Ionisation Cooling and Different Types of Cells

International
MUON Collider
Collaboration

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On behalf of the International Muon Collider Collaboration

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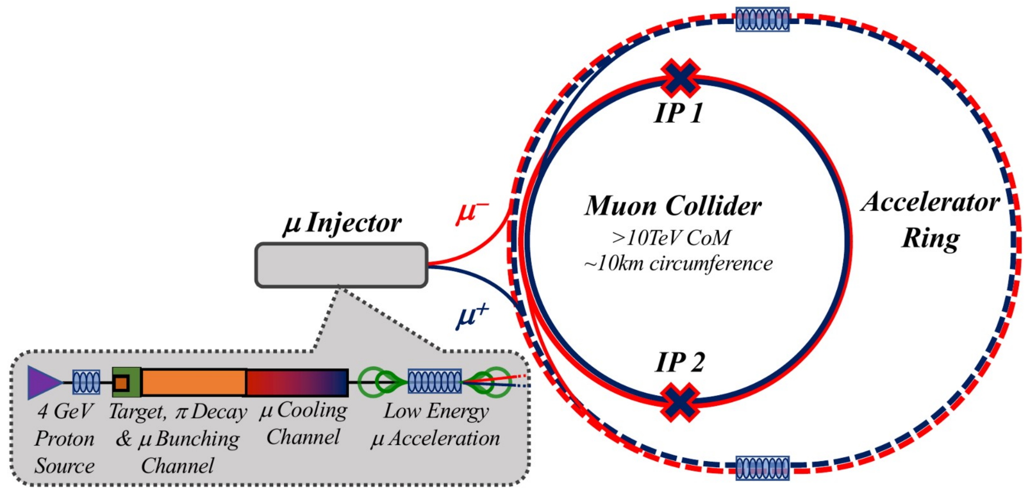
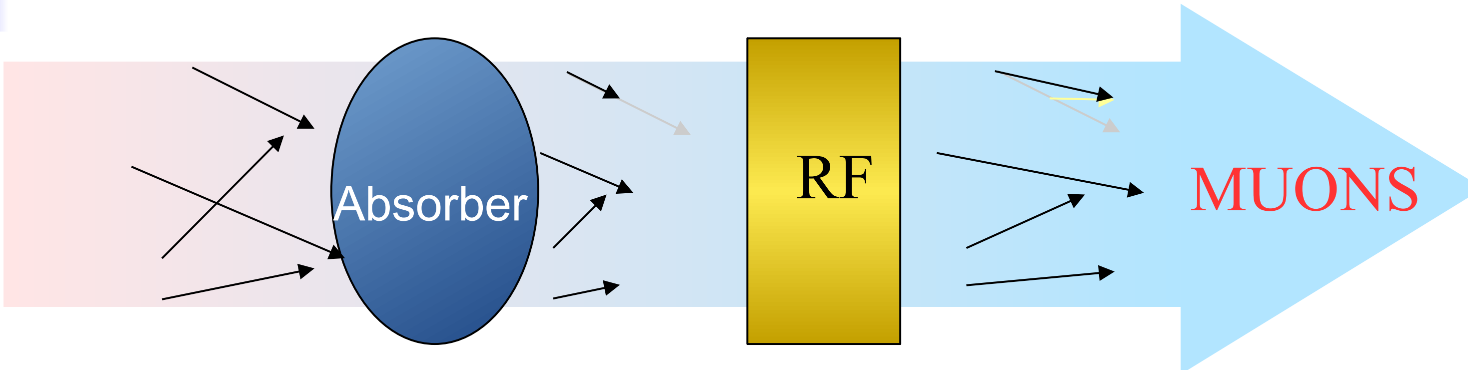
Muon Collider

- Muon collider → potential short cut to the energy frontier
  - Multi-TeV collisions in next generation facility
  - Combine precision potential of e^+e^- with discovery potential of pp
  - High-flux, TeV-scale neutrino beams for nuclear & BSM physics
- Bright muon beams are required
  - Protons onto a target to make pions
  - Pions are captured and decay to muons
  - Muon beam is cooled to get to high brightness
- Cooling time must be competitive with muon lifetime
  - Ionisation cooling



Muons/bunch	N	10^{12}	2.2
Repetition rate	f_r	Hz	5
Beam power	P_{coll}	MW	5.3
RMS longitudinal emittance	$\varepsilon_{\parallel}$	eVs	0.025
Norm. RMS transverse emittance	$\varepsilon_{\perp}$	µm	25

Ionisation Cooling



- Beam loses energy in absorbing material
  - Absorber removes momentum in all directions
  - RF cavity replaces momentum only in longitudinal direction
  - End up with beam that is more parallel
- Multiple Coulomb scattering from nucleus degrades the effect
  - Mitigate with tight focussing → low β
  - Mitigate with low-Z materials
  - Equilibrium emittance where MCS cancels the cooling
- Verified by the Muon Ionisation Cooling Experiment (MICE)

Transverse cooling - maths

- This can be expressed in terms of a change of emittance on passing through an absorber

$$\frac{d\epsilon_n}{dz} \approx \frac{1}{E}\left\langle\frac{dE}{dz}\right\rangle\epsilon_n + \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel}^3 E}.$$

dE/dz is negative!
Cooling

Heating

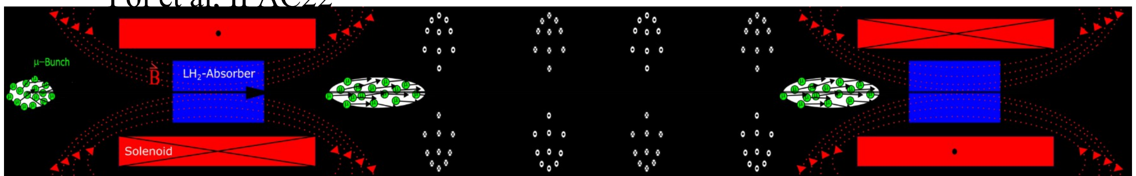
- There exists an equilibrium emittance where the two terms balance (no emittance change)

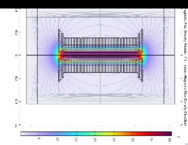
$$\epsilon_n(equilibrium) = \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel} < \frac{dE}{dz} >}$$

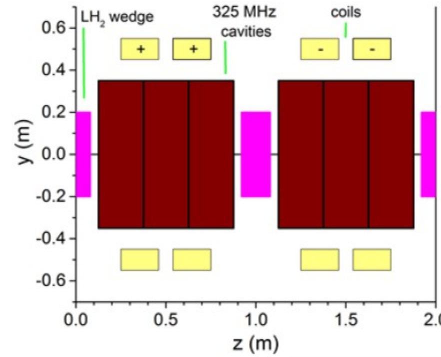
- Seek to minimise equilibrium emittance
  - Maximise radiation length L_R and energy loss *dE/dz*
  - Minimise focusing function $\beta_\perp$
  - Maximise acceptance – size of beam accepted in cooling channel

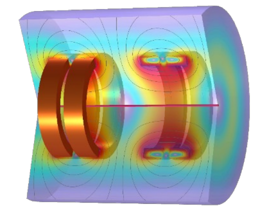
Muon Cooling

Sayed et al, PRSTAB 18, 2015
Fol et al, IPAC22



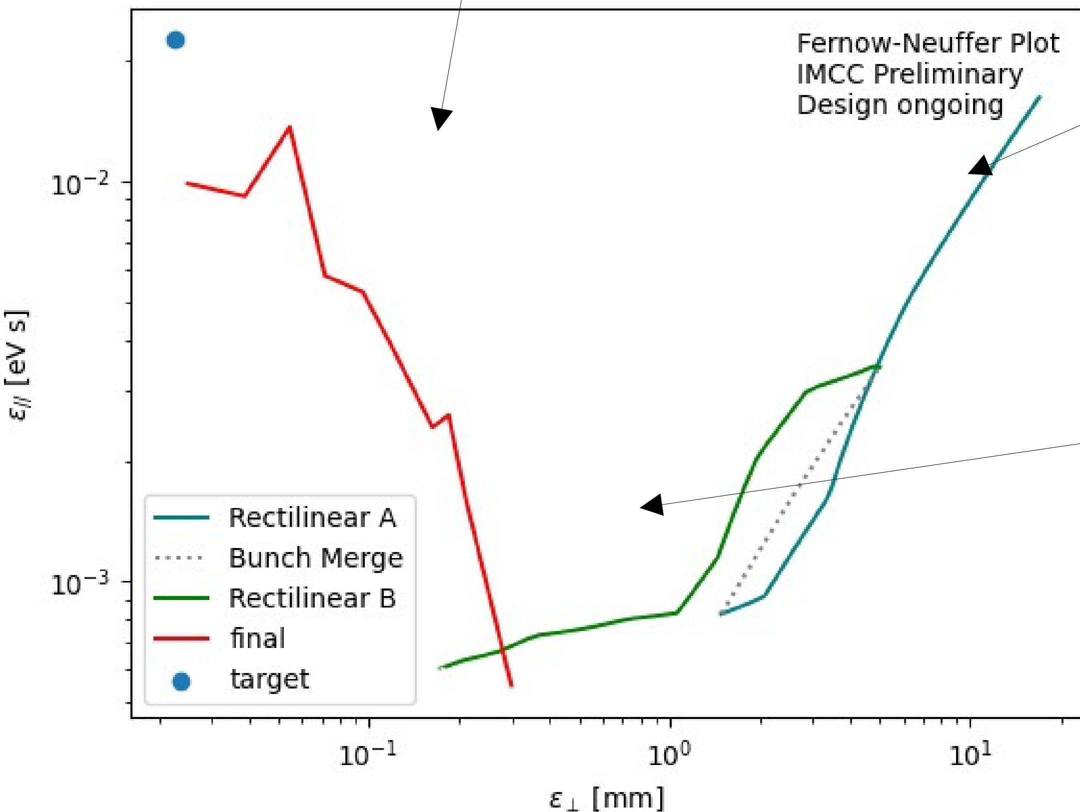




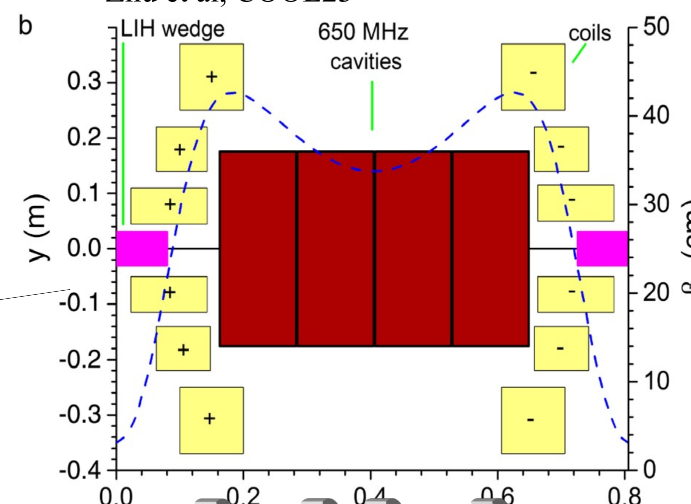


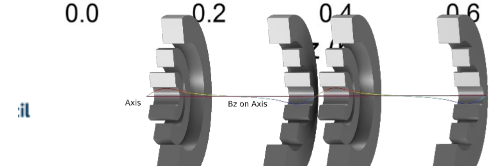
4D Final cooling

Rectilinear cooling



Stratakis et al, PRSTAB 18, 2015
Zhu et al, COOL23





Cooling Cells

- Solenoids typically have long fringe fields
  - Acceptance of the magnets is worse for short fringe fields
  - Consequence: thin lens approximation not a good model
- Consider instead the equation for focusing strength
  - (No canonical angular momentum)

$$2\beta_\perp \boxed{\beta''_\perp} - (\beta'_\perp)^2 + 4\beta_\perp^2 \boxed{\left(\frac{qB_z}{2p_z}\right)}^2 - 4 = 0$$

β = Twiss beta (~beam size)
β'= derivative in z
β''= second derivative in z

Magnet focusing strength
B_z is solenoid field on the z-axis
p_z is momentum in z direction
q is muon charge

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Constant solenoid solution

- Simplest solution – uniform solenoid

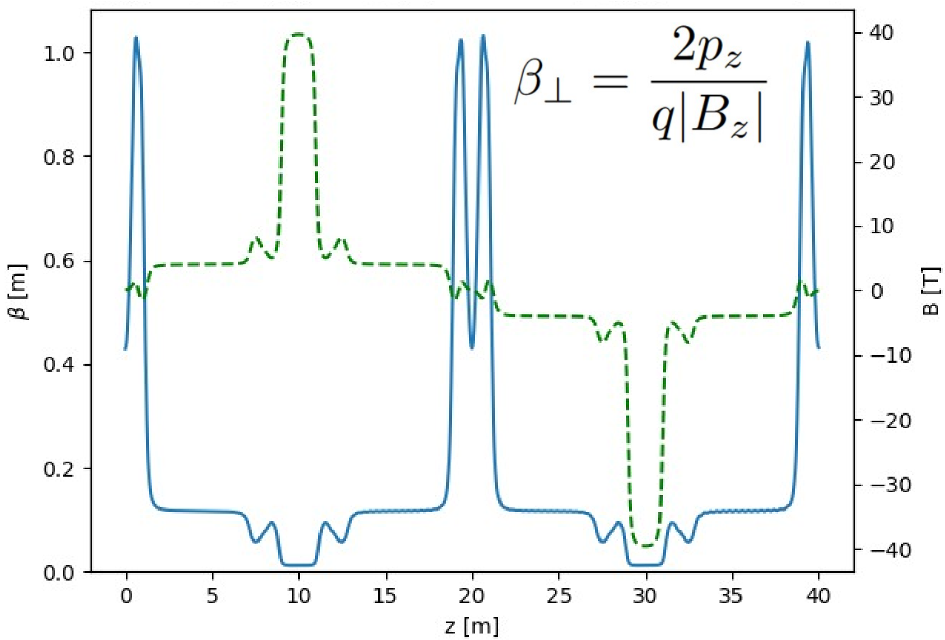
$$2\beta_\perp \cancelto{0}{\beta_\perp''} - (\cancelto{0}{\beta_\perp'})^2 + 4\beta_\perp^2 \left(\frac{qB_z}{2p_z}\right)^2 - 4 = 0$$

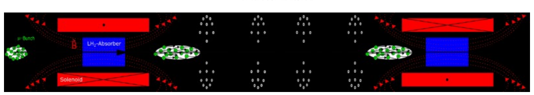
$$\beta_\perp = \frac{2p_z}{qB_z}$$

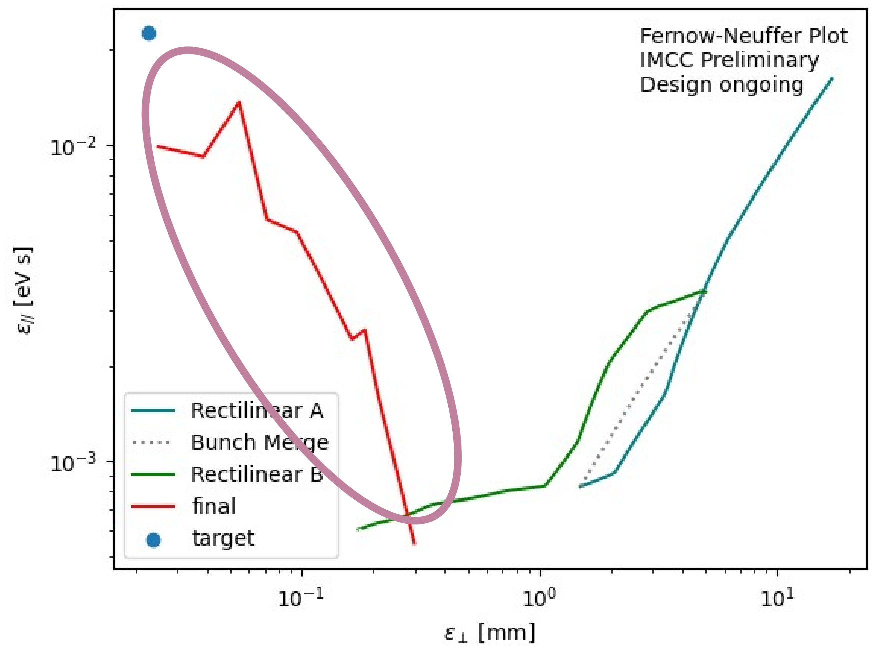
- Basic premise behind final cooling
  - Get Bz as high as possible → minimise $\beta_\perp$
  - Trim pz as emittance decreases → smaller Bz

Final cooling – example lattice

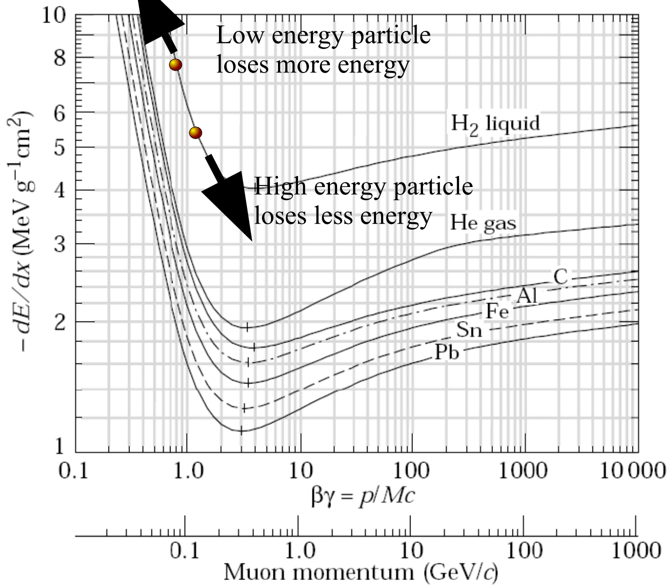
- Example lattice – final cooling
  - Note several phase advances in each solenoid – they are not thin lenses!
  - Excellent transverse cooling – but longitudinal heating

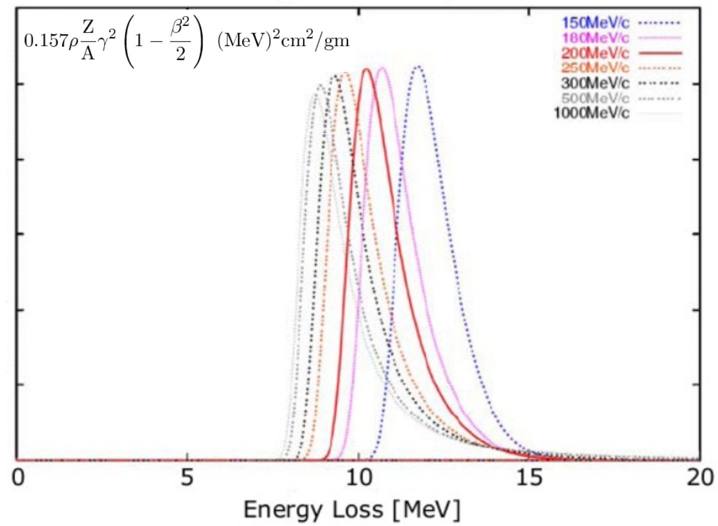






Longitudinal Heating



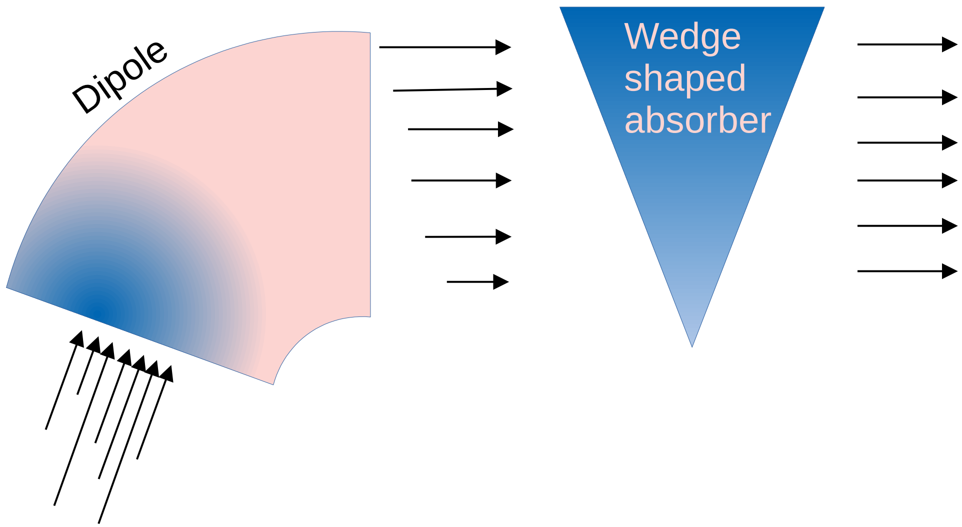


- In longitudinal phase space, the beam is usually heated
  - Heating due to random noise in the energy loss I.e. "straggling"
  - Heating due to curvature in energy loss (heating or weak cooling)

$$\frac{d<E^2>}{dz} = \left(2\frac{d}{dE}\frac{dE}{dz}\right) <E^2> + \left(\frac{d<E^2>}{dz}\right)_{Vlasov}$$

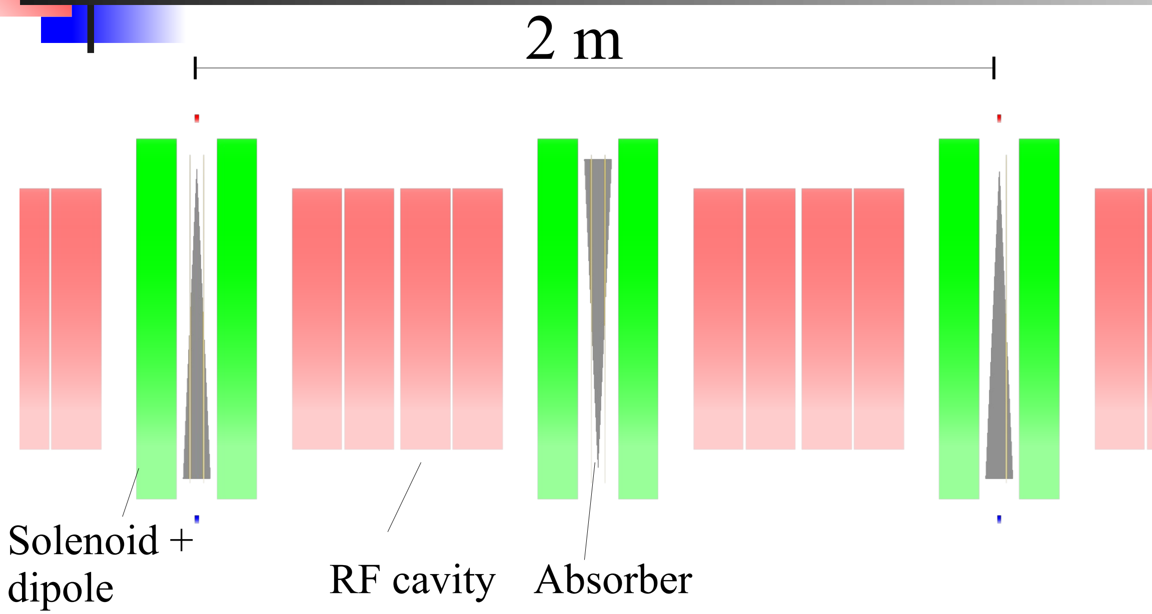
- Mitigate using emittance exchange
  - Move emittance from longitudinal to transverse phase space

Emittance exchange

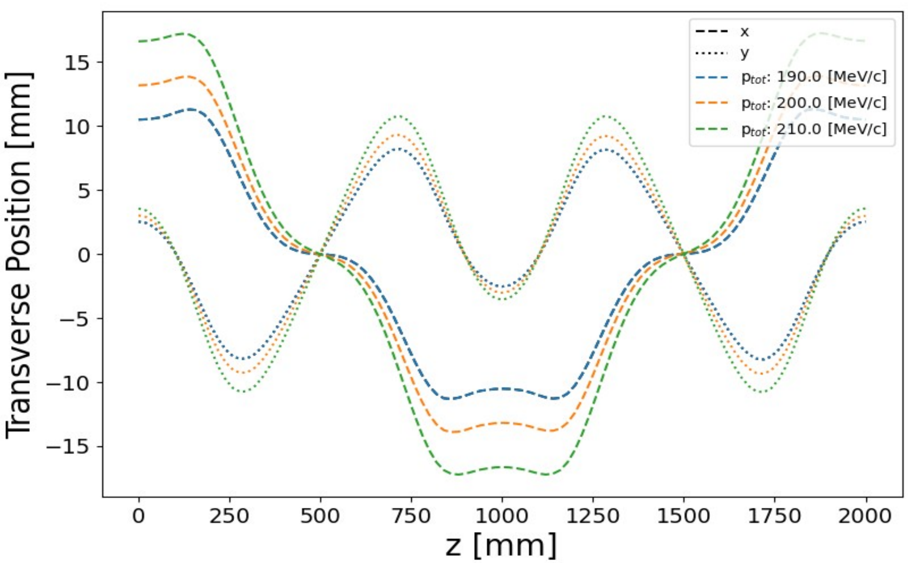


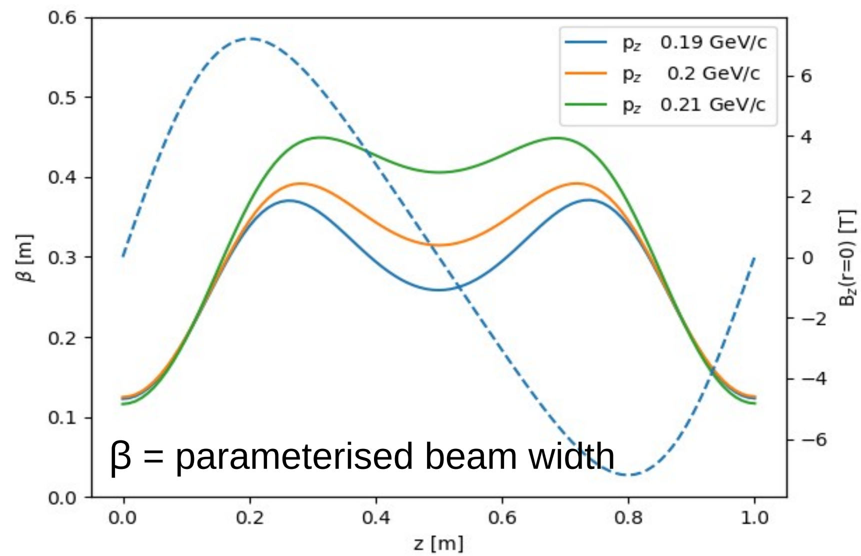
- Initial beam is narrow with some momentum spread
  - Low transverse emittance and high longitudinal emittance
- Beam follows curved trajectory in dipole
  - Higher momentum particles have higher radius trajectory
  - Beam leaves dipole wider with energy-position correlation
- Beam goes through wedge shaped absorber
  - Beam leaves wider without energy-position correlation
  - High transverse emittance and low longitudinal emittance

Emittance Exchange – Realisation?



Cooling System	
Cell length	2 m
Peak solenoid field on-axis	7.2 T
Dipole field	0.2 T
Dipole length	0.1 m
RF real estate gradient	22 MV/m
RF nominal phase	20°
RF frequency	704 MHz
Wedge thickness on-axis	0.0342 m
Wedge apex angle	5°
Wedge material	LiH





Rectilinear Cooling Optics

- Consider again the differential equation for focusing strength

$$2\beta_\perp \beta''_\perp - (\beta'_\perp)^2 + 4\beta_\perp^2 \left(\frac{qB_z}{2p_z}\right)^2 - 4 = 0$$

- Take a magnetic field that is a set of Fourier harmonics
  - Thin lens approximation is a bad one for solenoids!

$$B_z = \sum_{n=-\infty}^{\infty} b_n \exp\left(\frac{2\pi i n z}{L}\right)$$

- We expect solutions that are also a set of Fourier harmonics

$$\beta_\perp = \sum_{n=-\infty}^{\infty} \beta_n \exp\left(\frac{2\pi i n z}{L}\right)$$

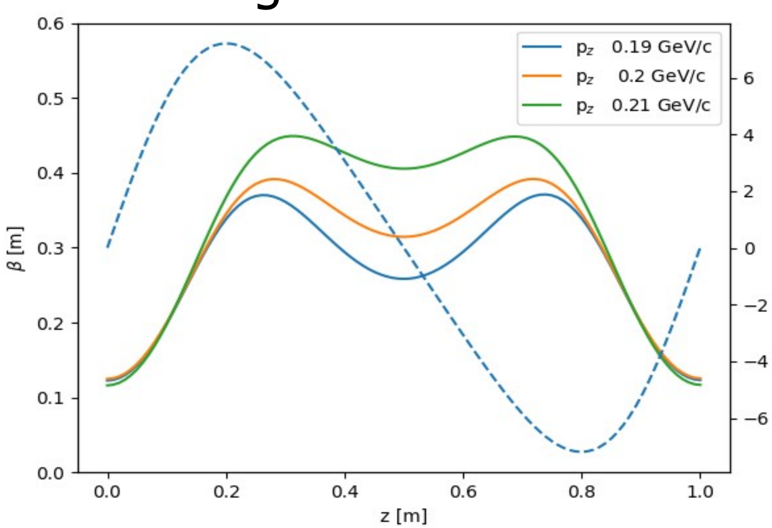
Stop Bands & Pass Bands

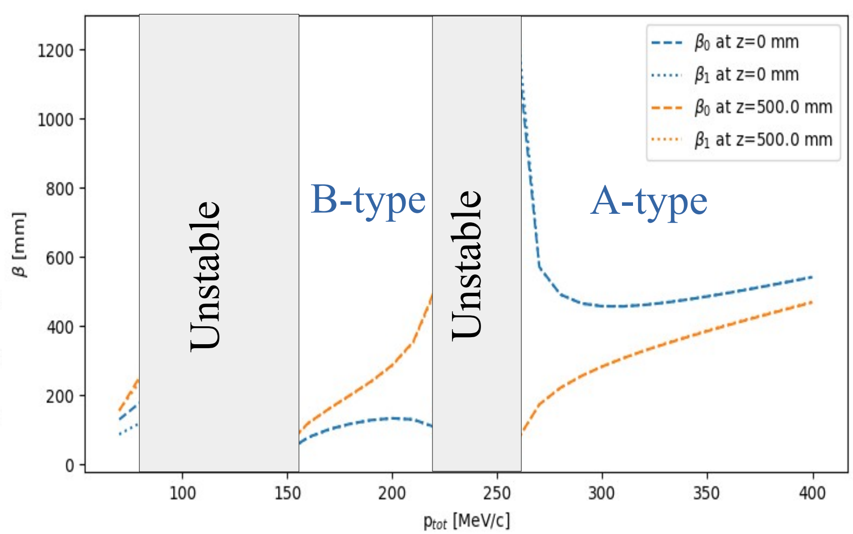
- What do solutions look like?

Wang & Kim, Phys. Rev. E 63, Recursive solution for beam dynamics of periodic focusing channels

$$\beta(s) = \frac{L}{\pi} \frac{\sin(\sqrt{\vartheta_0}\pi)}{\sqrt{\vartheta_0}\sin\mu}\left[1 + \sum_{n=1}^{\infty} \frac{\mathrm{Re}[\vartheta_n e^{i2n\pi s/L}]}{n^2 - \vartheta_0} +, \ldots, \right]$$

  - Θ_n are ~ Fourier harmonics of B_z
- Regions where solutions are stable and unstable





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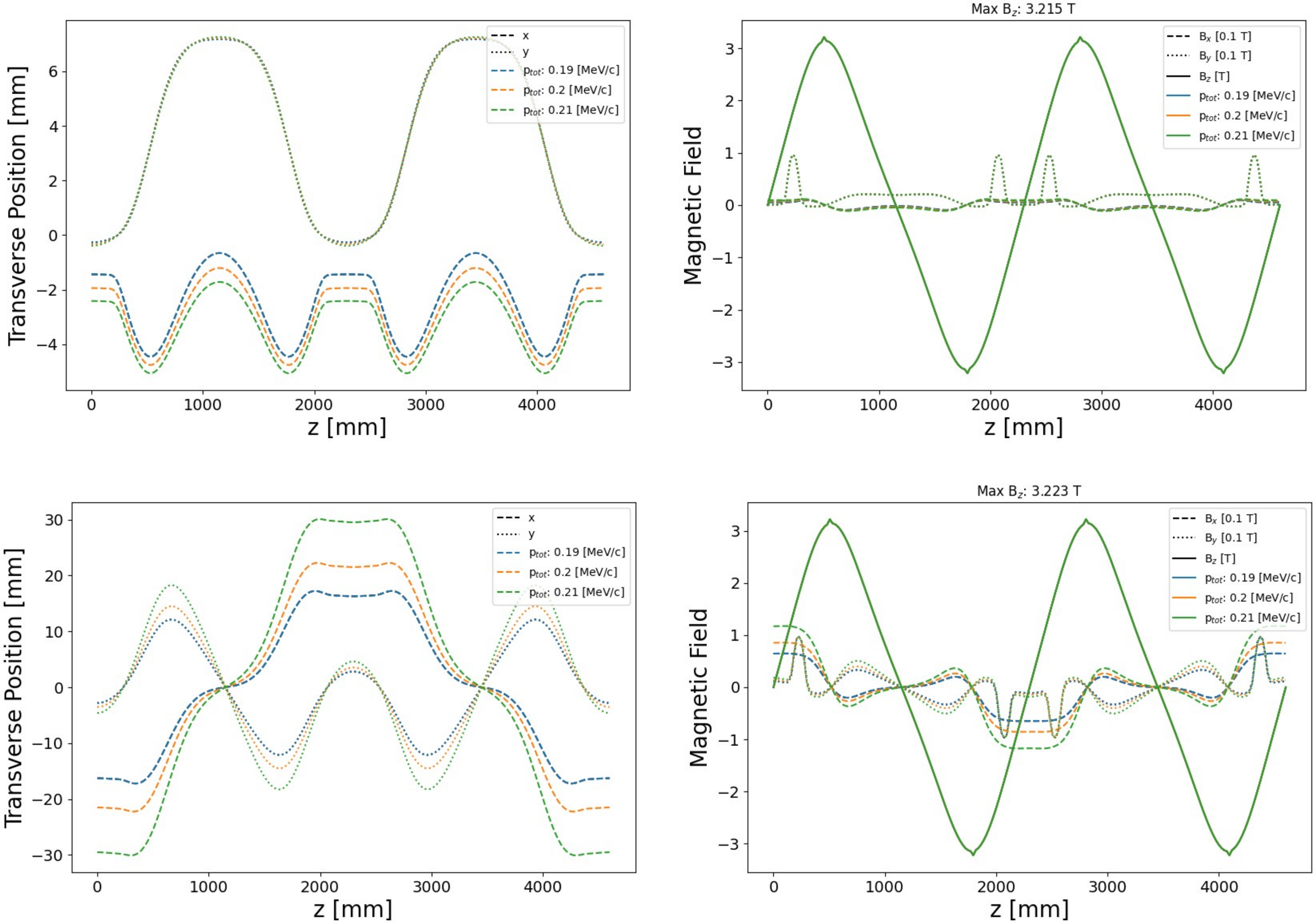
Dipole field – an extra dimension

- Separate to the transverse optics, dipole field is also important



Field off the axis

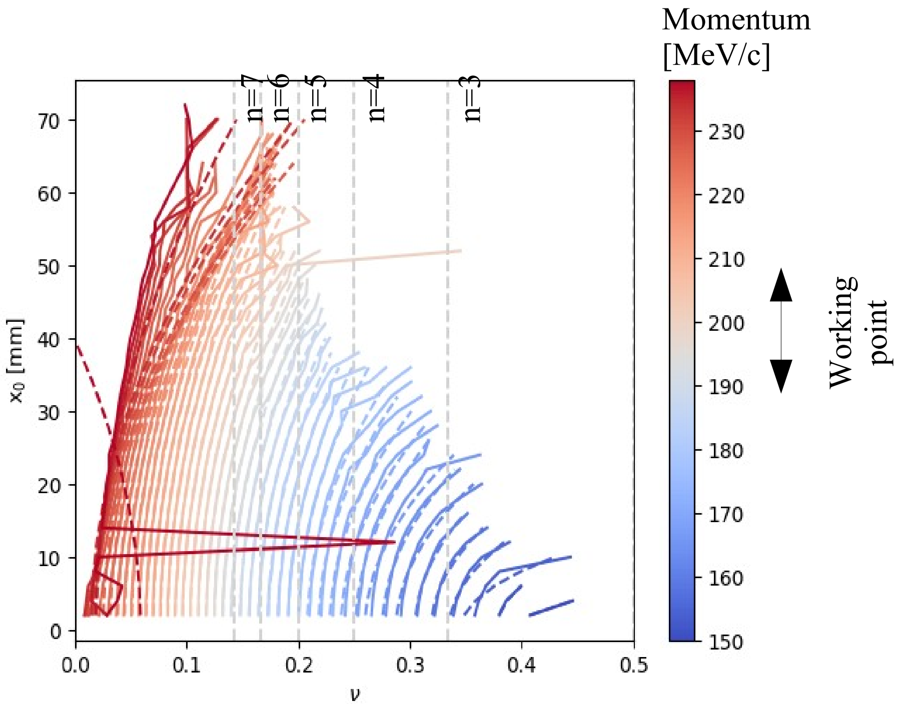
- Field off the axis can be expressed as derivative of solenoid on-axis field (consequence of Maxwell's equations)

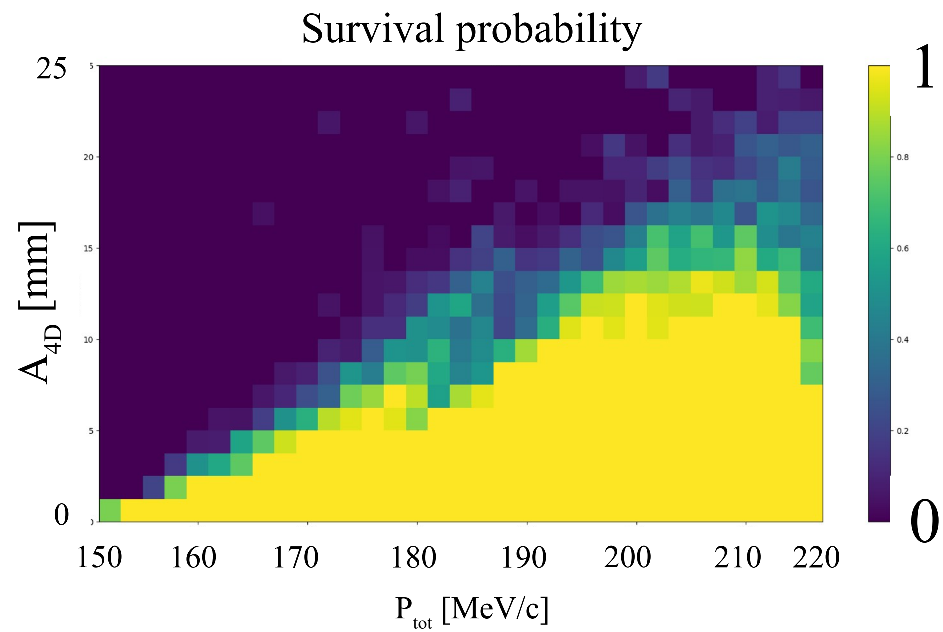
$$B_r(r,z) = \frac{(-1)^n}{n!(n-1)!}\left(\frac{r}{2}\right)^{2n-1}\partial_z^{2n-1}B_z(z, r=0)$$

$$B_z(r,z) = \frac{(-1)^n}{(n!)^2}\left(\frac{r}{2}\right)^{2n}\partial_z^{2n}B_z(z, r=0)$$

- Impact on Dynamic Aperture?
  - Dynamic Aperture = transverse region of the beam where the magnets are focusing
  - Qualitatively DA is worse for lower $\beta_\perp$
  - I know of no analytical evaluation
- Numerical discussion - below

Optics vs momentum





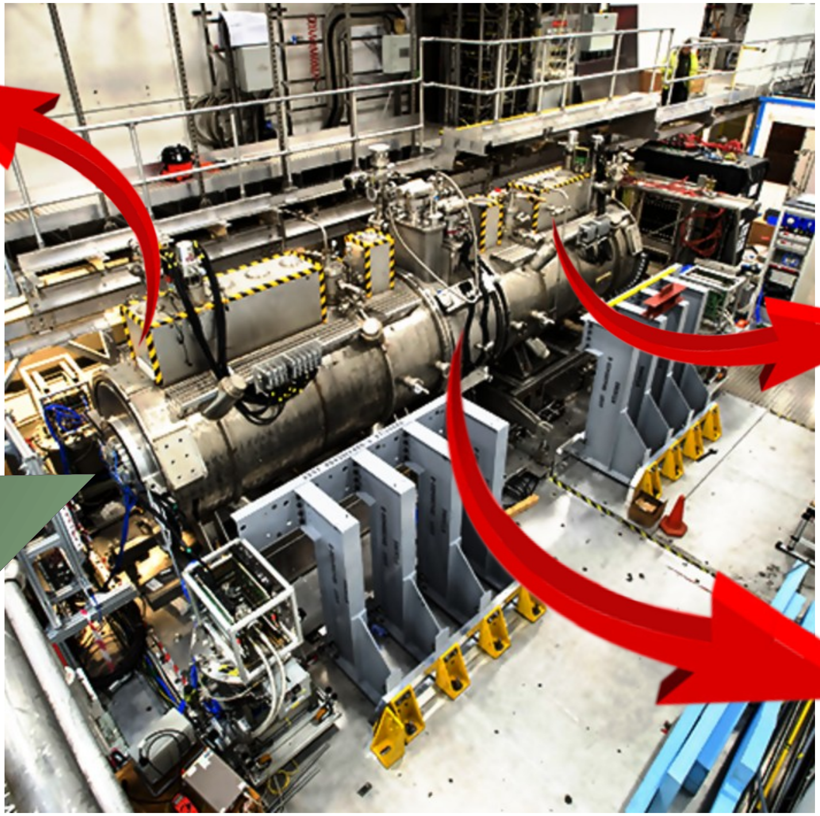
- Acceptance driven by tune consideration
  - Tune = number of focusing oscillations per magnetic cell
  - Acceptance for tune near to resonances

R&D Programme

- R&D Programme to test these ideas
  - MICE – check basic beam physics concept with ionisation cooling
  - MTA test stand – first ideas on RF cavities in magnetic fields
  - New? RF test stand – further development of RF cavity concepts
  - Cooling cell build – integrate RF, magnets and absorbers
  - Demonstrator – beam test
- Other desirables
  - Proton beam → collective effects
  - Final cooling test

MICE - Experimental set up

Measure muon position and momentum upstream



Beam

Measure muon position and momentum downstream

Cool the muon beam using LiH, LH_2, or polyethylene wedge absorbers

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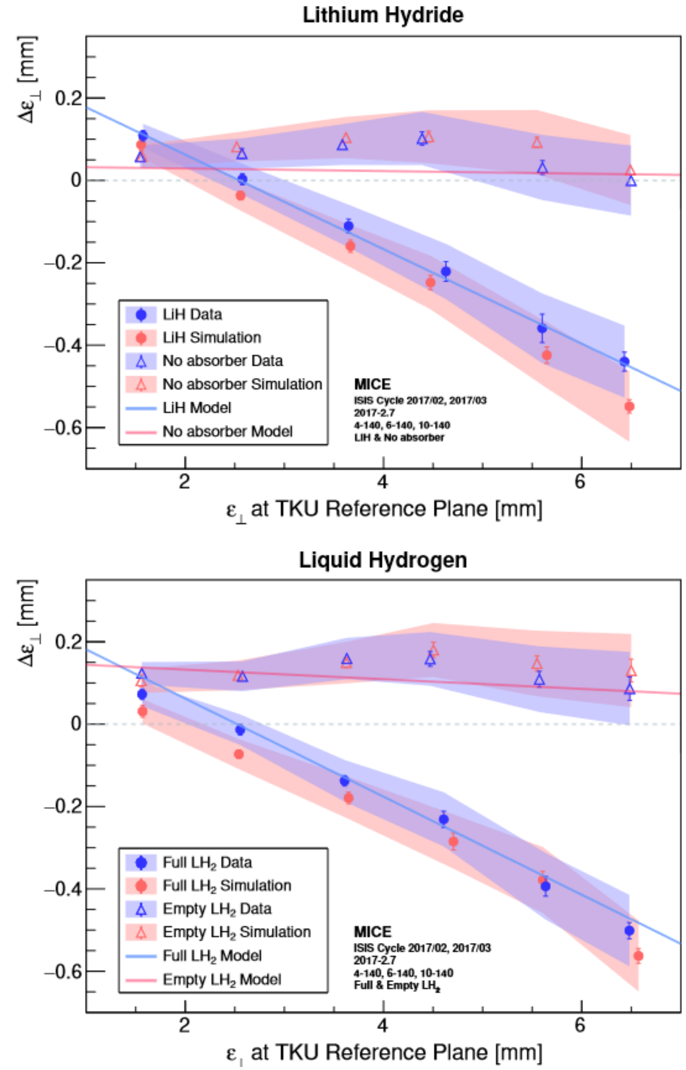
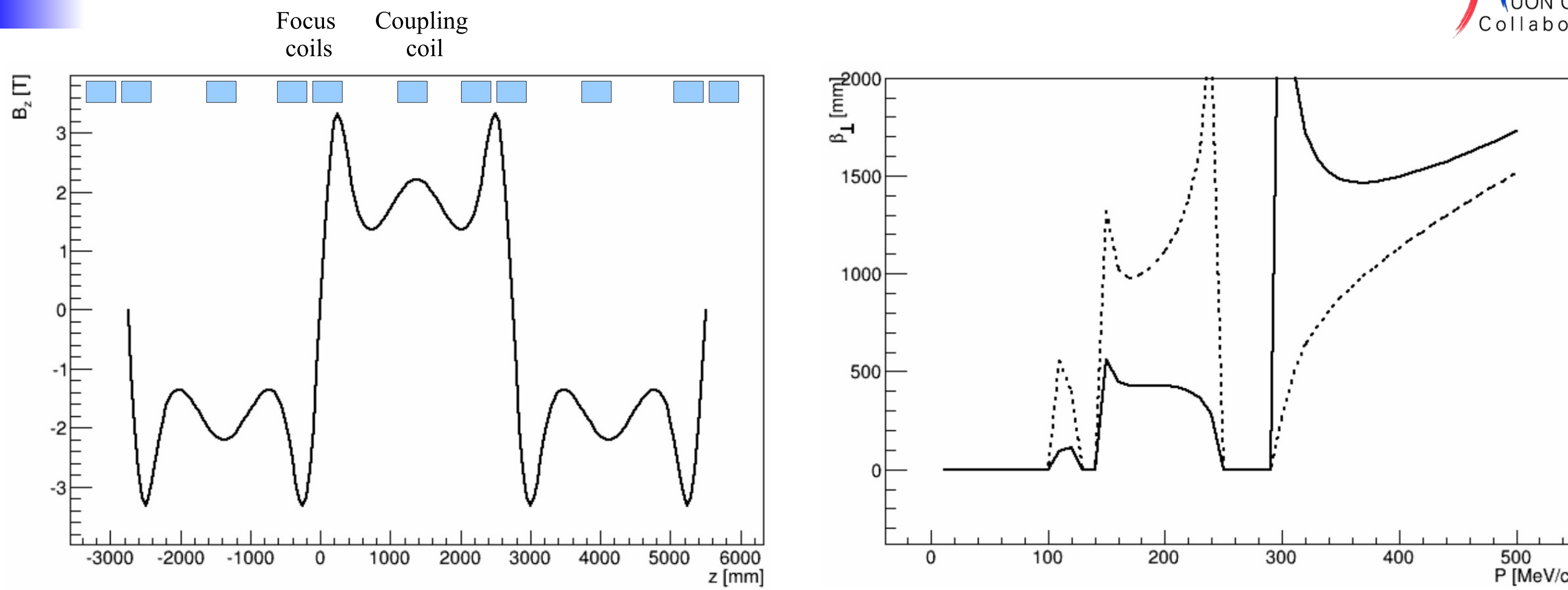
Emittance reduction

- When absorber installed:
  - Cooling above equilibrium emittance
  - Heating below equilibrium emittance
- When no absorber installed
  - Optical heating
  - Clear heating from Al window

P. Jurj o.b.o. MICE collaboration
Submitted to Nature Physics



MICE - lattice



- MICE lattice was a section of a full cell
- Full cell had similar sort of stop band structure that we propose in rectilinear lattice
  - Note beta is very flat with momentum
  - Also good acceptance and focusing performance
  - Awkward "Coupling Coil" interferes with RF

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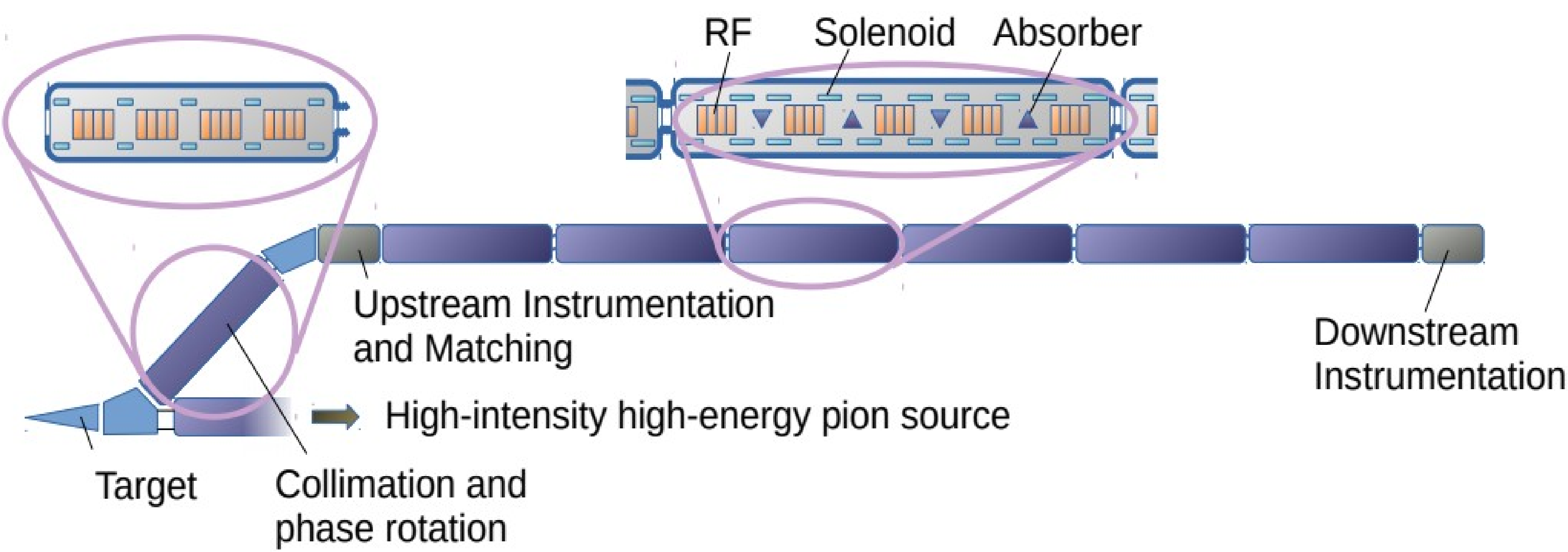
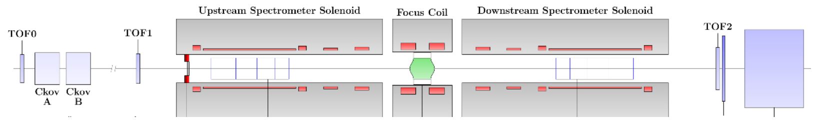
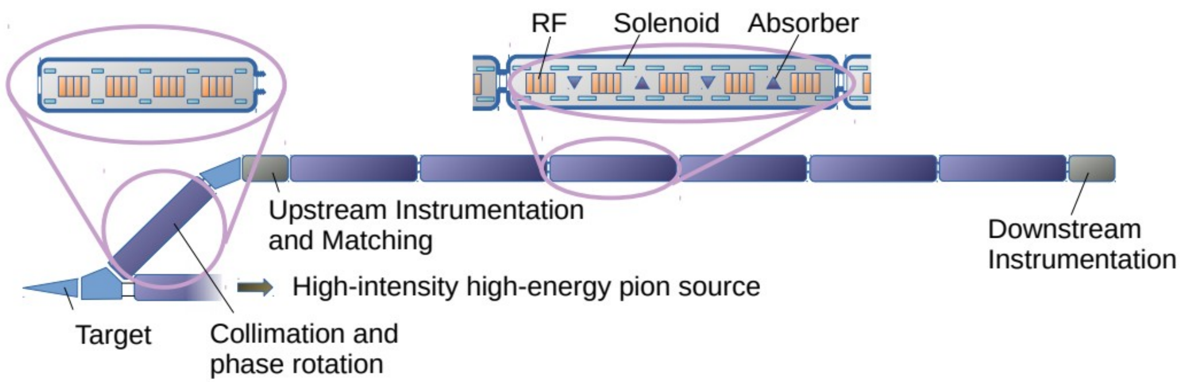
Cooling Demonstrator



Comparison with Existing Data





	MICE	Demonstrator
Cooling type	4D cooling	6D cooling
Absorber #	Single absorber	Many absorbers
Cooling cell	Cooling cell section	Many cooling cells
Acceleration	No reacceleration	Reacceleration
Beam	Single particle	Bunched beam
Instrumentation	HEP-style	Multiparticle-style

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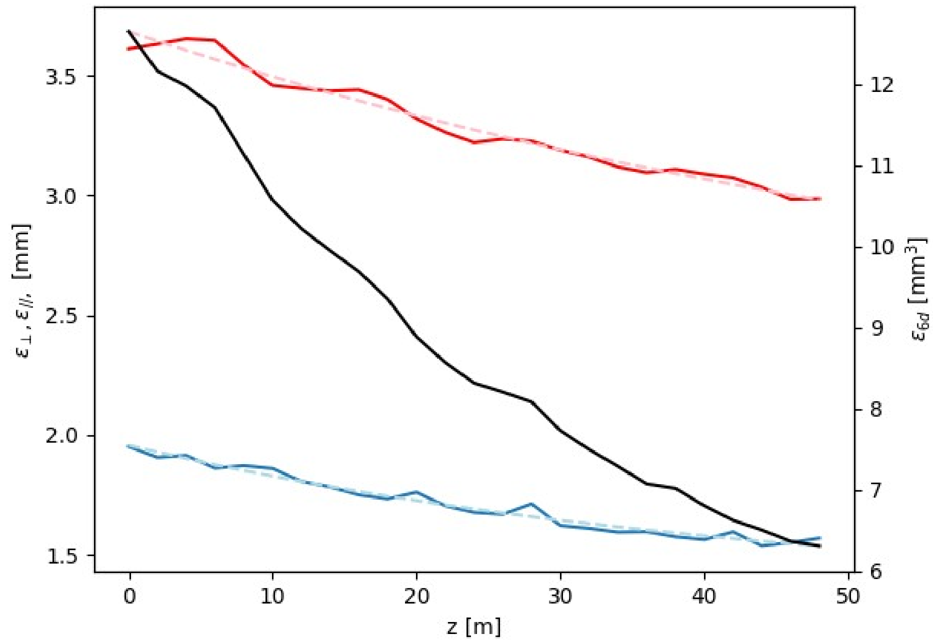
Be RF & LiH Performance

- Use Beryllium for RF cavity walls
- Use LiH in absorber
- Good cooling performance
  - Transverse and longitudinal emittance reduced by ~ 20 %
  - Approx factor two reduction in 6D emittance
- Optimisation ongoing
  - Assumes perfect matching for now
  - Assume LiH for now
    - Liquid Hydrogen performance likely better



Transmission losses	2.00%
Decay losses	4.00%
Trans ε in	1.95 mm
Trans ε out	1.57 mm
Long ε in	3.61 mm
Long ε out	2.99 mm
6D ε in	12.7 mm^3
6D ε out	6.3 mm^3

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Outlook

- Very exciting time for high brightness muon beam R&D
- I covered very basic aspects of currently studied cooling channels
- No time for
  - Helical cooling channels
  - Pure emittance exchange schemes
  - Parametric Resonance Ionisation Cooling
  - Quadrupole focused ionisation cooling
  - Cooling rings
  - …
- Aspects are in common
  - Need for extreme focusing
  - Need for large Dynamic Aperture
  - Tightly packed RF and focusing elements
- Need to prototype this equipment to show practical use