A nonperturbative test of nucleation calculations for strong phase transitions

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Want to know more?

INTRODUCTION

First order phase transitions in the early universe have long sustained interest as they play an important role when considering the following

- Baryogenesis
- Early universe stochastic gravitational wave (GW) background
- Analogue experiments
- Testing perturbative calculations.

The bubble nucleation rate – how quick the conversion between the metastable and stable phase is – can be written as a product of the dynamical and statistical information of the system in question. At high temperatures, the rate can be solved in classical field theory, but these perturbative methods depend on the semiclassical picture [1-12].

With the future GW detection projects, we need reliable predictions with controlled systematic uncertainties. We calibrate the accuracy of nucleation rate calculations in relativistic finite-temperature field theory at a simulation point in the real scalar theory [13], where the assumption has been that perturbation theory works well.

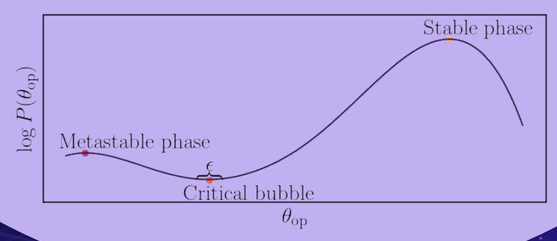
$$\Gamma = A_{\rm dyn} \times A_{\rm stat}$$
$$= A_{\rm dyn} \times \text{functional det} \times \left(\frac{S_b}{2\pi}\right)^{3/2} e^{-S_b}$$

METHODS

The calculation is done in two stages: [14,15]

1. Lattice Monte Carlo simulations using multicanonical method to simulate probability $P(\theta_{op})$ distribution and to draw configurations from the exponentially suppressed critical bubble $[\theta_c-\varepsilon/2, \theta_c+\varepsilon/2]$

2. Numerically evolve these critical bubble configurations in a thermal bath to determine if a configuration tunnels or not and calculate the rate of change of order parameter.



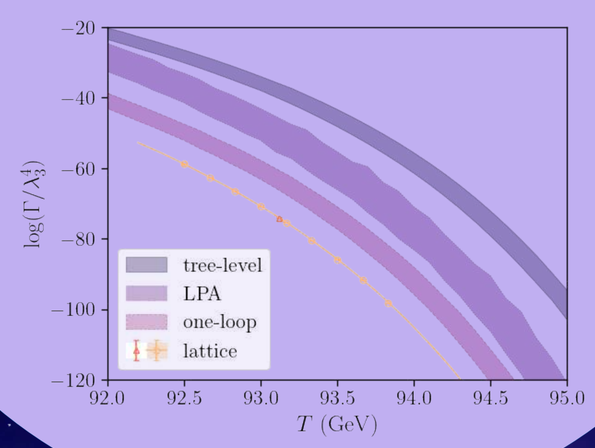
TAKE-HOME BUBBLE

Nucleation rate calculations are inaccurate in perturbation theory, lattice is significantly better.

RESULTS

The main result is the logarithm of the nucleation rate yielding -74.08(5). Comparing our result to the perturbative results – and reweighting our results to different temperatures and taking the infinite-volume limit – we see that the systematic uncertainty is much smaller on the lattice, compared to the perturbative methods. The perturbative methods reflect the different ways of approximating the dynamical prefactor and functional determinant.

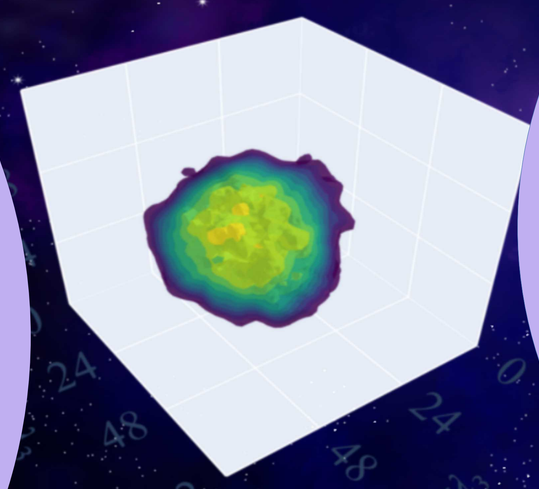
- Tree-level: T^4
- LPA: local potential approximation, drop imaginary parts
- One-loop: Full one-loop calculation
- Lattice: Triangle – simulation point, Circle – reweighted points



OUTLOOK

The simulated parameter point was chosen because it is known to work well in perturbation theory, with less than a 1% disagreement in the latent heat at one-loop and 10% at tree-level [13]. However, our results show a disagreement of 20% at one-loop and 100% at tree-level in |log Γ|.

- The discrepancy could be explained by the need for higher loop orders, but what if this isn't the case?
- Maybe there are other saddle points that are relevant for the nucleation rate calculation? [16, 17]
- What if the saddle point approximation breaks down? [18, 19]



BUBBLE, BUBBLE, PERTURB & TROUBLE

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