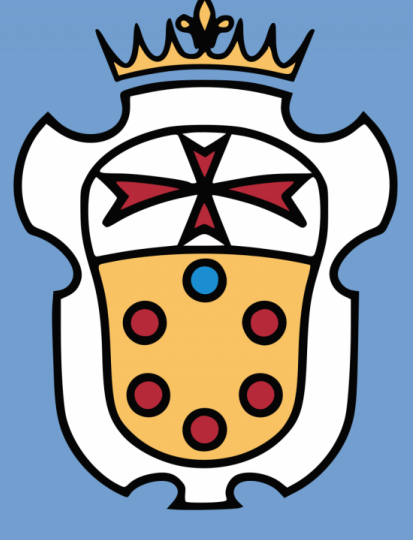


The Amplitudes of Quadratic Quasi-Normal Modes

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Summary

Two linear black hole quasi-normal modes can couple together and give rise to quadratic modes. Assuming the amplitudes of the linear modes are known, the amplitude of quadratic modes is a **prediction** of GR. We compute these amplitudes for any angular momenta of the QNM (ℓ, m, n)

SETUP

The Ringdown Phase of a Black Hole Merger is described by linearized perturbations called BH Quasi-Normal Modes (QNM), characterized by specific frequencies.

Due to the non-linearities of GR, two linear modes can couple together and give rise to **new** quadratic modes, whose frequency and amplitude are completely fixed (relative to the linear amplitudes).

METHODS

The metric is perturbed as $g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h_{\mu\nu}^{(1)} + \varepsilon^2 h_{\mu\nu}^{(2)}$

We define regularized RW and Zerilli Master Scalars

$$\Psi_{-}^{(i)} = \frac{r}{\lambda} \left[D_r \tilde{h}_{t-}^{(i)} - D_t \tilde{h}_{r-}^{(i)} - \frac{2}{r} \tilde{h}_{t-}^{(i)} \right] + g_{-}^{(i)}(r),$$

$$\Psi_{+}^{(i)} = \frac{2r}{\ell(\ell+1)} \left[\frac{\tilde{h}_{\theta}^{(i)}}{r^2} + \frac{1}{\lambda + 3M/r} \left(f^2(r) \tilde{h}_{rr}^{(i)} - r f(r) D_r \left(\frac{\tilde{h}_{\theta}^{(i)}}{r^2} \right) \right) \right] + g_{+}^{(i)}(r)$$

containing the graviton polarizations, where $g^{(2)}$ are functions that subtract asymptotically divergent terms.

The differential equation is $\frac{d^2 \Psi_{\pm}^{(i)}}{dr_*^2} + (\omega^2 - V_{\pm}) \Psi_{\pm}^{(i)} = S_{\pm}^{(i)}$

where $S^{(2)}$ is a source term. We solve it (imposing QNM boundary conditions) using the Leaver algorithm.

Finally, we reconstruct the metric in radiation gauge.

RESULTS

Fig. 1: $(220) \times (220) \rightarrow (44)$ and $(330) \times (220) \rightarrow (55)$ agree with the fit from Numerical Relativity (Zhu et Al. '24; Berti et Al. '23). New modes are found, some of which are suppressed in equal-mass mergers.

Fig. 2: GW190521-like merger, with $(220), (221), (210), (330), (440)$ linear modes (blue) derived from NR. Quadratic modes (green) are added using the ratios in *Fig.1*.

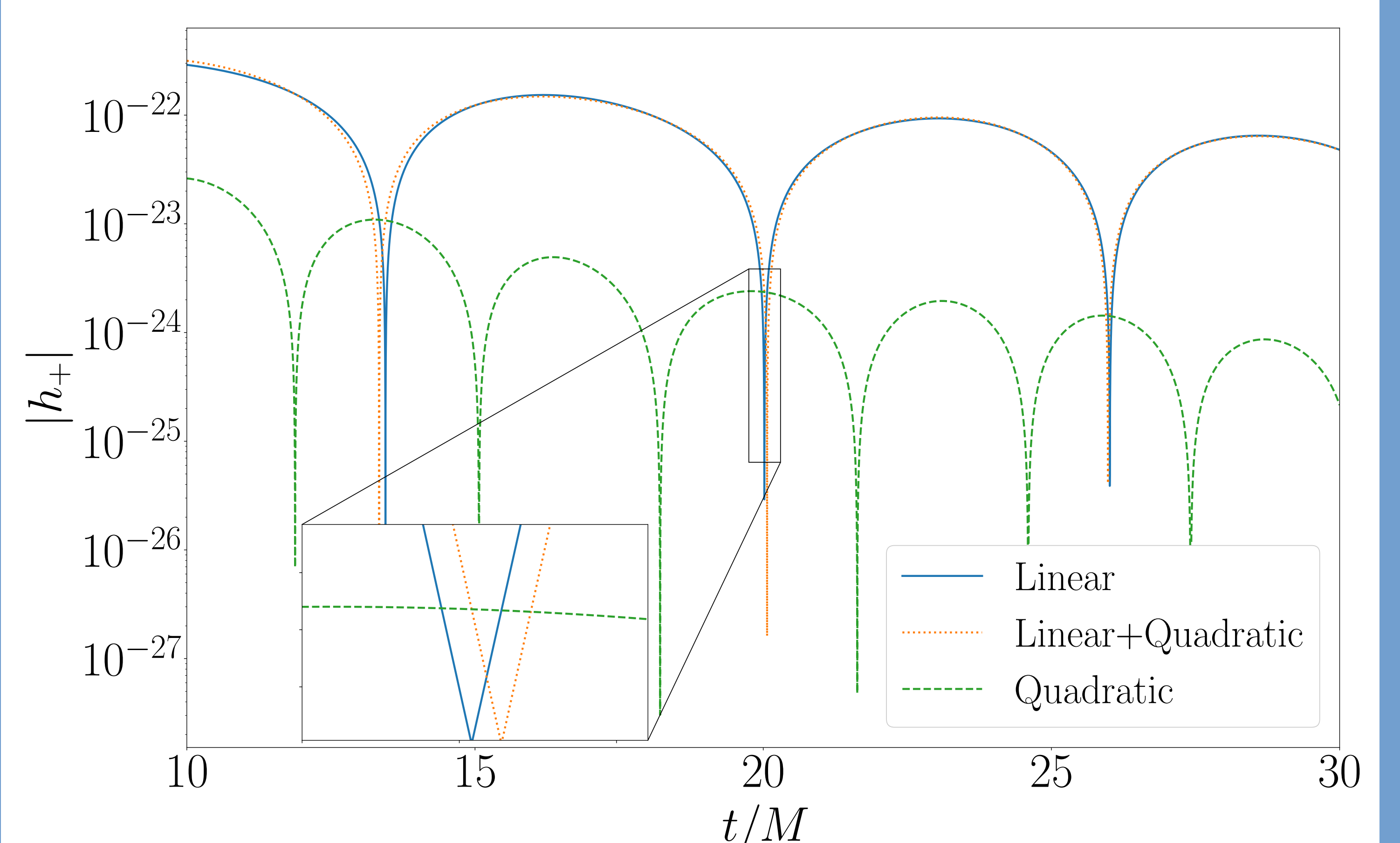


FIGURE 2: Linear and quadratic QNMs for realistic merger

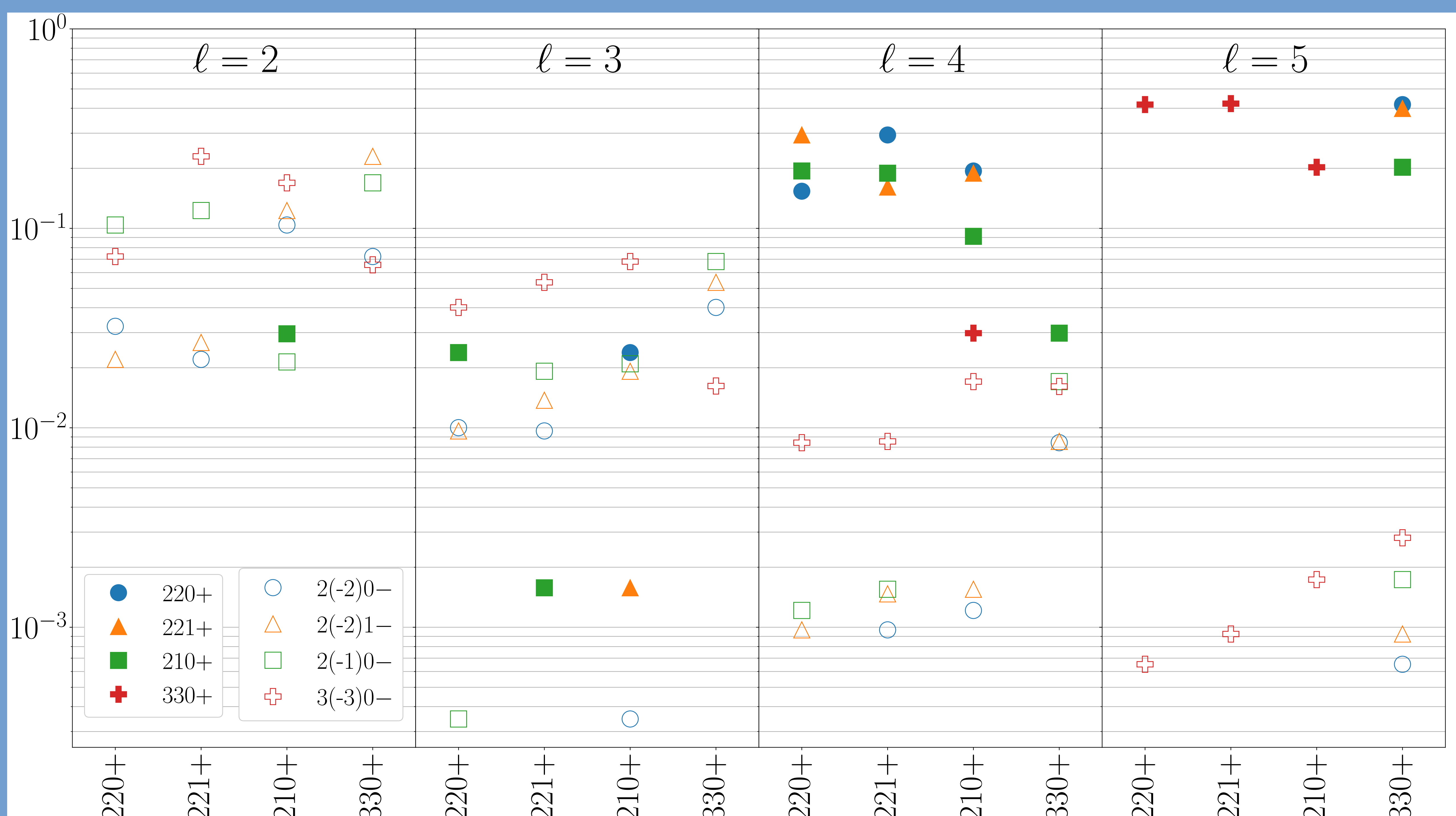


FIGURE 1: Ratio of quadratic amplitudes over product of two linear amplitudes