Timekeepers of the Universe: The recent GW observation by PTA and PBH

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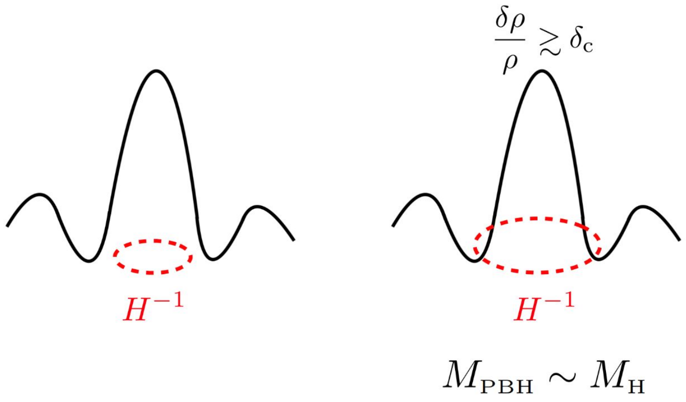
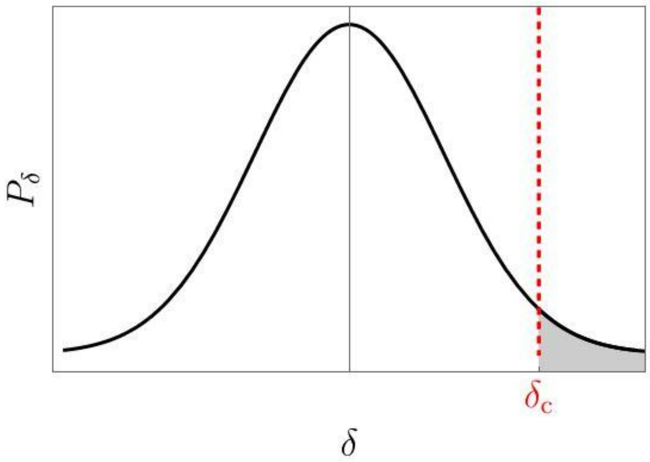
PBHs from collapse of large overdensities

REVIEW: LISA– arXiv:2310.19857



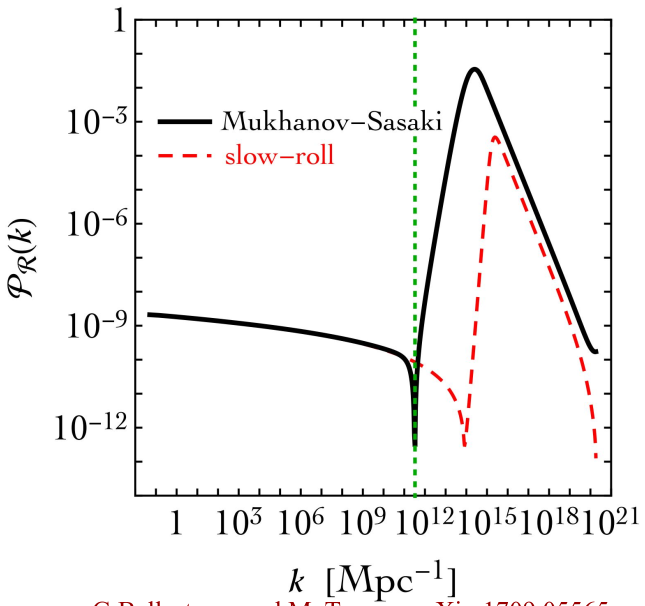


Mass distribution

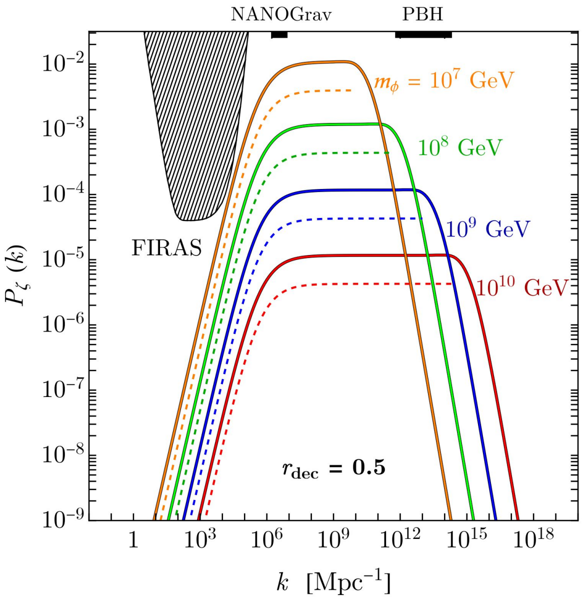
$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^{\gamma} \, \mathrm{P}_{\delta}(\delta) d\delta$$

Primordial Black Holes as DM candidates

What we can compute during inflation is the curvature perturbation field ζ (or R).
In order to get a sizeable amount of dark matter $P_\zeta \simeq 10^{-2}$-10^{-3}



G.Ballesteros and M. Taoso – arXiv:1709.05565



M. Kawasaki, N. Kitajima, and T. T. Yanagida – arXiv:1207.2550

PBH as Dark Matter:

- USR models
- Hybrid inflation
- Curvaton field
- exotic formation mechanisms (bubble collisions and so on)
- And etc etc…

$$M_H \simeq 17 M_\odot \left(\frac{g_\star}{10.75}\right)^{-1/6} \left(\frac{k/\kappa}{10^6 \mathrm{Mpc}^{-1}}\right)^{-2}$$

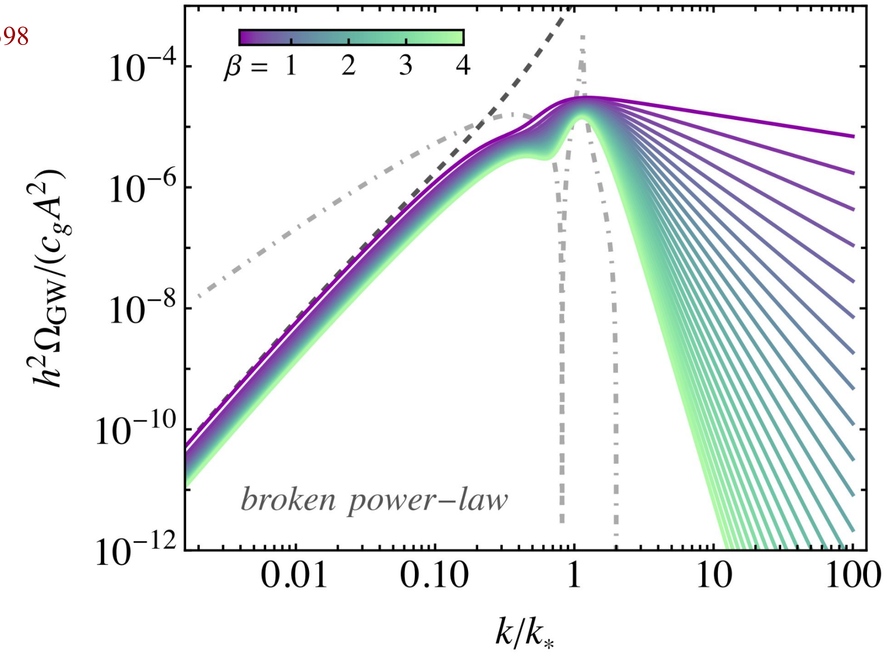
PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

$$h^2\Omega_{\rm GW}(k) = \frac{h^2\Omega_r}{24}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*s}}{g_{*s}^0}\right)^{-\frac{4}{3}}\mathcal{P}_h(k)$$

$$\mathcal{P}_h(k) \propto \mathcal{P}_\zeta^2(k)$$

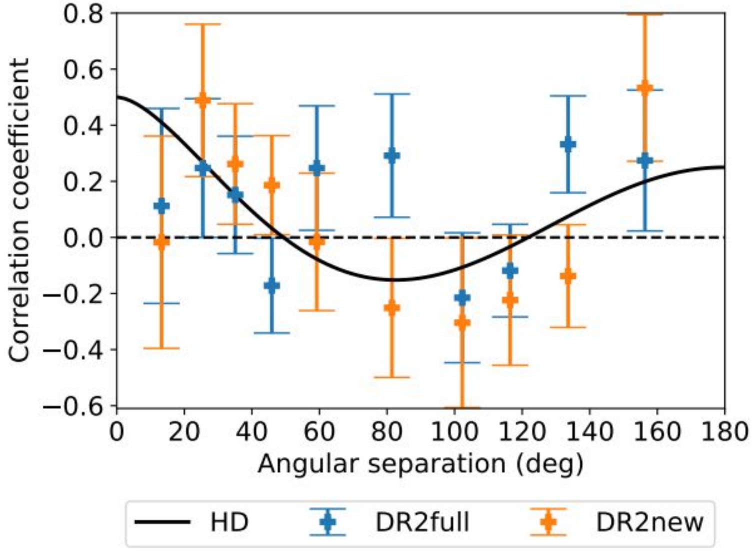
REVIEW G. Domenech– arXiv:2109.01398



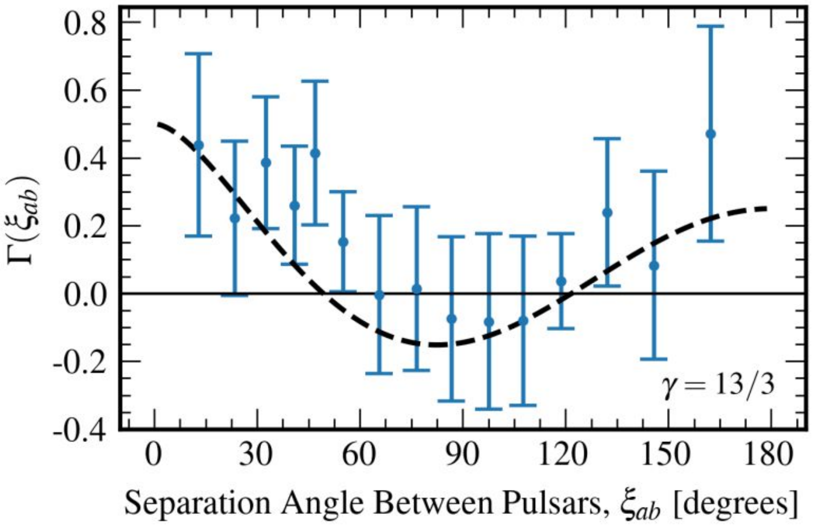
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

EPTA – arXiv:2306.16214



NANOGrav – arXiv:2306.16213
arXiv:2306.16219

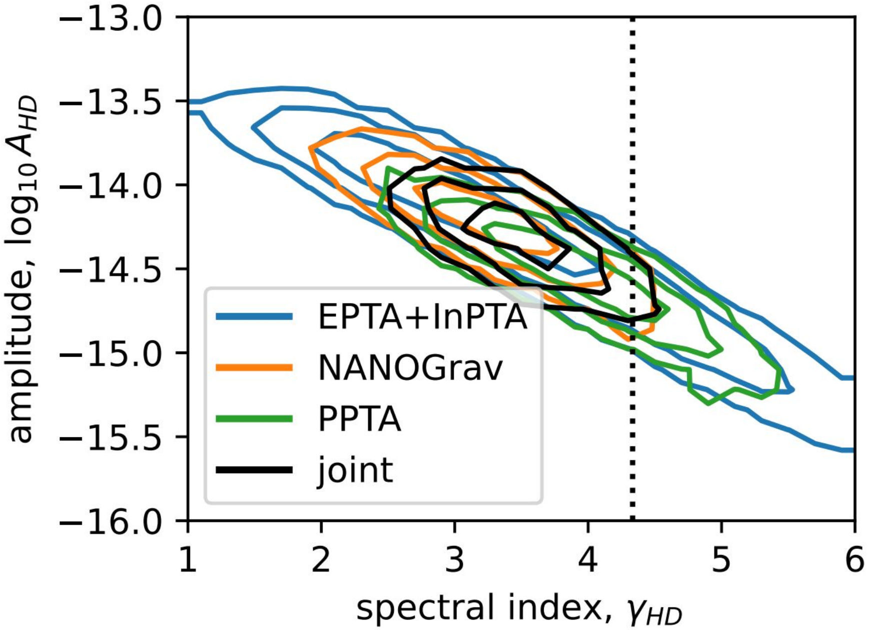


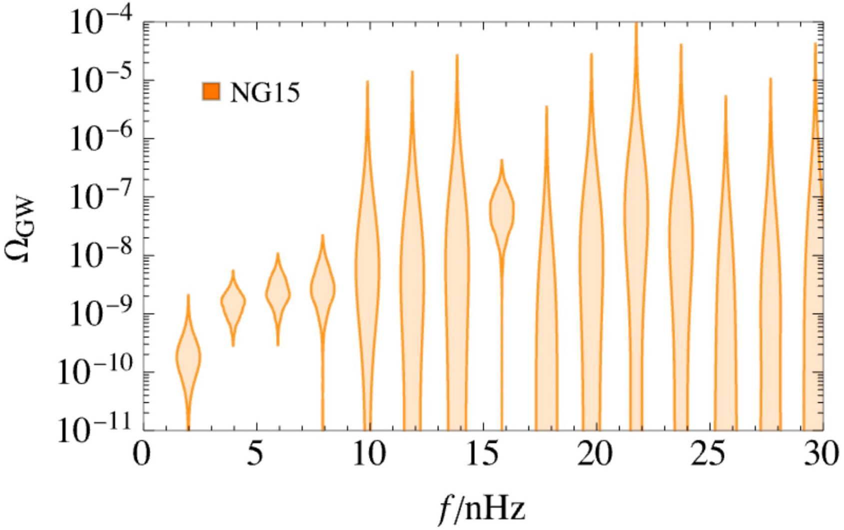
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693

NANOGrav – arXiv:2306.16213
arXiv:2306.16219





PBH and SGWB

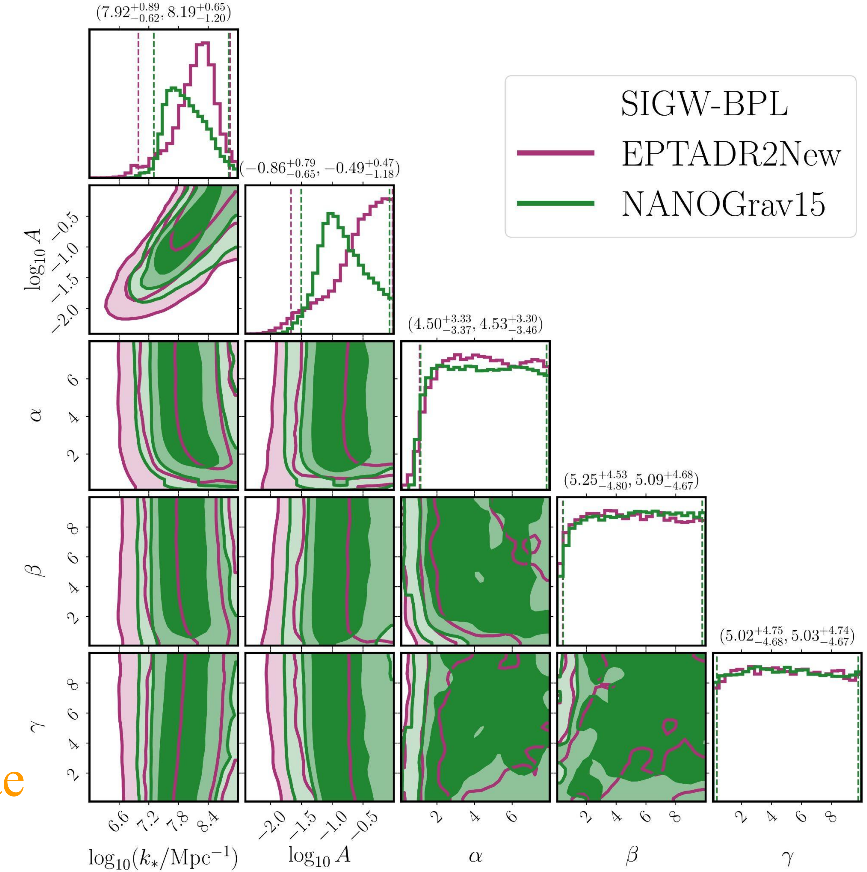
SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

Log-likelihood analysis

Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\mathrm{BPL}}(k) = A\frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma}+\alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$

$$\mathcal{P}_\zeta^{\mathrm{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$



arXiv:2306.17149

G.Franciolini, A.J.I., V. Vaskonen, H. Veermae

Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum <> Abundance <> GWs

- Non-Gaussianities in the abundance.

- Dependency of the PBH formation parameters on the PS shape.

- QCD impact on threshold.

Abundance of PBHs: The role of Non-Gaussianities (NG).

arXiv:2211.01728 G.Ferrante, G.Franciolini, A.J.I., A.Urbano

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

T. Harada, C. M. Yoo, T. Nakama and Y. Koga,.– arXiv:1503.03934

PRIMORDIAL NG IN $\zeta=F(\zeta_G)$

$$\zeta = F(\zeta_G)$$

power-series expansion [28–34, 36–39]

$$\zeta = \zeta_G + \tfrac{3}{5} f_{NL}\zeta_G^2 + \tfrac{9}{25} g_{NL}\zeta_G^3 + \dots$$

ultra slow-roll with an upward step [50]

$$\zeta = -\tfrac{2}{|h|}\left[\sqrt{1-|h|\zeta_G} - 1\right]$$

$$r_{dec} \equiv \left.\frac{3\rho_\phi}{3\rho_\phi + 4\rho_\gamma}\right|_{\text{curvaton decay}}$$

curvaton [48, 49]

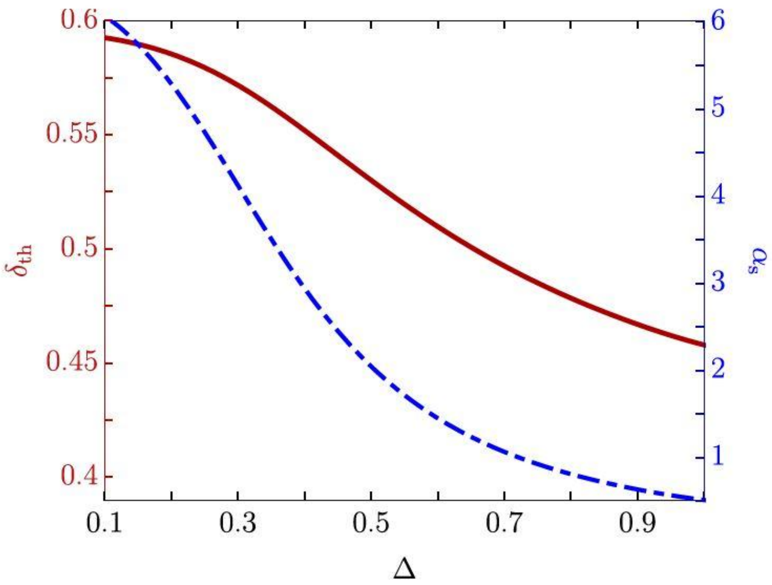
$$\zeta = \log\left[X(r_{dec}, \zeta_G)\right]$$

ultra slow-roll [42]

$$\zeta = -\left(\tfrac{6}{5} f_{NL}\right)^{-1}\log\left(1 - \tfrac{6}{5} f_{NL}\zeta_G\right)$$

Abundance of PBHs:
Shape dependencies

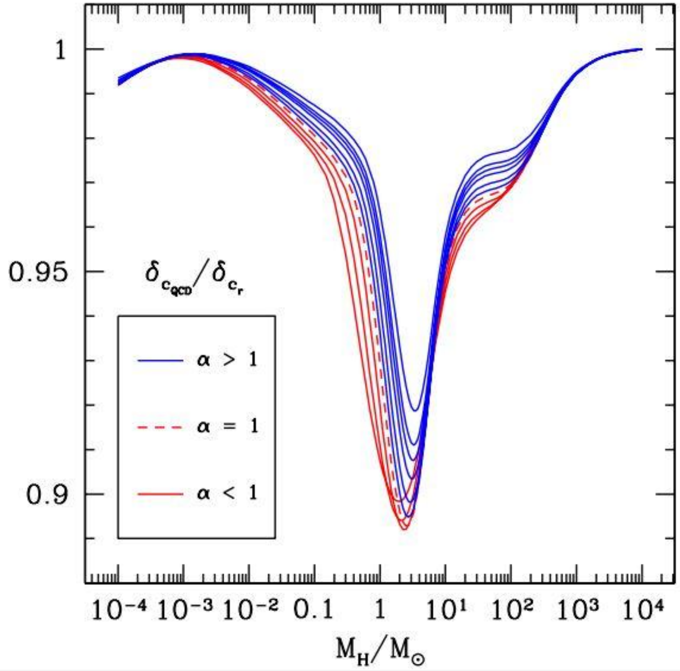
I. Musco,V. De Luca, G. Franciolini, A.Riotto.– arXiv:2011.03014



$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta}\,\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$

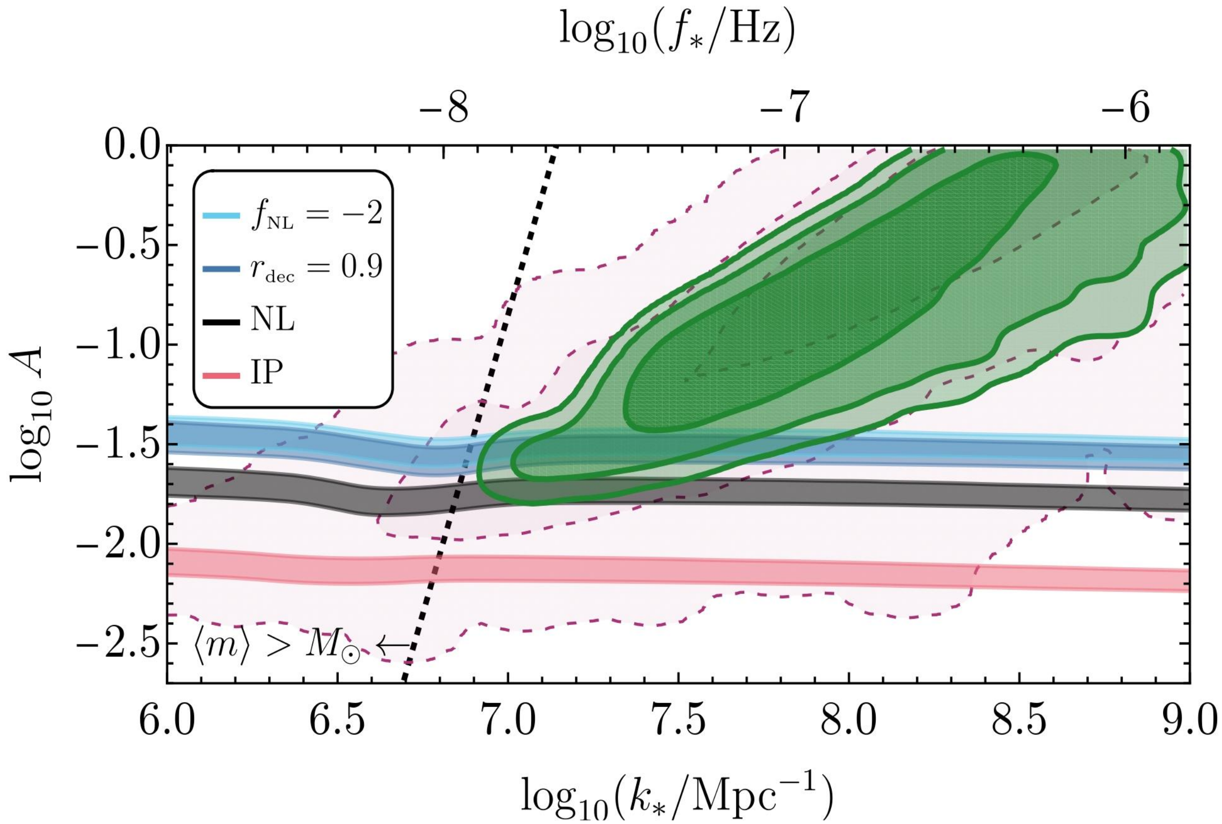
QCD phase transitions

I. Musco, K.Jedamzik, S.Young.– arXiv:2303.07980



Tension between NANOGrav and PBHs

$$\mathcal{P}_\zeta^{\rm BPL}(k) = A\frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma} + \alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$



Negative NGs to alleviate the tension between PTA and PBH overproduction.

Backup Slides

Mathematical formulation

By integrating δ over the radial coordinate *r* we get the compaction function *C*

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r)\left[1+\frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

Later on confirmed also by A.Gow et al arXiv:2211.08348

In the presence of NG C_1 takes the form

$$\mathcal{C}_1(r) = -2\Phi\, r\, \zeta'_{\rm G}(r)\,\frac{dF}{d\zeta_{\rm G}} = \mathcal{C}_{\rm G}(r)\,\frac{dF}{d\zeta_{\rm G}}\,, \qquad \text{with} \quad \mathcal{C}_{\rm G}(r) := -2\Phi\, r\, \zeta'_{\rm G}(r)$$

From the two-dimensional joint PDF of ζ_G and C_G, called P_G

NG PBH mass fraction adopting threshold statistics on the compaction function

$$\beta_{\rm NG} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C}-\mathcal{C}_{\rm th})^{\gamma}\,{\rm P_G}(\mathcal{C}_{\rm G},\zeta_{\rm G})\,d\mathcal{C}_{\rm G}\,d\zeta_{\rm G}\,, \tag{56}$$

$${\rm P_G}(\mathcal{C}_{\rm G},\zeta_{\rm G}) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}}\exp\left(-\frac{\zeta_{\rm G}^2}{2\sigma_r^2}\right)\exp\left[-\frac{1}{2(1-\gamma_{cr}^2)}\left(\frac{\mathcal{C}_{\rm G}}{\sigma_c}-\frac{\gamma_{cr}\zeta_{\rm G}}{\sigma_r}\right)^2\right]\,, \tag{57}$$

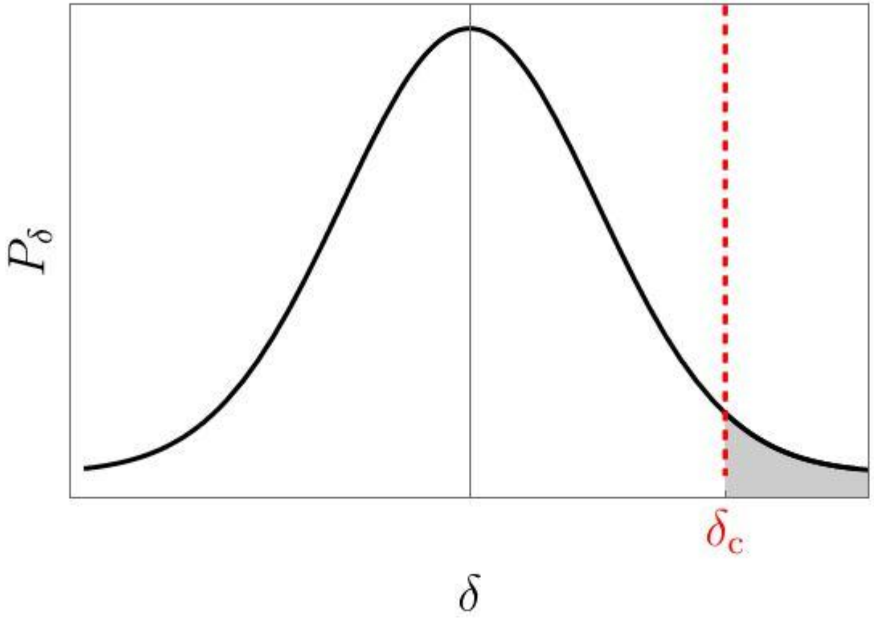
$$\mathcal{D} = \{\mathcal{C}_{\rm G},\,\zeta_{\rm G}\in\mathbb{R}\;:\;\mathcal{C}(\mathcal{C}_{\rm G},\zeta_{\rm G}) > \mathcal{C}_{\rm th}\;\wedge\;\mathcal{C}_1(\mathcal{C}_{\rm G},\zeta_{\rm G}) < 2\Phi\}\,, \tag{58}$$

arXiv:2211.01728 (Published on PRD) G.Ferrante, G.Franciolini, A.J.I., A.Urbano

Abundance of PBHs

Mass Fraction

$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^{\gamma} \, \mathrm{P}_{\delta}(\delta) d\delta$$



Mass distribution

$$f_{\mathrm{PBH}}(M_{\mathrm{PBH}}) \equiv \frac{1}{\Omega_{\mathrm{DM}}} \frac{d\Omega_{\mathrm{PBH}}}{d\log M_{\mathrm{PBH}}} \,, \qquad \Omega_{\mathrm{PBH}} = \int d\log M_H \left(\frac{M_{\mathrm{eq}}}{M_H}\right)^{1/2} \beta_{\mathrm{NG}}(M_H) \,,$$

PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

Log-likelihood analysis

Fitting the posterior distributions

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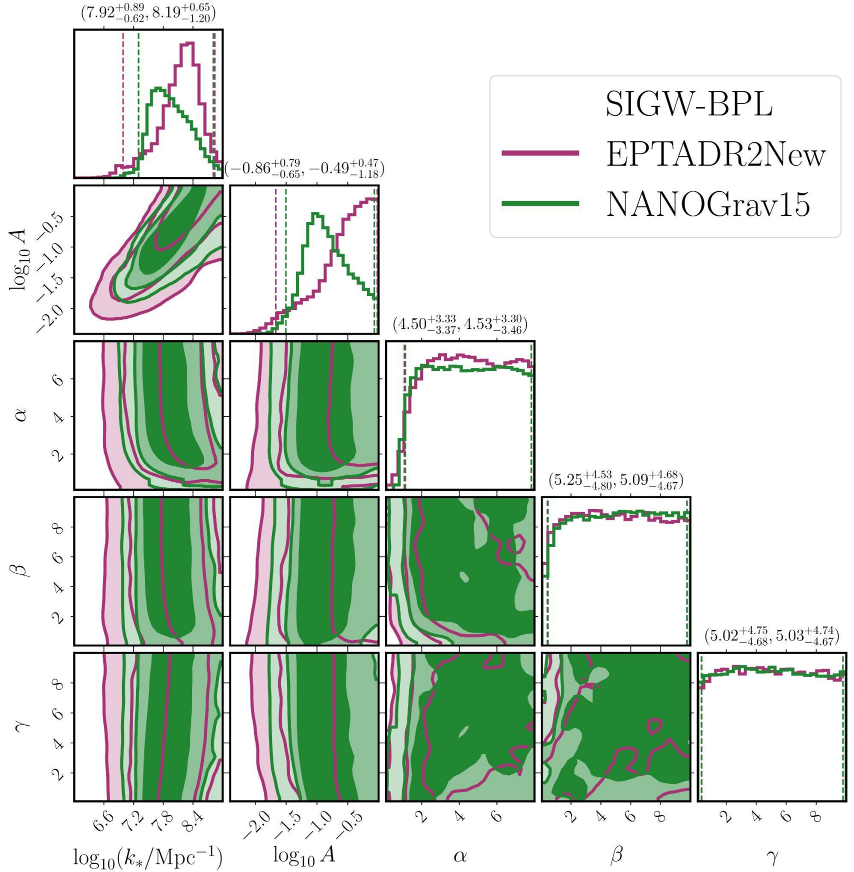

Results:

The casuality tail is not good:

$$\Omega_{\rm GW}(k \ll k_*) \propto k^3(1+\tilde{A}\ln^2(k/\tilde{k}))$$

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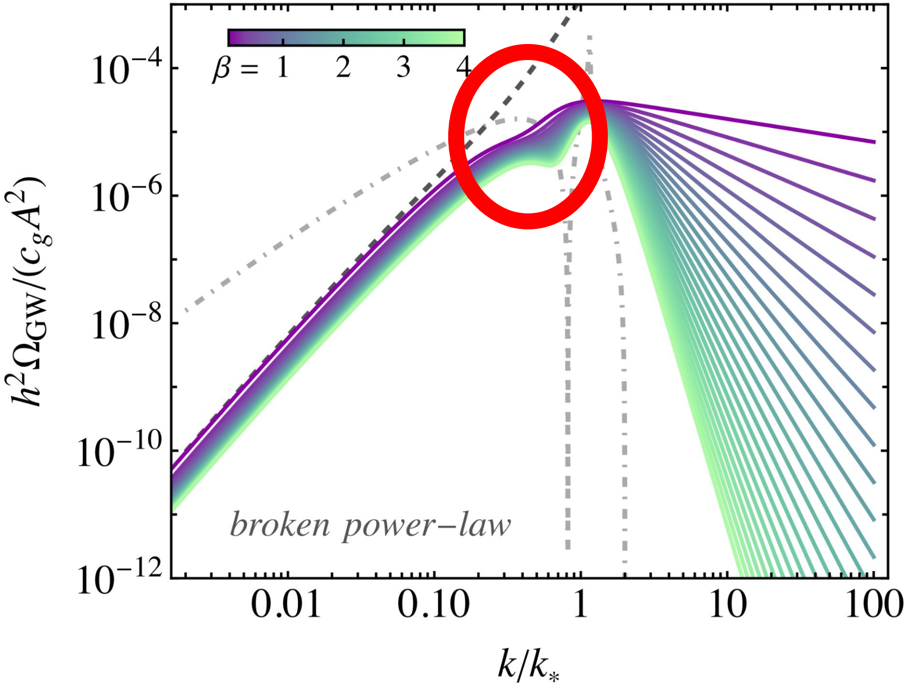

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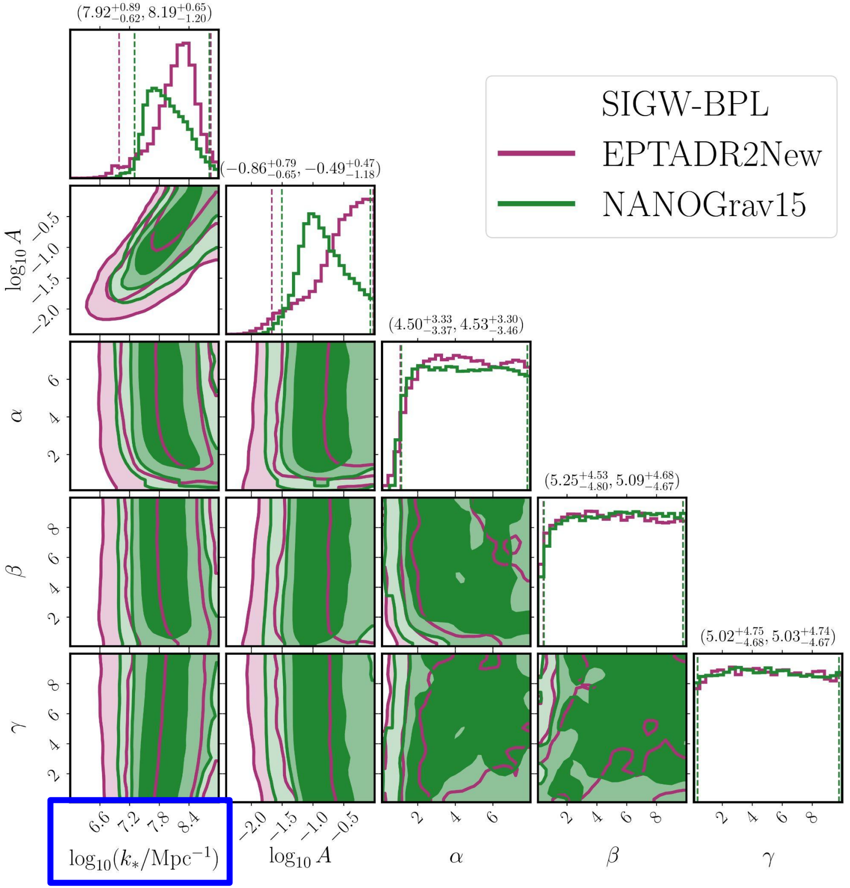
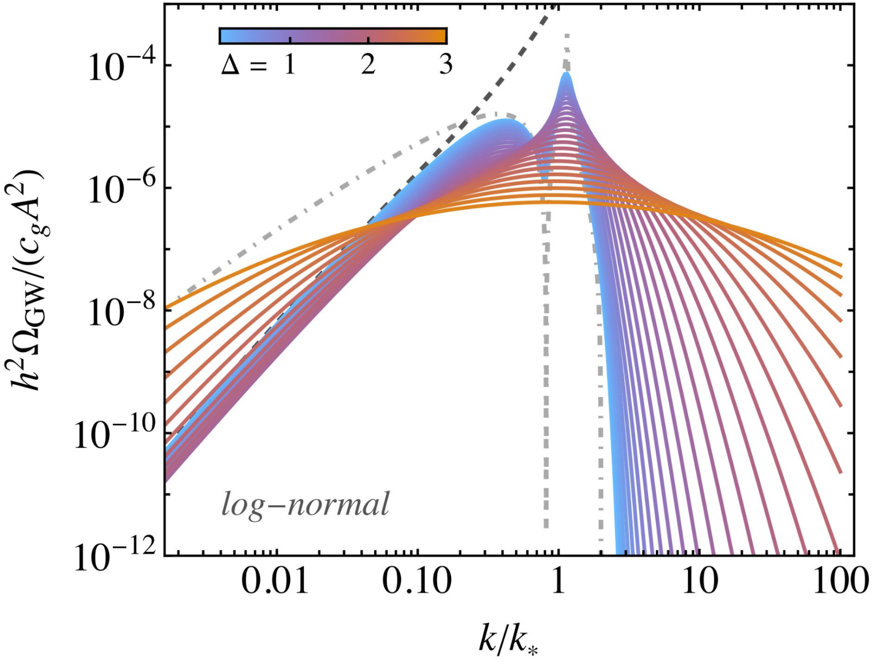
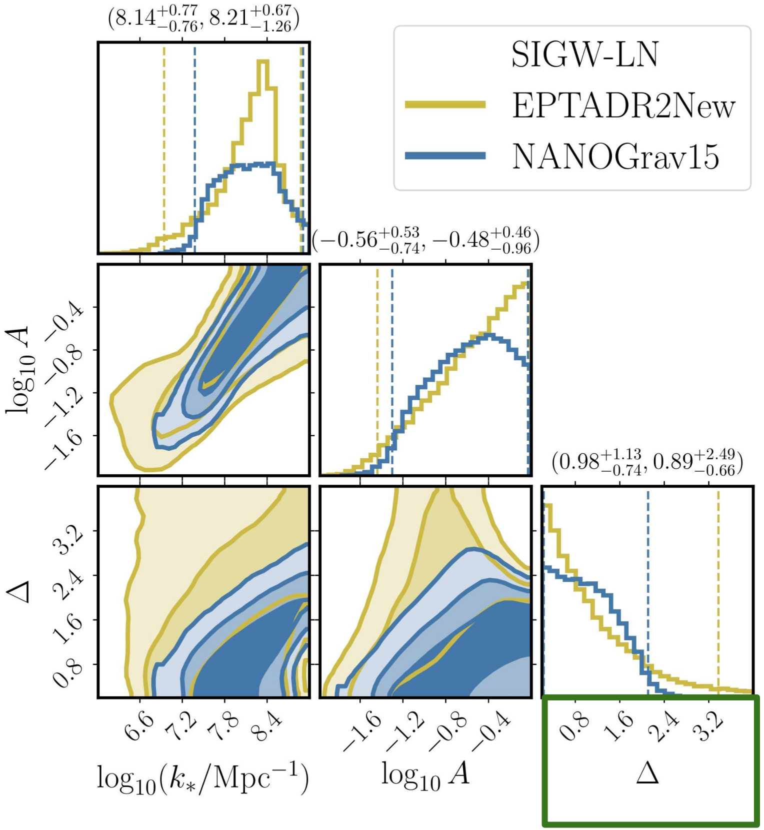
Results:

Position of the peak at higher frequencies.

Broad spectrum does not fit so well.







$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$

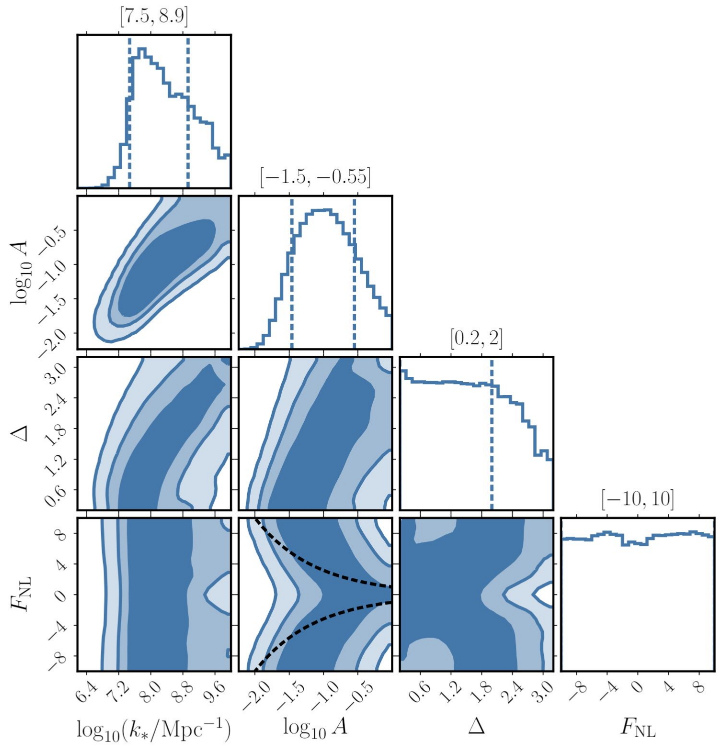
NGs directly in GWs

arXiv:2308.08546 (Accepted by PRD)
J. Ellis et al

HO corrections do not affect significantly the results showed before.

$$\Omega_{\rm GW}^{\rm NLO}/\Omega_{\rm GW} \propto A(3f_{\rm NL}/5)^2$$

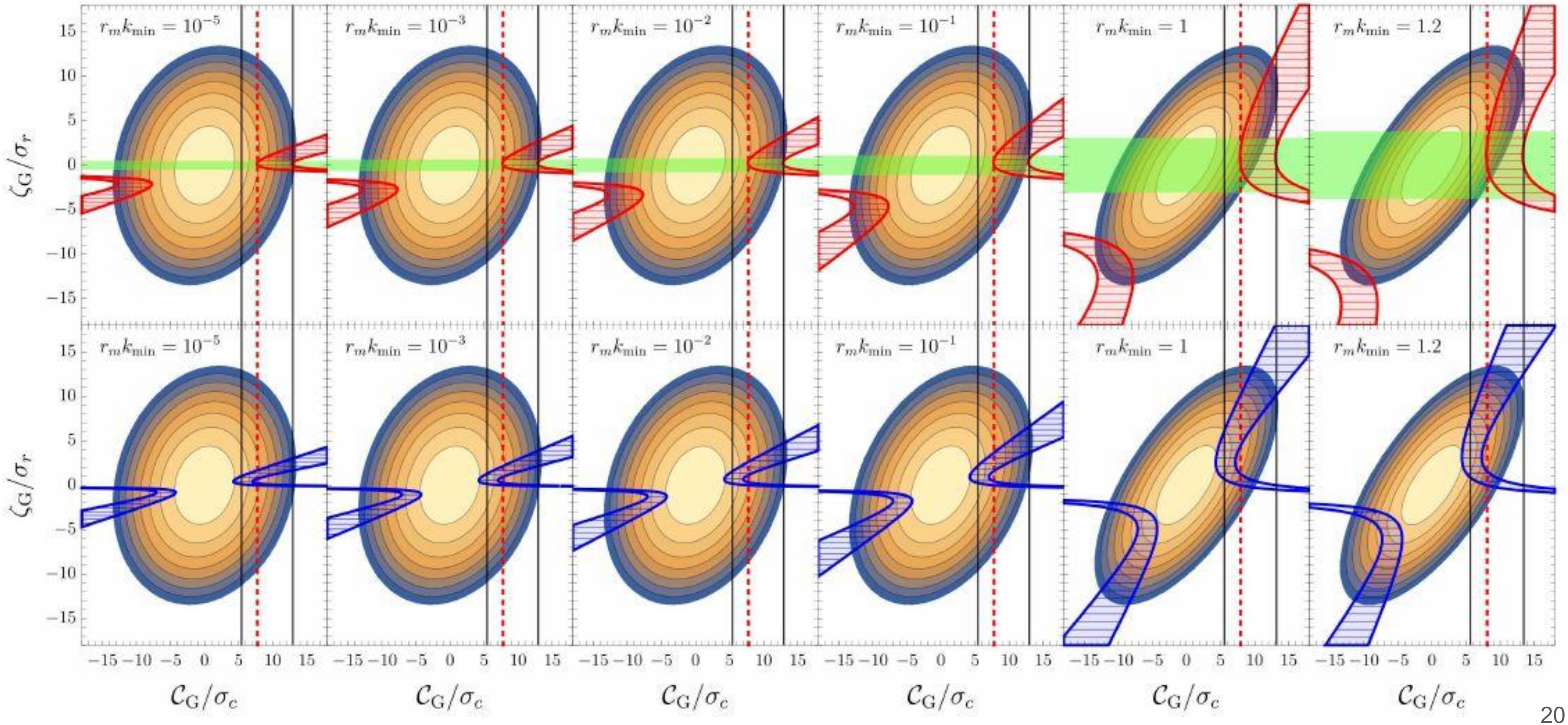
R. Cai, S. Pi, and M. Sasaki–arXiv:1810.11000
K. T. Abe, R. Inui, Y. Tada, and S. Yokoyama-arXiv:2209.13891



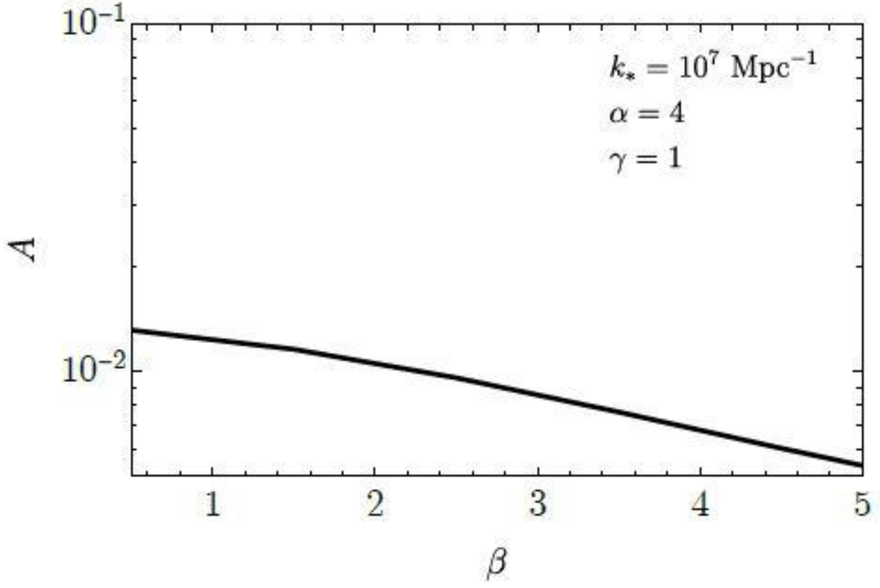
We cannot constrain the presence of NGs at PTA scales.
Large values of FNL are possible, provided the PS amplitude is sufficiently small.

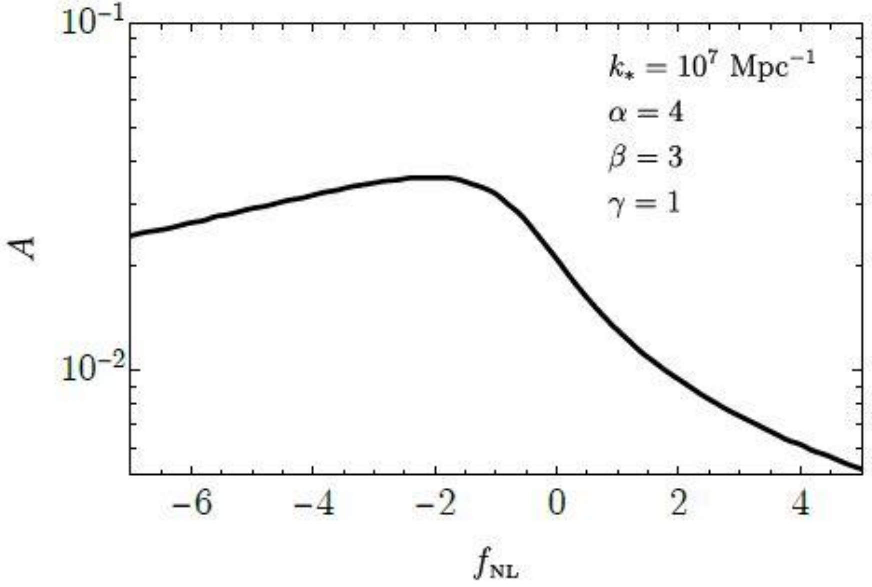
$$\frac{4(1-\sqrt{1-3\mathcal{C}_{\rm th}/2})}{3} < \mathcal{C}_{\rm G}\frac{dF}{d\zeta_{\rm G}} < \frac{4}{3}$$

Breaking of scale invariance



NG generic features





NG generic features

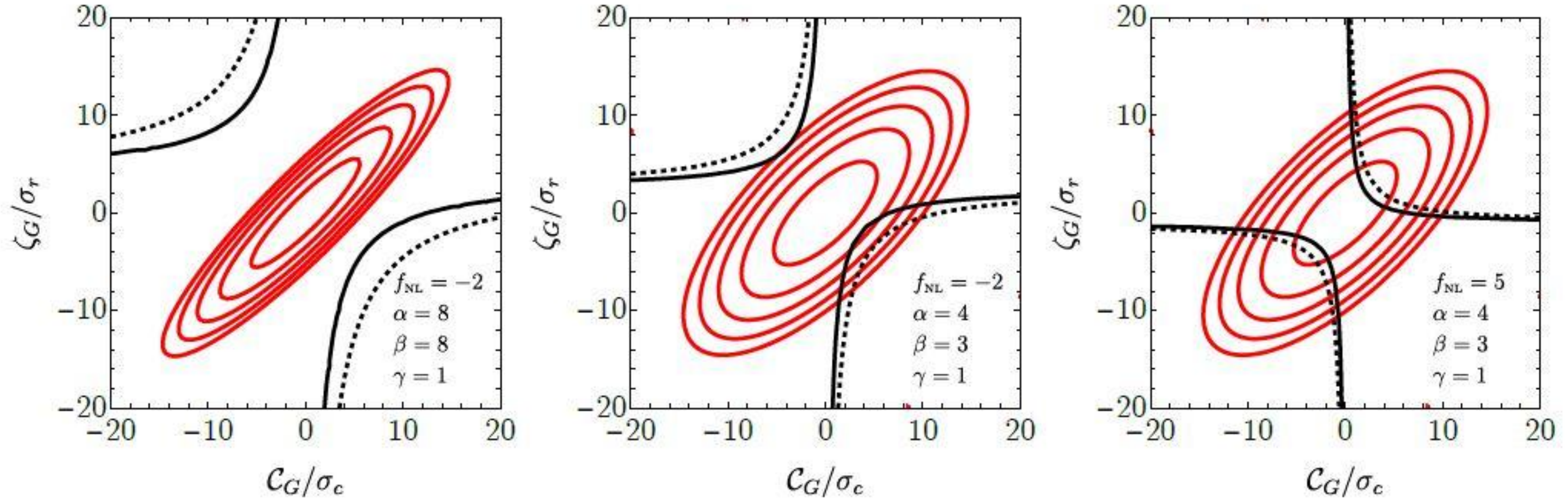
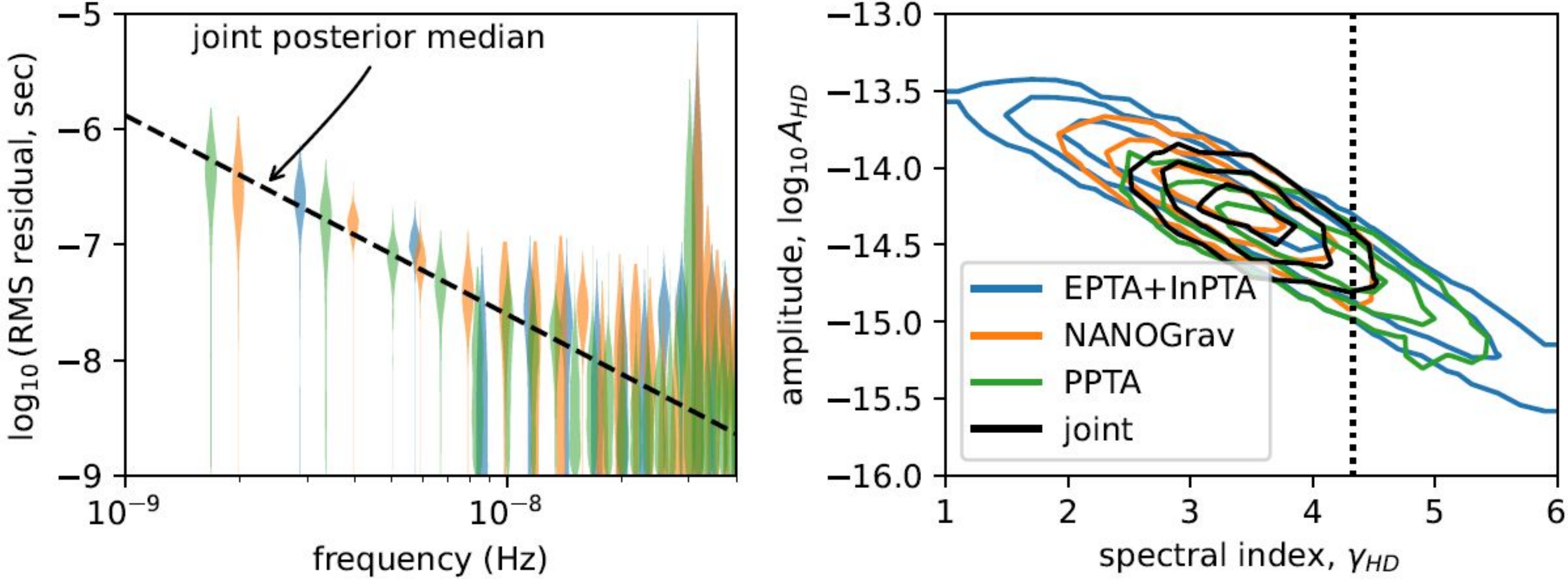


FIG. S4. Two dimensional PDF as a function of $(\mathcal{C}_{\rm G}, \zeta_{\rm G})$ compared to the over-threshold condition $\mathcal{C} > \mathcal{C}_{\rm th}$. In all panels, we considered the BPL power spectrum with an amplitude $A = 0.05$. The red lines indicates the contour lines corresponding to $\log_{10}(P_{\rm G}) = -45, -35, -25, -15, -5$. The collapse of type-I PBHs take place between the black solid and dashed lines (see more details in Ref. [195]). *Left panel:* Example of a very narrow power spectrum with $\alpha = \beta = 8$. The abundance is suppressed in the presence of negative $f_{\rm NL}$ by the strong correlation between $\mathcal{C}_{\rm G}$ and $\zeta_{\rm G}$ obtained for narrow spectra. *Center panel:* Example of negative non-Gaussianity and representative BPL spectrum. The PBH formation is sourced by regions of small $\zeta_{\rm G}$ and positive $\mathcal{C}_{\rm G}$ or both negative $\mathcal{C}_{\rm G}$ and $\zeta_{\rm G}$. *Right panel:* Example with positive $f_{\rm NL}$, showing the region producing PBHs populates the correlated quadrants of the plot, at odds with that is found in the other panels.

PBH and SGWB

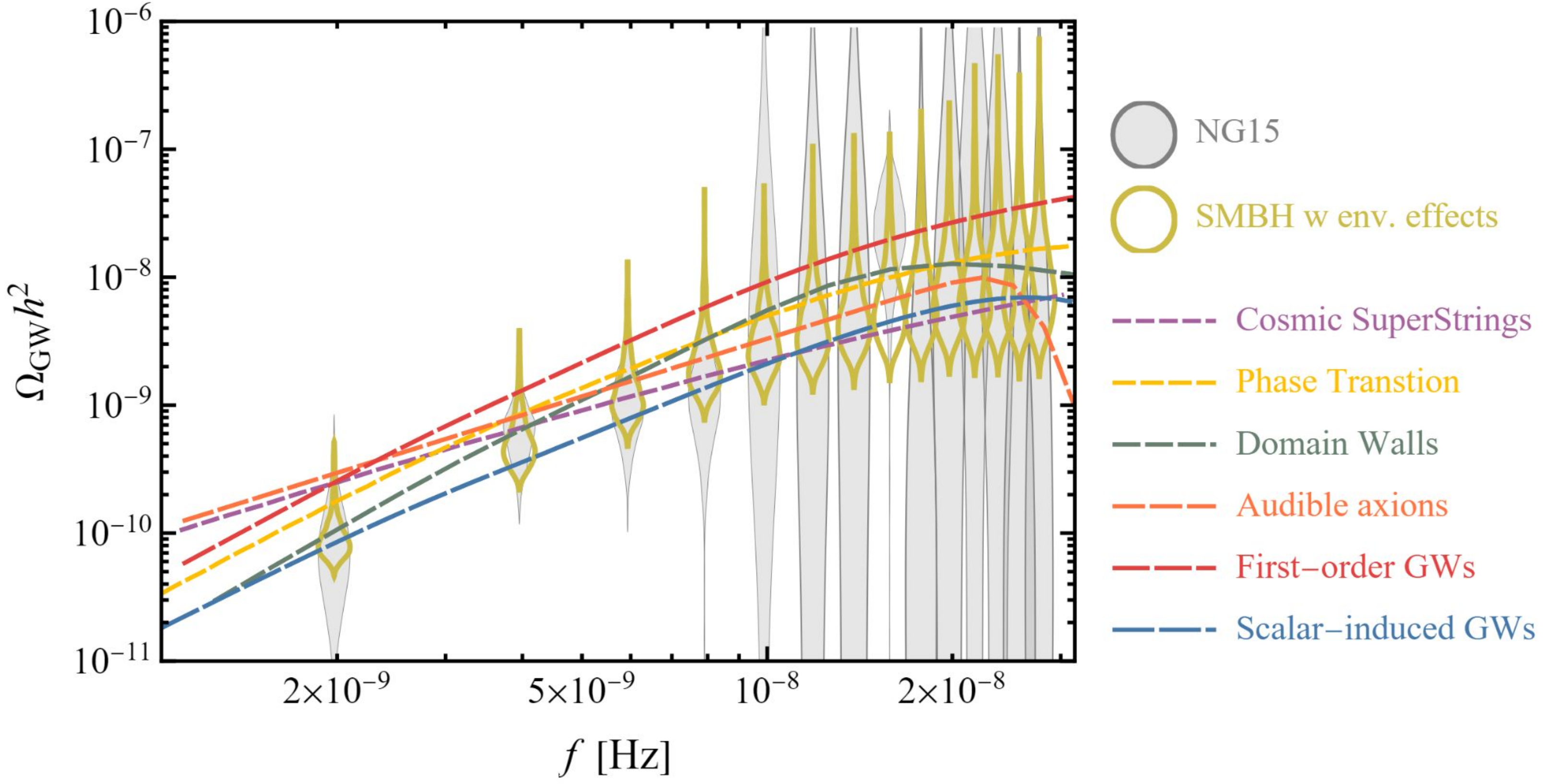
Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693



Are PBHs the end of the story?

All the PTA possible sources for NANOGrav: Astro vs Cosmo



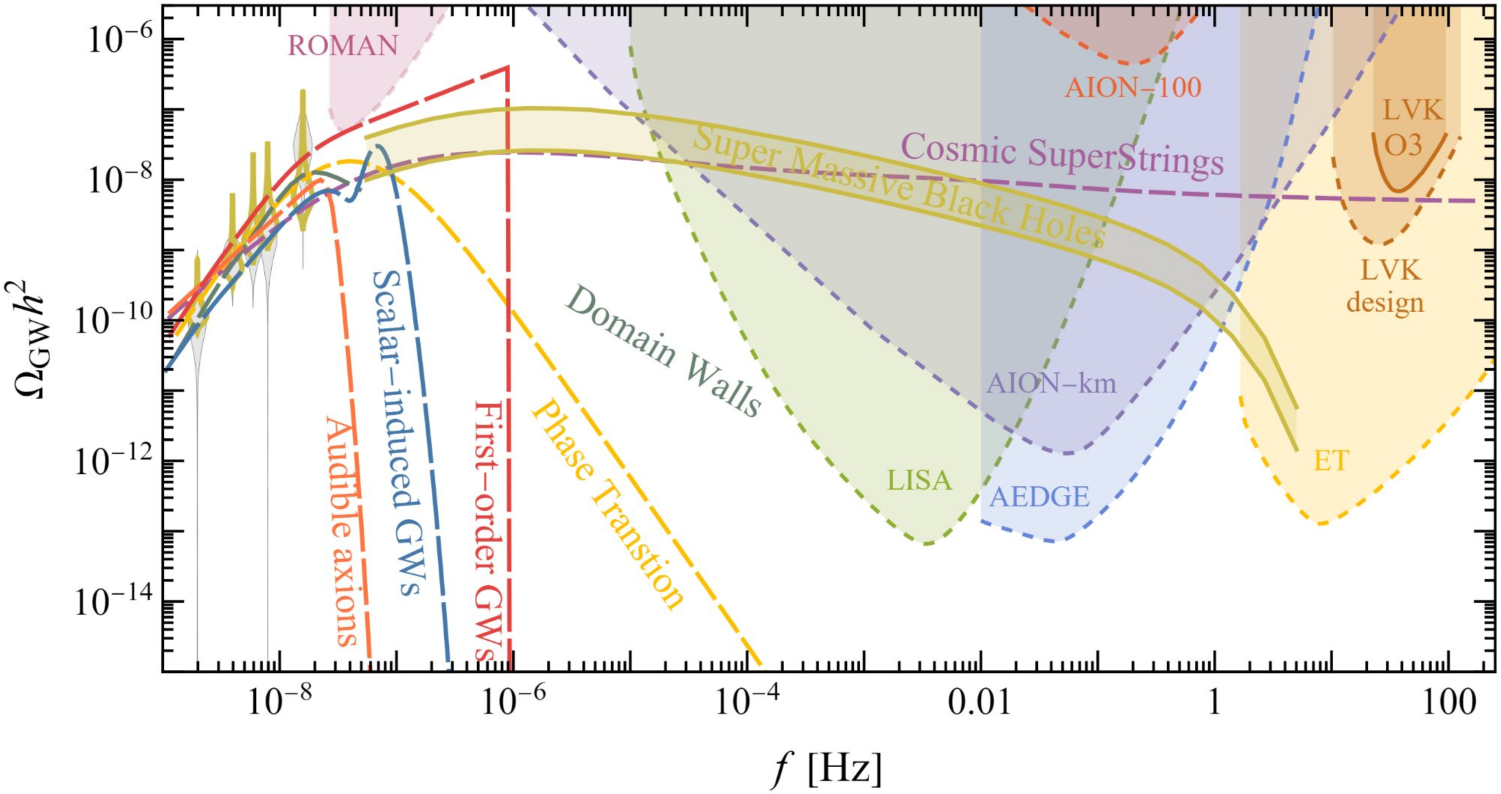
Results from Multi-Model Analysis (MMA)

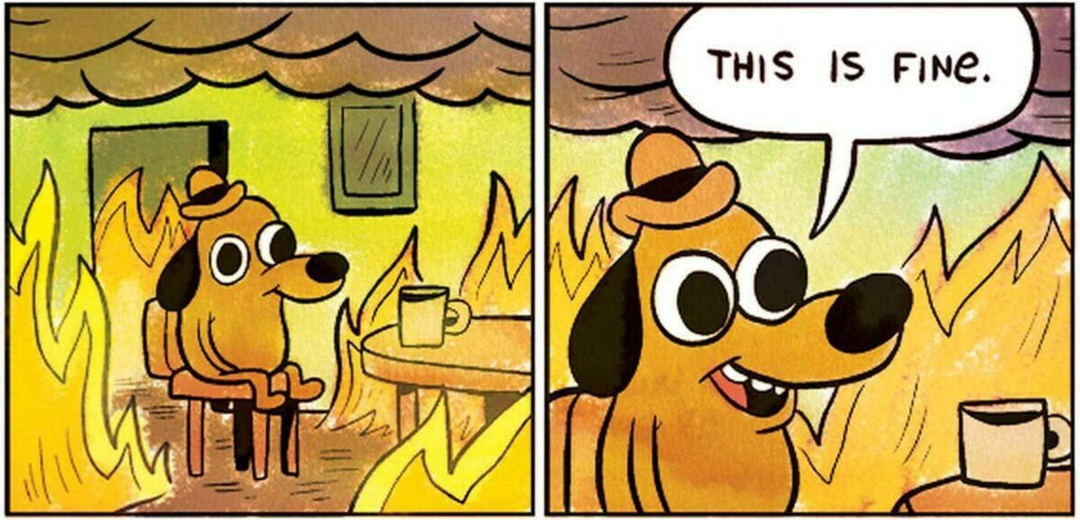
Scenario	Best-fit parameters	ΔBIC	Signatures
GW-driven SMBH binaries	$p_{\rm BH} = 0.07$	6.0	FAPS, LISA, mid-f, ~~LVK, ET~~
GW + environment-driven SMBH binaries	$p_{\rm BH} = 0.84$ $\alpha = 2.0$ $f_{\rm ref} = 34$ nHz	Baseline (BIC = 53.9)	FAPS, LISA, mid-f, ~~LVK, ET~~
Cosmic (super)strings (CS)	$G\mu = 2 \times 10^{-12}$ $p = 6.3 \times 10^{-3}$	-1.2 (4.6)	~~FAPS~~, LISA, mid-f, LVK, ET
Phase transition (PT)	$T_* = 0.34$ GeV $\beta/H = 6.0$	-4.9 (2.9)	~~FAPS, LISA, mid-f, LVK, ET~~
Domain walls (DWs)	$T_{\rm ann} = 0.85$ GeV $\alpha_* = 0.11$	-5.7 (2.2)	~~FAPS~~, LISA?, ~~mid-f, LVK, ET~~
Scalar-induced GWs (SIGWs)	$k_* = 10^{7.7}$/Mpc $A = 0.06$ $\Delta = 0.21$	-2.1 (5.8)	~~FAPS, LISA, mid-f, LVK, ET~~
First-order GWs (FOGWs)	$\log_{10} r = -14$ $n_{\rm t} = 2.6$ $\log_{10}(T_{\rm rh}/{\rm GeV}) = -0.67$	-2.0 (6.0)	~~FAPS, LISA, mid-f, LVK, ET~~
"Audible" axions	$m_a = 3.1 \times 10^{-11}$ eV $f_a = 0.87\, M_{\rm P}$	-4.2 (3.7)	~~FAPS, LISA, mid-f, LVK, ET~~

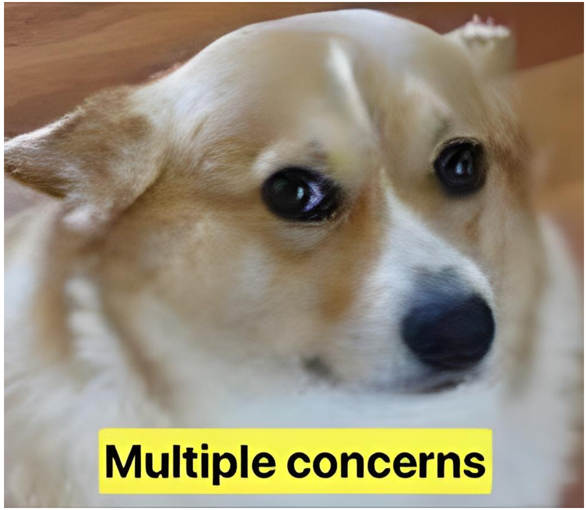
FAPS ≡ fluctuations, anisotropies, polarization, sources, mid-f ≡ mid-frequency experiment, e.g., AION [308], AEDGE [310], LVK ≡ LIGO/Virgo/KAGRA [161–163], ET ≡ Einstein Telescope [312] (or Cosmic Explorer [313]), ~~signature~~ ≡ not detectable

TABLE I. *The parameters of the different models are defined in the text. For each model, we tabulate their best-fit values, and the Bayesian information criterion* $BIC \equiv -2\ell + k \ln 14$*, where* k *denotes the number of parameters, relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the* ΔBIC *for the best-fit combined SMBH+cosmological scenario. The last column summarizes the prospective signatures.*

Other experiments?







A potential issue

Threshold values maybe are not correct? Different super-horizon threshold conditions may lead to an overestimation of the abundance, due to non-linear effects not included in the linear transfer function.

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r)\left[1 + \frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

V. De Luca, A. Kehagias, A. Riotto.– arXiv:2307.13633

Next step: finding a new prescription.