Signatures of ultralight bosons in the orbital eccentricity of binary black holes

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SUPERRADIANCE CRASHCOURSE

[1] Arvanitaki, et al. 2010
[2] Brito, Cardoso, and Pani, 2015
[3] Detweiler 1980

1) Take an ultralight boson φ w/ mass μ, rotating black hole w/ mass M, define $\alpha \equiv \frac{GM\mu}{\hbar c}\ (<1)$

2) EOM for ultralight boson in a Kerr metric for $r > r_s$:

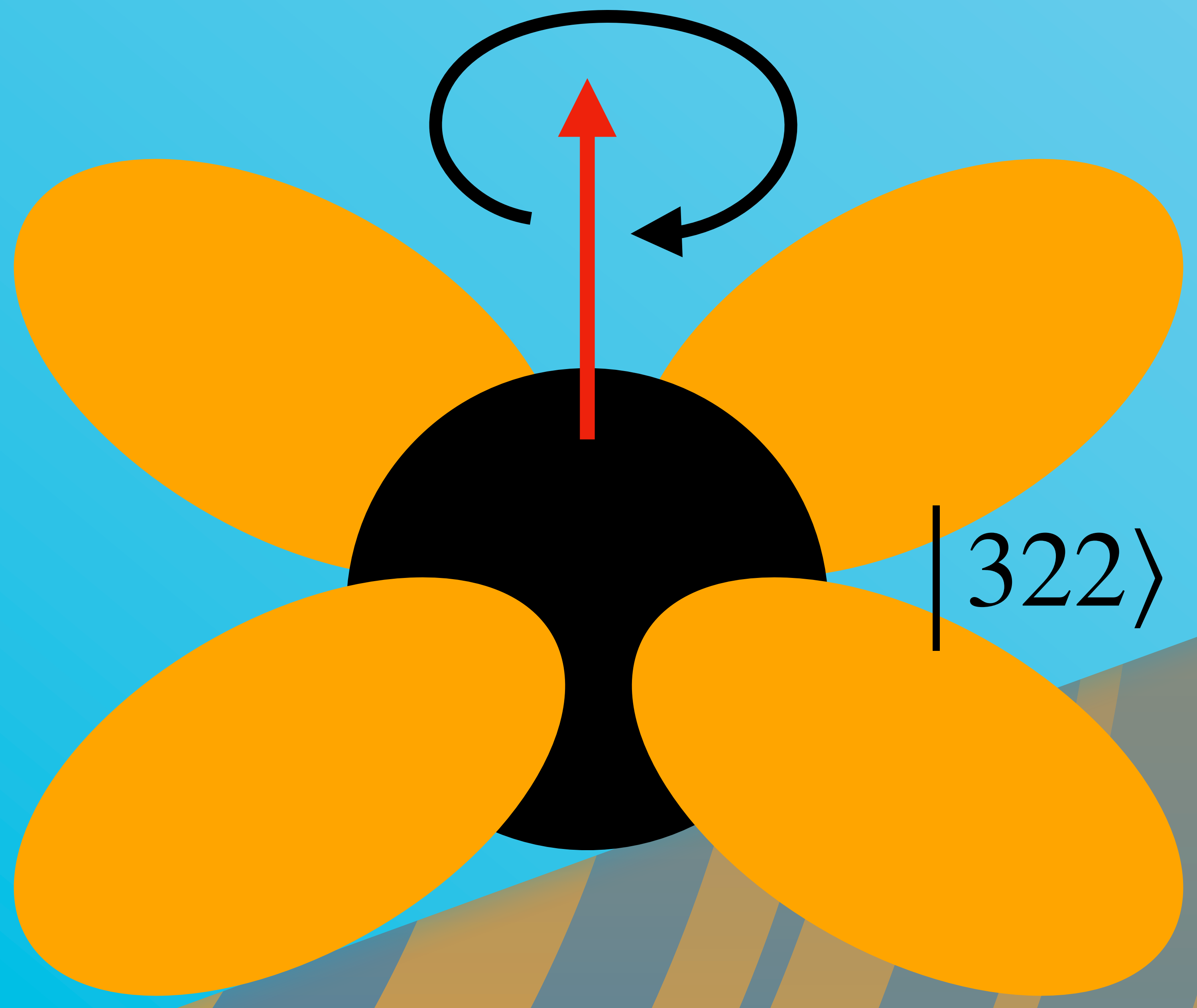
$$i\dot{\varphi} = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r}\right)\varphi + \mathcal{O}(\alpha^2)$$

→ Hydrogen-like states: **GRAVITATIONAL ATOM**

3) Horizon boundary conditions give

$\mathrm{Im}(\omega_{nlm}) \neq 0$ → **SUPERRADIANCE**

4) Fastest growing modes: $|211\rangle, |322\rangle$



CRASHCOURSE pt.2: BINARIES

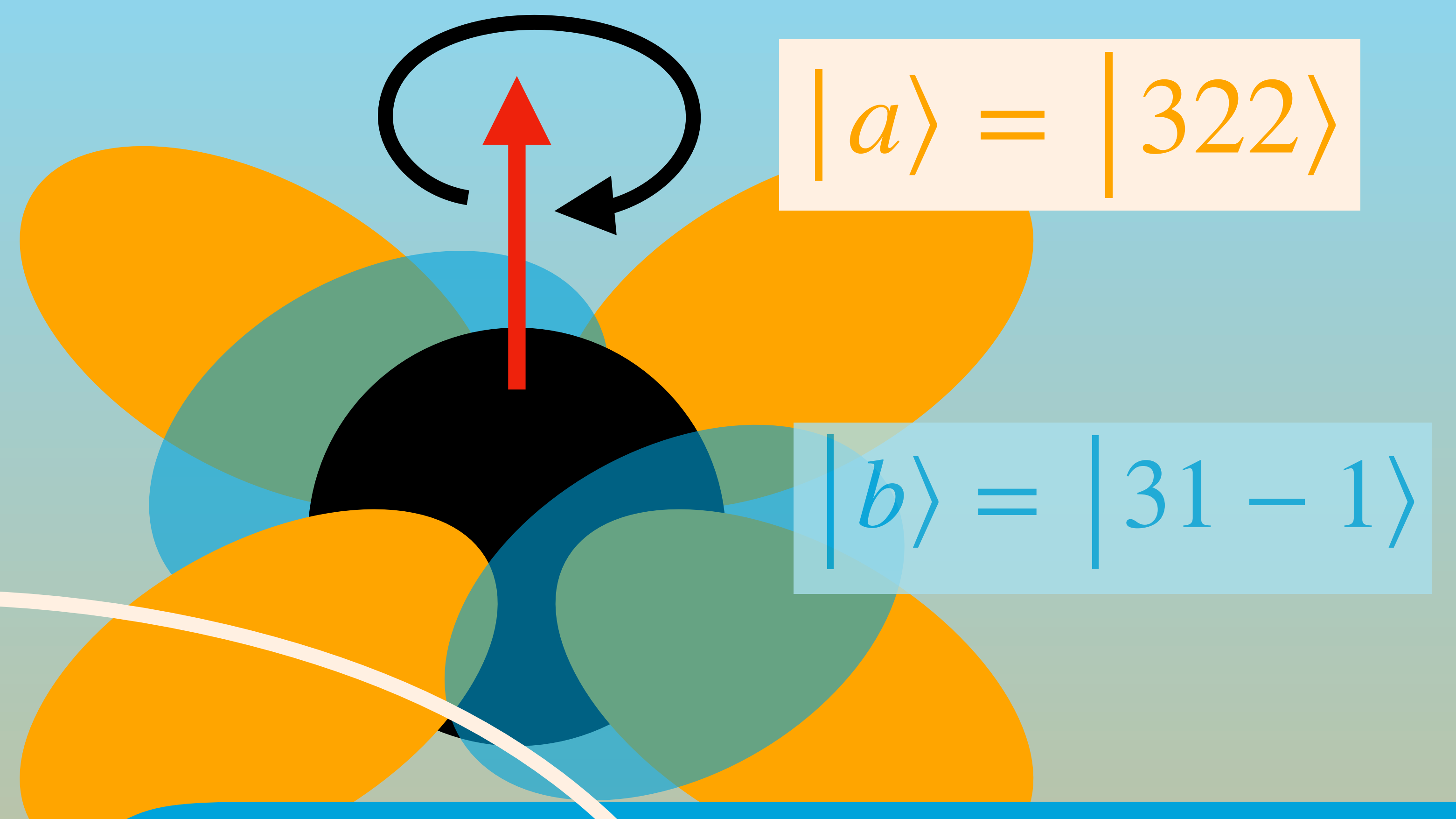
1) A binary companion induces a gravitational perturbation

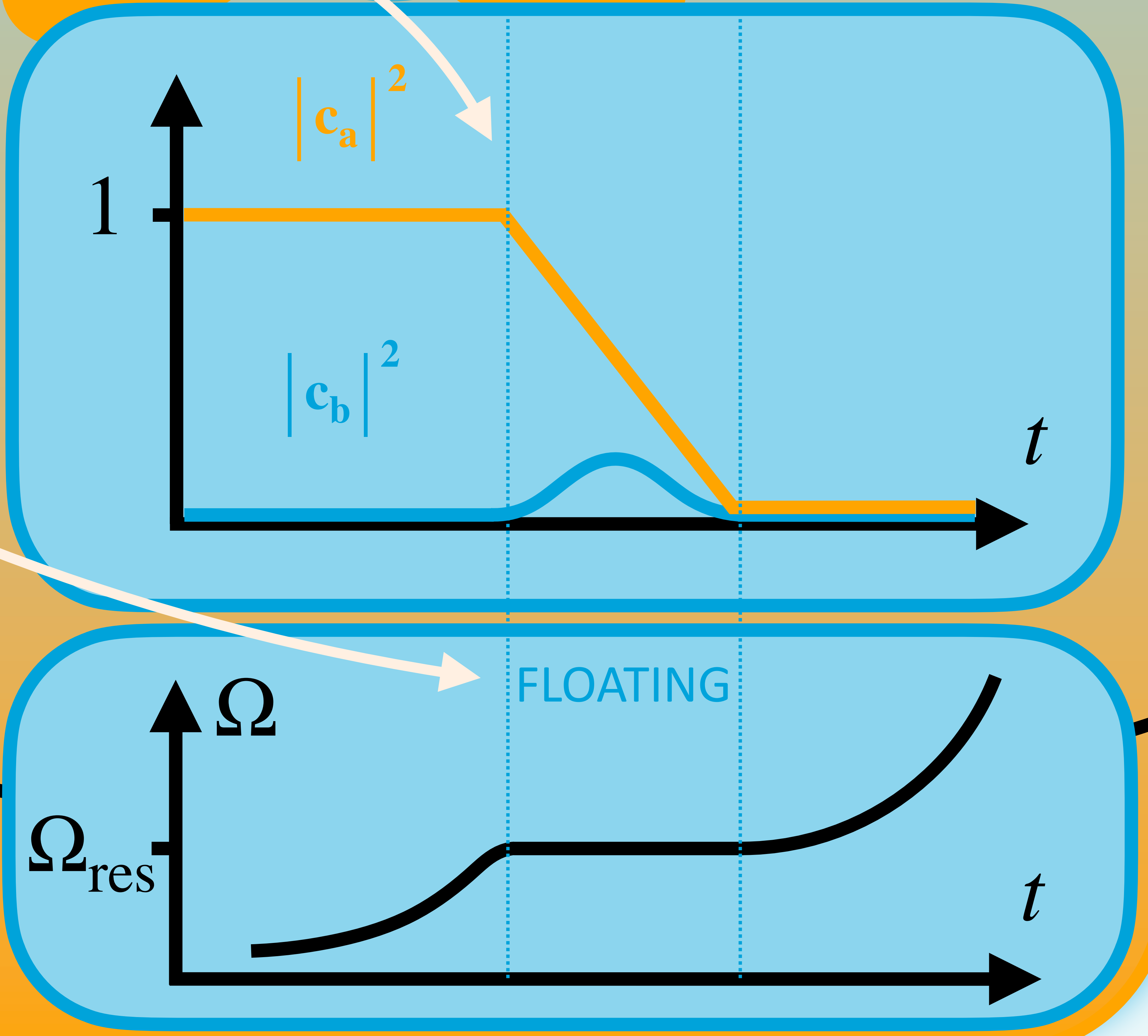
2) Schrödinger eqn. for a two-state system $|a\rangle$, $|b\rangle$:

$$i \begin{pmatrix} \dot{c}_a \\ \dot{c}_b \end{pmatrix} = \begin{pmatrix} -\frac{\Delta E}{2} & \eta(R_*)\mathrm{e}^{i\Delta m \varphi_*} \\ \eta(R_*)\mathrm{e}^{-i\Delta m \varphi_*} & \frac{\Delta E}{2} - i\Gamma_b \end{pmatrix} \cdot \begin{pmatrix} c_a \\ c_b \end{pmatrix}$$

- c_i: occupation densities
- ΔE, Δm: energy and angular momentum differences, respectively, Γ_b: decay rate of $|b\rangle$
- $\eta(R_*)$: mixing due to the perturbation
- φ_*: the true anomaly of the orbit

3) Landau-Zener (LZ) transition for $\Omega_{\mathrm{res}} \equiv \dot{\varphi}_* = \Delta E / \Delta m$.
Backreaction to the orbit → If the cloud loses energy, the orbit floats!





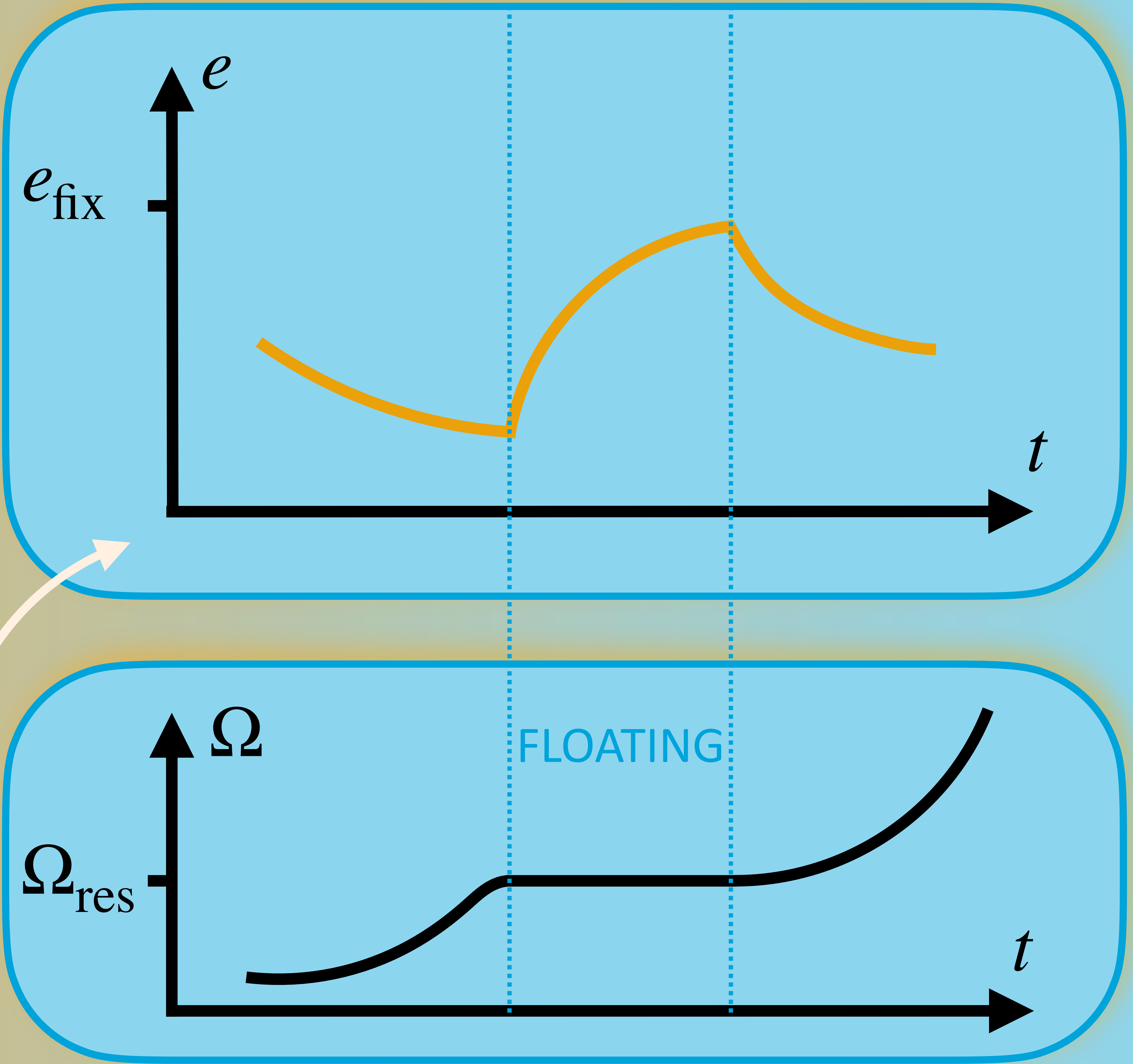
[4] Baumann, Chia, and Porto, 2019
[5] Baumann, et al., 2020

!Eccentricity Growth!

- For eccentricity e: $\varphi_* = \vartheta + 2e\sin\vartheta + \mathcal{O}(e^2)$
  (φ_*, ϑ the true and mean anomaly)
  $\rightarrow$ Jacobi-Anger expansion:
  $e^{i2e\sin\vartheta} = \sum_{k=-\infty}^{\infty} e^k/(|k|!)e^{ik\vartheta}$ in the mixing term.

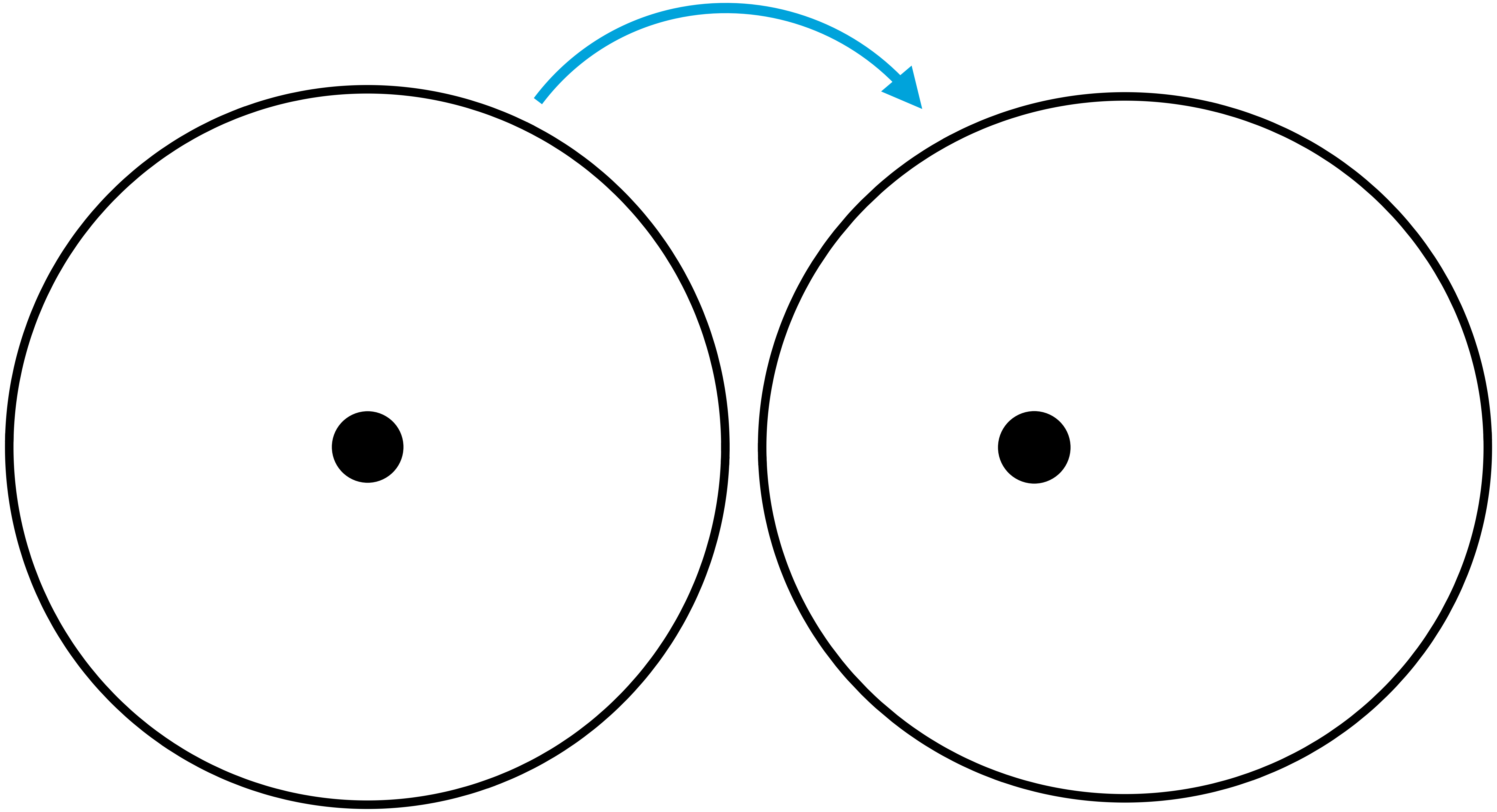
Resonance Condition: $\Omega_{\mathrm{res},k} \equiv \dot{\vartheta} = \Delta E/(\Delta m + k)$
$\rightarrow$ early and late resonances ($k \neq 0$).[6]

- During floating: orbital energy is constant, while for angular momentum: $\dot{L}_0 \propto \Omega_{\mathrm{res,k}} - \Omega_{\mathrm{res,0}} + \mathcal{O}(e^2)$.
- We have $d(L_0^2) \propto -d(e^2)$, and find that e can change towards a non-zero fixed point!
- For early resonances, the eccentricity grows.



- Frequency/semi-major axis stays constant, eccentricity grows:
- How does this change the orbit:

$$e \approx 0.001 \rightarrow e = 0.25$$



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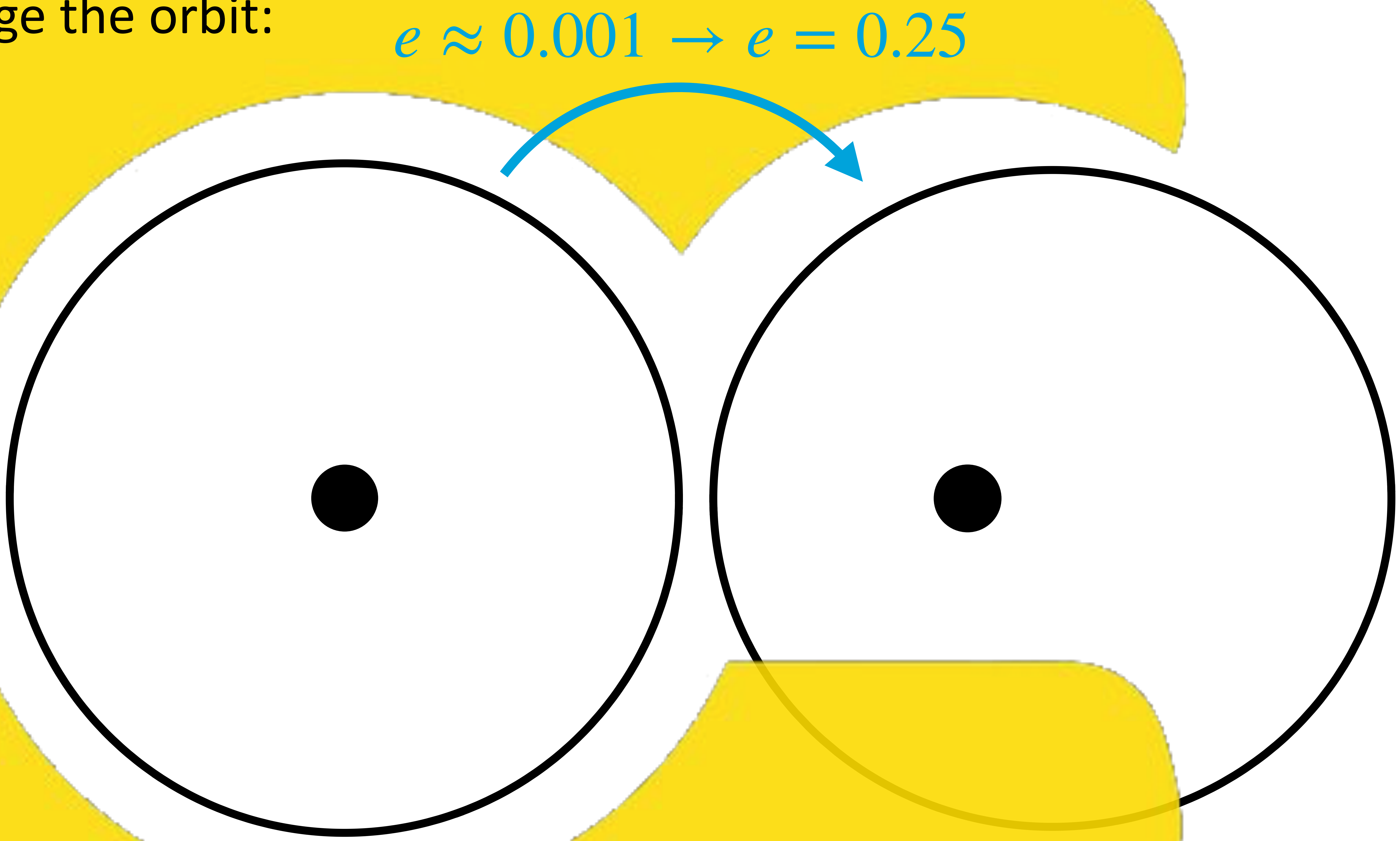


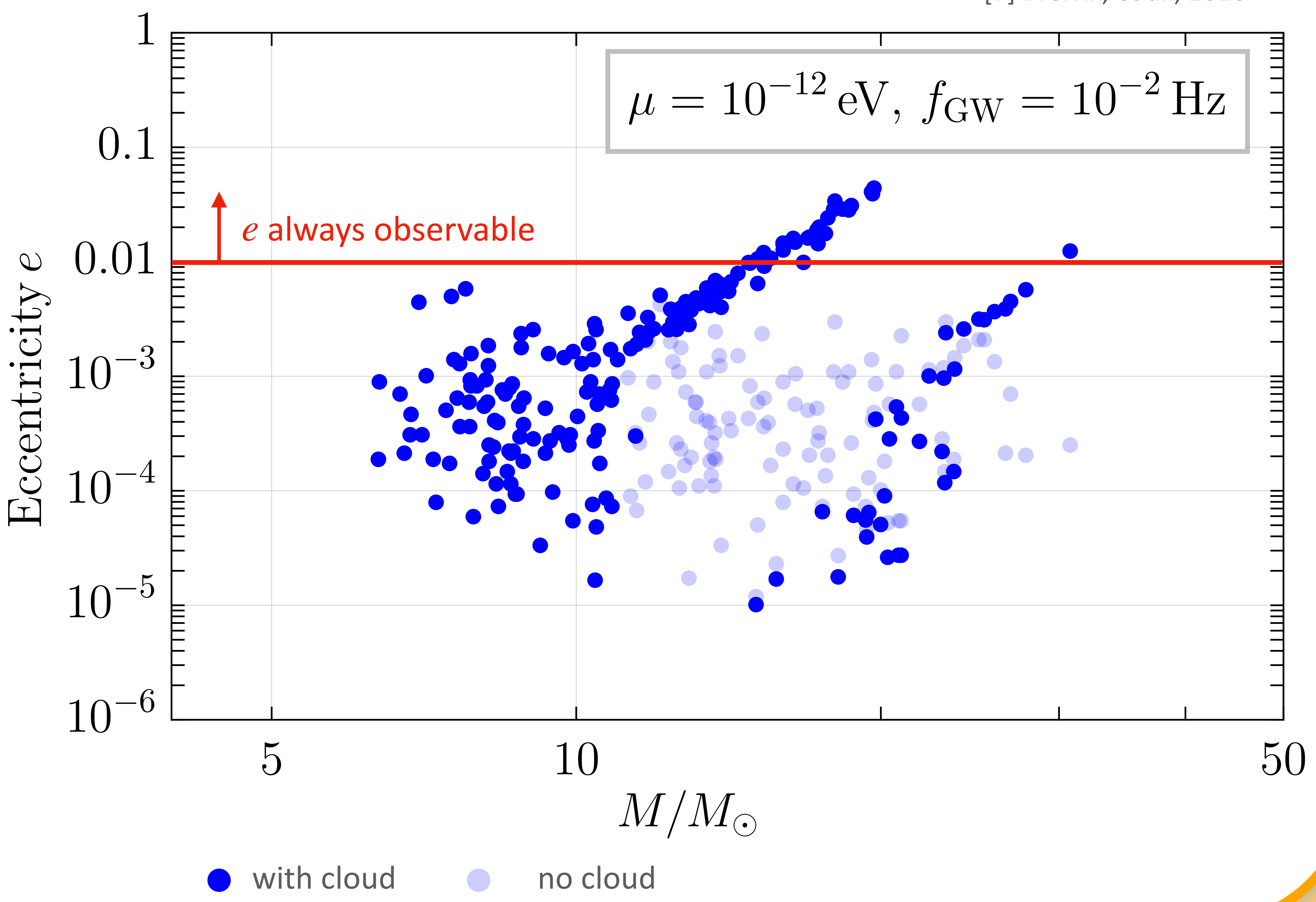
Binary Black Hole Populations in LISA

[6] Arun, et al. (LISA), 2022
[7] Breivik, et al., 2016

- population of binary BHs with chirp masses $< 10M_\odot$
- formed in isolation at an orbital frequency of $10^{-4}\pi$Hz.
- ultralight particle mass: of $10^{-12}\,$eV,

→ the binaries experience hyperfine and fine transitions, mostly $|322\rangle$ to $|31-1\rangle$.

- At the strongest early resonances, $k \leq -1$, the eccentricity can grow.

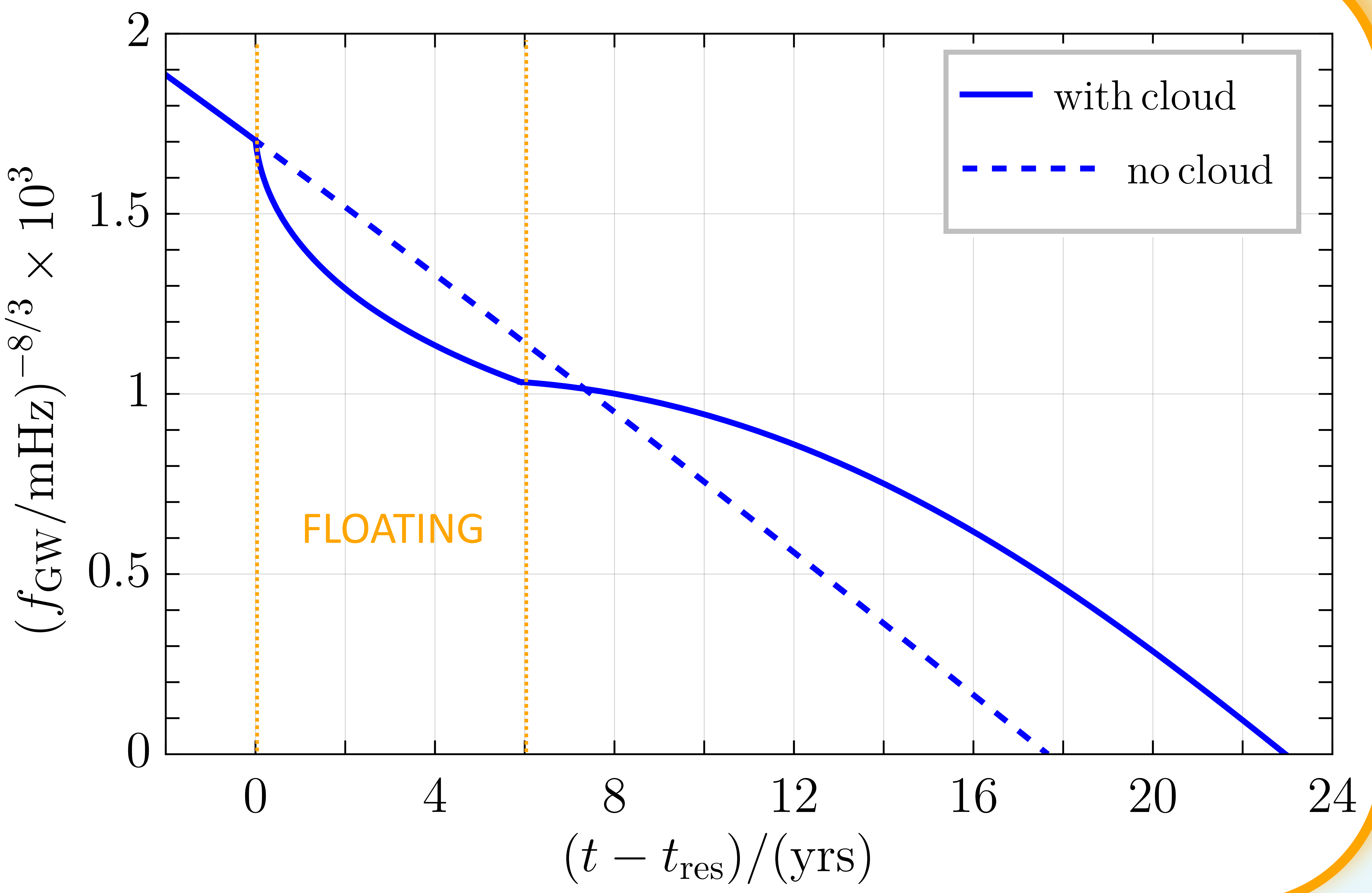


Eccentricity Growth in-band of LISA

The eccentricity growth can also fall into the LISA band, with floating times of $\mathcal{O}(\text{yrs})$.

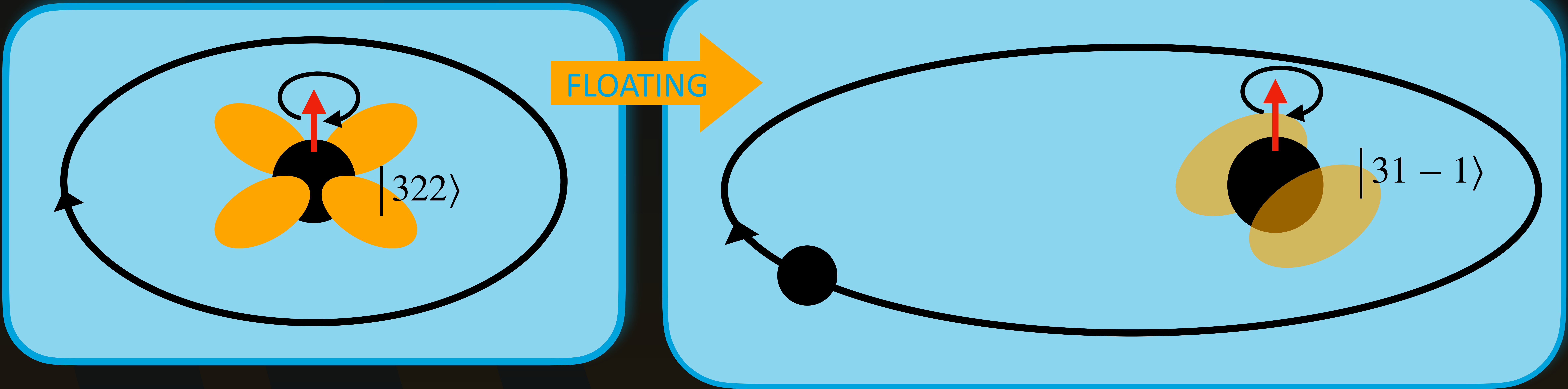
Example: $M_1 = 20\,M_\odot$, $\alpha = 0.21$, with companion $M_2 = 40\,M_\odot$.

Transition starts at $\approx 10\,\text{mHz}$.



Key Takeaways

- A perturbation on an eccentric orbit induces additional resonances.
- Backreaction to the orbit during a floating transition can significantly increase the eccentricity.
- Effects in populations as well as in single binaries can possibly be seen by LISA.



Note: Growth of eccentricity exaggerated. Results may vary.

Thank you for your attention!
Any questions?

References:

[1] A. Arvanitaki, et al., Phys. Rev. D 81, 123530 (2010)
[2] R. Brito, V. Cardoso, and P. Pani, Lect. Notes Phys. 906, pp.1 (2015)
[3] S. L. Detweiler, Phys. Rev. D 22, 2323 (1980)
[4] D. Baumann, H. S. Chia, and R. A. Porto, Phys. Rev. D 99, 044001 (2019)
[5] D. Baumann, et al., Phys. Rev. D 101, 083019 (2020)
[6] K. G. Arun, et al. (LISA), Living Rev. Rel. 25, 4 (2022)
[7] K. Breivik, et al., Astro- phys. J. Lett. 830, L18 (2016)

Backup-Slides

Balance Equations and Backreaction

(Many time dependencies not explicitly written)

Balance equations for orbit o, cloud c and BH energy E and angular momentum L, S:

$$\dot{E}_{\mathrm{o}} + \dot{E}_{\mathrm{c}} + \dot{M} = \mathcal{F}_{\mathrm{GW}} \equiv -\frac{32 f(e)}{5}\frac{M^5 q^2 (q+1)}{a^5}$$

$$\dot{L}_{\mathrm{o}} + s(\dot{L}_{\mathrm{c}} + \dot{S}) = \mathcal{T}_{\mathrm{GW}} \equiv \frac{\mathcal{F}_{\mathrm{GW}}}{\Omega}\frac{g(e)}{f(e)}$$

$$\dot{M} = 2\Gamma_b E_{\mathrm{c}(b)},\ \dot{S} = 2\Gamma_b L_{\mathrm{c}(b)}$$

Equations for Frequency and eccentricity backreaction:

$$\frac{d\Omega}{dt} = r\gamma_0 \mathfrak{f}^{11/3} f(e) \qquad \text{with } \mathfrak{f} \equiv \frac{\Omega}{\Omega_0}, q \equiv \frac{M}{M_*}$$

$$r \equiv \frac{\dot{E}_{\mathrm{o}}}{\mathcal{F}_{\mathrm{GW}}} = 1 - b\frac{\mathrm{sgn}(s\Delta m)\mathfrak{f}^{-11/6}}{\sqrt{f(e)\gamma_0 |\Delta m + k|}}\frac{d|c_a|^2}{dt}$$

$$\frac{de^2}{dt} = \frac{2}{3}\mathfrak{f}^{8/3}\frac{\gamma_0}{\Omega_0} f(e)\sqrt{1-e^2} \times \left[r\left(\mathfrak{f} - \sqrt{1-e^2}\right) - \mathfrak{f} + \frac{g(e)}{f(e)}\right]$$

$$b \equiv \frac{3M_{\mathrm{c},0}}{M}\frac{|\Delta m|\mathfrak{f}^{-3/2}}{|\Delta m + k|^{-1/2}}\frac{(1+q)^{1/3}}{\alpha q}\frac{\left(M\Omega_0\right)^{1/3}\Omega_0}{\sqrt{\gamma_0}}$$

Non-Linear Landau-Zener transition

Because of the backreaction on the orbital frequency, which controls the LZ transition, the problem becomes nonlinear.
It depends on the level-occupancy through the derivative of the parent state occupancy. In the parts of the parameter space where it has compact support, the backreaction simply renormalizes the parameters $(z_k, v_k) \to (\zeta_k, w_k)$ near the maximum value, as

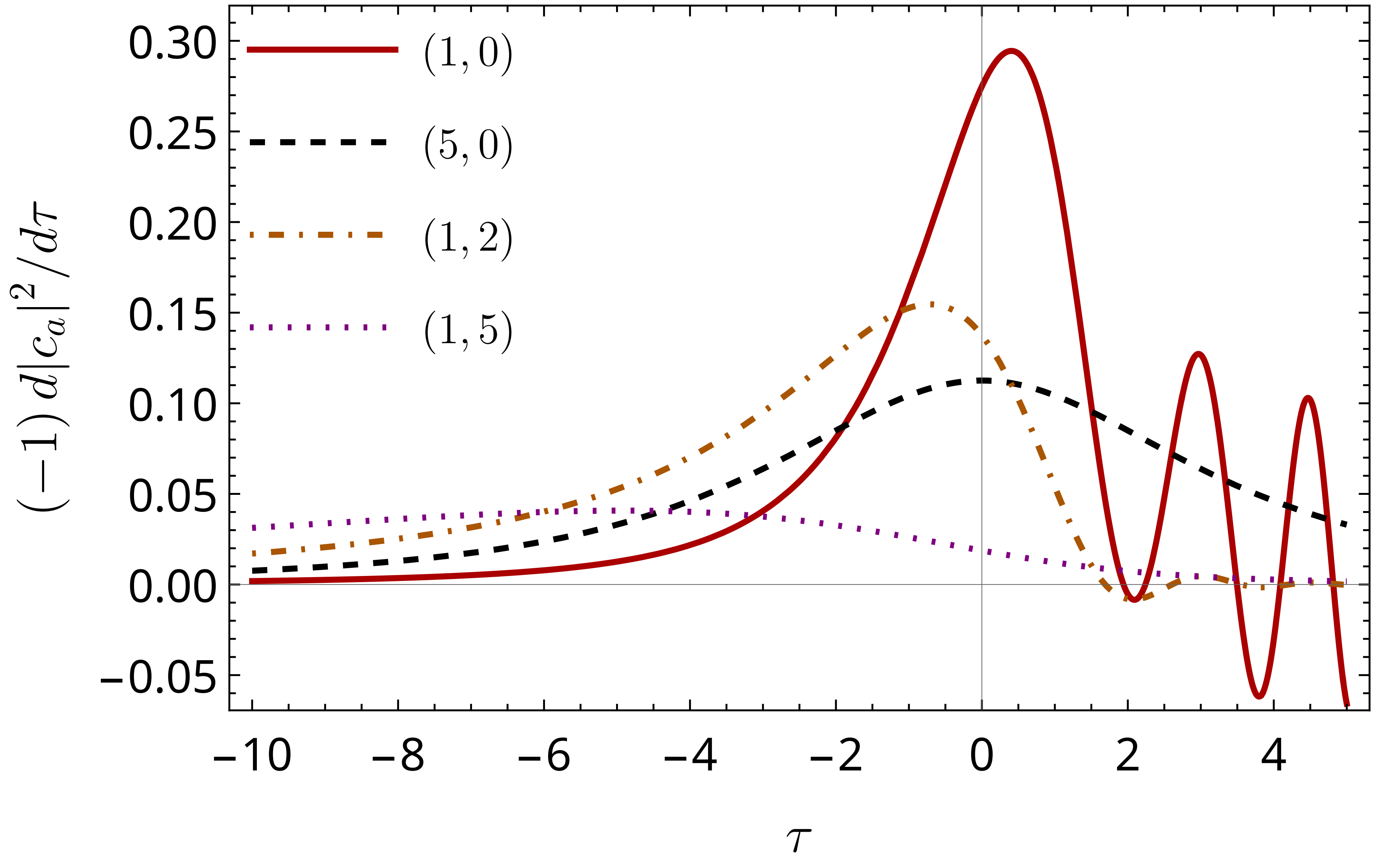
$$\zeta_k = \frac{\eta_k^2}{|\Delta m + k|\dot{\Omega}_k} = \frac{\eta_k^2}{|\Delta m + k|\gamma_k f(e) r} = \frac{z_k}{r_k(\zeta_k)}$$

and with the energy transfer itself depending on the level-occupancy of the cloud. The solution can, nonetheless, be found self-consistently in terms of the relevant parameters.

$$\left.\frac{d|c_a|^2}{d\tau}\right|_{\tau=-\zeta_k w_k} = \sqrt{\frac{z_k}{\zeta_k}}\psi_k(\zeta_k, w_k)$$

$$\psi_k \equiv \frac{1}{2}\exp(-\frac{1}{2}(\pi\zeta_k))\left[2w_k\left|\mathscr{C}H_{i\zeta}\left(\frac{\mathscr{C}w_k\Delta w_k}{2}\right)\right|^2 - \left(\mathscr{C}H_{1-i\zeta}\left(\frac{\mathscr{C}^*\Delta w_k^*}{2}\right)H_{i\zeta}\left(\frac{\mathscr{C}\Delta w_k}{2}\right) + \mathrm{c.c.}\right)\right]$$

with $\mathscr{C} = 1 + i$ and $\Delta w_k = w_k(1 - \zeta_k)$.



e.g. in the strong decay regime, we have

$$\psi_k \sim \frac{2\zeta_k e^{-\zeta_k[\pi - 2\arctan(\zeta_k)]}}{w_k(\zeta_k^2 + 1)}, r_k \sim 1 \pm 2b_k z_k \frac{e^{-\zeta_k[\pi - 2\arctan(\zeta_k)]}}{v_k(\zeta_k^2 + 1)}$$

Derivative of the parent state occupancy, evaluated for a linear LZ transition. Brackets in the legend correspond to (z_k, v_k). Note the decrease, shift to the left, of the amplitude, as well as the widening of the function, as the ratio v_k/z_kincreases.

Growth of Eccentricity

To calculate the evolution of the eccentricity towards the fixed point during floating, we change the time variable in the evolution equations from t to $e^2(t)$, yielding

$$\int_{e_{\rm in}^2}^{e_{\rm fin}^2} \frac{d(e^2)}{\sqrt{1-e^2}\left[\frac{g(e)}{f(e)} - \mathfrak{f}_k\right]} = \frac{2}{3}\frac{\sqrt{\gamma_0}}{\Omega_0}\frac{\mathfrak{f}_k^{5/6}}{|\Delta m + k|^{1/2}} b_k\left(1 - |c_a(\infty)|^2\right)$$

The duration of the floating period can be estimated as:

$$\Delta\tau_{\rm Fl} = \frac{1}{I_e}\int_{e_{\rm in}^2}^{e_{\rm fin}^2} d(e^2)\frac{1}{\sqrt{f(e)}\sqrt{1-e^2}\left[\frac{g(e)}{f(e)} - \mathfrak{f}_k\right]}$$