

INTRODUCTION

There is solid theoretical and observational motivation behind the idea of scale invariance as a fundamental symmetry of Nature.

We consider a recently proposed [1] gravity model featuring scale-invariance at the classical level — no explicit scale appears in the action — and study its inflationary predictions. A numerical analysis of the system allows us to corroborate earlier analytical findings and set robust constraints on the model's parameters using the latest Cosmic Microwave Background (CMB) data from *Planck* and *BICEP/Keck*.

SCALE-INVARIANCE AS A FUNDAMENTAL SYMMETRY OF NATURE

Why should gravity be scale-invariant in the Early Universe? [2]

- Theoretical principle beyond renormalizability;
- Naturalness problem;
- Flat inflationary potentials;
- Dynamical mass generation.

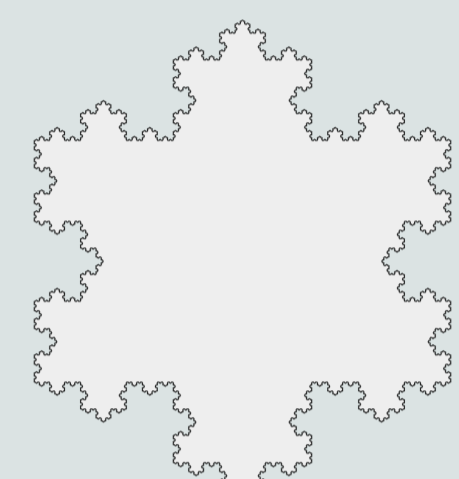


Figure 1: In the Koch snowflake scale invariance is realized as a self-similarity.

THE MODEL

The model: scale invariant and quadratic in curvature

$$\mathcal{L} = \sqrt{-g} \left[\frac{\alpha}{36} R^2 + \frac{\xi}{6} \phi^2 R - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda}{4} \phi^4 \right] \quad \alpha, \lambda, \xi > 0 \quad (1)$$

JORDAN FRAME

The field ϕ is subjected to an effective potential

$$V_{\text{eff}}(\phi) = -\frac{\xi}{6} \phi^2 R + \frac{\lambda}{4} \phi^4$$

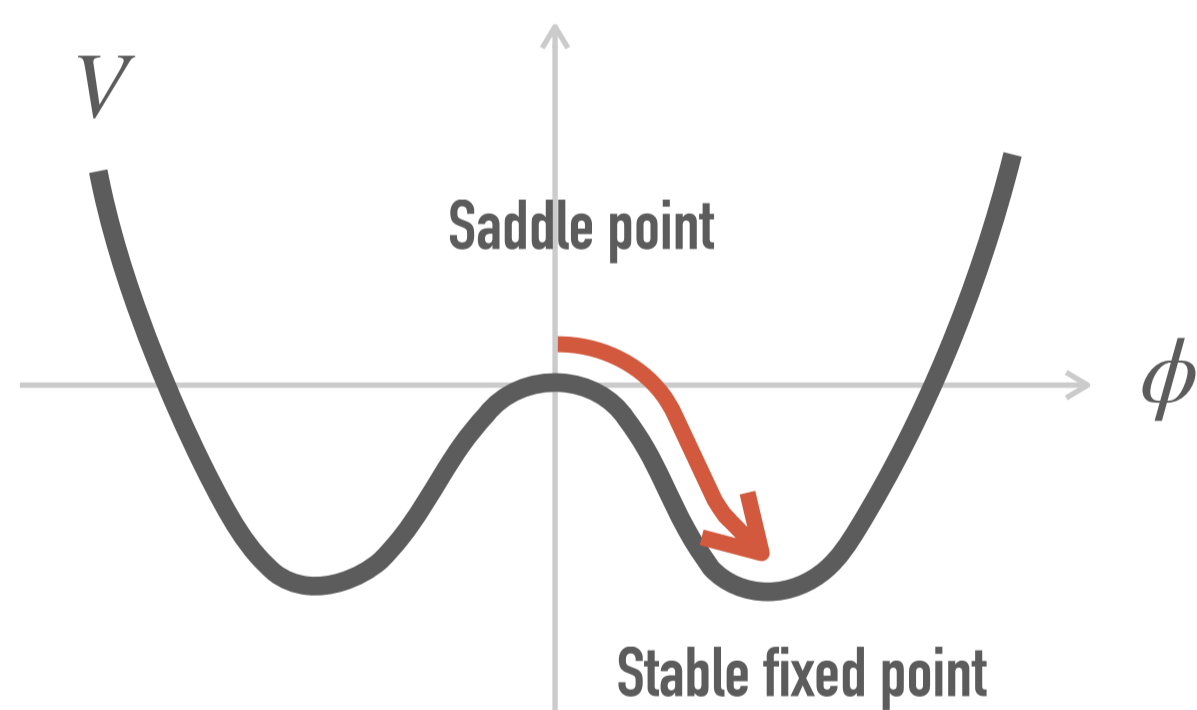


Figure 2: Effective potential $V(\phi)$ describing the Jordan-frame dynamics of the non-minimally coupled scalar ϕ .

Classical scale-symmetry breaking

The scalar field takes a non-zero VEV at the minimum

$$\langle \phi_0^2 \rangle = \frac{\xi R}{3\lambda}$$

Dynamical generation of a mass scale

Natural identification with the Planck mass

$$\frac{\xi}{6} \phi_0^2 R \equiv \frac{1}{2} M_{pl}^2 R$$

EINSTEIN FRAME

Two dynamical degrees of freedom: are we in multi-field inflation? Actually, Noether's current conservation has crucial consequences:

- The dynamics are constrained to an ellipse (Fig. 3);

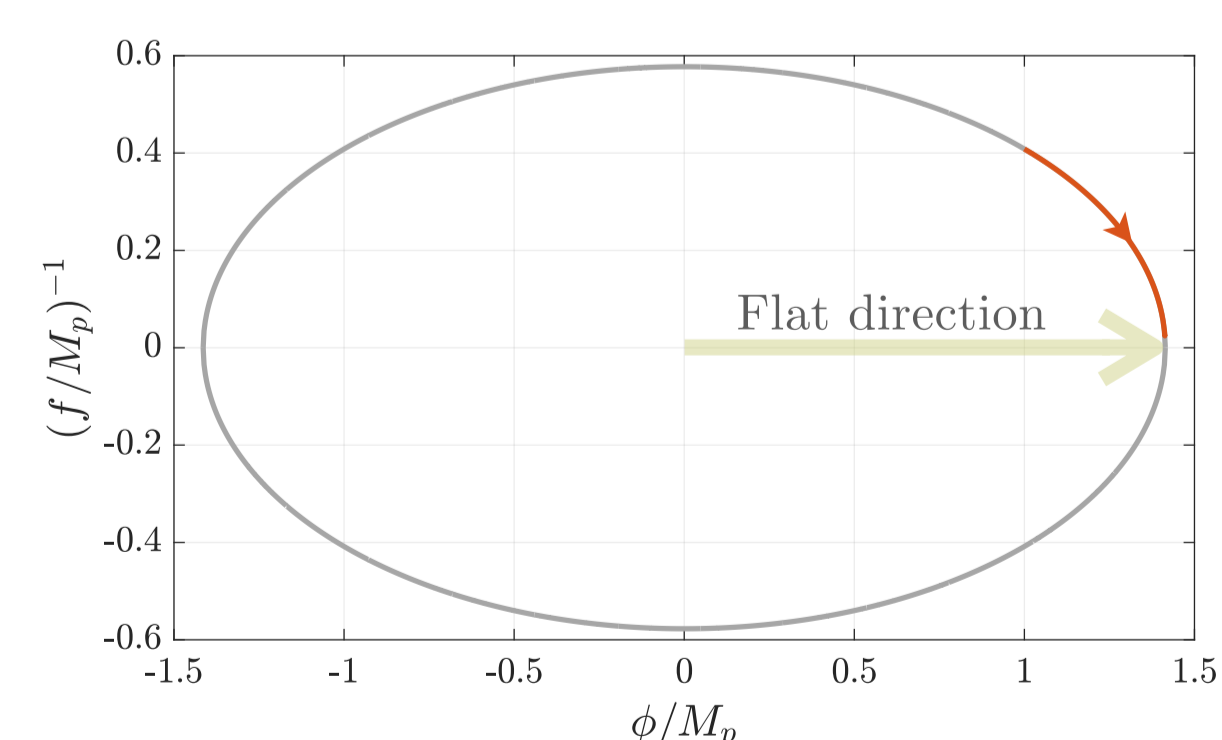
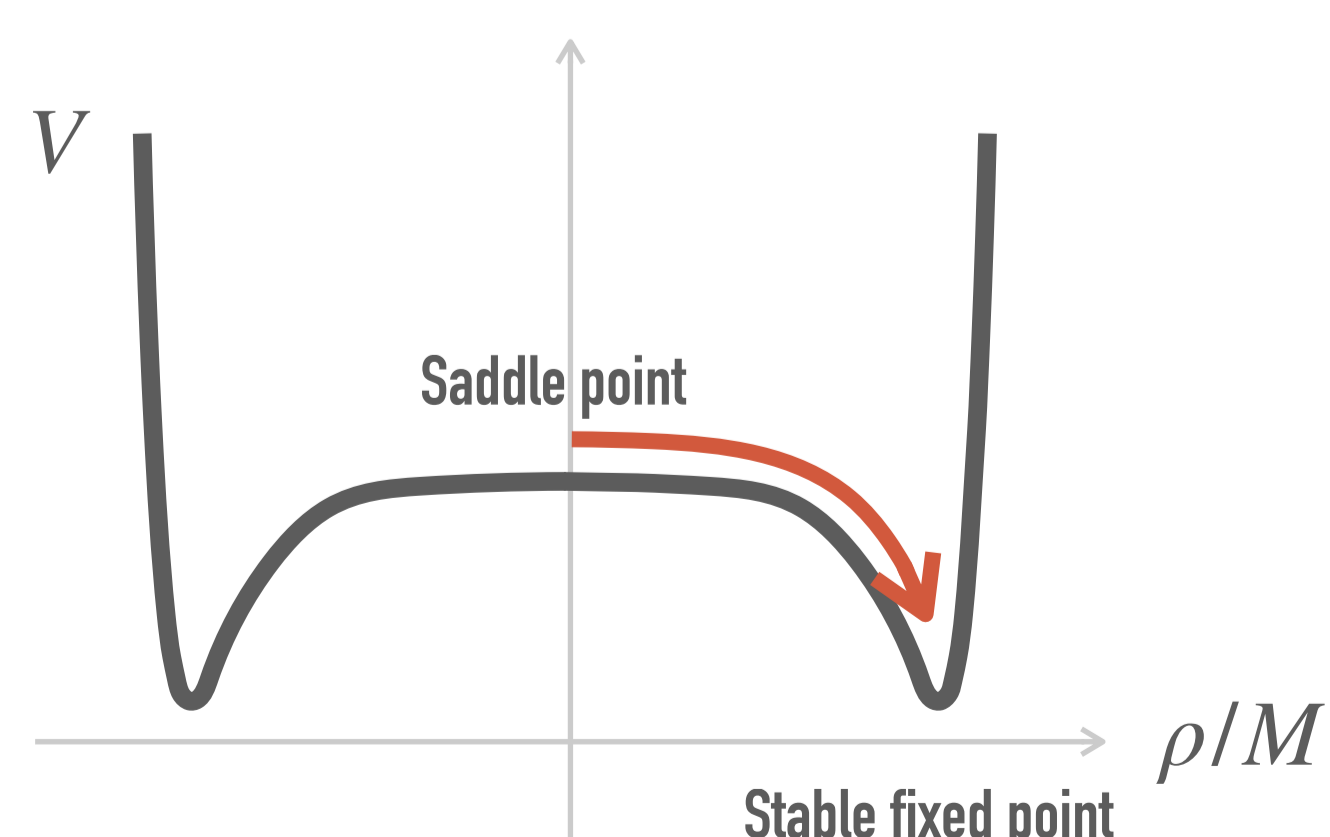


Figure 3: Ellipse constraining the dynamics in the two-fields plane. The arrow indicates the direction of motion.



- Effective single-field inflation: inflation is driven by the motion of the field ρ in a potential with a flat plateau (no fine-tuning needed!) (Fig. 4);

Figure 4: Mexican hat potential $V(\rho)$ describing the Einstein-frame dynamics of the inflaton ρ .

- The symmetry protects from geometrical destabilisation effects [3]: vanishing entropy perturbations ($\delta s = 0$).

NUMERICAL ANALYSIS

The integration scheme was presented in [4]:

Numerical integration up to the end of inflation ($|\epsilon| = 1$)

Sufficiently long inflation? → Discard

Compute A_s, n_s, α_s, r

Are they within some reasonably chosen ranges? → Discard

Implement CAMB and assign a likelihood based on how well the model agrees with CMB data

RESULTS

We derived robust constraints on the model's parameter that:

- Exclude conformal symmetry ($\xi = 1$) at high significance;
- Show an overall insensitivity to initial conditions;
- Predict a lower bound on the tensor-to-scalar ratio r , that will be testable from next generation CMB experiments.

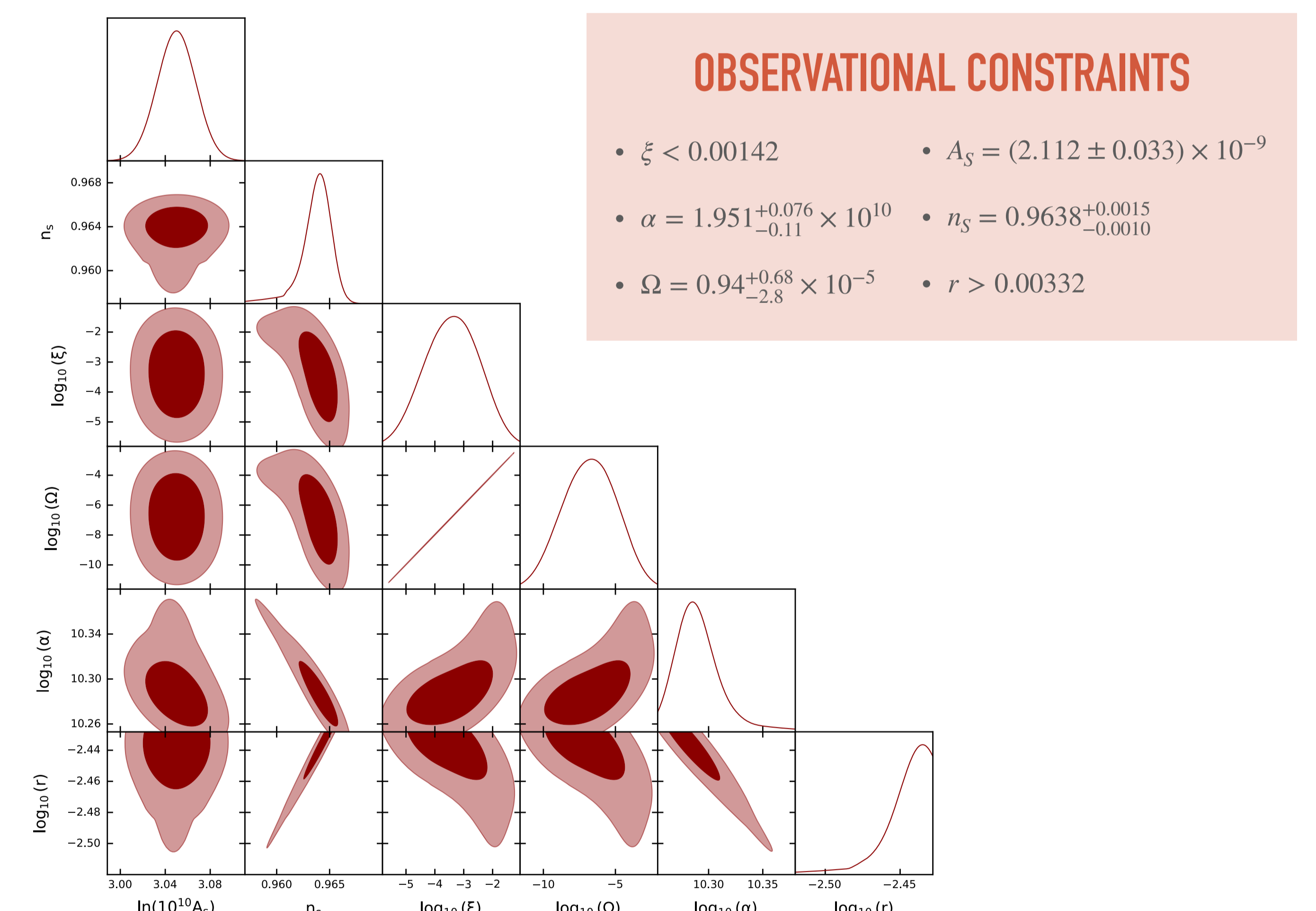


Figure 5: Triangular plot showing 2D joint and 1D marginalized posterior probability distributions for a selection of parameters. We define $\Omega \equiv \alpha\lambda + \xi^2$.

OBSERVATIONAL CONSTRAINTS

- $\xi < 0.00142$
- $A_s = (2.112 \pm 0.033) \times 10^{-9}$
- $\alpha = 1.951^{+0.076}_{-0.11} \times 10^{10}$
- $n_s = 0.9638^{+0.0015}_{-0.0010}$
- $\Omega = 0.94^{+0.68}_{-2.8} \times 10^{-5}$
- $r > 0.00332$

SCALE INVARIANCE AND STAROBINSKY'S INFLATION

Starobinsky's inflation itself becomes scale-invariant when the R^2 term dominates the inflationary dynamics. Does the model (1) lead to different predictions?

n_s and r are anti-correlated like in Starobinsky's model only at fixed ξ . Overall, they are correlated: it is potentially possible to discriminate between the two models!

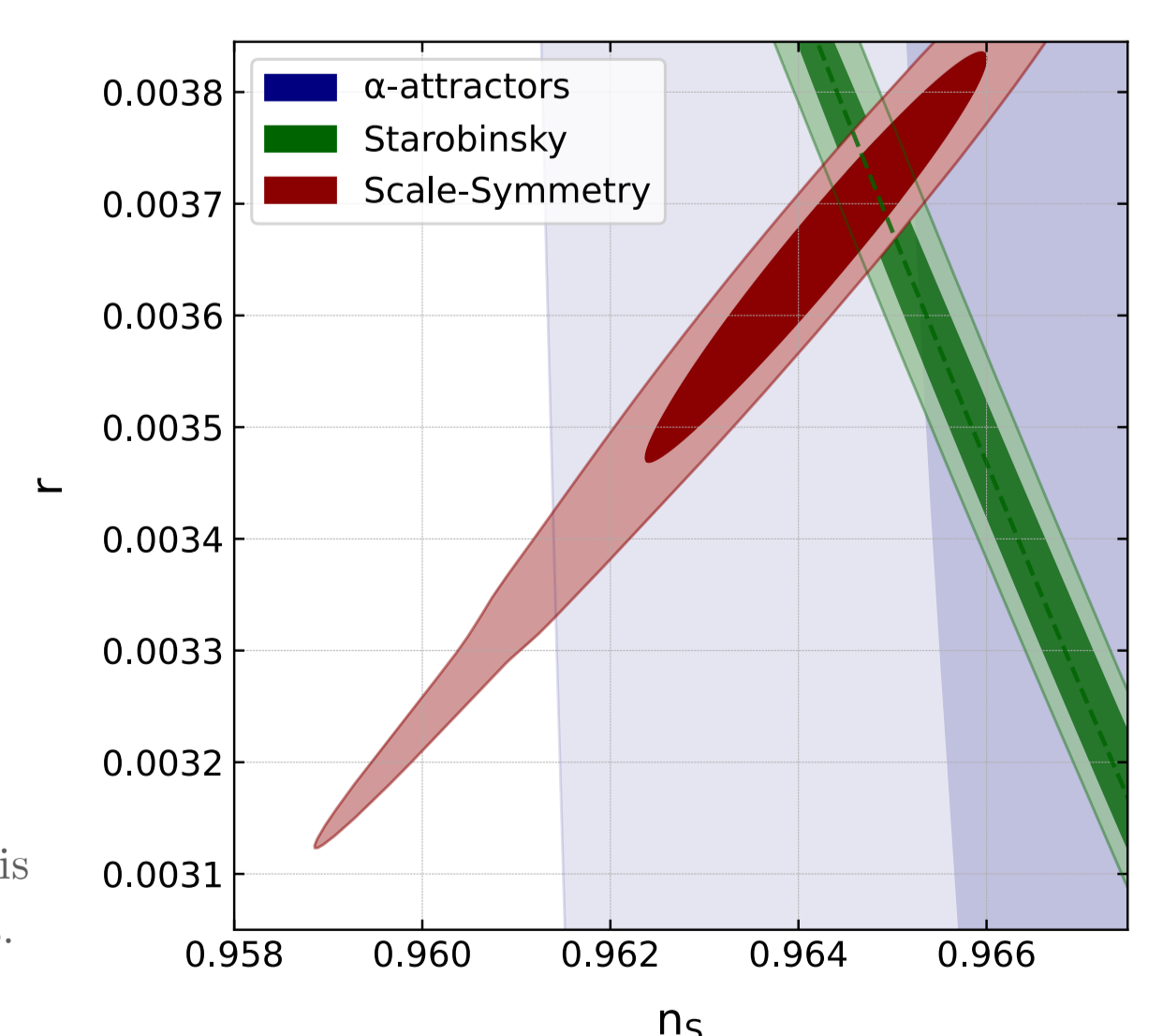


Figure 6: 2D contours for the scale-invariant model studied in this work, compared to those of Starobinsky inflation and α -attractors.

REFERENCES

- [1] M. Rinaldi & L. Vanzo, Phys. Rev. D **94** (2016) [arXiv:1512.07186]
- [2] Wetterich, Nucl. Phys. B, **964** (2021) 115326 [arXiv:2007.08805]
- [3] S. Renaux-Petel & K. Turzyński, Phys. Rev. Lett. **117** (2016) 141301 [arXiv:1510.01281]
- [4] W. Giarè, M. De Angelis, C. van de Bruck & E. Di Valentino, JCAP **12** (2023) [arXiv:2306.12414]