Analytic derivation of gravitational wave spectrum from expanding string loop on domain wall

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arXiv:2405.09599

w/ **Wakutaka Nakano (KEK)**

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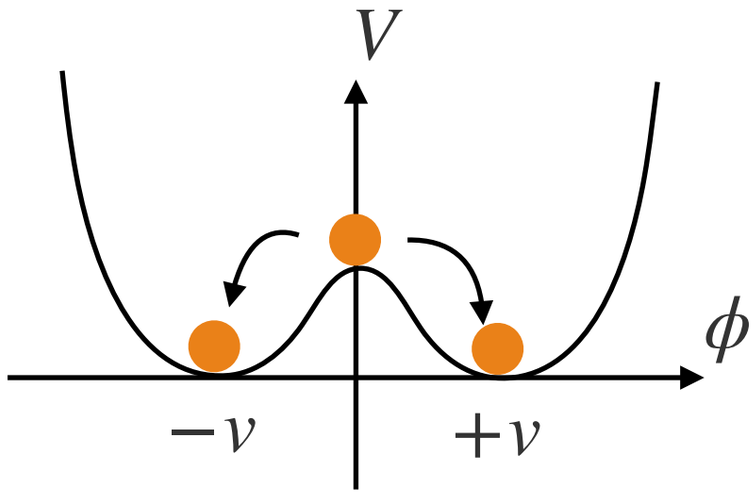
Domain wall

- **remnant of breaking of discrete sym.**



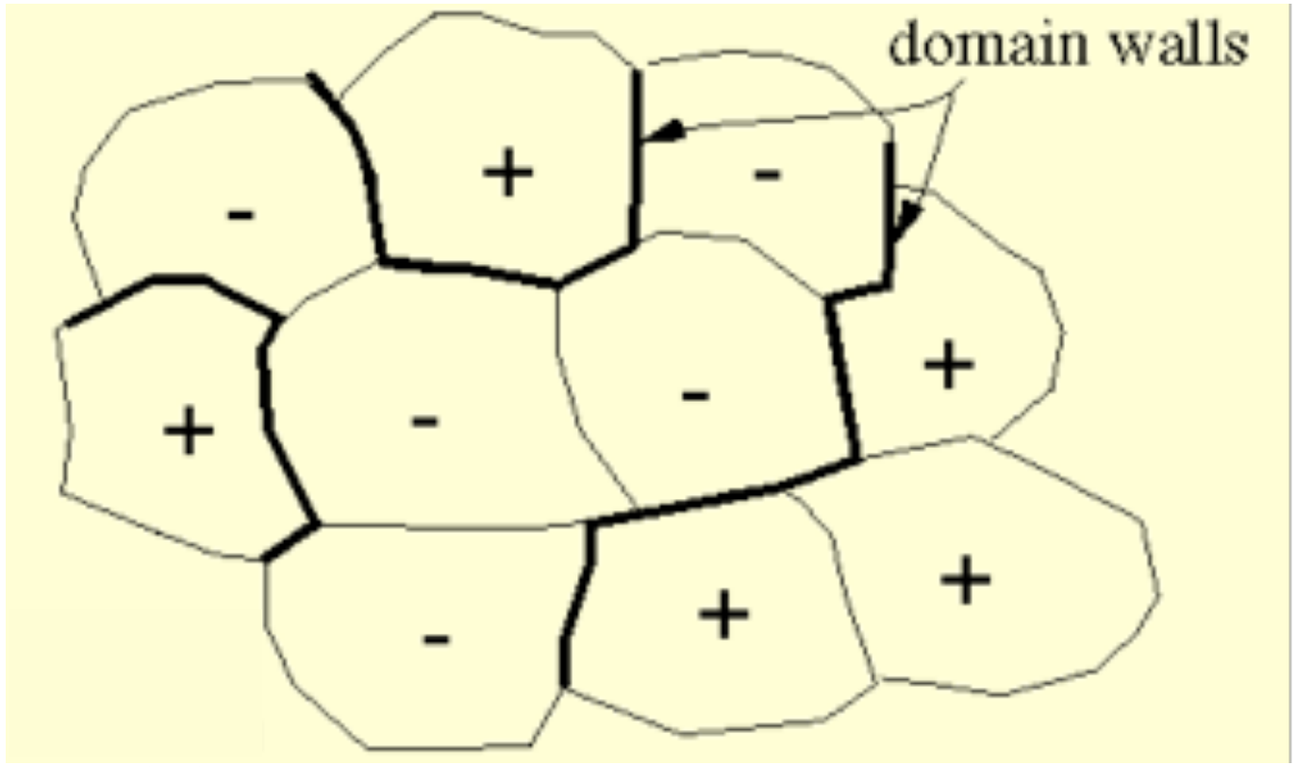


fig. from https://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_two.php

- easily dominates energy density of universe

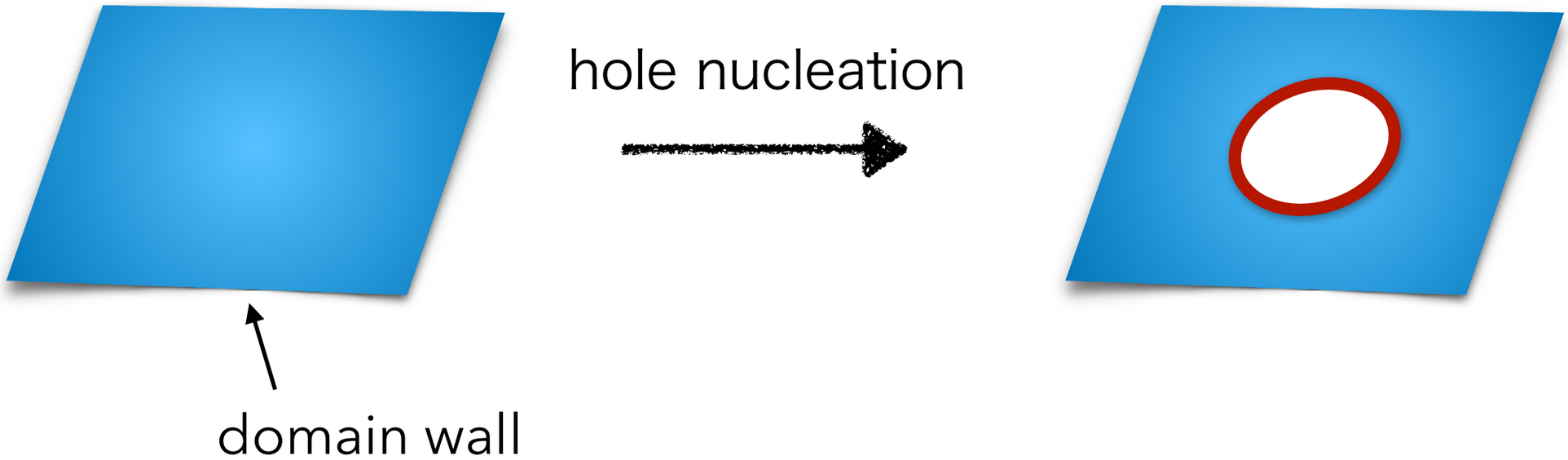
  → **dangerous object in cosmological sense**

  → **need to decay!**

Hole nucleation

[Kibble-Lazarides-Shafi '82, Everett-Vilenkin '82, Preskill-Vilenkin '92]

- one possible decay process: **hole nucleation**



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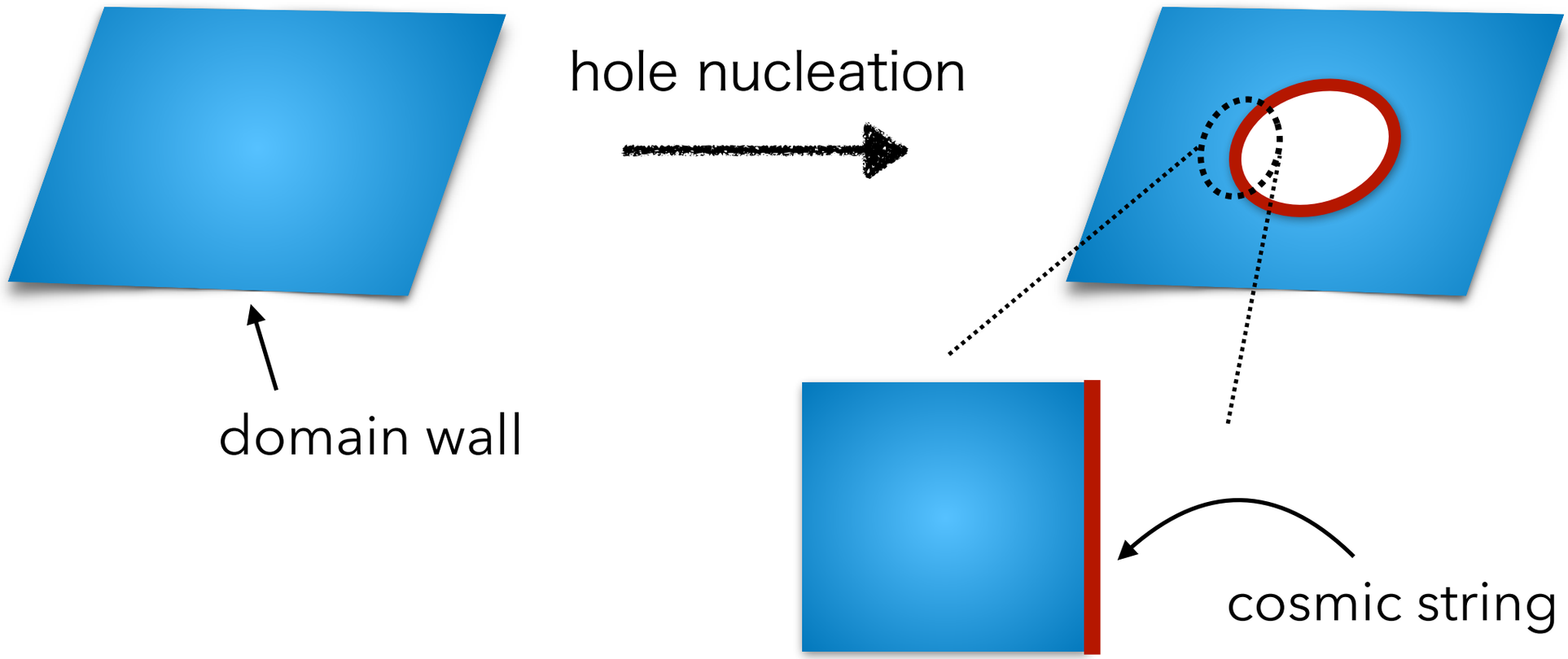

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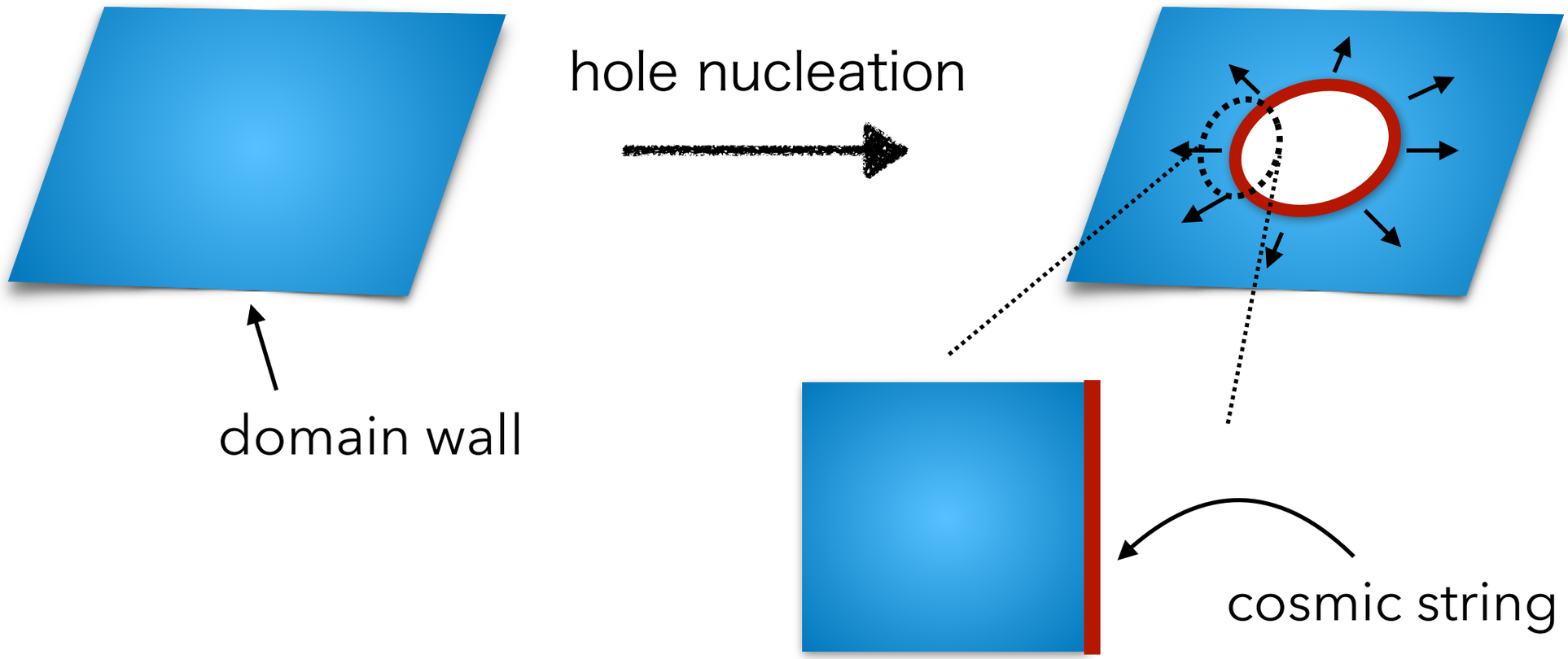

→ after nucleation, string loop expands on DW

→ **radiates gravitational wave!** [Dunsky+ '21]

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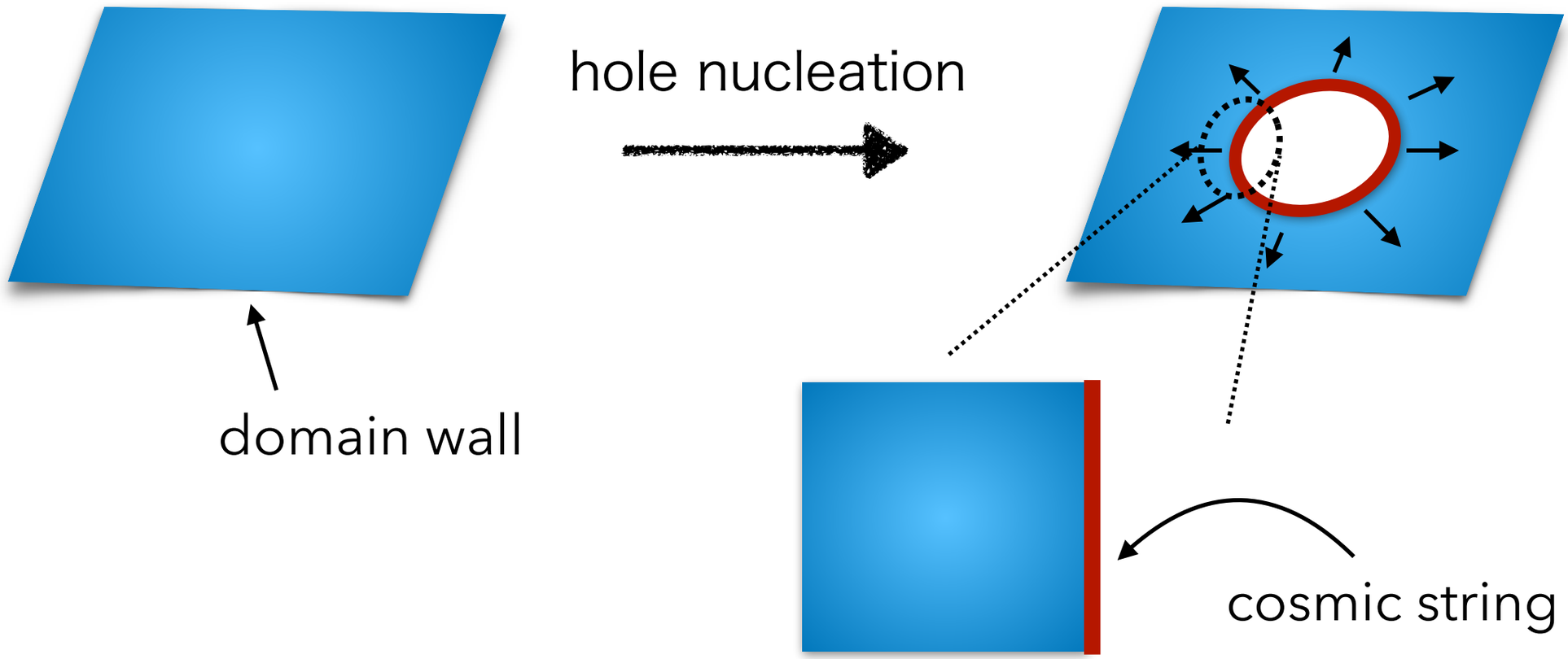

→ after nucleation, string loop expands on DW

→ **radiates gravitational wave!** [Dunsky+ '21]

- seems like **2D ver of bubble nucleation in 1st-order phase tr.**

Setup

- Formula of GW energy density

[cf. Jinno-Takimoto '16 & '17]

$$\rho_{\rm GW} \sim \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \cos(k(t_1 - t_2))\ \mathrm{F.T.} \langle T_{ij}^{\rm TT}(t_1, \vec{r}_1) T_{ij}^{\rm TT}(t_2, \vec{r}_2) \rangle_{\rm ens}$$

ensemble average of EM-tensor

Ensemble average

= average of nucleation points + average of DW configuration (planar)

Setup

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ensemble average of EM-tensor

Ensemble average
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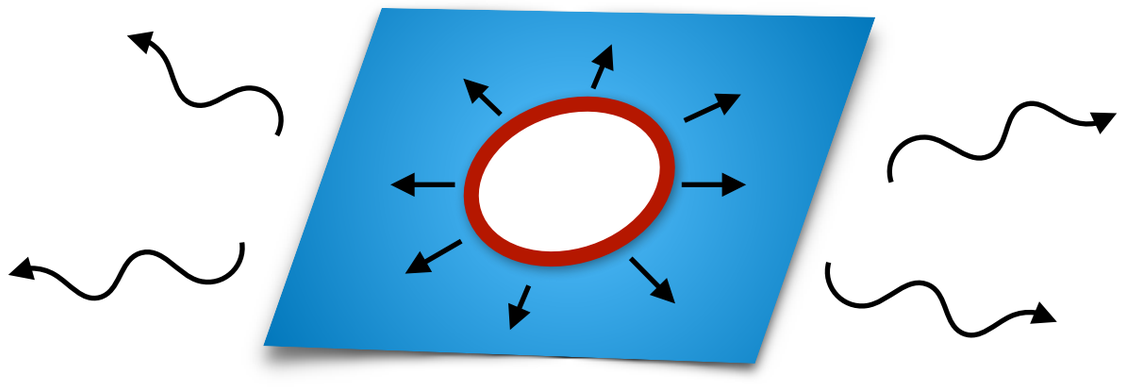
point:
The loops do radiate GWs even when they are circular-shape before collision!

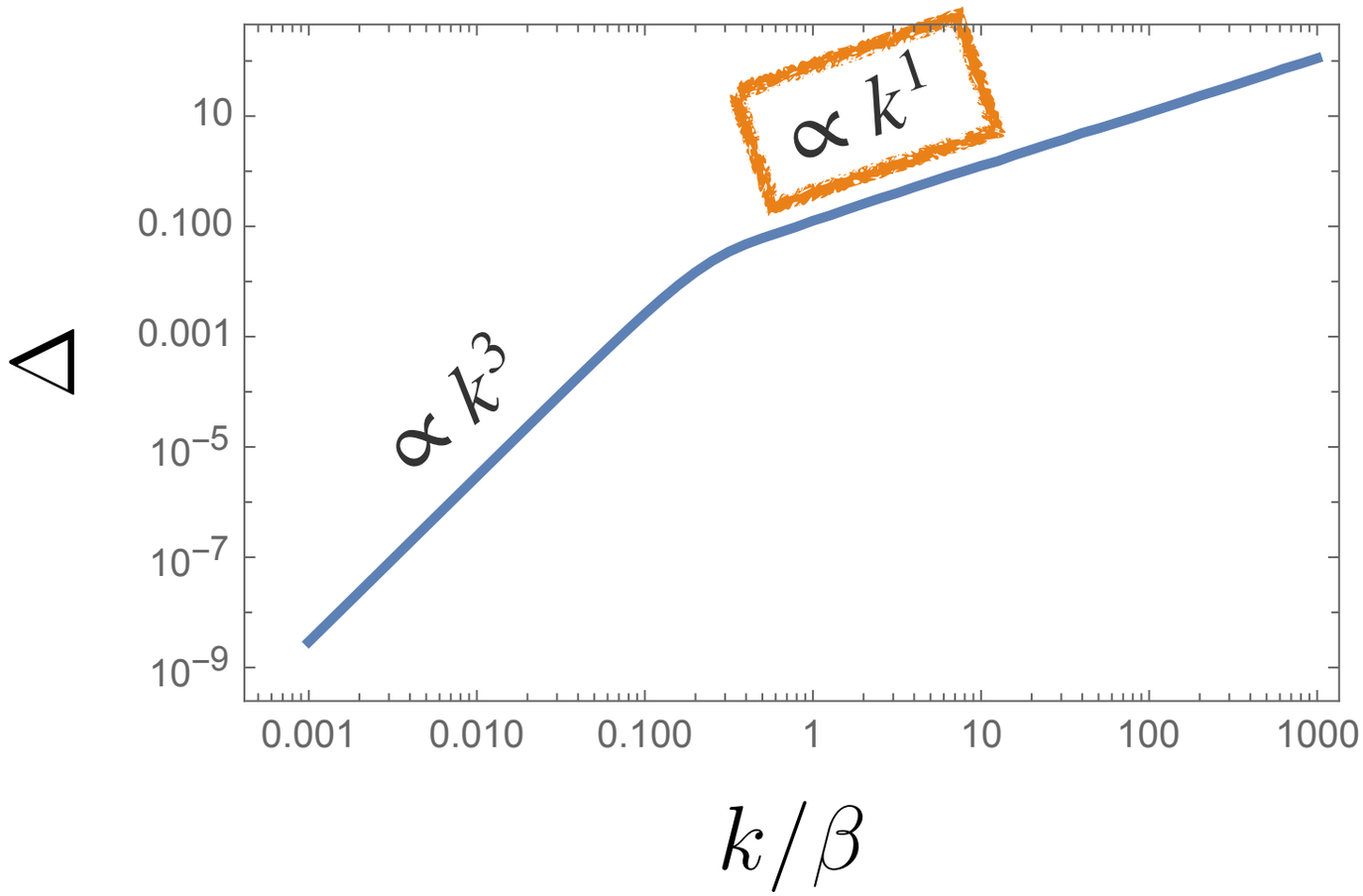


Result of GW spectrum $\Delta(k/\beta)$

[**YH**-Nakano 2405.09599]

$$\Omega_{\rm GW}(t_*, k_*) \propto \Delta(k_*/\beta)$$

see our paper for analytic expression of Δ!

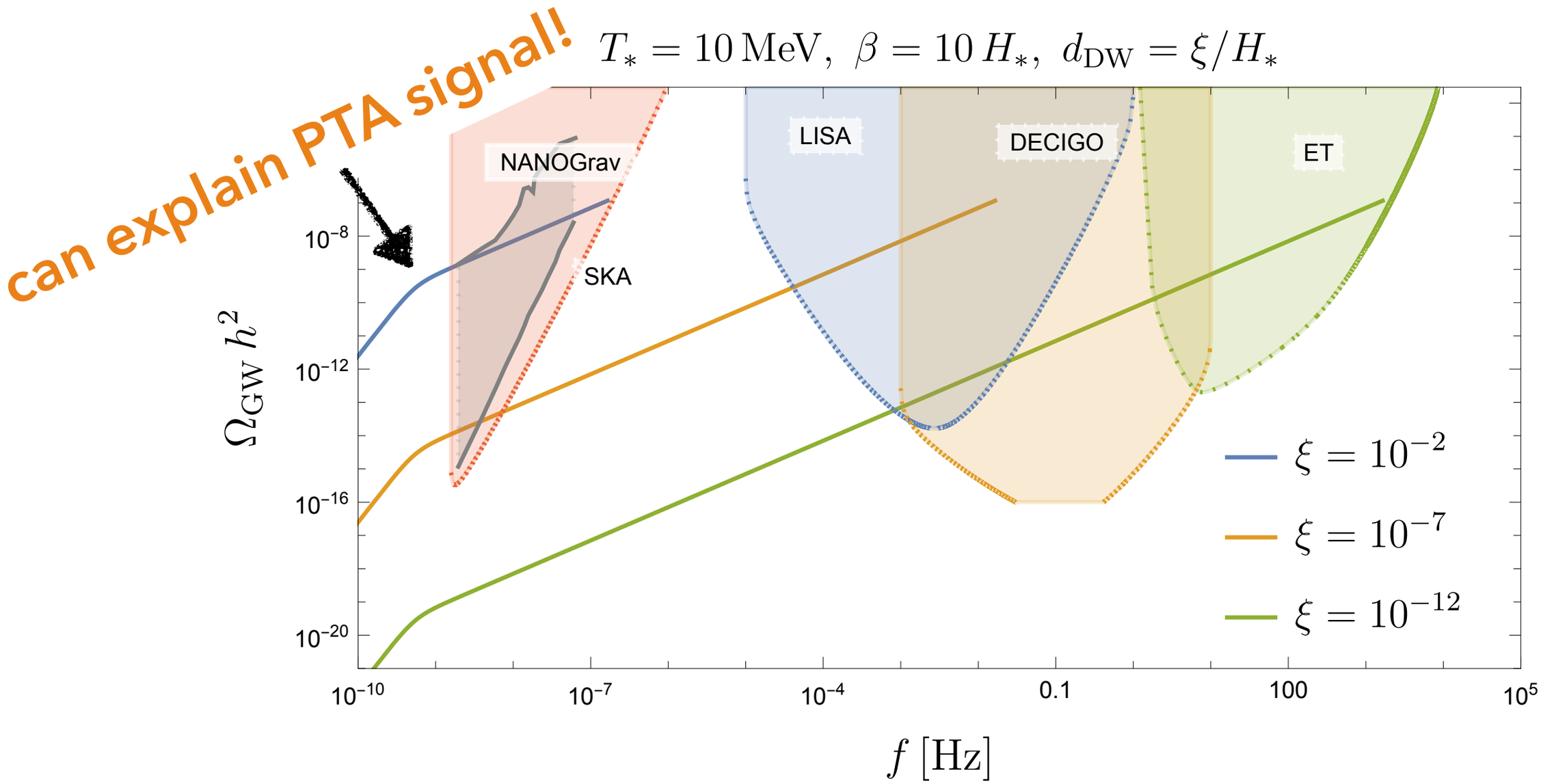


- **UV corresponds to small loops**

→ **not suppressed because they can radiate GW before collisions**

Present GW spectrum

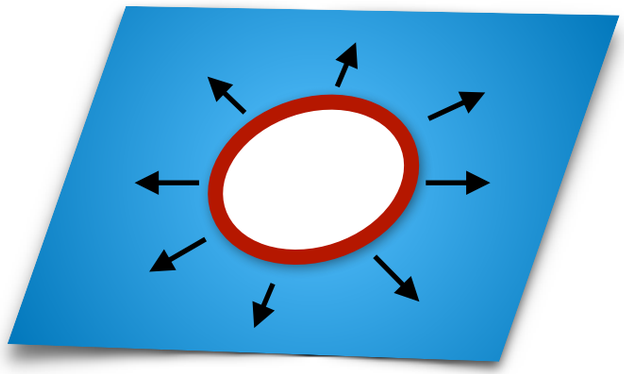
[**YH**-Nakano 2405.09599]

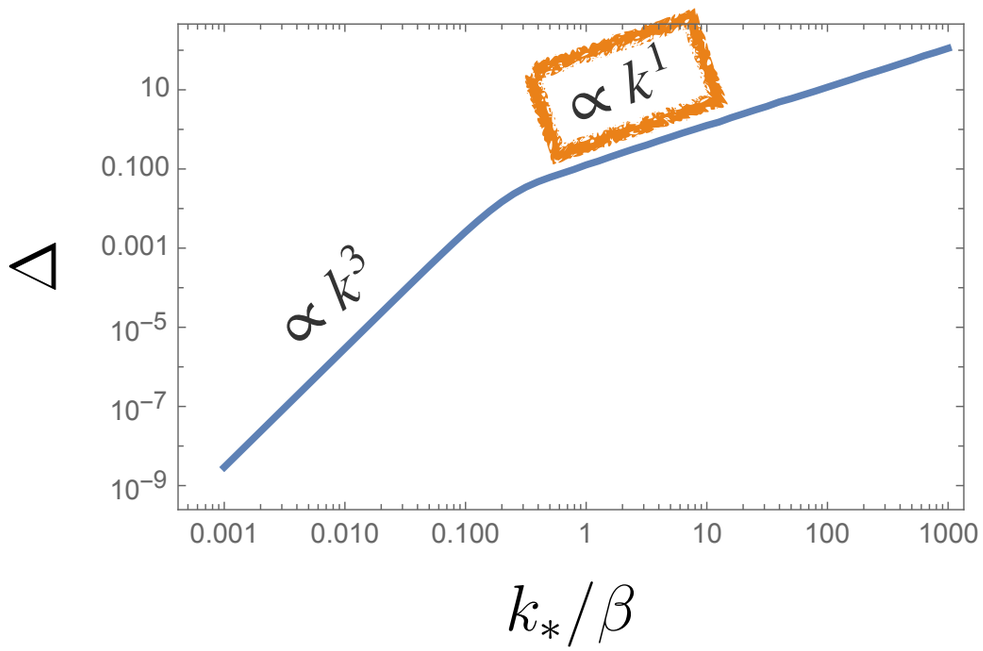


- Assumptions: $\begin{cases} \text{non-thermal production of DW, } d_{\mathrm{walls}} = d_H\big|_{\text{wall prod.}} \\ \text{DW not in scaling regime, } N_{\mathrm{walls}} = d_{H_*}/d_{\mathrm{walls}} \text{ per Hubble} \end{cases}$

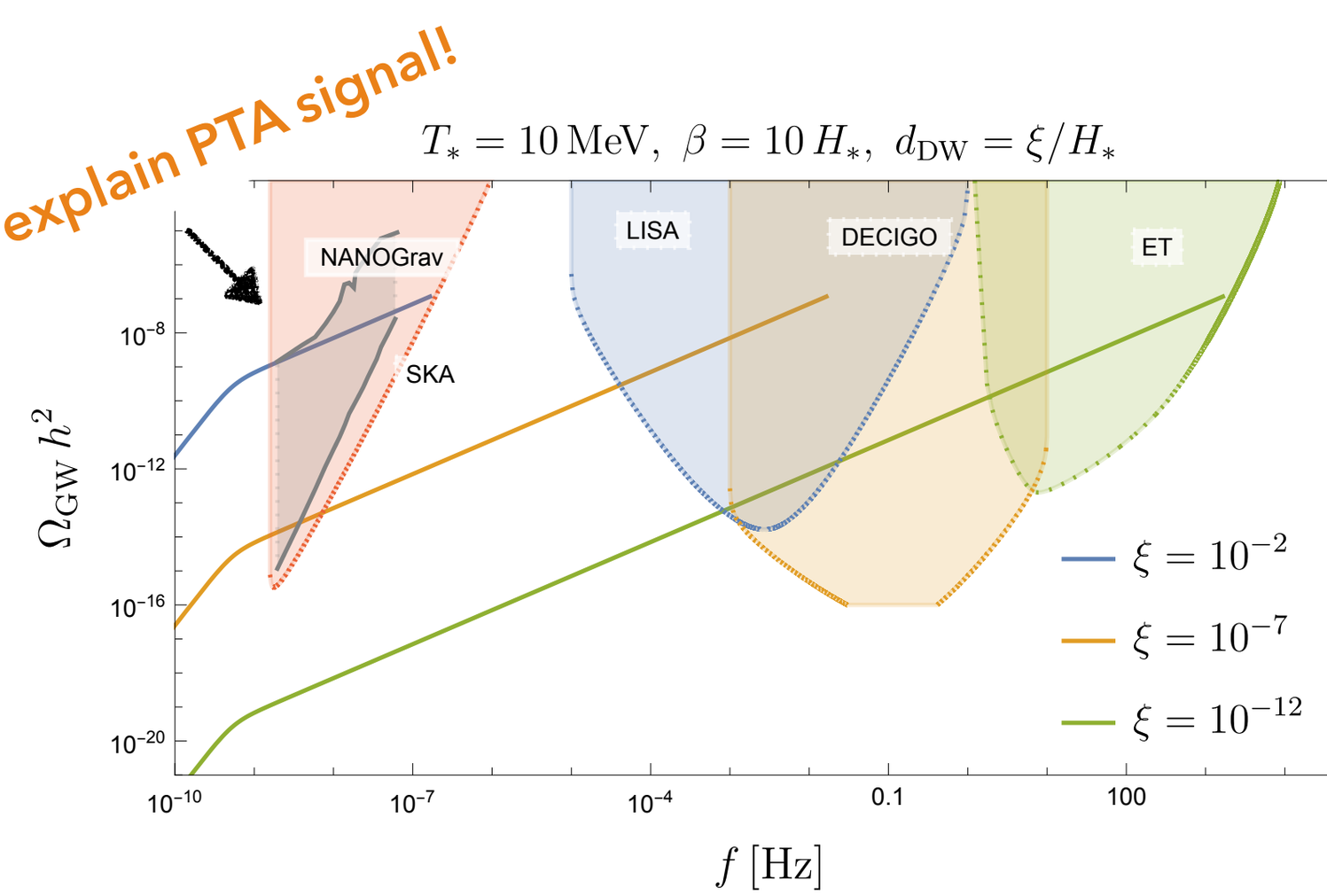
Summary

Expanding string loop on DW radiate GW with strange spectrum!





can explain PTA signal!



Backup

Quadrupole moment

- For single loop nucleated at $t = 0$ on the DW ($y = 0$):

$$Q_{ij} = \int d^3x\, \rho^s(x) \left[x_i x_j - \frac{1}{3}\delta_{ij}|\vec{x}|^2 \right]$$

$$= \frac{\kappa\rho_{\rm re}}{2} d_{\rm DW} t^4 \begin{pmatrix} +\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & +\frac{1}{6} \end{pmatrix}$$

- Quadrupole formula:

$$\frac{dE_{\rm GW}}{dt} = \frac{G}{5} \left(\dddot{Q}_{ij}\right)^2 \qquad \Delta E_{\rm GW} \sim \frac{G}{\beta} \left(\kappa\rho_{re}\, d_{\rm DW}\, t\right)^2$$

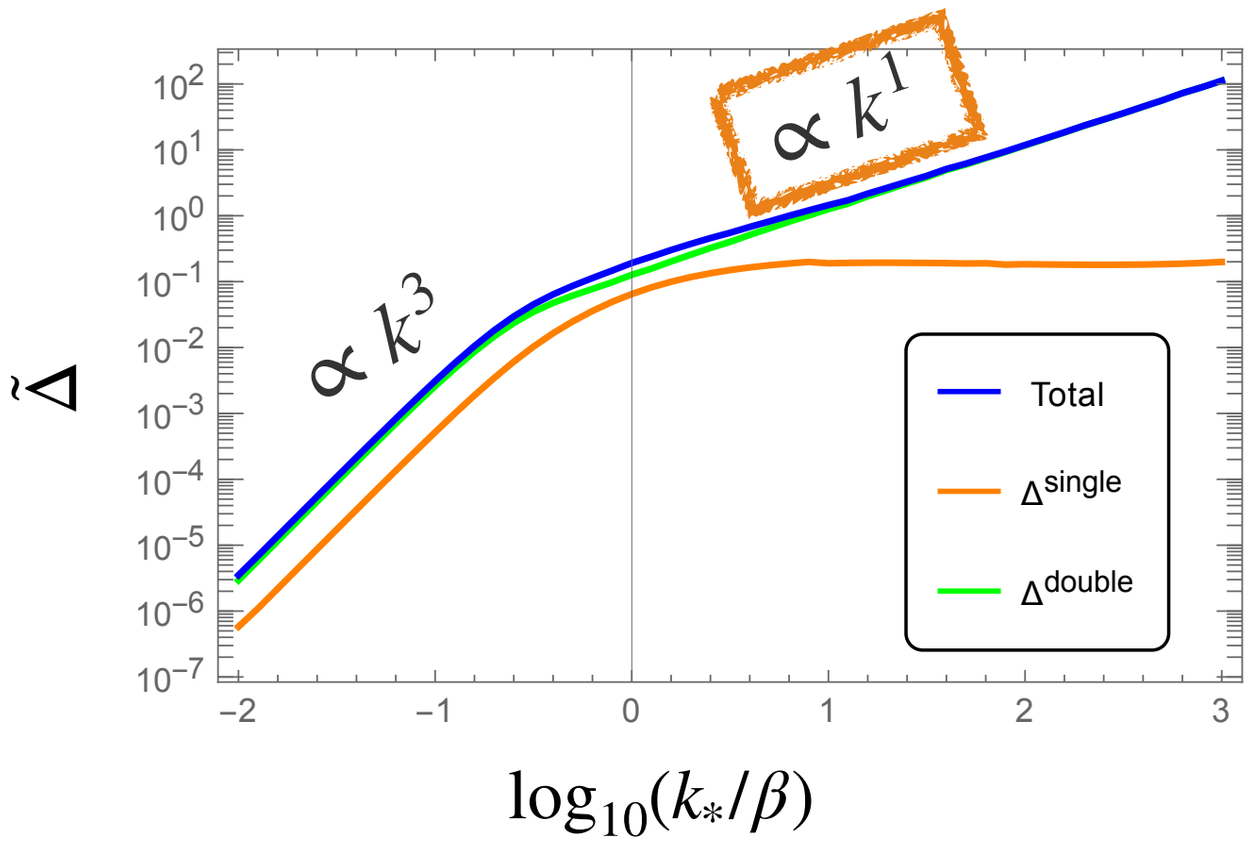
Result of GW spectrum

[**YH**-Nakano 2405.09599]

- GW energy density

see our paper for analytic expression of $\tilde{\Delta}$!

$$\Omega_{\rm GW}(t_*, k_*) = 10^{-5}\,\kappa^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha(t_*)}{1+\alpha(t_*)}\right)^2 N_{\rm walls}\, d_{\rm DW}^2 H_*\, \beta\, \underline{\tilde{\Delta}(k_*/\beta)}$$



β^{-1} : time duration of nucleation

$d_{\rm DW}$: width of DWs κ : efficiency

$N_{\rm walls}$: # of walls in Hubble patch

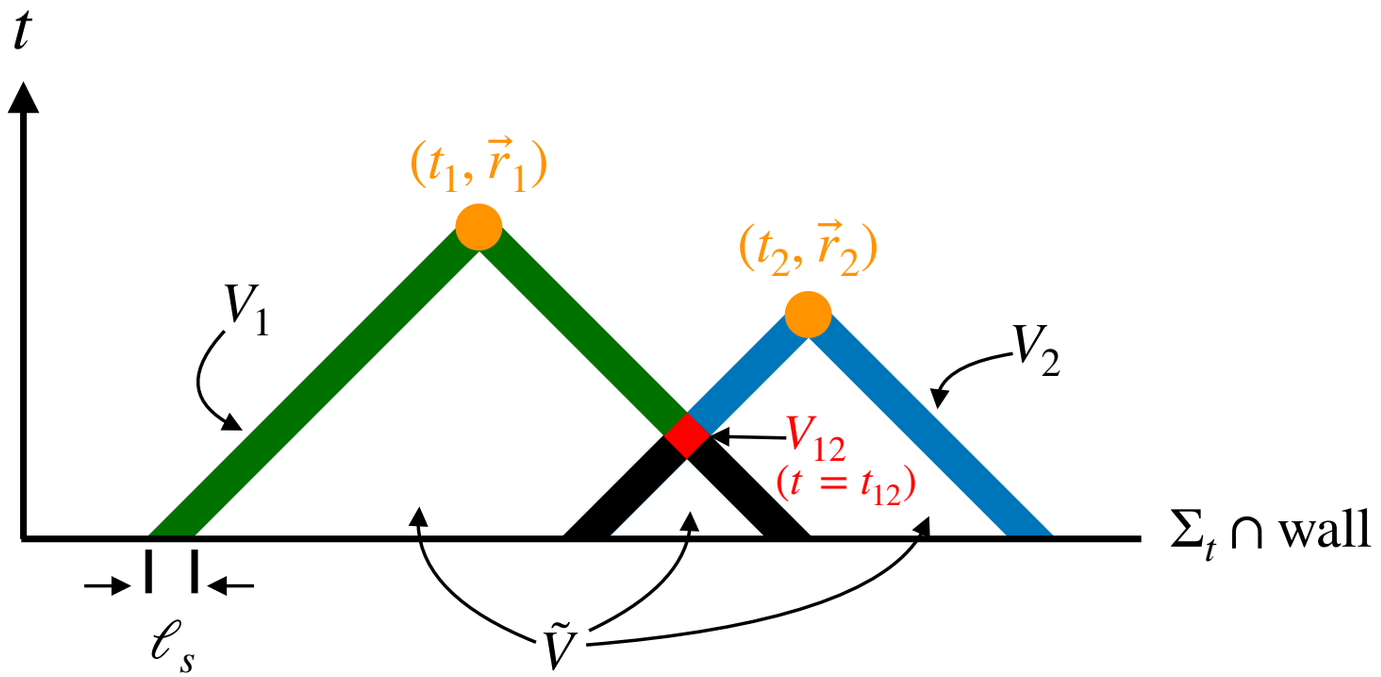
α_* : energy fraction of walls to plasma

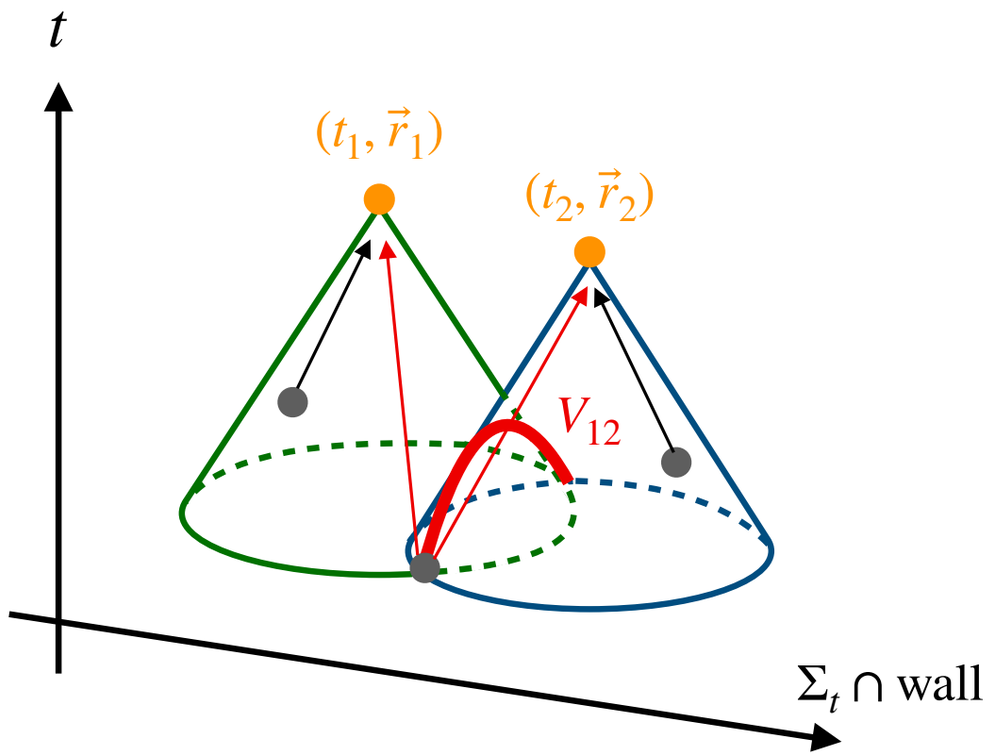
H_* : Hubble parameter

- **UV corresponds to small loops**

→ not suppressed because they can radiate GW before collisions

Light cone structure





Analytic formula of GW spectrum [**YH**-Nakano 2405.09599]

$$
\begin{aligned}
\Delta^{\rm double}(k/\beta) =& \frac{3}{64\pi}\beta k^3 \int_0^\infty dt_d \cos(kt_d) \int_{t_d}^\infty dr \frac{r^2 C_{\rm DW}(r)}{\mathcal{I}(r,t_d)^2} \\
& \times \left[\left(j_0(kr) - 2\frac{j_1(kr)}{kr} + 3\frac{j_2(kr)}{(kr)^2} \right) \right. \\
& \quad \times \left(g''_{a,\rm ov}(r,t_d) + g''_{a,\rm non}(r,t_d) \right) \left(g''_{a,\rm ov}(r,-t_d) + g''_{a,\rm non}(r,-t_d) \right) \\
& \quad + \left(j_0(kr) - 2\frac{j_1(kr)}{kr} - 5\frac{j_2(kr)}{(kr)^2} \right) \\
& \quad \times \left[\left(g''_{a,\rm ov}(r,t_d) + g''_{a,\rm non}(r,t_d) \right) g''_{b,\rm ov}(r,-t_d) + g''_{b,\rm ov}(r,t_d) \left(g''_{a,\rm ov}(r,-t_d) + g''_{a,\rm non}(r,-t_d) \right) \right] \\
& \left. + \left(j_0(kr) - 2\frac{j_1(kr)}{kr} + 19\frac{j_2(kr)}{(kr)^2} \right) g''_{b,\rm ov}(r,t_d) g''_{b,\rm ov}(r,-t_d) \right] .
\end{aligned}
$$

$$
C_{\rm DW}(r) = N_{\rm walls}\frac{d_{\rm DW}^2}{2r d_H}.
$$

$$
\begin{aligned}
\mathcal{I}(r,t_d) =& \frac{1}{2\beta^3}\left[8\pi\cosh\left(\frac{\beta t_d}{2}\right) + \beta^2 r\sqrt{r^2-t_d^2} K_1\left(\frac{\beta r}{2}\right) \right] \\
& - \frac{1}{4}\int_{-\infty}^{-r/2} dt_T\, e^{\beta t_T} \left[(2t_T - t_d)^2 \left(\pi - \cos^{-1}\left(\frac{r^2 - 2t_d t_T}{r(2t_T - t_d)} \right) \right) \right. \\
& \left. + (2t_T + t_d)^2 \left(\pi - \cos^{-1}\left(\frac{r^2 + 2t_d t_T}{r(2t_T + t_d)} \right) \right) \right] ,
\end{aligned}
$$

$$
\begin{aligned}
g''_{a,\rm ov}(r,t_d) &= \int_{-\infty}^{-r/2} dt_{1T} e^{\beta t_{1T}} g'_a(r,t_d,t_{1T}), \\
g''_{a,\rm non}(r,t_d) &= \int_{-r/2}^{t_d/2} dt_{1T} e^{\beta t_{1T}} g'_a(r,t_d,t_{1T}) \\
&= \frac{\pi}{\beta^3} e^{-\beta r/2} \left[8(-1 + e^{\beta(r+t_d)/2}) - \beta(r+t_d)\left\{ 4 + \beta(r+t_d) \right\} \right] \\
g''_{b,\rm ov}(r,t_d) &= \int_{-\infty}^{-r/2} dt_{1T} e^{\beta t_{1T}} g'_b(r,t_d,t_{1T}) \\
&= \frac{\sqrt{r^2 - t_d^2}}{\beta} \left[r K_1\left(\frac{\beta r}{2}\right) + t_d K_2\left(\frac{\beta r}{2}\right) \right] ,
\end{aligned}
$$

Setup

[**YH**-Nakano 2405.09599]

- Formula of GW energy density

$$\Gamma(t) \sim \Gamma_* e^{\beta(t-t_*)}$$

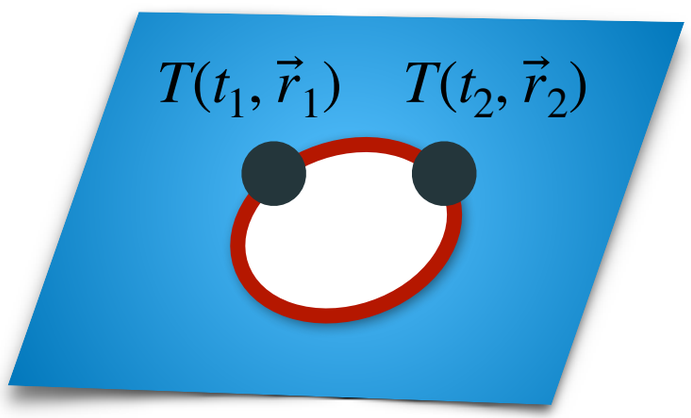
$$\rho_{\rm GW} \sim \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \cos(k(t_1-t_2))\ \mathrm{F.T.} \langle T_{ij}^{\rm TT}(t_1,\vec{r}_1) T_{ij}^{\rm TT}(t_2,\vec{r}_2)\rangle_{\rm ens}$$

ensemble average of 2pt. function

Ensemble average

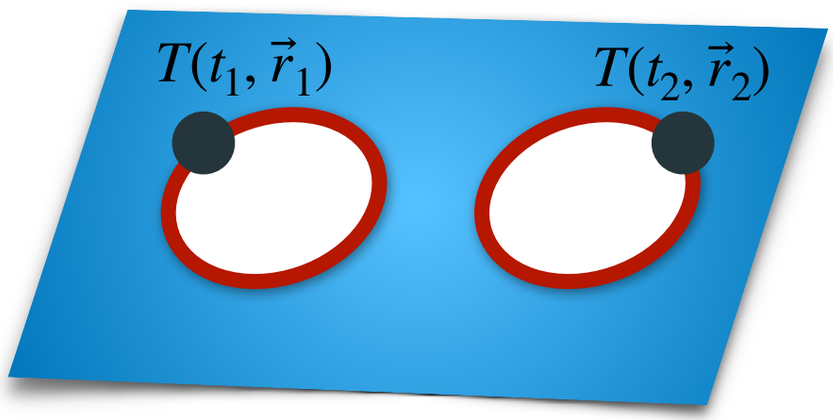
= average of nucleation points + average of DW configuration (planar)

$$\langle TT \rangle \simeq$$



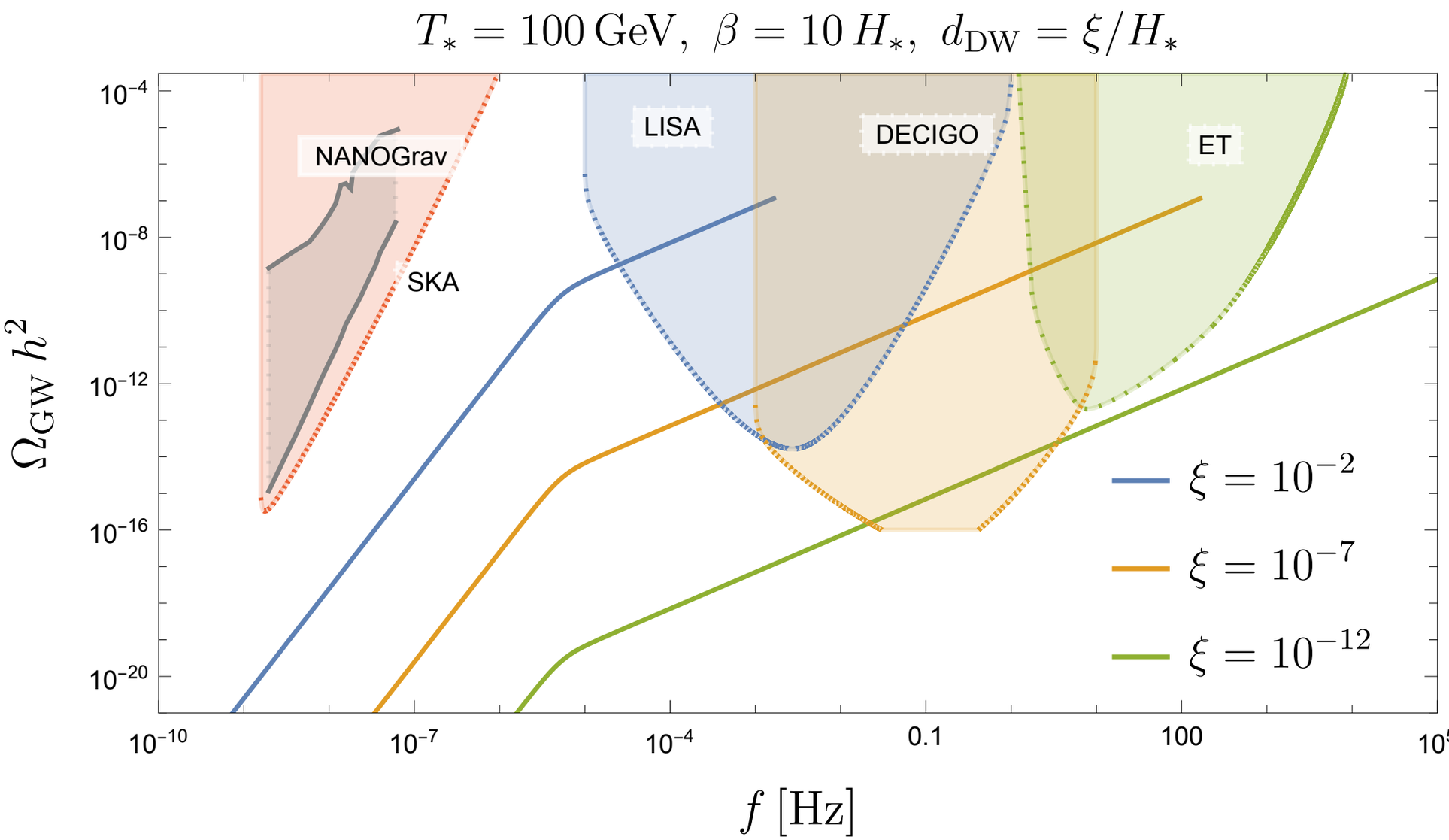
two points on one loop

$+$



two points on two loops

Present GW spectrum



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