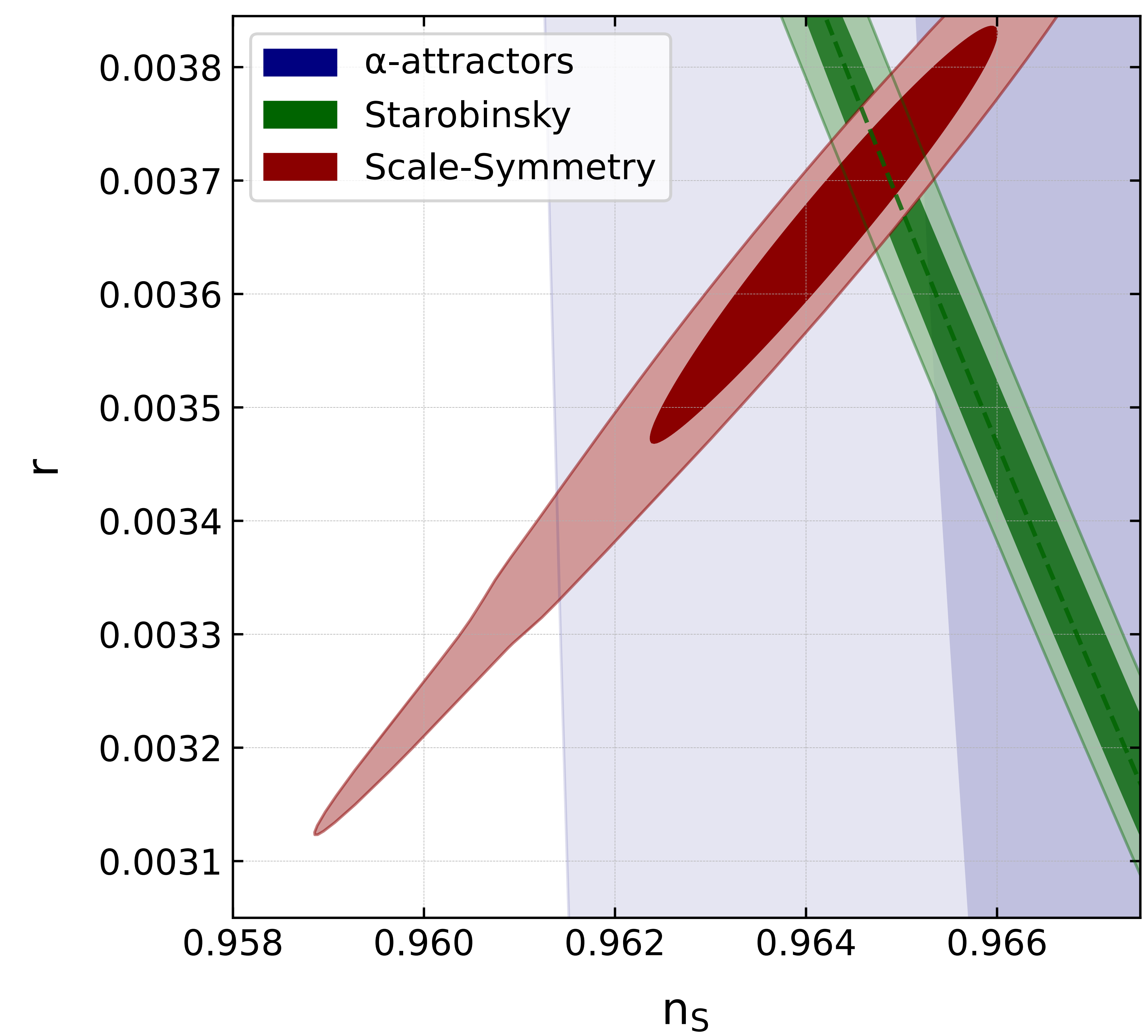
ANALYTICAL AND NUMERICAL ANALYSIS

- Lower bound on the tensor-to-scalar ratio: $r > 0.003$;
- Insensitivity to initial conditions;
- Vanishing entropy perturbations;
- Possibility to distinguish scale invariance from Starobinsky's inflation.



BACKUP SLIDES

FUNDAMENTAL SCALE INVARIANCE

C. Wetterich , Nuclear Physics B, 115326 (2021)
A. Strumia & A. Salvio, J High Energ Phys, 6 (2017)

Basic idea: a fundamental QFT does not involve any intrinsic parameter with dimension of mass or length

Following Wetterich, we can introduce an explicit mass scale k

Canonical field Dimension of a mass ← $\phi = k\,\tilde{\phi}$ → **Scale-invariant field** Dimensionless

The corresponding effective actions obey

$$k\partial_k \Gamma_k[\phi] = \zeta_k[\phi]$$

General solution

$$k\partial_k \Gamma_k[\tilde{\phi}] = 0$$

Particular, scaling solution holding when the canonical fields are expressed in terms of the scale-invariant ones

FUNDAMENTAL SCALE INVARIANCE

NATURALLY FLAT POTENTIALS FOR INFLATION

Scale-invariant theory non-minimally coupled to gravity

$$\mathscr{L}_J = \sqrt{-g}\left[\xi\,\phi^2 R - \lambda\phi^4 - \frac{1}{2}(\partial\phi)^2\right]$$

Weyl rescaling from the Jordan to the Einstein frame

$$\mathscr{L}_E = \sqrt{-\tilde{g}}\left[\frac{M_{pl}^2}{2}\tilde{R} - M_l^4\frac{\lambda}{\xi^2} - \frac{1}{2}(\partial\tilde{\phi})^2\right]$$

The potential is flat at tree-level: no fine-tuning. Scale symmetry breaking can occur from quantum corrections.

FUNDAMENTAL SCALE INVARIANCE

A CRITERION BEYOND RENORMALIZABILITY

For general renormalizable theories the effective action remains well defined in the continuum limit if one employs renormalized fields

$$\underbrace{\phi_{R,i}(x)}_{\text{Renormalized fields}} = k^{d_i} f_i(k)\, \underbrace{\tilde{\phi}_i(x)}_{\text{Scale-invariant field}}$$

Theories with fundamental scale invariance:

- ➤ Renormalizable
- ➤ For some choice of the fields $\tilde{\phi}$ the effective action becomes k-independent
- ➤ Exact scaling solutions: no free parameters.
  High predictive power

SCALE-INVARIANT QUADRATIC GRAVITY

WEYL CORRECTION

Squared Weyl curvature term: conformally-invariant, second order term. Why don't we add it to the action?

$$C^2 = 2R_{\mu\nu}R^{\mu\nu} - \frac{2}{3}R^2 + \mathcal{G}$$

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Background: The Weyl curvature term vanishes in a conformally flat spacetime
→ no contribution to the equations of motion

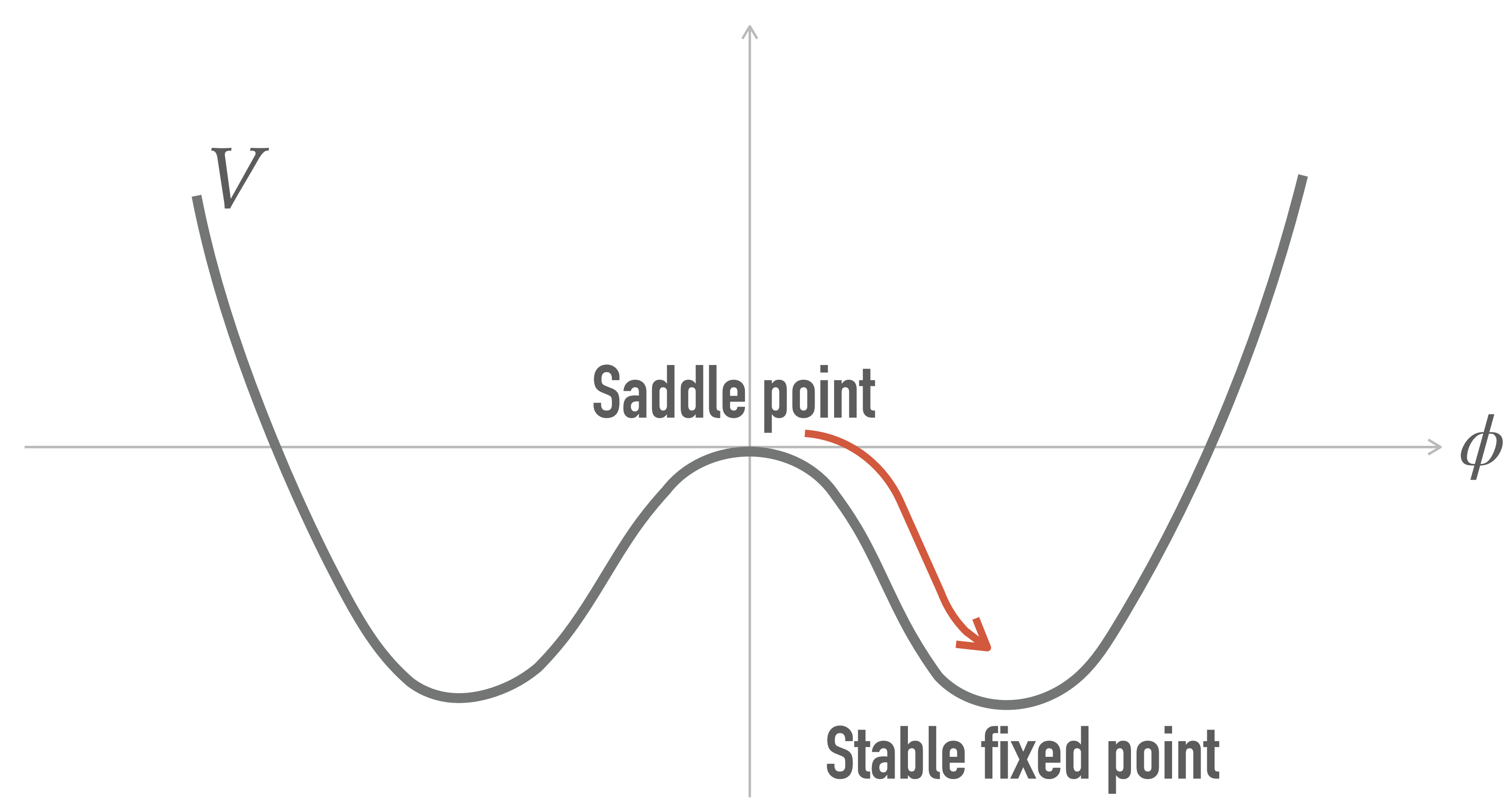
A. De Felice et al. Phys. Rev. D ***108****, 123524 (2023)*

Perturbations: Weyl-Starobinsky inflation is plagued by ghosts and classical instabilities
→ possible drawback also here (ongoing project)

SCALE-INVARIANT QUADRATIC GRAVITY

JORDAN FRAME

The field ϕ is subjected to an effective potential $V_{eff}(\phi) = -\frac{\xi}{6}\phi^2 R + \frac{\lambda}{4}\phi^4$



Classical scale-symmetry breaking

The scalar field takes a non-zero VEV at the minimum

$$\langle \phi_0^2 \rangle = \frac{\xi R}{3\lambda}$$

Dynamical generation of a mass scale

Natural identification with the Planck mass

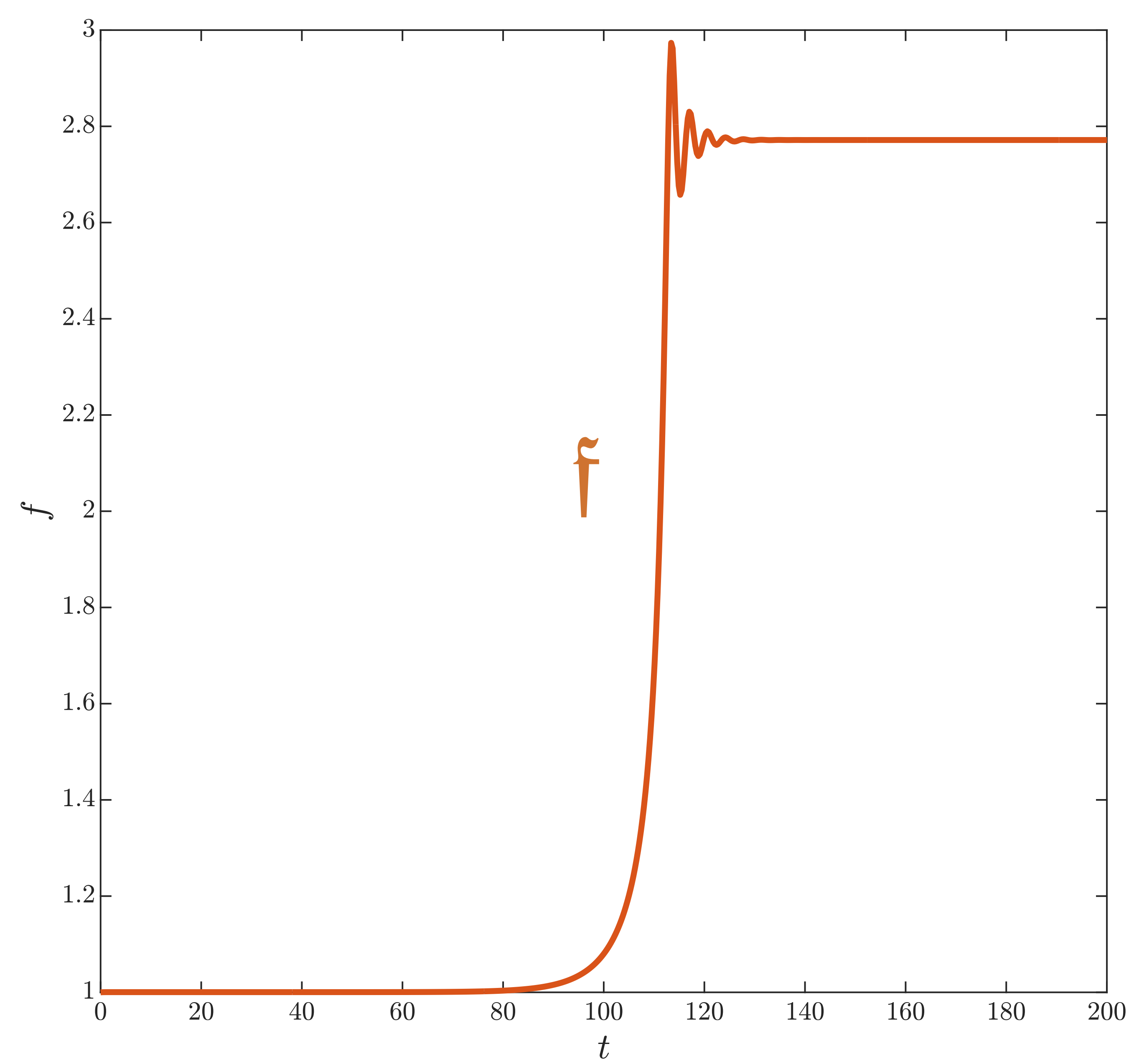
$$\frac{\xi}{6}\phi_0^2 R \equiv \frac{1}{2} M_{pl}^2 R$$

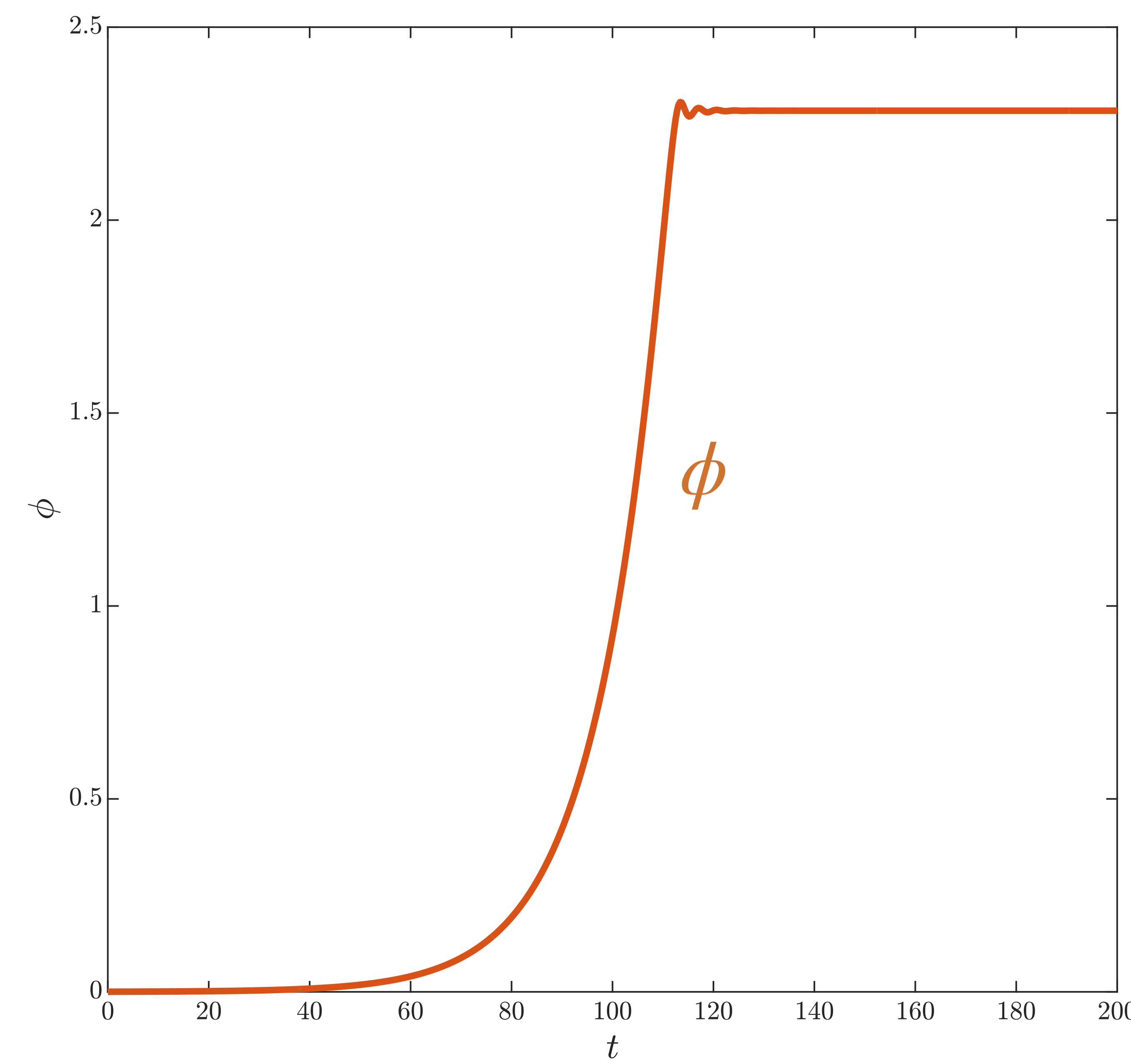
SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME $g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}$

Two dynamical degrees of freedom: are we in multi-field inflation?

$$\mathscr{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$





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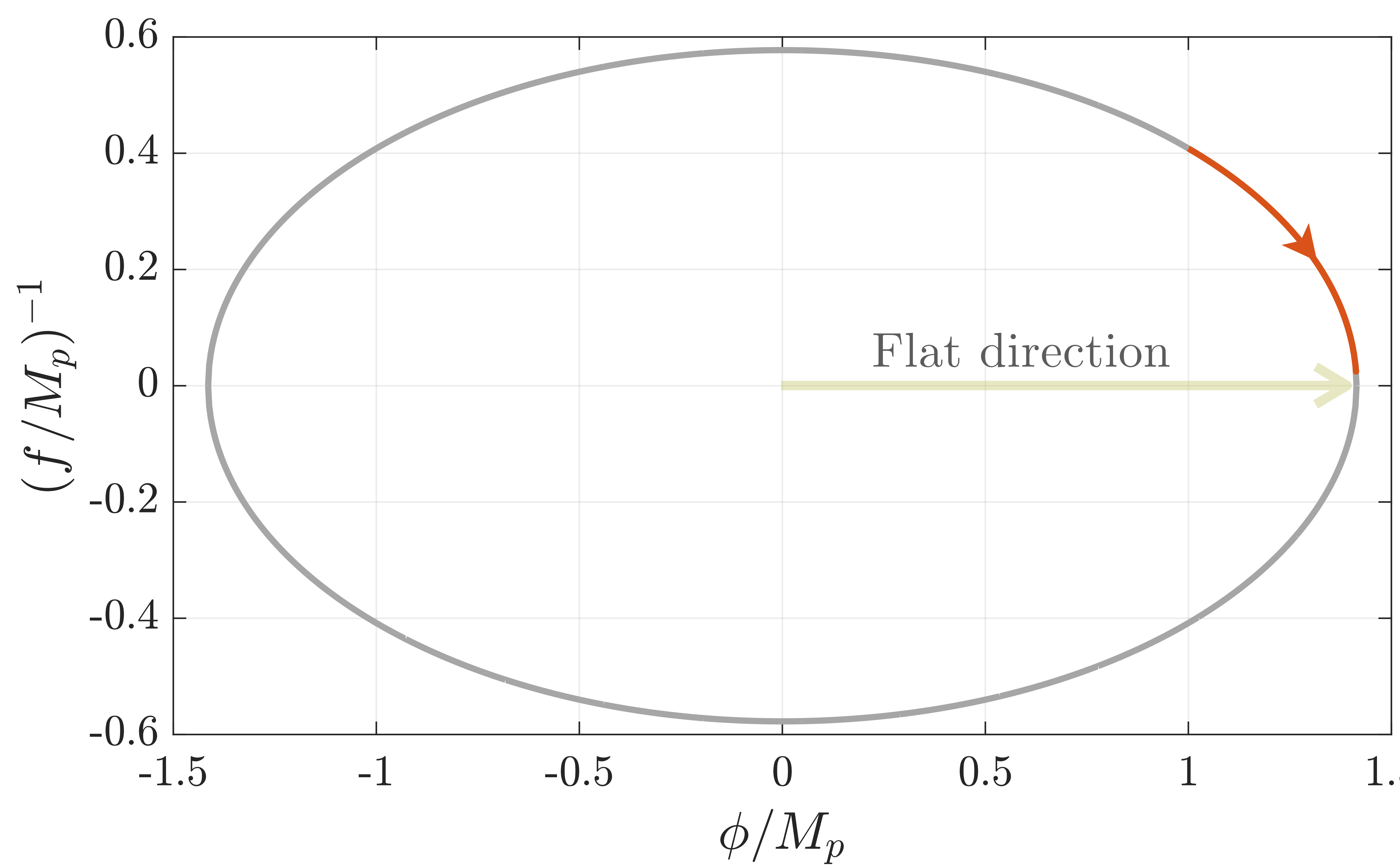
Noether's current conservation: constraint on the dynamics!



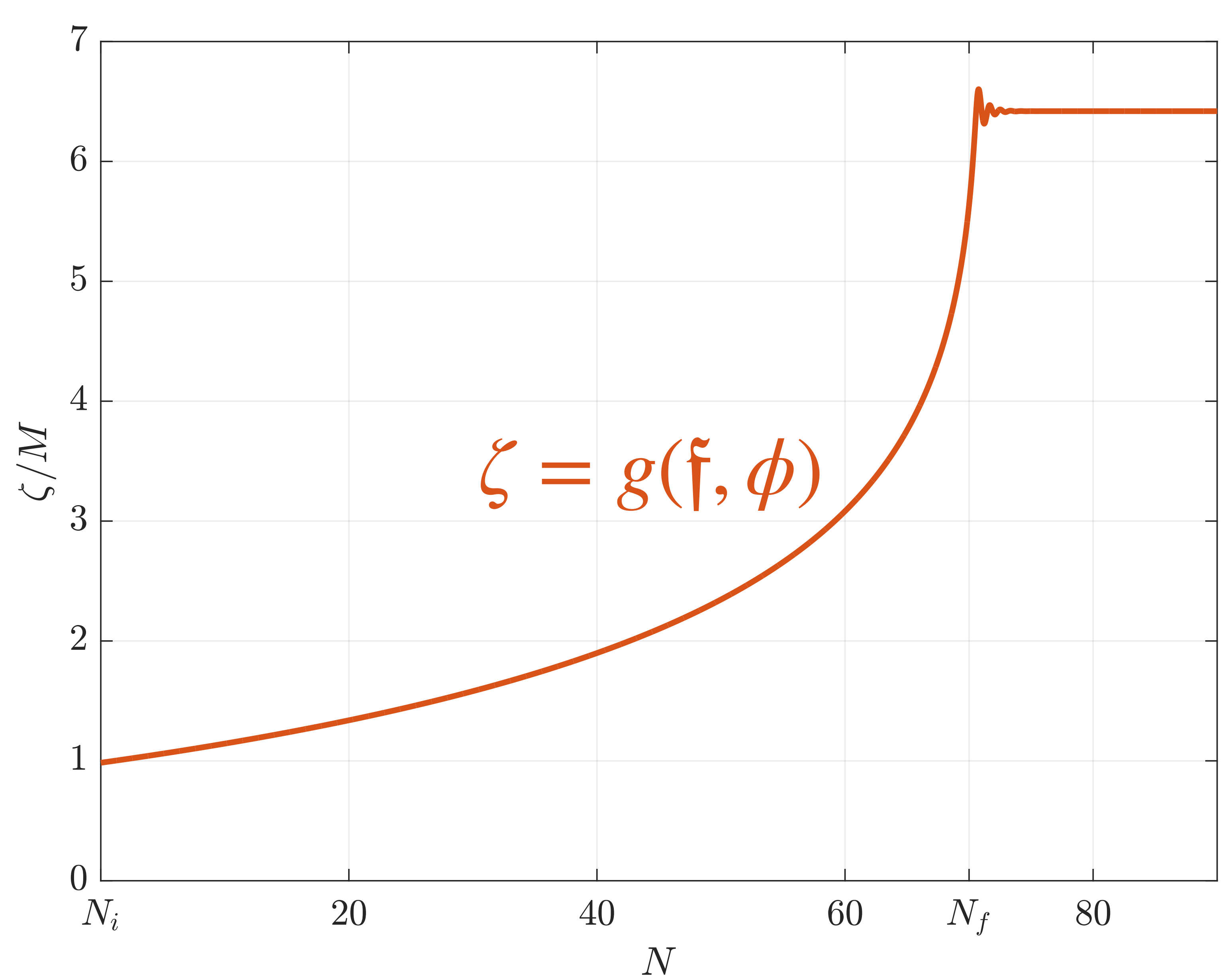
SCALE-INVARIANT QUADRATIC GRAVITY

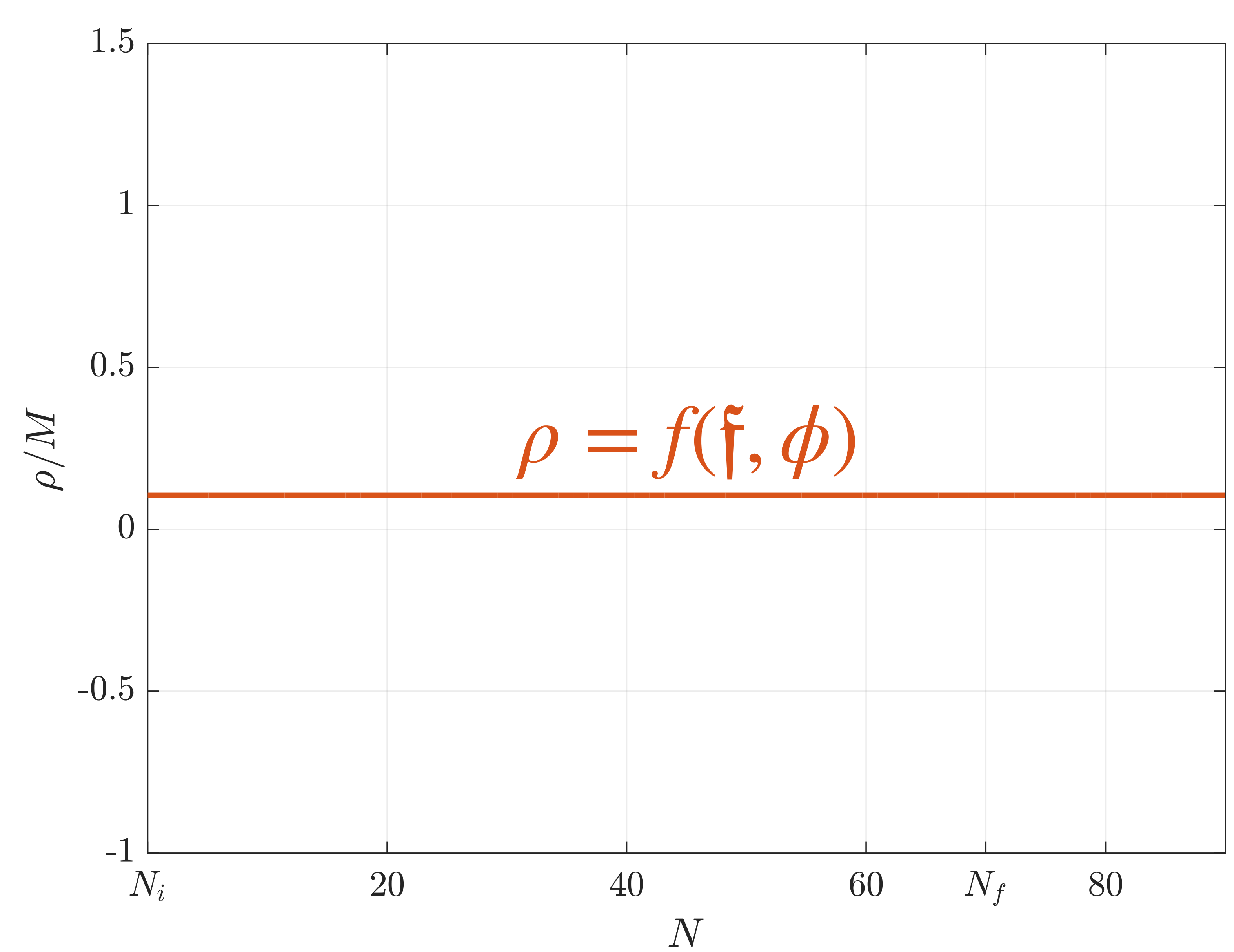
EINSTEIN FRAME: FIELDS' REDEFINITION

G. Tambalo & M. Rinaldi Gen Relativ Gravit 49 (2017)

Noether's current conservation can be employed to shift the dynamics on one field

$$\mathscr{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\zeta\,\partial^\mu\zeta - 3\mathrm{Cosh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2 \partial_\mu\rho\partial^\mu\rho - U(\zeta)\right)$$





INFLATIONARY PREDICTIONS

PRIMORDIAL SPECTRA

Even with non-zero initial velocity the Goldstone boson does not contribute

$$\rho'(N) \sim e^{-3N} \to 0$$

Single-field predictions are recovered, both in the Jordan and the Einstein frame

Scalar perturbations

$$\Delta_s^2(k) = \frac{1}{2M_{pl}^2 \epsilon} \left(\frac{H}{2\pi} \right)^2 \Bigg|_{k=aH}$$

$$n_s - 1 \approx -6\,\epsilon(\zeta_*) + 2\eta(\zeta_*)$$

Tensor perturbations

A. Ghoshal, D. Mukherjee, & M. Rinaldi JHEP ***5*** *(2023)*

$$\Delta_t^2(k) = \frac{2}{\pi^2} \left(\frac{H}{M_{pl}} \right)^2 \Bigg|_{k=aH}$$

$$r \approx -16\,\epsilon(\zeta_*)$$

INFLATIONARY PREDICTIONS

NUMERICAL ANALYSIS

W. Giarè, M. De Angelis, C. van de Bruck, & E. Di Valentino *JCAP 12(2023)014*

Numerical integration up to the end of inflation $(|\epsilon| = 1)$

↓

Sufficiently long inflation? → Discard

↓

Compute A_s, n_s, α_s, r

↓

Are they within some reasonably chosen ranges? → Discard

↓

Implement CAMB and assign a likelihood based on how well the model agrees with CMB data

INFLATIONARY PREDICTIONS

LIKELIHOOD

W. Giarè, M. De Angelis, C. van de Bruck, & E. Di Valentino JCAP 12(2023)014

MCMC analysis for
$\Lambda\text{CDM} + \alpha_s + r$

↓

Covariance matrix $\boldsymbol{\Sigma}$ and mean value of the parameters μ

↓

Analytical likelihood

DATA

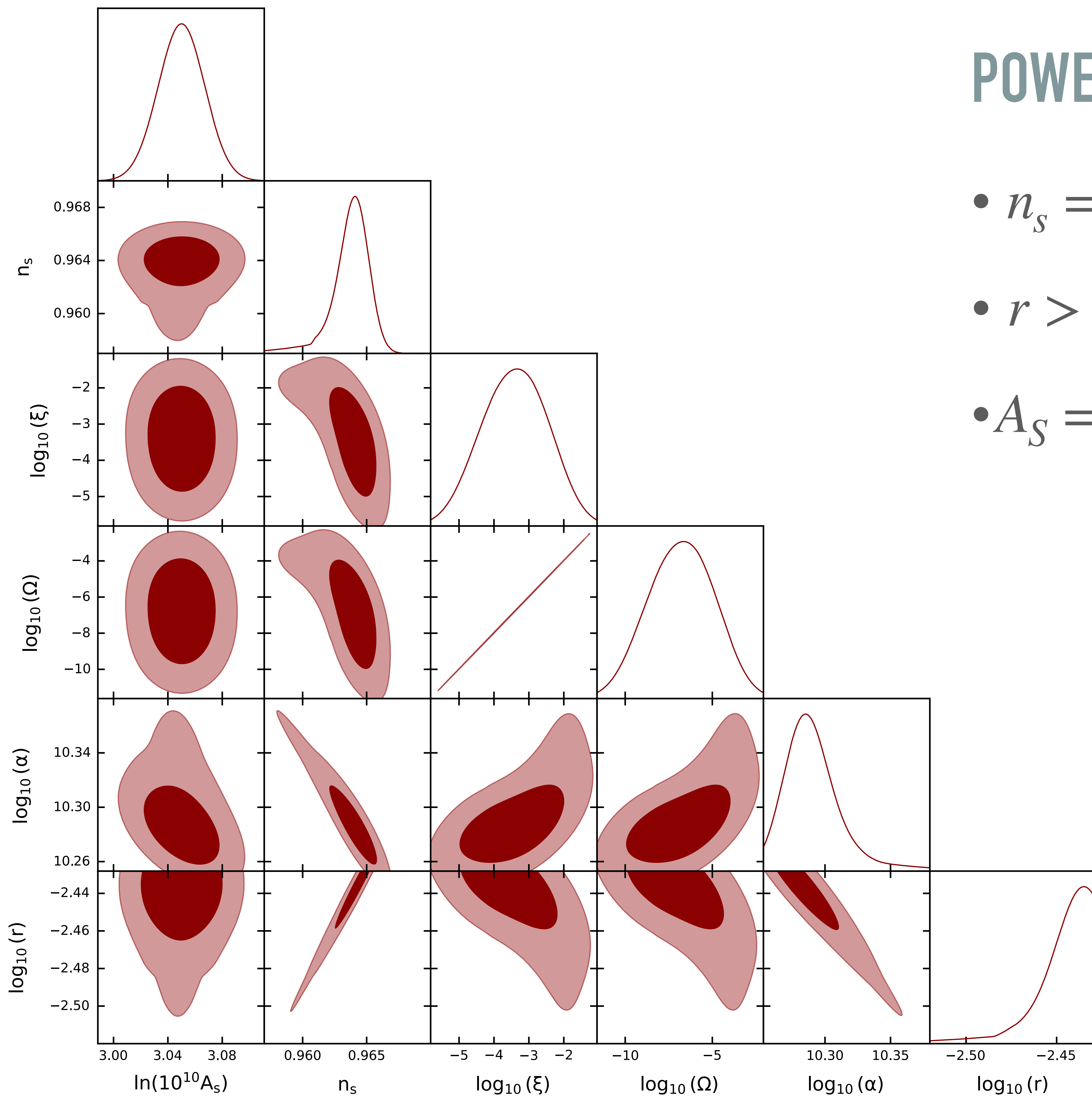
- Planck 2018 temperature and polarisation (TT TE EE) likelihood
- B-modes power spectrum likelihood cleaned for foreground contamination (Bicep/Keck Array Collaboration)

ANALYTICAL LIKELIHOOD

$$\mathscr{L} \propto \exp\left(-\frac{1}{2}\left(\mathbf{x}-\mu\right)^T \boldsymbol{\Sigma}^{-1}\left(\mathbf{x}-\mu\right)\right), \quad \mathbf{x} \equiv \left(A_s, n_s, \alpha_s, r\right)$$

INFLATIONARY PREDICTIONS

OBSERVATIONAL CONSTRAINTS



POWER SPECTRUM

- $n_s = 0.9638^{+0.0015}_{-0.0010}$
- $r > 0.00332$
- $A_S = (2.112 \pm 0.033) \times 10^{-9}$

PARAMETERS

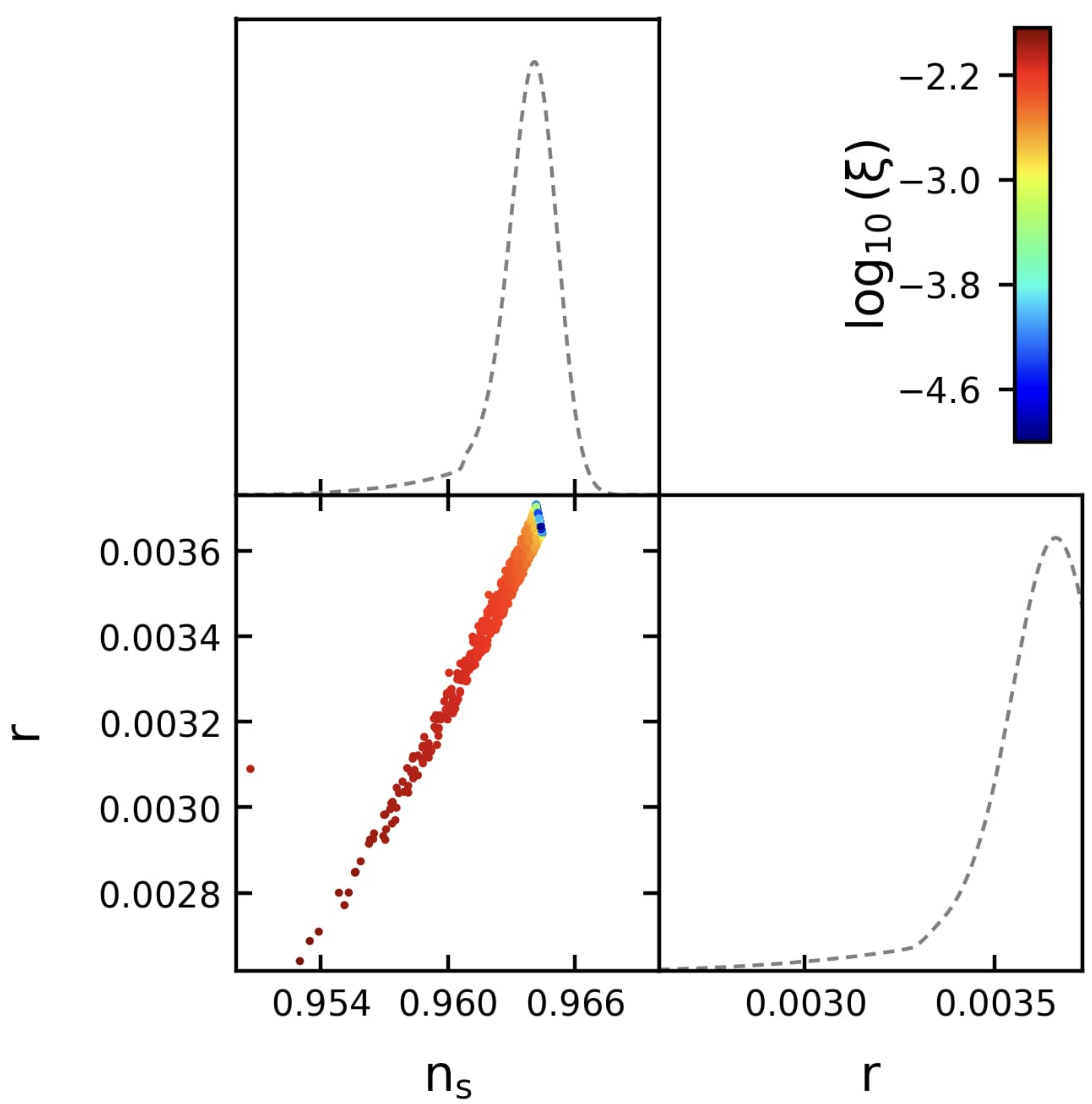
- $\xi < 0.00142$
- $\alpha = 1.951^{+0.076}_{-0.11} \times 10^{10}$
- $\Omega = 0.93^{+0.72}_{-2.8} \times 10^{-5}$

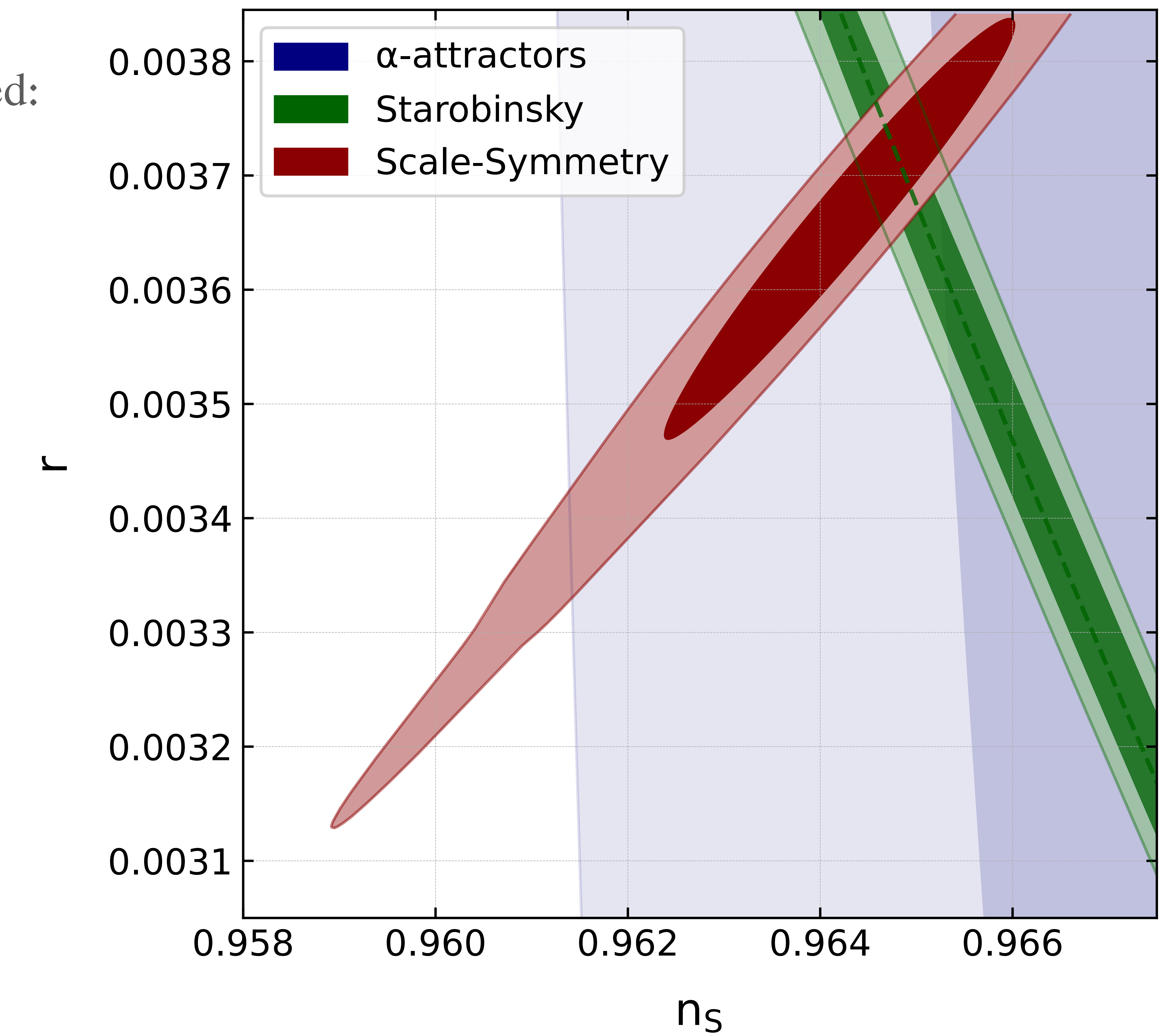
$$\Omega \equiv \alpha\lambda + \xi^2$$

INFLATIONARY PREDICTIONS

SCALE INVARIANCE VS STAROBINSKY

n_s and r are anti-correlated like in Starobinsky's model only at fixed ξ. Overall, they are correlated: it is potentially possible to discriminate between the two models!





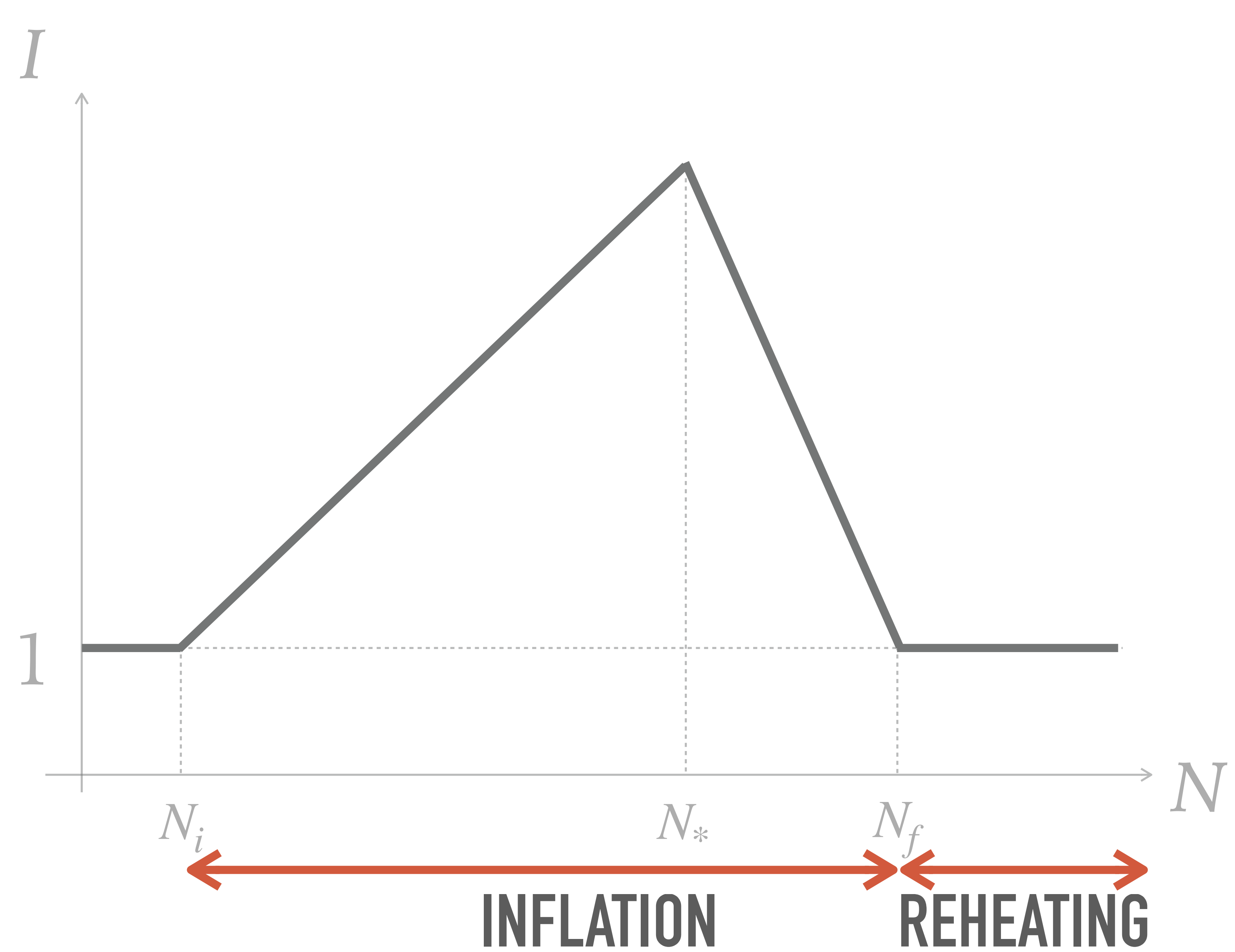
INFLATIONARY PREDICTIONS

MAGNETOGENESIS

*C. Cecchini & M. Rinaldi Phys Dar Univ **40** (2023)*

Modify the Maxwell's action and add helicity to generate primordial magnetic fields through a sawtooth coupling to the inflaton: EM conformal invariance is broken only during inflation → amplification of vector perturbations

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\, I^2[\zeta(t)]\left[F_{\mu\nu}F^{\mu\nu} - \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_E$$



$$I = \begin{cases} \mathscr{C}\left(\frac{a}{a_*}\right)^{\nu_1} & a_i > a > a_* \\ \mathscr{C}\left(\frac{a}{a_*}\right)^{-\nu_2} & a_* > a > a_f \end{cases}$$

INFLATIONARY PREDICTIONS

MAGNETOGENESIS

C. Cecchini & M. Rinaldi Phys Dar Univ ***40*** *(2023)*

Present-day magnetic field's amplitude and coherence length compatible with bounds on the IGM fields