Analytic derivation of gravitational wave spectrum from expanding string loop on domain wall

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arXiv:2405.

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Summary

- Domain walls (DWs) appear in various particle physics models whereas DWs must decay to avoid BBN constraint.

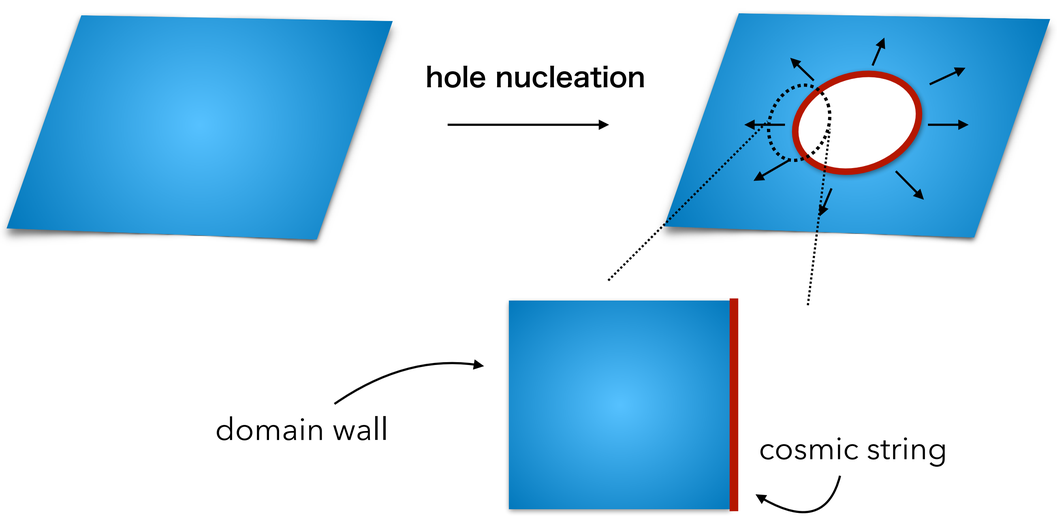
  → one popular way: **nucleation of holes bounded by cosmic strings**
- The nucleated holes expand and **radiate gravitational wave like cosmological first-order phase transition**

- We derive **an analytic formula of the stochastic gravitational wave (GW) spectrum.**
- **The GW spectrum $\Delta(k/\beta)$ is proportional to k^3 in IR and k^1 in UV**
- **This GW spectrum can explain recent PTA signal.**

Motivation

- A lot of particle physics models produce domain walls (DWs) when a discrete symmetry is spontaneously broken.
- In some models, **DWs decay by nucleation of loops of cosmic strings** [1-4]
  (cf.) axion-like model, GUT
- The string loops expand faster and faster by eating the DW.

→ looks like **2D version of bubble nucleation in 1st order phase transition**



These expanding loops radiate gravitational wave (GW)!

Setup

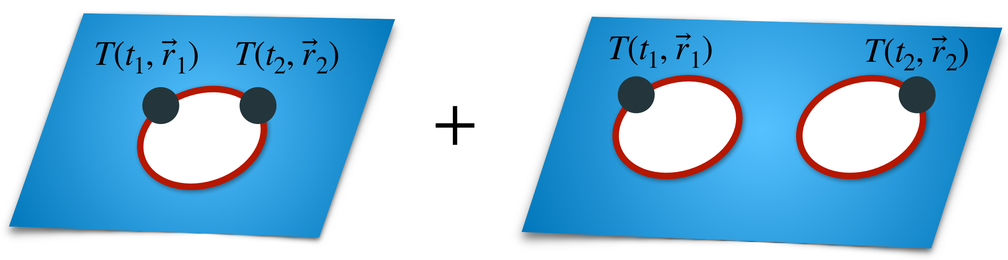
- Formula of GW energy density

EM-tensor T_{ij} lies only on DWs

$$\rho_{\rm GW} \sim \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \cos(k(t_1-t_2))\ \mathrm{F.T.}\ \langle T_{ij}^{\rm TT}(t_1,\vec{r}_1) T_{ij}^{\rm TT}(t_2,\vec{r}_2)\rangle_{\rm ens}$$

ensemble average of 2pt. function

two points on one loop **two points on two loops**

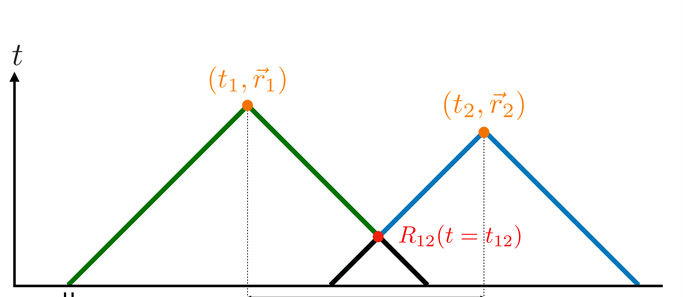


Most important: The loops do radiate GWs even before they collide to the other loops!

- **Ensemble average**
  = average of nucleation points + average of DW configuration

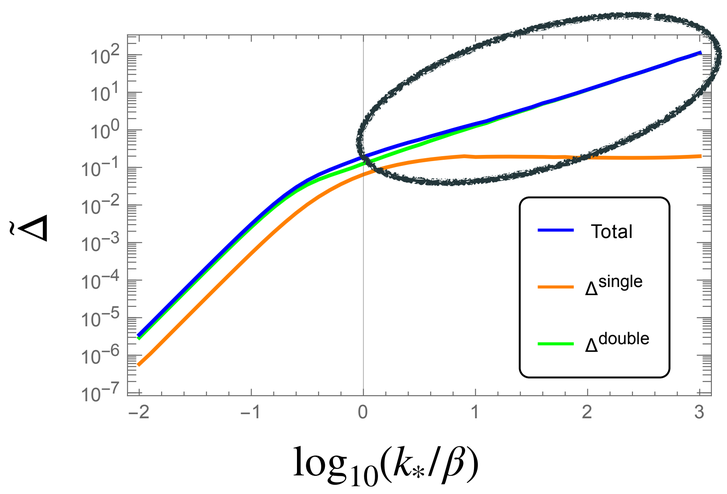
conditions to be considered [5,6]:

- Loops are nucleated on DWs with rate $\Gamma(t) = \Gamma_* e^{\beta(t-t_*)}$
- Initial size of nucleated loops is negligible.
- No loop inside the past light cone (envelope approx)
- Nucleation points should lie in { red region (R_{12}) for one-loop; green and blue regions for two-loop



Result of GW spectrum

- GW energy density $\Omega_{\rm GW}(t_*,k_*) = 10^{-5}\kappa^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha(t_*)}{1+\alpha(t_*)}\right)^2 \Delta\left(k_*/\beta\right)$
- Removing constant factors: $\Delta(k_*/\beta) = N_{\rm walls}\, d_{\rm DW}^2 H_*\, \beta\, \tilde{\Delta}(k_*/\beta)$



β^{-1} : typical size of collided loops

$d_{\rm DW}$: width of DWs

$N_{\rm walls}$: # of walls in Hubble patch

H_* : Hubble parameter

$$\tilde{\Delta} \propto \begin{cases} k^3 \text{ in IR regime} \\ k^1 \text{ in UV regime} \end{cases}$$

- **UV corresponds to small loops**

→ **not suppressed because they can radiate GW even before collisions.**

- We need UV cutoff, which is typically $k \sim d_{\rm DW}^{-1}$

Present GW spectrum

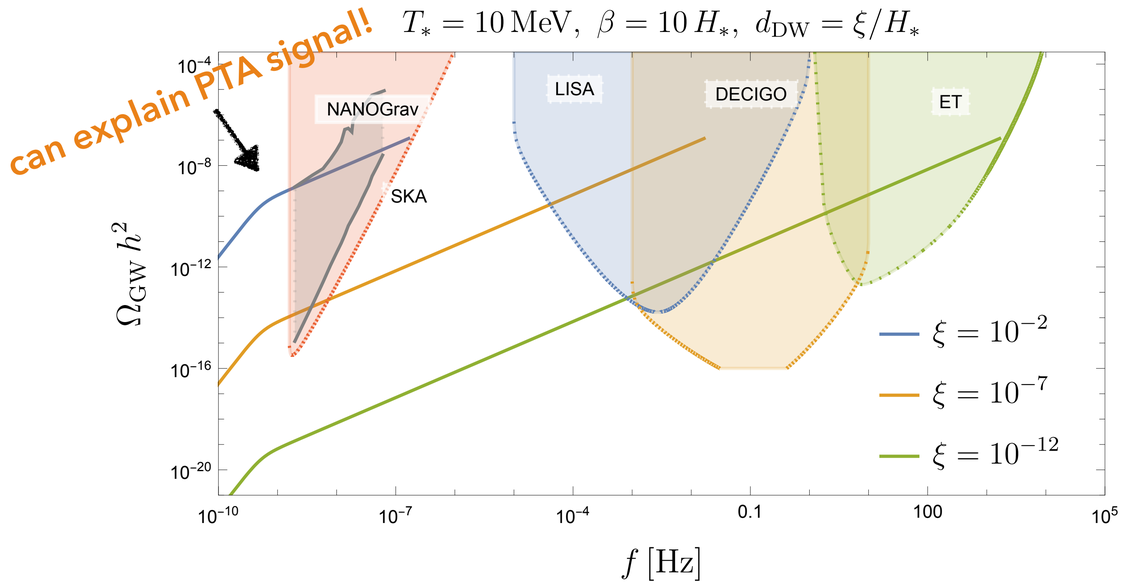
- Assumption1: DWs are produced non-thermally

  → DW width $d_{\rm DW} \simeq$ Hubble length at wall production $< d_{H_*}$
- Assumption2: String loops are nucleated before DWs get into scaling regime

  → $N_{\rm walls}$ is not necessarily $\mathcal{O}(1)$, but is taken as $N_{\rm walls} \sim d_{H_*}/d_{\rm DW}$ (most optimistic)

$\kappa, \alpha(t_*) = \mathcal{O}(1)$

$T_* = 10\,\mathrm{MeV},\ \beta = 10\,H_*,\ d_{\rm DW} = \xi/H_*$



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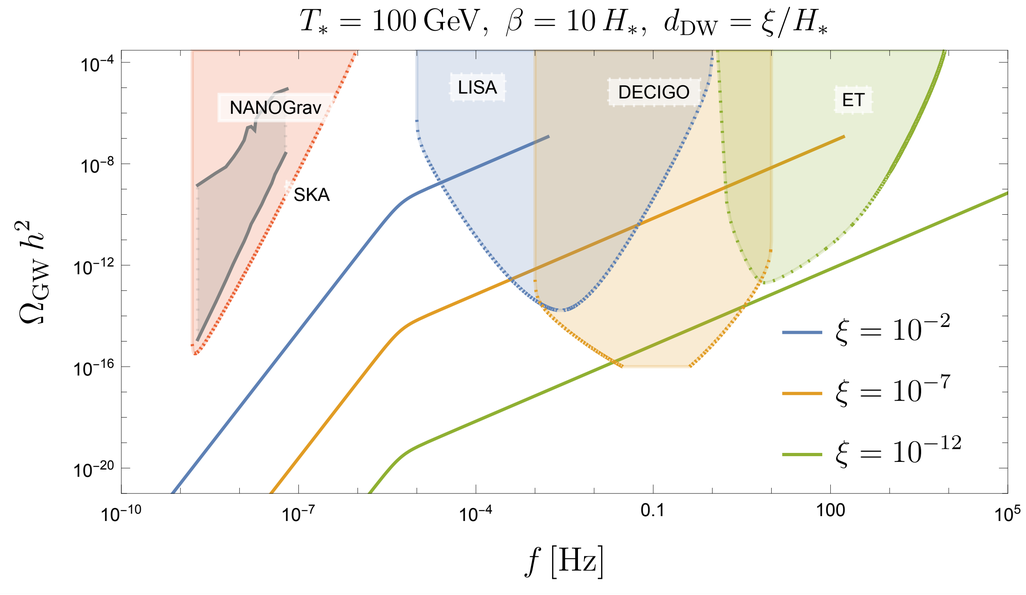

Future work

- **numerical simulation of GW spectrum**
- **derivation of β, α, T_* from microscopic models**

Refs.

[1] Kibble-Lazarides-Shafi '82 [2] Everett-Vilenkin '82
[3] Preskill-Vilenkin hep-ph/9209210 [4] Dunsky+ 2111.08750
[5] Jinno-Takimoto 1605.01403 [6] Jinno-Takimoto 1707.03111