Impact of theoretical uncertainties on model parameter reconstruction from gravitational wave signals sourced by cosmological phase transitions

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Gravitational wave (GW) pipeline.

Particle physics model: $\mathcal{L}_{\rm BSM}$

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Equilibrium thermodynamics: $V_{\rm eff}(\phi, T)$

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Bubble nucleation rate: $\Gamma(T)$

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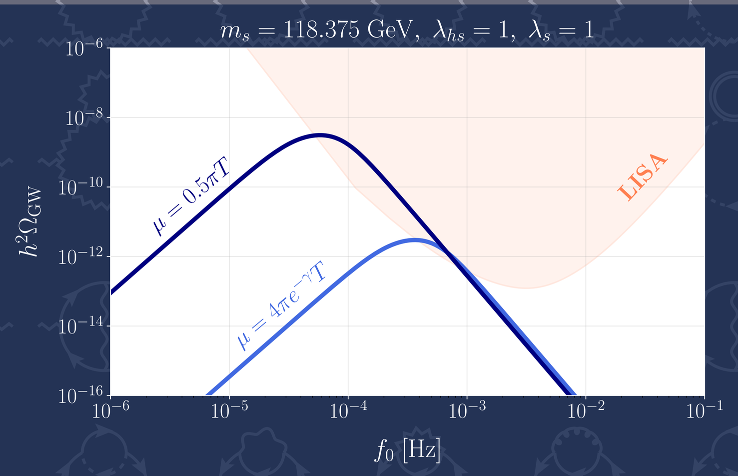
GW background: $h^2\Omega_{\rm GW}(\alpha, \beta_\star, v_w, H_\star)$



Huge theoretical uncertainties for single parameter points for leading order (LO) $V_{\rm eff}(\phi,T)$!

Standard model + scalar singlet: xSM.

- Motivation: dark matter, electroweak (EW) baryogenesis, ...
- Introduce scalar gauge singlet $S = x + s$ + SM Higgs $\Phi = \left(G^+, \frac{1}{\sqrt{2}}(v + h + iG^0)\right)^{\rm T}$

$$V_0(v,x) = \frac{1}{2}\mu_h^2 v^2 + \frac{1}{4}\lambda v^4 + \frac{1}{2}\mu_s^2 x^2 + \frac{1}{4}\lambda_s x^4 + \frac{1}{4}\lambda_{hs} v^2 x^2$$

- Generates two-step PT $(0,0) \xrightarrow{\rm step1} (0,x) \xrightarrow{\rm step2} (v_0, 0)$
- Second step: first-order EWPT ⟶ Observable GW background

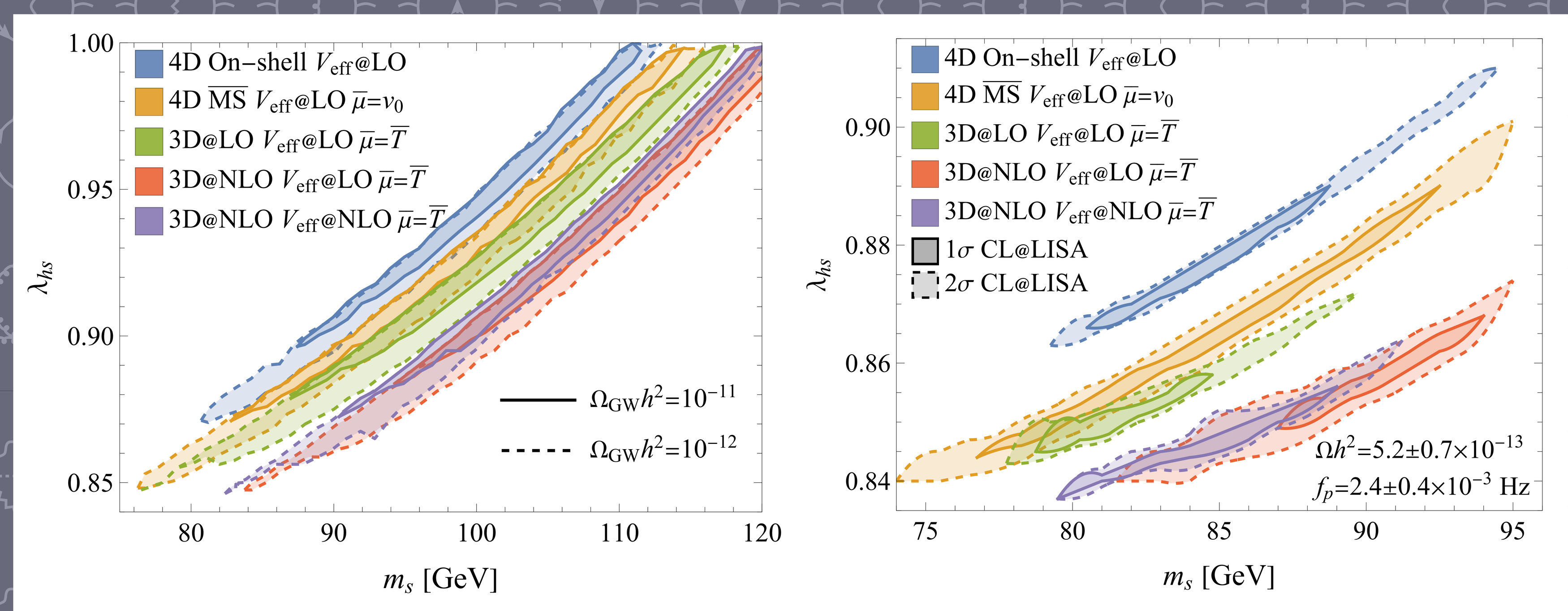
Thermal field theory: Linde problem.

- Primordial thermal bath induces corrections to effective potential $V_{\rm eff}(\phi) \to V_{\rm eff}(\phi, T)$
- Equilibrium finite temperature field theory: compactified time dimension with periodicity $\sim T^{-1}$

$$\longrightarrow \quad \phi(x) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \phi(\omega_n, \boldsymbol{p}) e^{i\omega_n \tau} e^{-i\boldsymbol{p}\boldsymbol{x}}$$

- Matsubara frequencies $\omega_n = \begin{cases} 2n\pi T\,, & \text{bosons} \\ (2n+1)\pi T\,, & \text{fermions} \end{cases}$
- Expansion parameter of bosonic zero mode $\epsilon \sim \dfrac{g^2 T}{\pi m}$ ⟶ non-perturbative $\mathcal{O}(1)$ for $m \to \dfrac{g^2 T}{\pi}$

What is the impact of computational diligence of the effective potential when reconstructing model parameters given a GW signal?



Parameter space featuring first-order PT extremely narrow. Small uncertainties matter!

High-temperature dimensional reduction (3D).

- Use scale hierarchy $\underbrace{\pi T}_{\rm hard} \gg \underbrace{gT}_{\rm soft} \gg \underbrace{g^{\frac{3}{2}}T}_{\rm supersoft} \gg \underbrace{g^2 T}_{\rm ultrasoft}$ to construct effective field theory (EFT)

A. Identify degrees of freedom (dofs) which live at the respective scales.

B. Construct most general 3D Lagrangian involving the dofs dynamical in the infrared.

C. Find effective Lagrangian parameters by matching correlation functions at desired order, e.g.,

$$\underbrace{p^2 + m_{\rm eff}^2 + \Pi^{(1)}_{\rm soft}(p=0; m^2_{\rm eff})}_{\rm 3D} = \underbrace{p^2 + m^2 + \Pi^{(1)}_{\rm soft}(p=0; m^2) + \Pi^{(1)}_{\rm hard}(p=0; m^2)}_{\rm 4D}$$

Corrections from hard modes then appear in EFT parameters.

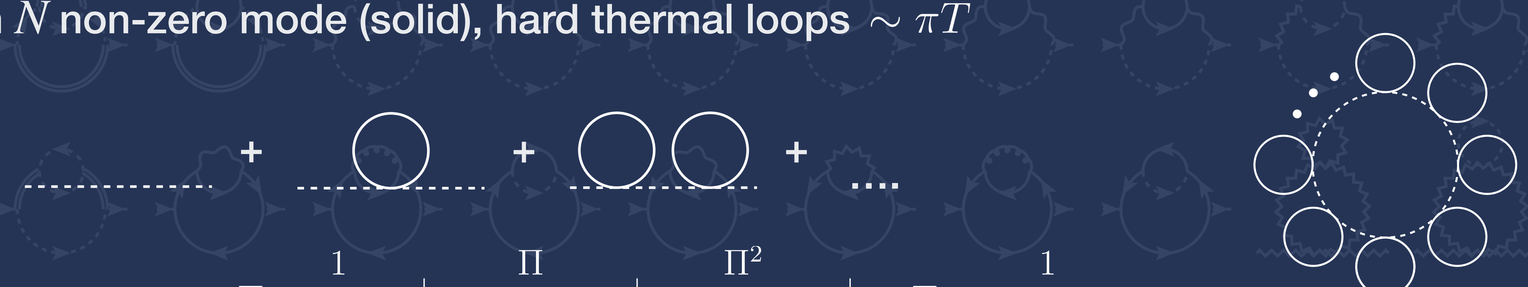
D. Compute $V_{\rm eff,3D}(\phi, T)$ beyond LO within 3D effective theory.

- EFT matching automatically includes resummations at any order in perturbation theory
- Higher orders obtained via `DRalgo` software package

EFT matching	1-loop $V_{\rm eff}$	2-loop $V_{\rm eff}$
LO	3D@LO $V_{\rm eff}$@LO	3D@LO $V_{\rm eff}$@NLO
NLO	–	3D@NLO $V_{\rm eff}$@NLO

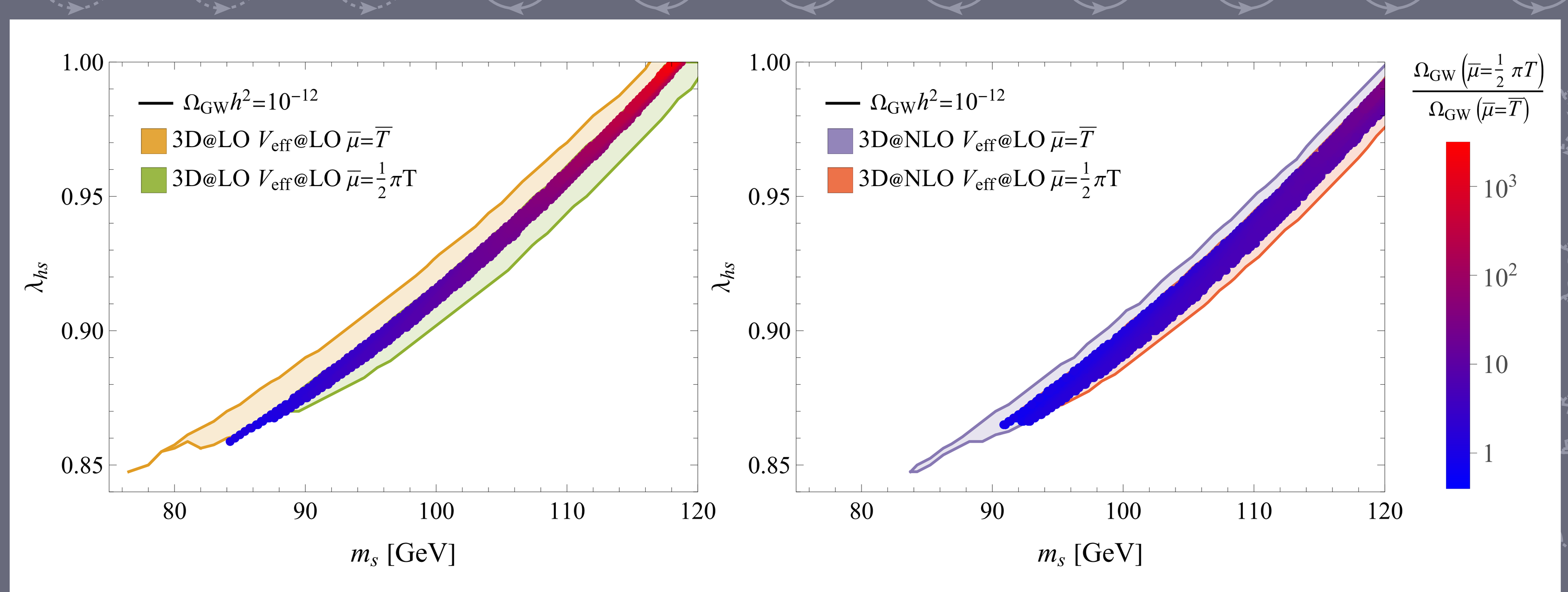
„Conventional" (4D) approach: Daisy resummation.

- Resummation of „Daisy" diagrams, where one zero mode loop (dashed) is dressed with N non-zero mode (solid), hard thermal loops $\sim \pi T$



$$= \frac{1}{p^2 - m^2} + \frac{\Pi}{(p^2-m^2)^2} + \frac{\Pi^2}{(p^2-m^2)^3} + \ldots = \frac{1}{p^2 - m^2 - \Pi}$$

- Amounts to replacing $m^2 \to m^2 + \Pi$, with thermal mass $\Pi \sim g^2 T^2$
- Expansion parameter $\epsilon \sim \dfrac{g}{\pi}$ for $m \to 0$ ⟶ perturbative!
- Corresponds to LO dimensional reduction
- Problems: incomplete LO resummation, no possibility to include higher-order corrections, strong dependence on renormalization group (RG) scale, ...



Higher-order EFT matching significantly decreases RG scale dependence.

Conclusions.

- Overall observable parameter space relatively robust
- Still: theoretical error from incomplete resummation dominates over experimental uncertainty for any signal visible by LISA
- Predictions from dimensionally reduced EFT converge quickly with order of Matsubara zero-mode loops
- RG scale dependence significantly reduced in NLO EFT