One-loop power spectrum in USR and implications for PBH DM

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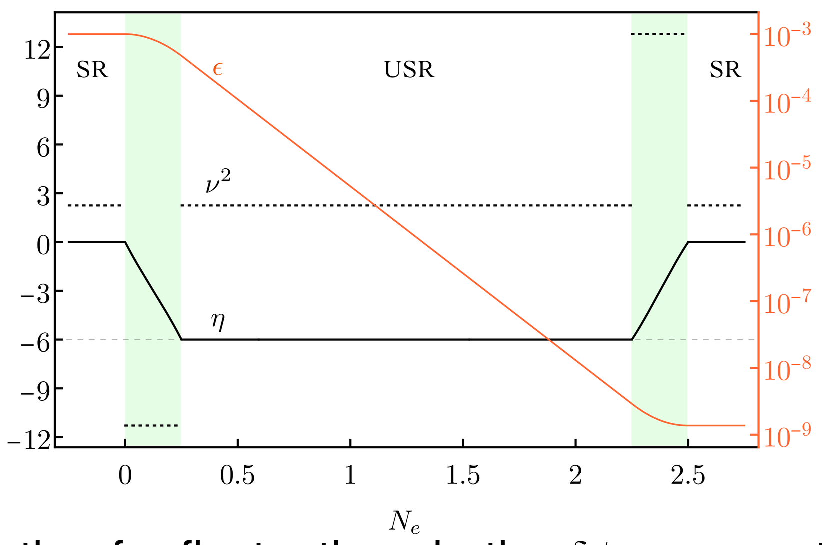
Introduction

Primordial black holes (PBHs) in the range from 10^{-16} to 10^{-12} Solar masses may constitute all the dark matter (DM) of the Universe. An inflationary model with an ultra slow-roll (USR) phase is capable of generating density fluctuations large and frequent enough, $\mathcal{P}_\zeta \sim 10^{-2}$, that will collapse into PBHs in later epochs –e.g. during the radiation epoch– constituting the totality of the DM . But, this values of $\mathcal{P}_\zeta$ almost beg the question of the validity of perturbation theory (PT). A paper by J. Kristiano and J. Yokoyama [1] studied this question considering the one-loop correction to $\mathcal{P}_\zeta$ in USR inflation. They concluded that such large values of ζ imply the breakdown of PT at CMB scales, which would severely threaten USR as a possibility for PBH formation. Other papers on the same issue have come to conflicting conclusions, leaving the question open, see e.g. [2, 3, 4].

We address the issue by improving on previous analysis: we include the full set of interactions and use a cutoff to regularize the ultraviolet divergences, absorbing them by introducing appropriate counterterms (cts). We find that whether PT breaks down depends on the duration of the transition between slow-roll (SR) inflation and USR inflation, and for well-motivated inflationary models considered in the literature, cosmological PT is valid.

Model of USR inflation

We consider a SR - USR - SR description of the inflationary dynamics. The transitions between SR and USR are not instantaneous, but have duration δN. The duration of the USR phase is ΔN.



The action for fluctuations in the $\delta\phi$-gauge, at leading order in $\epsilon = -\dot{H}/H^2$, is

$$S = \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x} \left[\frac{a^2}{2}\left((\partial_\tau \delta\phi)^2 - (\partial_i \delta\phi)^2 \right) - a^4 \sum_{n\geq 2} \frac{V_n \,\delta\phi^n}{n!} \right]$$

where $V_2 = -H^2(\nu^2 - 9/4)$, $V_n = \mathrm{d}^n V/\mathrm{d}\phi^n$ and

$$\nu^2 \equiv \frac{9}{4} + \frac{1}{2}\left(3\,\eta + \frac{\eta^2}{2} + \frac{\eta'}{aH} \right) .$$

We impose that ν^2 is piece-wise constant. Since ν is discontinuous, the self-interactions of $\delta\phi$ (proportional to $V_{3,4}$) are Dirac deltas centered on the moments of phase change. In the limit $\delta N \to 0$ (the model used in [1]), ν^2 satisfies $|\nu^2| \to 3/\delta N$ during the transitions; i.e. the interactions diverge.

In-In, regularization and cts

The power spectrum of ζ, $\mathcal{P}_\zeta$, calculated in the $\delta\phi$-gauge is, at late times,

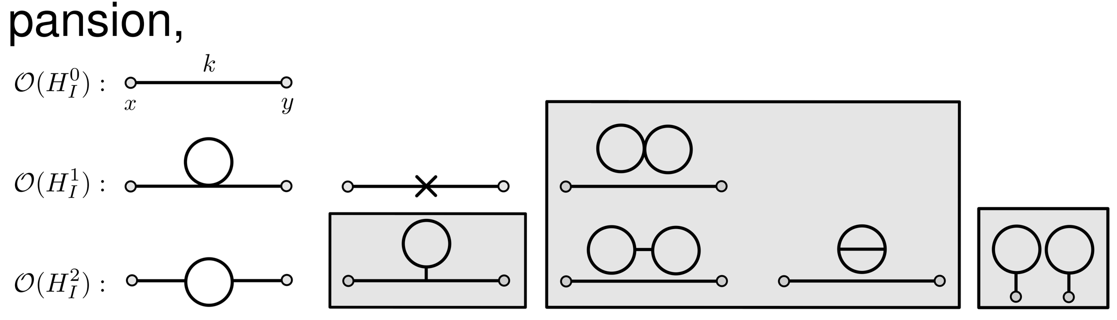
$$\mathcal{P}_\zeta(\tau,k) = \int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3} e^{-i\mathbf{k}\mathbf{r}} \frac{4\pi k^3}{2\epsilon(\tau) M_P^2} \langle \delta\phi(\mathbf{x}+\mathbf{r})\delta\phi(\mathbf{x}) \rangle \,.$$

We need to calculate the two-point correlation of $\delta\phi$, which in the in-in formalism is

$$\langle \delta\phi(\mathbf{x})\delta\phi(\mathbf{y}) \rangle = \left\langle \left(F(t,-\infty^-)\right)^\dagger \delta\phi(\mathbf{x})\delta\phi(\mathbf{y}) F(t,-\infty^-) \right\rangle ,$$

$$F(t,-\infty^-) = T \exp\left(-i \int_{-\infty(1-i\omega)}^{t} \mathrm{d}t' H_I(t') \right) .$$

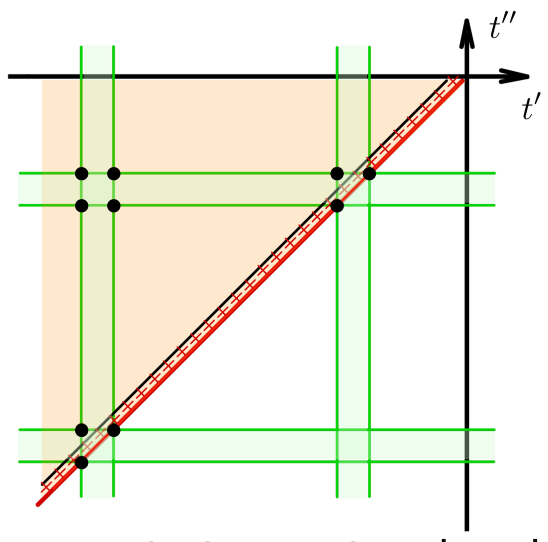
The two-point correlation admits a diagrammatic expansion,



The one-loop contribution gives an ultraviolet (UV) divergence that needs to be renormalized, for which we will use a UV cutoff Λ_{UV}. The cutoff goes into the momentum loops as usual,

$$\int^{\infty} \mathrm{d}^3\mathbf{p} \to \int^{\Lambda_{\mathrm{UV}}} \mathrm{d}^3\mathbf{p}$$

but its effect on the time integrals needs to be worked out: we remove from the integral a domain where the time intervals are smaller than those allowed by Λ_{UV},



We introduce counterterms to absorb the dependence of the loops on the regulator Λ_{UV}. For our purposes, it will only be necessary

$$H_I^{\mathrm{ct}}(\tau) \supset \int \mathrm{d}^3\mathbf{x}\, a^4\, \delta_V\, \delta\phi^2 \,.$$

δ_V comes from the renormalisation of the mass term and is a general function of time.

Analysis of the $\mathcal{P}_\zeta$ contributions

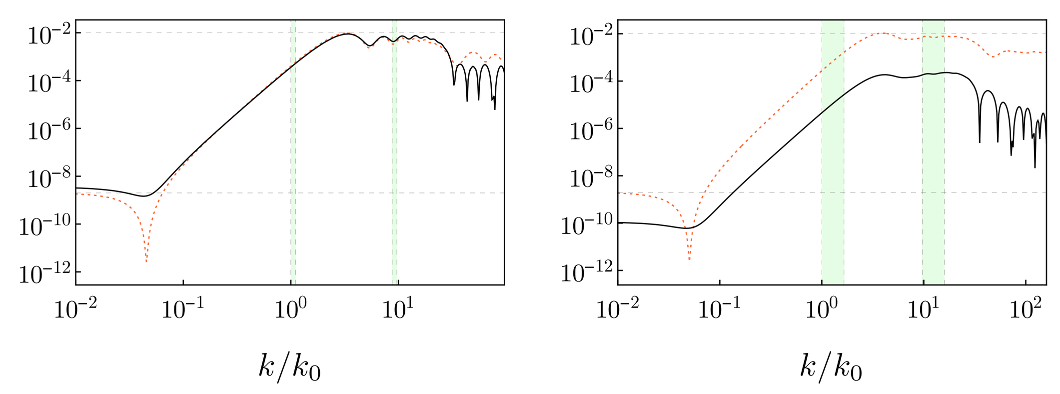
Tree-level: In our model, describes a scale invariant region in the CMB and an enhancement at the scales required to form PBHs,

$$\mathcal{P}_\zeta^{\mathrm{tl}}(\tau,k) = \frac{k^3}{4\pi^2 M_P^2 \epsilon(\tau)} |\delta\phi_k(\tau)|^2 \,.$$

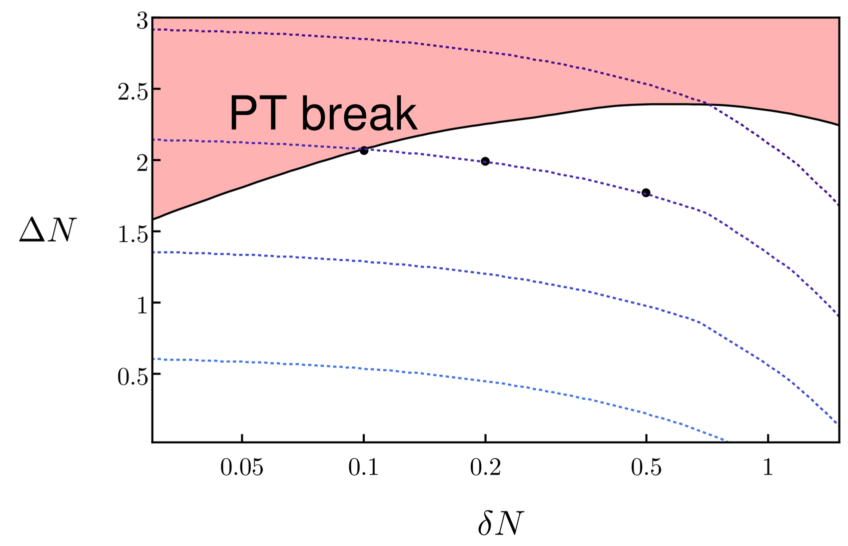
One-loop and cts: The divergent and finite part of the quartic contribution can be completely absorbed by the counterterms, so we do not include it.

In the cubic part, only the set of points $\tau' = \tau_i < \tau'' = \tau_j$ of the time integration region must be included, due to the regulator Λ_{UV} and the time-ordering T. Moreover, by properly dealing with the $-\infty(1-i\omega)$ prescription in the in-in, the final result of the cubic contribution does not show any UV divergence.

Discussion



A full renormalization procedure would require imposing a set of conditions on $\mathcal{P}_\zeta$ to extract the finite part of the counterterms. Although we have not performed the full renormalization, we can use our results to draw conclusions about the validity of perturbation theory, summarised in the condition $\mathcal{P}_\zeta^{\mathrm{ct}} \sim \mathcal{P}_\zeta^{\mathrm{1l}} \ll \mathcal{P}_\zeta^{\mathrm{tl}}$. We find that whether PT breaks or not depends on the values chosen for the two parameters, δN and ΔN, of the model.



When δN is made arbitrarily small, the $\mathcal{P}_\zeta^{\mathrm{1l}}$ diverges, invalidating any prediction based on PT through the in-in formalism: δN needs to be $\gtrsim 0.1$ for PT to hold. In realistic USR inflationary models this parameter is $\delta N \sim 0.4 - 0.5$, and perturbation theory does indeed appear to hold.

Comment on the δN dependence

Beyond the renormalization process, our work differs from previous results in the dependence of $\mathcal{P}_\zeta^{\mathrm{1l}}$ on δN. We obtain that, in the limit $\delta N \to 0$ (instantaneous transitions), the calculation at one-loop diverges. This result was missed in previous analyses.

In the ζ-gauge, the dominant term of the cubic action in USR is

$$S = \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x}\, M_P^2 a^2 \epsilon \frac{\eta'}{2} \zeta' \zeta^2 \,.$$

The interaction Hamiltonian in the interaction picture induced by this action is

$$H_I(\tau) = \int \mathrm{d}^3\mathbf{x}\, M_P^2 a^2 \epsilon \left(-\frac{\eta'}{2}\zeta'\zeta^2 + \frac{(\eta')^2}{16}\zeta^4 \right) .$$

The induced quartic interaction is usually neglected and it is precisely the one that gives rise to the divergence in the limit $\delta N \to 0$. Taking this term into account, we have shown that the power spectrum calculated through the $\delta\phi$-gauge and the ζ-gauge coincides in the limit $\delta N \to 0$.

References

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